Discussion Papers In Economics And Business

Old-age Social Security vs. Forward Intergenerational Public Goods Provision

Ryo Arawatari and Tetsuo Ono

Discussion Paper 11-26-Rev.2

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Old-age Social Security vs. Forward Intergenerational Public Goods Provision

Ryo Arawatari and Tetsuo Ono

Discussion Paper 11-26-Rev.2

August 2013

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Old-age Social Security vs. Forward Intergenerational Public Goods Provision*

Ryo Arawatari† Tetsuo Ono‡
Nagoya University Osaka University

August 1, 2013

Abstract

This paper introduces an overlapping-generations model with earnings heterogeneity and borrowing constraints. The labor income tax and the allocation of tax revenue between social security and forward intergenerational public goods are determined in a bidimensional majoritarian voting game played by successive generations. The political equilibrium is characterized by an ends-against-the-middle equilibrium where low- and high-income individuals form a coalition in favor of a lower tax rate and less social security while middle-income individuals favor a higher tax rate and greater social security. Government spending then shifts from social security to public goods provision if higher wage inequality is associated with the borrowing constraint and a high elasticity of marginal utility of youthful consumption.

Keywords: Borrowing constraint; Old-age social security; Forward intergenerational public goods; Ends-against-the-middle equilibrium; Wage inequality

JEL Classification: H41; H55; D72

---

*The authors would like to thank the Editor of this journal and three anonymous referees for their valuable comments and suggestions. Financial support of the Nomura Foundation is gratefully acknowledged.

†Graduate School of Economics, Nagoya University, Furo-cho, Chikusa-ku, Nagoya, Aichi 464-8601, Japan. E-mail: arawatari@soec.nagoya-u.ac.jp

‡Corresponding Author: Graduate School of Economics, Osaka University, 1-7, Machikaneyama, Toyonaka, Osaka 560-0043, Japan. E-mail: tono@econ.osaka-u.ac.jp

1 Introduction

1.1 Background and Motivation

Almost all Organization for Economic Cooperation and Development (OECD) countries have experienced some increase in wage inequality over the past few decades. Standard political economy theory suggests that higher wage inequality results in greater social security as the decisive voter becomes less affluent as wage inequality increases (Romer, 1975; Roberts, 1977; Meltzer and Richard, 1981). Given this theoretical prediction, it is natural to expect that higher wage inequality leads to a larger volume of social security.

Empirical evidence, however, does not necessarily support the above-mentioned theoretical prediction. For instance, OECD cross-country data shows that social security is negatively correlated with wage inequality (for example, Gottschalk and Smeeding, 1997; Chen and Song, 2009). In particular, the United Kingdom and the United States feature higher wage inequality and a smaller volume of social security whereas the Nordic countries display lower wage inequality and a larger volume of social security.

Several researchers have attempted to provide political economy models that explain this negative correlation (Benabou, 2000; Rodriguez, 2004; Chen and Song, 2009). In particular, some studies suggest that the presence of borrowing constraints is a key to demonstrate said negative correlation (Casamatta, Cremer and Pestieau, 2000; Bellettini and Berti Ceroni, 2007; Conde-Ruiz and Profeta, 2007; Cremer et al., 2007; Arawatari and Ono, 2013). However, they focus on a single policy issue, that is, social security that benefits the old at the expense of the young, and abstract away other policy issues that benefit the young. Because of this limitation, they fail to show how inequality within a generation affects political choices between conflicting expenditure demands by different generations.

The conflict over the distribution of government spending between different generations is considered from several viewpoints: income redistribution within and across generations (Conde-Ruiz and Galasso, 2005) and redistribution for the elderly vs. public goods provision for the young (Rangel, 2003; Levy, 2005; Bernasconi and Profeta, 2012). However, all these studies demonstrate the intergenerational conflict in the absence of borrowing constraint. In other words, they say nothing about how inequality within a generation affects the distribution of tax revenue across generations in the presence of borrowing constraint that produces a negative correlation between social security and inequality.

To resolve the limitation of the previous studies, we develop a model including borrowing constraints to demonstrate the empirically fitted correlation between inequality and the volume of old-age social security, and then add public goods provision that benefits the young into the model as alternative government spending.[1] Within this extended framework, we aim to provide theoretical predictions about the intergenerational distribution of tax revenue affected by wage inequality in the presence of borrowing constraint. In this respect, the present paper extends the two branches of literature mentioned above and fills a gap between them.

1.2 Analysis and Results

For the purpose of this analysis, we introduce an overlapping-generation economy with storage technology. In this economy, young workers are of three income categories: low, middle and high. Because they are not permitted to borrow in youth as a result of imperfect financial markets, lower-income individuals are more likely to be borrowing constrained. In youth, individuals decide how much to save and how much to consume. In old age, they retire and consume the return from saving and a pay-as-you-go (PAYG) social security benefit. Young workers pay a fixed proportion of their labor income to the government, and the tax revenue is divided into PAYG social security payments and public goods such as pure science and environmental maintenance. The former is enjoyed by the old. The latter, called forward intergenerational public goods, take a one-period lag to mature and thus benefit only the young.

The tax rate and the distribution of tax revenue between social security and public goods provision are determined in a bidimensional majoritarian voting game played by the young and the old. Voters cast a ballot on the labor income tax, which finances social security and public goods provision, and over the allocation of tax revenue between social security and public goods provision. Under this type of voting game, the existence of a Condorcet winner of the majority voting game is not necessarily guaranteed because of the multidimensionality of the issue space. To deal with this problem, we utilize the concept of a structure-induced equilibrium (Shepsle, 1979) with the notion of a once-and-for-all voting, which is applied to an overlapping generations framework by Conde-Ruiz and Galasso (2003, 2005).

Based on the above-mentioned concept of equilibrium, we consider the voting behavior of each type of individual. The preferences of the old are identical across all types of individuals because they owe no tax burden and receive the same level of social security benefit. In contrast, the preferences of the young depend on their income type because the tax burden differs across the board. In particular, the key to their preferences are the borrowing constraint and the elasticity of marginal utility of youthful consumption.

[1] The public good in the present model does not satisfy the non-rivalry property; per capita public spending for the young decreases as the number of the young increases. The good is classified as an impure public good in a strict sense. However, in the following, we call it as "a public good" for simplicity of description.

To understand the role of these two factors, consider first the case where a certain type of individual is borrowing unconstrained. A reduction in his wage decreases his marginal cost of taxation, thereby giving him an incentive to choose a higher tax rate. However, when he is borrowing unconstrained, there is an additional effect which works in the opposite direction. A decrease in consumption by one unit, which is caused by a rise in the tax rate, lowers the utility of consumption. This effect represents the marginal loss of utility, which becomes larger as his wage is decreased. Therefore, the additional loss of utility works to increase the marginal cost of taxation in response to a reduction in one's wage.

Which effect outweighs the other depends on the magnitude of the elasticity of marginal utility of consumption, denoted by σ. When $\sigma \leq 1$, the former effect outweighs the latter. Regardless of borrowing status, one's marginal cost of taxation decreases as his wage is decreased. Therefore, he prefers a higher tax rate in response to a reduction in his wage irrespective of whether he is borrowing constrained or not. However, when $\sigma > 1$, the latter effect outweighs the former when one is borrowing constrained. The borrowing-constrained individual prefers a higher tax rate in response to a reduction in his wage.

The result in the case of $\sigma > 1$ implies that there is a V-shaped relationship between the wage and the marginal cost of taxation. For a high-wage case where an individual is borrowing-unconstrained, a reduction in his wage decreases the marginal cost of taxation and thus increases his preferred tax rate. The opposite holds true for a low-wage case where he is borrowing-constrained: a borrowing-constrained individual prefers a lower tax rate in response to a reduction in his wage. The latter case entails a situation where a borrowing-constrained low-income individual prefers a lower tax rate than a borrowing-unconstrained middle-income individual. There is then an ends-against-the-middle equilibrium where low-income and high-income individuals form a coalition in favor of a low tax rate, and where middle-income individuals favor a high tax rate.

Given the characterization of political equilibrium, we investigate how the tax rate and the distribution of tax revenue are altered in response to changes in wage inequality. In particular, we consider a mean-preserving reduction of the decisive voter's wage in order to compare two groups of countries with similar per capita income levels but different levels of income inequality. We show that the standard theoretical result holds when the elasticity is below unity: an increase in wage inequality leads to a higher tax rate. We also show that a larger tax rate is associated with a larger fraction of old-age social security and a smaller fraction of forward intergenerational public goods provision in government expenditure.

However, when the elasticity is above unity, the mean-preserving reduction of the decisive voter's wage creates an inverse V-shaped relationship between the decisive voter's wage and the fraction of social security in government expenditure. The negative correlation arises when the decisive voter's wage is high and thus he is borrowing unconstrained, while the positive correlation arises when his wage is low and thus he is borrowing constrained. In particular, the latter case predicts that a higher level of inequality results in a lower tax rate, a smaller fraction of social security and a larger fraction of forward intergenerational public goods in government expenditure.

In the current framework, the negative correlation between inequality and the share of social security in government expenditure arises only in the equilibrium where the following two conditions hold: (i) the elasticity of marginal utility of youthful consumption is above unity; and (ii) the decisive voter is borrowing constrained. When one of the conditions fails to hold, the economy displays a positive correlation between inequality and the share of social security. Therefore, our analysis suggests that these factors are the keys to demonstrate the above-mentioned inverse V-shaped relationships. These relationships still hold even if we relax the assumption of the utility function.

The organization of this paper is as follows. Section 2 introduces the model and characterizes the economic equilibrium. Section 3 develops the political system, introduces the equilibrium concept of the voting game and demonstrates the voting behavior of each individual. Section 4 characterizes the political equilibrium. Section 5 examines how wage inequality affects the tax rate and the allocation of tax revenue between social security and forward intergenerational public goods. Section 6 briefly undertakes the analysis under a generalized utility function. Section 7 provides concluding remarks. Proofs of the propositions are provided in the appendix.

2 The Economic Environment

Consider a discrete time economy where time is denoted by $t = 0, 1, 2 \cdots$. The economy is made up of overlapping generations of individuals, each of whom lives two periods: youth and old age. The size of a generation born in period t, called generation t, is denoted by N_t. Population grows at a constant rate $n > 0 : N_{t+1} = (1+n)N_t$ for all $t \geq 0$. Within each generation, there are three types of agents according to ability to work, low, middle and high ($j = L, M, H$), whose proportions are respectively ρ^L, ρ^M and ρ^H, where $\sum_j \rho^j = 1$ and ρ^j satisfies the following assumption.

Assumption 1. $\frac{1+(1+n)}{2(1+n)} > \rho^j > \frac{n}{2(1+n)}$, $j = L, M, H$.

The first inequality of Assumption 1, $\{1+(1+n)\}/\{2(1+n)\} > \rho^j$, states that the proportion of type-k ($k = L, M, H$) young individuals, $\rho^j(1+n)$, must be less than half of the population, $(2+n)/2$. Otherwise, the type-k young individual becomes a decisive voter regardless of the preferences of others. The second inequality of Assumption 1,

$\rho^j > n/\{2(1+n)\}$, ensures that a young individual who prefers the highest tax rate among young individuals becomes the decisive voter.

To understand the argument stemming from the second inequality condition in Assumption 1, consider first the preferences of the old. As we explain below, the old choose a higher tax rate than young individual because they bear no tax burden but benefit from taxation via social security; the tax burden when young is viewed as a sunk cost for the old. In addition, the old have the same preferences over the policy because they benefit from the same social security.

Next, consider the preferences of the young. Suppose that a type-k ($k = L, M$ or H) prefers the highest tax rate. When the young and the old participate in voting, the sum of the type-k young and the old is given by $N_t\rho^k + N_{t-1}$, which is greater than half of the population in period t, $(N_t + N_{t-1})/2$, under the assumption of $\rho^k > n/2(1+n)$. This implies that the decisive voter becomes the old or the type-k young. However, the old cannot become the decisive voter because the population size of the old is smaller than that of the young under the assumption of $n > 0$. Therefore, the type-k young individual becomes the decisive voter. Figure 1 provides an example of preferences over the tax rate.

[Figure 1 about here.]

To assess the empirical plausibility of Assumption 1, let us suppose a generation to be 30 years in length. Assumption 1 becomes:

$$\frac{1+(1.0063)^{30}}{2(1.0063)^{30}} > \rho^j > \frac{(1.0063)^{30}-1}{2(1.0063)^{30}},$$

that is,

$$0.91414 > \rho^j > 0.085861,$$

where $1 + n = (1.0063)^{30}$ comes from the data by OECD (2013): the average annual population growth rate is 0.63% in sample OECD countries in 2010.[2]

2.1 Individuals

Each individual is assumed to receive utility from private consumption and publicly provided goods. The utility function of a type-j young individual in period t is specified by:

$$U_t^j = \frac{(c_t^{yj})^{1-\sigma}-1}{1-\sigma} + \eta\frac{(g_t)^{1-\sigma}-1}{1-\sigma} + \beta\cdot\left[c_{t+1}^{oj} + \eta\frac{(g_{t+1})^{1-\sigma}-1}{1-\sigma}\right],$$

[2]The assumption requires that the fraction of each type of the young must be more than 8.5% and less than 91.4% from the empirical viewpoint. This requirement is satisfied by the evidence reported by Ichino, Karabarbounis and Moretti (2011). They use the data from the World Value Surveys, and report that on average, 33%, 37% and 30% of the population are classified as poor, middle and rich, respectively, for twelve OECD countries.

where c_t^{yj} is consumption in youth, c_{t+1}^{oj} is consumption in old age, g_t is per capita public goods in period t, $\eta(> 0)$ is the parameter representing the preference for public goods, $\beta \in (0, 1]$ is the discount factor, and $\sigma(> 0)$ is the elasticity of marginal utility of youthful consumption or publicly provided goods. A lower σ implies a lower elasticity.

Following the literature (Conde-Ruiz and Galasso, 2005; Borck, 2007; Bethencourt and Galasso, 2008; Leroux, Pestieau and Racionero, 2011), we assume a quasi-linear utility function for analytical tractability. In Section 6, we will briefly investigate the case where the utility of old-age consumption is given by $\beta\{(c_{t+1}^{oj})^{1-\sigma} - 1\}/(1-\sigma)$ and show that the main result is not qualitatively unchanged under this alternative utility function.

Each individual works in his youth and retires in old age. The wage income is related to working ability. The wage of a type-j individual is given by $w^j (j = H, M, L)$, where w^j is constant over time and $w^L < w^M < w^H$. The average of the wage is denoted by $\bar{w} \equiv \rho^L w^L + \rho^M w^M + \rho^H w^H$.

Type-j's individual budget constraints in youth and old age are given respectively by:

$$\begin{aligned} c_t^{yj} + s_t^j &\leq (1 - \tau_t) w^j, \\ c_{t+1}^{oj} &\leq R s_t^j + b_{t+1}, \end{aligned}$$

where s_t^j is saving, τ_t is the income tax rate in period t, R is the gross interest rate, and b_{t+1} is the per capita social security benefit in old age. We impose the restriction of nonnegative savings as:

$$s_t^j \geq 0.$$

This rules out the possibility of borrowing in youth against future social security benefits (Diamond and Hausman, 1984; Conde-Ruiz and Profeta, 2007).

We assume that the economy is dynamically efficient.

Assumption 2. $R \geq 1 + n$.

The assumption implies that the rate of return from social security is lower than the private rate of return from saving. Nevertheless, low- and middle-income individuals may have an incentive to support this inferior system of intertemporal resource reallocation. This is because the current social security system involves an intragenerational redistribution component that transfers resources from the high to the low and the middle.

We also assume that (i) the interest rate is exogenous, and (ii) each individual receives the same amount of old age social security benefits regardless of contributions in their youth. The first assumption abstracts away the general equilibrium effect via the interest rate investigated by, for example, Cooley and Soares (1999) and Boldrin and Rustichini (2000). However, this simplification enables us to demonstrate more simply the analytical solution of the model. The second assumption abstracts away from the choice of social security systems (for example, Bismarckian vs. Beveridgean) as analyzed by Borck (2007), Conde-Ruiz and Profeta (2007) and Cremer et al. (2007). We adopt the second assumption to concentrate on the role of the borrowing constraint in the political determination of social security and public goods provision.

The representative type-j young individual maximizes his utility subject to the budget constraints and the restriction of nonnegative saving. When $s_t^j > 0$, the first-order condition for an interior solution is $(c_t^{yj})^{-\sigma} = \beta R$, and thus defines the optimal saving decision of a type-j individual given by $s_t^j = (1-\tau_t)w^j - (\beta R)^{-1/\sigma}$. By taking the borrowing constraint into account, the saving function of a type-j individual is:

$$s_t^j = \max\left\{0, (1-\tau_t)w^j - (\beta R)^{-1/\sigma}\right\}. \tag{1}$$

Eq. (1) indicates that the saving decision depends on the current tax rate τ_t, but is independent of the future tax rate τ_{t+1} and the proportion of tax revenues devoted to social security in old age, denoted by λ_{t+1}. This property comes from the assumption of a linear utility function of old-age consumption. Because of this property, we easily demonstrate the joint political determination of the tax rate τ and the proportion λ.

The saving function (1) implies that there is a critical rate of tax such that:

$$s_t^j > 0 \Leftrightarrow \tau_t < \hat{\tau}(w^j) \equiv 1 - \frac{1}{(\beta R)^{1/\sigma} w^j}. \tag{2}$$

A type-j individual chooses positive savings when the tax is below the critical rate. However, when the tax is above the critical rate, a type-j individual faces a borrowing constraint and can save nothing in youth. The critical rate of tax is higher when the wage income is larger because, given a tax rate common to all types of individuals, a more competent individual receives a higher level of disposable income.

2.2 The Government

In each period, the government collects tax revenue from the young by imposing an income tax. Following the conventions in the literature, we present the efficiency loss of taxation by assuming convex costs of collecting taxes (for example, Casamatta, Cremer, and Pestieau, 2000; Bellettini and Berti Ceroni, 2007; Cremer et al., 2007). Therefore, the actual tax revenue is given by $(1-\tau_t)\tau_t(\rho^L w^L + \rho^M w^M + \rho^H w^H) = (1-\tau_t)\tau_t \bar{w}$, where the term $(1-\tau_t)$ is the distortionary factor. The assumption of distortionary taxation is solely to ensure an interior solution to preferred tax rates and otherwise plays no role.

The government uses the tax revenue for old-age social security payments along with forward intergenerational public goods such as environmental preservation and pure science. The proportion $\lambda_t \in [0, 1]$ of tax revenue is devoted to old-age social security benefits and the remainder $(1-\lambda_t)$ is devoted to forward intergenerational public goods provision. The old-age social security is then an intergenerational transfer from the young to the old within a period. The budget constraint is $\lambda_t N_t(1-\tau_t)\tau_t\bar{w} = N_{t-1}b_t$. The per capita social security benefit in period t, b_t, is given by:

$$b_t = (1+n)\lambda_t(1-\tau_t)\tau_t\bar{w}.$$

The formation of public goods requires investment one period ahead of time. This assumption reflects the idea that pure science and investment in the environment do not obtain immediate results. Importantly, the current young generation can enjoy the outcomes of any investment in the future, while the current old generation cannot enjoy it while they are still alive. The budget constraint is $(1-\lambda_t)N_t(1-\tau_t)\tau_t\bar{w} = (N_t+N_{t+1})g_{t+1}$. The per capita public goods provision in period $t+1$, g_{t+1}, is given by:

$$g_{t+1} = \frac{1}{2+n}(1-\lambda_t)(1-\tau_t)\tau_t\bar{w}.$$

2.3 The Economic Equilibrium

We define the economic equilibrium as follows.

Definition 1. For a given sequence of tax rates and social security shares in government expenditure, $\{\tau_t, \lambda_t\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations, $\{c_t^{yj}, c_t^{oj}, s_t^j\}_{j=L,M,H}^{t=0,\cdots,\infty}$ with the initial condition $s_0^j(j = L, M, H)$, such that (i) in every period, a type-j individual maximizes his utility subject to the budget constraints and the nonnegativity constraint of saving, (ii) the social security budget and the public goods budget are balanced in every period, and (iii) the goods market clears every period.

From (1) and the private and government budget constraints, the consumption functions of a type-j individual in youth and old age are given respectively by:

$$c_t^{yj} = \begin{cases} (\beta R)^{-1/\sigma} & \text{if } \tau_t < \hat{\tau}(w^j) \\ (1-\tau_t)w^j & \text{if } \tau_t \geq \hat{\tau}(w^j) \end{cases}$$

$$c_{t+1}^{oj} = \begin{cases} R\{(1-\tau_t)w^j - (\beta R)^{-1/\sigma}\} + (1+n)\lambda_{t+1}(1-\tau_{t+1})\tau_{t+1}\bar{w} & \text{if } \tau_t < \hat{\tau}(w^j) \\ (1+n)\lambda_{t+1}(1-\tau_{t+1})\tau_{t+1}\bar{w} & \text{if } \tau_t \geq \hat{\tau}(w^j). \end{cases}$$

Because of the assumption of a quasi-linear utility function, the consumption in youth is type-independent and constant over time when the tax is below the critical rate.

The utility level obtained by individuals in economic equilibrium is represented by their indirect utility functions. We use the above-mentioned consumption functions to obtain an indirect utility function of a type-j young individual:

$$V_t^{yj} = \begin{cases} V_{t,s>0}^{yj} & \text{if } \tau_t < \hat{\tau}(w^j) \\ V_{t,s=0}^{yj} & \text{if } \tau_t \geq \hat{\tau}(w^j), \end{cases} \tag{3}$$

where:

$$V_{t,s>0}^{y,j} \equiv \frac{\left((\beta R)^{-1/\sigma}\right)^{1-\sigma}-1}{1-\sigma} + \beta\left[R\left\{(1-\tau_t)w^j-(\beta R)^{-1/\sigma}\right\}+(1+n)\lambda_{t+1}(1-\tau_{t+1})\tau_{t+1}\bar{w}\right]$$
$$+\eta\left\{\frac{(g_t)^{1-\sigma}-1}{1-\sigma}+\beta\frac{\left(\frac{1}{2+n}(1-\lambda_t)(1-\tau_t)\tau_t\bar{w}\right)^{1-\sigma}-1}{1-\sigma}\right\},$$

$$V_{t,s=0}^{y,j} \equiv \frac{((1-\tau_t)w^j)^{1-\sigma}-1}{1-\sigma}+\beta(1+n)\lambda_{t+1}(1-\tau_{t+1})\tau_{t+1}\bar{w}$$
$$+\eta\left\{\frac{(g_t)^{1-\sigma}-1}{1-\sigma}+\beta\frac{\left(\frac{1}{2+n}(1-\lambda_t)(1-\tau_t)\tau_t\bar{w}\right)^{1-\sigma}-1}{1-\sigma}\right\}.$$

$V_{t,s>0}^{y,j}$ denotes the indirect utility of a type-j young individual when he saves a portion of his income, and $V_{t,s=0}^{y,j}$ denotes the indirect utility when he is faced with a borrowing constraint and saves nothing. For each indirect utility function, the first term on the right-hand side shows the utility of consumption in youth, the second term shows the utility of consumption in old age and the third term shows the utility of public goods in old age. The public goods provision in period t, g_t, is omitted in the above expression because it is predetermined in period $t-1$ and thus, is politically irrelevant in period t.

For a type-j old individual in period t, the indirect utility function is:

$$V_t^{o,j} \equiv (1+n)\lambda_t(1-\tau_t)\tau_t\bar{w}+\eta\frac{(g_t)^{1-\sigma}-1}{1-\sigma}, \tag{4}$$

where the first-term on the right-hand side shows the social security benefits. The term Rs_{t-1}, representing the return from saving, is omitted in this expression because it is predetermined in period $t-1$. Old individuals have the same indirect utility function regardless of their type because their savings in youth are predetermined and the level of public goods they enjoy is predetermined one period in advance. Therefore, old individuals have the same preferences for the tax rate, τ, and the share of social security, λ.

3 The Political Institution and Voting

The tax rate τ and the proportion λ are determined by individuals through a political process of majoritarian voting. Elections take place every period and all young and old individuals cast a ballot over τ, the income tax, and λ, the share of social security in government expenditure. Individual preferences over the two issues are represented by the indirect utility functions at Eqs. (3) and (4) for the young and the old, respectively. Every individual has zero mass and thus, no individual vote can change the outcome of the election. Therefore, we assume individuals vote sincerely.

This majoritarian voting game has two significant characteristics. First, the issue space is bidimensional (τ and λ), and thus, the Nash equilibrium of a majoritarian voting game may fail to exist. To deal with this feature, we use the concept of issue-by-issue voting, or structure-induced equilibrium, as formalized by Shepsle (1979) and applied by Conde-Ruiz and Galasso (2003, 2005) for the framework of overlapping generations.

Second, the game is intrinsically dynamic because it describes the interaction among successive generations. To deal with this feature, we assume once-and-for-all voting (see, for example, Casamatta, Cremer, and Pestieau, 2000; Conde-Ruiz and Profeta, 2007). That is, period-t young individuals vote over the current and future taxes (τ_t and τ_{t+1}) and the allocation (λ_t and λ_{t+1}) given the static expectation that successive generations will make the same choice with their current choice persistent over time: $\tau_t = \tau_{t+1} = \tau$ and $\lambda_t = \lambda_{t+1} = \lambda$ for all t. The previous literature has shown that this type of behavior can be supported as a sub-game perfect equilibrium in a repeated voting if voters expect that any deviation from this static behavior will be punished by future generations (Conde-Ruiz and Galasso, 2003, 2005).[3]

Because of the above-mentioned assumption, the current model presents a static voting game. Therefore, the result in Shepsle (1979) can be applied to obtain the sufficient conditions for the existence of a structure-induced equilibrium. In particular, if preferences are single peaked along every dimension of the issue space, a sufficient condition for (τ^*, λ^*) to be an equilibrium of the voting game is that τ^* represents the outcome of majority voting over the jurisdiction τ when the other dimension is fixed at its level λ^*, and vice versa.

Preferences of the old are immediately shown to be single peaked along every dimension because they are given by $V_t^{o,j} \equiv (1+n)\lambda_t(1-\tau_t)\tau_t\bar{w}$ and satisfy $\partial^2 V^{o,j}/\partial\tau^2 < 0$ and $\partial^2 V^{o,j}/\partial\lambda^2 = 0$. The preferences of the young are also single peaked along every dimension. Nevertheless, the proof of this argument is not straightforward because the preferences of the young are kinked at the critical rate $\hat{\tau}(w^j)$. The formal proof is given in Appendix 8.1.

In what follows, we demonstrate preferences of the old and the young over policy.

3.1 Preferences of the Old Over Policy

The old choose τ to maximize $V^{o,j}$ in (4) given λ, and λ to maximize $V^{o,j}$ in (4) given τ. Their preferred tax rate and the share of social security are respectively given by:

$$\tau^{oj} = \frac{1}{2} \text{ and } \lambda^{oj} = 1 \text{ for all } j.$$

Maximization is realized when the tax rate is set to attain the top of the Laffer curve, $(1-\tau)\tau$. The old prefer to use the maximized tax revenue exclusively for social security

[3]The authors would like to thank one of the referees for pointing this out.

because they cannot benefit from the investment in public goods that take a one-period lag in formation.

3.2 Preferences of the Young Over the Tax Rate

Consider first the preferences of the young over τ. A type-j young individual chooses τ to maximize $V_{s>0}^{y,j}$ when he is borrowing unconstrained; he chooses τ to maximize $V_{s=0}^{y,j}$ when he is borrowing constrained. Thus, the tax rate τ chosen by the type-j young individual satisfies the following first-order condition:

$$\beta(1+n)\lambda(1-2\tau)\bar{w} + \beta\eta\left\{\frac{1}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right\}^{-\sigma}\frac{1}{2+n}(1-\lambda)(1-2\tau)\bar{w} \tag{5}$$
$$= \begin{cases} \beta R w^j & \text{if} \quad \tau < \hat{\tau}(w^j) \\ (1-\tau)^{-\sigma}(w^j)^{1-\sigma} & \text{if} \quad \tau \geq \hat{\tau}(w^j) \end{cases}$$

The first term on the left-hand side is the marginal benefit of social security, the second term on the left-hand side is the marginal benefit of public goods provision, and the right-hand side shows the marginal cost of taxation. The marginal cost is given by $\beta R w^j$ when a type-j young individual is borrowing unconstrained; and it is given by $(1-\tau)^{-\sigma}(w^j)^{1-\sigma}$ when he is borrowing constrained. The type-j young individual chooses the tax rate to equate marginal benefits and costs of taxation from the viewpoint of utility maximization.

Condition (5) shows that the marginal benefit of taxation depends on the average wage rather than each individual's wage; but the marginal cost depends on each individual's wage. In particular, a rise in type-i's wage increases his marginal cost of taxation when he is borrowing unconstrained. Therefore, a rise in his wage decreases his preferred tax rate as long as he is borrowing unconstrained. However, when he is borrowing-constrained, a rise in his wage may or may not increase his marginal cost of taxation depending on the elasticity of the marginal utility of youthful consumption, denoted by σ. The elasticity could have a crucial role in the determination of the preferred tax rate by a borrowing-constrained individual.

To understand the role of the elasticity in a borrowing-constrained case more precisely, consider the marginal cost of taxation for the type-i borrowing-constrained individual, denoted by MCT^j :

$$MCT^j \equiv (1-\tau)^{-\sigma}(w^j)^{1-\sigma} = w^j \cdot (c^{yj})^{-\sigma},$$

where the second equality comes from $c^{yj} = (1-\tau)w^j$. This expression shows that type-j's wage has two opposing effects on MCT^j. The first effect is expressed by the term w^j. An increase in the tax rate by one unit entails an increase in the tax burden by w^j units. In other words, a lower-income individual pays less tax than a higher-income individual in response to a marginal increase in the tax rate. Therefore, the term works to decrease the marginal cost of taxation in response to a reduction in one's wage.

The second effect is expressed by the term $(c^{yj})^{-\sigma}$. This term shows a marginal utility of consumption: a decrease in consumption by one unit, which is caused by a rise in the tax rate, leads to a decrease of utility of consumption by $(c^{yj})^{-\sigma}$ units. Given that $c^{yj} = (1-\tau)w^j$, this term becomes smaller as one's wage is decreased. Therefore, the term works to increase the marginal cost of taxation in response to a reduction of one's wage.

Which effect outweighs the other is likely to depend on the magnitude of the elasticity of marginal utility of consumption. When $\sigma \leq 1$, the former effect outweighs the latter. Regardless of borrowing status, one's marginal cost of taxation decreases as his wage is reduced (Panel (a) of Figure 2). However, the latter effect becomes more dominant as σ is increased. In particular, when $\sigma > 1$, the latter effect outweighs the former. There is then a V-shaped relationship between the wage and the marginal cost of taxation (Panel (b) of Figure 2). The latter case gives an insight into the mechanism of an inverse V-shaped relationship between inequality and policy variable, which will be investigated in detail in Section 5.

[Figure 2 about here.]

3.3 Preferences of the Young Over the Share of Social Security

Next, consider the preferences of the young over λ. The first derivative of $V^{y,j}$ with respect to λ is independent of the status of saving: $\partial V^{y,j}_{s>0}/\partial\lambda = \partial V^{y,j}_{s=0}/\partial\lambda$. Direct calculation leads to:

$$
\begin{aligned}
\frac{\partial V^{y,j}_{s>0}}{\partial\lambda} &= \frac{\partial V^{y,j}_{s=0}}{\partial\lambda} \\
&= \beta(1+n)(1-\tau)\tau\bar{w} - \beta\eta\left\{\frac{1}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right\}^{-\sigma}\frac{1}{2+n}(1-\tau)\tau\bar{w},
\end{aligned}
$$

where the first term on the right-hand side shows the marginal increase in the benefit of social security given by an increase in the share of social security, and the second term is the marginal loss of utility of public goods given by a decrease in the share of public goods provision. The share of social security, λ, is chosen to balance the marginal benefit and loss in terms of utility.

A noteworthy feature of the current model is that the preferred share by the young is type-independent. This is because (i) all types of young individuals enjoy the same level of forward intergenerational public goods, (ii) the utility of forward intergenerational public goods is separable from the utility of private goods, and (iii) the utility of old-age consumption is specified by a linear utility function. In Section 6, we employ a more generalized utility function and show that the preferred share becomes type-dependent but the main result still holds.

Another noteworthy feature is that for a low tax rate, the share λ could be zero: all tax revenue goes to public goods provision. When the tax revenue is low, the marginal utility of public goods is high even when all tax revenue is devoted to it. However, the marginal utility of old-age consumption is always constant because of a quasi-linear utility function.[4] Therefore, choosing $\lambda = 0$ is optimal for a young individual from the viewpoint of utility maximization when the tax rate is below the critical rate. The corner solution is not peculiar to the model with a quasi-linear utility function. As we demonstrate in Section 6, the qualitatively similar result also holds under a generalized utility function.

Based on the above argument, the preferred share of the young becomes:

$$\lambda = \begin{cases} 0 & \text{if } \tau \in [0, \underline{\tau}] \\ 1 - \frac{2+n}{(1-\tau)\tau\bar{w}} \left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma} & \text{if } \tau \in \left(\underline{\tau}, \frac{1}{2}\right] \end{cases}, \tag{6}$$

where:

$$\underline{\tau} \equiv \frac{1 - \sqrt{1 - \frac{4(2+n)}{\bar{w}}\left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma}}}{2}.$$

Figure 3 illustrates the graphs of (6). To proceed the analysis we make the following assumption.

[Figure 3 about here.]

Assumption 3.

$$\text{(i)} \quad 1 > \frac{4(2+n)}{\bar{w}}\left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma};$$

$$\text{(ii)} \; w^L > \frac{2}{(\beta R)^{1/\sigma}\left(1 + \sqrt{1 - \frac{4(2+n)}{\bar{w}}\left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma}}\right)}.$$

The first assumption implies that the preferred share in (6), which attains the highest value at $\tau = 1/2$, takes a positive value at $\tau = 1/2$. Therefore, the assumption ensures that a political equilibrium exists with $\lambda > 0$ for a range of $[\underline{\tau}, 1/2)$; otherwise, $\lambda = 0$ holds for any $\tau \in [0, 1/2)$, implying a trivial outcome of no provision of old-age social security. The range of τ is limited to $(0, 1/2)$ because the preferred tax rate by the old is equal to $1/2$ and that by the young is less than $1/2$. The second assumption is equivalent to $\underline{\tau} < \hat{\tau}\left(w^L\right)$. This assumption enables us to demonstrate cases of borrowing-unconstrained as well as borrowing-constrained type-L individuals in the presence of social security, $\lambda > 0$.

[4] We would like to thank one of the referees for suggesting this interpretation.

4 The Political Equilibrium

The previous section analyzed the voting behavior of each type of individual along the two dimensions of the issue space, τ and λ. Given that preferences are single peaked for each issue, we now apply Shepsle's (1979) result and characterize the structure-induced equilibrium of the game.

The structure-induced equilibrium outcome is found as follows. First, we determine the decisive voter over λ and calculate his most preferred share, denoted by $\lambda^{dec}(\tau)$, as a function of the tax rate τ, where the superscript "*dec*" indicates the decisive voter. Second, we determine the decisive voter over τ and calculate his most preferred tax rate, denoted by $\tau^{dec}(\lambda)$, as a function of the share parameter λ. Finally, we find the point where these reaction functions $\lambda^{dec}(\tau)$ and $\tau^{dec}(\lambda)$ cross. This point corresponds to the structure-induced equilibrium outcome of the voting game.

Consider the political determination of λ. The decisive voter over λ is a young individual because (i) the population size of the young is larger than that of the old, and (ii) all young individuals have the same preferences for λ regardless of their type. Therefore, from (6), the decisive voter's reaction function $\lambda^{dec}(\tau)$ is given by:

$$\lambda^{dec}(\tau) = \begin{cases} 0 & \text{if } \tau \in [0, \underline{\tau}] \\ 1 - \frac{2+n}{(1-\tau)\tau\bar{w}} \left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma} & \text{if } \tau \in \left(\underline{\tau}, \frac{1}{2}\right] . \end{cases} \tag{7}$$

Next, consider the political determination of τ. The decisive voter over τ belongs to the young generation because (i) young individuals choose lower tax rates than the old, and (ii) the population size of the young is larger than that of the old. In particular, to determine the type of decisive voter, we focus on the parameter σ representing the elasticity of marginal utility of youthful consumption and consider two cases separately: a low elasticity ($\sigma \leq 1$ in Subsection 4.1) and a high elasticity ($\sigma > 1$ in Subsection 4.2).

We adopt the above classification because the order of preferences for the tax rate critically depends on the degree of elasticity. For the case of $\sigma \leq 1$, a lower-income young individual prefers a higher tax rate. However, for the case of $\sigma > 1$, a low-income young individual may prefer a lower tax rate than middle-income (or middle- and high-income) individuals. For each case, we show the existence and uniqueness of a structure-induced equilibrium of the voting game and explain the mechanism underlying the result.

4.1 The Case of a Low Elasticity ($\sigma \leq 1$)

To determine the type of decisive voter over τ in the case of $\sigma \leq 1$, we consider the preferred tax rate of a type-j young individual given by (5). Figure 4 illustrates the condition (5) that determines the preferred tax rate by a type-j young ($j = L, M, H$)

individual. The left-hand side of (5), denoted by LHS, is decreasing in τ and is independent of the type of young individual. In contrast, the right-hand side of (5), denoted by RHS^j, is nondecreasing in τ, and dependent on the type of young individual and featured by $RHS^H \geq RHS^M \geq RHS^L$, where an equality holds if and only if $\sigma = 1$. The kink point of $\tau = \hat{\tau}(w^j)$ implies that a type-j young individual can save part of his income if $\tau < \hat{\tau}(w^j)$ and nothing if $\tau \geq \hat{\tau}(w^j)$. It is immediately observed from Figure 4 that given λ, a lower-income young individual prefers a higher tax rate: $\tau^{yH} < \tau^{yM} < \tau^{yL}$ for all $\lambda \in [0,1]$, where $\tau^{yj} (j = L, M, H)$ denotes the preferred tax rate of a type-j young individual.

[Figure 4 about here.]

Given the assumption of demographic structure (Assumption 1) and the fact that $\tau^{yH} < \tau^{yM} < \tau^{yL} < \tau^{oj}$, the decisive voter over τ is the one who prefers the highest tax rate among young individuals, that is, a type-L young individual. Therefore, the reaction function of $\tau, \tau^{dec}(\lambda)$, is implicitly given by (5) with $j = L$. To find the crossing point of the two reaction functions, $\lambda^{dec}(\tau)$ and $\tau^{dec}(\lambda)$, we substitute (7) into (5) with $j = L$ to obtain:

$$y(\tau; \bar{w}, n) = z(\tau; w^L),$$

where:

$$y(\tau; \bar{w}, n) = \begin{cases} \beta\eta \left\{\frac{1}{2+n}\bar{w}\right\}^{1-\sigma} \frac{1-2\tau}{((1-\tau)\tau)^{\sigma}} & \text{if } \tau \in [0, \underline{\tau}] \\ \beta(1+n)(1-2\tau)\bar{w} & \text{if } \tau \in (\underline{\tau}, 1/2] \end{cases}$$

$$z(\tau; w^L) = \begin{cases} \beta R w^L & \text{if } \tau < \hat{\tau}(w^L) \\ \frac{(w^L)^{1-\sigma}}{(1-\tau)^{\sigma}} & \text{if } \tau \geq \hat{\tau}(w^L). \end{cases}$$

The function $y(\tau; \bar{w}, n)$ represents the marginal benefit of taxation including the politically determined λ which is adjusted to a change in τ; and the function $z(\tau; w^L)$ represents the marginal cost of taxation for a type-L individual. Solving $y(\tau; \bar{w}, n) = z(\tau; w^L)$ for τ leads to the tax rate in a structure-induced equilibrium of the voting game. The corresponding λ is obtained by substituting the equilibrium τ into the reaction function λ^{dec} in (7).

Proposition 1. *Suppose that $\sigma \leq 1$ holds. There exists a unique structure-induced equilibrium of the voting game such that the decisive voter over τ is a type-L young individual.*

Proof. See Appendix 8.2.

There are two possible cases of the equilibrium. The first is the case where the wage of type-L individuals is high such that they can save part of their income in youth for future consumption. In this case, the equilibrium tax rate represented by the crossing point of $y(\tau;\bar{w},n)$ and $z(\tau;w^L)$ is below the critical rate of $\hat{\tau}(w^L)$. The second is the case where the wage of type-L individuals is low such that they save nothing in their youth. The equilibrium tax rate is given above the critical rate of $\hat{\tau}(w^L)$.

4.2 The Case of a High Elasticity ($\sigma > 1$)

Next, consider the case of a high elasticity such that $\sigma > 1$. The decisive voter over λ is equivalent to that in the previous case; the reaction function $\lambda^{dec}(\tau)$ is given by (7). However, the decisive voter over τ may differ from the previous case; the order of preferred tax rates may change depending on the value of λ.

To determine the decisive voter over τ, we recall the condition (5) that determines the tax rate preferred by a type-j young individual for a given λ. The graphs of (5) for the case of $\sigma > 1$ are illustrated in Figure 5. The main difference from the previous case is that RHS^i and $RHS^j (i \neq j)$ cross at tax rate $\tau \in (0, 1/2)$. This is because when a type-j individual is borrowing constrained, the slope of RHS^j becomes steeper as the elasticity σ increases. There are two critical values of τ, $\tilde{\tau}^{LM}$ and $\tilde{\tau}^{MH}$, such that RHS^L and RHS^M cross at $\tau = \tilde{\tau}^{LM}$ and RHS^M and RHS^H cross at $\tau = \tilde{\tau}^{MH}$. By direct calculation, we obtain:

$$\tilde{\tau}^{LM} \equiv 1 - \left(\frac{(w^L)^{1-\sigma}}{\beta R w^M}\right)^{1/\sigma} \text{ and } \tilde{\tau}^{MH} \equiv 1 - \left(\frac{(w^M)^{1-\sigma}}{\beta R w^H}\right)^{1/\sigma},$$

where $\hat{\tau}(w^L) < \tilde{\tau}^{LM} < \hat{\tau}(w^M) < \tilde{\tau}^{MH} < \hat{\tau}(w^H)$ (see Figure 5). The derivation of $\tilde{\tau}^{LM}$ and $\tilde{\tau}^{MH}$ is given in Appendix 8.3.

[Figure 5 about here.]

The tax rate preferred by a type-j young individual is determined by the crossing point of LHS and RHS of (5). RHS is independent of λ while LHS is strictly increasing in λ. The tax rate preferred by a type-j young individual depends on the size of λ. Overall, he prefers a higher tax rate when λ is higher.

The order of tax rates preferred by the three types of individuals is changed by the size of λ, as illustrated in Figure 5. First, when λ is low such that LHS of (5) crosses RHS of (5) with $j = L$ within the range $(0, \tilde{\tau}^{LM}]$, the tax rates preferred by the young are ordered by $\tau^{yH} < \tau^{yM} < \tau^{yL}$, where $\tau^{yj} (j = L, M, H)$ denotes the preferred tax rate by type-j young: the type-L young individual becomes the decisive voter. Second, when λ attains a middle value such that LHS of (5) crosses RHS of (5) with $j = M$ within the range $(\tilde{\tau}^{LM}, \tilde{\tau}^{MH}]$, the tax rates preferred by the young are ordered by $\tau^{yH} < \tau^{yL} < \tau^{yM}$ or $\tau^{yL} \leq \tau^{yH} < \tau^{yM}$: the decisive voter in this case is the type-M young individual. Finally, when λ is high such that LHS of (5) crosses RHS of (5) with $j = H$ within the range $[\tilde{\tau}^{MH}, 1/2]$, the tax rates preferred by the young are ordered by $\tau^{yL} < \tau^{yM} < \tau^{yH}$: the decisive voter becomes the type-H young individual.

Given the abovementioned feature, the reaction function of τ, $\tau = \tau^{dec}(\lambda)$, is now implicitly given by:

$$\beta(1+n)\lambda(1-2\tau)\bar{w} + (1+\beta)\eta\left\{\frac{1}{2+n}(1-\lambda)\bar{w}\right\}^{1-\sigma}\frac{1-2\tau}{((1-\tau)\tau)^{\sigma}} = \tilde{z}(\tau; w^L, w^M, w^H), \quad (8)$$

where $\tilde{z}(\tau; w^L, w^M, w^H) \equiv \min_j\{z(\tau, w^j)\}$. The graph of the function $\tilde{z}$ is illustrated by the bold curve in Figure 5.

We substitute the reaction function of $\lambda^{dec}(\tau)$, given by (7), into the left-hand side of (8) to obtain the condition that determines the equilibrium tax rate:

$$y(\tau; \bar{w}, n) = \tilde{z}(\tau; w^L, w^M, w^H),$$

where $y(\cdot)$ has been already defined in the previous subsection. Figure 6 illustrates the graphs of $y(\tau; \bar{w}, n)$ and $\tilde{z}(\tau; w^L, w^M, w^H)$. Solving $y(\tau; \bar{w}, n) = \tilde{z}(\tau; w^L, w^M, w^H)$ for τ leads to the tax rate in a structure-induced equilibrium of the voting game. The corresponding λ is obtained by substituting the equilibrium τ into the reaction function λ^{dec} in (7).

[Figure 6 about here.]

Proposition 2. *Suppose that $\sigma > 1$ holds. There exists a unique structure-induced equilibrium of the voting game such that the decisive voter over τ is:*

(i) *a type-L individual if* $-1 + 2\left\{(w^L)^{1-\sigma}/\beta R w^M\right\}^{1/\sigma} \leq Rw^M/(1+n)\bar{w}$;

(ii) *a type-M individual otherwise.*

Proof. See Appendix 8.4.

A noteworthy feature of Proposition 2 is that under certain conditions, the middle-income individuals prefer a higher tax rate than the low-income individuals. In particular, if the condition in statement (ii) of Proposition 2 holds, there exists an equilibrium, like an ends-against-the-middle equilibrium, where the low- and high-income young individuals form a coalition in favor of a low tax rate and the middle-income individual favoring a high tax rate becomes the decisive voter (see Figure 6).

The key factors in Proposition 2 are the borrowing constraints and the elasticity of marginal utility of youthful consumption. To understand the roles of these two factors, consider the case where the low-income individuals are faced with a borrowing constraint. Here, they wish to consume more in their youth, but cannot because of the borrowing constraint. In this situation, a higher tax rate produces two opposing effects: a negative effect that results in lower after-tax income and thus, the utility loss of taxation in youth, and a positive effect that produces higher social security benefit and thus, the utility gain in old age.

The net impact of taxation depends on the elasticity of marginal utility of youthful consumption. When the elasticity is low such that $\sigma < 1$, the positive effect outweighs the negative effect. The low-income individual then chooses the highest tax rate among the young and thus becomes the decisive voter. In contrast, when the elasticity is high such that $\sigma > 1$, the negative effect may outweigh the positive effect for low-income individuals. They choose a lower tax rate than the middle-income individuals, and this results in an equilibrium where the middle-income individual becomes the decisive voter.

Figure 7 illustrates the conditions that determine the decisive voter and his status of saving in a $w^L - w^M$ space. From the figure, we find that the decisive voter is a type-L individual when wage incomes levels of the two types of individuals are high such that the pair (w^L, w^M) is set within the area marked by $(j = L)$ in Figure 7. The order of preferred tax rates is the reverse of the wage rates. However, the order is changed when the wage income level of the type-L individual is sufficiently low such that (w^L, w^M) is set within the area marked by $(j = M)$ in Figure 7. The decisive voter becomes the type-M young individual. The equilibrium is featured by the situation that resembles the ends-against-the-middle equilibrium.

[Figure 7 about here.]

5 Effects of Inequality on Policy

Given the characterization of the political equilibrium in Section 4, we now investigate how the tax rate and the share of social security in government expenditure change in response to a change in inequality. In particular, we consider a mean-preserving reduction of the decisive voter's wage in order to compare two groups of countries with similar per capita income levels but different levels of income inequality. For this purpose, we suppose a reduction of the type-L's (or type-M's) wage associated with an increase in type-H's wage when the decisive voter is a type-L (or type-M) individual.

We focus on a non-trivial equilibrium with $\lambda > 0$ to observe the marginal effect on the share of social security in government expenditure. Under Assumption 3(ii), we can demonstrate cases of borrowing-unconstrained as well as borrowing-constrained type-L individuals in the presence of social security, $\lambda > 0$. Otherwise, the type-L young individual is always borrowing constrained when $\lambda > 0$.

Proposition 3. *Consider a political equilibrium with* $\lambda > 0$.

(i) *In an economy with $\sigma \leq 1$ where the decisive voter is a type-L young individual, the tax rate and the share of social security are nondecreasing in response to a mean-preserving reduction of w^L.*

(ii) *In an economy with $\sigma > 1$ where the decisive voter is a type-j ($j = L$ or M) young individual, a mean-preserving change in the decisive voter's wage (w^j) locally produces inverse V-shaped relationships between w^j and the tax rate (τ) and between w^j and the share of social security (λ).*

Proof. See Appendix 8.5.

Figure 8 illustrates the effects of a mean-preserving change in the decisive voter's wage on the equilibrium tax rate when a type-L individual is the decisive voter. Panel (a) is for the case of $\sigma \leq 1$; Panel (b) is for the case of $\sigma > 1$. Proposition 3 states that if the elasticity is low such that $\sigma \leq 1$, there is, in general, a monotone relationship between the decisive voter's wage and his preferred tax rate: the decisive voter prefers a higher tax rate as he becomes poorer. However, when the elasticity is high such that $\sigma > 1$, such a monotone relationship no longer holds. Once the decisive voter's wage falls below the threshold level that changes his status from unconstrained to constrained, he prefers a lower tax rate as he becomes poorer, as demonstrated in Subsection 3.2. Thus, there is an inverse V-shaped relationship between the decisive voter's wage and the preferred tax rate around the threshold level of wage, as illustrated in panel (c) of Figure 8. Given a positive correlation between the tax and the share of social security, there is also an inverse V-shaped relationship between the decisive voter's wage and the share of social security in government expenditure.

[Figure 8 about here.]

Two remarks are in order. First, a positive correlation between the tax and the share of social security arises even if we assume a representative individual and thus, no wage inequality within a generation. This statement is easily confirmed by looking at the preferred share of social security by the young, Eq. (7). However, when the assumption of wage inequality is dropped, we cannot investigate the effect of wage inequality on the distribution of government expenditure between different generations, which is the main objective of this paper.

Second, in the current framework, the negative correlation between inequality and the share of social security in government expenditure arises only in the equilibrium where the following two conditions hold: (i) the elasticity of marginal utility of youthful consumption, σ, is above unity; and (ii) the decisive voter is borrowing constrained. When one of the conditions fails to hold, the economy displays a positive correlation between inequality and the share of social security. Therefore, our analysis suggests that these factors are the keys in demonstrating the above-mentioned inverse V-shaped relationships. These relationships still hold even if the assumption of a quasi-linear utility function is dropped, as we briefly demonstrate in the next section.

6 A Generalized Utility Function

At this point, we have conducted an analysis assuming a quasi-linear utility function where the utility of old-age consumption is given by βc_{t+1}^{oj}. This specification enables us to illustratively show the existence and uniqueness of the political equilibrium. However, the specification also results in (i) a saving decision unaffected by social security; and (ii) type-independent preferences over the share of social security. We introduce a generalized utility function of old-age consumption to resolve these problems.

The main result of this section is that most of the previous results still hold true under the alternative utility function. That is, under a certain condition, there exists an equilibrium, like an ends-against-the-middle equilibrium, when the elasticity of marginal utility of youthful consumption is above unity and the decisive voter is borrowing constrained. In this equilibrium, a mean-preserving spread of income inequality results in a lower equilibrium tax rate and a lower share of social security in government expenditure.

For the purpose of analysis, we assume the following utility function:

$$U_t^j = \frac{(c_t^{yj})^{1-\sigma}-1}{1-\sigma} + \eta \frac{(g_t)^{1-\sigma}-1}{1-\sigma} + \beta \left[\frac{(c_{t+1}^{oj})^{1-\sigma}-1}{1-\sigma} + \eta \frac{(g_{t+1})^{1-\sigma}-1}{1-\sigma} \right].$$

The main difference from the previous model is that the utility of old-age consumption is given by $\beta\{(c_{t+1}^{oj})^{1-\sigma}-1\}/(1-\sigma)$ rather than βc_{t+1}^{oj}. The maximization of their lifetime utility under the budget constraints leads to the following saving function:

$$s_t^j = \max\left\{0, \frac{(\beta R)^{1/\sigma}}{(\beta R)^{1/\sigma}+R}\left[(1-\tau_t)w^j - \frac{b_{t+1}}{(\beta R)^{1/\sigma}}\right]\right\}.$$

Saving now depends on the social security benefit b_{t+1} that gives individuals a disincentive to save. We hereafter drop the time subscript because our focus is on the time-invariant policy.

We substitute the government budget constraint for social security $b = (1+n)\lambda\tau(1-\tau)\bar{w}$ into the above saving function to obtain the following condition that determines the saving behavior of a type-j individual:

$$s^j > 0 \Leftrightarrow \frac{w^j}{\bar{w}} \cdot \frac{(\beta R)^{1/\sigma}}{1+n} > \lambda\tau.$$

This inequality condition states that a type-j individual is borrowing unconstrained if his wage is high, the tax burden is low, and/or the share of social security in government expenditure is also low.

With the saving function and the government budget constraints, we give the consumption functions of a type-j individual in youth and old age as follows:

$$c_t^{yj} = \begin{cases} \frac{R}{(\beta R)^{1/\sigma}+R}\left[(1-\tau)w^j + \frac{(1+n)\lambda(1-\tau)\tau\bar{w}}{R}\right] & \text{if } \frac{w^j}{\bar{w}}\cdot\frac{(\beta R)^{1/\sigma}}{1+n} > \lambda\tau \\ (1-\tau)w^j & \text{if } \frac{w^j}{\bar{w}}\cdot\frac{(\beta R)^{1/\sigma}}{1+n} \leq \lambda\tau \end{cases}$$

$$c_{t+1}^{oj} = \begin{cases} \frac{(\beta R)^{1/\sigma}R}{(\beta R)^{1/\sigma}+R}\left[(1-\tau)w^j + \frac{(1+n)\lambda(1-\tau)\tau\bar{w}}{R}\right] & \text{if } \frac{w^j}{\bar{w}}\cdot\frac{(\beta R)^{1/\sigma}}{1+n} > \lambda\tau \\ (1+n)\lambda(1-\tau)\tau\bar{w} & \text{if } \frac{w^j}{\bar{w}}\cdot\frac{(\beta R)^{1/\sigma}}{1+n} \leq \lambda\tau. \end{cases}$$

Unlike the previous case, the consumption in youth is now type-dependent and is linearly related to lifetime income when individuals are borrowing unconstrained.

After some calculation, we can obtain indirect utility functions of type-j young and old individuals as follows:

$$V^{yj} = \begin{cases} V_{s>0}^{y,j} & \text{if } \frac{w^j}{\bar{w}}\cdot\frac{(\beta R)^{1/\sigma}}{1+n} > \lambda\tau \\ V_{s=0}^{y,j} & \text{if } \frac{w^j}{\bar{w}}\cdot\frac{(\beta R)^{1/\sigma}}{1+n} \leq \lambda\tau \end{cases}$$

$$V^{oj} = \begin{cases} V_{s>0}^{o,j} & \text{if } \frac{w^j}{\bar{w}}\cdot\frac{(\beta R)^{1/\sigma}}{1+n} > \lambda\tau \\ V_{s=0}^{o,j} & \text{if } \frac{w^j}{\bar{w}}\cdot\frac{(\beta R)^{1/\sigma}}{1+n} \leq \lambda\tau \end{cases}$$

where:

$$\begin{aligned} V_{s>0}^{y,j} &\equiv \frac{1}{1-\sigma}\left(\frac{R}{(\beta R)^{1/\sigma}+R}\right)^{-\sigma}\left[(1-\tau)w^j + \frac{(1+n)\lambda(1-\tau)\tau\bar{w}}{R}\right]^{1-\sigma} + \frac{\eta}{1-\sigma}(g)^{-\sigma} \\ &\quad + \frac{\beta\eta}{1-\sigma}\left[\frac{1}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right]^{1-\sigma}; \\ V_{s=0}^{y,j} &\equiv \frac{1}{1-\sigma}\left((1-\tau)w^j\right)^{1-\sigma} + \frac{\beta}{1-\sigma}\left[(1+n)\lambda(1-\tau)\tau\bar{w}\right]^{1-\sigma} + \frac{\eta}{1-\sigma}(g)^{-\sigma} \\ &\quad + \frac{\beta\eta}{1-\sigma}\left[\frac{1}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right]^{1-\sigma}; \\ V_{s>0}^{o,j} &\equiv \frac{1}{1-\sigma}\left[Rs_{-1}^j + (1+n)\lambda(1-\tau)\tau\bar{w}\right]^{1-\sigma}; \\ V_{s=0}^{o,j} &\equiv \frac{\beta\eta}{1-\sigma}\left[\frac{1+n}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right]^{1-\sigma}. \end{aligned}$$

The terms unrelated to political decisions are omitted from the above expressions. We can show that these preferences satisfy single-peaked properties by following the same manner as in the case of a quasi-linear utility function.

The policy preferences of the old are the same as for quasi-linear utility. That is, regardless of type and saving behavior, the old wish to maximize the tax revenue from the young and use it exclusively for social security: $\tau^{oj} = 1/2$ and $\lambda^{oj} = 1$ hold for all j. Accordingly, generalization of the utility function does not affect the policy preferences of the old.

We next consider the policy preferences of the young. Given λ, the preferred tax rate of a type-j young individual satisfies the following first-order condition with respect to τ:

$$LHS^y = RHS^{yj} \equiv \begin{cases} RHS^{yj}_{s>0} & \text{if } \tau < \tau^*(w^j, \lambda) \equiv \frac{w^j}{\bar{w}} \cdot \frac{(\beta R)^{1/\sigma}}{1+n} \cdot \frac{1}{\lambda} \\ RHS^{yj}_{s=0} & \text{if } \tau \geq \tau^*(w^j, \lambda) \end{cases}, \tag{9}$$

where:

$$LHS^y \equiv \beta\eta \left[\frac{1}{2+n}(1-\lambda)\tau\bar{w}\right]^{-\sigma} \frac{1}{2+n}(1-\lambda)(1-2\tau)\bar{w},$$

$$RHS^{yj}_{s>0} \equiv \left(\frac{R}{(\beta R)^{1/\sigma}+R}\right)^{-\sigma} \left[w^j + \frac{(1+n)\lambda\tau\bar{w}}{R}\right]^{-\sigma} \left[w^j - \frac{(1+n)\lambda(1-2\tau)\bar{w}}{R}\right], \text{and}$$

$$RHS^{yj}_{s=0} \equiv \left(w^j\right)^{1-\sigma} - \beta\left[(1+n)\lambda\tau\bar{w}\right]^{-\sigma}(1+n)\lambda(1-2\tau)\bar{w}.$$

LHS^y represents the marginal benefit of taxation in terms of the utility of public goods. This benefit is common to the three types of young agents because of the nature of public goods. RHS^{yj} represents the marginal cost of taxation plus the marginal benefit of social security in terms of the utility of consumption. The sum of these costs and benefits differs among individuals. In particular, the following properties hold (see Appendix 8.6 for the proof):

$$\begin{cases} RHS^{yL} \leq RHS^{yM} \leq RHS^{yH} & \text{if } \sigma \leq 1 \\ RHS^{yL}_{s=0} > RHS^{yM}_{s=0} > RHS^{yH}_{s=0} & \text{if } \sigma > 1, \end{cases} \tag{10}$$

where an equality in the first line holds if and only if $\sigma = 1$ and $s = 0$. Similar to the previous model, the order of $RHS^{yj}_{s=0}(j = L, M, H)$ critically depends on the degree of σ.[5]

Panel (a) of Figure 9 illustrates the graph of (9) when $\sigma \leq 1$ holds. The crossing point of LHS^y and RHS^{yj} determines the tax rate preferred by a type-j young individual. The figure shows that a lower-income young individual prefers a higher tax rate. Under the demographic structure assumption given in Assumption 1, a type-L young individual becomes the decisive voter over τ. That is, the ends-against-the-middle equilibrium never arises when the elasticity is low such that $\sigma \leq 1$.

[Figure 9 about here.]

Panel (b) of Figure 9 illustrates the graph of (9) when $\sigma > 1$ holds. A noteworthy feature is that lower-income young individuals prefer a lower tax rate when they are borrowing constrained. In particular, there may arise an equilibrium where the low- and the high-income young individuals form a coalition against the middle, as illustrated in Panel (b) of Figure 9. Therefore, the high elasticity and the borrowing constraint remain the keys to the existence of the ends-against-the-middle equilibrium.

[5] If $1/\sigma < 1$, the order of $RHS^{yj}_{s>0}(j = L, M, H)$ is ambiguous.

The determination of the share of social security λ is slightly different from that in the previous quasi-linear utility case. The preferred share of a type-j young is given by:

$$\lambda^{yj} = \begin{cases} 0 & \text{if} \quad \tau \leq \tilde{\tau}(w^j) \\ \dfrac{\frac{1}{2+n} - \left(\frac{\beta\eta R}{(2+n)(1+n)}\right)^{1/\sigma} \cdot \frac{R}{(\beta R)^{1/\sigma}+R} \cdot \frac{w^j}{\tau \bar{w}}}{\frac{1}{2+n} + \left(\frac{\beta\eta R}{(2+n)(1+n)}\right)^{1/\sigma} \frac{1+n}{(\beta R)^{1/\sigma}+R}} & \text{if } \tilde{\tau}(w^j) < \tau < \tau^*(w^j) \\ \dfrac{\frac{1}{2+n}}{\frac{1}{2+n} + \left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma}(1+n)} & \text{if} \quad \tau^*(w^j) \leq \tau \end{cases} \tag{11}$$

where

$$\tilde{\tau}(w^j) \equiv \left(\frac{\beta\eta R}{(2+n)(1+n)}\right)^{1/\sigma} \cdot \frac{R}{(\beta R)^{1/\sigma}+R} \cdot \frac{(2+n)w^j}{\bar{w}};$$

$$\tau^*(w^j) \equiv (\beta R)^{1/\sigma} \cdot \left\{\frac{1}{2+n} + \left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma} (1+n)\right\} \frac{(2+n)w^j}{(1+n)\bar{w}}.$$

The first two lines of the right-hand side in (11) represent the choice of λ when a type-j young is borrowing unconstrained; the third line represents the choice of λ when he is borrowing constrained. The derivation of (11) is given in Appendix 8.6.

When the tax burden is low, such that $\tau \leq \tilde{\tau}(w^j)$, a type-$j$ young individual can save much for his old-age consumption and thus, finds it unnecessary to use tax revenue for social security: $\lambda = 0$. However, when the tax is above $\tilde{\tau}(w^j)$, a type-j young individual finds it optimal to offset part of their tax-induced consumption loss with a social security benefit. In particular, a lower income agent prefers a higher share of social security.

A type-j young individual is borrowing constrained when the tax rate is high such that $\tau \geq \tau^*(w^j)$. Borrowing-constrained individuals choose the same share of social security regardless of their type. This is because they have the same level of old-age consumption that is equal to the lump-sum pension benefit. They then choose that share to equate the marginal utilities of old-age consumption and public goods, both of which are type-independent. This result is different from that under a quasi-linear utility function.

Panel (c) of Figure 9 illustrates the reaction function of λ for each type of an individual. The figure shows that $\lambda^{yH}(\tau) \leq \lambda^{yM}(\tau) \leq \lambda^{yL}(\tau) < \lambda^o$ holds for any τ. Thus, under the demographic structure in Assumption 1, a type-L individual agent becomes the decisive voter. We can derive the political equilibrium tax rate by substituting $\lambda = \lambda^{yL}$ into the decisive voter's first-order condition with respect to τ.

Given a brief characterization of the political equilibrium, we now compare the income inequality effects between the current and former models. In particular, we focus on the situation where the decisive voters over τ and λ are borrowing constrained. The decisive voter's choice of λ in the current framework is given by:

$$\lambda^{dec} = \frac{\frac{1}{2+n}}{\frac{1}{2+n} + \left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma}(1+n)},$$

which is independent of τ. We substitute this into the first-order condition with respect to τ, (9), for the case of $\tau \geq \tau^*(w^j, \lambda)$ and obtain the following condition that determines the equilibrium tax rate when the decisive voter is borrowing constrained:

$$\left(w^j\right)^{1-\sigma} = \beta\eta \left(\frac{\left(1-\lambda^{dec}\right)\bar{w}}{2+n}\right)^{1-\sigma} (\tau)^{-\sigma}(1-2\tau) \\ + \beta\left((1+n)\left(1-\lambda^{dec}\right)\bar{w}\right)^{1-\sigma}\left(\lambda^{dec}\right)^{1-\sigma}(\tau)^{-\sigma}(1-2\tau),$$

where the left-hand side shows the marginal cost of taxation, the first term on the right-hand side shows the marginal benefit of public goods, and the second term on the right-hand side shows the marginal benefit of social security. Given τ, the right-hand side is independent of w^j whereas the left-hand side is decreasing (increasing) in w^j if $\sigma > (<)1$. Thus, a mean-preserving reduction of the decisive voter's wage decreases (increases) the equilibrium tax rate if the elasticity is high (low) such that $\sigma > (<)1$. This result is qualitatively equivalent to that in the quasi-linear utility function model.

7 Conclusion

How does wage inequality affect the distribution of tax revenue between social security and forward intergenerational public goods provision in the presence of borrowing constraints? This paper develops a political economy model that addresses this question. Two features are crucial to our analysis and results: the elasticity of marginal utility of youthful consumption and the borrowing constraint. These features derive an ends-against-the-middle equilibrium where low- and high-income individuals form a coalition in favor of a low tax rate and middle-income individuals favor a high tax rate. In addition, higher wage inequality results in a lower level of social security and a lower share of social security (i.e., a higher share of public goods provision) in government expenditure when the decisive voter is borrowing constrained and the elasticity is above unity.

To obtain these results, we simplify the analysis by adopting a quasi-linear utility function. Because of this simplification, we can remove the link between saving and the allocation of tax revenue between social security and public goods provision. However, as shown in Section 6, we demonstrate that the main result is qualitatively unchanged under a generalized utility function. Thus, our analysis and result are almost robust to the assumption of a quasi-linear utility function.[6]

[6] The result established in this paper may fail to hold when we introduce intra-generational redistribution, i.e., income redistribution within young agents, into the model. Borrowing-constrained agents may prefer a higher, rather than a lower, tax rate in response to a reduction of their wage. We would like to thank one of the reviewers for pointing this out.

We exclude this possibility from the analysis because, as surveyed in Introduction, the empirical evidence shows the negative correlation between old-age social security and wage inequality in the presence

8 Appendix

8.1 Single-peakedness of Preferences

8.1.1 Single-peakedness of preferences over τ

The proof proceeds as follows. First, we show that both $V^{y,j}_{s>0}$ and $V^{y,j}_{s=0}$ are single peaked over τ. Then, we demonstrate that $\partial V^{y,j}_{s>0}/\partial\tau = \partial V^{y,j}_{s=0}/\partial\tau$ and $V^{y,j}_{s>0} = V^{y,j}_{s=0}$ hold at $\tau = \hat{\tau}(w^j)$, implying that $V^{y,j}$ has a unique local maximum over the whole range of τ and thus that $V^{y,j}$ is single peaked over τ.

The first and the second derivatives of $V^{y,j}_{s>0}$ and $V^{y,j}_{s=0}$ with respect to τ are:

$$
\begin{aligned}
\frac{\partial V^{y,j}_{s>0}}{\partial\tau} &= -\beta R w^j + \beta(1+n)\lambda(1-2\tau)\bar{w} \\
&\quad + \beta\eta\left(\frac{(1-\lambda)(1-\tau)\tau\bar{w}}{2+n}\right)^{-\sigma}\cdot\frac{(1-\lambda)(1-2\tau)\bar{w}}{2+n}; \\
\frac{\partial^2 V^{y,j}_{s>0}}{\partial\tau^2} &= (-2)\beta(1+n)\lambda\bar{w} + (-2)\beta\eta\left(\frac{(1-\lambda)(1-\tau)\tau\bar{w}}{2+n}\right)^{-\sigma}\frac{(1-\lambda)\bar{w}}{2+n} \\
&\quad + \beta\eta(-\sigma)\left(\frac{(1-\lambda)(1-\tau)\tau\bar{w}}{2+n}\right)^{-\sigma-1}\left\{\frac{(1-\lambda)(1-2\tau)\bar{w}}{2+n}\right\}^2 \\
&< 0; \\
\frac{\partial V^{y,j}_{s=0}}{\partial\tau} &= (-1)(1-\tau)^{-\sigma}(w^j)^{1-\sigma} + \beta(1+n)\lambda(1-2\tau)\bar{w} \\
&\quad + \beta\eta\left(\frac{(1-\lambda)(1-\tau)\tau\bar{w}}{2+n}\right)^{-\sigma}\cdot\frac{(1-\lambda)(1-2\tau)\bar{w}}{2+n}; \\
\frac{\partial^2 V^{y,j}_{s=0}}{\partial\tau^2} &= (-\sigma)(w^j)^{1-\sigma}(1-\tau)^{-\sigma-1} + (-2)\beta(1+n)\lambda\bar{w} \\
&\quad + (-2)\beta\eta\left(\frac{(1-\lambda)(1-\tau)\tau\bar{w}}{2+n}\right)^{-\sigma}\frac{(1-\lambda)\bar{w}}{2+n} \\
&\quad + \beta\eta(-\sigma)\left(\frac{(1-\lambda)(1-\tau)\tau\bar{w}}{2+n}\right)^{-\sigma-1}\left\{\frac{(1-\lambda)(1-2\tau)\bar{w}}{2+n}\right\}^2 \\
&< 0.
\end{aligned}
$$

$V^{y,j}_{s>0}$ and $V^{y,j}_{s=0}$ are single peaked over τ because the second derivatives are negative.

Next, we show that $\partial V^{y,j}_{s>0}/\partial\tau = \partial V^{y,j}_{s=0}/\partial\tau$ at $\tau = \hat{\tau}(w^j)$. By direct calculation, we have:

$$
\left.\frac{\partial V^{y,j}_{s>0}}{\partial\tau}\right|_{\tau=\hat{\tau}(w^j)} \gtreqless \left.\frac{\partial V^{y,j}_{s=0}}{\partial\tau}\right|_{\tau=\hat{\tau}(w^j)} \Leftrightarrow -\beta R w^j \gtreqless (-1)(w^j)^{1-\sigma}(1-\tau)^{-\sigma}.
$$

of intra-generational redistribution; and because our aim of this paper is to consider the allocation of government spending between old-age social security and forward intergenerational public goods provision in an empirically plausible situation.

At $\tau = \hat{\tau}(w^j) \equiv 1 - 1/\left(\beta R\right)^{1/\sigma} w^j$, the right-hand side of the above condition is rewritten as:

$$(-1)(w^j)^{1-\sigma}(1-\hat{\tau}(w^j))^{-\sigma} = -\beta R w^j,$$

implying that $\partial V_{s>0}^{y,j}/\partial\tau = \partial V_{s=0}^{y,j}/\partial\tau$ at $\tau = \hat{\tau}(w^j)$.

Finally, we show that $V_{s>0}^{y,j} = V_{s=0}^{y,j}$ hold at $\tau = \hat{\tau}(w^j)$. By direct calculation, we have:

$$\begin{aligned} & V_{s>0}^{y,j}\Big|_{\tau=\hat{\tau}(w^j)} \gtreqless V_{s=0}^{y,j}\Big|_{\tau=\hat{\tau}(w^j)} \\ \iff & \frac{\left((\beta R)^{-1/\sigma}\right)^{1-\sigma} - 1}{1-\sigma} + \beta R\left\{(1-\tau)w^j - (\beta R)^{-1/\sigma}\right\} = \frac{((1-\tau)w^j)^{1-\sigma} - 1}{1-\sigma}. \end{aligned}$$

At $\tau = \hat{\tau}(w^j) \equiv 1 - 1/\left(\beta R\right)^{1/\sigma} w^j$, the left-hand and right-hand sides of the above condition are reduced to, respectively:

$$\text{LHS} = \text{RHS} = \frac{\left((\beta R)^{-1/\sigma}\right)^{1-\sigma} - 1}{1-\sigma},$$

implying that $V_{s>0}^{y,j} = V_{s=0}^{y,j}$ hold at $\tau = \hat{\tau}(w^j)$.

8.1.2 Single-peakedness of preferences over λ

Before proceeding to the proof, we note that the status of saving is independent of λ because of the assumption of a quasi-linear utility function. Thus, it is sufficient to show that $\partial^2 V_{s>0}^{y,j}/\partial\lambda^2 < 0$ and $\partial^2 V_{s=0}^{y,j}/\partial\lambda^2 < 0$ for the proof.

The first and the second derivatives of $V_{s>0}^{y,j}$ and $V_{s=0}^{y,j}$ with respect to λ are:

$$\begin{aligned} \frac{\partial V_{s>0}^{y,j}}{\partial\lambda} &= \frac{\partial V_{s=0}^{y,j}}{\partial\lambda} = \beta(1+n)(1-\tau)\tau\bar{w} + (-1)\beta\eta\left(\frac{(1-\lambda)(1-\tau)\tau\bar{w}}{2+n}\right)^{-\sigma} \cdot \frac{(1-\tau)\tau\bar{w}}{2+n}; \\ \frac{\partial^2 V_{s>0}^{y,j}}{\partial\lambda^2} &= \frac{\partial^2 V_{s=0}^{y,j}}{\partial\lambda^2} = (-\sigma)\left(\frac{(1-\lambda)(1-\tau)\tau\bar{w}}{2+n}\right)^{-\sigma-1} \cdot \left(\frac{(1-\tau)\tau\bar{w}}{2+n}\right)^2 < 0. \end{aligned}$$

8.2 Proof of Proposition 1

As shown in the text, when $\sigma \le 1$, the decisive voter is a type-L individual and his preferred tax rate satisfies $y(\tau;\bar{w},n) = z(\tau;w^L)$. The functions $y(\tau;\bar{w},n)$ and $z(\tau;w^L)$ have the following properties: $\partial y(\tau;\bar{w},n)/\partial\tau < 0, \lim_{\tau\to 0} y(\tau;\bar{w},n) = \infty, y(1/2;\bar{w},n) = 0, \partial z(\tau;w^L)/\partial\tau \ge 0, z(0;w^L) = \max\{\beta R w^L, (w^L)^{1-\sigma}\} < \infty$, and $z(1/2;w^L) \in (0,\infty)$. These properties indicate that there exists a unique $\tau \in (0,1/2)$ that satisfies $y(\tau;\bar{w},n) = z(\tau;w^L)$. ■

8.3 The Derivation of $\tilde{\tau}^{LM}$ and $\tilde{\tau}^{MH}$

The derivation of $\tilde{\tau}^{LM}$ is as follows. For the range of $(\hat{\tau}(w^L), \hat{\tau}(w^M))$, the right-hand side of (5), denoted by RHS^j, is given by:

$$RHS^j = \begin{cases} RHS^L = (w^L)^{1-\sigma}/(1-\tau)^\sigma & \text{for } j = L \\ RHS^M = \beta R w^M & \text{for } j = M. \end{cases}$$

$RHS^L < RHS^M$ holds at $\tau = \hat{\tau}(w^L)$; $RHS^L > RHS^M$ holds at $\tau = \hat{\tau}(w^M)$. Thus, there exists a unique τ, denoted by $\tilde{\tau}^{LM} \in (\hat{\tau}(w^L), \hat{\tau}(w^M))$, that satisfies $RHS^L = RHS^M$ because RHS^L is continuous and strictly increasing in τ whereas RHS^M is independent of τ. We can derive $\tilde{\tau}^{LM}$ by solving $(w^L)^{1-\sigma}/(1-\tau)^\sigma = \beta R w^M$ for τ.

Similarly, the tax rate that satisfies $RHS^M = RHS^H$ for the range of $(\hat{\tau}(w^M), \hat{\tau}(w^H))$ is derived by solving $(w^M)^{1-\sigma}/(1-\tau)^\sigma = \beta R w^H$ for τ. The solution is denoted by $\tilde{\tau}^{MH}$.

8.4 Proof of Proposition 2

8.4.1 Existence and uniqueness of the equilibrium

As shown in the text, when $\sigma > 1$, the decisive voter's preferred tax rate satisfies $y(\tau; \bar{w}, n) = \tilde{z}(\tau; w^L, w^M, w^H)$. The functions $y(\tau; \bar{w}, n)$ and $\tilde{z}(\tau; w^L, w^M, w^H)$ have the following properties:

$$\begin{aligned} \partial y(\tau; \bar{w}, n)/\partial \tau &< 0, \\ \lim_{\tau \to 0} y(\tau; \bar{w}, n) &= \infty, \\ y(1/2; \bar{w}, n) &= 0, \\ \partial \tilde{z}(\tau; w^L, w^M, w^H)/\partial \tau &\geq 0, \\ \tilde{z}(0; w^L, w^M, w^H) &= \max\{\beta R w^L, (w^L)^{1-\sigma}\} < \infty, \\ \tilde{z}(1/2; w^L, w^M, w^H) &\in (0, \infty). \end{aligned}$$

These properties indicate that there exists a unique $\tau \in (0, 1/2)$ satisfying $y(\tau; \bar{w}, n) = \tilde{z}(\tau; w^L, w^M, w^H)$.

8.4.2 The determination of the decisive voter

Suppose that the type-H young individual is the decisive voter. He/she is borrowing unconstrained under Assumption 3(ii). Then, from Figure 6, it must hold that $y(\tilde{\tau}^{MH}; \bar{w}, n) = \beta(1+n)(1-2\tilde{\tau}^{MH})\bar{w} > \tilde{z}(\tilde{\tau}^{MH}; w^L, w^M, w^H) = \beta R w^H$ at $\tau = \tilde{\tau}^{MH}$, that is:

$$(1+n)(1-2\tilde{\tau}^{MH})\bar{w} > R w^H.$$

This condition never holds under the assumptions of $R \geq 1+n$ (Assumption 2) and $w^H > \bar{w}$. Therefore, the decisive voter is a type-L or type-M young individual. From Figure 6, the type-L young individual becomes the decisive voter if $\beta(1+n)(1-2\tilde{\tau}^{LM})\bar{w} \leq \beta R w^M$, that is, if:

$$-1+2\cdot\left(\frac{(w^L)^{1-\sigma}}{\beta R w^M}\right)^{1/\sigma} \leq \frac{R}{(1+n)\bar{w}}w^M.$$

Otherwise, the decisive voter is a type-M young individual. ■

8.5 Proof of Proposition 3

(i) For the case of $\sigma \leq 1$, the decisive voter is a type-L young individual and the equilibrium tax rate satisfies $y(\tau;\bar{w},n) = z(\tau;w^L)$, as shown in Subsection 4.1. When the mean-preserving change in w^L is considered, $y(\tau;\bar{w},n)$ is unchanged while $z(\tau;w^L)$ is nonincreasing with reductions of w^L. Therefore, the equilibrium tax rate satisfying $y(\tau;\bar{w},n) = z(\tau;w^L)$ is nondecreasing in response to a mean-preserving reduction of w^L. Given that $\lambda^{dec}(\tau)$ is increasing in τ for $\tau \in (0,1/2)$, λ is also nondecreasing in response to a mean-preserving decrease in w^L.

(ii) For the case of $\sigma > 1$, the decisive voter is a type-j ($j = L$ or M) individual depending on parameter values, as shown in Proposition 2. To simplify the presentation, suppose that a type-L individual is the decisive voter. Note that the following argument applies for the case where a type-M is the decisive voter.

Assume that the equilibrium tax rate is given by $\tau^{equil} = \hat{\tau}(w^L)$: a type-$L$ individual is indifferent between saving and not saving. Under this situation, the decisive voter's wage w^L satisfies $\beta(1+n)(1-2\hat{\tau}(w^L))\bar{w} = \beta R w^L$, or:

$$R(w^L)^2 + (1+n)\bar{w}w^L - 2(1+n)\frac{\bar{w}}{(\beta R)^{1/\sigma}} = 0.$$

Solving this equation for w^L, we obtain:

$$w^L = \hat{w}^L \equiv \frac{-(1+n)\bar{w} + \sqrt{\{(1+n)\bar{w}\}^2 + 8R(1+n)\bar{w}/(\beta R)^{1/\sigma}}}{2R}.$$

Therefore, the equilibrium tax rate is given by $\tau^{equil} = \hat{\tau}(w^L)$ when a type-L individual with $w^L = \hat{w}^L$ is the decisive voter.

We now consider a mean-preserving change of w^L around $\hat{w}^L$. As shown in Subsection 4.2, the equilibrium tax rate satisfies $y(\tau;\bar{w},n) = \tilde{z}(\tau;w^L,w^M,w^H)$ if $\sigma > 1$. In particular, around $w^L = \hat{w}^L$, there exists a positive real number ε such that the equilibrium tax rate satisfies the following condition:

$$\tilde{z}(\tau;w^L,w^M,w^H) = \begin{cases} \beta R w^L & \text{for } w^L \in (\hat{w}^L-\varepsilon, \hat{w}^L], \\ \frac{(w^L)^{1-\sigma}}{(1-\tau)^\sigma} & \text{for } w^L \in [\hat{w}^L, \hat{w}^L+\varepsilon). \end{cases}$$

We focus on the range $(\hat{w}^L-\varepsilon, \hat{w}^L+\varepsilon)$ and consider a mean-preserving change of w^L around $\hat{w}^L$. The right-hand side of the above equation is increasing in w^L within the range $(\hat{w}^L - \varepsilon, \hat{w}^L)$ and decreasing in w^L within the range $(\hat{w}^L, \hat{w}^L + \varepsilon)$. This property implies that the equilibrium tax rate attains the highest value at $w^L = \hat{w}^L$ within the range $(\hat{w}^L - \varepsilon, \hat{w}^L + \varepsilon)$. Therefore, there is an inverse V-shaped relationship between the decisive voter's wage and the equilibrium tax rate around $w^L = \hat{w}^L$. Given $\lambda^{dec}(\tau)$ is increasing in τ for $\tau \in (0, 1/2)$, there is also an inverse V-shaped relationship between the decisive voter's wage and the equilibrium share of social security around $w^L = \hat{w}^L$. ■

8.6 Supplementary Explanation for Section 6

8.6.1 Derivation of (10)

In order to establish that condition (10) holds, we first investigate the property of $RHS^{yj}_{s>0}$. The first derivative of $RHS^{yj}_{s>0}$ with respect to w^j leads to:

$$
\begin{aligned}
&\left(\frac{R}{(\beta R)^{1/\sigma} + R}\right)^{-\sigma} \cdot \frac{\partial RHS^{yj}_{s>0}}{\partial w^j} \\
&= \left(w^j + \frac{(1+n)\lambda\tau\bar{w}}{R}\right)^{-\sigma-1} \cdot \left[(1-\sigma)w^j + \frac{(1+n)\lambda\bar{w}}{R}\left\{\sigma(1-2\tau) + \tau\right\}\right],
\end{aligned}
$$

where the term $\{\sigma(1-2\tau)+\tau\}$ is positive provided that $\tau > 1/2$. Thus, $\partial RHS^{yj}_{s>0}/\partial w^j > 0$ holds if $\sigma \leq 1$: this implies that $RHS^{yL}_{s>0} < RHS^{yM}_{s>0} < RHS^{yH}_{s>0}$ if $\sigma \leq 1$.

Next, we investigate the property of $RHS^{yj}_{s=0}$. Direct calculation leads to:

$$
\begin{aligned}
RHS^{yL}_{s=0} \gtreqless RHS^{yM}_{s=0} &\Leftrightarrow \left(w^L\right)^{1-\sigma} \gtreqless \left(w^M\right)^{1-\sigma}; \\
RHS^{yM}_{s=0} \gtreqless RHS^{yH}_{s=0} &\Leftrightarrow \left(w^M\right)^{1-\sigma} \gtreqless \left(w^H\right)^{1-\sigma}.
\end{aligned}
$$

Therefore, we obtain:

$$
RHS^{yL}_{s=0} \gtreqless RHS^{yM}_{s=0} \gtreqless RHS^{yM}_{s=0} \Longleftrightarrow 1/\sigma \lesseqgtr 1.
$$

An equality holds if and only if $\sigma = 1$.

8.6.2 Derivation of (11)

Suppose first that the type-j young agent is borrowing unconstrained. The first-order condition for the maximization of $V^{y,j}_{s>0}$ with respect to λ is given by:

$$
\frac{\partial V^{y,j}_{s>0}}{\partial \lambda} = 0 \Leftrightarrow \lambda = \lambda^{yj}_{s>0} \equiv \frac{\frac{1}{2+n} - \left(\frac{\beta\eta R}{(2+n)(1+n)}\right)^{1/\sigma} \cdot \frac{R}{(\beta R)^{1/\sigma}+R} \cdot \frac{w^j}{\tau\bar{w}}}{\frac{1}{2+n} + \left(\frac{\beta\eta R}{(2+n)(1+n)}\right)^{1/\sigma} \frac{1+n}{(\beta R)^{1/\sigma}+R}} (< 1).
$$

Taking into account the corner solution $\tau = 0$, we obtain:

$$
\lambda^{yj}_{s>0} \equiv \max\left\{0, \frac{\frac{1}{2+n} - \left(\frac{\beta\eta R}{(2+n)(1+n)}\right)^{1/\sigma} \cdot \frac{R}{(\beta R)^{1/\sigma}+R} \cdot \frac{w^j}{\tau\bar{w}}}{\frac{1}{2+n} + \left(\frac{\beta\eta R}{(2+n)(1+n)}\right)^{1/\sigma} \frac{1+n}{(\beta R)^{1/\sigma}+R}}\right\}.
$$

The preferred share λ is increasing in τ and is positive if and only if $\tau = \tau^*(w^j)$.

$$\frac{\partial V^{y,j}_{s>0}}{\partial \lambda} = 0 \Leftrightarrow \lambda = \lambda^{yj}_{s=0} \equiv \frac{\frac{1}{2+n}}{\frac{1}{2+n} + \left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma}(1+n)} (< 1),$$

where $\lambda^{yj}_{s=0}$ is constant and independent of τ. The equality holds between $\lambda^{yj}_{s>0}$ and $\lambda^{yj}_{s=0}$ at $\tau = \tau^*(w^j)$. Therefore, the preferred share λ by a type-j young agent is given as (11).

References

[1] Arawatari, R., and Ono, T., 2013. The Political Economy of Social Security in a Borrowing-constrained Economy. *Journal of Public Economic Theory* forthcoming.

[2] Bellettini, G., and Berti Ceroni, C., 2007. Income Distribution, Borrowing Constraints and Redistributive Policies. *European Economic Review* 51, 625–645.

[3] Benabou, R., 2000. Unequal Societies: Income Distribution and the Social Contract. *American Economic Review* 90, 96–129.

[4] Bernasconi, M., and Profeta, P., 2012. Public Education and Redistribution When Talents Are Mismatched. *European Economic Review* 56, 84–96.

[5] Bethencourt, C., and Galasso, V., 2008. Political Complements in the Welfare State: Health Care and Social Security" *Journal of Public Economics* 92, 609–632.

[6] Boldrin, M., and Rustichini, A., 2000. Political Equilibria with Social Security. *Review of Economic Dynamics* 3, 41–78.

[7] Borck, P., 2007. On the Choice of Public Pensions When Income and Life Expectancy Are Correlated. *Journal of Public Economic Theory* 9, 711–725.

[8] Casamatta, G., Cremer, H., and Pestieau, P., 2000. The Political Economy of Social Security. *Scandinavian Journal of Economics* 102, 503–522.

[9] Chen, K., and Song, Z., 2009. Markovian Social Security in Unequal Societies. Available at SSRN: http://ssrn.com/abstract=1367476

[10] Conde-Ruiz, J.I., and Galasso, V., 2003. Early Retirement. *Review of Economic Dynamics* 6, 12–36.

[11] Conde-Ruiz, J.I., and Galasso, V., 2005. Positive Arithmetic of the Welfare State. *Journal of Public Economics* 89, 933–955.

[12] Conde-Ruiz, J.I., and Profeta, P., 2007. The Redistributive Design of Social Security Systems. *Economic Journal* 117, 686–712.

[13] Cooley, T.F., and Soares, J., 1999. A Positive Theory of Social Security Based on Reputation. *Journal of Political Economy* 107, 135–160.

[14] Cremer, H., De Donder, P., Maldonado, D., and Pestieau, P., 2007. Voting Over Type and Generosity of a Pension System When Some Individuals Are Myopic. *Journal of Public Economics* 91, 2041–2061.

[15] Diamond, P., and Hausman, J.A., 1984. Individual Retirement and Savings Behavior. *Journal of Public Economics* 23, 81–114.

[16] Gottschalk, P., and Smeeding, T.M., 1997. Cross-national Comparisons of Earnings and Income Inequality. *Journal of Economic Literature* 35, 633–687.

[17] Ichino, A., Karabarbounis, L., and Moretti, E., 2011. The Political Economy of Intergenerational Income Mobility. *Economic Inquiry* 49, 47–69.

[18] Leroux, M-L., Pestieau, P., and Racionero, M., 2011. Voting on Pensions: Sex and Marriage. *European Journal of Political Economy* 27, 281–296.

[19] Levy, G., 2005. The Politics of Public Provision of Education. *Quarterly Journal of Economics* 120, 1507–1534.

[20] Meltzer, A., and Richard, S., 1981. A Rational Theory of the Size of Government. *Journal of Political Economy* 89, 914–927.

[21] OECD, 2013. *OECD Factbook 2013: Economic, Environmental and Social Statistics.* OECD, Paris.

[22] Rangel, A., 2003. Forward and Backward Intergenerational Goods: Why is Social Security Good for the Environment? *American Economic Review* 93, 813–834.

[23] Roberts, K.W.S., 1977. Voting Over Income Tax Schedules. *Journal of Public Economics* 8, 329–340.

[24] Rodriguez, F., 2004. Inequality, Redistribution, and Rent-seeking. *Economics and Politics* 16, 287–320.

[25] Romer, T., 1975. Individual Welfare, Majority Voting, and the Properties of a Linear Income Tax. *Journal of Public Economics* 4, 163–185.

[26] Shepsle, K.A., 1979. Institutional Arrangements and Equilibrium in Multidimensional Voting Models. *American Journal of Political Science* 23, 27–59.

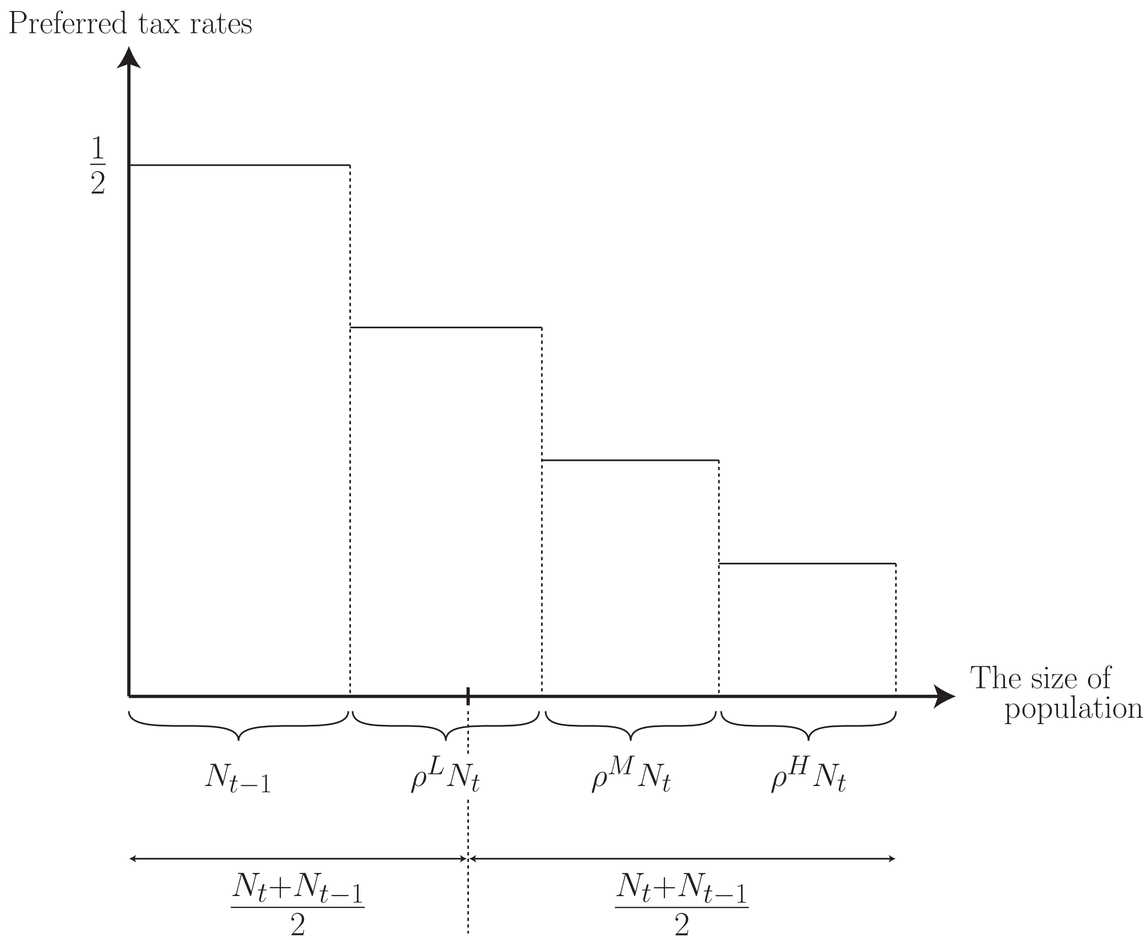

Figure 1: This figure illustrates an example of the tax rates preferred by the old and the young. In this example, a type-L young individual becomes a decisive voter.

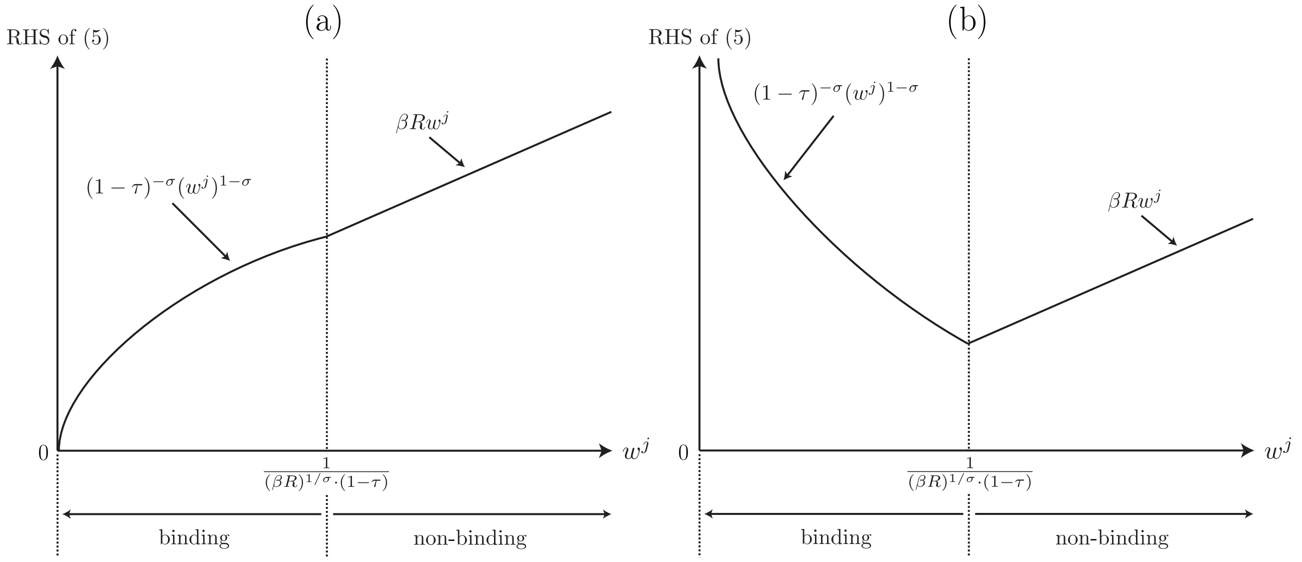

Figure 2: Marginal cost of taxation. Panel (a) is the case of $\sigma \leq 1$; Panel (b) is the case of $\sigma > 1$.

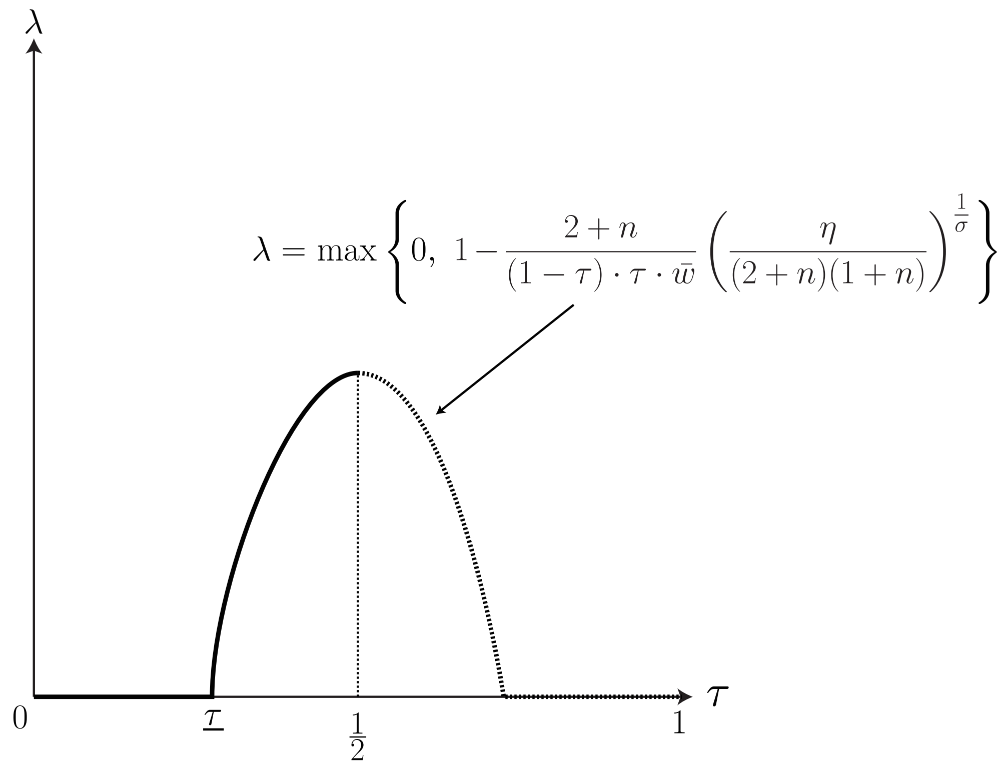

Figure 3: This figure illustrates the share of social security (λ) preferred by the young in response to a change in the tax rate (τ).

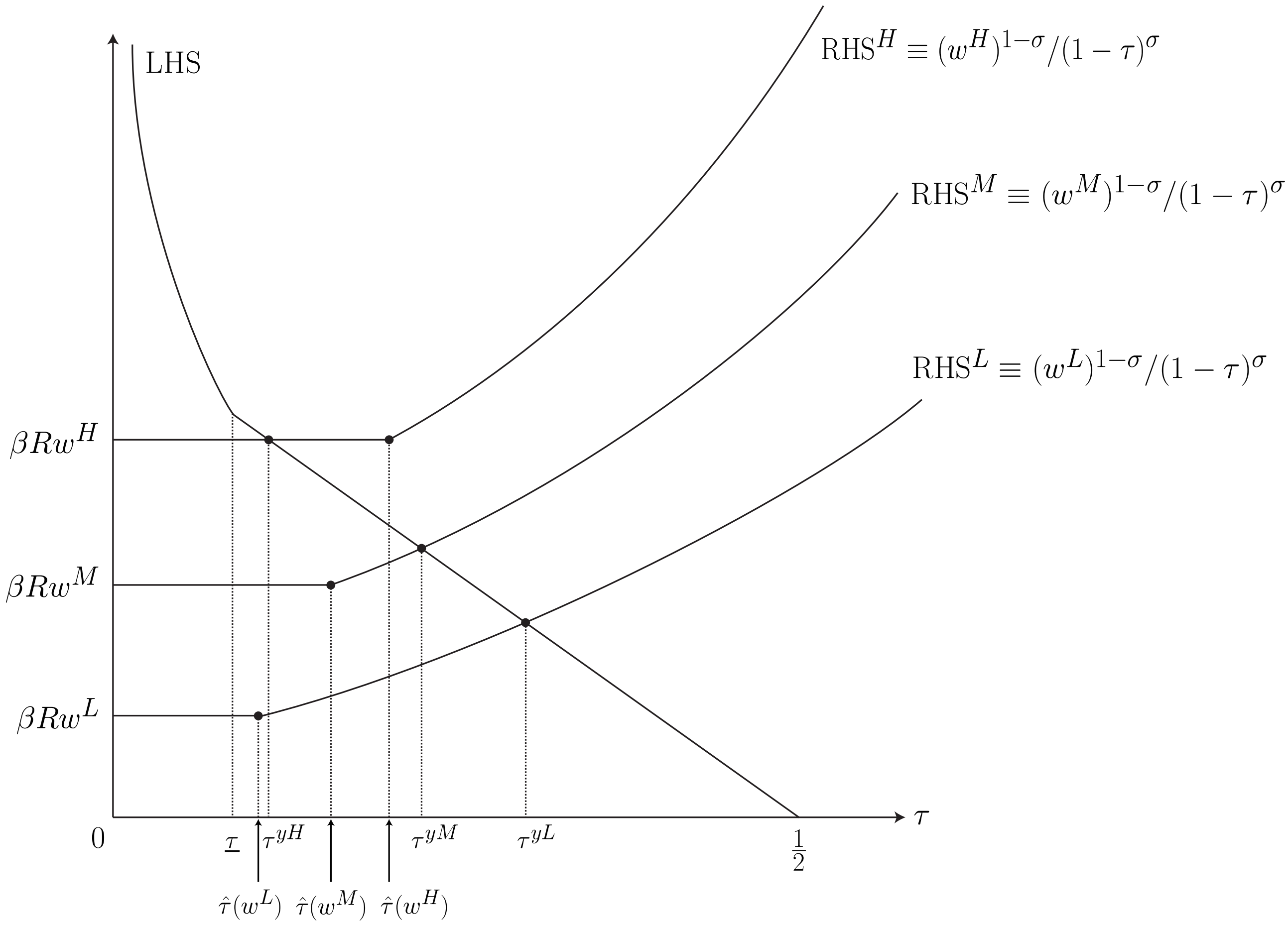

Figure 4: The tax rates preferred by the three types of young individuals in the case of $\sigma \leq 1$.

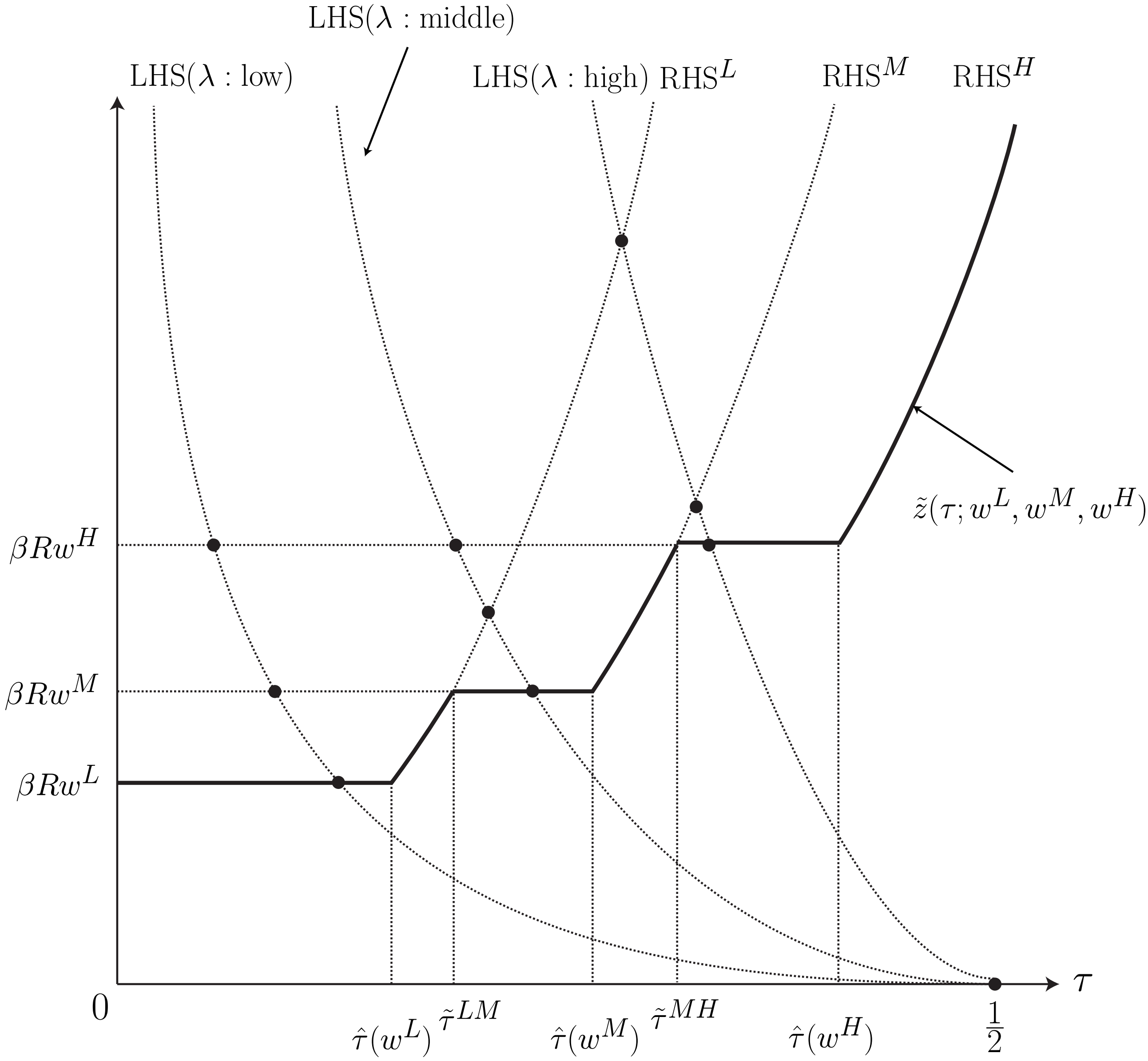

Figure 5: The tax rate preferred by a type-j young individual in the case of $\sigma > 1$. The bold curve illustrates the graph of $\widetilde{z}$ in (8).

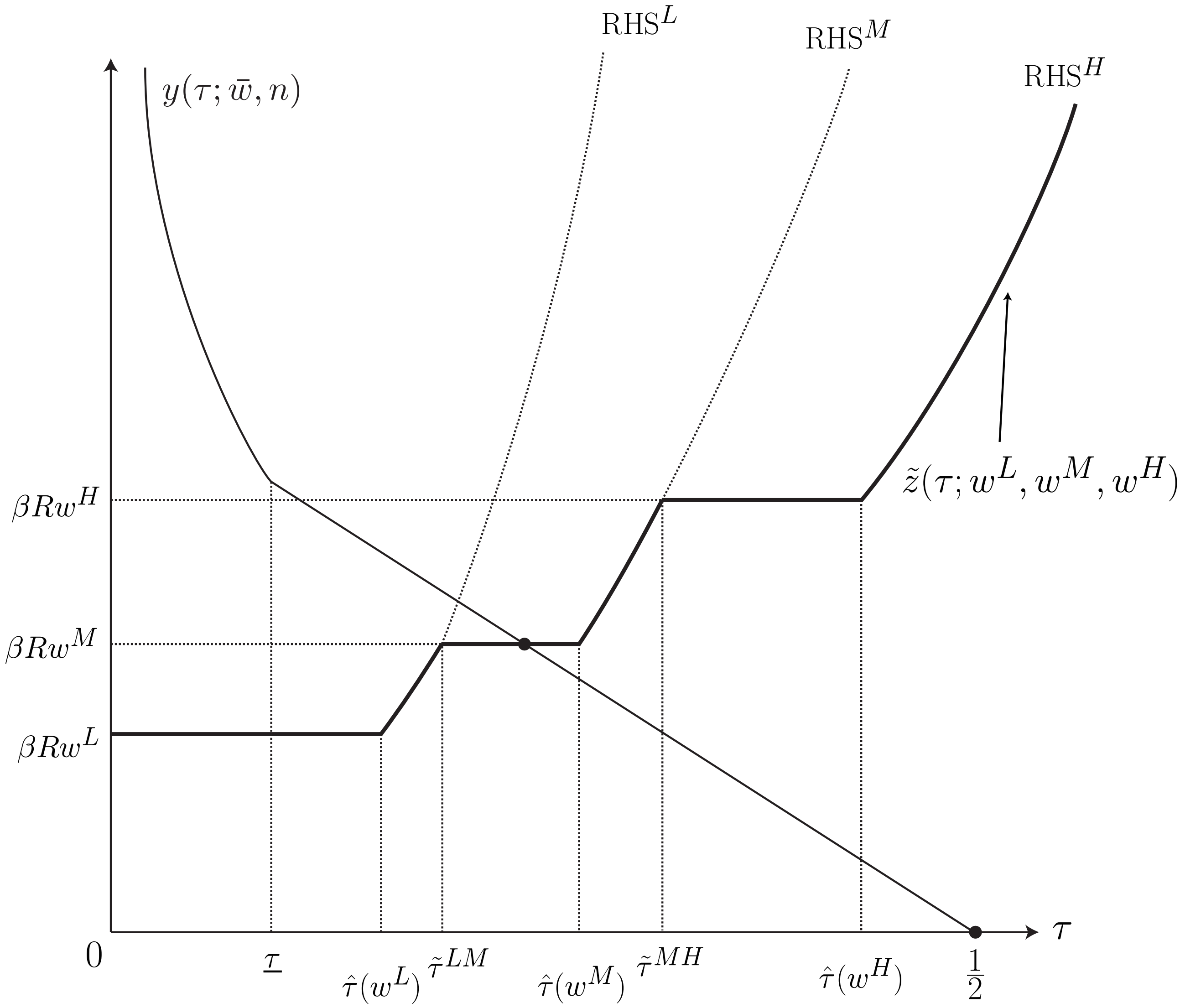

Figure 6: The determination of the tax rate in the case of $\sigma > 1$. The figure illustrates the case where the decisive voter is a type-M young individual.

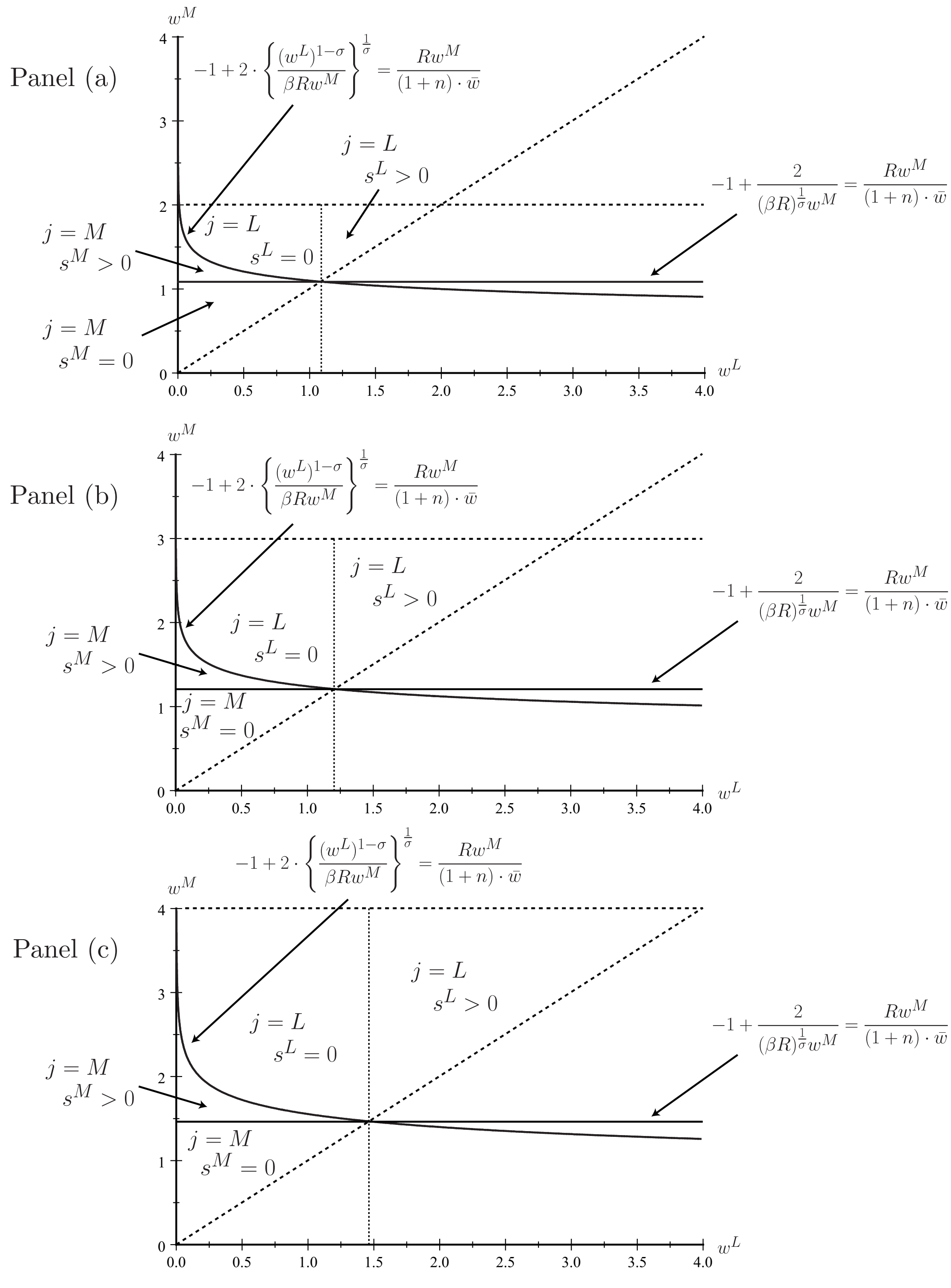

Figure 7: The conditions that determine the decisive voter and his status of saving in a $w^L - w^M$ space. To illustrate the figure, we assume a generation to be 30 years in length. Our selection of parameters is $1+n=(1.01)^{30}, \beta=(0.98)^{30}, R=(1.015)^{30}$ and $\sigma=1.2$. We illustrate the set of parameters for the following three cases: $\overline{w}=2.0$ (panel (a)), $\overline{w}=3.0$ (panel (b)), and $\overline{w}=4.0$ (panel (c)). The triangular area surrounded by the vertical axis, the 45-degree line and $w^M=\bar{w}$ line covers the set of wages (w^L, w^M) relevant for the analysis. The wage of the middle, w^M, is assumed to be below the average: this assumption reflects a typical right-skewed income distribution employed in the literature.

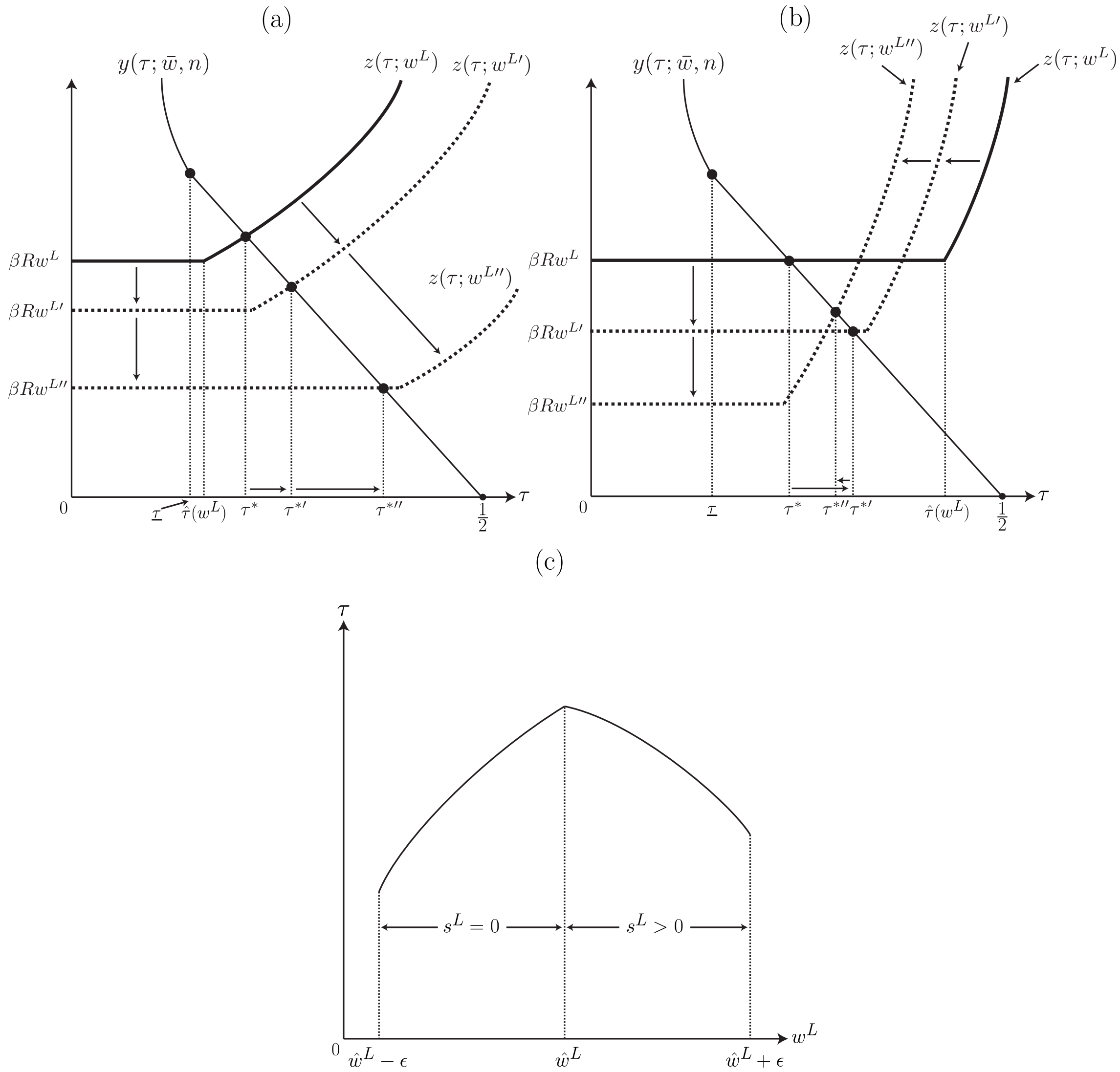

Figure 8: Panels (a) and (b) depict the graphs of the equation $y = z$ that determines the equilibrium tax rate when the decisive voter is a type-L individual for the cases of $\sigma < 1$ and $\sigma > 1$, respectively. The three graphs of the function z are associated to the three levels of type-L's wage income, $w^L, w^{L\prime}$ and $w^{L\prime\prime}$ where $w^L > w^{L\prime} > w^{L\prime\prime}$. Panel (c) illustrates the relation between the decisive voter's (i.e., type-L's) wage and the equilibrium tax rate around the critical value of type-L's wage, $\hat{w}^L$, in the case of $\sigma > 1$.

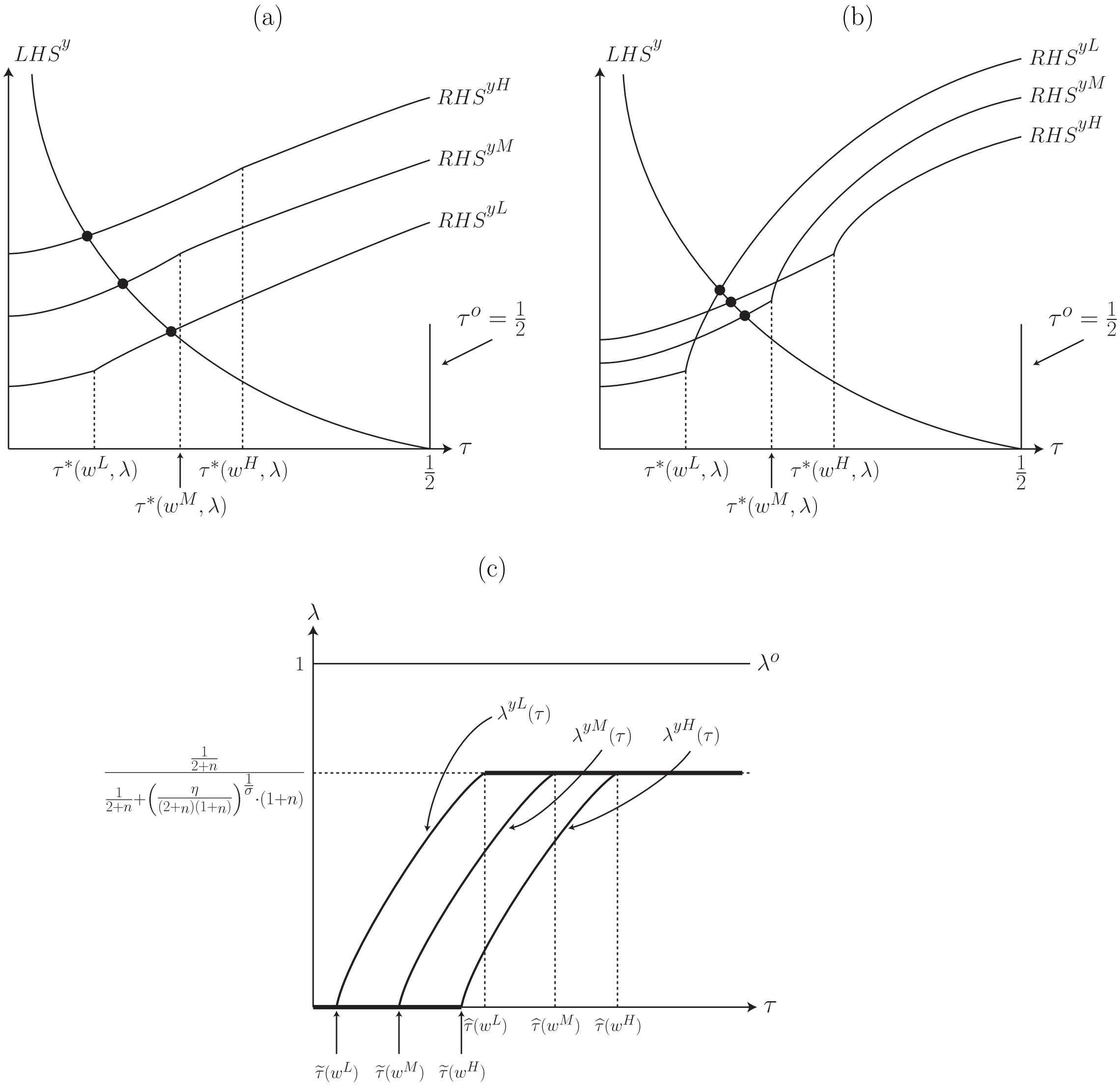

Figure 9: Panels (a) and (b) illustrate the condition expressing the preferred tax rates by the young in cases of $\sigma \leq 1$ and $\sigma > 1$, respectively. Panel (c) illustrates the condition of the preferred shares of social security by three types of young individuals.