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Abstract

In this paper, we present a dynamic general equilibrium model to investigate how different contracting modes based on formal and relational enforcements endogenously emerge and are dynamically linked with the process of economic development. Formal contracts are enforced by third-party institutions (courts), while relational contracts are self-enforcing agreements without any third-party involvement. The novel feature of our model is that it demonstrates the co-evolution of these different enforcement modes and market equilibrium conditions, all of which are jointly determined. We then characterize the equilibrium paths of such dynamic processes and show the time structure of relational contracting in the endogenous process of economic development. In particular, we show that relational contracting fosters the emergence of a market-based economy in low-development stages but its role declines as the economy grows and enters high-development stages.

Keywords: dynamic general equilibrium, economic development, arm's length contract, relational contract
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1 Introduction

Informal contract arrangements, which we call *relational contracting* in this paper, are common during the developing stages of economies. These arrangements are not based on formally written contracts, but rather on long-term relationships, implicit agreements, and a reputation mechanism via personal ties and connections, as typically observed in tribal and ancient societies[1] as well as in emerging and transition economies.[2]

Among other informal contract arrangements, one well-documented example is relationship (insider) lending based on personal relationships between borrowers and lenders, typically banks. Lamoreaux (1994) reported that during the early 19th century, the New England banks lent a large portion of their funds to the board of directors and those who had close personal ties with these banks. Lamoreaux then found evidence that such relationship lending contributed to the economic growth in New England during that period.[3] A related historical fact is that major German banks such as Commerzbank, Dresdner, and Deutsche grew rapidly in the 19th century by developing long-term relationships with industry enterprises by offering low interest rates to them and being represented as board directors on these firms. Such close and lasting relationships between large banks and firms contributed to the rapid expansion of the German economy between the late 19th century and the First World War (Allen (2001)). Maurer and Haber (2007) also provide related empirical evidence about Mexican banks during similar periods (1880-1913): these bankers were engaged in relationship lending by responding to information asymmetry and a large enforcement cost but they did not loot their own banks.

These facts pose a positive view that relational contracting is not a substitute but a complement to the market economy in that the former fosters the latter.[4] On the other hand, there is also a negative view on relational contracting that it plays a less important role as the economy grows and reaches more developed stages. For example, the New England banks that lent to closely-related persons (for example, directors of these banks) in the early 19th century eventually had begun to lend to 'outside' borrowers, whom they did not personally know well, as the economy changed from capital-poor to capital-rich, thus expanding the anonymous credit market in the late

[1]See Levi-Strauss (1969), Malinowski (1961), and Mauss (1967) for anthropological studies on reciprocal exchange and gift exchange in tribal societies. See Greif (2006) and Milgrom, North and Weingast (1990) for a discussion on how the merchant trade system functioned as a reputation device in medieval times.

[2]See McMillan and Woodruff (1999) for a discussion of trade credits in Vietnam and Johnson, McMillan and Woodruff (2002) for a discussion of relational contracting in Russia.

[3]As Lamoreaux (1994) emphasized, insider lending was a phenomenon observed not only in New England but also in other U.S. states during the early 19th century before the U.S. markets expanded.

[4]See Lamoreaux (1994), Allen (2001), and Maurer and Haber (2007) discussed above. For the positive role of other informal institutions, see the papers contained in Aoki and Hayami (2000). For example, Greif (2000) discusses the roles of the community trading system in pre-modern Europe and Fafchamps (2000) provides evidence about business networks in Sub-Saharan Africa.

19th century (Lamoreaux (1994)). A related argument is that the relationship-based financial system was dominant in Asian countries such as Korea and Japan after World War II but it has been recently changing to the market-based financial system (see Rajan and Zingales (2000)). Demirgüç-Kunt and Levine (2004) provided the related evidence that the ratio of bank finance relative to equity finance is negatively associated with per capita GDP levels across countries, suggesting that bank finance, which is often characterized as a long-term lending relationship between a particular bank and a firm, becomes less important and is replaced by market-based finance in developed countries.

The main objective of our paper is to provide a theoretical framework for understanding how and when relational contracting contributes to the process of economic development. In particular, we characterize the time structures of relational contracting (self-enforcing agreements) that change in the different phases of economic development. A novel feature of our model is to embed relational contracts that are supported by long-term relationships into a dynamic general equilibrium model. Long-term relationships have been mostly analysed in the partial equilibrium framework in the literature of repeated games (see Mailath and Samulaeson (2006)): in a typical repeated game, players play the stage games repeatedly over time by assuming that the outside markets are exogenously given. However, despite much historical evidence that shows the important roles of long-term relationships in the process of economic development, there are few theoretical studies that consider the macroeconomic implications of long-term relationships. These are what we address in this paper. The important departure of our paper from the literature of repeated games is that we investigate how long-term relationships affect and are affected by the growth path of a macro-economy which is endogenous as well.

In our model economy, producers who finance their capital investment have the incentive to default after they borrow the funds from lenders. We consider the two means which prevent producers from defaulting. The first type is the competitive credit market, in which everyone can borrow and lend at a given market interest rate in the spot manner. However, when lenders lend their funds to the credit market, they must incur some enforcement cost in order to write a formally enforceable contract which prevents borrowers from defaulting. We call this type of contract an *arm's length or formal contract* and call producers (respectively, lenders) who engage in an arm's length contract *A-producers* (respectively, *A-lenders*). The enforcement cost incurred by A-lenders includes costly activities such as collecting evidence about accounting data, hiring lawyers and accounting professionals, and using the public enforcement agency (courts).

The other way to protect lenders from default is to use implicit and informal contracting arrangements. Specifically, we suppose that there is a local community in our model economy where each producer in the community has a personal connection with a particular lender within the same community. The producer and the lender form a long-term relationship over successive generations. We call this type of producer and lender a *R-producer* and a *R-lender*, respectively. Each R-producer can engage in relational contracting with a R-lender for financing capital investment without using the outside credit market. This type of contract is what we call a *relational contract*. Because they interact with each other over time, each relationship pair of a R-producer and a R-lender can avoid the strategic default problem via a self-enforcing agreement.

The advantage of relational contracts over arm's length contracts is the saving in the enforcement cost which is inevitable under the latter while its disadvantage is that relational contracts must be self-enforceable, that is, R-producers must be given the incentive not to renege on the agreed upon repayment schedules.

As is well known from the repeated games literature, an implicit agreement is self-enforceable if each party's deviation from honouring the agreement results in future losses larger than the one-time gains obtained by the deviation. The novelty of our model is how it relates the self-enforceability of relational contracting to the endogenous process of economic development in a dynamic general equilibrium framework. The profit of an arm's length contract, which becomes the deviation payoff for each R-producer when quitting the current relationship, is endogenously determined and affects the self-enforcing condition of relational contract. In turn, the change in the self-enforcing condition creates a feedback effect on the equilibrium determination of the profit of arm's length contract through changes in the market prices, such as wage and interest rates. These two-way interactions between the self-enforcing condition of relational contracts and the market equilibrium conditions jointly lead to the macroeconomic dynamics of the development process.

We then characterize the equilibrium paths of the model economy which involve the dynamic transformation from a relationship-based system relying on relational contracts to a market-based system relying on arm's length (formal) contracts. In any equilibrium path, there exists a unique switching period before which the self-enforcing constraint becomes less stringent so that each R-producer invests and produces more than each A-producer, but after which it becomes more stringent so that each A-producer invests and produces more than each R-producer. In particular, we show that the output level of a R-producer relative to that of an A-producer, which we call the relative output of the R-producer, declines over time from the initial period until it hits some critical value. Thus, in the early stages of development, relational contracting contributes more to the expansion of the economy than the arm's length contract, while the former contributes less than the latter in the matured stages of development. We also show that the economies which start with less reliance on relational contracting switch more early from the relationship-based system to the market-based system than those which start with more reliance on relational contracting.

The key behind these results is the dynamic general equilibrium interaction between the self-enforcing condition of relational contracting and the determination of market prices such as the interest and wage rates. Since relationship pairs of R-producers and R-lenders do not incur the enforcement cost associated with capital investment, they face a lower opportunity cost to raise capital for production than A-producers do. Thus, R-producers can invest more in capital than A-producers do when the self-enforcing condition becomes less stringent. In the early stages of development, the income level is so low that the small saving limits the credit supply, and hence, the market interest rate becomes high, resulting in the low capital investment of A-producer. Then, the profit of an arm's length contract, which becomes the outside option for each R-producer who quits a relationship, becomes lower so that the self-enforcing condition of a relational contract becomes more likely to be sustained. This leads to a larger capital investment of R-producers than A-producers. Since R-producers have lower opportunity cost to raise capital than A-producers, the output expansion of the former relative to the latter contributes to higher development of the economy as a whole. However, as the market interest rates fall during the course of development, the self-enforcing condition becomes more stringent so that R-producers eventually switch to invest less in capital than A-producers do.

In such way we show that the economy exhibits the dynamic switching pattern such that it relies more on relational contracting in the early stages of development but it switches to rely more on arm's length contracts in the competitive market in the later stages of development. This result is related to an argument made by Polanyi (1947) that Western societies experienced the 'great transformation' from nonmarket systems to market-based systems when their economies expanded rapidly in the 19th century. Moreover, R-producers contribute to economic development by producing more in the early development stages in which the market economy is so immature that A-producers produce less. Thus relational contracting plays a positive role in fostering economic growth during low-development stages and in promoting the emergence of a market-based economy. These results are consistent with the aforementioned historical evidence that relational contracting complements the rise of a market-based economy (Aoki and Hayami (2000) and Lamoreaux (1994)). In addition, our result also confirms the historical fact that the relationship-based system declines as the economy grows and expands more (see Lamoreaux (1994), Rajan and Zingales (2000) and Demirgüç-Kunt and Levine (2004) as we have already mentioned).

Related literature. Although several papers address relational contracting in partial equilibrium frameworks, [5] few studies have attempted to examine its macroeconomic implications via dynamic general equilibrium models. Some papers compare informal contracting enforcement, such as reputation, with formal and legal enforcement in random matching environments. Kranton (1996) focuses on market-based monetary exchange and relational (self-enforcing) contracts that emerge in a Kiyotaki-Wright-type monetary search model. Dhilon and Rigolini (2006) compare reputation and legal enforcement by endogenizing the quality of enforcement institutions. Francois (2011) investigates the evolution of endogenous institutions, but his analysis focuses on the roles of social norms formed through the change in endogenous preferences. Francois and Roberts (2003) examine how relational contracting affects long-run economic growth in an R&D-based endogenous growth model. However, they do not focus on the choice between arm's length and relational contracts. They also conduct a steady-state

[5] See, for example, Baker, Gibbons and Murphy (2002), Itoh and Morita (2007), Levin (2003), MacLeod and Malcomson (1998), and Ramey and Watson (2003).

analysis of the long-run growth rate and study the macroeconomic effects of productivity shocks on relational contracting between firms and workers. Fafchamps (2003) addresses the dynamic issue of how markets spontaneously emerge in a repeated game setting.

Our paper is also related to the models of competitive economies with endogenous debt constraints that prevent borrowers from defaulting (see, for example, Jeske (2006), Kehoe and Levine (1993), and Hellwig and Lorenzoni (2009) for the models of international borrowing and lending with default risk).

Our work here differs from all the research cited above because our main concern is the dynamic change in the contracting modes and its relation to the process of economic development which have been not addressed in the existing studies. Our new insight is that both the sustainability of relational contracting and the evolution of economic development are endogenous and jointly determined through dynamic general equilibrium effects. Specifically, we address the hitherto unaddressed issues of how and when relationship-based economic systems change to market-based systems during the endogenous process of economic development.

The remaining sections are organized as follows. In Section 2, we provide a partial equilibrium model with a choice between arm's length and relational contracts by taking all the market prices as exogenously given. In Section 3, we turn to the full model of a dynamic general equilibrium and define an equilibrium of the model economy. In Section 4, we characterize the set of equilibrium paths and show that relational contracting contributes more to economic growth in the early stages of development but that its role becomes more limited as the economy enters the mature stages of development. In Section 5, we discuss an extension of the model which allows endogenous formation of relationship pairs. All proofs are relegated to the Appendix (with some given in the Online Appendix).

2 Model: Partial Equilibrium

Before describing the full model of a dynamic general equilibrium, we will begin with a partial equilibrium model by taking all the market prices as exogenously given. This will be helpful for understanding the basic structure of relational contracting, which will be combined in a general equilibrium model later.

2.1 Economic Environment

We consider an overlapping generations (OLG) economy with discrete time $t = 0, 1, 2, ...$ Every period, a continuum of one unit mass of individuals is newly born; each individual lives for two periods: young and old. In each generation, one young individual is born from each old individual, and we use notation i to denote both individual i and the dynasty to which individual i belongs. For simplicity, we assume that each individual is concerned with only his or her consumption when old.

The newly born individuals consist of lenders and producers (who become borrowers). We use the masculine pronoun for producers and the feminine pronoun for lenders. Each young lender born in period t is endowed with her income w_t in terms of the numèraire good (which corresponds to the final good in the full model in Section 3). Because every young lender is concerned with her consumption level when old, she will lend all the income w_t to borrowers and consume all the saved income. In the full model of a general equilibrium in Section 3, we will determine the income of young lender w_t in the labour market equilibrium.

In contrast, producer i can produce a specific good i (which corresponds to intermediate good i in the full model below) by investing in capital one period in advance. Specifically, a producer can produce one unit of his specific good when old if he invests in one unit of capital when young. We assume that capital fully depreciates after one period and that young producers are not endowed with any income so that they need to finance their investments when young.

2.2 Preference

We assume that individual i (lender or producer) has an altruistic preference over the consumption level of his or her child. [6] The reason why we introduce the altruistic preference is to ensure that each individual has a reputational concern as we will see in more detail below. If an individual reneges on an implicitly agreed upon contract, his or her child may be punished by not having a better trading opportunity and hence may consume less in the future period, which becomes a utility loss for the deviating individual in the current period. Without an altruistic concern about the child's consumption, any individual never honours implicit promises as will be apparent in the following analysis.

More specifically, we consider an individual in dynasty i who was born in period $t-1$ and whose consumption when old (in period t) is denoted by $C_t^{t-1}(i) \geq 0$. Then, we assume that the utility $U^{t-1}(i)$ of an individual in dynasty i, born in period $t-1$, depends not only on his/her own consumption level when old in period t, $C_t^{t-1}(i)$, but also on the consumption level of his/her child in period $t+1$, $C_{t+1}^{t}(i)$, as follows:

$$U^{t-1}(i) \equiv C_t^{t-1}(i) + \delta C_{t+1}^{t}(i), \tag{1}$$

where $\delta \in (0,1)$ represents the parameter value measuring the degree to which each individual is altruistic about his/her child. We will often call δ the *discount factor* when

[6] One might think why we use the framework of two-period-lived OLG model with altruistic preference instead of using an infinitely-lived agent model. The main reason for this is to simplify the saving decision of each individual which makes the credit market equilibrium condition easier to be handled. In an infinitely-lived agent model, we need to keep track of the Euler equation of saving decision as an additional dynamic equation which would make the dynamic analysis of self-enforcing agreement more complicated. Our OLG model of a two-period-lived agent with altruistic preference can avoid such complication while allowing us to incorporate a self-enforcing agreement into a dynamic general equilibrium in a simpler way.

no confusion arises because it plays a similar role to the discount factor in repeated games.

To keep the model simple, we assume that there is no bequest transfer across generations in each dynasty. Then, each individual consumes all of his/her old income for himself/herself such that $C_t^{t-1}(i)$ is equal to the lifetime income level of individual i born in period $t-1$. We will discuss an extension of the basic model to allow bequests in the Conclusion (see the Online Appendix).

2.3 Strategic Default and Contracting Modes

Each producer wants to finance his capital investment for future production from lenders but there exists a strategic default problem such that any producer can run away and deny any repayment after he borrows from a lender. Anticipating such a strategic default, the lenders never lend to the producers.

In this paper, we consider two alternative means to alleviate this problem.

Arm's Length Contract. In the economy, there is a competitive credit market where borrowing and lending are made at a given interest rate. The credit market is competitive in the sense that everyone takes the market interest rate $r_t \geq 0$ as given in any period t. [7] However, the credit market is not perfectly competitive in the sense that it involves the strategic default problem we have assumed above. This can be avoided in the credit market as follows.

In the credit market, each young lender must spend $1-\lambda$ units of the numèraire good ($0 < \lambda < 1$) for lending one unit to a borrower and preventing him from committing strategic default. Here, $1-\lambda$ is called the *enforcement cost* per unit lending in the credit market. For example, $1-\lambda$ includes the costs of making the information disclosure credible, collecting hard evidence, hiring professionals such as lawyers and accountants who help write formal contracts, and using outside institutions such as courts. [8] Then, when a young lender lends w_t to the credit market in period t, she needs to spend $(1-\lambda)w_t$ for contract enforcement and will thus earn the interest income $r_{t+1}\lambda w_t$ in the next period $t+1$ by lending the remaining amount of λw_t.

As long as lenders incur the enforcement cost, the credit market works as the standard competitive market where lenders and borrowers trade at a given market interest rate r_t in the anonymous and spot fashion. We call such a transaction made in the anonymous credit market an *arm's length contract*. We also call a producer

[7] Here, r_t denotes the gross interest rate for a spot transaction in the credit market, which specifies that one unit of borrowing in period $t-1$ must result in the repayment of r_t units in the next period t. Thus, by the nature of spot transactions in the credit market, no borrower can roll a debt obligation forward in period $t-1$ to his child who would repay it in period $t+1$.

[8] We can also allow borrowers to incur some costs preventing themselves from committing strategic default. For example, each borrower makes some investment in the enforcement technology such that he can commit himself not to default. Since such a possibility cannot alter the main results substantially, we will not pursue such a case in what follows.

(borrower) and a lender who are engaged in the arm's length contract in the credit market, an *A-producer* and an *A-lender*, respectively.

We let $x_t \geq 0$ denote the capital investment (equivalently, the production level) of an A-producer. We also let $p_t \geq 0$ denote the price for a specific good an A-producer produces. Then, the profit of an old A-producer in period t is given by $\pi_t \equiv p_t x_t - r_t x_t$. In this section, we take the capital investment choice of A-producers and their profit π_t as exogenously fixed; these will be endogenously determined in the next section.

Relational Contract. Relational contracting becomes an alternative to avoid the strategic default problem as follows. In the economy, there is a local community where l ($l < 1$) producers and l lenders reside. Thus, $1 - l$ producers and $1 - l$ lenders are outside the local community and engage in arm's length contracts in the credit market. The producers and lenders in the community are matched with each other in the initial period $t = 0$ in a one-to-one manner. Then, each of the matched l pairs forms a personal connection and relationship. We assume that such relationships formed in the initial period can be inherited over successive generations. We call a producer in the community a *R-producer* and a lender in the community a *R-lender* respectively.

In the basic model, we will focus on the case that the number of relationship pairs in the local community does not increase from its initial value $l \in (0, 1)$ by assuming that nobody can enter the local community in order to seek a relationship partner. Further, as we will see below, all R-producers and R-lenders never quit their relationships in equilibrium, ensuring that the number of relationship pairs is never decreased from its initial value $l \in (0, 1)$ as well. Then, the number of relationship pairs becomes constant at $l \in (0, 1)$ over time. The fact that the membership of those who engage in relational contracting is constant is observed in several places where relational contracting is based on closed community memberships. [9] In Section 5, we will drop this assumption and turn to the case where the number of relationship pairs evolves over time because the producers and lenders freely enter the matching market to seek relationship partners.

Any R-producer and R-lender in each pair can always exercise the option to quit their relationship (called the *quitting option*) at any time by leaving the community. We will then assume that if a R-producer (respectively, a R-lender) quits the relationship, his (respectively, her) child cannot also form a relationship with the child of his (respectively, her) partner in the next period. In such a case, not only the current R-producer and R-lender but also their children have no choice but to engage in an arm's length contract in the anonymous credit market. [10]

In what follows, we let $z_t \geq 0$ denote the capital investment (equivalently, the production level) of a R-producer. Each young R-producer, born in period $t - 1$, can

[9] For example, see Fafchamps (2000) for the case of the business community in Zimbabwe.

[10] However, we do not assume permanent dissolution here; that is, we do not assume that once a R-producer and an R-lender dissolve their relationship, all their descendants cannot re-form the previous relationships forever. We only assume that it takes at least one period for a dissolved relationship to be re-formed. We later show that the assumption of one-period dissolution of a relationship is sufficient for each relationship pair to honour the agreed upon relational contracts over time.

directly finance his investment z_t from his partner, a young R-lender, in exchange for making repayment R_t in period t. Since the repayment R_t is not secured due to the strategic default of the R-producer, such an agreement, $\{z_t, R_t\}$, must be implicit and self-enforceable. We call this type of contract a *relational contract*.

2.4 Timing within Each Period

Events in each period proceed in the local community as follows (see Figure 1 for the time line). First, at the beginning of each period, the old R-producer and R-lender of each relationship pair decide whether to exercise the quitting option. When they exercise the quitting option, their relationship is dissolved and their children will have no choice but to engage in an arm's length contract in the next period. When an old R-producer and an old R-lender in a relationship do not exercise the quitting option, in the same period, their children (the young R-producer and R-lender) simultaneously decide whether to exercise the quitting option. When they do not exercise the quitting option, they agree on a relational contract, $\{z_t, R_t\}$, which specifies the capital investment level z_t and the repayment R_t to the R-lender. Because the producer cannot commit himself to repay R_t, such a relational contract must be self-enforceable.

By exercising the quitting option, the young R-producer and R-lender obtain their outside payoffs: any young R-producer who quits a relationship in period $t-1$ obtains the outside profit π_t while any young R-lender who quits a relationship in period $t-1$ lends her endowment w_{t-1} to the outside credit market and earns the interest income $\lambda r_t w_{t-1}$. For the time being, we will treat the outside profit π_t of a producer and the market interest rate r_t to be exogenous, although we will endogenize these variables in the next section.

2.5 Initial Period ($t = 0$)

In the initial period ($t = 0$), each old producer (irrespective of an A-producer or a R-producer) owns an initial capital stock, x_0, which is assumed to be historically given. [11] Because the old producers in the initial period do not need to raise funds for capital investment, they can produce the goods of x_0 without any production costs. There is also one unit mass of old lenders in the initial period.

2.6 Relational Contracts: Self-Enforcing Conditions

Consider any dynasty of relationships that consist of R-producers and R-lenders in the successive generations who implicitly agree to enforce a sequence of relational contracts $\{z_t, R_t\}_{t=0}^{\infty}$ from the initial period $t = 0$ onward. Here, a contract $\{z_{t+1}, R_{t+1}\}$ is designed for t-th generation of the dynasty born in period t. Then we will consider the

[11] We here assume that all old producers in the initial period own the same amount of initial capital stock, $z_0 = x_0$, for simplifying the analysis. Our results do not substantially change even when we allow $z_0 \neq x_0$.

conditions under which such sequence of relational contracts is self-enforceable by all the generations of the dynasty.

There are three constraints to be satisfied for a relational contract $\{z_t, R_t\}$ to be self-enforceable. The first constraint is the *incentive compatibility* (IC) condition, according to which the old R-producer has no incentive to renege on the agreed upon repayment R_t. The second constraint is the *individual rationality condition for R-producer* (IRP), according to which the young R-producer is weakly better off by agreeing to a relational contract instead of exercising the quitting option. The third constraint is the *individual rationality condition for R-lender* (IRL), according to which the young R-lender is weakly better off agreeing to a relational contract instead of exercising the quitting option.

Incentive Compatibility (IC). Under a relational contract $\{z_t, R_t\}$, the young R-producer promises to repay R_t to the R-lender matching him when old (in period t) in exchange for borrowing z_t directly from her. For the time being, we suppose that $w_{t-1} \geq z_t$; this will be shown to be true in any equilibrium (see Lemma 2 below). After the generation in period $t-1$ follows the relational contract, the next generation born in period t will also agree to a relational contract $\{R_{t+1}, z_{t+1}\}$ and enforce it where the R-producer (who is the child of the old R-producer in period t) does not exercise the quitting option but honours the contracted agreement R_{t+1} when old in period $t+1$.

Anticipating the outcome in period $t+1$ described above, the old R-producer in period t makes the repayment R_t to the old R-lender and does not exercise the quitting option only if the following IC constraint, (IC_t), is satisfied:

$$p_t z_t - R_t + \delta\{p_{t+1} z_{t+1} - R_{t+1}\} \geq p_t z_t + \delta \pi_{t+1}, \tag{IC_t}$$

where $p_{t+1} > 0$ denotes the price of a specific good an old R-producer produces in period $t+1$.

We now explain IC_t in detail. The right-hand side of IC_t denotes the payoff the old R-producer could obtain if he reneged on repayment R_t and exercised the quitting option. By doing so, he can save on repayment R_t and capture the whole revenue $p_t z_t$ from capital investment z_t, but in the next period, $t+1$, his child faces the dissolution of the relationship with the child of the current lender. In such a case, the child of such a deviating producer would obtain his outside profit π_{t+1}. This child's future payoff is evaluated using the altruistic parameter $\delta > 0$ from the viewpoint of the current producer. Thus, the sum of these payoffs can be guaranteed by the old R-producer when he reneges on the repayment R_t. In contrast, the left-hand side of IC_t denotes the payoff of the old R-producer when he makes contracted repayment R_t to the R-lender matching him in period t expecting that the relationship is inherited by the next generation, in which case his child also makes repayment R_{t+1} in period $t+1$. This future payoff is also evaluated using δ from the viewpoint of the current old producer in period t. Thus, IC_t is necessary for the old R-producer not to renege on repayment R_t in period t.

Individual Rationality (IR). In addition to the above, the following individual rationality constraint, IRL_t, of each young R-lender must be satisfied: [12]

$$R_t + \lambda r_t(w_{t-1} - z_t) + \delta\{R_{t+1} + \lambda r_{t+1}(w_t - z_{t+1})\} \geq \lambda r_t w_{t-1} + \delta\lambda r_{t+1} w_t. \qquad (\text{IRL}_t)$$

Otherwise, the young R-lender would exercise the quitting option in period $t-1$; by exercising the quitting option, the lender and her child lend their entire wage income to the credit market so as to earn the interest income corresponding to the payoffs on the right-hand side of IRL_t. Here, $(1-\lambda)w_t$ must be spent for the enforcement cost of arm's length contract. Further, the interest income of her child, $\lambda r_{t+1} w_t$, is evaluated by the altruistic parameter δ from the viewpoint of the current old lender. However, if the R-lender does not exercise the quitting option, she would obtain contracted repayment R_t in period t, which appears as the first term on the left-hand side of IRL_t, in addition to interest income $\lambda r_t(w_{t-1} - z_t)$ from the saving on the remaining income $w_{t-1} - z_t$ after lending z_t to the R-producer. (Note here that we are assuming that $w_{t-1} \geq z_t$.) Here, $(1-\lambda)(w_{t-1} - z_t)$ must be spent for the enforcement cost of arm's length contract in the credit market. The child of the lender is then paid the contracted amount R_{t+1} in addition to the interest income $\lambda r_{t+1}(w_t - z_{t+1})$ when old in period $t+1$. Here, the payoff of the lender's child is evaluated by the altruistic parameter $\delta > 0$ again from the viewpoint of the current old lender.

Combining IC_t with IRL_t, we derive the following modified IC condition, denoted by IC_t^*:

$$\delta\{p_{t+1}z_{t+1} - \lambda r_{t+1} z_{t+1} - \pi_{t+1}\} \geq \lambda r_t z_t. \qquad (\text{IC}_t^*)$$

This condition is necessary for a relational contract to be self-enforceable.

Next, we consider the individual rationality constraint of each young R-producer (IRP_t):

$$p_t z_t - R_t + \delta\{p_{t+1}z_{t+1} - R_{t+1}\} \geq \pi_t + \delta\pi_{t+1}. \qquad (\text{IRP}_t)$$

The condition IRP_t ensures that each young R-producer prefers continuing the relationship to dissolving it. Suppose that some young R-producer exercises the quitting option when young. Then, he earns the outside profit π_t and his child earns the outside profit π_{t+1}. The latter is evaluated by δ from the viewpoint of the current producer. The sum of these payoffs corresponds to the payoff each A-producer obtains via an arm's length contract, which thus becomes his outside option. However, the left-hand side of IRP_t denotes the payoff of the young R-producer who does not exercise the quitting option when young; by doing so, he earns $p_t z_t - R_t$ when old (period t) by making repayment R_t to the relationship lender. Following this, his child also continues the relationship and earns profit $p_{t+1}z_{t+1} - R_{t+1}$ by making the contracted repayment

[12] As long as IRL_t is satisfied, any old R-lender has no incentive to renege on the repayment R_t as well when $R_t < 0$. This follows from the following fact: since the right-hand side of IRL_t is larger than $\lambda r_t(w_{t-1} - z_t) + \delta\lambda r_{t+1} w_t$, IRL_t implies that $R_t + \delta\{R_{+1} + \lambda r_{t+1}(w_t - z_{t+1})\} \geq \delta\lambda r_{t+1} w_t$, which means that every old R-lender wants to make the repayment $R_t < 0$ and continue a relationship to the next generation rather than quitting the relationship, provided she has already lent z_t to the R-producer from her income w_{t-1} when young.

R_{t+1} to the R-lender when old (period $t+1$). Thus, the young R-producer obtains the sum of these payoffs by continuing the relationship.

By subtracting the right-hand sides of IRP_t and IRL_t from their left-hand sides, the net total surplus of a relationship pair of a young R-producer and a young R-lender born in period $t-1$ is defined as

$$TS_t \equiv p_t z_t - \lambda r_t z_t - \pi_t + \delta\{p_{t+1}z_{t+1} - \lambda r_{t+1}z_{t+1} - \pi_{t+1}\}$$

for $t \geq 1$.

Here, we assume that the inverse demand function for a specific good is given by $p_t = \alpha A \eta_t^{\alpha-1}$ for $\eta_t = z_t$ or $\eta = x_t$, where $A > 0$ and $\alpha \in (0,1)$ are the parameters, which we will derive by using monopolistic competition in the general equilibrium model in Section 3. Then, we can define the net gain from relational contracting in period t as

$$p_t z_t - \lambda r_t z_t - \pi_t = \alpha A(z_t^\alpha - x_t^\alpha) + r_t(x_t - \lambda z_t).$$

This net gain increases when the interest rate r_t goes up, provided the R-producer's investment, z_t, is less than $(1/\lambda)x_t$, which will actually be the case in equilibrium (see Lemma 3 below). This is because a rise in the interest rate reduces the profit of an arm's length contract π_t more than the increase in the opportunity cost of relational contracting $\lambda r_t z_t$ when the R-producer's investment is less than $(1/\lambda)x_t$. We will see later that the increase in the interest rate makes relational contract easier to be satisfied so that it becomes more sustainable in the low developed stages in which the interest rates are high. We will then endogenize the interest rate and show how the endogenous change in the interest rate plays an important role for determining the self-enforcing condition of relational contracting, and as a result, the dynamic process of the economy.

In the initial period $t = 0$, each initial old R-producer earns the profit $\pi_0 \equiv \alpha A x_0^\alpha$ because he owns the initial capital x_0 while each initial old R-lender earns nothing. Thus, their joint profit is π_0. Since each initial old R-producer also has the outside option to earn π_0, the net total surplus TS_0 in the initial period $t = 0$ becomes $TS_0 \equiv \delta\{\alpha A z_1^\alpha - \lambda r_1 z_1 - \pi_1\}$. [13]

Then, for a relationship pair of a young R-producer and a young R-lender born in period $t-1$ to agree on a relational contract $\{z_t, R_t\}$, it must satisfy both IRP_t and IRL_t; that is, the total net surplus of each relationship pair must be non-negative:

$$TS_t \geq 0, \quad \text{for any } t \geq 0 \qquad (\text{TS}_t).$$

We can then readily show that there exists a sequence of repayments $\{R_t\}_{t=1}^\infty$ that satisfy IC_t, IRL_{t-1}, and IRP_{t-1} for all $t \geq 1$ as long as IC_t^* and TS_{t-1} are satisfied for all $t \geq 1$.[14]

Finally, we denote by

$$J_t \equiv p_t z_t + \lambda r_t(w_{t-1} - z_t)$$

[13] Here, we have IRP_0: $\pi_0 - R_0 + \delta\{p_1 z_1 - R_1\} \geq \pi_0 + \delta\pi_1$, and IRL_0: $R_0 + \delta\{R_1 + \lambda r_1(w_0 - z_1)\} \geq \delta\lambda r_1 w_0$.

[14] It is sufficient to set $R_t = \lambda r_t z_t$ for each $t \geq 1$ and R_0 such that $\pi_0 \geq R_0 \geq 0$.

and

$$V_t \equiv J_t + \delta J_{t+1}$$

the joint profit and the joint payoff of a relationship pair born in period $t-1$, respectively.

Although there are many possible equilibria sustained by different relational contracts as known in the Folk Theorem of repeated game theory, we will focus on the equilibrium called the *best relational contracting equilibrium (BRCE)*, in which the initial generation of relationship pairs in each dynasty chooses a sequence of all future relational contracts $\{z_t, R_t\}_{t=1}^{\infty}$ so as to maximize the weighted sum of joint payoffs of all generations in the same dynasty $\sum_{t=0}^{\infty} \beta^t V_t$ for some weight $\beta \in (0,1)$, subject to the above constraints $\{(\text{IC}_t^*), (\text{TS}_{t-1})\}_{t=1}^{\infty}$, given the future paths of all the market prices $\{r_t, w_t\}_{t=0}^{\infty}$.[15] [16] [17] Thus, the initial generation in each dynasty puts the welfare weight β^t on the t-th generation of the same dynasty when solving the optimal relational contracts. Then, since $\sum_{t=0}^{\infty} \beta^t V_t = J_0 + (\delta+\beta)\sum_{t=1}^{\infty} \beta^{t-1} J_t$ and since $J_0 = \pi_0$ is exogenously given, the above optimization problem is equivalent to maximizing $\sum_{t=1}^{\infty} \beta^{t-1} J_t$ subject to IC_t^* and TS_{t-1} for all $t \geq 1$. The results we establish in what follows do not depend on the particular values of the welfare weight $\beta \in (0,1)$.[18]

Then, we can show the following result on the optimal relational contracts.

Lemma 1. *(i) Each relationship pair born in period $t-1$ agrees to a relational contract $\{z_t, R_t\}$ and follows it if and only if IC_s^* and TS_s are satisfied for all $s \geq t$. (ii) In the optimal relational contract, $z_t \leq \hat{\lambda} x_t$ holds in any period t where $\hat{\lambda} \equiv \lambda^{1/(\alpha-1)} > 1$, and $\hat{\lambda} x_t$ maximizes the joint profit J_t. (iii) The optimal relational contract involves a downward distortion of capital investment $z_t < \hat{\lambda} x_t$ as compared to the investment level maximizing the joint profit J_t only if IC_t^* is binding.*

Lemma 1 (i) implies that a sequence of relational contracts $\{z_t, R_t\}_{t=1}^{\infty}$ is self-enforceable if and only if IC_t^* and TS_{t-1} are satisfied for all $t \geq 1$. Among all relational contracts that satisfy this requirement, the initial generation of relationship pairs in each dynasty chooses the relational contracts that maximize the weighted sum of the

[15] See Acemoglu, Golosov and Tsyvinski (2008) for a related treatment of self-enforcing agreements in a dynamic general equilibrium model, although they consider a different model from ours.

[16] In addition, we need to impose the condition $p_t z_t \geq R_t \geq -\lambda r_t (w_{t-1} - z_t)$ for all $t \geq 1$ and $\pi_0 \geq R_0 \geq 0$. This ensures that the consumption levels of each old R-producer and each old R-lender are non-negative in each period; this is called condition NNC_t. However, we can solve the optimal relational contracts without NNC_t, and then, can check that the optimal relational contract satisfies NNC_t later by adjusting the repayment R_t appropriately and using the other conditions such as IC_t^* and TS_t. Also, IC_t^* and TS_0 imply that $p_t z_t + \lambda r_t (w_{t-1} - z_t) \geq \lambda r_t w_{t-1} + \pi_t$ for all t which then ensures that every relationship cannot gain from engaging in arm's length contract in the credit market in each period t on the equilibrium path in which IC_t and TS_0 are satisfied for all $t \geq 1$.

[17] This formulation is equivalent to maximizing the sum of R-producers' payoffs in each dynasty subject to IC_t, IRP_{t-1}, and IRL_{t-1} for all $t \geq 1$: $\max_{\{R_t, z_t\}} \sum_{t=0}^{\infty} \beta^t \{p_t z_t - R_t\}$ subject to IC_t, IRP_{t-1} and IRL_{t-1} for $t = 1, 2, ...$

[18] If one might think that it is reasonable to suppose that the initial generation has the same welfare weight as his or her altruistic preference parameter δ, we can set $\beta = \delta$.

joint profits of all generations in the same dynasty $\sum_{t=1}^{\infty} \beta^{t-1} J_t$. Lemmas 1 (ii) and (iii) then show that the optimal relational contract involves a lower capital investment than the one maximizing the joint profit J_t without IC_t^*; that is, $z_t < \hat{\lambda} x_t$. This occurs only if IC_t^* is binding.

One of the key variables which affect the optimal relational contract is the interest rate r_t in the anonymous credit market. As we have already mentioned, high interest rates make relational contract easier to be self-enforceable so that R-producers invest more than A-producers do when $z_t \leq (1/\lambda) x_t$ holds, which will be shown to be true in any equilibrium below. This implies that relational contract becomes less constrained in the early stages of development in which the anonymous credit market is immature so that the interest rates are high but it becomes more constrained in the well-developed stages with low interest rates. In the next section, we will address this issue by turning to the full model of a dynamic general equilibrium in which all the market prices such as $\{w_t, r_t\}_{t=0}^{\infty}$ are endogenously determined together with the optimal relational contract we have shown above. The novel part of our model is that we investigate how these market prices affect and are affected by the self-enforcing conditions of relational contracts, resulting in the dynamic process of the whole economy.

3 Model: General Equilibrium

In this section we provide a dynamic general equilibrium model which is kept as simple as possible in order to incorporate the market equilibrium conditions into the partial equilibrium model presented in the previous section. To this end, we need at least three things: (i) the income level each lender earns w_t is endogenously determined, (ii) the interest rate r_t is endogenously determined, and (iii) the price of each specific good p_t is endogenous and some positive profit for the production of that good can be guaranteed because otherwise the total surplus of relational contract cannot be positive and hence relational contract will never become sustainable. The first and second parts are addressed by introducing the credit and labour market clearing conditions. The last part is addressed by considering a monopolistic competition for specific goods (modelled as intermediate goods below) which ensures a positive profit for each producer who obtains monopolistic power over a differentiated good he produces.

3.1 Markets

In the economy, there is a single final good, taken as a numèraire, which is used for both consumption and investment. The final good Y_t is produced by a continuum of intermediate goods, each of which is indexed by $i \in [0, 1]$, and labour L_t in the following manner:

$$Y_t = A L_t^{1-\alpha} \int_0^1 \eta_t(i)^\alpha di, \tag{2}$$

where $\alpha \in (0, 1)$, $A > 0$, and $\eta_t(i)$ denotes the input demand for intermediate good i in period t (which is equivalent to its output).

Each intermediate good i is produced by producer i, either an A-producer or a R-producer, who possesses the specific knowledge to produce that intermediate good. The specific good we have considered in the partial equilibrium model in Section 2 is interpreted as an intermediate good in the general equilibrium model in this section. Here, as we have assumed in the partial equilibrium model, producer i (an A-producer or a R-producer) can produce one unit of intermediate good i when old by investing one unit of capital when young.

We also assume that each young lender, born in each period t, is endowed with one unit of labour and inelastically supplies it to the labour market in order to earn the wage income w_t. The labour supplied by these young lenders (workers) is used for producing the final good of the economy.

We assume that there is perfect competition in the final good market. Then, the final good firm chooses the demand for labour L_t and intermediate inputs $\eta_t(i)$ to maximize its profit:

$$AL_t^{1-\alpha}\int_0^1 \eta_t(i)^\alpha di - w_t L_t - \int_0^1 p_t(i)\eta_t(i)di, \tag{3}$$

where the wage rate w_t and price of intermediate good i, $p_t(i)$, are taken as given. The corresponding first-order conditions are as follows:

$$A\alpha L_t^{1-\alpha}\eta_t(i)^{\alpha-1} = p_t(i) \tag{4}$$

and

$$A(1-\alpha)L_t^{-\alpha}\int_0^1 \eta_t(i)^\alpha di = w_t. \tag{5}$$

Equation (4) corresponds to the inverse demand function for a specific good we have used in the partial equilibrium model in the previous section.

We also assume that the labour market is perfectly competitive and we let $w_t \geq 0$ denote the competitive market wage in period t in the labour market.

Finally, as we have explained in the previous section, there exists the credit market where a producer and a lender can enter into a formal credit contract, which we call the *arm's length contract*, in the spot manner. However, here, the lender must incur the enforcement cost to prevent the borrower from defaulting: each lender must spend $1-\lambda \in (0,1)$ unit of the final good as the enforcement cost for lending one unit to the credit market.

3.2 Arm's Length Contract Producers

Because every A-producer faces the same demand function, (4), for his intermediate good, we will omit notation i hereafter.

We then obtain π_t, the profit of an intermediate good producer (an A-producer) who finances his investment x_t via an arm's length contract in period t, as follows:

$$\begin{aligned} \pi_t &\equiv p_t x_t - r_t x_t \\ &= AL_t^{1-\alpha}\alpha x_t^\alpha - r_t x_t, \end{aligned} \tag{6}$$

where the price of an intermediate good, p_t, is given by (4) and $\eta_t(i) = x_t$ for each A-producer i. Because $L_t = 1$ holds in the labour market equilibrium, we set $L_t = 1$ in what follows.

Each young A-producer born in period $t-1$ chooses the capital investment level x_t to maximize his payoff $\pi_t + \delta\pi_{t+1}$. Here, note that his consumption level C_t^{t-1} is equal to the profit π_t he earned in period t, whereas the consumption level of his child is $C_{t+1}^t = \pi_{t+1}$. Thus, the payoff of a young A-producer is given by $C_t^{t-1} + \delta C_{t+1}^t = \pi_t + \delta\pi_{t+1}$. Since each A-producer takes the market prices such as w_t and r_t as given and his capital investment x_t affects only his own profit π_t, we will focus on the equilibrium in which a young A-producer chooses his capital investment x_t so as to maximize only his own profit π_t.[19]

We then define such optimal investment by $x_t = x(r_t)$ as a function of the interest rate r_t, which satisfies the following first-order condition:

$$A\alpha^2 x_t^{\alpha-1} = r_t. \tag{7}$$

Thus, we have

$$\max_{x_t} \pi_t = A\alpha(1-\alpha)x_t^{\alpha} \tag{8}$$

where x_t satisfies (7). In what follows, we will often omit the argument r_t from $x(r_t)$ and simply write x_t for $x_t = x(r_t)$ when no confusion arises.

3.3 Definition of Equilibrium Paths

Now, we provide a formal definition of an equilibrium path in this model economy.

Definition. *A sequence $\{x_t, z_t, w_t, r_t\}_{t=0}^{\infty}$ is said to be an equilibrium path of the economy if the following conditions are satisfied.*

(i) *Each young A-producer born in period $t-1$ chooses his capital investment x_t so as to maximize his own profit π_t, which satisfies the optimality condition (7).*

(ii) *The initial generation of the relationship pairs in each dynasty chooses a sequence of future relational contracts $\{z_t, R_t\}_{t=1}^{\infty}$ so as to maximize the weighted sum of the joint payoffs of all generations in the same dynasty $\sum_{t=0}^{\infty}\beta^t V_t$ subject to $\{(IC_t^*), (TS_{t-1})\}_{t=1}^{\infty}$.*

(iii) *The labour market equilibrium (LME_t): Setting $L_t = 1$ in (5) determines the market wage w_t as*

$$w_t = A(1-\alpha)(lz_t^{\alpha} + (1-l)x_t^{\alpha}).$$

[19] However, in general, there may exist strategic interactions between successive generations in the same dynasty of A-producers in that they may make the current capital investment choices contingent on the observed history about the capital investments chosen in the past periods in the same dynasty. However, we can show that no generality is lost by confining our attention to the equilibrium in which each A-producer chooses x_t in order to maximize only his own profit π_t in any period t no matter what histories are observed up to period t (see the Online Appendix).

(iv) *The credit market equilibrium (CME_{t-1}):*

$$\lambda w_{t-1} = \lambda l z_t + (1-l)x_t.$$

Here, the initial capital $z_0 = x_0$ is given.

Condition (i) yields the optimal capital choice of each young A-producer taking market prices w_t and r_t as given. Condition (ii) is the optimal choice with regard to relational contract $\{z_t, R_t\}$ for every relationship pair, as we have explained in Section 2. Condition (iii) gives LME_t which is the labour market clearing condition: market wage w_t is determined by clearing the labour market ($L_t = 1$) using the labour demand function (5) with the labour supply equal to one unit mass of young workers in every period. Condition (iv) gives CME_{t-1} which is the credit market clearing condition in period $t-1$. Each young R-producer lends $\lambda(w_{t-1} - z_t)$ to the anonymous credit market in period $t-1$ after she makes a relationship lending z_t to the young R-producer matching her. Here, note that we are supposing $w_{t-1} \geq z_t$ and that each R-lender must spend $(1-\lambda)(w_{t-1} - z_t)$ for the enforcement cost of the arm's length contract. In addition to this supply of credit, $1-l$ young A-lenders outside the community have nothing but to lend λw_{t-1} each to the credit market. Thus, the total credit supply in the anonymous credit market is given by $l\lambda(w_{t-1} - z_t) + (1-l)\lambda w_{t-1}$. Conversely, $1-l$ young A-producers finance their capital investment x_t each from the credit market. Thus, the total demand in the credit market is given by $(1-l)x_t$. Then, the credit market of period $t-1$ clears if CME_{t-1} holds.

In this model economy, the initial condition is given by initial capital stock $z_0 = x_0$ owned by each initial old intermediate good producer irrespective of a R-producer or an A-producer. Thus, every initial old intermediate good producer produces $z_0 = x_0$ without any production costs. Then, the market wage in the initial period ($t = 0$) is determined by the labour market equilibrium LME_0: $w_0 = A(1-\alpha)x_0^\alpha$.

4 Characterization of Equilibrium Dynamics: From Relationships to Markets

In this section, we show that relational contracting contributes to economic growth in low-development stages while its value declines as the economy grows and enters high-development stages.

4.1 Preliminary Results

We begin by showing the preliminary results that will be useful for characterizing the equilibrium paths.

First, we have thus far assumed that each R-producer does not invest in capital more than the funds available to the R-lender matching him, i.e., $w_{t-1} \geq z_t$, in any equilibrium. Now we show that this is actually the case in any equilibrium. If this is not the case in some period t, we have $z_t > w_{t-1}$ and hence each R-producer needs to finance the remaining amount $z_t - w_{t-1}$ from the credit market after borrowing w_{t-1} directly from the R-lender matching him. Then, the joint profit of the relationship pair is changed to $J_t = p_t z_t - r_t(z_t - w_{t-1})$ which is decreasing in the interest rate r_t. If each R-producer demands high capital investment $z_t > w_{t-1}$ in the credit market, the equilibrium interest rate r_t must go up in order to clear the credit market. This however reduces the joint profit J_t and lowers the capital investment of the R-producer z_t until it restores the condition $w_{t-1} \geq z_t$ in the credit market equilibrium. We thus obtain the following.

Lemma 2. *In any equilibrium path $z_t \leq w_{t-1}$ must hold in any period t.*

Second, we show that IC_t^* is always binding in any equilibrium path.

Lemma 3. *In any equilibrium, $z_t \leq (1/\lambda)x_t$ holds in any period t. Then, the IC condition IC_t^* becomes binding in any period t in any equilibrium.*

Recall that the net gain from relational contracting in period t is defined as $\alpha A(z_t^\alpha - x_t^\alpha) + r_t(x_t - \lambda z_t)$ which is decreasing in r_t when $z_t > (1/\lambda)x_t$. If each R-producer invests more than $(1/\lambda)x_t$ (i.e., $z_t > (1/\lambda)x_t$) in some period t, the interest rate must go up in order to meet such high capital investment demand in the credit market equilibrium (CME_{t-1}). This in turn reduces the net gain from a relational contract, lowering the capital investment of the R-producer so that as a consequence $z_t \leq (1/\lambda)x_t$ is restored. When $z_t \leq (1/\lambda)x_t$ holds, we have that $z_t < \hat{\lambda}x_t$ so that (IC_t^*) must be binding in equilibrium due to Lemma 1.

4.2 Rise and Fall of Relational Contracts

Next, we characterize equilibrium paths. In particular, we investigate how relationship lending contributes to the process of economic development relative to market lending based on arm's length contracts. Then, we show that each R-producer who enters into relational contract invests and hence produces more than each A-producer who enters into arm's length contract in the early stages of development, but the former invests less than the latter in later development stages.

Thanks to Lemma 2 and 3, any equilibrium path of the economy can be described as a sequence $\{z_t, x_t, r_t, w_t\}_{t=0}^{\infty}$ which satisfies binding IC_t^* and CME_t:

$$\delta\{\alpha A z_{t+1}^\alpha - \lambda r_{t+1} z_{t+1} - \pi_{t+1}\} = \lambda r_t z_t, \quad t = 1, 2, ..., \qquad (\mathrm{IC}_t^*)$$

$$\lambda w_t = \lambda l z_{t+1} + (1-l)x_{t+1}, \quad t = 1, 2, ..., \qquad (\mathrm{CME}_t)$$

where $r_t = \alpha^2 A x_t^{\alpha-1}$ (see (7)), $w_t = A(1-\alpha)(l z_t^\alpha + (1-l)x_t^\alpha)$ (see LME_t), and

$$\lambda w_0 = \lambda l z_1 + (1-l)x_1 \qquad (\mathrm{CME}_0)$$

for a given initial market wage $w_0 = (1-\alpha)Ax_0^\alpha$.

By substituting $r_t = \alpha^2 A x_t^{\alpha-1}$ into IC_t^*, using $\pi_t = A\alpha(1-\alpha)x_t^\alpha$ (see (8)), and defining a new variable $y_t \equiv z_t/x_t$ for the relative output of a R-producer which measures the ratio between capital investments (output) of relational and arm's length contracts, we can re-write the above equilibrium conditions as follows:

$$\left(\frac{x_{t+1}}{x_t}\right)^\alpha \delta\{y_{t+1}^\alpha - \lambda\alpha y_{t+1} - (1-\alpha)\} = \lambda\alpha y_t, \quad t = 1, 2, ..., \tag{9}$$

$$\lambda(1-\alpha)Ax_t^\alpha(ly_t^\alpha + (1-l)) = x_{t+1}(\lambda l y_{t+1} + (1-l)), \quad t = 1, 2, ..., \tag{10}$$

and

$$\lambda w_0 = x_1(\lambda l y_1 + (1-l)). \tag{11}$$

Note that $y_t \leq \lambda^{-1}$ holds in any period t due to Lemma 3.

Since w_0 is exogenously given, the above equations (9) and (10) fully determine an equilibrium path $\{x_t, y_t\}_{t=1}^\infty$ once we fix the value of (x_1, y_1) in the first period $t = 1$. The first period values of (x_1, y_1) can be freely chosen as long as they satisfy the initial condition (11). Thus, there are multiple equilibrium paths even though the initial condition x_0 and hence w_0 are exogenously fixed.

We first show that the relative output of R-producer y_t declines over time until it hits some critical value and that there exists a *switching period* T before which $y_t > 1$ holds but after which $y_t < 1$ holds. Such a switching period T is unique for a given y_1. Thus, the economy relies more on relational contracts in the early stages of development (for $t \leq T$) but more on arm's length contracts in the subsequent stages of development (for $t > T$) in the sense that each young R-producer invests more than each young A-producer in the former stages but not in the latter stages.

To derive this result, we introduce some notations and make conditions on the primitives of the model.

We first define the following output of A-producers:

$$\overline{x} \equiv \left[\frac{\lambda(1-\alpha)A(\lambda^{-\alpha}l + (1-l))}{1-l}\right]^{\frac{1}{1-\alpha}} \tag{12}$$

and we then show that this becomes the upper bound for capital investment (production level) which A-producers can attain in any equilibrium path in which they do not stagnate below the initial investment level x_0. To see why, note that using CME_t for $t \geq 1$, $1/\lambda \geq y_t$ and $z_{t+1} \geq 0$, we have that

$$\lambda(1-\alpha)Ax_t^\alpha(\lambda^{-\alpha}l + (1-l)) \geq (1-l)x_{t+1} \tag{13}$$

in any period $t \geq 1$. Further, CME_0 and $\lambda < 1$ imply that $\lambda(1-\alpha)Ax_0^\alpha(\lambda^{-\alpha}l+(1-l)) \geq \lambda w_0 = x_1(ly_1 + (1-l)) \geq (1-l)x_1$. Thus, the above inequality (13) must be satisfied for all periods $t \geq 0$. When we consider the development process of the economy in which each A-producer invests more in capital than the initial level x_0, it is necessary to have $\overline{x} \geq x_0$ because otherwise we can show that $x_t \leq x_0$ holds for all $t \geq 1$. [20] This implies that the market economy never emerges in the sense that A-producers always stagnate below the production level in the initial period, which is contrary to our interest for considering the process of economic development. Thus, we need the following condition for investigating the process such that the economy takes off from the initial development state.

Assumption 1. $\overline{x} \geq x_0$.

Given Assumption 1, we can readily see from (13) that $x_t \leq \overline{x}$ is satisfied in any period $t \geq 0$, that is, $\overline{x}$ becomes an upper bound for capital investment which each A-producer chooses in any period.[21]

Next, we define the following value of the altruistic preference parameter (the discount factor), which plays a critical role for determining equilibrium paths.

$$\tilde{\delta} \equiv \left(\frac{\alpha\lambda}{\lambda^{1-\alpha} - \lambda}\right)\left(\frac{l\lambda^{-\alpha} + (1-l))}{(1-l)^2}\right). \tag{14}$$

Then, we make the following assumption. [22]

Assumption 2. $\delta > \tilde{\delta}$.

Assumption 2 can ensure a large enough gain from relational contracting (the left-hand side of IC_t^*) which makes it possible for a relationship pair to invest more in capital than an A-producer when the interest rate is high. Since the interest rates are high in the early stages of development in which wage income and hence saving is low, each young R-producer has more incentive to choose a higher investment than each young A-producer in the early stages but not in the well-developed stages under Assumption 2.

More formally, we show the following main result of this paper.

Proposition 1. *Suppose that Assumptions 1 and 2 hold. Then, any equilibrium path has the following features: if $y_1 > 1$, then*

- *the relative output of R-producer y_t decreases over time until it hits some critical value less than one, denoted by $\hat{y} < 1$, and thus, $y_1 > y_2 > \cdots > \hat{y}$;*

[20] Suppose that $x_0 > \overline{x}$. Then, by the definition of $\overline{x}$, we have $\lambda(1-\alpha)A(\lambda^{-\alpha}l+(1-l))x_0^\alpha \leq (1-l)x_0$. Then, since CME_0 implies that $\lambda(1-\alpha)A(\lambda^{-\alpha}l+(1-l))x_0^\alpha \geq (1-l)x_1$, we have $x_0 \geq x_1$. Next, using CME_t, we have $(1-l)x_0 \geq \lambda(1-\alpha)A(\lambda^{-\alpha}l+(1-l))x_0^\alpha \geq \lambda(1-\alpha)A(\lambda^{-\alpha}l+(1-l))x_1^\alpha \geq (1-l)x_2$, implying that $x_0 \geq x_2$. Repeating this, we obtain $x_t \leq x_0$ for all $t \geq 1$.

[21] Note that $\overline{x} = \overline{x}^{1-\alpha}\overline{x}^\alpha \geq \overline{x}^{1-\alpha}x_0^\alpha = [\lambda(1-\alpha)A(\lambda^{-1}l+(1-l))/(1-l)]x_0^\alpha \geq x_1$ due to CME_0. Repeating this, we get $\overline{x} \geq x_t$ for all $t \geq 0$.

[22] Here, for $1 > \delta$ to be satisfied, Assumption 2 requires that $1 > \tilde{\delta}$, which is more likely to be satisfied when the number of relationship pairs l is small and the enforcement cost $1-\lambda$ is large. More precisely, if we let $l \to 0$ in $\tilde{\delta}$, then we can show that $1 > \tilde{\delta}$ holds when $\lambda < (1/(1+\alpha))^{1/\alpha}$.

- *there exists a switching period T such that $y_t > 1$ holds for all $t \leq T$ but $y_t < 1$ holds for all $t > T$;*
- *the switching period T is unique for a given y_1.*

Proposition 1 states that relational contracting contributes to the process of economic development relative to arm's length contracts more in the early stages than the later stages of development in that the relative output of R-producer y_t declines over time until it hits some critical value $\hat{y} < 1$. In particular, there exists a unique switching period T before which relationship lending yields more capital investment and output than the market lending based on arm's length contracts, but after which the former yields smaller capital investment and output than the latter (see Figure 2). Proposition 1 thus confirms the historical argument that the Western society drastically changed from a non-market system, which relies more on personal connections, kinship networks, and community, to a market system in the 19th century as the Western economies grew faster (Polanyi (1947)).

Proposition 1 supports the positive view of relational contracting that the non-market system (the relationship-based system) is not impediment to economic growth in the early periods but has a positive effect of promoting the emergence of a market system in subsequent stages. Specifically, the expansion of the relative output of R-producer y_t can save the enforcement cost associated with arm's length contracts in the whole society, contributing to economic growth. Lamoreaux (1994) reports supportive evidence that the relationship-based lending became effective in financing and promoting economic growth in New England in the early 19 century where the financial market was so immature that market-based financing was difficult. Another related evidence is the positive role of large German banks such as Commerzbank, Dresdner, and Deutsche in having financed the rapid expansion of German industries between the late 19th century and the early 20th century. These large banks formed close and lasting relationships with industry enterprises by offering low interest rates and being represented as board directors on these firms (Allen (2011)). Maurer and Haber (2007) also empirically support the positive view of relationship lending in that Mexican bankers during 1888–1913 were largely involved in relationship lending because they optimally responded to a large enforcement cost but they did not loot their own banks at the expense of outside shareholders. [23]

Proposition 1 also shows the other side of relational contracting: the relationship-based system eventually declines and has serious limitations as the economy becomes richer. Specifically, our result shows that each R-producer who engages in relationship lending eventually switches to invest less than each A-producer who engages in market lending in the later stages of development. Lamoreaux (1994) found evidence that insider lending practices, which had served as an important financing device in New

[23] Fafchamps (2000) also discusses that business networks based on personal relationships perform a variety of valuable functions in Sub-Saharan Africa.

England in the early 19th century, became less important and were eventually replaced by market-based finance as the United States transitioned from a capital-poor economy to a capital-rich economy in the late 19th century. Demirgüç-Kunt and Levine (2004) also reported a related fact that bank finance, which is sometimes characterized as a long-term relationship between particular banks and firms, becomes less popular relative to market-based finance, such as equity, in a more-developed country (see Rajan and Zingales (2000) for a related argument).

The intuition behind Proposition 1 is understood as follows. The income levels in the early stages of development are so low that total saving is limited and hence the market interest rates are high. Since each relationship pair extracts the net gain from relational contracting defined as $\alpha A(z_t^\alpha - x_t^\alpha) + r_t(x_t - \lambda z_t)$ in period t and its investment z_t is less than $(1/\lambda)x_t$ (Lemma 3), a rise in the interest rate leads to a large net gain from relational contracting, which makes the IC condition (IC_t^*) more likely to be satisfied. This leads to higher capital investment by a R-producer over an A-producer in the low-development stages in which the interest rates are expected to be high in the credit market. However, as the economy develops well in the course during which relationship pairs save on enforcement cost, the anonymous credit market expands and the interest rates fall over time. Thus, each R-producer who expects low interest rates and tight IC_t^* invests less in capital and produces less than an A-producer in the subsequent stages in which lower interest rates make arm's length contracts more profitable.

As we have already mentioned, equilibrium paths are not unique because there are multiple candidates for the equilibrium values of y_1 and x_1 in the first period as long as they satisfy (11): $\lambda w_0 = x_1(\lambda l y_1 + (1-l))$. Thus, the differences in the equilibrium output in the first period (x_1, y_1) cause the different evolution patterns of the whole economy with different switching periods. Which economies with larger or smaller relative output of R-producer y_1 in the first period switch more early from the relationship-based system to the market-based system? We now address this issue, and give the following result.

Proposition 2. *Suppose that Assumptions 1 and 2 hold. Take any two equilibrium paths $\{y_t'\}_{t=1}^\infty$ and $\{y_t''\}_{t=1}^\infty$ with the corresponding switching periods T' and T'' where $y_1' \geq y_1''$. Then, $y_t' \geq y_t''$ holds in any period $t \geq 1$ so that $T' \geq T''$.*

Proposition 2 shows that the economy which starts with less reliance on relational contracts (smaller y_1) in the first period continues to rely less on relational contracts in every future period and switches more early from the relationship-based phase to the market-based phase than the economy which starts with more reliance on relational contracts in the first period (see Figure 3). This result has the implication that the degree to which the market-based system is currently dominant in a country depends on the extent to which that country has relied on it in the early development phases.

We can also investigate how the switching period T changes with exogenous variables of the model economy such as the discount factor of individuals δ and the enforcement cost of an arm's length contract $1-\lambda$.

Suppose for example that δ increases, that is, individuals become more altruistic about their children. This change raises the gain from relational contracting so that IC_t^* becomes more likely to be satisfied. However, since IC_t^* always binds in any period t (Lemma 3), such an increase in the gain from relational contracting must be offset by the reduction of the relative output y_{t+1} of a R-producer in a future period in order to keep IC_t^* binding in equilibrium. Thus, as long as the relative output y_1 in the first period does not rise by an increase in the discount factor δ, the relative output in the second period y_2 must decrease in order to keep IC_1^* binding. Repeating this, the relative outputs of R-producers y_t in all the future periods must decrease, resulting in a decrease in the switching period T. We can also see a similar effect of the rise in the enforcement cost $1-\lambda$. Suppose that λ decreases. Then, the relationship pairs face lower opportunity costs to invest in capital such that IC_t^* becomes more easily satisfied. Again, such an increase in the gain from relational contracting must be offset by the reduction in the relative output y_{t+1} in the future period in order to keep IC_t^* binding in equilibrium.

More formally, by letting $\zeta \equiv (\delta, 1-\lambda)$, we use $\{y_t(\zeta)\}_{t=1}^{\infty}$ to denote an equilibrium path of the relative output of R-producer by making its dependence of δ and λ explicit. Let $T(\zeta)$ be the corresponding switching period. Then, we show that the economy more early switches from a relationship-based system to a market-based system when the discount factor and the enforcement cost increase as long as the first period relative output of R-producers y_1 does not increase by such a change. [24]

Proposition 3. *Suppose that Assumptions 1 and 2 hold. Let $\zeta' > \zeta''$. Then, $T(\zeta') \leq T(\zeta'')$ if $y_1(\zeta') \leq y_1(\zeta'')$.*

4.3 Long-Run Behaviour

We have shown that the economy switches from the development stages which rely more on the relationship-based system to the development stages which rely more on the market-based system. We have also shown the time structure of relational contracting such that its value relative to an arm's length contract declines over time until the relative output of the R-producer y_t hits some critical value $\hat{y} < 1$. However, Proposition 1 does not say anything about how the relative output y_t behaves over time after it goes below the critical value $\hat{y}$.

To investigate the long-run behaviour of equilibrium dynamics, we define the steady state of the economy, denoted by $(\tilde{y}, \tilde{x})$, as $y_{t+1} = y_t = \tilde{y}$ and $x_{t+1} = x_t = \tilde{x}$ in (9) and

[24] Recall that there is a freedom to choose the relative output y_1 in the first period $t=1$ for determining an equilibrium path. By such indeterminacy, we cannot know how y_1 changes with ζ. If y_1 increases with ζ, it may be possible that we have $T(\zeta') > T(\zeta'')$ even when $\zeta' > \zeta''$.

(10) which satisfy

$$\delta\{\tilde{y}^{\alpha} - \lambda\alpha\tilde{y} - (1-\alpha)\} = \lambda\alpha\tilde{y}, \tag{15}$$

$$\lambda(1-\alpha)A\tilde{x}^{\alpha}(l\tilde{y}^{\alpha} + (1-l)) = \tilde{x}(\lambda l\tilde{y} + (1-l)). \tag{16}$$

Recall that our model economy is characterized by a two-dimensional system of non-linear difference equations (9) and (10). Then, we show that the economy eventually reaches the steady state where each A-producer invests in more capital and hence produces more than each R-producer does.

Proposition 4. *Suppose that Assumptions 1 and 2 hold. Then, (i) the steady sate of the economy* $(\tilde{x}, \tilde{y})$ *exists and is unique. (ii) The economy eventually converges to the steady state* $(\tilde{y}, \tilde{x})$ *in the long run where* $\tilde{y} < 1$.

Proposition 4 states that the relationship-based system declines in the long run as the economy develops well, although it contributes to economic development more than the market-based system does in the early development stages.

We can also conduct a comparative statics exercise about the steady state of the economy $(\tilde{y}, \tilde{x})$.

Corollary. *Suppose that Assumption 2 holds. Then, the relative output of R-producer* $\tilde{y}$ *in the steady state is decreasing in the discount factor* δ *and the enforcement cost* $1-\lambda$*. Suppose also that* λ *is so small that* $\lambda \leq \alpha(1-l)/(1-\alpha)$*. Then, the output of A-producer* $\tilde{x}$ *is also decreasing in* δ *and* $1-\lambda$.

When individuals become more altruistic about their children (δ increases), the net gain of relational contracting (the left-hand side of (15)) increases as well. However, since IC_t^* always binds in the steady state, the relative output of a R-producer $\tilde{y}$ must then go down in order to make IC_t^* binding. Furthermore, the decrease in the relative output of a R-producer reduces not only the credit demand (the right-hand side of (16)) but also saving (the left-hand side of (16)) because the wage income becomes lower. When the enforcement cost $1-\lambda$ is large (λ is small), the former effect is dominated by the latter effect so that the net credit supply decreases and thus the output of an A-producer $\tilde{x}$ becomes smaller as well. The comparative statics result about the change in the enforcement cost λ is similarly explained as well.

5 Formation of New Relationships

5.1 Matching and Separation

We have thus far focused on the case where the measure of those who engage in relational contract, denoted by $l \in (0, 1)$, is exogenously fixed and does not change over time. By restricting to such a case, we have investigated how the investment (output) levels of A-producers and R-producers change over time. Put differently, our analysis has been limited to the changes in the 'extensive' margin of relational and arm's length contracts by assuming that its 'intensive' margin, which is the relative measures of those who engage in these contracting modes, is exogenous.

In this section, we will extend our basic model to allow producers and lenders to freely find their partners and form relationship pairs. Then, we will investigate the changes in both the intensive and extensive margins of the two contracting modes together.

For this purpose, we introduce two twists into the basic model as follows. First, we abandon the assumption that there is a local community where particular producers and lenders have personal relationships. Instead, we allow every individual, irrespective of a producer or a lender, to enter the matching market for seeking a partner and forming a relationship pair. The producers and lenders who enter the matching market are randomly matched with each other (we will specify the matching function below). Second, each relationship pair is resolved with exogenous probability, in which case the separated producer and lender enter the matching market for meeting another partners.

More specifically, we denote by l_t^s the measure of relationship pairs of young R-producers and young R-lenders whose predecessors have started the relationships from period s and are not still dissolved in the beginning of period $t \geq s$. Each of those l_t^s relationship pairs will continue to be matched and carried over the next generation in period $t+1$ with probability $\gamma_t \in (0,1)$, while the relationship will be dissolved with probability $1-\gamma_t$. We also introduce an uncertainty into the exogenous separation probability $1-\gamma_t \in (0,1)$ such that the set of possible separation shocks is given by $\Gamma \equiv \{\gamma^1, ..., \gamma^n\}$ and $\gamma_t = \gamma^i$ is realized with probability $q^i \in (0,1)$ where $\sum_{i=n} q^i = 1$ and $\gamma^1 < \cdots < \gamma^n \equiv 1$ (here we set $\gamma^n = 1$ without loss of generality). Such uncertainty allows us to generate an endogenous process that relationship pairs can be heterogeneous with respect to the exogenous separation shock $\gamma_t \in \Gamma$ they face when making relational contracts. Then, we can address the dynamic issue of how each relationship switches from engaging in relational contract to arm's length contract in some periods although it relies on the former in other periods.[25]

We assume that a young R-producer and a young R-lender of relationship pair, born in period $t-1$, know how much likely is their relationship to continue to their children in the next period t, that is, they know the realization of γ_t before making a capital investment decision. Thus a relational contract in period t can be contingent on the realization of the exogenous separation shock γ_t. However, we assume that they do not know how likely is it that the relationship in the next generation will be resolved, that is, they do not know the exogenous separation shock γ_{t+1} such that the relationship of their children will be resolved.

The events in a period proceeds as follows (see Figure 4 for the time line). When a

[25]Without any such uncertainty about γ_t, every formed relationship pair engages in either relational contract or arm's length contract from the outset when the relationship starts. Then we cannot address the dynamic issue of how the relationships engaging in relational contracts endogenously switch to arm's length contracts over time.

relationship was not dissolved in the previous period and the separation shock of γ_t was observed to them, a young R-producer and a young R-lender of the relationship, born in period $t-1$, have the following options: one is to continue the relationship by implementing the agreed upon relational contract and the other is to go to the anonymous credit market for earning the profits of arm's length contract without dissolving the relationship.[26] When a R-producer and a R-lender become old, they have the following options: one is to go to the matching market for seeking another partners for their children (the details of the matching market will be given below) and the other is to keep the relationship carried over the next generation. All the relationship pairs of old R-producers and old R-lenders who were fully separated have nothing but to go to the matching market as well as all old A-producers and old A-lenders do so. Then, if a match between an old producer and an old lender in the matching market is successful, their children will start new relationships. Otherwise, their children must engage in arm's length contract in the anonymous credit market.

The matching market is modelled as follows: when there are $l_{p,t}$ old producers and $l_{l,t}$ old lenders in the matching market for seeking relationship partners, we assume that they are randomly matched with each other according to the matching function $M(l_{p,t}, l_{l,t})$ where M exhibits the constant returns to scale. Since every existing relationship is dissolved exogenously with the same average probability $\mu \equiv \sum_i q^i \gamma^i$,[27] $l_{p,t} = l_{l,t}$ always holds in any period t in any equilibrium. Thus, each old producer (respectively, lender) is matched with an old lender (respectively, producer) in the matching market with probability $M(l_{p,t}, l_{l,t})/l_{i,t} = m \equiv M(1,1)$ for $i = p, l$.

We denote by $\{z_t^s(\gamma_t), R_t^s(\gamma_t), d_t^s(\gamma_t)\}_{t \geq s}$ a sequence of relational contracts from period t onward, which is agreed by a relationship pair matched in period s (see Figure 5 for the time line). Here, capital investment $z_t^s(\gamma_t)$ and repayment $R_t^s(\gamma_t)$ depend on the realization of the separation shock $\gamma_t \in \Gamma$ because, by the above assumption, the young R-producer and the young R-lender, born in period $t-1$, know the separation shock γ_t in period t before they make a capital investment decision in period $t-1$. $d_t^s(\gamma_t) \in \{0, 1\}$ is the indicator function which takes one only when the young relationship pair decides to engage in a relational contract instead of going to the outside credit market after having observed the separation shock γ_t.

5.2 Equilibrium with Formation of New Relationships

Given the above modification of the basic model, we show that relational contracting contributes to economic development in the early stages but it declines in the later stages in terms of not only its extensive margin but also its intensive margin: each

[26] There is also the third option that the young R-producer and young R-lender fully dissolve the relationship and engage in arm's length contract in the anonymous credit market. However, we can show that this case never happens on equilibrium path because they find it optimal to continue the relationship rather than voluntarily dissolve it.

[27] Also all relationships are never voluntarily dissolved in any equilibrium as we have noted in footnote 26.

R-producer invests less than each A-producer does over time as well as the measure of the relationships who engage in relational contracts becomes smaller over time.

In this subsection we will give an informal and intuitive argument for this result (see Appendix for more detailed analysis).

The fraction of the relationship pairs $\sum_{s=0}^{t}\sum_{i=1}^{n} q^i d_t^s(\gamma^i) l_t^s$ that engage in relational contract rather than go to the outside credit market changes over time because the decision to implement a relational contract $d_t^s(\gamma) \in \{0,1\}$ is an endogenous object. Then we will see how such implementation decisions $\{d_t^s(\gamma)\}_{t\geq s}$ evolve over time by modifying the incentive compatibility condition as follows: let J_t^s denote the sum of expected profits of an old R-producer and an old R-lender in period t whose predecessors have started the relationship from period s. If their parents decided to go to the matching market instead of continuing the relationship in period $t-1$, they would obtain the sum of the expected profits in period t as $mJ_t^t+(1-m)(\lambda r_t w_{t-1}+\pi_t)$ because in the matching market each of their parents finds a different partner with matching probability m in which case they would obtain the sum of expected profits J_t^t by starting a new relationship formed in period t while each of their parents fails to match a new partner with probability $1-m$ in which case they would obtain the sum of profits $\lambda r_t w_{t-1} + \pi_t$ by engaging in arm's length contract in the credit market. Thus, the net gain from relational contracting in period $t \geq s$ becomes $J_t^s - mJ_t^t - (1-m)(\lambda r_t w_{t-1} + \pi_t)$ for the relationship that has lasted from period s.

Then, we can show that the previous IC constraint IC_t^* is replaced by the new one, called A-$\text{IC}_t^s(\gamma)$, for a relationship that has lasted from period s and faces the exogenous separation shock $\gamma \in \Gamma$ in period $t \geq s$ as follows:

$$\gamma\delta\{J_{t+1}^s - mJ_{t+1}^{t+1} - (1-m)(\pi_{t+1} + \lambda r_{t+1} w_t)\} \geq \lambda r_t z_t^s(\gamma). \qquad (\text{A-IC}_t^s(\gamma))$$

As in the previous IC, the gain from relational contracting, captured by the left hand side of the above inequality, must be at least as large as the opportunity cost of investing in capital $\lambda r_t z_t^s(\gamma)$ under relational contract, captured by its right hand side. The main difference from the original IC_t^* is here that the net gain from honouring the relational contract (the left-hand side of A-$\text{IC}_t^s(\gamma)$) is multiplied by the probability of the relationship being continued $\gamma \in \Gamma$ and the probability of meeting a new partner in the matching market m. Since the relationship is resolved with exogenous probability $1-\gamma$, the net gain from relational contracting is realized with probability γ. Further, since the R-producer and R-lender could obtain the sum of expected profits $mJ_{t+1}^{t+1} + (1-m)(\pi_{t+1}+\lambda r_{t+1} w_t)$ even if they separated from each other, such outside gain should be subtracted from the net surplus the relationship pair can extract from the current match.

Here, the net gain from relational contracting $J_{t+1}^s - mJ_{t+1}^{t+1} - (1-m)(\lambda r_{t+1} w_t + \pi_{t+1})$ depends on how many separation shocks $\gamma_{t+1} \in \Gamma$ will result in the decision to implement the agreed upon relational contract in period $t+1$. When many separation shocks cause the future decisions to go to the outside credit market, i.e., $d_{t+1}^s(\gamma) = 0$ for many $\gamma \in \Gamma$, such gain from relational contracting becomes so small that it becomes difficult to enforce the agreed upon relational contract in the current period.

Also, as we have seen in the basic model, since the interest rates fall over time as the outside credit market expands, the gain from relational contracting tends to be small during the development process. By combining these effects, A-IC$_t^s(\gamma)$ becomes more stringent for lower γ as the economy proceeds to develop well over time. Eventually, the relationships facing more severe separation shocks (lower γ) decide not to implement relational contract but to switch to arm's length contract in the credit market in the later stages of development.

More formally, we can show the following result.

Proposition 5. *Suppose that $q^n > m$. Then, there exist some $\hat{y} \in (0, 1/\lambda)$ and $\overline{\lambda} \in (0,1)$ such that, for all $\lambda \in (0, \overline{\lambda})$, in any equilibrium path the relational contract agreed by each relationship pair formed in period $s \geq 0$ has the following dynamic pattern:*

(i) Intensive Margin: there exists a sequence of the cut-off periods $\{T^s(\gamma^i)\}_{i=1}^k$ for some k ($1 \leq k \leq n$) such that $d_t^s(\gamma^i) = 0$ for all $t \geq T^s(\gamma^i)$ where $T^s(\gamma^1) \leq \cdots \leq T^s(\gamma^k)$.

(ii) Extensive Margin: the relative output of R-producer, defined by $y_t^s(\gamma^i) \equiv z_t^s(\gamma^i)/x_t$ for $i \leq k$, decreases over time as long as the relationship chooses a higher relative output $y_t^s(\gamma^i)$ than the cut-off value $\hat{y}$, i.e., as long as $y_t^s(\gamma^i) > \hat{y}$.

Proposition 5 shows that relational contracting contributes to economic development in the early stages but its role declines over time in both intensive and extensive margins: the relationships that face a separation shock γ^i switch to engage in arm's length contract after the cut-off period $T^s(\gamma^i)$ and such switching becomes more early for the relationships that face more severe separation shocks (i.e., the cut-off period $T^s(\gamma)$ decreases with γ). Also, in the development stages in which each R-producer invests more than an A-producer, its relative output declines over time as shown in Proposition 1.

6 Conclusion

In this paper, we have investigated a dynamic general equilibrium model that takes into account the dynamic change in the contract enforcement modes from relational contracts to arm's length contracts over time. We have shown that relational contracting plays an important role in sustaining production in the early stages of the development process in which arm's length contracting does not function well to support capital investment. In subsequent periods, producers find it profitable to use arm's length contracts because the economy is so well-developed that the market size is large and the interest rate falls. Thus, as the economy enters its mature stages, relationship-based systems decline and may be partially replaced by market-based systems. We have focused on relational contracting between borrowers and lenders, which becomes valuable for relating our theoretical results to the historical evidence on relationship-based financing. This is one of the modelling choices that capture relational contracting in dynamic general equilibrium frameworks. It is important to investigate how the development process is dynamically linked with long-term relationships in different contexts such as firm-worker relationships, inter-firm relationships, and government-public relationships.

We conclude the paper by briefly discussing the role of bequest transfers between successive generations. In the main text, we have assumed that each old individual has no technology to give his or her child bequest. One might think that old individuals will bequeath their children when bequest is available because they care about the consumption levels of their children. The possibility of bequest allows each producer to finance a part of capital investment from the bequest he has received from his parent. Thus, A-producers can reduce the enforcement cost and relationship pairs are more likely to satisfy IC_t^* by using bequest to finance capital investment. However, we can show that no old individuals bequeath to their children at all, as in our basic setting, as long as the altruistic parameter value δ belongs to a certain small range (a more detailed analysis is relegated to the Online Appendix).

7 Appendix: Proofs for Lemmas and Propositions

In this section we will give the formal proofs for the lemmas and propositions presented in the main text.

Proof of Lemma 1.

(i) For a relationship pair born in period $t-1$ to be inherited by the next generation, IC_s^* and TS_s must be satisfied for all $s \geq t$. Otherwise, $\text{IC}_{\tilde{t}}^*$ or $\text{TS}_{\tilde{t}}$ is violated in some period $\tilde{t} \geq t$, which implies that at least one of $\text{IC}_{\tilde{t}}$, $\text{IRL}_{\tilde{t}}$ and $\text{IRP}_{\tilde{t}}$ is violated so that the relationship pair born in period $\tilde{t}-1$ is dissolved. By anticipating this and using the backward induction argument, every relationship pair born in any period before $\tilde{t}-2$ would not have the incentive to maintain the relationship. On the other hand, if both IC_s^* and TS_s are satisfied for all $s \geq t$, then we can show that there exists a subgame perfect equilibrium in which every relationship pair born in period $s \geq t-1$ honours the relational contract $\{z_s, R_s\}$, which satisfies all IC_s, IRP_s, and IRL_s for all $s \geq t$. To see this, assume that IC_s^* and TS_s hold for all $s \geq t$. Then, we can find a sequence of relational contracts $\{z_s, R_s\}_{s=t}^{\infty}$ such that all IC_s, IRP_s, and IRL_s are satisfied.[28]

Take a sequence of relational contracts $\{z_s, R_s\}_{s=t}^{\infty}$ which satisfies IC_s, IRP_s and IRL_s for all $s \geq t$. Then, we can show that the following strategies played by R-producers and R-lenders can constitute a subgame perfect equilibrium in a continuation

[28]For example, we can set $R_s = \lambda r_s z_s$ in every period $s \geq t \geq 1$ and $R_0 \in (0, \pi_0)$, which satisfy all IC_s, IRP_s, and IRL_s with some z_s in every period $s \geq t \geq 0$. Also, note that IC_t^* implies that $p_t z_t > \lambda r_t z_t$ for $t \geq 2$ and TS_0 implies that $p_1 z_1 > \lambda r_1 z_1$. Thus we can find R_t to ensure non-negative consumption NNC_t that $p_t z_t \geq R_t \geq -\lambda r_t (w_{t-1} - z_t)$ for all $t \geq 1$ as well as $\pi_0 \geq R_0 \geq 0$.

equilibrium from period t onward:

- Period $s \geq t$:
  - The young R-producer and young R-lender of each relationship agree to the relational contract $\{z_{s+1}, R_{s+1}\}$[29] if their parents agreed and honoured the relational contract $\{z_s, R_s\}$ in the previous period. Otherwise, they simultaneously exercise the quitting option.
- Period $s + 1$:
  - The old R-producer honours to make the repayment R_{s+1} and does not exercise the quitting option, provided his and the R-lender's parents did not exercise the quitting option in the previous period.
  - The old R-lender does not exercise the quitting option, provided her and the R-producer's parents did not exercise the quitting option in the previous period.

The above strategies specify the trigger strategy-like feature as follows: if a young R-producer and a young R-lender agree to the relational contract $\{z_{s+1}, R_{s+1}\}$ in period s and they actually behave according to that contract in period $s + 1$, their children will decide to continue the relationship in period $s + 1$ and follow the agreement of the relational contract $\{z_{s+2}, R_{s+2}\}$ in the next period $s + 2$. Suppose that this is not the case. That is, they agree to a different contract $\{z'_{s+1}, R'_{s+1}\} \neq \{z_{s+1}, R_{s+1}\}$ or they agree to the specified contract but some of them does not implement it (e.g., the R-producer reneges on the repayment R_{s+1}) in period $s+1$. If this were the case, their children would play a continuation equilibrium in which they simultaneously decide to exercise the quitting option in period $s + 1$. Here, note that we always have a continuation equilibrium in which any R-producer and any R-lender of any relationship pair simultaneously exercise the quitting option because the relationship pair is resolved as long as at least one partner decides to exercise the quitting option.

Then, because IC_s, IRL_s, and IRP_s are satisfied in any period $s \geq t$, the R-producer and R-lender never make any profitable deviation from agreeing the specified relational contract $\{z_{s+1}, R_{s+1}\}$ and implementing it, ensuring that the relationship is inherited by the next generation.

(ii) First, the optimal relational contract should solve the following problem:

$$\max_{\{z_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} J_t$$

subject to IC_t^* and TS_{t-1}, $t = 1, 2, ...$ Here, IC_t^*, $t \geq 1$, implies that TS_t holds for all $t \geq 2$. Also, TS_1 is satisfied when TS_0 and IC_1^* hold. Thus only the relevant TS_t is TS_0.

[29]For example, we can suppose that a young R-producer offers $\{z_{s+1}, R_{t+1}\}$ to the young R-lender matching him.

This shows that the optimal relational contract should maximize $\sum_{t=0}^{\infty} \beta^t J_t$ subject to IC_t^* for $t \geq 1$ and TS_0.

Second, note that $\hat{\lambda}x(r_t)$ maximizes $\alpha A z_t^\alpha - \lambda r_t z_t$ over $z_t \geq 0$. Suppose now that $z_t > \hat{\lambda}x(r_t)$ in some period t. Then, if we can slightly decrease z_t, we can still keep IC_t^*, IC_{t-1}^* and TS_0. Note that z_t affects only IC_t^*, IC_{t-1}^* and possibly TS_0. The slight decrease of z_t makes IC_t^* easier to be satisfied while it increases the left hand side of IC_{t-1}^* and TS_0 (when considering the choice of z_1) because $\hat{\lambda}x(r_t)$ maximizes the left hand side of IC_{t-1}^* and TS_0 (when considering z_1). But then such change increases the joint profit $J_t = \alpha A z_t - \lambda r_t z_t + \lambda r_t w_{t-1}$ due to the definition of $\hat{\lambda}x(r_t)$. Thus $z_t \leq \hat{\lambda}x(r_t)$ holds.

(iii) Suppose that $z_t < \hat{\lambda}x(r_t)$ but IC_t^* is not binding in some period t. Then, if we can slightly increase z_t toward $\hat{\lambda}x(r_t)$, we can still keep IC_t^* and IC_{t-1}^*. The slight increase of z_t does not violate IC_t^* while it increases the left hand side of IC_{t-1}^* because of $z_t < \hat{\lambda}x(r_t)$. This does not violate TS_0 as well. But then such change increases the joint profit J_t due to the definition of $\hat{\lambda}x(r_t)$. Q.E.D.

Proof of Lemma 2.

Suppose contrary to the claim that $z_s > w_{s-1}$ holds in some period s in some equilibrium path. In such equilibrium each R-producer must finance the remaining capital investment $z_s - w_{s-1} > 0$ from the credit market after he borrows w_{s-1} from the R-lender matching him.

There are two cases: (i) $z_{s-1} \leq w_{s-2}$ and (ii) $z_{s-1} > w_{s-2}$.

Case (i): $z_{s-1} \leq w_{s-2}$. In case (i), the R-producer in period $s-2$ does not finance from the credit market. Thus, in case (i), IC_{s-1} for R-producer and IRL_{s-1} for R-lender should be modified to

$$p_{s-1}z_{s-1} - R_{s-1} + \delta\{p_s z_s - R_s - r_s(z_s - w_{s-1})\} \geq p_{s-1}z_{s-1} + \delta\pi_s \qquad (\text{IC}'_{s-1})$$

and

$$R_{s-1} + \lambda r_{s-1}(w_{s-2} - z_{s-1}) + \delta R_s \geq \lambda r_{s-1}w_{s-2} + \delta\lambda r_s w_{s-1} \qquad (\text{IRL}'_{s-1})$$

Combining these conditions, IC_{s-1}^* must be changed to

$$\delta\{\alpha A z_s^\alpha - r_s z_s - \pi_s + (1-\lambda)r_s w_{s-1}\} \geq \lambda r_{s-1}z_{s-1}. \qquad (\text{IC}_{s-1}^*\text{-(i)})$$

Also, in case (i) the joint profit in period s is given by $J_s = \alpha A z_s^\alpha - r_s z_s + r_s w_{s-1}$ because capital investment $z_s - w_{s-1} > 0$ must be financed from the credit market.

Case (ii): $z_{s-1} > w_{s-2}$. In case (ii), the R-producer in period $s-2$ finances the remaining amount $z_{s-1} - w_{s-2}$ from the credit market after borrowing w_{s-2} directly from the R-lender. Thus, in case (ii), IC_{s-1} for R-producer and IRL_{s-1} for R-lender should be modified to

$$\begin{aligned} &p_{s-1}z_{s-1} - R_{s-1} - r_{s-1}(z_{s-1} - w_{s-2}) + \delta\{p_s z_s - R_s - r_s(z_s - w_{s-1})\} \\ &\geq p_{s-1}z_{s-1} - r_{s-1}(z_{s-1} - w_{s-2}) + \delta\pi_s \end{aligned}$$

and

$$R_{s-1} + \delta R_s \geq \lambda r_{s-1} w_{s-2} + \delta \lambda r_s w_{s-1} \qquad (\mathrm{IRL}'_{s-1})$$

Combining these conditions, IC^*_{s-1} must be changed to

$$\delta\{\alpha A z_s^\alpha - r_s z_s - \pi_s + (1-\lambda) r_s w_{s-1}\} \geq \lambda r_{s-1} w_{s-2}. \qquad (\mathrm{IC}^*_{s-1}\text{-(ii)})$$

In case (ii) the joint profit in period s is given by $J_s = \alpha A z_s^\alpha - r_s z_s + r_s w_{s-1}$ again.

In both cases (i) and (ii) the credit market equilibrium in period $s-1$ is modified as

$$(1-l)\lambda w_{s-1} = l(z_s - w_{s-1}) + (1-l)x_s \qquad (\mathrm{CME}')$$

because l R-producers require the capital demand $z_s - w_{s-1} > 0$ each in the credit market in addition to the credit demand x_s of A-producers each while $(1-l)$ A-lenders supply the capital w_{s-1} each. Then CME′ is written by $(l+(1-l)\lambda)w_{s-1} = lz_s + (1-l)x_s$. Since $z_s > w_{s-1}$ by our supposition, this yields $(l+(1-l)\lambda)z_s > (l+(1-l)\lambda)w_{s-1} = lz_s + (1-l)x_s$, which in turn implies that $z_s > \lambda z_s > x_s$.

However, the joint profit of the relationship pair in period s is given by $J_s = \alpha A z_s^\alpha - r_s z_s + r_s w_{s-1}$ which is maximized at $x_s = x(r_s)$. Then J_s can increase by reducing z_s slightly because $z_s > x_s = x(r_s)$. Since $\mathrm{IC}^*_{s-1}-$(i), $\mathrm{IC}^*_{s-1}-$(ii) and TS_0 are more likely to be satisfied when z_s decreases toward $x(r_s)$, the original relational contract cannot be optimal as long as the incentive compatibility constraint in period s is not violated when z_s is slightly decreased.

We verify this last point.

Case (1): $z_{s+1} \leq w_s$. In this case, by using a similar argument and noting that $z_s > w_{s-1}$ by our supposition, we can show that the incentive compatibility constraint in period s is modified as

$$\delta\{\alpha A z_{s+1}^\alpha - \lambda r_{s+1} z_{s+1} - \pi_{s+1}\} \geq \lambda r_s w_{s-1}$$

Case (2): $z_{s+1} > w_s$. In this case we modify the incentive compatibility constraint in period $s+1$ as

$$\delta\{\alpha A z_{s+1}^\alpha - r_{s+1} z_{s+1} - \pi_{s+1} + (1-\lambda) r_{s+1} w_s\} \geq \lambda r_s w_{s-1}$$

Either case (1) or (2) above is not affected by z_s. Thus a slight decrease in z_s does not violate these constraints. Thus we prove that $z_s > w_{s-1}$ never happens in any period in any equilibrium. Q.E.D.

Proof of Lemma 3.

By CME_t we have

$$\lambda l(w_t - z_{t+1}) + \lambda(1-l)w_t = (1-l)x_{t+1}$$

where $w_t \geq z_{t+1}$ due to Lemma 2. Thus, the above equality implies that $\lambda(1-l)w_t \leq (1-l)x_{t+1}$ and hence $\lambda w_t \leq x_{t+1}$. Since $z_{t+1} \leq w_t$ holds by Lemma 2 again, we have $\lambda z_{t+1} \leq \lambda w_t \leq x_{t+1}$ which shows that $x_{t+1} \geq \lambda z_{t+1}$.

Since $\hat{\lambda} > \lambda^{-1}$, we then obtain $\hat{\lambda} x_{t+1} > \lambda^{-1} x_{t+1} \geq z_{t+1}$ which implies that $\hat{\lambda} x_{t+1} > z_{t+1}$. Then ($IC^*_{t+1}$) is binding due to Lemma 1. Q.E.D.

Proof of Proposition 1.
Suppose that Assumption 1 and 2 are satisfied. By Assumption 1, we know that $x_t \leq \overline{x}$ holds in any period t in any equilibrium path.

By using (10) (i.e., CME_t) in the main text, we obtain

$$\begin{aligned} \left(\frac{x_{t+1}}{x_t}\right)^{\alpha} &= x_{t+1}^{\alpha-1} \frac{\lambda(1-\alpha)A(ly_t^{\alpha} + (1-l))}{\lambda l y_{t+1} + (1-l)} \\ &\geq \overline{x}^{\alpha-1} \frac{\lambda(1-\alpha)A(ly_t^{\alpha} + (1-l))}{\lambda l y_{t+1} + (1-l)} \\ &= \frac{1-l}{l\lambda^{-\alpha} + (1-l)} \frac{ly_t^{\alpha} + (1-l)}{\lambda l y_{t+1} + (1-l)} \\ &\geq \frac{(1-l)^2}{l\lambda^{-\alpha} + (1-l)} \end{aligned}$$

where the first inequality follows from the fact that $x_t \leq \overline{x}$ for all t and the second inequality from $0 \leq y_t \leq \lambda^{-1}$ (Lemma 2) respectively.

Then, by using the above fact, we can write (9) (i.e., IC^*_t) in the main text as

$$\begin{aligned} \lambda \alpha y_t &= \left(\frac{x_{t+1}}{x_t}\right)^{\alpha} \delta\{y_{t+1}^{\alpha} - \lambda \alpha y_{t+1} - (1-\alpha)\} \\ &\geq \frac{(1-l)^2}{l\lambda^{-\alpha} + (1-l)} \delta\{y_{t+1}^{\alpha} - \lambda \alpha y_{t+1} - (1-\alpha)\}. \end{aligned}$$

Now, we will show that the right hand side of this inequality, denoted by $F(y_{t+1})$, is greater than its left hand side $\lambda \alpha y_t$ when $y_{t+1} = y_t = 1$. To see this, define

$$f(y) \equiv \frac{y^{\alpha} - \lambda \alpha y - (1-\alpha)}{y}$$

over $[\underline{y}, 1/\lambda]$ where $\underline{y} > 0$ is defined as y which satisfying $y^{\alpha} - \lambda \alpha y - (1-\alpha) = 0$. In any equilibrium $y_t \geq \underline{y}$ must be satisfied. Then we can show that $f(\underline{y}) = 0$ and $f(\lambda^{-1}) = \lambda^{1-\alpha} - \lambda > 0$. Also we can verify that $f'(y) = (1-\alpha) y^{-2} (1 - y^{\alpha})$ so that $f' = 0$ at $y = 1$ and $f' > (<) 0$ for $y < (>) 1$. Then we can show that $f(y) > f(\lambda^{-1})$ for all $y \in [1, 1/\lambda)$.

Since Assumption 2 ($\delta \geq \tilde{\delta}$) implies that $\delta \left(\frac{(1-l)^2}{(\lambda^{-\alpha} l + (1-l))}\right) f(\lambda^{-1}) > \lambda \alpha$, we have $\delta \left(\frac{(1-l)^2}{(\lambda^{-\alpha} l + (1-l))}\right) f(y) > \delta \left(\frac{(1-l)^2}{(\lambda^{-\alpha} l + (1-l))}\right) f(\lambda^{-1}) > \lambda \alpha$ for all $y \in [1, 1/\lambda)$ so that $F(y) > \lambda \alpha y$ for all $y \in [1, 1/\lambda)$. Then, if the economy starts with $y_1 > 1$, any equilibrium path $\{y_t\}_{t=1}^{\infty}$ which must satisfy $F(y_{t+1}) \leq \lambda \alpha y_t$ decreases over time until it hits $\hat{y} < 1$ such that $F(\hat{y}) = \lambda \alpha \hat{y}$. Here, such $\hat{y}$ is unique and $\hat{y} < 1$ holds due to $\delta \geq \tilde{\delta}$. Also, since y_t monotonically decreases over time until it hits $\hat{y} < 1$, there exists a switching period T

such that $y_t > 1$ for all $t < T$ while $y_t < 1$ for all $t > T$.

Uniqueness: finally, we show that the switching period T defined above is unique once we fix the relative output of R-producer y_1 in the first period ($t = 1$). To see this, we show that equilibrium path $\{y_t\}_{t=1}^{\infty}$ of the relative output is uniquely determined for a given y_1. Then, since any equilibrium path $\{y_t\}_{t=1}^{\infty}$ is decreasing over time when $y_t < \hat{y}$, the switching period T must be unique for a given y_1.

Fix a (x_1, y_1) which satisfies CME$_0$: $\lambda w_0 = x_1(\lambda l y_1 + (1-l))$. By using (9), we obtain

$$\left(\frac{x_{t+1}}{x_t}\right)\delta^{1/\alpha}[y_{t+1}^{\alpha} - \lambda\alpha y_{t+1} - (1-\alpha)]^{1/\alpha} = (\lambda\alpha)^{1/\alpha} y_t^{1/\alpha}. \tag{A1}$$

Also we can re-write (10) as

$$\frac{x_{t+1}}{x_t} = \lambda(1-\alpha)A\frac{l y_t^{\alpha} + (1-l)}{\lambda l y_{t+1} + (1-l)} x_t^{\alpha-1}$$

which we substitute into (x_{t+1}/x_t) of (A1) in order to obtain

$$\lambda^{1-1/\alpha}(1-\alpha)A\delta^{1/\alpha}\frac{[y_{t+1}^{\alpha} - \lambda\alpha y_{t+1} - (1-\alpha)]^{1/\alpha}}{\lambda l y_{t+1} + (1-l)} = \frac{\alpha^{1/\alpha} x_t^{1-\alpha}}{l y_t^{\alpha-1/\alpha} + (1-l) y_t^{-1/\alpha}}. \tag{A2}$$

We denote by $g(y_{t+1}; \delta)$ the left hand side of (A2) and by $\psi(y_t, x_t)$ the right hand side of (A2) respectively. Then we can show that

$$\begin{aligned} \frac{\partial g}{\partial y_{t+1}} &\propto (1-l)(y_t^{\alpha-1} - \lambda) + \lambda(1-\alpha)l(1-\lambda y_t) \\ &> 0 \end{aligned}$$

due to $y_t \leq \hat{\lambda} \equiv \lambda^{1/(\alpha-1)}$.

By using CME$_{t-1}$, i.e., $\lambda w_{t-1} = x_t(\lambda l y_t + (1-l))$, and substituting $x_t = \lambda w_{t-1}/(\lambda l y_t + (1-l))$ into $\psi(y_t, x_t)$, we obtain

$$\tilde{\psi}(y_t, w_{t-1}) \equiv \frac{\alpha^{1/\alpha}(\lambda w_{t-1})^{1-\alpha}}{(l y_t^{\alpha-1/\alpha} + (1-l) y_t^{-1/\alpha})(\lambda l y_t + (1-l))^{1-\alpha}}$$

where we can verify that $\tilde{\psi}$ is increasing in y_t and w_{t-1}.

Also, by using LME$_t$ and CME$_t$, we also obtain

$$\begin{aligned} w_t &= \lambda(1-\alpha)A x_t^{\alpha}(l y_t^{\alpha} + (1-l)) \\ &= \lambda(1-\alpha)A\left(\frac{\lambda w_{t-1}}{\lambda l y_t + (1-l)}\right)^{\alpha}(l y_t^{\alpha} + (1-l)) \\ &\equiv \tilde{w}(y_t, w_{t-1}) \end{aligned}$$

Here we can veriy that $\tilde{w}$ is increasing in y_t:

$$\frac{\partial \tilde{w}}{\partial y_t} \propto (1-l)l\alpha(y_t^{\alpha-1} - \lambda) > 0$$

due to $y_t < \hat{\lambda}$. $\tilde{w}$ is also increasing in w_{t-1} as well. From this, we can show the following: define $w_1 = \hat{w}(y_1) \equiv \tilde{w}(y_1, w_0)$ which is uniquely determined once we fix y_1 because w_0 is exogenously given. Next we define $w_2 = \hat{w}(y_2, y_1) \equiv \tilde{w}(y_2, w_1) = w(y_2, \hat{w}(y_1))$ where $\hat{w}(y_2, y_1)$ is increasing in both y_1 and y_2 because $\tilde{w}(y_2, w_1)$ is increasing in y_2 and w_1 as well as $w_1 = \hat{w}(y_1)$ is increasing in y_1. Thus $\hat{w}(y_2, y_1)$ is uniquely determined once we fix y_2 and y_1. Then we can define $w_t = \hat{w}(y_t, y_{t-1}, ..., y_1)$ in the similar way where $\hat{w}$ is increasing in each argument.

By substituting $w_{t-1} = \hat{w}(y_{t-1}, y_{t-2}, ..., y_1)$ into $\tilde{\psi}(y_t, w_{t-1})$, we have

$$\phi(y_t, y_{t-1}, ..., y_1) \equiv \tilde{\psi}(y_t, \hat{w}(y_{t-1}, ..., y_1))$$

which is increasing in each argument because $\tilde{\psi}$ is increasing. Then the equilibrium path $\{y_t\}_{t=1}^{\infty}$ recursively satisfies $g(y_{t+1}; \delta) = \phi(y_t, ..., y_1)$ for a given y_1. Since $g(y_2; \delta)$ is increasing with y_2 and $g(\underline{y}; \delta) = 0$, the equilibrium value of y_2 which satisfies $g(y_2; \delta) = \phi(y_1)$ must be unique for a given y_1. Also, the equilibrium value of y_3 which satisfies $g(y_3; \delta) = \phi(y_2, y_1)$ must be also unique. Repeating this, it must be that y_{t+1} which satisfies $g(y_{t+1}; \delta) = \phi(y_t, ..., y_1)$ must be unique for a given y_1. Q.E.D.

Proof of Proposition 2.
In the proof of Proposition 1 we have shown that an equilibrium path $\{y_t\}_{t=1}^{\infty}$ is determined by solving $g(y_{t+1}; \delta) = \phi(y_t, y_{t-1}, ..., y_1)$ recursively. Here recall that ϕ is increasing in each argument and g is increasing in y_{t+1}. Take any two equilibrium paths $\{y'_t\}_{t=1}^{\infty}$ and $\{y''_t\}_{t=1}^{\infty}$ with $y'_1 \geq y''_1$. Then, since $g(y_2; \delta) = \phi(y_1)$ holds, we have $y'_2 \geq y''_2$ when $y'_1 \geq y''_1$. Also, since $g(y_3; \delta) = \phi(y_2, y_1)$ and $y'_t \geq y''_t$ for $t = 1, 2$, we have $y'_3 \geq y''_3$. Repeating this, we can see from $g(y_{t+1}; \delta) = \phi(y_t, y_{t-1}, ..., y_1)$ that $y'_{t+1} \geq y''_{t+1}$ because $y'_s \geq y''_s$ for all $s \leq t-1$.

Since $y'_t \geq y''_t$ holds in any period t and these equilibrium paths decrease over time until they hit the same critical value $\hat{y} < 1$, the corresponding switching periods T' and T'' must have the property that $T' \geq T''$. Q.E.D.

Proof of Proposition 3.
First, suppose that $\delta' > \delta''$ for a given λ. Then, let $\zeta' \equiv (\delta', 1-\lambda) > \zeta'' \equiv (\delta'', 1-\lambda)$. Note that $g(y; \delta)$ which was defined in the proof of Proposition 1 is increasing in both y and δ. Thus, since $g(y_2(\zeta); \delta) = \phi(y_1(\zeta))$ and ϕ is increasing, we have $y_2(\zeta') \leq y_2(\zeta'')$ if $y_1(\zeta') \leq y_1(\zeta'')$. In the recursive way we can show that $y_{t+1}(\zeta') \leq y_{t+1}(\zeta'')$ holds for any $t \geq 1$ because $g(y_{t+1}(\zeta); \delta) = \phi(y_t(\zeta), ..., y_1(\zeta))$ holds, ϕ is increasing in each argument and $y_s(\zeta') \leq y_s(\zeta'')$ for all $s \leq t$.

Next, suppose that $\lambda' < \lambda''$ for a given δ. Then, let $\zeta' \equiv (\delta, 1-\lambda') > \zeta'' \equiv (\delta, 1-\lambda'')$. We verify that $\tilde{\psi}$ is increasing in λ while g is decreasing in λ. We can also show that $\hat{w}(y_t, ..., y_1)$ is increasing in λ for all $t \geq 1$. Thus ϕ is increasing in λ as well. Since g is decreasing and ϕ is increasing in λ, $y_t(\zeta') \leq y_t(\zeta'')$ holds for all $t \geq 1$ when $y_1(\zeta') \leq y_1(\zeta'')$. Q.E.D.

Proof of Proposition 4.

We first show that there exists a unique steady state. For this purpose, it suffices to show that there exists a unique $\tilde{y}$ which satisfies (15) because then there exists a unique $\tilde{x}$ satisfying (16). Let $D(y) \equiv \delta\{y^\alpha - \lambda\alpha y - (1-\alpha)\} - \lambda\alpha y$ over the domain $y \in [\underline{y}, 1/\lambda]$. Then we can show that $D(\underline{y}) = -\lambda\alpha\underline{y} < 0$ and that $D(1) = \alpha(\delta(1-\lambda) - \lambda) > 0$ due to Assumption 2. Also $D(\lambda^{-1}) = \delta(\lambda^{-1} - 1) - \alpha > 0$ due to Assumption 2 and $D'' = \alpha(\alpha-1)\delta y^{\alpha-2} < 0$. Thus D is strictly concave with $D(\underline{y}) < 0$, $D(1) > 0$ and $D(\lambda^{-1}) > 0$. Then there exists a unique $\tilde{y} \in (\underline{y}, 1/\lambda)$ such that $D(\tilde{y}) = 0$ where $\tilde{y} < 1$ holds.

Next, we show that the economy eventually converges to a unique steady state $(\tilde{y}, \tilde{x})$ defined in (15) and (16) in the main text under Assumption 1 and 2.

We already know from Proposition 1 that $y_t < 1$ holds for all large t. Thus in what follows we suppose that $y_t < 1$ without loss of generality.

By substituting the following CME_t

$$\lambda(1-\alpha)Ax_t^\alpha(ly_t^\alpha + (1-l)) = x_{t+1}(\lambda y_{t+1} + (1-l)) \tag{CME_t}$$

into the following IC_t^*

$$\left(\frac{x_{t+1}}{x_t}\right)^\alpha \delta\{y_{t+1}^\alpha - \lambda\alpha y_{t+1} - (1-\alpha)\} = \lambda\alpha y_t, \tag{IC_t^*}$$

we obtain

$$\lambda(1-\alpha)Ax_{t+1}^{\alpha-1}\frac{\delta\{y_{t+1}^\alpha - \lambda\alpha y_{t+1} - (1-\alpha)\}}{\lambda l y_{t+1} + (1-l)} = \frac{\lambda\alpha y_t}{ly_t^\alpha + (1-l)}. \tag{A3}$$

We define

$$h(y, y'') \equiv \left[\frac{\lambda(1-\alpha)A(ly^\alpha + (1-l))}{\lambda l y'' + (1-l)}\right]^{\frac{1}{1-\alpha}}.$$

From CME_t together with $\underline{y} < y_t < 1$, we have $\lambda(1-\alpha)Ax_t^\alpha(l\underline{y}^\alpha + (1-l)) \leq x_{t+1}(\lambda l + (1-l))$ which implies that x_t is eventually larger than $h(\underline{y}, 1)$:

$$x_t \geq h(\underline{y}, 1) \equiv \left[\frac{\lambda(1-\alpha)A(l\underline{y}^\alpha + (1-l))}{\lambda l + (1-l)}\right]^{\frac{1}{1-\alpha}} \tag{A4}$$

for all large t. Then (A3) implies that for all large t:

$$\lambda(1-\alpha)Ah(\underline{y}, 1)^{\alpha-1}\frac{\delta\{y_{t+1}^\alpha - \lambda\alpha y_{t+1} - (1-\alpha)\}}{\lambda l y_{t+1} + (1-l)} \geq \frac{\lambda\alpha y_t}{ly_t^\alpha + (1-l)}$$

which is equivalent to

$$\left(\frac{\lambda l + (1-l)}{l\underline{y}^\alpha + (1-l)}\right)\frac{\delta\{y_{t+1}^\alpha - \lambda\alpha y_{t+1} - (1-\alpha)\}}{\lambda l y_{t+1} + (1-l)} \geq \frac{\lambda\alpha y_t}{ly_t^\alpha + (1-l)}. \tag{A5}$$

for all large t. Letting $H(y_{t+1})$ denote the left hand side of (A5), we can verify that $H(1) > \lambda\alpha$ that is equivalent to $\delta > (l\underline{y}^\alpha + (1-l))\lambda/(1-\lambda)$ implied by $\delta > \tilde{\delta}$. Also,

$H(\underline{y}) = 0 < \lambda\alpha\underline{y}$. Thus there exists some y, denoted by $y^*(1)$, such that $y^*(1) \in (\underline{y}, 1)$ and $H(y^*(1)) = \lambda\alpha y^*(1)$ (if such $y^*(1)$ are multiple, we take the smallest one). Then (A5) shows that $y_t \geq y^*(1)$ eventually holds for all large enough t.

Next, by using CME_t and $\underline{y} < y_t < 1$, we have

$$\lambda(1-\alpha)Ax_t^{\alpha} \geq x_{t+1}(\lambda l\underline{y} + (1-l))$$

which implies that

$$x_{t+1} \leq h(1, \underline{y}) = \left[\frac{\lambda(1-\alpha)A}{\lambda l\underline{y} + (1-l)}\right]^{\frac{1}{1-\alpha}} \tag{A6}$$

for all large t. By combining this with (A3), we obtain

$$\lambda(1-\alpha)Ah(1,\underline{y})^{\alpha-1}\frac{\delta\{y_{t+1}^{\alpha} - \lambda\alpha y_{t+1} - (1-\alpha)\}}{\lambda l y_{t+1} + (1-l)} \leq \frac{\lambda\alpha y_t}{l y_t^{\alpha} + (1-l)}.$$

which is equivalent to

$$(\lambda l\underline{y} + (1-l))\frac{\delta\{y_{t+1}^{\alpha} - \lambda\alpha y_{t+1} - (1-\alpha)\}}{\lambda l y_{t+1} + (1-l)} \leq \frac{\lambda\alpha y_t}{l y_t^{\alpha} + (1-l)} \tag{A7}$$

for all large t. When we set $y_{t+1} = y_t = 1$, the left hand side of (A7) is equal to $\delta\left(\frac{\alpha(1-\lambda)(\lambda l\underline{y}+(1-l))}{\lambda l+(1-l)}\right)$ which is larger than $\lambda\alpha$ due to $\delta > \tilde{\delta}$. Thus there exists some y, denoted by $y^{**}(1)$, such that $y^{**}(1) \in (\underline{y}, 1)$ and the both sides of (A7) are equal at $y_{t+1} = y_t = y^{**}(1)$ (if such $y^{**}(1)$ are multiple, we take the largest one). Then we can verify from (A7) that $y_t \leq y^{**}(1)$ holds for large enough t.

We have thus established that $y^*(1) \leq y_t \leq y^{**}(1)$ holds for all large enough t.

Next, since $y^*(1) \leq y_t \leq y^{**}(1)$ for all large t, we can verify that $x_t \geq h(y^*(1), y^{**}(1))$ and $x_t \leq h(y^{**}(1), y^*(1))$ for all large t. Then, by using the similar step to (A5), we can show that

$$\frac{\lambda l y^{**}(1) + (1-l)}{l(y^*(1))^{\alpha} + (1-l)}\frac{\delta\{y_{t+1}^{\alpha} - \lambda\alpha y_{t+1} - (1-\alpha)\}}{\lambda l y_{t+1} + (1-l)} \geq \frac{\lambda\alpha y_t}{l y_t^{\alpha} + (1-l)} \tag{A8}$$

for all large t. Here the left hand side of (A8) is less than its right hand side at $y_{t+1} = y_t = y^*(1)$ if and only if

$$\frac{\lambda l y^{**}(1) + (1-l)}{l(y^*(1))^{\alpha} + (1-l)} < \frac{\lambda l + (1-l)}{l\underline{y}^{\alpha} + (1-l)}$$

which holds because of $y^{**}(1) < 1$ and $y^*(1) > \underline{y}$. Also the left hand side of (A8) is larger than its right hand side at $y_{t+1} = y_t = 1$ if and only if

$$\left(\frac{\lambda l y^{**}(1) + (1-l)}{l(y^*(1))^{\alpha} + (1-l)}\right)\left(\frac{\delta\alpha(1-\lambda)}{\lambda l + (1-l)}\right) > \lambda\alpha \tag{A9}$$

which holds due to $\delta > \tilde{\delta} \geq \left(\frac{l\lambda^{-\alpha}+(1-l)}{(1-l)^2}\right)\left(\frac{\lambda}{1-\lambda}\right)$, $y^{**}(1) > 0$ and $y^*(1) < 1 < \lambda^{-1}$. Thus there exists some $y^*(2) \in (y^*(1), 1)$ such that the left hand side of (A8) is equal to its right hand side at $y_{t+1} = y_t = y^*(2)$ (if such $y^*(2)$ are multiple, we take the smallest one). Then we obtain from (A8) the result that $y_t \geq y^*(2)$ for all large enough t.

Since $x_t \leq h(y^{**}(1), y^*(1))$ for all large t, by a similar argument to (A7), we can also show that

$$\frac{\lambda l y^*(1) + (1-l)}{l(y^{**}(1))^\alpha + (1-l)} \frac{\delta\{y_{t+1}^\alpha - \lambda\alpha y_{t+1} - (1-\alpha)\}}{\lambda l y_{t+1} + (1-l)} \leq \frac{\lambda\alpha y_t}{l y_t^\alpha + (1-l)} \tag{A10}$$

for all large t. The left hand side of (A10) is larger than its right hand side at $y_{t+1} = y_t = y^{**}(1)$ if and only if

$$\frac{\lambda l y^*(1) + (1-l)}{l(y^{**}(1))^\alpha + (1-l)} > \frac{\lambda l \underline{y} + (1-l)}{l + (1-l)}$$

which is satisfied because of $y^*(1) > \underline{y}$ and $1 > y^{**}(1)$. Thus there exists some $y^{**}(2) \in (\underline{y}, y^{**}(1))$ such that the left hand side of (A10) is equal to its right hand side at $y_{t+1} = y_t = y^{**}(2)$. Thus (A10) implies that $y_t \leq y^{**}(2)$ holds for all large t.

We have thus shown that $y^*(2) \leq y_t \leq y^{**}(2)$ holds for all large enough t.

Repeating this process, we can find a sequence $\{y^*(m), y^{**}(m)\}_{m=0}^\infty$ such that $y^*(m) < y^{**}(m)$, where $y^*(0) = \underline{y}$ and $y^{**}(0) = 1$, and

$$\frac{\lambda l y^{**}(m-1) + (1-l)}{l(y^*(m-1))^\alpha + (1-l)} \frac{\delta\{(y^*(m))^\alpha - \lambda\alpha y^*(m) - (1-\alpha)\}}{\lambda l y^*(m) + (1-l)} = \frac{\lambda\alpha y^*(m)}{l(y^*(m))^\alpha + (1-l)} \tag{A11}$$

and

$$\frac{\lambda l y^*(m-1) + (1-l)}{l(y^{**}(m-1))^\alpha + (1-l)} \frac{\delta\{(y^{**}(m))^\alpha - \lambda\alpha y^{**}(m) - (1-\alpha)\}}{\lambda l y^{**}(m) + (1-l)} = \frac{\lambda\alpha y^{**}(m)}{l(y^{**}(m))^\alpha + (1-l)} \tag{A12}$$

for each $m = 1, 2, ...$ Here, $y^*(m) \leq y^*(m+1)$ while $y^{**}(m+1) \leq y^{**}(m)$ for each $m \geq 1$. Then, for each $m \geq 1$, there exists some $T(m)$ such that $y^*(m) \leq y_t \leq y^{**}(m)$ holds for all $t \geq T(m)$.

We now define the limit point of $(y^*(m), y^{**}(m))$, denoted by (y^*, y^{**}), satisfying (A11) and (A12) at $y^*(m) = y^*(m-1) = y^*$ and $y^{**}(m-1) = y^{**}(m) = y^{**}$:

$$\frac{\lambda l y^{**} + (1-l)}{\lambda l y^* + (1-l)} \delta\{(y^*)^\alpha - \lambda\alpha y^* - (1-\alpha)\} = \lambda\alpha y^* \tag{A13}$$

and

$$\frac{\lambda l y^* + (1-l)}{\lambda l y^{**} + (1-l)} \delta\{(y^{**})^\alpha - \lambda\alpha y^{**} - (1-\alpha)\} = \lambda\alpha y^{**}. \tag{A14}$$

Here, note that, since $y^*(m)$ is increasing and bounded above, the sequence $\{y^*(m)\}$ converges to y^*. Also, note that, since $y^{**}(m)$ is decreasing and bounded below, the sequence $\{y^{**}(m)\}$ converges to y^{**}.

Let $S \equiv \{(y^*, y^{**}) \in [\underline{y}, 1/\lambda]^2 \mid \text{(A13) and (A14) are satisfied.}\}$. Then $S \neq \emptyset$ because the steady state of the economy $(\tilde{y}, \tilde{y})$ satisfies (A13) and (A14) by its definition.

Now we show that $\lim_{t\to\infty} y_t \in [y^*, y^{**}]$ for any $(y^*, y^{**}) \in S$. Take any $(y^*, y^{**}) \in S$. Take also any $\varepsilon > 0$. Then, since $(y^*(m), y^{**}(m))$ is convergent, there exists some large m'' such that $y^* - y^*(m'') \leq \varepsilon$ and $y^{**}(m'') - y^{**} \leq \varepsilon$. For such m'', we can find a large $T(m'')$ such that for all $t \geq T(m'')$, $y^*(m'') \leq y_t \leq y^{**}(m'')$, which implies that $y^* - \varepsilon \leq y_t \leq y^{**} + \varepsilon$. Since $\varepsilon > 0$ is arbitrary, we have $\lim_{t\to\infty} y_t \in [y^*, y^{**}]$. Since $(y^*, y^{**}) \in S$ is arbitrary, by taking $(\tilde{y}, \tilde{y}) \in S$ we establish that $\lim_{t\to\infty} y_t = \tilde{y}$. Q.E.D.

Proof of Corollary.
First, we consider the effect of δ on $\tilde{y}$ and $\tilde{x}$.

We can see from (15) that

$$\frac{\partial \tilde{y}}{\partial \delta} = -\frac{\tilde{y}^\alpha - \lambda\alpha\tilde{y} - (1-\alpha)}{\delta\alpha(\tilde{y}^{\alpha-1} - \lambda) - \lambda\alpha} < 0$$

because $\tilde{y}$ is the smallest root to (15) and thus the denominator is positive.

Next we define

$$\rho(y) \equiv \frac{ly^\alpha + (1-l)}{\lambda l y + (1-l)}$$

where we verify that ρ' has the same sign as $\lambda l y^\alpha(\alpha - 1) + (1-l)(\alpha y^{\alpha-1} - \lambda)$ and that $\rho'' < 0$ at any y satisfying $\rho'(y) = 0$. Thus ρ is strictly quasi-concave function. Also $\rho'(0) = +\infty$ and $\rho'(1) \geq 0$ if and only if $\lambda \leq \alpha(1-l)/(l(1-\alpha) + (1-l))$ as we have assumed. Thus $\rho' > 0$ for all $y \in [0, 1]$ under our assumption.

Then, we can show from (16) that $\partial\tilde{x}/\partial\delta < 0$ because the increase in δ reduces $\tilde{y}$.

Second, we consider the effect of λ. We can readily see from (15) that

$$\frac{\partial \tilde{y}}{\partial \lambda} = \frac{\alpha\tilde{y} + \delta\alpha\tilde{y}}{\delta\alpha(\tilde{y}^{\alpha-1} - \lambda) - \lambda\alpha} > 0$$

Also, from (16) we have $\tilde{x} = [\lambda(1-\alpha)A\rho(\tilde{y})]^{1/(1-\alpha)}$. Then we can compute that

$$\frac{d\tilde{x}}{d\lambda} = \frac{\partial \tilde{x}}{\partial \lambda} + \frac{\partial \tilde{x}}{\partial \tilde{y}}\frac{\partial \tilde{y}}{\partial \lambda} > 0$$

where $\partial\tilde{x}/\partial\lambda$ has the same sign as $\partial/\partial\lambda(\lambda/(\lambda l\tilde{y} + (1-l))) > 0$, $\partial\tilde{x}/\partial y > 0$ if $\lambda \leq \alpha(1-l)/(l(1-\alpha) + (1-l))$, and $\partial\tilde{y}/\partial\lambda > 0$. Q.E.D.

Proof of Proposition 5.
Fix any equilibrium path. In the equilibrium each relationship pair matched in period s agrees to a sequence of relational contracts $\{z_t^s(\gamma), R_t^s(\gamma), d_t^s(\gamma)\}_{t\geq s}$ from period s onward and its t-th generation implements the corresponding agreement $\{z_t^s(\gamma), R_t^s(\gamma), d_t^s(\gamma)\}$ for the realization of exogenous separation shock $\gamma \in \Gamma$ in period t.

We define by $l_t \equiv \sum_{s=0}^{t} l_t^s$ the total measure of the relationship pairs of young R-producers and young R-lenders in period t whose parents were not separated in the previous period. Thus there are $1 - l_t$ remaining young R-producers (or young R-lenders) who engage in arm's length contracts in the credit market in period t. Then we can derive the law of motion governing the process of l_t. Since l_t is the sum of all the relationships with different birth dates ($0 \le s \le t$) which are not dissolved in period t, we have $l_t = \sum_{s=0}^{t} l_t^s$. Here $l_t^s = \sum_i q^i \gamma^i l_{t-1}^s$ for $t \ge s+1$ is the measure of the relationships which have been carried over from period s and l_t^t is the measure of newly matched relationships in period t. In the beginning of period t there are $1 - l_{t-1}$ old A-producers and $1 - l_{t-1}$ old A-lenders who engaged in arm's length contracts in the credit market and $\sum_i q^i(1-\gamma^i)l_{t-1}$ relationships are exogenously separated. All these old producers and lenders enter the matching market in period t for finding the new partners for their children. Since each old producer (respectively, lender) is matched with an old lender (respectively, producer) with probability $m \in (0,1)$ in the matching market, $l_t^t = m(1 - l_{t-1} + \sum_i q^i(1-\gamma^i)l_{t-1})$ relationships of old R-producers and old R-lenders are newly matched in period t. Their children will start the new relationships from period t. Then, by using the fact that $l_{t-1} = \sum_{s=0}^{t-1} l_{t-1}^s$, we obtain $\mu l_{t-1} = \sum_{s=0}^{t-1} \mu l_{t-1}^s = \sum_{s=0}^{t-1} l_t^s$ because $l_t^s = \mu l_{t-1}^s$. Thus, we can derive

$$
\begin{aligned}
l_t &= \sum_{s=0}^{t-1} l_t^s + l_t^t \\
&= \mu l_{t-1} + l_t^t \\
&= \mu l_{t-1} + m(1 - \mu l_{t-1}) \\
&= m + (1-m)\mu l_{t-1}
\end{aligned}
$$

which converges to a unique steady state $l^* \equiv \frac{m}{1-(1-m)\mu}$. Then we define $\bar{l} \equiv \max\{l_0, l^*\}$, which implies that $l_t \le \bar{l}$ for all t.

Next, we derive the necessary conditions for equilibrium relational contract to be sustainable. Take a relationship pair of a young R-producer and a young R-lender matched in period s who agree on a sequence of relational contracts $\{z_t^s(\gamma), R_t^s(\gamma), d_t^s(\gamma)\}_{s \ge t}$ from period s onward. Here, recall that $d_t^s(\gamma) \in \{0,1\}$ and that $d_t^s(\gamma) = 1$ holds when they implement the period t-relational contract $(z_t^s(\gamma), R_t^s(\gamma))$ and $d_t^s(\gamma) = 0$ holds when they decide to go to the outside credit market respectively.[30] Let $\Gamma_t^s \equiv \{\gamma \in \Gamma \mid d_t^s(\gamma) = 1\}$ be the set of the exogenous separation shocks which induce the implementation of the relational contract in period t instead of going to the outside credit market.

We define by

$$
u_{p,t}^s(\gamma) \equiv p_t z_t^s(\gamma) - R_t^s(\gamma) - r_t \max[z_t^s(\gamma) - w_{t-1}, 0],
$$

the profit of an old R-producer in period t when $d_t^s(\gamma) = 1$ where the term $r_t \max[z_t^s(\gamma) - w_{t-1}, 0]$ reflects the case that the R-producer invests $z_t^s(\gamma)$ more than w_{t-1} owned by

[30] In any equilibrium any R-producer and R-lender never voluntarily dissolve their relationship as we will see below.

the R-lender, in which case he must borrow the extra amount $z_t^s(\gamma) - w_{t-1}$ from the outside credit market. Also, we define by

$$u_{l,t}^s(\gamma) \equiv R_t^s(\gamma) + \lambda r_t \max[w_{t-1} - z_t^s(\gamma), 0]$$

the profit of an old R-lender in period t when $d_t^s(\gamma) = 1$ where the term $\lambda r_t \max[w_{t-1} - z_t^s(\gamma), 0]$ reflects the case that the young R-lender lends the remaining amount $w_{t-1} - z_t^s(\gamma)$ to the credit market after making the relationship lending $z_t^s(\gamma)$ to the young R-producer. Then we also define by

$$V_{p,t}^s \equiv \sum_{i=1}^n q^i \{d_t^s(\gamma^i) u_{p,t}^s(\gamma^i) + (1 - d_t^s(\gamma^i))\pi_t\}$$

and

$$V_{l,t}^s \equiv \sum_{i=1}^n q^i \{d_t^s(\gamma^i) u_{l,t}^s(\gamma^i) + (1 - d_t^s(\gamma^i))\lambda r_t w_{t-1}\}$$

the expected profits of an old R-producer and an old R-lender respectively.

Suppose that $\gamma \in \Gamma_t^s$ is realized where $d_t^s(\gamma) = 1$. Then, by following the agreed upon relational contract, the old R-producer obtains the following payoff:

$$U_{p,t}^s(\gamma) \equiv u_{p,t}^s(\gamma) + \gamma\delta V_{p,t+1}^s + (1-\gamma)\delta\{mV_{p,t+1}^{t+1} + (1-m)\pi_{t+1}\}$$

when facing the separation shock $\gamma \in \Gamma$ in period t. With probability γ, the relationship continues to the next period in which case his child will obtain the expected profit $V_{p,t+1}^s$. With probability $1-\gamma$, the relationship is exogenously separated in which case the old R-producer go to the matching market for seeking a different old lender. Then, with probability m, match is successful so that his child can start a new relationship and obtain the expected profit $V_{p,t+1}^{t+1}$ while with probability $1-m$ match is not successful so that his child has nothing but to go to the outside credit market and earn the arm's length contract profit π_{t+1}.

On the other hand, if the old R-producer reneged on the repayment $R_t^s(\gamma)$ and quitted the relationship (and then went to the matching market) in period t, he would obtain the following deviation payoff:

$$\tilde{U}_{p,t}^t(\gamma) \equiv p_t z_t^s(\gamma) - r_t \max[z_t^s(\gamma) - w_{t-1}, 0] + \delta\{mV_{p,t+1}^{t+1} + (1-m)\pi_{t+1}\}.$$

Thus, the following incentive compatibility constraint must be satisfied:

$$U_{p,t}^s(\gamma) \geq \tilde{U}_{p,t}^s(\gamma) \qquad (\text{IC}_t^s(\gamma))$$

Second, for any $\gamma \in \Gamma$, the young R-producer must be not better off by quitting the relationship, which yields the profit π_t, and going to the matching market for seeking a new lender when old, which yields the expected profit $mV_{p,t+1}^{t+1} + (1-m)\pi_{t+1}$ to his child. This individual rationality constraint is given by

$$d_t^s(\gamma)u_{p,t}^s(\gamma) + (1 - d_t^s(\gamma))\pi_t \geq \pi_t + \delta\{mV_{p,t+1}^{t+1} + (1-m)\pi_{t+1}\} \qquad (\text{IRP}_t^s(\gamma))$$

where the left hand side denotes the profit of the young R-producer who obtains $u^s_{p,t}(\gamma)$ for $\gamma \in \Gamma^s_t$ but π_t for $\gamma \notin \Gamma^s_t$.

Third, a similar individual rationality constraint for the young R-lender must be satisfied: for all $\gamma \in \Gamma$,

$$d^s_t(\gamma)u^s_{l,t}(\gamma) + (1 - d^s_t(\gamma))\lambda r_t w_{t-1} \geq \lambda r_t w_{t-1} + \delta\{mV^{t+1}_{l,t+1} + (1-m)\lambda r_{t+1} w_t\} \quad (\text{IRL}^s_t(\gamma))$$

By combining $\text{IC}^s_t(\gamma)$ with $\text{IRL}^s_t(\gamma)$ for $\gamma \in \Gamma^s_t$ (thus $d^s_t(\gamma) = 1$), we obtain the modified incentive compatibility condition:

$$\gamma\delta\{J^s_{t+1} - mJ^{t+1}_{t+1} - (1-m)(\lambda r_{t+1} w_t + \pi_{t+1})\} \geq \lambda r_t \min[z^s_t(\gamma) - w_{t-1}, 0] + \lambda r_t w_{t-1} \quad (\text{A-IC}^s_t(\gamma))$$

where $J^s_{t+1} \equiv V^s_{p,t+1} + V^s_{l,t+1}$.

Also, by combining $\text{IRL}^s_t(\gamma)$ with $\text{IRP}^s_t(\gamma)$ for any $\gamma \in \Gamma$ (thus any $d^s_t(\gamma) \in \{0, 1\}$), the total net surplus of relationship must be non-negative:

$$\begin{aligned} TS^s_t(\gamma) \equiv\ & d^s_t(\gamma)(u^s_{p,t}(\gamma) + u^s_{l,t}(\gamma)) + (1 - d^s_t(\gamma))(\lambda r_t w_{t-1} + \pi_t) - (\lambda r_t w_{t-1} + \pi_t) \\ & + \gamma\delta\{J^s_{t+1} - mJ^{t+1}_{t+1} - (1-m)(\lambda r_{t+1} w_t + \pi_{t+1})\} \\ & \geq 0 \qquad (\text{TS}^s_t(\gamma)) \end{aligned}$$

Note that the young R-producer (respectively, the young R-lender) obtains his (her) expected payoff of period t, which takes into account his (her) concern about the child's consumption in period $t + 1$, as $V^s_{p,t} + \mu\delta V^s_{p,t+1} + (1 - \mu)\{mV^{t+1}_{p,t+1} + (1 - m)\pi_{t+1}\}$ (respectively, $V^s_{l,t} + \mu\delta V^s_{l,t+1} + (1 - \mu)\{mV^{t+1}_{l,t+1} + (1 - m)\lambda r_{t+1} w_t\}$). Let $\tilde{V}^s_t$ be the sum of these expected payoffs. Then, as in the basic model, we suppose that each relationship pair formed in period s chooses a sequence of relational contracts $\{z^s_t(\gamma), R^s_t(\gamma), d^s_t(\gamma)\}_{t \geq s}$ from period s onward in order to maximize the weighted sum of the expected payoffs in the future generations of the same dynasty, $W_s = \tilde{V}^s_s + \mu\beta W_{s+1} + (1 - \mu)\beta \times Constant$ where the last term $Constant$ denotes the future payoff attained when the relationship is separated, which is not affected by the choice of relational contracts $\{z^s_t(\gamma), R^s_t(\gamma), d^s_t(\gamma)\}_{t \geq s}$ by the relationship formed in period s and hence we can ignore it. Here, some remarks on the set of constraints are in order. First, as we have noted, any young R-producer and any young R-lender of any relationship never fully dissolve their relationship in which case they must engage in arm's length contract and go to the matching market for seeking another partners when old. This is because they can always attain the same payoffs under full dissolution by using a relational contract without dissolution.[31] Second, $\text{TS}^s_t(\gamma)$ can be always satisfied

[31] Suppose that they dissolve relationship for some $\gamma \in \Gamma$ in period t (thus going to the credit market when young in period t and going to the matching market when old in period $t + 1$). Then, the R-producer (respectively, R-lender) obtains $\pi_t + \delta\{mV^{t+1}_{p,t+1} + (1 - m)\pi_{t+1}\}$ (respectively, $\lambda r_t w_{t-1} + \delta\{mV^{t+1}_{l,t+1} + (1-m)\lambda r_{t+1} w_t\}$). However, they can always attain at least these payoffs without dissolving the relationship by using a relational contract which specifies $d^s_t(\gamma) = 0$ and mimics the equilibrium contract used by the relationship matched in period $t + 1$, $\{z^{t+1}_\tau(\gamma), R^{t+1}_\tau(\gamma), d^{t+1}_\tau(\gamma)\}_{\tau \geq t+1}$ if $J^{t+1}_{t+1} \geq \lambda r_{t+1} w_t + \pi_{t+1}$ and $d^s_{t+1}(\gamma) = 0$ (i.e., going to the outside credit market) otherwise.

because $d_t^s(\gamma) = 1$ should be chosen only when $u_{p,t}^s(\gamma) + u_{l,t}^s(\gamma) \geq \lambda r_t w_{t-1} + \pi_t$. Thus we consider only A-IC$_t^s(\gamma)$ for $\gamma \in \Gamma_t^s$ as the relevant constraint.

Finally, the credit market and labour market equilibrium conditions are modified as follows:

$$\lambda(1-l_t)w_t + \lambda \sum_{s=0}^{t} \left(\sum_i q^i d_t^s(\gamma^i) l_t^s \max[w_t - z_{t+1}^s(\gamma^i), 0] + \sum_i q^i (1 - d_t^s(\gamma^i)) l_t^s w_t \right)$$
$$= (1-l_t)x_{t+1} + \sum_{s=0}^{t} \left(\sum_i q^i d_t^s(\gamma^i) l_t^s \max[z_{t+1}^s(\gamma^i) - w_t, 0] + \sum_i q^i (1 - d_t^s(\gamma^i)) l_t^s x_{t+1} \right) \tag{A-CME$_t$}$$

and

$$w_t = (1-\alpha)A \left((1-l_{t-1})x_t^\alpha + \sum_{s=0}^{t-1} \left(\sum_i q^i d_{t-1}^s(\gamma^i) l_{t-1}^s z_t^s(\gamma^i)^\alpha + \sum_i q^i (1 - d_{t-1}^s(\gamma^i)) l_{t-1}^s x_t^\alpha \right) \right) \tag{A-LME$_t$}$$

Here the left hand side of A-CME$_t$ is the total credit supply while its right hand side is the total credit demand in the credit market. There are $(1 - l_t)$ young A-lenders (respectively, A-producers) who supply w_t each (respectively, demand x_{t+1} each) while R-lenders (respectively, R-producers) who implement the relational contracts supply the credit of $\max[w_t - z_{t+1}^s(\gamma), 0]$ each (respectively, demand $\max[z_{t+1}^s(\gamma) - w_t, 0]$ each). All other R-lenders (respectively, R-producers) behave as A-lenders (respectively, A-producers) do so, i.e., R-lenders (respectively, R-producers) lend (respectively, borrow) all their wage income w_t (respectively, the investment demand x_{t+1}).

Then, by using A-IC$_t^s(\gamma)$, A-CME$_t$ and A-LME$_t$ together, we can show the following claims (the formal proofs are given in Online Appendix).

Lemma A1. *Suppose that $\lambda \leq 1 - \bar{l}$. Then $w_t \geq z_{t+1}^s(\gamma)$ for all s, all $t \geq s$ and all $\gamma \in \Gamma_t^s$.*

Lemma A2. *Suppose that $\lambda \leq 1 - \bar{l}$. Then, $z_t^s(\gamma) \leq (1/\lambda)x_t$ for all $\gamma \in \Gamma_t^s$ so that A-IC$_t^s(\gamma)$ always binds for $\gamma \in \Gamma_t^s$.*

Lemma A3. *(i) If $d_t^s(\gamma) = 1$, then $d_t^s(\gamma') = 1$ for all $\gamma' > \gamma$. (ii) Suppose that $\delta > \frac{\lambda\alpha}{(1-\alpha)(1-m)}$. Then, $d_t^s(\gamma^n) = 1$ must be satisfied for all $t \geq T$ for some period $T \geq s$*

Lemma A4. *In any equilibrium path*

$$\left(\frac{x_{t+1}}{x_t}\right) \geq \lambda(1-\alpha)A(1-\bar{l})x_t^{\alpha-1}$$

and $x_t \leq \max\{x_0, \overline{X}\}$ for all $t \geq 0$ where

$$\overline{X} \equiv \lambda \left(\frac{(1-\alpha)A}{1-\bar{l}}\right)^{\frac{1}{1-\alpha}}.$$

Lemma A1 shows that any R-producer never invests more than the fund owned by the R-lender matching him as we have seen in Lemma 2. Thus A-IC$_t^s(\gamma)$ is simplified to the condition derived in the main text:

$$\gamma\delta\{J_{t+1}^s - mJ_{t+1}^{t+1} - (1-m)(\lambda r_{t+1}w_t + \pi_{t+1})\} \geq \lambda r_t z_t^s(\gamma) \qquad (\text{A-IC}_t^s(\gamma))$$

where $J_{t+1}^s = \sum_i q^i \max[p_{t+1}z_{t+1}^s(\gamma^i) - \lambda r_{t+1}z_{t+1}^s(\gamma^i), \pi_{t+1}]$ by optimal choice of $d_{t+1}^s(\gamma) \in \{0,1\}$. Lemma A2 states that such A-IC$_t^s(\gamma)$ becomes always binding for $\gamma \in \Gamma_t^s$ which implies that $y_t^s(\gamma^i)/\gamma^i = y_t^s(\gamma^j)/\gamma^j$ for all $\gamma^i, \gamma^j \in \Gamma_t^s$. Then, it follows from Lemma A2 that $z_t^s(\gamma)/x_t \leq 1/\lambda$ for all $\gamma \in \Gamma_t^s$. In addition, by using $r_t = \alpha^2 A x_t^{\alpha-1}$, we have $p_t z_t^s(\gamma) - \lambda r_t z_t^s(\gamma) \leq \alpha A x_t^\alpha\{\lambda^{-\alpha} - \alpha\}$. This shows that $J_t^t \leq \alpha A x_t(\lambda^{-\alpha} - \alpha)$ due to $\lambda^{-\alpha} - \alpha > 1 - \alpha$. Lemma A3 (i) says that the relationships facing lower separation probability (higher γ) implement the relational contracts more likely than those facing higher separation probability. Lemma A3 (ii) also shows that every relationship eventually implements the relational contract when facing the least severe separation shock $\gamma^n = 1$. Finally, Lemma A4 states that the output growth rate of A-producers x_{t+1}/x_t is bounded below and that the output level of each A-producer x_t is bounded above.

Now we assume that $\lambda \leq \lambda''$ where $\delta = \lambda''\alpha/(1-m)(1-\alpha)$.

By Lemma A3, $d_t^s(\gamma^n) = 1$ for all $t \geq$ for some period $T \geq s$. This implies that A-IC$_t^s(\gamma^n)$ is satisfied for all $t \geq T$ and that $\Gamma_t^s \neq \emptyset$ for all $t \geq T$. Then, due to Lemma A2, A-IC$_t^s(\gamma)$ must hold as equality for all $\gamma \in \Gamma_t^s$ for all $t \geq T$.

Recall that $J_{t+1}^{t+1} \leq \alpha A x_{t+1}^\alpha(\lambda^{-\alpha} - \alpha)$, $\pi_t = \alpha A(1-\alpha)x_t^\alpha$ and $r_t = \alpha^2 A x_t^{\alpha-1}$. Then, by using the fact that A-IC$_t^s(\gamma)$ binds for all $\gamma \in \Gamma_t^s$ and all $t \geq T$ for some $T \geq s$, we obtain

$$\delta\left(\frac{x_{t+1}}{x_t}\right)^\alpha \{J_{t+1}^s - m(\lambda^{-\alpha} - \alpha) - (1-m)(1-\alpha)\} \leq \lambda\alpha z_t^s(\gamma)/\gamma$$

for all $\gamma \in \Gamma_t^s$ and all $t \geq T$. Note that $J_{t+1}^s = \sum_i q^i \max[\alpha A z_{t+1}^s(\gamma^i)^\alpha - \lambda r_{t+1}z_{t+1}^s(\gamma^i), \pi_{t+1}]$ and $z_{t+1}^s(\gamma^i)/\gamma^i = z_{t+1}^s(\gamma^j)/\gamma^j$ for $i \neq j$. Define $\tilde{y}_t \equiv z_t^s(\gamma)/\gamma$ for all $\gamma \in \Gamma_t^s$ where $\tilde{y}_t \leq 1/\lambda$ by Lemma A2. Here we omit superscript s from $\tilde{y}_t$ to denote the birth date of a relationship becasue our result can be applied to any relationship which starts from any period.

Then, the above inequality can be written by

$$\lambda\alpha\tilde{y}_t \geq \delta[\lambda(1-\alpha)A(1-\bar{l})(\max\{x_0, \overline{X}\})^{\alpha-1}]^\alpha \times \left\{\sum_i q^i \max[(\gamma^i)^\alpha \tilde{y}_{t+1}^\alpha - \lambda\alpha\tilde{y}_{t+1}, 1-\alpha] - m(\lambda^{-\alpha} - \alpha) - (1-m)(1-\alpha)\right\}. \qquad (\text{A15})$$

Let $K(\tilde{y}_{t+1})$ be the right hand side of A15 which is a function of $\tilde{y}_{t+1} \in [0, 1/\lambda]$. Then we can readily verify that $K(1/\lambda) \geq \delta[\lambda(1-\alpha)A(\max\{x_0, \overline{X}\})^{\alpha-1}]^\alpha\{(q^n - m)(\lambda^{-\alpha} - 1)\} > 0$ due to $q^n > m$ and hence that there exists some $\lambda' \in (0,1)$ such that $K(1/\lambda) > \alpha\lambda$

holds for all $\lambda \in (0, \lambda')$. Then define $\overline{\lambda} \equiv \min\{\lambda'', \lambda', 1-\bar{l}\}$ and let $\lambda \in (0, \overline{\lambda})$. Thus, Lemma A1-A3 can be applied for all $\lambda \in (0, \overline{\lambda})$ as well as $K(1/\lambda) > \alpha\lambda$. Then, since $K(0) < 0$ and $K(1/\lambda) > \alpha\lambda$ for $\lambda \in (0, \overline{\lambda})$, there exists some $\hat{y} \in (0, 1/\lambda)$ such that $K(\hat{y}) = \lambda\alpha\hat{y}$. If $\tilde{y}_t > \hat{y}$, $\tilde{y}_t$ decreases over time according to A15 and eventually $\tilde{y}_t \leq \hat{y}$ holds.

Also, we can define $\overline{y}^i \in [0, 1/\lambda]$, $i = 1, 2, ..., n$, such that $f_i(y) \equiv (\gamma^i)^\alpha y^\alpha - \gamma^i\lambda\alpha y = 1 - \alpha$ as long as $f_i(1/\lambda) > 1 - \alpha$. Since $f_i(y) > f_j(y)$ for all $i > j$, $\overline{y}^i < \overline{y}^j$ for all $i > j$. Let $I \equiv \{i = 1, 2, ..., n \mid f_i(1/\lambda) \geq 1 - \alpha\}$. Then we can verify that for all $i \in I$, $f_i(y) \geq 1 - \alpha$ if and only if $y \in [\overline{y}^i, 1/\lambda]$. On the other hand, $f_i(y) < 1 - \alpha$ for all $y \in [0, 1/\lambda]$ for all $i \notin I$. Then define k such that $\overline{y}^{k-1} \geq \hat{y} \geq \overline{y}^k$ (define $k = 1$ if $\hat{y} < \overline{y}^n$ and $k = n$ if $\hat{y} > \overline{y}^1$).

Then, we can find some period $T^s(\gamma^i) \geq T$ for $i \geq k - 1$ such that $\tilde{y}_t$ decreases over time when $\tilde{y}_t > \hat{y}$ according to the dynamic constraint A15 and eventually $\tilde{y}_t \leq \overline{y}^i$ for all $t \geq T^s(\gamma^i)$ so that $d_t^s(\gamma^i) = 0$ for all $t \geq T^s(\gamma^i)$. Since $\overline{y}^i > \overline{y}^j$ for all $i < j$, we have $T^s(\gamma^j) \leq T^s(\gamma^j)$ for $i > j$ where $k - 1 \geq i > j$. If $\tilde{y}_t < \hat{y}$ for all $t \geq s$, then define $T^s(\gamma^i) = s$ for all $i \leq k - 1$.

Finally, we show that $y_t(\gamma^i)$ decreases over time whenever $y_t(\gamma^i) > \hat{y}$. By the above result, we know that $K(\tilde{y}_t) \leq \lambda\alpha\tilde{y}_t$ for all $\tilde{y}_t > \hat{y}$ which implies that $\tilde{y}_t$ decreases over time as long as $\tilde{y}_t > \hat{y}$. This gives the desired result. Q.E.D.

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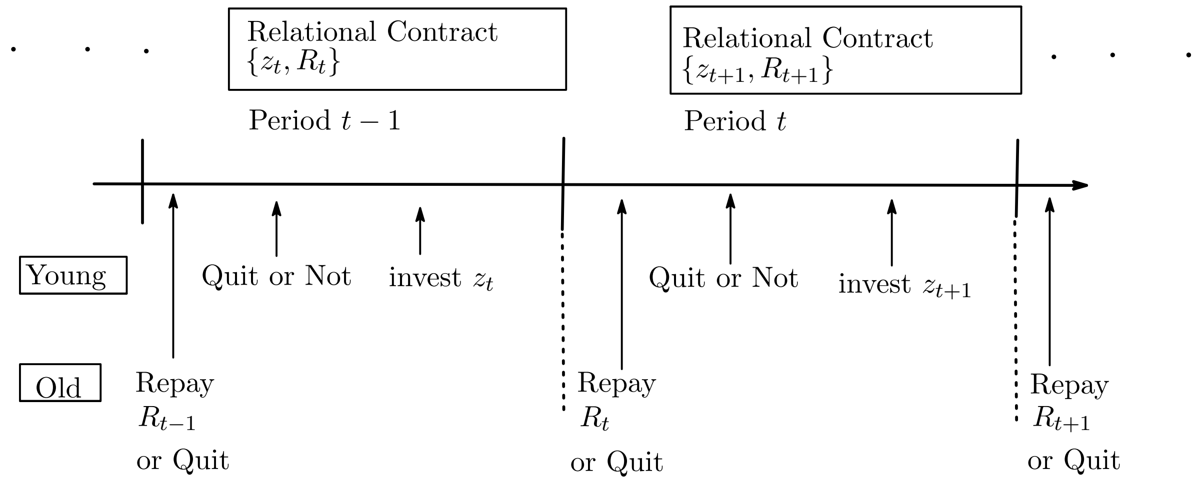

Figure 1: Time Line

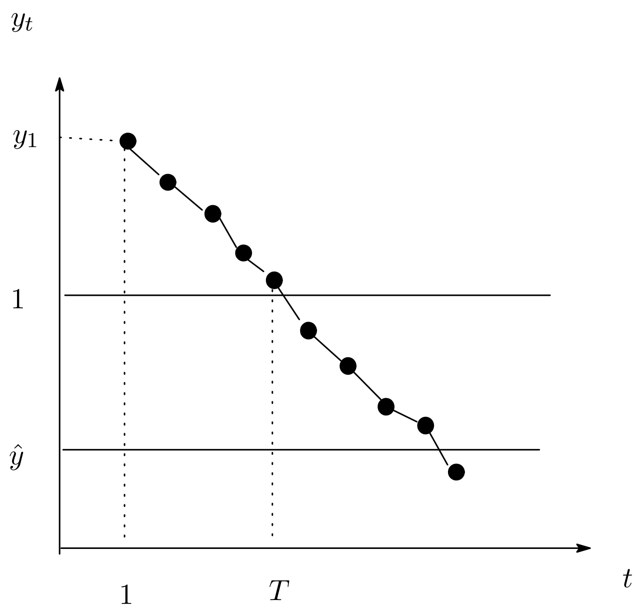

Figure 2: Equilibrium path of the relative output of R-producer y_t

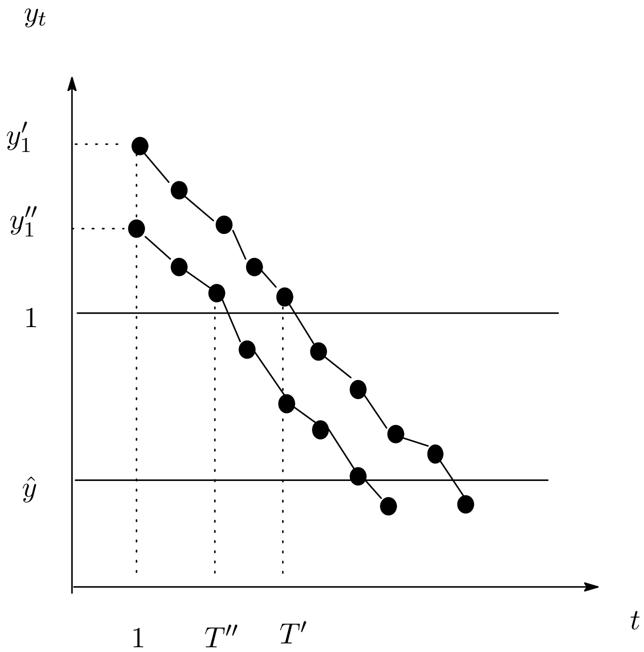

Figure 3: Different equilibrium paths with different first period values y_1

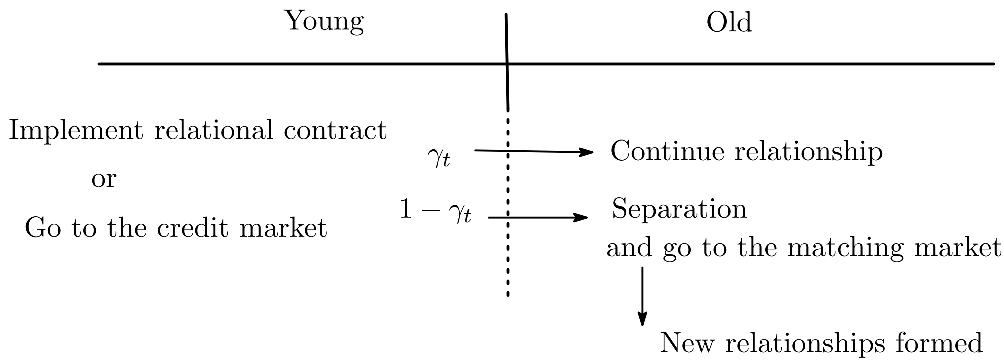

Figure 4: Matching and Separation

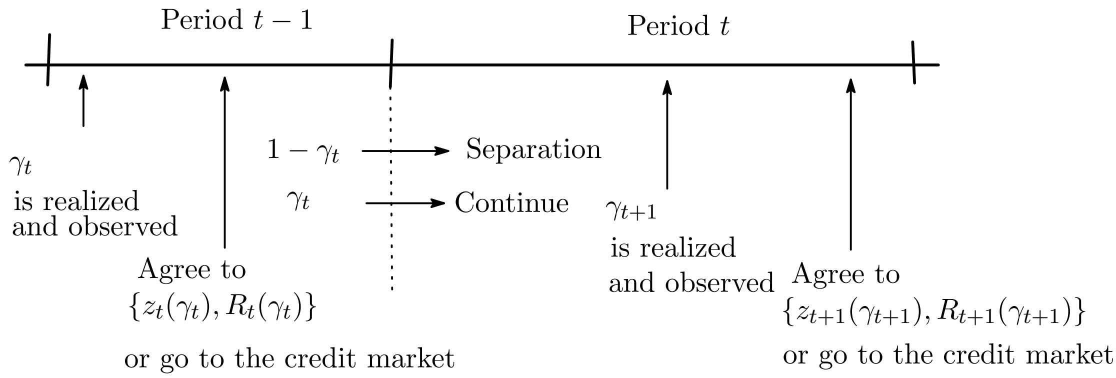

Figure 5: Time Line for Relational Contract

Online Appendix (Not for Publication)

Appendix B: Equilibrium capital investment by A-producers

In the main text we have so far assumed that each young A-producer born in period $t-1$ chooses a capital investment x_t so as to maximize only his own profit π_t. Because the utility function of each A-producer depends on his child's consumption level, one might think that there may be intergenerational strategic interactions between an A-producer's choice of capital investment x_t and his child's capital investment choice x_{t+1}. However, the following claim shows that it is sufficient to focus only on the equilibrium in which each A-producer acts to maximize only his own profit π_t no matter what histories are observed.

Suppose that each young A-producer in each dynasty, born in period t, can observe all the capital investments $\{x_1, x_2, ..., x_{t-1}\}$ chosen in the same dynasty and all the market prices $\{w_0, ..., w_{t-1}, r_1, ..., r_{t-1}, p_0, ..., p_{t-1}\}$ in the past periods. Then we let h_t be such a history observed up to period t. Let H_t be also the set of all these histories observed to a young A-producer up to period t.

Proposition B1. *In any equilibrium, every A-producer born in period* $t-1$ *chooses capital investment level* x_t *to maximize only his own profit* π_t *no matter the history observed up to period* t*.*

Proof. Take an equilibrium path in which each A-producer uses a strategy $\sigma_t : H_t \to [0, \infty)$ which maps from, H_t, the set of all previous observed histories up to period t to the current capital investment level $x_t \geq 0$.

Then, by recalling that $x(r_t)$ maximizes the per period profit π_t of an A-producer, we will show that every A-producer acts to maximize his own profit π_t and hence chooses $x_t = x(r_t)$ in every period irrespective of observed histories $h_t \in H_t$. We denote by $\overline{\sigma}_t$ such strategy defined as $\overline{\sigma}(h_t) = x(r_t)$ for all $h_t \in H_t$.

To show this, suppose that there exists an equilibrium with $\sigma_t \neq \overline{\sigma}_t$ for some A-producer in some period t. Thus $\sigma(h_t) \neq x(r_t)$ for some $h_t \in H_t$. We then denote by $\{x_s\}_{s=t}^{\infty}$ the equilibrium sequence of capital investments of A-producers from period t onward according to the equilibrium strategies $\{\sigma_s\}_{s=t}^{\infty}$.

In the proof of the claim, we will denote by $\pi(x_t)$ the profit of an A-producer who chooses x_t in period t.

Since the A-producer could always choose $x(r_t)$ in period t, it must be the case that

$$\pi(x_t) + \delta\pi(x_{t+1}) \geq \pi(x(r_t)) + \delta\pi(x'_{t+1})$$

where his child will choose x'_{t+1} in period $t+1$ following the choice of his parent $x(r_t)$ according to his equilibrium strategy σ_{t+1}. The above inequality yields

$$\delta\{\pi(x_{t+1}) - \pi(x'_{t+1})\} \geq \pi(x(r_t)) - \pi(x_t). \qquad \text{(B1)}$$

Also, for x'_{t+1} to be the optimal choice by the young A-producer in period $t+1$ following $x(r_t)$ chosen by his parent, we must have

$$\pi(x'_{t+1}) + \delta\pi(x''_{t+2}) \geq \pi(x(r_{t+1})) + \delta\pi(x'_{t+2})$$

because he could always choose $x(r_{t+1})$ following his parent choice $x(r_t)$ where x''_{t+2} denotes the choice of his child in period $t+2$ following x'_{t+1}. Here x'_{t+2} is the choice in period $t+2$ following $x(r_{t+1})$. This then yields

$$\delta\{\pi(x''_{t+2}) - \pi(x'_{t+2})\} \geq \pi(x(r_{t+1})) - \pi(x'_{t+1}).$$

Since $\pi(x(r_{t+1})) \geq \pi(x_{t+1})$ for any $x_{t+1} \neq x(r_{t+1})$, we have

$$\delta\{\pi(x''_{t+2}) - \pi(x'_{t+2})\} \geq \pi(x_{t+1}) - \pi(x'_{t+1}). \tag{B2}$$

Combining inequalities (B1) with (B2), we have

$$\delta^2\{\pi(x''_{t+2}) - \pi(x'_{t+2})\} \geq \pi(x(r_t)) - \pi(x_t). \tag{B3}$$

In period $t+2$ the young A-producer in the dynasty in question must choose x'_{t+2} following the choice $x(r_{t+1})$ by his parent in period $t+1$. Thus, since he could always choose x''_{t+2} instead of x'_{t+2} in period $t+2$, we must have

$$\pi(x'_{t+2}) + \delta\pi(x''_{t+3}) \geq \pi(x''_{t+2}) + \delta\pi(x'_{t+3})$$

where x''_{t+3} and x'_{t+3} denote the choices by the A-producer in period $t+3$ following x'_{t+2} and x''_{t+2} respectively. Thus

$$\delta\{\pi(x''_{t+3}) - \pi(x'_{t+3})\} \quad \geq \quad \pi(x''_{t+2}) - \pi(x'_{t+2})$$

Combining this with (B3), we have

$$\delta^3\{\pi(x''_{t+3}) - \pi(x'_{t+3})\} \geq \pi(x(r_t)) - \pi(x_t)$$

Repeating this argument for all periods $s+t \geq t$, we obtain

$$\delta^s\{\pi(x''_{t+s}) - \pi(x'_{t+s})\} \geq \pi(x(r_t)) - \pi(x_t), \ \ \forall\, s \geq 0.$$

Since $\pi(x(r_t)) \neq \pi(x_t)$ by our supposition ($\sigma(h_t) = x_t \neq x(r_t)$), there exists some $\varepsilon > 0$ such that $\pi(x(r_t)) - \pi(x_t) \geq \varepsilon$. The left hand side of the above inequality is bounded above by $\delta^s\pi(x(r_{t+s}))$ because $\pi(x_{t+s}) \geq 0$ and $\pi(x(r_{t+s})) = \max_x \pi(x)$ given r_{t+s}. Note here that $\pi(x_{t+s}) \geq 0$ because, by the spot transaction nature of arm's length contract in the credit market, each A-producer must make the repayment $r_{t+s}x_{t+s}$ from what he earns $p_{t+s}x_{t+s}$ when he is old.

Then, if $\pi(x(r_{t+s}))$ is bounded above, the left hand side of the above inequality goes to zero by letting $s \to \infty$ and noting $\delta < 1$, which is a contradiction. Thus it suffices to show that $\pi(x(r_t)) < +\infty$ for any equilibrium interest rate $r_t \geq 0$. This is equivalent to the condition that $r_t \geq \underline{r}$ for all t for some $\underline{r} > 0$ because then $\pi(x(r)) \leq \max_x \alpha(Ax)^\alpha - \underline{r}x < +\infty$. Suppose that $r_t = 0$ for some period t or $r_t \to 0$. In either case $x(r_t)$ must go to infinity. Since $\pi(x_t) + \delta\pi(x_{t+1}) \geq \pi(x(r_t)) + \delta\pi(x'_{t+1})$ must hold in period t (see (B1) above), either $\pi(x_t) \to \infty$ or $\pi(x_{t+1}) \to \infty$ or both must hold due to $\pi(x(r_t)) \to \infty$ and $\pi(x'_{t+1}) \geq 0$, which implies $x_t \to \infty$ or $x_{t+1} \to \infty$ with $r_{t+1} = 0$. In the former case ($x_t \to \infty$) CME$_{t-1}$ must imply $\lambda w_{t-1} = \lambda l z_t + (1-l)x_t \to \infty$, which holds only when $w_{t-1} \to \infty$ so that $x_{t-1} \to \infty$ or $z_{t-1} \to \infty$. We can deal with the latter case $x_{t+1} \to \infty$ in a similar way. However, then $w_{t-2} \to \infty$ by CME$_{t-2}$ in period $t-1$. Repeating this, $w_0 \to \infty$ which is however impossible. Thus $r_t \geq \underline{r}$ must hold in any period t for some $\underline{r} > 0$. This then establishes the claim. Q.E.D.

Appendix C: Proofs for Lemma A1-A4

In this subsection we will give the proofs for Lemma A1-A4 which are used to prove Proposition 5 in the main text.

Lemma A1. *Suppose that $\lambda \leq 1 - \bar{l}$. Then $w_t \geq z^s_{t+1}(\gamma)$ for all s, all $t \geq s$ and all $\gamma \in \Gamma^s_t$.*

Proof. Suppose contrary to the claim that $z^s_t(\gamma) > w_{t-1}$ for some period t and some $\gamma \in \Gamma^s_t$. Then, the relationship's joint profit of period t becomes $p_t z^s_t(\gamma) - r_t(z^s_t(\gamma) - w_{t-1})$. Also the right hand side of A-IC$^s_t(\gamma)$ must be $\lambda r_t w_{t-1}$ which is independent of $z^s_t(\gamma)$. Thus the choice of $z^s_t(\gamma)$ does not affect A-IC$^s_t(\gamma)$ but changing $z^s_t(\gamma)$ slightly toward x_t can increase the joint profit of period t, $p_t z^s_t(\gamma) - r_t(z^s_t(\gamma) - w_{t-1})$, as long as $z^s_t(\gamma) \neq x_t$. Thus $z^s_t(\gamma) = x_t$ must hold. On the other hand, A-CME$_{t-1}$ can be written by

$$\lambda(1 - l_{t-1})w_{t-1} + S^R_{t-1} = (1 - l_{t-1})x_t + D^R_t$$

where

$$S^R_t \equiv \lambda \sum_{s=0}^{t-1} \left(\sum_i q^i l^s_{t-1}[d^s_{t-1}(\gamma^i) \max[w_{t-1} - z^s_t(\gamma^i), 0] + (1 - d^s_{t-1}(\gamma^i))w_{t-1}] \right\}$$

and

$$D^R_t \equiv \sum_{s=0}^{t-1} \left\{ \sum_i q^i [d^s_{t-1}(\gamma^i) \max[z^s_t(\gamma^i) - w_{t-1}, 0] + (1 - d^s_{t-1}(\gamma^i))x_t] \right\}.$$

Since $S^R_{t-1} \leq \lambda l_{t-1} w_{t-1}$ and $D^R_t \geq 0$, A-CME$_{t-1}$ implies that $\lambda w_{t-1} \geq (1 - l_{t-1})x_t$. Since $l_t \leq \bar{l}$ for all t, $\lambda w_{t-1} \geq (1 - \bar{l})x_t$. But, by assumption that $\lambda \leq 1 - \bar{l}$, we have $\lambda w_{t-1} \geq (1 - \bar{l})x_t \geq \lambda x_t$ which shows that $w_{t-1} \geq x_t$, contradicting to $z^s_t(\gamma) = x_t > w_{t-1}$. Q.E.D.

Lemma A2. *Suppose that $\lambda \leq 1 - \bar{l}$. Then, $z^s_t(\gamma) \leq (1/\lambda)x_t$ for all $\gamma \in \Gamma^s_t$ so that A-IC$^s_t(\gamma)$ always binds for all $\gamma \in \Gamma^s_t$.*

Proof. Suppose that A-IC$_t^s(\gamma)$ is not binding for some period t and some $\gamma \in \Gamma_t^s$. Lemma A1 shows that $w_{t-1} \geq z_t^s(\gamma)$ holds for all $\gamma \in \Gamma_t^s$ and for all $t \geq s$. Then the relationship's joint profit of period t becomes $p_t z_t^s(\gamma) - \lambda r_t(z_t(\gamma) - w_{t-1})$ for such γ. This profit is maximized at $z_t^s(\gamma) = \hat{\lambda} x_t$. Thus, as long as A-IC$_t^s(\gamma)$ is slack for some γ at t, $z_t^s(\gamma) = \hat{\lambda} x_t$ must hold by the optimality of relational contract: otherwise, a slight change of $z_t^s(\gamma)$ toward $\hat{\lambda} x_t$ can increase the joint profit without violating all the constraints.

Due to Lemma A1, A-CME$_{t-1}$ can be written by

$$\begin{aligned}
&\lambda(1-l_{t-1})w_{t-1} + \lambda \sum_{s=0}^{t-1} \sum_i q^i d_{t-1}^s(\gamma^i) l_{t-1}^s (w_{t-1} - z_t^s(\gamma^i)) \\
&+ \sum_{s=0}^{t-1} \sum_i q^i (1 - d_{t-1}^s(\gamma^i)) l_{t-1}^s (\lambda w_{t-1} - x_t) \\
&= (1-l_{t-1}) x_t
\end{aligned}$$

Now we show that $x_t \geq \lambda w_{t-1}$. Suppose not. Then, the left hand side of the above A-CME$_{t-1}$ cannot be less than $\lambda(1-l_{t-1})w_{t-1}$ so that $\lambda(1-l_{t-1})w_{t-1} \leq (1-l_{t-1})x_t$ must hold, implying that $x_t \geq \lambda w_{t-1}$, a contradiction. Thus $x_t \geq \lambda w_{t-1}$ must hold. But then $(1/\lambda)x_t \geq w_{t-1} \geq z_t^s(\gamma)$ which shows that $\hat{\lambda} x_t \geq (1/\lambda) x_t > z_t^s(\gamma) = \hat{\lambda} x_t$, a contraction. Q.E.D.

Lemma A3. *(i) If $d_t^s(\gamma) = 1$, then $d_t^s(\gamma') = 1$ for all $\gamma' > \gamma$. (ii) Suppose that $\delta > \frac{\lambda\alpha}{(1-\alpha)(1-m)}$. Then, $d_t^s(\gamma^n) = 1$ must be satisfied for all $t \geq T$ for some period $T \geq s$*

Proof. (i) Suppose that $d_t^s(\gamma') = 1$ but $d_t^s(\gamma'') = 0$ for some $\gamma'' > \gamma'$ in some period t. Then, modify the original equilibrium contract only for such period t and γ'' as $\tilde{d}_t^s(\gamma'') = 1$ and $\tilde{z}_t^s(\gamma'') = z_t^s(\gamma') + \varepsilon$ for $\varepsilon \geq 0$. Note that $d_t^s(\gamma') = 1$ implies that $p_t z_t^s(\gamma') - \lambda r_t z_t^s(\gamma') \geq \pi_t > 0$. Thus $z_t^s(\gamma') > 0$ and hence A-IC$_t^s(\gamma')$ implies that $J_{t+1}^s - mJ_{t+1}^{t+1} - (1-m)(\lambda r_{t+1} w_t + \pi_{t+1}) > 0$. Then, since A-IC$_t^s(\gamma')$ holds and $\gamma'' > \gamma'$, we can see that $\gamma''\delta\{J_{t+1}^s - mJ_{t+1}^{t+1} - (1-m)(\lambda r_{t+1} w_t + \pi_{t+1})\} > \gamma'\delta\{J_{t+1}^s - mJ_{t+1}^{t+1} - (1-m)(\lambda r_{t+1} w_t + \pi_{t+1})\} \geq \lambda r_t z_t^s(\gamma')$. Thus A-IC$_t^s(\gamma'')$ can be slack at $\tilde{z}_t^s(\gamma'')$ for a small enough $\varepsilon \geq 0$.

Case 1: $w_{t-1} > z_t^s(\gamma')$. Then, take $\varepsilon > 0$ that satisfies $\tilde{z}_t^s(\gamma'') \leq w_{t-1}$. Thus $\tilde{z}_t^s(\gamma'') > z_t^s(\gamma')$. But, then we obtain

$$\begin{aligned}
p_t \tilde{z}_t^s(\gamma'') + \lambda r_t(w_{t-1} - \tilde{z}_t^s(\gamma'')) &> p_t z_t^s(\gamma') + \lambda r_t(w_{t-1} - z_t^s(\gamma')) \\
&\geq \lambda r_t w_{t-1} + \pi_t
\end{aligned}$$

where the first inequality follows from that $z_t^s(\gamma') < \hat{\lambda} x_t$ and $p_t z_t - \lambda r_t z_t$ is increasing in $z_t \leq \hat{\lambda} x_t$, and the second inequality from $d_t^s(\gamma') = 1$. This shows that the modified contract can improve the joint surplus of period t, a contradiction to $d_t^s(\gamma'') = 0$.

Case 2: $w_{t-1} = z_t^s(\gamma')$. Then, $\tilde{z}_t^s(\gamma'') = w_{t-1} + \varepsilon \geq w_{t-1}$ for $\varepsilon \geq 0$ in which case the joint profit of period t becomes

$$\begin{aligned}\Phi_t(\varepsilon) &\equiv p_t\tilde{z}_t^s(\gamma'') + r_t(w_{t-1} - \tilde{z}_t^s(\gamma'')) \\ &= p_t(w_{t-1} + \varepsilon) - r_t\varepsilon \\ &= \alpha A(w_{t-1} + \varepsilon)^\alpha - r_t\varepsilon\end{aligned}$$

where $\Phi_t(0) = p_t w_{t-1} \geq \lambda r_t w_{t-1} + \pi_t$ due to $d_t^s(\gamma') = 1$. Thus, if $p_t w_{t-1} > \lambda r_t w_{t-1} + \pi_t$, then we set $\varepsilon = 0$ which shows that the joint surplus of period t, $\Phi_t(0)$, can be positive. Next, suppose that $p_t w_{t-1} = \lambda r_t w_{t-1} + \pi_t$ which is re-written by $p_t w_{t-1} - \lambda r_t w_{t-1} = \pi_t$ so that $\alpha A w_{t-1}^\alpha - \lambda r_t w_{t-1} = \pi_t \equiv \max_z \alpha A z^\alpha - r_t z$. This implies that $w_{t-1} < x_t$ because x_t maximizes $\alpha A x^\alpha - r_t x$ over $x \geq 0$. However, then $\Phi_t'(0) = \alpha^2 A w_{t-1}^{\alpha-1} - r_t = \alpha^2 A[w_{t-1}^{\alpha-1} - x_t^{\alpha-1}] > 0$. We can thus set a small enough $\varepsilon > 0$ yielding a posituve joint surplus of period t, $\Phi_t(0) > 0$, contradicting to $d_t^s(\gamma'') = 0$ again.

(ii) Suppose that $\delta > \frac{\lambda\alpha}{(1-m)(1-\alpha)}$.

Suppose contrary to the claim that $d_t^s(\gamma^n) = 0$ for all $t \geq s$ for some period s. Then, by above (i), $d_t^s(\gamma) = 0$ for all $\gamma \in \Gamma$ for all $t \geq s$. Then, we will show that this cannot be an equilibrium becasue there are the deviation contracts which are profitable for some relationships.

Case 1: $p_t z_t^\tau(\gamma) - \lambda r_t z_t^\tau(\gamma) \leq \pi_t$ for all $\tau > s$, for all $t \geq \tau$ and for all $\gamma \in \Gamma$. Then, $J_t^t = \lambda r_t w_{t-1} + \pi_t$ for all $t \geq s$, which implies that there exist no relationships engaging in relational contracts at all from period s onward in all the future periods. Thus, A-CME$_t$ implies that $\lambda w_t = x_{t+1}$ for all $t \geq s$, which shows that x_t converges to $x^* \equiv [\lambda(1-\alpha)A]^{1/(1-\alpha)}$ as $t \to \infty$. We will however show that Case 1 never happens. If not, set a new contract offered at period s as follows: $\tilde{z}_t^s(\gamma) \equiv x_t + \varepsilon$ ($\varepsilon > 0$) and $\tilde{d}_t^s(\gamma) = 1$ for all $t \geq s$ and for all γ satisfying $\delta \geq \frac{\lambda\alpha}{\gamma(1-m)(1-\alpha)}$ (such γ exists by assumption). Set $\tilde{d}_t^s(\gamma) = 0$ for all other γ. Let $\tilde{\Gamma} \equiv \left\{\gamma \in \Gamma \mid \delta \geq \frac{\lambda\alpha}{\gamma(1-m)(1-\alpha)}\right\}$. Then, since $\lambda w_{t-1} = x_t$ for all $t > s$, we have $w_{t-1} > x_t$ for all $t > s$. Thus $\tilde{z}_t^s(\gamma) \leq w_{t-1}$ holds for all $\gamma \in \tilde{\Gamma}$ if we take small enough $\varepsilon > 0$. However, then we obtain

$$\begin{aligned}p_t\tilde{z}_t^s(\gamma) - \lambda r_t\tilde{z}_t^s(\gamma) &> p_t x_t - \lambda r_t x_t \\ &\geq p_t x_t - r_t x_t \\ &= \pi_t\end{aligned}$$

for all $\gamma \in \tilde{\Gamma}$ where the first inequality follows from $p_t z - \lambda r_t z$ is increasing in $z < \hat{\lambda} x_t$ and $\tilde{z}_t^s(\gamma) = x_t + \varepsilon$. Thus the expected joint profit of period $t \geq s$ that is attained by the new contract becomes $\tilde{J}_t^s \equiv \sum_{\gamma^i \in \tilde{\Gamma}}[p_t\tilde{z}_t^s(\gamma^i) + \lambda r_t(w_{t-1} - \tilde{z}_t^s(\gamma^i))] + \sum_{\gamma^i \notin \tilde{\Gamma}}[\lambda r_t w_{t-1} + \pi_t]$ which can be larger than the equilibrium profit $\lambda r_t w_{t-1} + \pi_t$ for all $t \geq s$. This can be then a profitable deviation if the new contract is feasible. The feasibility will be shown for all large $t \geq s$, i.e., A-IC$_t^s(\gamma)$ for $\gamma \in \tilde{\Gamma}$ is satisfied for all large enough $t \geq s$. To see this, note that $x_t \to x^*$ as $t \to \infty$ and that $\delta \geq \frac{\lambda\alpha}{\gamma(1-m)(1-\alpha)}$ implies that, when $t \to \infty$, we have $\gamma\delta\{\tilde{J}_{t+1}^s - mJ_{t+1}^{t+1} - (1-m)(\lambda r_{t+1}w_t + \pi_{t+1})\} > \gamma\delta(1-m)\pi_t \geq \lambda r_t(x_t + \varepsilon)$ for a small enough ε where note that $J_{t+1}^{t+1} = \lambda r_{t+1}w_t + \pi_{t+1}$. Thus A-IC$_t^s(\gamma)$ holds for all $\gamma \in \Gamma_t^s$ for all large enough $t \geq s$. Thus the new contract becomes a profitable deviation for all large $t \geq s$.

Case 2: $\pi_t z_t^\tau(\gamma'') - \lambda r_t z_t^\tau(\gamma'') > \pi_t$ for some $\tau \geq s$, some $t \geq \tau$ and some $\gamma'' \in \Gamma$. Then, consider the new contract in such period s by mimicking the equilibrium contract used by the relationship pair formed in such future period $\tau > s$ as follows: i) $d_t^s(\gamma) = 0$ for all t where $s \leq t \leq \tau - 1$, ii) $\{z_t^\tau(\gamma), R_t^\tau(\gamma), d_t^\tau(\gamma)\}_{t \geq \tau}$ where set $d_t^s(\gamma) = 1$ if $p_t z_t^\tau(\gamma) - \lambda r_t z_t^\tau(\gamma) > \pi_t$ and set $d_t^s(\gamma) = 0$ if $p_t z_t^\tau(\gamma) - \lambda r_t z_t^\tau(\gamma) \leq \pi_t$. Note that there exist some $\gamma \in \Gamma$ and some period t for which $p_t z_t^\tau(\gamma) - \lambda r_t z_t^\tau(\gamma) > \pi_t$ by assumption. However, then the above modified contract is feasible and strictly improves the joint profit in any period t whenever $p_t z_t^\tau(\gamma) - \lambda r_t z_t^\tau(\gamma) > \pi_t$. This is a contradiction. Q.E.D.

Lemma A4. *In any equilibrium path*

$$\left(\frac{x_{t+1}}{x_t}\right) \geq \lambda(1-\alpha)A(1-\bar{l})x_t^{\alpha-1}$$

and $x_t \leq \max\{x_0, \overline{X}\}$ *for all* t *where*

$$\overline{X} \equiv \lambda\left(\frac{(1-\alpha)A}{1-\bar{l}}\right)^{\frac{1}{1-\alpha}}.$$

Proof. By using Lemma A1 ($w_t \geq z_{t+1}^s(\gamma)$ for all $\gamma \in \Gamma_{t+1}^s$), A-CME$_t$ implies that $\lambda w_t \geq (1-l_t)x_{t+1}$. Then, by using A-LME$_t$ and Lemma A2, we obtain

$$\begin{aligned}
\lambda w_t &= \lambda(1-\alpha)A\left((1-l_{t-1}x_t^\alpha + \sum_{s=0}^{t-1}\left(\sum_i q^i l_{t-1}^s[d_t^s(\gamma^i)z_t^s(\gamma^i)^\alpha + (1-d_t^s(\gamma^i))x_t^\alpha]\right)\right) \\
&\leq \lambda(1-\alpha)Ax_t^\alpha\left((1-l_{t-1}) + \sum_{s=0}^{t-1}\left(\sum_i q^i l_{t-1}^s[d_t^s(\gamma^i)(1/\lambda)^\alpha + (1-d_t^s(\gamma^i))]\right)\right) \\
&\leq \lambda(1-\alpha)Ax_t^\alpha\left((1-l_{t-1} + \sum_{s=0}^{t-1}\sum_i q^i l_{t-1}^s(1/\lambda)^\alpha)\right) \\
&= \lambda(1-\alpha)Ax_t^\alpha((1-l_{t-1}) + (1/\lambda)^\alpha l_{t-1}) \\
&\leq \lambda(1-\alpha)A(1/\lambda)^\alpha x_t^\alpha \\
&= \lambda^{1-\alpha}(1-\alpha)Ax_t^\alpha
\end{aligned}$$

which in turn implies that $\lambda^{1-\alpha}(1-\alpha)Ax_t^{\alpha} \geq \lambda w_t \geq (1-\bar{l})x_{t+1}$ for all t. Thus,

$$x_t \leq \overline{X} \equiv \lambda \left(\frac{(1-\alpha)A}{1-\bar{l}} \right)^{\frac{1}{1-\alpha}}$$

for all t when $x_0 \leq \overline{X}$. When $x_0 > \overline{X}$, $x_t \leq x_0$ for all t. In either case we have $x_t \leq \max\{x_0, \overline{X}\}$ for all t.

Second, by using Lemma A1 and Lemma A2 ($z_{t+1}^s(\gamma) \leq (1/\lambda)x_{t+1}$ for all $\gamma \in \Gamma_{t+1}^s$), A-CME$_t$ shows that

$$\begin{aligned} \lambda w_t &= \lambda \sum_{s=0}^{t} \sum_i q^i d_t^s(\gamma^i) l_t^s z_{t+1}^s(\gamma^i) + (1-l_t)x_{+1} + \sum_{s=0}^{t} \sum_i q^i (1-d_t^s(\gamma^i)) l_t^s x_{t+1} \\ &\leq \lambda \sum_{s=0}^{t} \sum_i q^i l_t^s (1/\lambda) x_{t+1} + \sum_{s=0}^{t} \sum_i q^i l_t^s x_{t+1} + (1-l_t)x_{t+1} \\ &= x_{t+1} \end{aligned}$$

which yields $x_{t+1} \geq \lambda w_t$. Then, we obtain $x_{t+1} \geq \lambda w_t = \lambda(1-\alpha)A(1-l_{t-1})x_t^{\alpha} \geq \lambda(1-\alpha)A(1-\bar{l})x_t^{\alpha}$ because $l_t \leq \bar{l}$ for all t. Thus,

$$\left(\frac{x_{t+1}}{x_t} \right) \geq \lambda(1-\alpha)A(1-\bar{l})x_t^{\alpha-1}.$$

Q.E.D.

Appendix D: Bequest transfer

In the main text we have assumed that any individual cannot give his/her child bequest at all. We now allow each individual to bequeath his/her child when old. The introduction of bequest may affect the analysis because the bequest can be used to finance capital investment. However, in what follows we will show that individuals have no incentives to bequeath their children in *any* equilibrium when the altruistic preference parameter $\delta > 0$ is in some small range. Then we can show the conditions under which the results we have already established in the main text remain true even when we introduce the bequest technology into the basic model as long as δ is in a certain small range.

First, consider a young A-producer born in $t-1$ who receives bequest b_{t-1} from his parent and wants to invest x_t in capital. Suppose also that he gives his child bequest $b_t \geq 0$. Then let (x_t, b_t) be a choice of an A-producer born in period $t-1$. Given this, the profit of an old A-producer is given as follows:

$$\pi(x_t, b_t | b_{t-1}) = \begin{cases} p_t x_t + \lambda(b_{t-1} - x_t)r_t - b_t & \text{if } b_{t-1} \geq x_t \\ p_t x_t - (x_t - b_{t-1})r_t - b_t & \text{if } b_{t-1} < x_t \end{cases} \qquad \text{(D1)}$$

When $b_{t-1} \geq x_t$, the bequest b_{t-1} can be used for financing x_t and the remaining amount $b_{t-1} - x_t$ can be saved to earn $\lambda r_t(b_{t-1} - x_t)$ in the credit market. Note here that lending of one unit in the credit market must be accompanied with the enforcement cost $1-\lambda$. When $b_{t-1} < x_t$, the A-producer needs to use the external financing and incur the borrowing cost $r_t(x_t - b_{t-1})$.

We suppose that each individual observes the past capital investments and bequest transfers made in the dynasty to which he or she belongs as well as he or she observes the past market prices. Let H_t be a set of all these histories observed to an A-producer up to period t. A strategy of a young A-producer is then defined as a mapping from H_t the set of observed histories up to period t to capital investment level $x_t \geq 0$ and bequest transfer to his child $b_t \geq 0$. Let $\sigma_t = (x_t, b_t) : H_t \to [0, \infty)^2$ be such strategy.

We will denote by $\pi(\sigma_t | b_{t-1})$ the profit of an A-producer who uses a strategy $\sigma_t = (x_t, b_t)$ in period t, given the bequest b_{t-1} received from his parent. Then a young A-producer born in period $t-1$ obtains his payoff as $\pi(\sigma_t|b_{t-1}) + \delta\pi(\sigma_{t+1}|b_t)$ and chooses his strategy σ_t to maximize this payoff subject to $b_t \geq 0$, given the previous period choice b_{t-1}.

In what follows we will impose the upper bound on the market interest rate r_t, which we denote by $d > 0$. In order to make exposition simple, for the time being, we will maintain the condition of $r_t \leq d$ in any equilibrium path. We will give more precise conditions on the primitives of the model which ensure this restriction later.

Then we show the following result:

Proposition D1. *Suppose that $\delta < 1/(1+d)$. Then in any equilibrium every A-producer born in period $t-1$ chooses $\sigma_t = \overline{\sigma}_t \equiv (x_t, 0)$ which gives his child no bequest $b_t = 0$ and makes capital investment x_t to maximize only his own current profit $\pi(x_t, 0|b_{t-1})$ given any bequest received from his parent b_{t-1} no matter the history.*

Proof. Suppose contrary to the claim that there exists an equilibrium with $\{\sigma_t\}_{t=1}^{\infty}$ in which some A-producer born in some period $t-1$ chooses $\sigma_t \neq \overline{\sigma}_t$. For σ_t to be an equilibrium choice, it must be that

$$\pi(\sigma_t|b_{t-1}) + \delta\pi(\sigma_{t+1}|b_t) \geq \pi(\overline{\sigma}_t|b_{t-1}) + \delta\pi(\sigma'_{t+1}|0) \quad \text{(D2)}$$

where σ'_{t+1} denotes the strategy chosen by the child of the A-producer who deviates from his equilibrium play σ_t to $\overline{\sigma}_t = (x_t, 0)$ where x_t maximizes $\pi(x_t, 0|b_{t-1})$.

Let $\hat{\sigma}_t \equiv (x_t, 0)$ be the strategy defined by setting bequest transfer equal to zero in the original strategies $\sigma_t = (x_t, b_t)$. Let also $\hat{\sigma}'_{t+1} \equiv (x'_{t+1}, 0)$.

Then, by using (D1), (D2) can be written by

$$\pi(\hat{\sigma}_t|b_{t-1}) - b_t + \delta\{\pi(\hat{\sigma}_{t+1}|b_t) - b_{t+1}\} \geq \pi(\overline{\sigma}_t|b_{t-1}) + \delta\{\pi(\hat{\sigma}'_{t+1}|0) - b'_{t+1}\}.$$

Since $\pi(\hat{\sigma}_{t+1}|b_t) \leq \pi(\hat{\sigma}_{t+1}|0) + r_{t+1}b_t$ due to $\lambda < 1$, the above inequality implies that

$$\begin{aligned} -(1-\delta r_{t+1})b_t - \delta b_{t+1} \;\geq\; & \{\pi(\overline{\sigma}_t|b_{t-1}) - \pi(\hat{\sigma}_t|b_{t-1})\} \\ & + \delta\{\pi(\hat{\sigma}'_{t+1}|0) - \pi(\hat{\sigma}_{t+1}|0)\} - \delta b'_{t+1} \\ \geq\; & \{\pi(\overline{\sigma}_t|b_{t-1}) - \pi(\hat{\sigma}_t|b_{t-1})\} \\ & + \delta\{\pi(\hat{\sigma}'_{t+1}|0) - \pi(\overline{\sigma}_{t+1}|0)\} - \delta b'_{t+1} \\ =\; & \Delta_t - \delta\Delta_{t+1} - \delta b'_{t+1} \end{aligned}$$

where $\Delta_t \equiv \pi(\overline{\sigma}_t|b_{t-1}) - \pi(\hat{\sigma}_t|b_{t-1}) \geq 0$ and $\Delta_{t+1} \equiv \pi(\overline{\sigma}_{t+1}|0) - \pi(\hat{\sigma}'_{t+1}|0) \geq 0$. Since $\sigma_t \neq \overline{\sigma}_t$, $\Delta_t > 0$ or/and $b_t > 0$ must hold.

Since $b_{t+1} \geq 0$, the above inequality implies

$$-(1-\delta r_{t+1})b_t - \Delta_t \geq -\delta\Delta_{t+1} - \delta b'_{t+1} \tag{D3}$$

For σ'_{t+1} to be the strategy chosen by the child of the A-producer after the latter chose $\overline{\sigma}_t$, it must be that

$$\pi(\sigma'_{t+1}|0) + \delta\pi(\sigma''_{t+2}|b'_{t+1}) \geq \pi(\overline{\sigma}_{t+1}|0) + \delta\pi(\sigma'_{t+2}|0) \tag{D4}$$

where σ''_{t+2} and σ'_{t+2} denote the strategies chosen by the child of the A-producer in period $t+1$ following σ'_{t+1} and $\overline{\sigma}_{t+1}$ respectively. Here the left hand side of (D4) is bounded above by

$$\pi(\hat{\sigma}'_{t+1}|0) - b'_{t+1} + \delta\{\pi(\overline{\sigma}_{t+2}|0) + r_{t+2}b'_{t+1} - b''_{t+2}\}$$

due to $\pi(\sigma''_{t+2}|b'_{t+1}) = \pi(\hat{\sigma}''_{t+2}|b'_{t+1}) - b''_{t+2} \leq \pi(\hat{\sigma}''_{t+2}|0) + r_{t+2}b'_{t+1} - b''_{t+2} \leq \pi(\overline{\sigma}_{t+2}|0) + r_{t+2}b'_{t+1} - b''_{t+2}$.

Then, by a similar argument to (D3), we can show that

$$-(1-\delta r_{t+2})b'_{t+1} - \Delta_{t+1} \geq -\delta\Delta_{t+2} - \delta b'_{t+2}. \tag{D5}$$

Similarly, for σ'_{t+2} to be chosen after the deviation $\overline{\sigma}_{t+1}$, we must have

$$-(1-\delta r_{t+3})b'_{t+2} - \Delta_{t+2} \geq -\delta\Delta_{t+3} - \delta b'_{t+3} \tag{D6}$$

which implies that

$$-b'_{t+2} \geq \frac{1}{1-\delta r_{t+3}}\{\Delta_{t+2} - \delta\Delta_{t+3} - \delta b'_{t+3}\} \tag{D7}$$

because of $1 > \delta d \geq \delta r_{t+3}$.

By applying the same condition to period $t+4$, we obtain

$$-b'_{t+3} \geq \frac{1}{1-\delta r_{t+4}}\{\Delta_{t+3} - \delta\Delta_{t+4} - \delta b'_{t+4}\}.$$

Then we show that

$$\begin{aligned} -\Delta_{t+3} - b'_{t+3} &\geq -\Delta_{t+3} + \frac{1}{1-\delta r_{t+4}}\{\Delta_{t+3} - \delta\Delta_{t+4} - \delta b'_{t+4}\} \\ &\geq \frac{\delta r_{t+4}}{1-\delta r_{t+4}}\Delta_{t+3} - \frac{\delta}{1-\delta r_{t+4}}(\Delta_{t+4} + b'_{t+4}) \\ &\geq -\frac{\delta}{1-\delta r_{t+4}}(\Delta_{t+4} + b'_{t+4}) \end{aligned}$$

due to $1 < \delta r_{t+4} \leq \delta d$, $b'_{t+4} \geq 0$ and $\Delta_{t+i} \geq 0$, $i = 3, 4$. Then (D7) must imply that

$$-b'_{t+2} \geq \frac{1}{1-\delta r_{t+3}}\left\{\Delta_{t+2} - \frac{\delta^2}{1-\delta r_{t+4}}(\Delta_{t+4} + b'_{t+4})\right\}.$$

By substituting this into b'_{t+2} in (D5), using the similar argument repeatedly and noting that $r_t \leq d$ for all t, we can show that

$$\begin{aligned} -(1-\delta r_{t+2})b'_{t+1} - \Delta_{t+1} &\geq -\delta\Delta_{t+2} - \delta b'_{t+2} \\ &\geq -\delta\Delta_{t+2} + \frac{\delta}{1-\delta r_{t+3}}\left\{\Delta_{t+2} - \frac{\delta^2}{1-\delta r_{t+4}}(\Delta_{t+4} + b'_{t+4})\right\} \\ &= \frac{\delta^2 r_{t+3}}{1-\delta r_{t+3}}\Delta_{t+2} - \delta\left(\frac{\delta^2}{(1-\delta r_{t+3})(1-\delta r_{t+4})}\right)(\Delta_{t+4} + b'_{t+4}) \\ &\geq -\delta\left(\frac{\delta}{1-\delta d}\right)^T(\Delta_{t+T+1} + b'_{t+T+1}) \end{aligned}$$

for arbitrary integer T.

Since T is arbitrary and Δ_s and b'_s are bounded above for all $s \geq 0$,[32] the right hand side converges to zero by taking $T \to \infty$, given $\delta/(1-\delta d) < 1$ which is equivalent to $\delta < 1/(1+d)$. Thus the right hand side of (D5) is bounded below from zero. However, since the left hand side of (D5) is strictly negative because $1 > \delta d \geq \delta r_{t+1}$ and $b_t > 0$ or/and $\Delta_t > 0$, this is a contradiction. Q.E.D.

We also show that every A-lender has no incentives to give their children positive bequest in any equilibrium when $\delta < 1/(1+d)$.

Proposition D3. *Suppose that $\delta < 1/(1+d)$. Then A-lenders leave no bequest to their children at all in any equilibrium.*

Proof. Suppose that $\delta < 1/(1+d)$. Suppose also contrary to the claim of that there exists some equilibrium in which some A-lender in some dynasty gives a positive bequest

[32] Note here that $b'_s \leq \max_{x_s \geq 0} \pi(x_s, 0|0) < +\infty$ and $\Delta_s \leq \max_{x_s} \pi(x_s, 0|0) < +\infty$.

$b_t > 0$ in some period t where $b_t \in [0, w_t]$. For this to be an equilibrium choice, the equilibrium payoff of that lender in period t

$$\lambda r_t(w_{t-1} + b_{t-1}) - b_t + \delta\{\lambda r_{t+1}(w_t + b_t) - b_{t+1}\}$$

must not be less than

$$\lambda r_t(w_{t-1} + b_{t-1}) + \delta\{\lambda r_{t+1}w_t - b'_{t+1}\}$$

which can be obtained by offering no bequest $b'_t = 0$ which is then followed by the choice of his child b'_{t+1} in the next period $t+1$. This condition thus yields

$$-(1 - \delta\lambda r_{t+1})b_t - \delta b_{t+1} \geq -\delta b'_{t+1} \tag{D8}$$

For b'_{t+1} to be an optimal choice by the child of the deviating lender in period $t+1$, we must have that

$$\begin{aligned} &\lambda r_{t+1}w_t - b'_{t+1} + \delta\{\lambda r_{t+2}(w_{t+1} + b'_{t+1}) - b'_{t+2}\} \\ \geq\ &\lambda r_{t+1}w_t + \delta\{\lambda r_{t+2}w_{t+1} - b''_{t+2}\} \end{aligned}$$

where b'_{t+2} is the bequest choice by the lender in $t+2$ following his parent's choice b'_{t+1} whereas b''_{t+2} is the choice by the same lender following no bequest given by his parent. This is simplified to

$$-(1 - \delta\lambda r_{t+2})b'_{t+1} \geq -\delta b''_{t+2} \tag{D9}$$

By combining (D8) with (D9), we obtain

$$\begin{aligned} -(1 - \delta\lambda r_{t+1})b_t &\geq -\delta b'_{t+1} \\ &\geq -\frac{\delta^2}{1 - \delta\lambda r_{t+2}} b''_{t+2} \end{aligned}$$

where the first inequality follows from $b_{t+1} \geq 0$ and the second inequality from (D9) respectively where $1 > \delta d\delta \geq \delta\lambda r_{t+2}$ due to $1/(1+d) > \delta$ and $1 > \lambda$. Repeating the similar argument over period $t+3$, the right hand side of the above inequality is bounded below from

$$-\frac{\delta^3}{(1 - \delta\lambda r_{t+2})(1 - \delta\lambda r_{t+3})} b''_{t+3} \tag{D10}$$

Repeating this process for any period $t + T \geq t$, (D10) is bounded below from

$$-\frac{\delta^T}{\Pi_{i=1}^{T-1}(1 - \delta\lambda r_{t+i})} b''_{t+T}$$

which is further bounded below from

$$-(1 - \delta\lambda d)\left(\frac{\delta}{1 - \delta\lambda d}\right)^T b''_{t+T}$$

because of $d \geq r_{t+i}$, $i \geq 1$. Since T can be arbitrary, $b''_{t+T} \leq w_{t+T} < +\infty$, and $\delta < 1 - \delta\lambda d$, this converges to zero by taking $T \to \infty$. Thus the right hand side of (D8) must converge to zero but its left hand side is strictly negative because of $b_t > 0$, which is a contradiction. Q.E.D.

Next we consider how the opportunity of bequest transfer affects the choice of the optimal relational contracts in equilibrium. When a young R-producer receives a bequest b^p_{t-1} from his parent in period $t-1$, he can use this to invest z_t in capital. When a young R-lender receives a bequest b^l_{t-1} from her parent in period $t-1$, she will have the total fund $w_{t-1} + b^l_{t-1}$ which will be invested in capital investment for the R-producer matching her.

We denote by V^p_t and V^l_t the profits of a R-producer and a R-lender without repayment R_t and bequest transfer b^i_t, $i = p, l$, leaving to their children respectively.

There are three possible cases which may occur in equilibrium.

Case (1). $b^p_{t-1} \geq z_t$. Then a R-producer invests z_t in capital by himself and saves the remaining amount $b^p_{t-1} - z_t$. Then we have $V^p_t = p_t z_t + \lambda r_t(b^p_{t-1} - z_t)$ and $V^l_t = \lambda r_t(w_{t-1} + b^l_{t-1})$.

Case (2). $z_t > b^p_{t-1}$. There are two sub-cases:

Case (2-1). $w_{t-1} + b_{t-1} \geq z_t$ where $b_{t-1} \equiv b^p_{t-1} + b^l_{t-1}$. In this case a young R-producer borrows $z_t - b^p_{t-1}$ from the R-lender matching him for capital investment z_t. Then we have $V^p_t = p_t z_t$ and $V^l_t = \lambda r_t(w_{t-1} + b_{t-1} - z_t)$.

Case (2-2). $z_t > w_{t-1} + b_{t-1}$. Then the R-producer needs to borrow $z_t - w_{t-1} - b_{t-1}$ from the credit market after receiving $w_{t-1} + b^l_{t-1}$ from the R-lender and using his own fund b^p_{t-1}. Thus we have $V^p_t = p_t z_t + r_t(w_{t-1} + b_{t-1} - z_t)$ and $V^l_t = 0$.

Thus the incentive compatibility condition for an old R-producer becomes

$$V^p_t - R_t - b^p_t + \delta\{V^p_{t+1} - R_{t+1} - b^p_{t+1}\} \geq V^p_t - \hat{b}^p_t + \delta\pi(x_{t+1}, \hat{b}^p_{t+1}|\hat{b}^p_t) \qquad (\text{BIC}_t)$$

where $\hat{b}^p_t$ denotes the bequest transfer of an old R-producer after quitting from the relationship and engaging in arm's length contracts, and $\pi(x_{t+1}, \hat{b}^p_{t+1}|\hat{b}^p_t)$ denotes the profit of an A-producer (defined by (D1)). The individual rationality constraint for a young R-lender becomes

$$V^l_t + R_t - b^l_t + \delta\{V^l_{t+1} + R_{t+1} - b^l_{t+1}\} \geq \lambda r_t(w_{t-1} + b^l_{t-1}) - \hat{b}^l_t + \delta\{\lambda r_{t+1}(w_t + \hat{b}^l_t) - \hat{b}^l_{t+1}\} \qquad (\text{BIRL}_t)$$

where $\hat{b}^l_t$ denotes the bequest transfer of a R-lender after quitting relationship. Thus, by combining these conditions together, we have the modified incentive compatibility condition:

$$\begin{aligned} & - b_t + V^l_t + \delta\{J_{t+1} - b_{t+1} - \pi(x_{t+1}, \hat{b}^p_{t+1}|\hat{b}^p_t) - [\lambda r_{t+1}(w_t + \hat{b}^p_t) - \hat{b}^l_{t+1}]\} \\ & \geq \lambda r_t(w_{t-1} + b^l_{t-1}) - \hat{b}_t \end{aligned} \qquad (\text{BIC}^*_t)$$

where $b_t \equiv b^p_t + b^l_t$ and $\hat{b}_t \equiv \hat{b}^p_t + \hat{b}^l_t$ and $J_{t+1} \equiv V^p_{t+1} + V^l_{t+1}$.

Also the individual rationality constraint for a young R-producer is

$$V_t^p - R_t - b_t^p + \delta\{V_{t+1}^p - R_{t+1} - b_{t+1}^p\} \geq \pi(x_t, \hat{b}_t^p | b_{t-1}^p) + \delta\pi(x_{t+1}, \hat{b}_{t+1}^p | \hat{b}_t^p) \quad (\text{BIRP}_t)$$

By combining BIRL_t with BIRP_t, we obtain the net total surplus condition:

$$\begin{aligned} J_t - b_t + \delta\{J_{t+1} - b_{t+1}\} \;&\geq\; \pi(x_t, \hat{b}_t^p | b_{t-1}^p) + \delta\pi(x_{t+1}, \hat{b}_{t+1}^p | \hat{b}_t^p) \\ &+\; \lambda r_t(w_{t-1} + b_{t-1}^l) - \hat{b}_t^l + \delta\{\lambda r_{t+1}(w_t + \hat{b}_t^l) - \hat{b}_{t+1}^l\} \quad (\text{BTS}_t) \end{aligned}$$

Let $V_t \equiv J_t - b_t + \delta\{J_{t+1} - b_{t+1}\}$ denote the joint payoff of a young R-producer and a young R-lender born in period $t-1$. Then, each relationship pair chooses a sequence $\{z_t, b_t^p, b_t^l\}_{t=0}^{\infty}$ in order to maximize the weighted sum of the joint payoffs $\sum_{t=0}^{\infty} \beta^t V_t = J_0 - b_0 + (\delta + \beta)\sum_{t=1}^{\infty} \beta^{t-1}(J_t - b_t)$ subject to BIC_t^* and BTS_{t-1} for $t = 1, 2, ...$

We first show the following lemma.

Lemma D1. *In any equilibrium path $z_t \leq \hat{\lambda} x_t$ holds in any period t.*

Proof. We consider the equilibrium choice of z_t. If above Case (1) is applied to the equilibrium choice of z_t in period t, then z_t does not affect BIC_t^*. z_t affects only the left hand sides of BIC_{t-1}^*, BTS_t and BTS_{t-1} through the term J_t. Since $J_t = p_t z_t + \lambda r_t(w_{t-1} + b_{t-1} - z_t)$ in Case (1), z_t must maximize J_t subject to $b_t^p \geq z_t$. Thus $z_t \leq \hat{\lambda} x_t$ must be satisfied. If Case (2-1) is applied to z_t, then z_t affects the left hand side of BIC_t^* through the term $\lambda r_t z_t$ as well as the left hand sides of BIC_{t-1}, BTS_t and BTS_{t-1} through the term J_t. Since $J_t = p_t z_t + \lambda r_t(w_{t-1} + b_{t-1} - z_t)$ in Case (2-1), the equilibrium choice of z_t must maximize J_t subject to BIC_t^*. Thus $z_t \leq \hat{\lambda} x_t$ must be satisfied as well. Finally, if Case (2-2) is applied to z_t, then z_t affects only BTS_t, BTS_{t-1} and BIC_{t-1}^* through the term J_t. Thus, since $J_t = p_t z_t + r_t(w_{t-1} + b_{t-1} - z_t)$ in Case (2-2), z_t must maximize J_t in equilibrium, resulting in $z_t = x_t < \hat{\lambda} x_t$.

Thus we have established that $z_t \leq \hat{\lambda} x_t$ in any period t. Q.E.D.

We then show the following lemma.

Lemma D2. *Let $\tilde{x}$ be defined as*

$$\tilde{x} \equiv \left(\frac{\lambda(1-\alpha)A(1-l)}{l\hat{\lambda} + (1-l)} \right)^{1/(1-\alpha)}.$$

Then, it must be that $r_t \leq d \equiv \alpha^2 A[\min\{\tilde{x}, x_0\}]^{\alpha-1}$ in any period t in any equilibrium path.

Proof. There are three distinct cases of the credit market equilibrium depending on whether or not each producer invests more than what he owns.

Case (i): $w_{t-1} + b_{t-1} \geq z_t$ and $x_t > \hat{b}^p_{t-1}$. In this case the credit market equilibrium becomes

$$\lambda l(w_{t-1} + b_{t-1} - z_t) + \lambda(1-l)(w_{t-1} + \hat{b}^l_{t-1}) = (1-l)(x_t - \hat{b}^p_{t-1})$$

which implies that $\lambda w_{t-1} \leq \lambda l z_t + (1-l)x_t \leq (\lambda l\hat{\lambda} + (1-l))x_t$ due to Lemma D1 ($z_t \leq \hat{\lambda}x_t$).

Note that both $w_{t-1}+b_{t-1} \geq z_t$ and $x_t \leq \hat{b}^p_{t-1}$ are never compatible with each other in the credit market equilibrium.[33] Thus, the remaining cases are the following:
Case (ii): $z_t > w_{t-1} + b_{t-1}$ and $x_t > \hat{b}^p_{t-1}$. In this case the credit market equilibrium becomes

$$\lambda(1-l)(w_{t-1} + \hat{b}^p_{t-1}) = l(z_t - w_{t-1} - b_{t-1}) + (1-l)(x_t - \hat{b}^p_{t-1})$$

which implies that $(\lambda(1-l) + (1-l))w_{t-1} \leq (l\hat{\lambda} + (1-l))x_t$.
Case (iii): $z_t > w_{t-1} + b_{t-1}$ and $x_t \leq \hat{b}^p_{t-1}$. In this case the credit market equilibrium becomes

$$\lambda(1-l)(w_{t-1} + \hat{b}^l_{t-1}) + \lambda(1-l)(\hat{b}^p_{t-1} - x_t) = l(z_t - w_{t-1} - b_{t-1})$$

which implies that $(\lambda(1-l) + l)w_{t-1} \leq (l\hat{\lambda} + (1-l)\lambda)x_t$.

By Case (i)-(iii) above, it must be that $\lambda w_{t-1} \leq (l\hat{\lambda}+(1-l))x_t$ which together with LME$_t$ implies that $\lambda(1-\alpha)A(1-l)x^{\alpha}_{t-1} \leq (l\hat{\lambda}+(1-l))x_t$. Thus $x_t \geq \min\{\tilde{x}, x_0\}$ must be satisfied. Since $r_t = \alpha^2 A x_t^{\alpha-1}$, we conclude that $r_t \leq d$ in any period t. Q.E.D.

In what follows we fix δ such that $\delta < 1/(1+d)$ for d being defined in Lemma D2. Then we show that introducing bequest transfer never expands the set of incentive compatible relational contracts under the following condition.

Assumption D1. $\min\{x_0, \tilde{x}\} \geq (1/(1-\alpha))^{1/\alpha}(\delta\alpha^2 A)^{1/(1-\alpha)}$.

In particular, we show that under Assumption D1, the equilibrium relational contract which maximizes the weighted sum of expected payoffs of all the generations in each dynasty uses no bequest transfers at all. Recall that $\sum_{t=0}^{\infty}\beta^t V_t$ is the weighted sum of joint payoffs of generations in a dynasty where $\sum_{t=0}^{\infty}\beta^t V_t = J_0 - b_0 + (\delta + \beta)\sum_{t=1}^{\infty}\beta^{t-1}J_t$.

Proposition D4. *Suppose that $\beta \leq \delta < \min\{1/(1+d), 1/d - \beta\}$ and that Assumption D1 holds. Then the optimal relational contract involves no bequest transfers at all even when bequest transfer is allowed.*

[33] If this is the case, credit demand is zero while credit supply is positive.

Proof. Suppose that $\delta < 1/(1+d)$. Then Proposition D2 and D3 show that $\hat{b}_t^p = \hat{b}_t^l = 0$ in any period t. Thus, (BIC$_t^*$) is simplified to

$$-b_t + V_t^l + \delta\{J_{t+1} - b_{t+1} - \pi(x_{t+1}, 0|0) - \lambda r_{t+1} w_t\} \geq \lambda r_t(w_{t-1} + b_{t-1}^l) \qquad (\text{BIC}_t^*)$$

Here, we have

$$-b_t + \delta\{J_{t+1} - b_{t+1} - \pi(x_{t+1}, 0|0) - \lambda r_{t+1} w_t\} \geq 0 \qquad (\text{BIC}_t^* \text{ in Case (1)})$$

$$-b_t + \delta\{J_{t+1} - b_{t+1} - \pi(x_{t+1}, 0|0) - \lambda r_{t+1} w_t\} \geq \lambda r_t(z_t - b_{t-1}^p) \quad (\text{BIC}_t^* \text{ in Case (2-1)})$$

$$-b_t + \delta\{J_{t+1} - b_{t+1} - \pi(x_{t+1}, 0|0) - \lambda r_{t+1} w_t\} \geq \lambda r_t(w_{t-1} + b_{t-1}^l) \quad (\text{BIC}_t^* \text{ in Case (2-2)})$$

Next we show that BTS$_t$ for $t \geq 1$ is implied by other constraints in any period $t \geq 1$. First note that BIC$_{t-1}^*$ implies that $J_t - b_t \geq \pi(x_t, 0|0) + \lambda r_t w_t$. Also we can show the following claim:

Claim 1. The choice of $b_t \geq 0$ which maximizes $-b + \delta\pi(x_t, 0|b)$ becomes zero ($b = 0$).
Proof. Recall that $\pi(x_t, 0|b) = p_t x_t - r_t(x_t - b)$ if $x_t > b$ and $\pi(x_t, 0|b) = p_t x_t + \lambda r_t(b - x_t)$ if $b \geq x_t$ respectively. Here x_t must be chosen in order to maximize $\pi(x, 0|b)$ over $x \geq 0$ in equilibrium due to Proposition D1. Let $x(r_t)$ be the maximizer of $p_t x_t - r_t x_t$ and $x(\lambda r_t)$ be the maximizer of $p_t x_t - \lambda r_t x_t$. Then $x_t \in \{x(r_t), b, x(\lambda r_t)\}$ must be satisfied in any equilibrium. Thus $\pi(x_t, 0|b)$ varies with b only when $x(r_t) \leq b \leq x(\lambda r_t)$. Since $-1 + \delta\alpha^2 A b^{\alpha-1} \leq -1 + \delta r_t \leq -1 + \delta d < 0$ over $b \in [x(r_t), x(\lambda r_t)]$. Thus $b = 0$ is the optimal choice to maximize $-b + \delta\pi(x_t, 0|b)$. Q.E.D.

By using this claim, we can show that for any $b' \geq 0$ and $b'' \geq 0$, $\pi(x_{t+1}, 0|0) + \lambda r_{t+1} w_t \geq \pi(x_{t+1}, 0|b') - (1/\delta)b' + \lambda r_{t+1}(w_t + b'') - (1/\delta)b''$ holds. Thus, $\pi(x_{t+1}, 0|0) + \lambda r_{t+1} w_t \geq \pi(x_{t+1}, 0|b_t^p) - (1/\delta)b_t^p + \lambda r_{t+1}(w_t + b_t^l) - (1/\delta)b_t^l$. Then, BIC$_t^*$ implies that $-b_t + V_t^l + \delta\{J_{t+1} - b_{t+1} - \pi(x_{t+1}, 0|b_t^p) - \lambda r_{t+1}(w_t + b_t^l)\} \geq \lambda r_t(w_{t-1} + b_{t-1}^l)$ which shows that $J_{t+1} - b_{t+1} \geq \pi(x_{t+1}, 0|b_t^p) + \lambda r_{t+1}(w_t + b_t^l)$ for all $t \geq 1$. Since BIC$_{t+1}^*$ also implies that $J_{t+2} - b_{t+2} \geq \pi(x_{t+2}, 0|0) + \lambda r_{t+2} w_{t+1}$, BTS$_t$ is satisfied in any period $t \geq 2$.

Finally we show that BTS$_1$ holds as well. Since $J_2 - b_2 \geq \pi(x_2, 0|0) + \lambda r_2 w_1$, BTS$_1$ is satisfied if $J_1 - b_1 \geq \pi(x_1, 0|b_0^p) + \lambda r_1(w_0 + b_0^l)$. To see this, consider BTS$_0$: $J_0 - b_0 + \delta\{J_1 - b_1\} \geq \pi_0 + \delta\pi(x_1, 0|0) + \delta r_1 w_0$. Here any initial old lender has no incomes at all and any initial old producer earns the profit π_0. Thus $J_0 = \pi_0$. Since $J_0 = \pi_0 \geq b_0 \geq b_0^p$ and $-b + \delta\pi(x_1, 0|b)$ is maximized at $b = 0$ again (Claim 1 above), BTS$_0$ implies that $-b_0 + \delta\{J_1 - b_1\} \geq -b_0^p + \delta\pi(x_1, 0|b_0^p) + \delta r_1 w_0$ which in turn implies that

$$\begin{aligned}
\delta\{J_1 - b_1 - \pi(x_1, 0|b_0^p)\} &\geq b_0^l + \delta\lambda r_1 w_0 \\
&\geq \delta r_1 b_0^l + \delta\lambda r_1 w_0 \\
&= \delta\lambda r_1(w_0 + b_0^l)
\end{aligned}$$

because $1 > \delta d \geq \delta r_1$. Thus $J_1 - b_1 \geq \pi(x_1, 0|b_0^p) + \lambda r_1(w_0 + b_0^l)$ and BTS_t is satisfied for all $t \geq 1$.

Thus we have established that only relevant constraint of BTS_t is BTS_0:

$$-b_0 + \delta\{J_1 - b_1\} \geq \delta\pi(x_1, 0|0) + \delta r_1 w_0 \qquad (\text{BTS}_0)$$

where $\hat{b}_0^p = 0$ and $J_0 = \pi_0$.

Recall that $V_t \equiv J_t - b_t + \delta(J_{t+1} - b_{t+1})$ be the joint payoff of a R-producer and a R-lender in t-th generation. Then the equilibrium relational contract $\{z_t, b_t^p, b_t^l\}_{t=0}^{\infty}$ must maximize the weighted sum of expected payoffs of all future generations $\sum_{t=0}^{\infty} \beta^t V_t = J_0 - b_0 + (\delta + \beta) \sum_{t=1}^{\infty} \beta^{t-1}(J_t - b_t)$ subject to BIC_t^*, $t \geq 1$, and BTS_0.

First, observe that $b_0^p = b_0^l = 0$ must hold because $-b_0 + \delta J_1$ is maximized at $b_0 = 0$ due to $1 > \delta d \geq \delta r_t$ and BIC_1^* is relaxed by reduction of b_0^l. Since $\delta < 1/d - \beta$ also holds by assumption, $-b_0 + (\delta + \beta)J_1$ is maximized at $b_0 = 0$. Thus $b_0 = 0$ becomes optimal. Second, $b_t^l = 0$ must be also satisfied for $t \geq 1$. To see this, note that reduction of b_t^l from any positive level makes all the relevant constraints BIC_t^*, BIC_{t-1}^*, BIC_{t+1}^* and BTS_0 satisfied because $-b_t^l + \delta J_{t+1}$ is increased due to $1 > \delta d$ and $r_t \leq d$. Since the joint payoff $-b_t^l + \beta J_{t+1}$ is improved by reducing $b_t^l > 0$ due to $1 \geq \beta d$ and $d \geq r_t$, $b_t^l = 0$ becomes optimal.

Finally, consider the equilibrium choice of b_t^p for $t \geq 1$. We first show the following claim.

Claim 2. Case (2-2) ($z_t > w_{t-1} + b_{t-1}$) never happens in any period t.
Proof. Suppose that Case (2-2) occurs in some period t. In this case the equilibrium choice of z_t must maximize $J_t = p_t z_t + r_t(z_t - w_{t-1} - b_{t-1})$ because this makes all the relevant constraints relaxed while increasing the joint payoff. Thus $z_t = x_t$ must be satisfied. However, then CME_t implies that $\lambda(1-l)w_{t-1} = l(z_t - w_{t-1} - b_{t-1}) + (1-l)x_t$ where $\hat{b}_{t-1}^l = \hat{b}_{t-1}^p = 0$ and hence $\lambda w_{t-1} > x_t$. Thus we must have $w_{t-1} > x_t = z_t$ which however contradicts to $z_t > w_{t-1} + b_{t-1}$. Q.E.D.

Case (2-1) is only the case that $b_t^p > 0$ may happen in equilibrium because $b_t^p > 0$ relaxes BIC_t^* in Case (2-1). Now, we use the change of variable $\tilde{z}_{t+1} \equiv z_{t+1} - b_t^p$. Then, note that b_t^p affects only $-b_t^p + \delta J_{t+1}$ in all the relevant constraints through the term $p_{t+1}(b_t^p + \tilde{z}_{t+1})$. If Case (1) or (2-1) is applied to period $t+1$, then $J_{t+1} = p_{t+1}(b_t^p + \tilde{z}_{t+1}) + \lambda r_{t+1}(w_t - \tilde{z}_{t+1})$. Since maximizing $-b_t^p + \delta J_{t+1}$ and choosing $b_t^p = 0$ can relax all the relevant constraints while maximizing $-b_t^p + \beta J_{t+1}$ improves the objective function, it suffices to show that $b_t^p = 0$ maximizes $-b_t^p + \delta J_{t+1}$ and $-b_t^p + \beta J_{t+1}$. Taking the first order derivative of the former, we obtain $-1 + \delta\alpha^2 A(b_t^p + \tilde{z}_{t+1})^{\alpha-1}$. If this is positive for some period $t+1$, we have $\tilde{z}_{t+1} + b_t^p < (\delta\alpha^2 A)^{1/(1-\alpha)}$. Then we obtain

$$\begin{aligned} & p_{t+1}(\tilde{z}_{t+1} + b_t^p) - \pi_{t+1} \\ \leq\ & \alpha A(\delta\alpha^2 A)^{\alpha/(1-\alpha)} - (1-\alpha)\alpha A x_{t+1}^{\alpha} \\ \leq\ & 0 \end{aligned}$$

because $x_{t+1} \geq (1/(1-\alpha))^{1/\alpha}(\delta\alpha^2 A)^{1/(1-\alpha)}$ which holds due to Assumption D1 and the fact that $x_t \geq \min\{x_0, \tilde{x}\}$ for all $t \geq 1$. However, then the net total surplus $J_{t+1} - b_{t+1} - \pi_{t+1} - \lambda r_{t+1} w_t$ becomes strictly negative so that BIC_t^* never holds, a contradiction. Thus $b_t^p = 0$ which maximizes $-b_t^p + \delta J_{t+1}$ must be satisfied in any period t in any equilibrium. Also, since $\beta \leq \delta$ as we assumed, $-b_t^p + \beta J_{t+1}$ is maximized at $b_t^p = 0$. Thus, the equilibrium relational contract must involve $b_t^p = 0$ for all $t \geq 1$ as well. Q.E.D.

Finally we make the remark on the conditions we have so far made above: $\beta \leq \delta < 1/(1+d)$, $\delta + \beta < 1/d$ and Assumption D1. We show that there is an open set of the parameter values for which these conditions hold together with Assumption 1 and 2 made in the main text. To see this, assume that x_0 is small so that $x_0 < \tilde{x}$. Then $d = \alpha^2 A x_0^{\alpha-1}$. Then Assumption D1 is equivalent to $\delta \leq (1-\alpha)^{(1-\alpha)/\alpha}/d$. Since d is independent of λ and l, Assumption 2 can be consistent with $\delta < 1/(1+d)$, $\delta + \beta < 1/d$, $\beta \leq \delta$ and Assumption D1 if λ and l are both small. Assumption 1 also holds when x_0 is small.