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Abstract

This paper analyzes the determinants of government debt and social security for the old in a closed-economy, overlapping-generation model. Under the probabilistic voting, the model presents (i) an intergenerational link of resource allocation via debt and social security; (ii) multiple political equilibria; and (iii) a negative correlation between tax and debt. These three results are robust to the introduction of public goods as an alternative government expenditure or to the introduction of income heterogeneity within a generation.

Key words: Government debt; Social security; Overlapping generations; Probabilistic voting

JEL Classification: D72, H55, H63

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1 Introduction

Many OECD countries have experienced the aging of their population over the past two decades (OECD, 2010). Aging increases the share of old voters in population, providing an incentive for politicians to expand expenditure on the old (Galasso, 2006). Given the budgetary constraint, politicians might choose to finance increased expenditure by issuing more government debt. This choice shifts the fiscal burden from current to future generations, resulting in an increase in the debt-to-GDP ratio (Roseveare et al., 1996; Yashiro, 1997).

Evidence seems to suggest that this scenario is playing out in Japan. Over the past two decades, the share of the old over 65 years old has increased by 50%, the ratio of social security expenditure to GDP has increased by 63%, and the debt-to-GDP ratio has almost doubled, from 86% in 1995 to 172% in 2008 (OECD, 2010). However, the scenario does not necessarily hold for other OECD countries. For the last two decades, the debt-to-GDP ratio has always been over 100% in Greece and Italy. In contrast, the ratio decreased by half in Australia, Denmark, Sweden, and New Zealand. In addition, the ratio has been low and stable at approximately 50% in Norway (OECD, 2010). The evidence suggests the need to consider another factor that explains the differences in debt-to-GDP ratio between countries.

Song, Storesletten and Zilibotti (2011) provided a new insight, that is, the differences between countries' public goods preferences, into the problem of the debt-to-GDP ratio. They developed a small, open, overlapping-generation model with many countries in which countries differ in their public goods preferences. In this framework, they employed probabilistic voting on public goods provision for which, in each period, the amount of public goods is determined to maximize the weighted sum of the utilities of young and old generations (see, for example, Grossman and Helpman (1998), Hassler et al. (2005) and Song (2011), which adopt the probabilistic voting in an overlapping-generation model).

Under the aforementioned voting mechanism, Song, Storesletten and Zilibotti (2011) characterized a Markovian political equilibrium in which policy in each period is conditioned on a payoff-relevant state variable (i.e., government debt). In this equilibrium, a country with a strong preference for public goods attains a tight fiscal policy with low tax and low debt accumulation, whereas the other countries experience loose fiscal policy with high tax and high debt accumulation. Specifically, debt is accumulated up to the natural debt limit in the latter group of countries when the labor supply is inelastic.

Song, Storesletten and Zilibotti (2011) considered Scandinavian countries in the former category and Greece and Italy in the latter. However, this interpretation does not fit the empirical evidence in terms of taxes. The data suggest that Scandinavian countries implement higher tax rates than do Greece and Italy (OECD, 2010). In other words, the positive correlation between tax and the debt-to-GDP ratio, which was shown in Song, Storesletten and Zilibotti (2011), does not well explain the empirical tax and debt evidence from OECD countries.

Röhrs (2010) extended the model of Song, Storesletten and Zilibotti (2011) by assuming a closed economy in which government debt is owned only by residents. In her framework, debt level is below the debt limit regardless of the public goods preference. Therefore, a closed-economy assumption resolves the accumulation of debt to the upper bound level observed in the model of Song, Storesletten and Zilibotti (2011). However, her closed-economy model breaks an intertemporal link via debt: there is no effect of government debt on intertemporal resource allocation. In addition, her model still shows a positive correlation between tax and debt.

The aim of this paper is to reconsider the models of Song, Storesletten and Zilibotti (2011) and Röhrs (2010). Specifically, our analysis is based on the model by Röhrs (2010) but differs from hers in that we introduce a social security payment to the old as a government expenditure (see, for example, Grossman and Helpman, 1998; Azariadis and Galasso, 2002; Conde-Ruiz, Galasso and Profeta, 2011). Our assumption is motivated by the rapid increase of social security's contribution to government expenditure due to population aging (OECD, 2011). Under this alternative framework, we demonstrate the following three results, which were not shown in Röhrs (2010): (i) an intertemporal link of resource allocation via debt; (ii) multiple political equilibria; and (iii) a negative correlation between tax and debt.

The mechanism behind the first result is as follows. In the framework of Röhrs (2010), the government expenditure is limited to a public goods provision that is enjoyed by both the young and the old: there is no intergenerational resource reallocation via debt politics. In contrast, in the current framework with social security, a higher level of social security payment to the initial old is financed at the expense of successive generations' loss of consumption: there is an intergenerational transfer of private goods from the young to the old. The introduction of social security into the framework of Röhrs (2010) restores an intertemporal link of resource allocation via government debt.

The second and the third results are obtained in the following way. In our closed-economy framework, one unit of government bond is equivalent to one unit of saving. Because old-age consumption is the sum of saving and social security, government bond repayment and social security payment are perfect substitutes for households. Given this feature, probabilistic voting on fiscal policy results in a Markovian social security policy function that produces a one-to-one trade-off between government debt and social security. With this policy function, the government expenditure, comprising debt repayment and social security payment, becomes constant over time; that is, for financing expenditure, an elected government sees no difference between tax and government debt issue. This property produces the second result, the multiplicity of equilibria, and the third result, a negative correlation between tax and debt.

We introduce two extensions to the basic model to determine the robustness of our result. The first extension is to introduce a public goods provision as an alternative means of government spending; the second extension is to allow income heterogeneity including two types of agents, the rich and the poor. We show that the three results in the basic model still hold under these extended frameworks. Our analysis and results therefore suggest that old-age social security is a crucial and strong factor in characterizing Markovian political equilibrium in an overlapping-generation model with government debt.

In addition to Song, Storesletten and Zilibotti (2011) and Röhrs (2010), our work is related to several others in the literature including Grossman and Helpman (1998), Azariadis and Galasso (2002), Hassler et al. (2003), Hassler et al. (2005), Hassler, Storesletten and Zilibotti (2007), Gonzalez-Eiras and Niepelt (2008, 2011), Arawatari and Ono (2009), D'Amato and Galasso (2010) and Conde-Ruiz, Profeta and Galasso (2011). The current work is similar to these studies in that we focus on Markovian social security policy in voting but differs from them in that we consider the case of an unbalanced budget, as commonly observed in OECD countries.

The organization of this paper is as follows. Section 2 develops the basic model: the preferences of agents are specified by the utility function with a constant intertemporal elasticity of substitution. Section 3 characterizes political equilibria in the basic model and provides the main result of this paper. Section 4 considers two extended models and shows that the main result in the basic model still holds under these two extensions. Section 5 returns to the basic model but assumes a generalized utility function and shows that the main result in Section 2 is not affected by this generalization. Section 6 provides concluding remarks. All the proofs are given in Appendix.

2 The Model and Economic Equilibrium

Consider a discrete-time closed economy that starts at $t = 0$. The economy is populated by overlapping generations of two-period-lived homogeneous agents who work in youth (the first period of life) and retire in old age (the second period of life). Each agent has one child, implying a constant population.

Agents consume private goods in both periods of life. The lifetime utility of a young agent born in period t is specified by

$$\max \frac{(c_t^y)^{1-\sigma} - 1}{1-\sigma} + \beta \cdot \frac{(c_{t+1}^o)^{1-\sigma} - 1}{1-\sigma},$$

where c_t^y is consumption in youth, c_{t+1}^o is consumption in old age, $\beta \in (0,1)$ is a discount factor, and $\sigma > 0$ is the inverse of the interest-rate elasticity of consumption. A lower $1/\sigma$ implies a lower interest-rate elasticity of consumption.

Agents work in youth and obtain a constant wage, $y(> 0)$. They consume a part of their after-tax income in youth and save the rest for their old-age consumption. They store income from youth to old age by buying one-period government bonds. In old age, agents obtain the return from government bonds as well as old-age social security provided by the government and consume them.

Individual budget constraints in youth and old age are given by, respectively,

$$\begin{aligned} c_t^y + p_t b_t^d &\leq (1-\tau_t)\cdot y, \\ c_{t+1}^o &\leq b_t^d + z_{t+1}, \end{aligned}$$

where b_t^d is the demand for one-period government bonds, p_t is the price of a government bond sold at time t, $\tau_t \in [0,1]$ is a period-t labor income tax rate, and z_{t+1} is old-age social security benefit.

Each agent maximizes his/her utility subject to the budget constraints. Solving the utility maximization problem yields the consumption Euler equation that determines the trade-off between consumption in youth and old age:

$$c_t^y = \left(\frac{p_t}{\beta}\right)^{\frac{1}{\sigma}} \cdot c_{t+1}^o.$$

With the budget constraints and the Euler equation, we obtain the demand function for government bond,

$$b_t^d = \frac{1}{p_t + (p_t/\beta)^{\frac{1}{\sigma}}} \cdot \left\{ (1-\tau_t)\cdot y - \left(\frac{p_t}{\beta}\right)^{\frac{1}{\sigma}} \cdot z_{t+1} \right\}; \tag{1}$$

and consumption functions in youth and old age, respectively,

$$c_t^y = \frac{(p_t/\beta)^{\frac{1}{\sigma}}}{p_t + (p_t/\beta)^{\frac{1}{\sigma}}} \cdot \{(1-\tau_t)\cdot y + p_t z_{t+1}\}, \tag{2}$$

$$c_{t+1}^o = \frac{1}{p_t + (p_t/\beta)^{\frac{1}{\sigma}}} \cdot \{(1-\tau_t)\cdot y + p_t z_{t+1}\}. \tag{3}$$

The revenue of the government in period t consists of newly issued one-period bonds and a labor income tax, $p_t b_t^s + \tau_t y$, where b_t^s is the supply of bonds that pay one unit of goods in period $t+1$ and $\tau_t y$ is the tax revenue from the young born in period t. The expenditure of the government in period t consists of social security payments, z_t, and the repayment of government bonds issued in period $t-1$, b_{t-1}^s. Therefore, the government budget constraint is given by

$$p_t b_t^s + \tau_t y = z_t + b_{t-1}^s \quad \forall t \geq 0. \tag{4}$$

Economic Equilibrium

An economic equilibrium is defined as follows.

Definition 1. For a given sequence of policies, $\{\tau_t, z_t, b_t^s\}_{t=0}^{\infty}$ with an initial condition b_{-1}, an *economic equilibrium* is a sequence of allocations $\{c_t^y, c_t^o, b_t\}_{t=0}^{\infty}$ and prices $\{p_t\}_{t=0}^{\infty}$ such that

(i) the utility maximization problem is solved for each generation t; that is, agents maximize utility with respect to their demand for government bond, b_t^d, subject to the budget constraints;

(ii) the sequence of policies satisfies the government budget constraint (4) in every period;

(iii) asset market clears at all dates: $b_t^s = b_t^d = b_t \ \forall t \geq 0$, where b_t denotes the equilibrium level of government bond.

The asset-market-clearing condition becomes

$$b_t = \frac{1}{p_t + (p_t/\beta)^{\frac{1}{\sigma}}} \cdot \left[(1-\tau_t)\cdot y - \left(\frac{p_t}{\beta}\right)^{\frac{1}{\sigma}} \cdot z_{t+1}\right],$$

where the left-hand and right-hand sides correspond to the supply and the demand of government bond, respectively. The price that clears the market is implicitly given by

$$p_t = P(b_t,\ (1-\tau_t)\cdot y,\ z_{t+1}).$$

With this equilibrium price, the clearing condition is rewritten as

$$\left(\frac{p_t}{\beta}\right)^{\frac{1}{\sigma}} = \frac{(1-\tau_t)\cdot y - P(b_t,\ (1-\tau_t)\cdot y,\ z_{t+1})\cdot b_t}{b_t + z_{t+1}}. \tag{5}$$

With (2), (3), and (5), we can write the consumption functions as

$$c_t^y = (1-\tau_t)\cdot y - P(b_t,\ (1-\tau_t)\cdot y,\ z_{t+1})\cdot b_t, \tag{6}$$

$$c_{t+1}^o = b_t + z_{t+1}. \tag{7}$$

The indirect utility functions of the young and the old in period t, denoted by V_t^y and V_t^o, are given by, respectively,

$$V_t^y = \frac{[(1-\tau_t)\cdot y - P(b_t,\ (1-\tau_t)\cdot y,\ z_{t+1})\cdot b_t]^{1-\sigma}-1}{1-\sigma} + \beta\cdot\frac{(b_t+z_{t+1})^{1-\sigma}-1}{1-\sigma}, \quad (8)$$

$$V_t^o = \frac{(b_{t-1}+z_t)^{1-\sigma}-1}{1-\sigma}. \quad (9)$$

3 Political Equilibrium

The current paper assumes probabilistic voting in the demonstration of the political mechanism. In each period, the government in power maximizes a political objective function. Formally, the political objective function in period t is given by $\Omega_t = \omega V_t^o + (1-\omega)V_t^y$, where $\omega \in [0,1]$ is the relative weight of old agents. An explicit microfoundation for this modeling is explained in Persson and Tabellini (2000, Chapter 3) and Acemoglu and Robinson (2005, Appendix).

The present paper restricts its attention to Markov-perfect equilibria, as described by Krusell et al. (1997). Voters condition their strategies only on payoff-relevant state variables. In the current framework, the government bond, denoted by b, is the only payoff-relevant state variable. We specifically restrict our methods to stationary Markov perfect equilibria of the voting game so that the policy function is time invariant.

Hereafter, we omit time indexes and use recursive notation. Let b' denote the government bond issued in the current period and redeemed in the next period; let z' denote the social security payment in the next period. We denote b^{DL} as the debt limit, which is formally defined below, and focus on the case of $b \geq 0$ throughout the paper.

Definition 2. A *stationary Markov-perfect political equilibrium* is a three-tuple $\langle B, T, Z\rangle$, where B: $\left[0, b^{DL}\right] \to \left[0, b^{DL}\right]$ is a debt rule, $T : [0, b^{DL}] \to [0,1]$ is a tax rule, and $Z : [0, b^{DL}] \to [0, y]$ is a social security rule, such that

$$\langle B, T, Z\rangle = \arg\max_{\{b'\in[0,b^{DL}],\tau\in[0,1],z\in[0,y]\}} \Omega,$$

subject to the government budget constraint

$$P(b',\ (1-\tau)\cdot y,\ z')\cdot b' + \tau y = z + b,$$

where $z' = Z(b')$.

We substitute the government budget constraint into the political objective function to obtain the following unconstrained problem:

$$\max_{\{z,b'\}} \omega \cdot \frac{(b+z)^{1-\sigma}-1}{1-\sigma} + (1-\omega)\cdot\left[\frac{(y-z-b)^{1-\sigma}-1}{1-\sigma} + \beta\cdot\frac{(b'+Z(b'))^{1-\sigma}-1}{1-\sigma}\right].$$

The tax rate τ disappears from the objective function because the tax revenue from the young is returned to them via the purchase of government bonds.

The first-order conditions with respect to z and b' are

$$\begin{aligned} &z : \omega\cdot(b+z)^{-\sigma} - (1-\omega)\cdot(y-z-b)^{-\sigma} = 0,\\ &b' : 1 + Z'(b') = 0. \end{aligned}$$

An analytical solution of z can be obtained by solving the first-order condition with respect to z:

$$z = Z(b) \equiv \frac{y}{1+\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}} - b. \tag{10}$$

The function in (10) results in $Z'(b) = -1$, implying that the first-order condition with respect to b' is satisfied for any b.

To obtain equilibrium tax and debt policy functions, we need to determine the price that clears the asset market. We substitute the social security function in (10) into the government budget constraint (4) with $b^s = b$ to obtain

$$pb + \tau y = \frac{y}{1+\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}}.$$

We substitute this condition and $z' = y/\left\{1+\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}\right\} - b'$ into the asset-market clearing condition (5) to obtain $(p/\beta)^{\frac{1}{\sigma}} = ((1-\omega)/\omega)^{\frac{1}{\sigma}}$, or

$$p = \beta\cdot\frac{1-\omega}{\omega}. \tag{11}$$

This expression represents the equilibrium price in the asset market. The following assumption ensures that the price is below unity:

Assumption 1. $\beta\cdot\frac{1-\omega}{\omega} < 1$.

Given the equilibrium price in (11), the government budget constraint in (4) with $b = b^s$ becomes

$$\beta\cdot\frac{1-\omega}{\omega}\cdot b' + \tau y = z + b. \tag{12}$$

The debt limit, denoted by b^{DL}, is defined as the debt level satisfying the government budget constraint with $\tau = 1$ and $z = 0$. With $p = \beta(1-\omega)/\omega$, b^{DL} satisfies $(\beta(1-\omega)/\omega)\cdot$

$b^{DL} + 1 \cdot y = 0 + b^{DL}$, or

$$b^{DL} = \frac{y}{1 - \beta \cdot \frac{1-\omega}{\omega}}. \tag{13}$$

The tax and debt policy functions, denoted by $\tau = T(b) \in [0,1]$ and $b' = B(b) \in [0, b^{DL}]$, are such that they satisfy the government budget constraint (12). Specifically, we focus on a linear debt policy function, which is given by

$$B(b) = B_0 + B_1 \cdot b, \tag{14}$$

where B_0 and B_1 are the parameters. The tax policy function must be specified such that the government budget constraint (12) is satisfied:

$$T(b) = \frac{1}{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}} - \beta \cdot \frac{1-\omega}{\omega y} \cdot (B_0 + B_1 b). \tag{15}$$

Equilibrium policy functions are given by (10), (14), and (15). Let $\bar{z}$, $\bar{b}$, and $\bar{\tau}$ denote the steady-state values of z, b, and τ, respectively. The remaining task is to determine a set of parameters that ensures a stable steady-state political equilibrium with $\bar{z} \in [0, y], \bar{b} \in [0, b^{DL}]$ and $\bar{\tau} \in [0, 1]$. The following proposition presents the set.

Proposition 1: *Suppose that the following conditions hold:*

$$B_0 \geq 0 \text{ and } B_1 \in \left(-1,\ 1 - \frac{1}{y} \cdot \left\{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}\right\} \cdot B_0\right].$$

There exists a stable political equilibrium with $\bar{z} \in [0, y], \bar{b} \in [0, b^{DL}]$ *and* $\bar{\tau} \in [0, 1]$ *described by the following linear Markov strategy:*

$$\begin{aligned} Z(b) &= \frac{y}{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}} - b, \\ B(b) &= B_0 + B_1 b, \\ T(b) &= \frac{1}{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}} - \frac{1}{y} \cdot \beta \cdot \frac{1-\omega}{\omega} \cdot (B_0 + B_1 b). \end{aligned}$$

Proof. See the Appendix.

The political equilibrium in the present economy has the following features. First, consumption levels in youth and old-age are constant across generations. Second, a one-unit government bond issue reduces social security payment by one unit. Third, there are multiple political equilibria indexed by the free parameters B_0 and B_1. Because this work addresses a stationary equilibrium, B_0 and B_1 are set to be constant over time.

The intuition behind the first feature is as follows. The aggregate resource in the current economy is given by y: this value is constant over time. The scramble for limited resources occurs between the young and the old in every period via the political process. Because the political powers of the young and the old are constant over time, the fractions of resources allocated to the young and the old are also constant over time. The consumption levels in youth and old-age are constant across generations, except for the initial old agents. Specifically, the consumption of the initial old is $c_0^o = b_{-1} + z_0$; the consumption levels in youth and old age after generation 0 are, respectively,

$$c^y = \frac{\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}} \cdot y}{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}} \text{ and } c^o = \frac{y}{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}}.$$

The second feature is due to that government bonds and social security are perfect substitute for funding old-age consumption. Recall that the old-age consumption is constant as mentioned above. Given this property, the budget constraint in old age, $c_t^o = b + z$, implies that the sum of government bond repayment and social security payment is constant over time. Therefore, one unit of government bond repayment is replaced by one unit of social security payment.

Finally, to consider the third feature, notice that the government expenditure, $b + z$, is constant over time:

$$b + z = Z(b) + b = \frac{1}{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}} \cdot y$$

. This constancy implies that the government revenue is also constant over time. The constant revenue comprises tax revenue from the young and the issue of government bonds to the young. The ratio of tax to government bonds issued for funding expenditure might depend on the preferences of the young. However, these preferences are independent of tax and government bonds issued because young's consumption level in the next period (i.e., old age) is independent of the funding mechanism, as demonstrated in the first feature. Therefore, there are multiple political equilibria, depending on the government expenditure funding mechanism.

Technically, a multiplicity of equilibria arises from the lack of a first-order condition with respect to τ. The choice of the tax rate does not affect the objective function of the government, as demonstrated in the unconstrained problem. In this situation, the government must set three policy variables, τ, b', and z, by using only two first-order conditions with respect to b' and z. Therefore, the government can freely choose the tax rate and the issue of government bonds as long as they satisfy the government budget constraint.

The abovementioned features create a negative correlation between tax and government debt along the equilibrium path that displays monotone convergence: a lower tax rate is associated with a higher level of government debt if $B_1 > 0$. The result of the negative correlation is opposite to the result of the positive correlation demonstrated in Song, Storesletten and Zilibotti (2011) in a small open economy framework and in Röhrs (2010) in a closed economy framework. However, our result is consistent with the recent empirical evidence in OECD countries.

In closing this section, we consider the political power of the old and its impact on utility across generations. Specifically, we consider the utility of the initial old, denoted by V_{-1}^{old}, and the lifetime utility of each young generation in period $t \geq 1$, denoted by V^{young}, which are given by, respectively,

$$V_{-1}^{old} = \frac{\left(\frac{y}{1+\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}} \right)^{1-\sigma} - 1}{1-\sigma},$$

$$V^{young} = \frac{1}{1-\sigma} \cdot \left\{ \frac{y}{1+\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}} \right\}^{1-\sigma} \cdot \left[\left(\frac{1-\omega}{\omega} \right)^{\frac{1-\sigma}{\sigma}} + \beta \right] - \frac{1+\beta}{1-\sigma},$$

where b_{-1} is the initial government debt. Direct calculation leads to

$$\frac{\partial V^{young}}{\partial \omega} < 0, \frac{\partial V_{-1}^{old}}{\partial \omega} > 0.$$

The details of the calculation are given in the Appendix. The result suggests that an increase in the political power of the old benefits the initial old at the expense of successive generations.

4 Extensions

To this point, we have analyzed the politics of government debt in the framework in which (i) government spending is limited to old-age social security and (ii) each generation is composed of homogeneous agents. In this section, we attempt to relax these assumptions and investigate how the analysis and the results of the basic model are modified by this relaxation.

In the first part of this section, we keep the assumption of homogeneous agents but introduce public goods as an alternative form of government spending. Under this extended framework, we consider how the tax revenue is allocated between public goods and social security via politics. In the second part of this section, we limit the government spending to social security, as in the basic model, but allow for two types of agents: the rich and the poor. We demonstrate how income is reallocated between two types of agents via politics.

4.1 Public Goods vs. Social Security

Let g_t denote the public goods provision in period t. The utility function of an agent in generation t is specified by

$$\frac{(c_t^y)^{1-\sigma}-1}{1-\sigma}+\theta\cdot\frac{(g_t)^{1-\sigma}-1}{1-\sigma}+\beta\cdot\left[\frac{(c_{t+1}^o)^{1-\sigma}-1}{1-\sigma}+\theta\cdot\frac{(g_{t+1})^{1-\sigma}-1}{1-\sigma}\right],$$

where $\theta(>0)$ is a parameter representing the public goods preferences. An agent maximizes this utility subject to the budget constraints in youth and old age given in Section 2. The consumption functions and the demand function for government bonds are the same as those under the basic model in Section 2 because public goods enter into the utility function in such a way that they have no effect on consumption and saving decisions. The asset-market-clearing condition is implicitly given by $p_t = P(b_t,\ (1-\tau_t)\cdot y,\ z_{t+1})$, or (5).

The government budget constraint is now modified to

$$p_t b_t^s + \tau_t y = z_t + g_t + b_{t-1}^s \quad \forall t \geq 0.$$

Because of the nature of public goods, the per capita level of public goods is equivalent to the aggregate level of public goods. We substitute the government budget constraint, the asset-market-clearing condition, and the consumption functions into the utility function. Then we obtain the following indirect utility functions of the young and the old:

$$\begin{aligned} V^y &= \frac{[(1-\tau)\cdot y - P(b',\ (1-\tau)\cdot y,\ z')\cdot b']^{1-\sigma}-1}{1-\sigma}+\beta\cdot\frac{(b'+z')^{1-\sigma}-1}{1-\sigma} \\ &\quad +\theta\cdot\frac{(g)^{1-\sigma}-1}{1-\sigma}+\beta\theta\cdot\frac{(g')^{1-\sigma}-1}{1-\sigma}, \qquad (16) \\ V^o &= \frac{(b+z)^{1-\sigma}-1}{1-\sigma}+\theta\cdot\frac{(g)^{1-\sigma}-1}{1-\sigma}. \qquad (17) \end{aligned}$$

The objective function of the government is $\Omega = \omega V^o + (1-\omega)V^y$. We substitute the government budget constraint and the asset-market-clearing condition (5) into Ω to obtain the objective function in the unconstrained maximization problem:

$$\Omega = \omega \cdot \frac{(b+z)^{1-\sigma}-1}{1-\sigma} + (1-\omega)\cdot\left[\frac{(y-z-g-b)^{1-\sigma}-1}{1-\sigma} + \beta\cdot\frac{(b'+z')^{1-\sigma}-1}{1-\sigma}\right]$$
$$+\,\theta\cdot\frac{(g)^{1-\sigma}-1}{1-\sigma} + (1-\omega)\cdot\beta\theta\cdot\frac{(g')^{1-\sigma}-1}{1-\sigma}.$$

The tax rate disappears from the objective function in this extended framework for the same reason it did in the basic model.

To determine the equilibrium policy functions, we solve the unconstrained optimization problem and obtain the first-order conditions with respect to z, g, and b':

$$z : \omega\cdot(b+z)^{-\sigma} - (1-\omega)\cdot(y-z-g-b)^{-\sigma} = 0,$$
$$g : (1-\omega)\cdot(y-z-g-b)^{-\sigma} - \theta\cdot(g)^{-\sigma} = 0,$$
$$b' : (b'+z')^{-\sigma}\cdot\left(1+\frac{\partial z'}{\partial b'}\right) + \theta\cdot(g')^{-\sigma}\cdot\frac{\partial g'}{\partial b'} = 0.$$

Analytical solutions of z and g are obtained by solving the first-order conditions with respect to z and g for z and g, respectively,

$$z = Z(b) = \phi\cdot\left(\frac{1-\omega}{\theta}\right)^{\frac{1}{\sigma}}\cdot y - b, \tag{18}$$
$$g = G(b) = \phi\cdot\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}\cdot y, \tag{19}$$

where

$$\phi \equiv \frac{1}{\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}\left\{1+\left(\frac{1-\omega}{\theta}\right)^{\frac{1}{\sigma}}\right\} + \left(\frac{1-\omega}{\theta}\right)^{\frac{1}{\sigma}}}.$$

The functions in (18) and (19) result in $Z'(b) = -1$ and $G'(b) = 0$, implying that the first-order condition with respect to b' holds for any b.

The remaining task is to determine the functional forms of tax and debt policy functions in the same manner as in the previous section. First, we compute the price that clears the asset market. We substitute the social security policy function (18) and the government budget constraint into the asset-market-clearing condition given by (5). Then, we obtain $p = \beta\cdot\frac{1-\omega}{\omega} < 1$, where $p < 1$ holds under Assumption 1. The price in the current economy is equivalent to that in the basic model because here public goods have no direct effect on economic decisions. With $p = \beta\cdot(1-\omega)/\omega$, the government budget constraint is reduced to

$$\beta\cdot\frac{1-\omega}{\omega}\cdot b' + \tau y = \phi\cdot\left\{\left(\frac{1-\omega}{\theta}\right)^{\frac{1}{\sigma}} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}\right\}\cdot y.$$

Next, we focus on a debt policy specified by a linear function, $B(b) = B_0 + B_1 \cdot b$. The equilibrium tax policy function must be set to satisfy the abovementioned government budget constraint:

$$T(b) = \phi \cdot \left\{ \left(\frac{1-\omega}{\theta} \right)^{\frac{1}{\sigma}} + \left(\frac{1-\omega}{\omega} \right)^{\frac{1}{\sigma}} \right\} - \beta \cdot \frac{1-\omega}{\omega y} \cdot (B_0 + B_1 b).$$

Following the same procedure as in the previous section, we identify a set of parameters, B_0 and B_1, that ensure a stable steady-state political equilibrium with $\bar{z} \in [0, y], \bar{b} \in [0, b^{DL}]$ and $\bar{\tau} \in [0, 1]$.

Proposition 2. *Suppose that the following conditions hold:*

$$B_0 \geq 0 \text{ and } B_1 \in \left(-1,\ 1 - \frac{1}{y} \cdot \frac{1}{\phi \cdot \left(\frac{1-\omega}{\theta}\right)^{\frac{1}{\sigma}}} \cdot B_0 \right].$$

There exists a stable political equilibrium with $\bar{z} \in [0, y], \bar{b} \in [0, b^{DL}]$ and $\bar{\tau} \in [0, 1]$ described by the following linear Markov strategy:

$$\begin{aligned}
Z(b) &= \phi \cdot \left(\frac{1-\omega}{\theta} \right)^{\frac{1}{\sigma}} \cdot y - b, \\
G(b) &= \phi \cdot \left(\frac{1-\omega}{\omega} \right)^{\frac{1}{\sigma}} \cdot y, \\
B(b) &= B_0 + B_1 b, \\
T(b) &= \phi \cdot \left\{ \left(\frac{1-\omega}{\theta} \right)^{\frac{1}{\sigma}} + \left(\frac{1-\omega}{\omega} \right)^{\frac{1}{\sigma}} \right\} - \frac{1}{y} \cdot \beta \cdot \frac{1-\omega}{\omega} \cdot (B_0 + B_1 b).
\end{aligned}$$

Proof. See the Appendix.

The result in Proposition 2 indicates that an introduction of public goods does not qualitatively affect the property of the political equilibrium in the basic model. The extended model still has the following features, which were embodied in the basic model: (i) a multiplicity of equilibria depending on the parameters B_0 and B_1; (ii) a one-to-one trade-off between government bond issue and social security payment; and (iii) a negative correlation between tax and government debt along the equilibrium path that displays monotone convergence.

In addition to these features, a notable feature of the extended model is that there remains an intertemporal link such that the debt policy function depends on the previous level of government debt. This result counters the argument by Röhrs (2010). In the closed economy model with public goods but without old-age social security, she demonstrated that the policy function for debt is flat and thus independent of the previous level of government debt. Based on this result, she argued that the result of a flat level of debt is a generic feature of the closed economy model. However, our result indicates that an introduction of old-age social security into her model breaks an intertemporal dichotomy.

The result in Proposition 2 shows that the level of public goods provision is constant and independent of the free parameters B_0 and B_1. Thus, we can perform a comparative statics analysis to examine the effect of the political power of the old on public goods provision. The following corollary summarizes the result. Proof is given in the Appendix.

Corollary 1. *Consider an increase in ω that implies an increase in the political power of the old.*

(i) *Suppose that $1/\sigma > 1$. There is an inverse U-shaped relationship between ω and the equilibrium public goods provision: $\partial\bar{g}/\partial\omega \gtreqless 0$ if and only if $\omega \lesseqgtr 1/2$.*

(ii) *Suppose that $1/\sigma < 1$. There is a U-shaped relationship between ω and the equilibrium public goods provision: $\partial\bar{g}/\partial\omega \gtreqless 0$ if and only if $\omega \gtreqless 1/2$.*

(iii) *Suppose that $1/\sigma = 1$. There is no effect of ω on the public goods provision:*

To investigate the result in Corollary 1, we examine the first-order condition with respect to g in the unconstrained optimization problem, given by

$$\underbrace{(1-\omega)\cdot(y-z-g-b)^{-\sigma}}_{\text{Marginal cost}} = \underbrace{\theta\cdot(g)^{-\sigma}}_{\text{Marginal benefit}},$$

where the left-hand side and right-hand side show the marginal cost and benefit of public goods provision, respectively. The condition states that an increase in ω has the following two opposing effects. First, given z and b, such an increase lowers the marginal cost of public goods provision, giving the government an incentive to choose a higher level of public goods provision to equate the marginal cost and benefit. This effect has a positive impact on public goods provision. Second, an increase in ω gives the government an incentive to choose a higher level of social security, resulting in a larger marginal cost of public goods provision given g and b. The government chooses a lower level of public goods provision to balance the marginal cost and benefit of public goods provision.

Which effect overcomes the other depends on the interest-rate elasticity, $1/\sigma$. If the elasticity is high ($1/\sigma > 1$), the positive effect overcomes the negative one when the initial political power of the old is low ($\omega < 1/2$). However, the negative effect overcomes the positive one when the initial political power of the old is high ($\omega > 1/2$). The two opposing effects offset each other at $\omega = 1/2$. Therefore, there is an inverse U-shaped relationship between ω and g if $1/\sigma > 1$. The result is reversed when the elasticity is low ($1/\sigma < 1$): there is a U-shaped relationship between ω and g. The two effects are always offset each other if $1/\sigma = 1$.

4.2 Rich vs. Poor

Suppose that there are two types of agents, the rich and the poor. The proportion of the rich in each generation is $1-\pi \in [0,1]$, and the proportion of the poor is π. The proportion is stationary across generations.

The rich work in youth and retire in old age; the economic behavior of the rich is similar to that in the basic model. Therefore, the demand function for government bonds is given by (1), and consumption functions in youth and old-age are

$$\begin{aligned} c_t^{yr} &= \frac{(p_t/\beta)^{\frac{1}{\sigma}}}{p_t + (p_t/\beta)^{\frac{1}{\sigma}}} \cdot \left[(1-\tau_t)\cdot y + p_t z_{t+1}\right], \\ c_{t+1}^{or} &= \frac{1}{p_t + (p_t/\beta)^{\frac{1}{\sigma}}} \cdot \left[(1-\tau_t)\cdot y + p_t z_{t+1}\right], \end{aligned}$$

where c_t^{yr} and c_{t+1}^{or} denote consumption of the rich in youth and in old age, respectively.

The poor are assumed to be unemployed or unable to work in youth. They receive social security benefits in both periods of life. In addition, because of the lack of labor income, they are assumed to be unable to access financial markets: they consume their social security benefits within each period. Under these two assumptions, the utility of the poor in generation t becomes

$$V_t^{yp} = \frac{(v_t)^{1-\sigma} - 1}{1-\sigma} + \beta \cdot \frac{(v_{t+1})^{1-\sigma} - 1}{1-\sigma},$$

where v_t is the per capita social security payments to the poor in period t. We allow for the possibility that the old-age social security for the poor, v_{t+1}, could be different from that for the rich, z_{t+1}.

The government budget constraint is

$$p_t b_t^s + (1-\pi)\cdot \tau_t y = (1-\pi)\cdot z_t + 2\pi v_t + b_{t-1}^s.$$

On the left-hand side, the second term, $(1-\pi)\cdot\tau_t y$, is the tax revenue from the rich young agent. On the right-hand side, the first term, $(1-\pi)\cdot z_t$, is the social security payments to the rich old agents; the second term, $2\pi v_t$, is the social security payments to the poor young and old.

The asset-market-clearing condition is $b_t^s = (1-\pi)\cdot b_t^d$. With $b_t^s = b_t$ and the demand function in (1), the clearing condition is implicitly given by $p_t = P\left((1-\tau_t)\cdot y,\ b_t,\ z_{t+1}\right)$, or

$$\left(\frac{p_t}{\beta}\right)^{\frac{1}{\sigma}} = \frac{(1-\pi)(1-\tau_t)\cdot y - P(b_t,\ (1-\tau_t)\cdot y,\ z_{t+1})\cdot b_t}{b_t + (1-\pi)\cdot z_{t+1}}. \tag{20}$$

With $p_t = P\left((1-\tau_t)\cdot y,\ b_t,\ z_{t+1}\right)$ and the private and government budget constraints, we can write the indirect utility functions of the rich young and the rich old as follows:

$$V_t^{yr} = \frac{\left\{(1-\tau_t)\cdot y - \frac{1}{1-\pi}\cdot P\left((1-\tau_t)\cdot y,\ b_t,\ z_{t+1}\right)\cdot b_t\right\}^{1-\sigma} - 1}{1-\sigma} + \beta\cdot\frac{\left(\frac{1}{1-\pi}\cdot b_t + z_{t+1}\right)^{1-\sigma} - 1}{1-\sigma},$$

$$V_t^{or} = \frac{\left(\frac{1}{1-\pi}\cdot b_{t-1} + z_t\right)^{1-\sigma} - 1}{1-\sigma}.$$

The political objective function is a weighted average of the indirect utility functions

$$\Omega_t = \omega\cdot\left\{\pi V_t^{op} + (1-\pi)\cdot V_t^{or}\right\} + (1-\omega)\cdot\left\{\pi V_t^{yp} + (1-\pi)\cdot V_t^{yr}\right\}.$$

The task of the government in period t is to maximize Ω_t subject to the government budget constraint:

$$P\left((1-\tau_t)\cdot y,\ b_t,\ z_{t+1}\right)\cdot b_t + (1-\pi)\cdot\tau_t y = (1-\pi)\cdot z_t + 2\pi v_t + b_{t-1}. \tag{21}$$

We substitute the government budget constraint into Ω_t to obtain the objective function in the unconstrained problem with recursive notation:

$$\begin{aligned}
\Omega = {} & \omega(1-\pi)\cdot\frac{\left(\frac{1}{1-\pi}\cdot b + z\right)^{1-\sigma} - 1}{1-\sigma} \\
& + (1-\omega)(1-\pi)\left[\frac{\left(y - z - \frac{2\pi}{1-\pi}\cdot v - \frac{1}{1-\pi}\cdot b\right)^{1-\sigma} - 1}{1-\sigma} + \beta\cdot\frac{\left(\frac{1}{1-\pi}\cdot b' + z'\right)^{1-\sigma} - 1}{1-\sigma}\right] \\
& + \pi\cdot\frac{(v)^{1-\sigma} - 1}{1-\sigma} + (1-\omega)\pi\beta\cdot\frac{(v')^{1-\sigma} - 1}{1-\sigma}.
\end{aligned}$$

By replacing v with g, we observe that the current objective function has a qualitatively similar form as that in Subsection 4.1. Therefore, we can determine the equilibrium policy functions and price following the same procedure as in Subsection 4.1 (See Appendix for the calculation details).

Proposition 3. *Suppose that the following conditions hold:*

$$B_0 \geq 0 \text{ and } B_1 \in \left(-1,\ 1 - \frac{1}{\widetilde{\phi}} \cdot \frac{1}{\frac{1-\pi}{2\pi} \cdot \{2 \cdot (1-\omega)\}^{\frac{1}{\sigma}} \cdot y} \cdot B_0\right].$$

There exists a stable political equilibrium with $\bar{z} \in [0, y], \bar{b} \in \left[0, b^{DL}\right]$ *and* $\bar{\tau} \in [0,1]$ *featured by the following linear Markov strategy:*

$$\begin{aligned}
Z(b) &= \widetilde{\phi} \cdot \frac{1}{2\pi} \cdot \{2 \cdot (1-\omega)\}^{\frac{1}{\sigma}} \cdot y - \frac{1}{1-\pi} \cdot b, \\
V(b) &= \widetilde{\phi} \cdot \frac{1}{2\pi} \cdot \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}} \cdot y \\
B(b) &= B_0 + B_1 b, \\
T(b) &= \widetilde{\phi} \cdot \frac{1}{1-\pi} \cdot \left\{\frac{1-\pi}{2\pi} \cdot \{2 \cdot (1-\omega)\}^{\frac{1}{\sigma}} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}\right\} \\
&\quad - \frac{1}{(1-\pi) \cdot y} \cdot \beta \cdot \frac{1-\omega}{\omega} \cdot (B_0 + B_1 b),
\end{aligned}$$

where

$$\widetilde{\phi} \equiv (1-\pi) \cdot \frac{1}{\frac{1-\pi}{2\pi} \cdot \{2 \cdot (1-\omega)\}^{\frac{1}{\sigma}} \cdot \left\{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}\right\} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}}.$$

Proof. See the Appendix.

The political equilibrium in the current model shows the same property as does the equilibrium in Subsection 4.1 because the objective function here is qualitatively similar to that in Subsection 4.1. Therefore, the current model still has the three features embodied in the basic model: (i) a multiplicity of political equilibria; (ii) a one-to-one trade-off between government bond issue and social security payment; and (iii) a negative correlation between tax and government debt along the equilibrium path that displays monotone convergence.

In the current framework with the rich and the poor, the size of social security payment to the poor is constant and independent of the free parameters B_0 and B_1. Thus, we can perform a comparative statics analysis to examine the effects of the political power of the old, ω, and the share of the poor, π, on the size of social security payment to the poor, v. The following two corollaries summarize the effects of ω and π on v. Proof is given in the Appendix.

Corollary 2. *Consider an increase in ω that implies an increase of the political power of the old.*

(i) *Suppose that $1/\sigma > 1$. There is an inverse U-shaped relationship between ω and the* *size of social security payment to the poor:* $\partial \bar{v}/\partial \omega \gtreqless 0$ *if and only if* $\omega \lesseqgtr 1/2$.

(ii) *Suppose that* $1/\sigma < 1$. *There is a U-shaped relationship between* ω *and the size of social security payment to the poor:* $\partial \bar{v}/\partial \omega \gtreqless 0$ *if and only if* $\omega \gtreqless 1/2$.

(iii) *Suppose that* $1/\sigma = 1$. *There is no effect of* ω *on the size of social security payment to the poor:* $\partial \bar{v}/\partial \omega = 0$.

Corollary 3. *An increase in the share of the poor results in a smaller size of the social security payment to the poor:* $\partial \bar{v}/\partial \pi < 0$.

The result in Corollary 2 is qualitatively similar to that in Corollary 1. The similarity is due to that social security payment to the poor in the current model plays the same role as do public goods in the former model at the stage of government optimization. We can apply the interpretation for the result in Corollary 1 to the result in Corollary 2.

Corollary 3 states that an increase in the share of the poor results in a decrease, rather than an increase, in the size of social security payment to the poor. The mechanism behind this counterintuitive result is simple: in an economy with a fixed endowment, an increase in the share of the poor yields a decrease in the tax base while it increases recipients. Therefore, the benefit per person decreases in response to an increase in the poor.

5 Discussion

The analysis to this point has been based on the individual preferences specified by the utility function with a constant intertemporal elasticity of substitution. Under this specification with the notion of stationary Markov perfect equilibrium, we obtained the following three properties of political equilibrium: (i) a multiplicity of equilibria, (ii) a one-to-one trade-off between government bond issue and social security payment, and (iii) a negative correlation between tax and government debt along the equilibrium path that displays monotone convergence.

From a robustness perspective, we next investigate whether these properties still hold under a generalized utility function. Our focus is on the economy with the social security and public goods considered in Subsection 4.1. We assume the utility function of the form

$$u(c_t^y) + \theta \cdot \psi(g_t) + \beta \cdot \left\{ u(c_{t+1}^o) + \theta \cdot \psi(g_{t+1}) \right\},$$

where $u(\cdot)$ and $\psi(\cdot)$ are strictly increasing, strictly concave, and twice continuously differentiable functions with $\lim_{c \to 0}(du/dc) = +\infty$ and $\lim_{g \to 0}(d\psi/dg) = +\infty$ and where $\theta(> 0)$ is an exogenous parameter that represents the public goods preference.

The objective function of the government is $\Omega = \omega V^o + (1-\omega)V^y$. After some manipulation, we obtain the objective function Ω in the unconstrained optimization problem in a recursive form:

$$\begin{aligned}\Omega &= \omega \cdot u\,(b+z) + (1-\omega)\cdot[u(y-z-g-b) + \beta\cdot u\,(b'+z')]\\ &\quad + \theta\cdot\psi(g) + (1-\omega)\cdot\beta\cdot\theta\cdot\psi(g').\end{aligned}$$

Derivation of this function is given in the Appendix.

The first-order conditions with respect to z, g, and b' are, respectively,

$$\begin{aligned}z &: \omega\cdot u'\,(b+z) - (1-\omega)\cdot u'(y-z-g-b) = 0,\\ g &: \theta\cdot\psi'(g) - (1-\omega)\cdot u'(y-z-g-b) = 0,\\ b' &: u'\,(b'+z')\cdot\left(1+\frac{\partial z'}{\partial b'}\right) + \theta\cdot\psi'(g')\cdot\frac{\partial g'}{\partial b'} = 0,\end{aligned}$$

where $u'(\cdot)$ and $\psi'(\cdot)$ represent first derivatives.

Suppose that the policy functions of social security and public goods are given by

$$\begin{cases} z = Z(b) \equiv \bar{Z} - b,\\ g = G(b) \equiv \bar{G}, \end{cases} \tag{22}$$

where $\bar{Z} \in (0,y)$ and $\bar{G} \in (0,y)$ are determined by the first-order conditions with respect to z and g:

$$\begin{aligned}\omega\cdot u'\left(\bar{Z}\right) - (1-\omega)\cdot u'(y-\bar{Z}-\bar{G}) &= 0,\\ \theta\cdot\psi'(\bar{G}) - (1-\omega)\cdot u'(y-\bar{Z}-\bar{G}) &= 0.\end{aligned} \tag{23}$$

Under the assumption of u and ψ, there is a unique pair of $\bar{Z} \in (0,y)$ and $\bar{G} \in (0,y)$ that satisfy the above two conditions. The guess in (22) results in a constant objective function that is independent of b, b', and τ. In addition, the first-order conditions with respect to b' are also satisfied under the guess in (22) because (22) results in $1+\partial z'/\partial b' = 1-1 = 0$ and $\partial g'/\partial b' = 0$. Therefore, the two functions in (22) are the equilibrium policy functions of social security and public goods.

The next task is to show that the equilibrium price is $p = \beta\cdot(1-\omega)/\omega$. We utilize the government's first-order condition with respect to z and the policy functions in (23) to write the ratio of marginal utilities in youth and old age as

$$\frac{u'(c^o)}{u'(c^y)} = \frac{u'(\bar{Z})}{u'(y-\bar{Z}-\bar{G})} = \frac{1-\omega}{\omega}.$$

We substitute this into the consumption Euler equation to obtain:

$$p = \beta \cdot \frac{u'(c^o)}{u'(c^y)} = \beta \cdot \frac{1-\omega}{\omega} < 1$$

where an inequality holds under Assumption 1.

The final task is to determine the equilibrium tax and debt policy functions. With $p = \beta \cdot (1-\omega)/\omega$, the government budget constraint is

$$\beta \cdot \frac{1-\omega}{\omega} \cdot b' + \tau y = \bar{Z} + \bar{G}.$$

We focus on a linear debt policy function specified by

$$B(b) = B_0 + B_1 b. \tag{24}$$

Given this debt policy function, the tax policy function is set to satisfy the above government budget constraint:

$$T(b) = \frac{\bar{Z}}{y} + \frac{\bar{G}}{y} - \frac{\beta}{y} \cdot \frac{1-\omega}{\omega} \cdot (B_0 + B_1 b)\,. \tag{25}$$

The equilibrium policy functions are given in (22), (24) and (25). We can verify that the three properties described at the beginning of this section still hold under the assumption of a generalized utility function.

6 Conclusion

This paper reconsidered the models of Song, Storesletten and Zilibotti (2011) and Röhrs (2010) who investigated intergenerational conflict over government debt, tax, and public goods. Specifically, our analysis is based on the closed-economy model by Röhrs (2010) but differs from the previous two studies in that, instead of public goods, we introduce social security payment to the old as government expenditure. Under this alternative framework, we demonstrate the following three results: (i) an intertemporal link of resource allocation via debt; (ii) multiple political equilibria; and (iii) a negative correlation between tax and debt.

The first result, which was missed in Röhrs (2010), can be restored by the introduction of social security. The second and the third results come from the one-to-one trade-off between government debt and social security: this property is specific to our closed-economy framework with social security payment to the old. Specifically, the third result, which is opposite to that in Song, Storesletten and Zilibotti (2011) and Röhrs (2010), is consistent with the empirical evidence from OECD countries. These three results are robust to the introduction to public goods as additional government expenditure or to the income heterogeneity within a generation.

The main caveat of our analysis lies in the assumption of a closed economy: the government debt is solely owned by the residents. This assumption approximates the situation, in Japan where the proportion of government debt held by non-residents is approximately 5%. However, the proportion is more than 40% in Italy and Spain, which also face a massive budget deficit, as does Japan (Artus, 2010). An introduction of social security payment to the old into the small open economy framework of Song, Storesletten and Zilibotti (2011) will provide more insight to the politics of government debt; this extension is left to future work.

7 Appendix

7.1 Proof of Proposition 1

Proof of Proposition 1.

First, we define a set (B_0, B_1) that ensures a stable steady-state equilibrium with $\bar{b} \in [0, b^{DL}]$. From the equilibrium debt policy function in (14), $\bar{b}$ is given by

$$\bar{b} = \frac{B_0}{1 - B_1}. \tag{26}$$

Expression (14) requires that $B_1 \in (-1, 1)$ for the stability of the equilibrium path of b. Under the assumption of $B_1 \in (-1, 1)$, we obtain

$$\bar{b} \geq 0 \;\Leftrightarrow\; B_0 \geq 0.$$

With (13), which defines b^{DL}, we can rewrite $\bar{b} < b^{DL}$ as

$$\bar{b} \leq b^{DL} \;\Leftrightarrow\; B_1 \leq 1 - \frac{1}{y} \cdot \left\{1 - \beta \cdot \frac{1-\omega}{\omega}\right\} \cdot B_0.$$

Therefore, there exists a stable $\bar{b} \in \left[0, b^{DL}\right]$ if B_0 and B_1 satisfy the following:

$$B_0 \geq 0, \tag{27}$$

$$B_1 \in \left(-1,\; 1 - \frac{1}{y} \cdot \left\{1 - \beta \cdot \frac{1-\omega}{\omega}\right\} \cdot B_0\right]. \tag{28}$$

Second, we determine a set (B_0, B_1) that ensures $\bar{z} \in [0, y]$. Under the conditions of (27) and (28) for B_0 and B_1, it always holds that $\bar{z} \leq y$; and $\bar{z} \geq 0$ is rewritten as

$$\bar{z} \geq 0 \;\Leftrightarrow\; B_1 \leq 1 - \frac{1}{y} \cdot \left\{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}\right\} \cdot B_0. \tag{29}$$

Third, we determine a set (B_0, B_1) that ensures $\bar{\tau} \in [0, 1]$. Under the conditions of (27) and (28), $\bar{\tau} \geq 0$ and $\bar{\tau} \leq 1$ are rewritten as follows:

$$\bar{\tau} \geq 0 \;\Leftrightarrow\; B_1 \leq 1 - \frac{1}{y} \cdot \left\{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}\right\} \cdot \beta \cdot \frac{1-\omega}{\omega} \cdot B_0; \tag{30}$$

$$\bar{\tau} \leq 1 \;\Leftrightarrow\; -\frac{\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}}{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}} \leq \beta \cdot \frac{1-\omega}{\omega y} \cdot \frac{B_0}{1 - B_1}. \tag{31}$$

The condition (31) always holds because the left-hand side is negative and the right-hand side is positive under the conditions of (27) and (28).

The analysis so far indicates that the existence of a stable steady-state equilibrium with $\bar{z} \in [0, y]$, $\bar{b} \in \left[0, b^{DL}\right]$, and $\bar{\tau} \in [0, 1]$ requires (27), (28), (29), and (30). These conditions are summarized in Proposition 1.

■

7.2 Effect of ω on Utility

Direct calculation leads to

$$\frac{\partial V_{-1}^{old}}{\partial \omega} > 0;$$

$$\frac{\partial V^{young}}{\partial \omega} = \left\{ \frac{y}{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}} \right\}^{1-\sigma} \cdot \frac{1}{\sigma} \cdot \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}-1} \cdot \left(-\frac{1}{\omega} - \frac{1-\omega}{\omega^2}\right)$$
$$\times \left[-\frac{1}{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}} \cdot \left\{ \left(\frac{1-\omega}{\omega}\right)^{\frac{1-\sigma}{\sigma}} + \beta \right\} + \frac{\omega}{1-\omega} \right].$$

The derivative $\partial V^{young}/\partial \omega$ implies

$$\frac{\partial V^{young}}{\partial \omega} \gtreqless 0 \;\Leftrightarrow\; 1 \lesseqgtr \beta \cdot \frac{1-\omega}{\omega}.$$

Under Assumption 1, we obtain $\partial V^{young}/\partial \omega < 0$.

■

7.3 Proof of Proposition 2

The debt policy function $B(b) = B_0 + B_1 b$ requires $B_1 \in (-1, 1)$ for the stability of the steady-state equilibrium. The steady-state level of b, denoted by $\bar{b}$, is $\bar{b} = B_0/(1 - B_1)$. Therefore, $\bar{b} \geq 0$ holds if and only if $B_0 \geq 0$ under the assumption of $B_1 \in (-1, 1)$.

The debt limit is still given by b^{DL} in (13). Thus $\bar{b} \leq b^{DL}$ requires

$$\bar{b} \leq b^{DL} \;\Leftrightarrow B_1 \leq 1 - \frac{1}{y} \cdot \left\{ 1 - \beta \cdot \frac{1-\omega}{\omega} \right\} \cdot B_0.$$

A pair (B_0, B_1) must satisfy the following for the existence of a stable path of $\{b\}$ that converges to the steady state with $\bar{b} \in [0, b^{DL}]$:

$$\begin{cases} B_0 \geq 0, \\ B_1 \in \left(-1,\ 1 - \frac{1}{y} \cdot \left\{1 - \beta \cdot \frac{1-\omega}{\omega}\right\} \cdot B_0\right]. \end{cases} \tag{32}$$

Next, we determine a set (B_0, B_1) that ensures $\bar{z} \in [0, y]$. Under the abovementioned conditions in (32), it always holds that $\bar{z} < y$; and $\bar{z} > 0$ is rewritten as

$$\bar{z} \geq 0 \ \Leftrightarrow\ B_1 \leq 1 - \frac{1}{y} \cdot \frac{1}{\phi \cdot \left(\frac{1-\omega}{\theta}\right)^{\frac{1}{\sigma}}} \cdot B_0.$$

Third, we determine a set (B_0, B_1) that ensures $\bar{\tau} \in [0, 1]$. With the debt policy function given in Proposition 2 and $\bar{b} = B_0/(1 - B_1)$, the steady-state level of τ is

$$\bar{\tau} = \phi \cdot \left\{ \left(\frac{1-\omega}{\theta}\right)^{\frac{1}{\sigma}} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}} \right\} - \beta \cdot \frac{1-\omega}{\omega y} \cdot \frac{B_0}{1 - B_1}.$$

We obtain

$$\bar{\tau} > 0 \ \Leftrightarrow\ B_1 < 1 - \frac{1}{y} \cdot \frac{1}{\phi \cdot \left\{ \left(\frac{1-\omega}{\theta}\right)^{\frac{1}{\sigma}} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}} \right\}} \cdot \beta \cdot \frac{1-\omega}{\omega} \cdot B_0$$

and

$$\bar{\tau} < 1 \Leftrightarrow \ \frac{\left\{ \left(\frac{1-\omega}{\theta}\right)^{\frac{1}{\sigma}} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}} \right\}}{\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}} \cdot \left\{1 + \left(\frac{1-\omega}{\theta}\right)^{\frac{1}{\sigma}}\right\} + \left(\frac{1-\omega}{\theta}\right)^{\frac{1}{\sigma}}} - 1 < \beta \cdot \frac{1-\omega}{\omega y} \cdot \frac{B_0}{1 - B_1}.$$

The second condition always holds because the left-hand side is negative and the right-hand side is positive for any feasible B_0 and B_1. The conditions derived so far are summarized as in Proposition 2.

■

7.4 Proof of Corollary 1

The level of public goods is

$$
\begin{aligned}
g &= \phi \cdot \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}} \cdot y \\
&= \frac{\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}}{\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}} \cdot \left\{1 + \left(\frac{1-\omega}{\theta}\right)^{\frac{1}{\sigma}}\right\} + \left(\frac{1-\omega}{\theta}\right)^{\frac{1}{\sigma}}} \cdot y \\
&= \frac{1}{1 + \left(\frac{1}{\theta}\right)^{\frac{1}{\sigma}} \cdot \left\{(1-\omega)^{\frac{1}{\sigma}} + (\omega)^{\frac{1}{\sigma}}\right\}} \cdot y.
\end{aligned}
$$

Differentiation of the term $\left\{(1-\omega)^{\frac{1}{\sigma}} + (\omega)^{\frac{1}{\sigma}}\right\}$ with respect to ω leads to

$$
\frac{d\left\{(1-\omega)^{\frac{1}{\sigma}} + (\omega)^{\frac{1}{\sigma}}\right\}}{d\omega}
\begin{cases}
\lesseqgtr 0 \quad \Leftrightarrow \begin{cases} \omega \lesseqgtr \frac{1}{2} & \text{if } \frac{1}{\sigma} > 1, \\ \omega \gtreqless \frac{1}{2} & \text{if } \frac{1}{\sigma} > 1, \end{cases} \\
= 0 & \text{if } \frac{1}{\sigma} = 1.
\end{cases}
$$

Given this result, we obtain the result in Corollary 1.

7.5 Proof of Proposition 3

To determine the equilibrium policy functions, we solve the unconstrained optimization problem and obtain the following first-order conditions with respect to z, v, and b':

$$
\begin{aligned}
\Omega &= \omega(1-\pi) \cdot \frac{\left(\frac{1}{1-\pi} \cdot b + z\right)^{1-\sigma} - 1}{1-\sigma} \\
&+ (1-\omega)(1-\pi)\left[\frac{\left(y - z - \frac{2\pi}{1-\pi} \cdot v - \frac{1}{1-\pi} \cdot b\right)^{1-\sigma} - 1}{1-\sigma} + \beta \cdot \frac{\left(\frac{1}{1-\pi} \cdot b' + z'\right)^{1-\sigma} - 1}{1-\sigma}\right] \\
&+ \pi \cdot \frac{(v)^{1-\sigma} - 1}{1-\sigma} + (1-\omega)\pi\beta \cdot \frac{(v')^{1-\sigma} - 1}{1-\sigma}.
\end{aligned}
$$

$$
\begin{aligned}
&z : \omega \cdot \left(\frac{1}{1-\pi} \cdot b + z\right)^{-\sigma} - (1-\omega) \cdot \left(y - z - \frac{2\pi}{1-\pi} \cdot v - \frac{1}{1-\pi} \cdot b\right)^{-\sigma} = 0, \\
&v : 2(1-\omega) \cdot \left(y - z - \frac{2\pi}{1-\pi} \cdot v - \frac{1}{1-\pi} \cdot b\right)^{-\sigma} - (v)^{-\sigma} = 0, \\
&b' : (1-\pi) \cdot \left(\frac{1}{1-\pi} \cdot b' + z'\right)^{-\sigma} \cdot \left(\frac{1}{1-\pi} + \frac{\partial z'}{\partial b'}\right) + \pi \cdot (v')^{-\sigma} \cdot \frac{\partial v'}{\partial b'} = 0.
\end{aligned}
$$

The first-order conditions with respect to z and v lead to the following analytical solutions:

$$z = Z(b) \equiv \frac{\frac{1-\pi}{2\pi} \cdot \{2 \cdot (1-\omega)\}^{\frac{1}{\sigma}}}{\frac{1-\pi}{2\pi} \cdot \{2 \cdot (1-\omega)\}^{\frac{1}{\sigma}} \cdot \left\{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}\right\} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}} \cdot y - \frac{1}{1-\pi} \cdot b, \tag{33}$$

$$v = V(b) \equiv \frac{\frac{1-\pi}{2\pi} \cdot \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}}{\frac{1-\pi}{2\pi} \cdot \{2 \cdot (1-\omega)\}^{\frac{1}{\sigma}} \cdot \left\{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}\right\} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}} \cdot y. \tag{34}$$

The functions in (33) and (34) result in $Z'(b') = -1$ and $V'(b') = 0$, respectively. These results imply that the first-order condition with respect to b' is satisfied for any b.

The next task is to determine the functional forms of the tax and debt policy functions. First, we compute the price that clears the asset market. With the social security policy functions, (33) and (34), and the government budget constraint, (21), the asset-market-clearing condition (20) is reduced to $p = \beta \cdot (1-\omega)/\omega < 1$, where $p < 1$ holds under Assumption 1.

Given (33) and (34) and the equilibrium price, the government budget constraint is now reduced to

$$\beta \cdot \frac{1-\omega}{\omega} \cdot b + (1-\pi) \cdot \tau y = \widetilde{\phi} \cdot \left\{ \frac{1-\pi}{2\pi} \cdot \{2 \cdot (1-\omega)\}^{\frac{1}{\sigma}} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}} \right\} \cdot y,$$

where

$$\widetilde{\phi} \equiv (1-\pi) \cdot \frac{1}{\frac{1-\pi}{2\pi} \cdot \{2 \cdot (1-\omega)\}^{\frac{1}{\sigma}} \cdot \left\{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}\right\} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}}.$$

We focus on a debt policy function specified by a linear policy function, $B(b) = B_0 + B_1 \cdot b$. The equilibrium tax policy function must be set to satisfy the aforementioned government budget constraint:

$$\begin{aligned} \tau = T(b) \equiv & \frac{\widetilde{\phi}}{1-\pi} \cdot \left\{ \frac{1-\pi}{2\pi} \cdot \{2 \cdot (1-\omega)\}^{\frac{1}{\sigma}} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}} \right\} \\ & - \frac{1}{(1-\pi) \cdot y} \cdot \beta \cdot \frac{1-\omega}{\omega} \cdot (B_0 + B_1 b). \end{aligned}$$

In what follows, we seek to determine a set of parameters, B_0 and B_1, that ensure a stable steady-state political equilibrium with $\bar{z} \in [0, y], \bar{b} \in \left[0, b^{DL}\right]$, $\bar{\tau} \in [0, 1]$, and $\bar{v} \in [0, y]$.

The debt policy function, $B(b) = B_0 + B_1 \cdot b$, requires $B_1 \in (-1, 1)$ for the stability of the steady-state equilibrium. The steady-state level of b, denoted by $\bar{b}$, is $\bar{b} = B_0/(1 - B_1)$. Therefore, $\bar{b} \geq 0$ holds if and only if $B_0 \geq 0$ under the assumption of $B_1 \in (-1, 1)$.

The debt limit, denoted by b^{DL}, is derived by setting $\tau = 1$ and $z = v = 0$ in the government budget constraint (21). With $p = \beta \cdot (1-\omega)/\omega$, b^{DL} becomes

$$b^{DL} \equiv \frac{(1-\pi)y}{1-\beta \cdot \frac{1-\omega}{\omega}};$$

and $\bar{b} \le b^{DL}$ requires

$$\bar{b} \le b^{DL} \Leftrightarrow \; B_1 \le 1 - \frac{1}{(1-\pi)\cdot y} \cdot \left\{1 - \beta \cdot \frac{1-\omega}{\omega}\right\} \cdot B_0.$$

A pair (B_0, B_1) must satisfy the following for the existence of a stable path of $\{b\}$ that converges to the steady state with $\bar{b} \in \left[0, b^{DL}\right]$:

$$\begin{cases} B_0 \ge 0 \\ B_1 \in \left(-1, 1 - \frac{1}{(1-\pi)\cdot y} \cdot \left\{1 - \beta \cdot \frac{1-\omega}{\omega}\right\} \cdot B_0\right] \end{cases} \tag{35}$$

Next, we derive the condition that ensures $\bar{z} \in [0, y]$. Under the assumption of (35), it always holds that $\bar{z} < y$. With (33) and $\bar{b} = B_0/(1-B_1)$, we have

$$\bar{z} \ge 0 \Leftrightarrow \; B_1 \le 1 - \frac{\frac{1-\pi}{2\pi} \cdot \{2\cdot(1-\omega)\}^{\frac{1}{\sigma}} \cdot \left\{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}\right\} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}}{(1-\pi)\cdot \frac{1-\pi}{2\pi} \cdot \{2\cdot(1-\omega)\}^{\frac{1}{\sigma}} \cdot y} \cdot B_0.$$

Third, we determine a set (B_0, B_1) that ensures $\bar{\tau} \in [0, 1]$. With $\bar{b} = B_0/(1-B_1)$, the steady-state τ becomes

$$\bar{\tau} = \frac{\widetilde{\phi}}{1-\pi} \cdot \left\{\frac{1-\pi}{2\pi} \cdot \{2\cdot(1-\omega)\}^{\frac{1}{\sigma}} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}\right\} - \frac{1}{(1-\pi)\cdot y} \cdot \beta \cdot \frac{1-\omega}{\omega} \cdot \frac{B_0}{1-B_1},$$

which leads to

$$\begin{aligned} \bar{\tau} \ge 0 \Leftrightarrow B_1 &< 1 - \frac{1}{y} \cdot \beta \cdot \frac{1-\omega}{\omega} \\ &\times \frac{\frac{1-\pi}{2\pi} \cdot \{2\cdot(1-\omega)\}^{\frac{1}{\sigma}} \cdot \left\{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}\right\} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}}{(1-\pi)\cdot \left\{\frac{1-\pi}{2\pi} \cdot \{2\cdot(1-\omega)\}^{\frac{1}{\sigma}} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}\right\}} \cdot B_0; \text{ and} \\ \bar{\tau} \le 1 \Leftrightarrow &\; \frac{\frac{1-\pi}{2\pi} \cdot \{2\cdot(1-\omega)\}^{\frac{1}{\sigma}} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}}{\frac{1-\pi}{2\pi} \cdot \{2\cdot(1-\omega)\}^{\frac{1}{\sigma}} \cdot \left\{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}\right\} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}} - 1 \\ &< \frac{1}{(1-\pi)\cdot y} \cdot \beta \cdot \frac{1-\omega}{\omega} \cdot \frac{B_0}{1-B_1}, \end{aligned}$$

where $\bar{\tau} \le 1$ holds for any pair (B_0, B_1) satisfying (35). The conditions derived so far are summarized as in Proposition 3.

■

7.6 Proof of Corollaries 2 and 3

The social security payment to the poor is

$$
\begin{aligned}
v &= \widetilde{\phi} \cdot \frac{1}{2\pi} \cdot \left(\frac{1-\omega}{\omega} \right)^{\frac{1}{\sigma}} \cdot y, \\
&= \frac{1-\pi}{2\pi} \cdot \frac{1}{\frac{1-\pi}{2\pi} \cdot \{2 \cdot (1-\omega)\}^{\frac{1}{\sigma}} \cdot \left\{ 1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}} \right\} + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\sigma}}} \cdot \left(\frac{1-\omega}{\omega} \right)^{\frac{1}{\sigma}} \cdot y \\
&= \frac{y}{(2)^{\frac{1}{\sigma}} \cdot \left\{ (\omega)^{\frac{1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} \right\} + \frac{2\pi}{1-\pi}}.
\end{aligned}
$$

Because ω has an effect on v through the term $\left\{ (\omega)^{\frac{1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} \right\}$, we can apply the result in the proof of Corollary 1. The effect of π on v is observed from the term $2\pi/(1-\pi)$ of the denominator in the third line.

■

7.7 Derivation of the government objective function in Section 5

An agent maximizes his/her utility

$$
u(c_t^y) + \theta \cdot \psi(g_t) + \beta \cdot \left\{ u(c_{t+1}^o) + \theta \cdot \psi(g_{t+1}) \right\}
$$

subject to the budget constraints in youth and old age:

$$
\begin{aligned}
c_t^y + p_t b_t^d &\leq (1-\tau_t) \cdot y, \\
c_{t+1}^o &\leq b_t^d + z_{t+1}.
\end{aligned}
$$

Solving the problem yields the consumption Euler equation

$$
\beta \cdot u'(c_{t+1}^o) = p_t \cdot u'(c_t^y).
$$

We substitute the budget constraints into the above consumption Euler equation to obtain the demand function of government bond:

$$
b_t^d = b^d \left((1-\tau_t) \cdot y, z_{t+1}, p_t \right).
$$

The asset-market-clearing condition, $b_t^s = b_t^d = b_t$, gives an equilibrium price implicitly given by

$$p_t = P(b_t,\ (1 - \tau_t) \cdot y,\ z_{t+1}). \tag{36}$$

The government budget constraint is

$$p_t b_t^s + \tau_t y = z_t + g_t + b_{t-1}^s. \tag{37}$$

With (36) and (37) with $b_t^s = b_t$, the consumption in youth becomes

$$\begin{aligned} c_t^y &= (1 - \tau_t) \cdot y - P(b_t,\ (1 - \tau_t) \cdot y,\ z_{t+1}) \cdot b_t \\ &= y - z_t - g_t - b_{t-1}, \end{aligned} \tag{38}$$

where the second line comes from (37), and the consumption in old age is

$$c_{t+1}^o = b_t + z_{t+1}, c_t^o = b_{t-1} + z_t. \tag{39}$$

By the use of (38) and (39), we can derive the objective function in the unconstrained problem, as demonstrated in Section 5.

■

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