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Tetsuo Ono†
Osaka University

Abstract

This paper analyzes the political economy of public education and in-cash transfer in an overlapping generations model of a two-class society in which the dynamics of inequality is driven by the accumulation of human capital. The two redistributive policies are determined by voting, while private education that supplements public education is purchased individually. The model, which includes two-dimensional voting, demonstrates either of the following two types of stable steady-state equilibria, which are in line with the evidence: a high-inequality equilibrium with government expenditure favoring lump-sum transfer, or a low-inequality equilibrium with that favoring public education.

- JEL Classification Numbers: D72, D91, I24

  Key words: Public education, political economy, inequality

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†Tetsuo Ono, Graduate School of Economics, Osaka University, 1-7, Machikaneyama, Toyonaka, Osaka 560-0043, Japan. E-mail: tono@econ.osaka-u.ac.jp

1 Introduction

In most developed countries, redistribution is carried out through in-kind transfer programs such as public education for successive generations, as well as through in-cash transfer programs such as social security and welfare budgets within the current generation. The size and the composition of redistributive policies in democratic countries are determined via voting, and redistributive expenditures affect human capital formation and thus the next generation's income distribution, which in turn have an effect on future voting over in-cash and in-kind redistribution programs. Therefore, it is natural to expect some correlation between inequality and the size and composition of redistributive expenditures.

Figure 1 illustrates a scatter plot of the Gini index and the ratio of in-cash transfer to public education expenditure in OECD (Organization of Economic Cooperation and Development) countries. The figure shows a positive correlation between inequality and the ratio. High-inequality countries are associated with a large share of in-cash transfer in government expenditure, whereas low-inequality countries are associated with a large share of public education in government expenditure. The aim of this study is to develop a model that explains this cross-country difference, and to clarify the role of private education and family backgrounds in policymaking tackling inequality and redistribution.

[Figure 1 here.]

For the purpose of analysis, we use the framework in which Gradstein and Justman (1996) investigate the role of private education as an alternative to public education. The present model differs from theirs in the following two aspects. First, Gradstein and Justman (1996) consider public education as only a means of redistribution, while the present model allows for in-cash, lump-sum transfer as an alternative to public education. The presence of lump-sum transfer might incentivize some agents to prefer lump-sum transfer to public education, and to make use of the transfer benefits for private education for their children.

Second, Gradstein and Justman (1996) assume that the marginal productivity of education is constant, while the present model assumes that it is dependent on human capital. In particular, the present paper assumes that the marginal productivity of private education increases with the parents' human capital level. This assumption reflects the family background effect: educated parents can provide their children a better environment for learning (Gertler and Glewwe, 1990).

The present model, which includes the above two aspects, works as follows. There are two types of family dynasties classified according to their endowed level of human capital: low and high. An agent in each type of family enters adulthood with a stock of human capital invested by his/her parents, earns after-tax labor income, and receives lump-sum transfer benefits form the government. He/she decides the allocation of the disposable income between current consumption and private investment in his/her child's further education. The private educational investment combined with public education determines his/her child's human capital level, which determines the child's income.

Every adult agent votes over the tax rate as well as the allocation of tax revenue between public education and lump-sum transfer. Given the bidimensional issue space, the Nash equilibrium of a majority voting game may fail to exist. To deal with this, we use the concept of issue-by-issue voting, that is, the notion of a structure-induced Nash equilibrium voting game formalized by Shepsle (1979) and applied by Conde-Ruiz and Galasso (2003, 2005) for the framework of overlapping generations. Throughout the paper, the low-type individuals are the majority in the economy. We compute the low-type's preferred allocation for a given tax rate and his/her preferred tax rate for a given allocation, and find the point where these two reaction functions cross.

Based on the notion of structure-induced Nash equilibrium voting, we first focus on some period t and demonstrate the period-t political equilibrium outcome for a given inequality level. When inequality is low, the economy attains an equilibrium with government expenditure favoring public education. The low-type individuals, as the majority, prefer public education to lump-sum transfer because their income is not far from the average and they have a certain level of income. When inequality is high, the economy attains an equilibrium with government expenditure favoring lump-sum transfer. The low-type individuals feel the need for support via lump-sum transfer to compensate for their low income.

The characterization of the period-t political equilibrium enables us to show the effect of inequality on redistribution policy, but not the mutual interaction between inequality and redistribution policy. To illustrate the interaction, we analyze the dynamics of inequality and redistribution policies across periods, and show that the family backgrounds do affect the joint determination of inequality and redistribution policy. In particular, we show that the economy with weaker family background effect attains a more equal state with government expenditure favoring public education, and that the economy with the stronger family background effect attains a less equal state with government expenditure favoring lump-sum transfer. The result provides one possible explanation for the cross-country difference in inequality and redistribution policy observed in Figure 1.

The remainder of this paper is as follows. We first present a literature review. Thereafter, Section 2 sets up the model and characterizes an economic equilibrium. Section 3 considers voting behavior of agents and characterizes political equilibria in each period. Sections 4 shows the existence and stability of a steady-state equilibrium, and clarifies the role of family backgrounds in the joint determination of inequality and redistribution policy. Section 5 checks the robustness of the result under alternative assumptions. Some proofs are relegated to Appendix.

1.1 Literature Review

In the recent decades, there has been an increasing amount of literature on the political economy of inequality and redistribution (see, e.g., Persson and Tabellini, 2000 and Borck, 2007, for a survey). However, most of these focus on either redistribution in cash through lump-sum transfer (e.g., Romer, 1975; Roberts, 1977; Meltzer and Richard, 1981; Alesina and Rodrik, 1994; Persson and Tabellini, 1994; Benabou, 2000) or redistribution in kind through public education (e.g., Stiglitz, 1974; Glomm and Ravikumar, 1992, 1996, 1998, 2001; Epple and Romano, 1996; Gradstein and Justman, 1996, 1997; Hoyt and Lee, 1998; Cardak, 2004; Glomm, 2004; Bearse, Glomm, and Patterson, 2005; de la Croix and Doepke, 2009). These studies do not demonstrate the effect of current income inequality on the allocation of tax revenue between the two types of redistribution via voting, and that of the allocation on the inequality within the next generation through human capital accumulation.

Some recent studies provide partial answers to the above questions. Bearse, Glomm, and Janeba (2001) and Creedy and Moslehi (2009) consider an alternative to lump-sum transfer, namely, public education (Bearse, Glomm and Janeba, 2001) and public goods provision (Creedy and Moslehi, 2009). They consider voting on the composition of government expenditure, and successfully demonstrate how the composition is affected by inequality of voters. However, in their models, the tax rate is considered exogenous to ensure voting over one dimension. In addition, there is no dynamic interaction between inequality and redistributive policy because pre-tax income distribution is exogenously given in their static frameworks.

Bernasconi and Profeta (2012) overcome the limitations in the above mentioned studies by developing a two-period overlapping generations model of a two-class society with human capital accumulation. They consider probabilistic voting over public education and lump-sum transfer within a generation. However, private education as an alternative to public education, which occurs to a significant extent in many developed countries (OECD, 2013), is exempted from their analysis, because their focus is on the role of public education as a device to provide for recognizing the talent of poor-born children.[1]

The literature review thus far suggests that the following questions remain unresolved. How do the politics of public education and lump-sum transfer affect economic decision

[1] Arcalean and Schiopu (2010) study the interaction between public and private education expenditure. However, they abstract away from voting and consider education policy as given. Their focus is on the effect of change in education policy on human capital accumulation and growth.

over private education of agents? How does the decision in turn affect the inequality among agents and their preferences over the size and the composition of redistributive expenditures? What are the dynamic, long-run consequences of this interaction for inequality and redistributive policy? What causes the difference in inequality and redistribution policy across countries? The contribution of this paper is to answer these unresolved issues in the previous studies, and to show that private education and family background play crucial roles in answering these issues.

2 The Model and Economic Equilibrium

We consider a discrete-time overlapping generations economy that starts at time 0. The economy is populated by two types of family dynasties, indexed by $i \in \{L, H\}$, of agents who live in two periods, youth and adulthood. A type-i adult agent in period 0 is endowed with h_0^i units of human capital, where $0 < h_0^L < h_0^H$. Thus, period-0 type-L and type-H agents are endowed with low and high human capital, respectively.

Each adult agent produces one offspring; hence, the population remains constant in every generation. The fraction of type-i agents within each generation is given by $\phi^i \in (0, 1)$, where ϕ^i is constant across generations and satisfies $0 < \phi^H < 0.5 < \phi^L < 1$ with $\sum_i \phi^i = 1$. The assumption implies that in every period, type-L agents are the majority in the economy. This assumption reflects the right-skewed income distribution in the real world. In Section 5, we undertake a brief analysis of an alternative case where the type-H agents are the majority in the economy.

2.1 Preferences and Budget Constraints

A type-i adult agent at time t is endowed at the time he/she enters adulthood with a stock of human capital h_t^i, which also defines his/her effective labor capacity. He/she receives a lump-sum transfer from the government, b_t. Given the income tax τ_t and the transfer b_t, a type-i adult decides the allocation of disposable income between current consumption, c_t^i, and private investment in his/her child's further education, z_t^i, subject to the budget constraint,

$$c_t^i + z_t^i \leq (1 - \tau_t) h_t^i + b_t.$$

A type-i adult of generation t derives utility from current consumption, c_t^i, and from his/her child's anticipated future income, h_{t+1}^i. A type$-i$'s preferences are specified by the following utility function

$$u_t^i = (1 - \theta) \ln c_t^i + \theta \ln h_{t+1}^i,$$

where $\theta \in (0, 1)$ is a common parameter reflecting the bequest motive. A higher θ implies a greater incentive for educational investment. We employ this logarithmic utility function for tractability of analysis. In Section 5, it is shown that the result is qualitatively unchanged when we employ a constant elasticity-of-substitution utility function.

2.2 Human Capital Formation

The level of offspring's education, h^i_{t+1}, is determined by public schooling, e_t, as well as by privately purchased supplementary education, z^i_t. We assume that the individual level of education is determined by the human capital production function, $h^i_{t+1} = A^i \cdot \left[\left(\bar{h}_t\right)^{\mu} e_t + (h^i_t)^{\mu} z^i_t\right]$, or

$$h^i_{t+1} = A^i \left(\bar{h}_t\right)^{\mu} \cdot \left[e_t + \left(\rho^i_t\right)^{\mu} z^i_t\right],$$

where

$$\bar{h}_t \equiv \phi^L h^L_t + \phi^H h^H_t \text{ and } \rho^i_t \equiv \frac{h^i_t}{\bar{h}_t}.$$

The variable $\bar{h}_t$ denotes the average human capital in period t, and the variable ρ^i_t denotes the ratio of type-i's human capital to the average human capital. The parameters A^i and μ are assumed to satisfy $A^i > 0$ and $\mu \in (0,1)$. The term $e_t + (\rho^i_t)^{\mu} z^i_t$ is the sum of the effective public and private educational investments.

The function above has the following two features. First, the marginal productivity of public education depends on the average human capital representing, for example, the quality of teachers in public schools (de la Croix and Doepke, 2004). On the other hand, the marginal productivity of private education depends on the parents' human capital level, implying that"educated parents can provide an environment conductive to better learning, such as directly helping children with schoolwork, which will also raise the human capital received per year by the child" (Gertler and Glewwe, 1990). This feature is further discussed in Section 5.

Second, the parameter $A^i(> 0)$ represents a durable productive asset handed from generation to generation, such as genetic ability, technology transfer, and business succession (Gradstein and Justman, 1996). The distribution of A^i is assumed to be stationary over time and to be positively correlated to human capital, h^i_t

$$A^H = A > 0 \text{ and } A^L = \alpha A \text{ where } \alpha \in (0,1).$$

This assumption implies that on average, children born in higher-income families are endowed with a higher productivity of human capital, and thus, a higher learning technology (see, e.g., Huggett, Ventura and Yaron, 2006).[2]

[2] A possible extension is to assume that children have the same genetic ability with a probability q. For example, children born in higher-income families have high genetic ability, A^H, with a probability q, while they have low genetic ability, A^L, with a probability $1-q$. Bernasconi and Profeta (2012) assume that this genetic probability of talent transmission, q, is not generally known in public, thereby resulting in the talent mismatch. The current paper abstracts away from the talent transmission and mismatch; instead, it focuses on the interaction between public and private education choice.

2.3 Government Budget Constraint

In each period, the government raises tax revenue to finance the provision of uniform public schooling for all children, e_t, as well as lump-sum transfer, b_t. The fraction $\lambda_t \in [0,1]$ of the tax revenue is devoted to lump-sum transfer; the rest is devoted to public schooling. Thus, the government budget constraint is given by

$$b_t = \lambda_t(1-\tau_t)\tau_t\bar{h}_t;$$
$$e_t = (1-\lambda_t)(1-\tau_t)\tau_t\bar{h}_t,$$

where the term $\bar{h}_t$ is the average human capital in period t, which is equivalent to the aggregate income in that period. The term $(1-\tau_t)$ denotes the distortionary factor that represents efficiency loss of taxation. This assumption, which is common in the political economy literature (see, e.g., Conde-Ruiz and Galasso, 2004; Conde-Ruiz and Profeta, 2007; Bethencourt and Galasso, 2008), is solely to ensure an interior solution to preferred tax rates and otherwise plays no role.

The timing of events in period t is as follows. First, adult agents vote on the tax rate τ_t as well as the fraction of tax revenue devoted to lump-sum transfer λ_t by majority vote. Second, subject to the budget constraint, each agent decides on the allocation of disposable income between consumption and private education. We solve the model by backward induction.

2.4 Economic Equilibrium

Given a sequence of tax rates and the sizes of redistribution and public education, $\{\tau_t, b_t, e_t\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations, $\{z_t^i, c_t^i, h_t^i\}_{i=L,H}^{t=0,\ldots,\infty}$ with the initial condition h_0^i $(i = L, H)$, such that (i) in every period, a type-i agent maximizes his/her utility subject to the budget constraint and the non-negativity constraint of investment in private education and (ii) the government budget is balanced in every period.

Recall that $\rho_t^i \equiv h_t^i/\bar{h}_t(i = L, H)$ denotes the ratio of type-i's private human capital to the average human capital. By definition, the relation of $h_t^L < \bar{h}_t < h_t^H$ holds for all t, and thus, $\rho_t^i(i = L, H)$ satisfies the following property:

$$0 < \rho_t^L \equiv \frac{h_t^L}{\bar{h}_t} < 1 < \rho_t^H \equiv \frac{h_t^H}{\bar{h}_t}.$$

The variable ρ_t^L, on which we will focus in the following sections, captures the extent of income inequality in the economy; a higher ρ_t^L implies less inequality.

The utility maximization problem of a type-i agent in period t is as follows:

$$\max_{z_t^i \in \left[0,(1-\tau_t)h_t^i + b_t\right]} (1-\theta)\ln c_t^i + \theta \ln h_{t+1}^i$$

subject to

$$c_t^i + z_t^i \leq (1-\tau_t)h_t^i + b_t,$$
$$h_{t+1}^i = A^i \left(\bar{h}_t\right)^\mu \cdot \left[e_t + \left(\rho_t^i\right)^\mu z_t^i\right],$$

given $\tau_t, h_t^i, \rho_t^i, b_t, e_t$ and $\bar{h}_t$.

Solving the utility maximization problem of a type-i agent leads to the following private education decision:

$$z_t^i = \max\left\{0, \theta\left\{(1-\tau_t)h_t^i + b_t\right\} - \frac{1-\theta}{\left(\rho_t^i\right)^\mu} e_t\right\}. \tag{1}$$

Equation (1) states that the investment decision depends on an adult's human capital h_t^i as well as government policy variables, τ_t, b_t and e_t. In particular, an agent chooses to invest in private education if his/her human capital is high, the tax rate is low, the size of redistribution is large, and/or the level of public education is low; otherwise, he/she chooses no private investment in education and consumes all of his/her disposable income.

With the use of (1) and the budget constraint, we can write the consumption function as

$$c_t^i = \min\left\{(1-\tau_t)h_t^i + b_t, \quad (1-\theta)\cdot\left((1-\tau_t)h_t^i + b_t + \frac{e_t}{\left(\rho_t^i\right)^\mu}\right)\right\}.$$

The sum of the effective public and private educational investments, $e_t + \left(\rho_t^i\right)^\mu z_t^i$, becomes

$$e_t + \left(\rho_t^i\right)^\mu z_t^i = \max\left\{e_t, \quad \theta\cdot\left[e_t + \left(\rho_t^i\right)^\mu\left\{(1-\tau_t)h_t^i + b_t\right\}\right]\right\}.$$

The utility obtained by agents in economic equilibrium is represented by their indirect utility functions. We use the above mentioned investment and consumption functions to obtain an indirect utility function of a type-i agent as follows:

$$V_t^i = \begin{cases} V_{t,z>0}^i \equiv \ln\left[(1-\tau_t)h_t^i + b_t + \frac{e_t}{\left(\rho_t^i\right)^\mu}\right] + (1-\theta)\ln(1-\theta) + \theta\ln A^i\left(\bar{h}_t\right)^\mu \theta\left(\rho_t^i\right)^\mu & \text{if } z_t^i > 0; \\ V_{t,z=0}^i \equiv (1-\theta)\ln\left[(1-\tau_t)h_t^i + b_t\right] + \theta\ln A^i\left(\bar{h}_t\right)^\mu e_t & \text{if } z_t^i = 0, \end{cases}$$

where $V_{t,z>0}^i$ denotes the indirect utility of a type-i agent when he/she invests some portion of his/her income in private education, and $V_{t,z=0}^i$ denotes the indirect utility when he/she does not invest in private education.

With the use of the government budget constraints $b_t = \lambda_t(1-\tau_t)\tau_t\bar{h}_t$ and $e_t = (1-\lambda_t)(1-\tau_t)\tau_t\bar{h}_t$, we can rewrite the condition that determines investment decisions as follows:

$$z_t^i > 0 \Leftrightarrow \lambda_t > \left(\frac{1-\theta}{\theta}\cdot\frac{1}{\left(\rho_t^i\right)^\mu} - \frac{\rho_t^i}{\tau_t}\right)\cdot\left(1 + \frac{1-\theta}{\theta}\cdot\frac{1}{\left(\rho_t^i\right)^\mu}\right)^{-1}. \tag{2}$$

This inequality condition is more likely to be satisfied as the tax rate τ_t is lower and the share of redistribution in government expenditure λ_t is higher. A lower tax rate and a higher share of redistribution in government expenditure produce a larger income effect, thereby giving an agent an incentive to invest in private education.

With the use of condition (2) and the government budget constraints, we can write the indirect utility function in terms of the tax rate τ_t and the fraction λ_t as follows:

$$V_t^i = \begin{cases} V_{t,z>0}^i \equiv \ln(1-\tau_t)\left[\rho_t^i + \lambda_t\tau_t + \frac{1}{\left(\rho_t^i\right)^\mu}(1-\lambda_t)\tau_t\right] + X_{Z>0}\left(\bar{h}_t, \rho_t^i\right) \\ \qquad \text{if } \lambda_t > \left(\frac{1-\theta}{\theta}\cdot\frac{1}{\left(\rho_t^i\right)^\mu} - \frac{\rho_t^i}{\tau_t}\right)\cdot\left(1+\frac{1-\theta}{\theta}\cdot\frac{1}{\left(\rho_t^i\right)^\mu}\right)^{-1}; \\ V_{t,z=0}^i \equiv \ln(1-\tau_t) + (1-\theta)\ln\left(\rho_t^i + \lambda_t\tau_t\right) + \theta\ln(1-\lambda_t)\tau_t + X_{Z=0}\left(\bar{h}_t, \rho_t^i\right) \\ \qquad \text{if } \lambda_t \le \left(\frac{1-\theta}{\theta}\cdot\frac{1}{\left(\rho_t^i\right)^\mu} - \frac{\rho_t^i}{\tau_t}\right)\cdot\left(1+\frac{1-\theta}{\theta}\cdot\frac{1}{\left(\rho_t^i\right)^\mu}\right)^{-1}, \end{cases} \tag{3}$$

where $X_{Z>0}\left(\bar{h}_t, \rho_t^i\right)$ and $X_{Z=0}\left(\bar{h}_t, \rho_t^i\right)$ are the terms unrelated to political decisions and defined as follows:

$$X_{Z>0}\left(\bar{h}_t, \rho_t^i\right) \equiv \ln\bar{h}_t + (1-\theta)\ln(1-\theta) + \theta\ln A^i\left(\bar{h}_t\right)^\mu\theta\left(\rho_t^i\right)^\mu;$$
$$X_{Z=0}\left(\bar{h}_t, \rho_t^i\right) \equiv (1-\theta)\ln\bar{h}_t + \theta\ln A^i\left(\bar{h}_t\right)^{\mu+1}.$$

3 Period$-t$ Political Equilibrium

In each period t, the tax rate τ_t and the proportion λ_t are determined by period-t adult agents through a political process of majority voting. Type-L and type-H adult agents cast a ballot over τ_t, the income tax rate, and λ_t, the share of lump-sum transfer in government expenditure. Individual preferences over the two issues are represented by the indirect utility function in (3) for $i = L, H$. Every agent has zero mass, and thus, no individual vote can change the outcome of the election. Therefore, we assume that agents vote sincerely.

The current majority voting game is characterized by a bidimensional issue space, τ and λ. Thus, a Nash equilibrium may not exist within the majority voting game. To deal with this, we use the concept of issue-by-issue voting, or the structure-induced Nash equilibrium, as formalized by Shepsle (1979) and applied by Conde-Ruiz and Galasso (2003, 2005) to the framework of overlapping generations. Under the concept of the structure-induced Nash equilibrium, a sufficient condition for (τ_t^*, λ_t^*) to be a *period-t political equilibrium* of the voting game is that τ_t^* represents the outcome of majority voting over τ_t when the other dimension is fixed at its level λ_t^*, and vice versa, provided that preferences are single peaked along every dimension of the issue space.[3]

[3] Probabilistic voting is also useful to demonstrate two-dimensional voting (see, e.g., Persson and

Under the current framework, type-L agents are the majority for each issue, and the preferences of type-L agents, specified in Eq. (3), are singled-peaked for each issue. We can apply the concept of the structure-induced Nash equilibrium to the current framework. Let $\lambda_t^L(\tau)$ denote type-L's most preferred share as a function of the tax rate τ_t, and let $\tau_t^L(\lambda)$ denote type-L's most preferred tax rate as a function of λ_t. The point where these two reaction functions cross corresponds to the structure-induced Nash equilibrium outcome of the voting game. Given the assumption that type-L agents are the majority for each issue, we investigate two cases, a case of $z_t^L > 0$ and a case of $z_t^L = 0$, respectively.

3.1 Type-L's Preferred Policy When $z_t^L > 0$

Suppose that the type-L agent invests a part of his/her income in education. The condition $z_t^L > 0$ in (2) is rewritten in terms of ρ_t^L as follows:

$$z_t^L > 0 \Leftrightarrow \lambda_t > \left(\frac{1-\theta}{\theta} \cdot \frac{1}{(\rho_t^L)^\mu} - \frac{\rho_t^L}{\tau_t}\right) \cdot \left(1 + \frac{1-\theta}{\theta} \cdot \frac{1}{(\rho_t^L)^\mu}\right)^{-1}. \tag{4}$$

Under the condition in (4), the type-L agent, as a decisive voter, chooses τ_t to maximize his/her indirect utility, $V_{t,z>0}^L$.

The first derivative of $V_{t,z>0}^L$ with respect to τ_t is

$$\frac{\partial V_{t,z>0}^L}{\partial \tau_t} = \frac{-1}{1-\tau_t} + \frac{\lambda_t + (1-\lambda_t)/\left(\rho_t^L\right)^\mu}{\rho_t + \lambda_t \tau_t + (1-\lambda_t)\tau_t/\left(\rho_t^L\right)^\mu}.$$

The first term on the right-hand side shows the marginal cost of taxation; the second term shows the marginal benefit of taxation. The above equation indicates that the marginal cost is independent of the share of redistribution in government expenditure, λ_t, while the marginal benefit is increasing in λ_t. Therefore, there is a critical value of λ_t, and type$-L$ agents will find it optimal to bear no tax burden when λ_t is below this critical value, while they find it optimal to bear some tax burden when λ_t is above the critical value. The optimal choice of τ_t by type-L agents is summarized as follows:

$$\tau_t = \tau_{z>0}^L(\lambda_t) \equiv \max\left\{0, \frac{1}{2} \cdot \left(1 - \frac{\left(\rho_t^L\right)^{1+\mu}}{1 - \lambda_t \cdot \left(1 - (\rho_t^L)^\mu\right)}\right)\right\}, \tag{5}$$

where the superscript and the subscript in $\tau_{z>0}^L(\cdot)$ denote the type-L and $z_t^L > 0$, respectively.

Next, consider the choice of λ_t by type$-L$ agents when $z_t^L > 0$. As we can see in Eq. (3), a marginal increase in λ_t results in an increase in redistribution by $(1-\tau_t)\tau_t$ units, whereas it results in a decrease in public education by $(1-\tau_t)\tau_t/\left(\rho_t^L\right)^\mu$ units. The

Tabellini, 2000). The present study employs issue-by-issue voting rather than probabilistic voting because analytical solutions of the present model are not available in the latter voting approach.

net benefits are positive or negative depending on the relative magnitude between them. That is,

$$\frac{\partial V^L_{t,z>0}}{\partial \lambda_t} \gtrless 0 \text{ if and only if } \rho^L_t \gtrless 1,$$

where $\rho^L_t < 1$ always holds. Therefore, the type-L's choice of λ_t when $z^L_t > 0$ is given by

$$\lambda_t = \lambda^L_{z>0}(\tau_t) \equiv 0.$$

[Figure 2 here.]

The period-t political equilibrium when $z^L_t > 0$ is the point where the two reaction functions, $\tau_t = \tau^L_{z>0}(\lambda_t)$ and $\lambda_t = \lambda^L_{z>0}(\tau_t)$, cross as demonstrated in Figure 2. Substituting $\lambda_t = \lambda^L_{z>0}(\tau_t) \equiv 0$ into $\tau_t = \tau^L_{z>0}(\lambda_t)$ in (5), we can compute the period-t equilibrium policy when $z^L_t > 0$ as $(\tau_t, \lambda_t) = \left(\frac{1}{2}\left(1 - \left(\rho^L_t\right)^{1+\mu}\right), 0\right)$. We substitute the policy into the condition $z^L_t > 0$ in (4) to find the range of ρ^L_t that is consistent with the condition $z^L_t > 0$ as $\rho^L_t > \left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)}$. We can also find that $z^H_t > 0$ holds in the current equilibrium because $z^H_t > z^L_t$ always holds by the definition of ρ^L_t. The analysis thus far is summarized in the following lemma.

- **Lemma 1.** *There exists a period-t political equilibrium with $z^L_t > 0$ and $z^H_t > 0$ if*

$$\rho^L_t \in \left(\left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)}, 1\right).$$

The corresponding policy is

$$(\tau_t, \lambda_t) = \left(\frac{1}{2}\left(1 - \left(\rho^L_t\right)^{1+\mu}\right), 0\right).$$

The condition in Lemma 1, $\rho^L_t \in \left(\left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)}, 1\right)$, determines the investment decision by the type-L agents. The condition says that the type-L's income is not far from the average. They have a certain level of income and thus can afford to invest in private education. However, they feel the need for public support toward education because the marginal productivity of public education is higher than that of the private education financed by the lump-sum transfer. Therefore, they prefer public education to lump-sum transfer and thus want to finance education both privately and publicly.

3.2 Type-L's Preferred Policy When $z^L_t = 0$

Suppose that the type-L agent does not invest in private education: $z^L_t = 0$. The condition $z^L_t = 0$ in terms of ρ^L_t is

$$z^L_t = 0 \Leftrightarrow \lambda_t \leq \left(\frac{1-\theta}{\theta} \cdot \frac{1}{(\rho^L_t)^\mu} - \frac{\rho^L_t}{\tau_t}\right) \cdot \left(1 + \frac{1-\theta}{\theta} \cdot \frac{1}{(\rho^L_t)^\mu}\right)^{-1}. \qquad (6)$$

Under the condition in (6), the type-L agent, as a decisive voter, chooses τ_t to maximize his/her indirect utility $V_{t,z=0}^L$.

The first derivative of $V_{t,z=0}^L$ with respect to τ_t is

$$\frac{\partial V_{t,z=0}^L}{\partial \tau_t} = \frac{-1}{1-\tau_t} + (1-\theta)\cdot\frac{\lambda_t}{\rho_t^L + \lambda_t\tau_t} + \frac{\theta}{\tau_t}.$$

The first term on the right-hand side shows the marginal cost of taxation; the second term shows the marginal benefit of taxation via redistribution; and the third terms shows the marginal benefit of taxation via public education. Corner solutions, $\tau_t = 0$ and 1, are not optimal for type$-L$ agents because $\partial V_{t,z=0}^L/\partial\tau_t\big|_{\tau_t=0} = +\infty > 0$ and $\partial V_{t,z=0}^L/\partial\tau_t\big|_{\tau_t=1} = -\infty < 0$ hold. Thus, the optimal solution satisfies $\partial V_{t,z=0}^L/\partial\tau_t = 0$, which results in

$$\tau_t = \tau_{z=0}^L(\lambda_t) \Leftrightarrow \lambda_t = \frac{(1+\theta)-\theta/\tau_t}{1-2\tau_t}\rho_t^L.$$

Next, consider the choice of λ_t by the type$-L$ agents when they do not invest in private education, $z_t^L = 0$. The first derivative of $V_{t,z=0}^L$ with respect to λ_t is

$$\frac{\partial V_{t,z=0}^L}{\partial \lambda_t} = (1-\theta)\cdot\frac{\tau_t}{\rho_t^L + \lambda_t\tau_t} - \frac{\theta}{1-\lambda_t}.$$

The first term on the right-hand side shows the marginal benefit from an increase in redistribution, and the second term shows the marginal cost from a decrease in spending on public education. When the tax rate τ_t is low, the latter effect overcomes the former one; type$-L$ agents prefer no redistribution, $\lambda_t = 0$. However, when the tax rate is high, the two opposing effects are offset at some level of $\lambda_t \in (0,1)$. In this case, the optimal share satisfies $\partial V_{t,z=0}^L/\partial\lambda_t = 0$. Therefore, the preferred share λ_t for type$-L$ agents is summarized as

$$\lambda_t = \lambda_{z=0}^L(\tau_t) \equiv \max\left\{0, (1-\theta) - \frac{\theta\rho_t^L}{\tau_t}\right\}.$$

[Figure 3 about here.]

The two reaction functions, $\tau_t = \tau_{z=0}^L(\lambda_t)$ and $\lambda_t = \lambda_{z=0}^L(\tau_t)$, are illustrated in Figure 3. The crossing points of the two reaction functions may correspond to the period-t structure-induced Nash equilibrium of the voting game when $z_t^L = 0$. We substitute the solution into the condition $z_t^L = 0$ in (6) to derive the range of ρ_t^L that is consistent with the condition $z_t^L = 0$. We can also show that $z_t^H > 0$ holds in the current equilibrium. The analysis thus far is summarized in the following lemma.

- **Lemma 2.** *There exists a period-t political equilibrium with* $z_t^L = 0$ *and* $z_t^H > 0$ *if*

$$\rho_t^L \in \left(0, \left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)}\right].$$

The corresponding policy is

$$(\tau_t, \lambda_t) = \begin{cases} \left(\frac{\theta}{1+\theta}, 0\right) & \text{if} \quad \rho_t^L \in \left[\frac{1-\theta}{1+\theta}, \left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)}\right]; \\ \left(\frac{1}{2}(1-\rho_t^L), (1+\theta) - \frac{2\theta}{1-\rho_t^L}\right) & \text{if} \quad \rho_t^L < \frac{1-\theta}{1+\theta}. \end{cases}$$

Proof. See Appendix A.1. ■

The condition in Lemma 2, $\rho_t^L \in \left(0, \left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)}\right]$, implies that the type-$L$'s income level is too low to pay for private education. Because of this negative income effect on private education, they prefer public education to private education regardless of the relative efficiency of private education. Therefore, they choose $e_t > 0$ for voting and $z_t^L = 0$ for economic decisions.

The remaining condition determines the type-L's preferences for lump-sum transfer. First, consider the case in which the type-L's income level is moderately high: $\rho_t^L \in \left[\frac{1-\theta}{1+\theta}, \left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)}\right]$. In this case, they feel no need for government support via lump-sum transfer, and find it optimal to use all the tax revenue for public education. Therefore, they choose a set of policies distinguished by no provision of lump-sum transfer. The corresponding policy is $(\tau_t, \lambda_t) = \left(\frac{\theta}{1+\theta}, 0\right)$, as demonstrated in Panel (a) of Figure 3.

Alternatively, consider the case in which the type-L's income level is too low such that $\rho_t^L < \frac{1-\theta}{1+\theta}$ holds. Because of their very low income level, the type-L agents feel the need for support via lump-sum transfer. However, this support is not enough to finance private education; they also need support for public education. Therefore, they choose a set of policies including the provision of both public education and lump-sum transfer. The corresponding policy is $(\tau_t, \lambda_t) = \left(\frac{1}{2}(1-\rho_t^L), (1+\theta) - \frac{2\theta}{1-\rho_t^L}\right)$, as demonstrated in Panel (b) of Figure 3.

3.3 Period-t Political Equilibrium

With the use of the results in Lemmas 1 and 2, we are now able to show the period-t political equilibrium policy for a given ρ_t^L.

Proposition 1. *The period-t political equilibrium policy is given as follows:*

$$(\tau_t, \lambda_t) = \begin{cases} \left(\frac{1}{2}(1-\rho_t^L), (1+\theta) - \frac{2\theta}{1-\rho_t^L}\right) & \text{if} \quad 0 < \rho_t^L < \frac{1-\theta}{1+\theta}; \\ \left(\frac{\theta}{1+\theta}, 0\right) & \text{if} \quad \frac{1-\theta}{1+\theta} \le \rho_t^L \le \left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)}; \\ \left(\frac{1}{2}\left(1-\left(\rho_t^L\right)^{1+\mu}\right), 0\right) & \text{if} \quad \left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)} < \rho_t^L < 1. \end{cases}$$

Proposition 1 states that the economy attains an equilibrium distinguished by the provision of both public education and lump-sum transfer when inequality is high such that $\rho_t^L < (1-\theta)/(1+\theta)$; it attains an equilibrium distinguished by the provision of public education only when inequality is low such that $\rho_t^L \ge (1-\theta)/(1+\theta)$. The result implies that the model shows a positive correlation between inequality and the ratio of lump-sum transfer to public education. This correlation is in line with the empirical evidence in OECD countries depicted in Figure 1.

To understand the mechanism behind the result, let us recall the type-L's human capital production function given by

$$h_{t+1}^L = A^L \left(\bar{h}_t\right)^\mu \cdot \left[e_t + \left(\rho_t^L\right)^\mu z_t^L\right].$$

For type-L agents, one unit of investment in public education produces a better return than that in private education because $\left(\rho_t^L\right)^\mu < 1$ holds. Thus, type-L agents prefer public education to private one from the viewpoint of optimality.

The preferences for the lump-sum transfer b_t and the corresponding economic decision on private investment in education z_t^L are not straightforward; they are dependent on the inequality represented by ρ_t^L. When the inequality is low such that $\left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)} < \rho_t^L < 1$, type-$L$ agents are endowed with moderately high income, they can afford to invest in private education and thus choose $z_t^L > 0$ from the viewpoint of utility maximization. In contrast, when the inequality is moderate such that $\frac{1-\theta}{1+\theta} \leq \rho_t^L \leq \left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)}$, type-$L$ agents are endowed with moderately low income and cannot afford the expense of spending in private education; they choose $z_t^L = 0$. In both cases, the type-L agents have certain level of income for consumption, and therefore, there is no need to require lump-sum transfer. They prefer public education to lump-sum transfer and thus choose $b_t = 0$ (i.e., $\lambda_t = 0$).

Finally, consider the case where the type-L's income level is considerably lower such that $0 < \rho_t^L < \frac{1-\theta}{1+\theta}$. Type-$L$ agents choose $z_t^L = 0$ because of their low income level. Given $z_t^L = 0$, the type-L's problem for choosing λ_t becomes

$$\max_{\lambda_t} \ln(1-\tau_t) + (1-\theta)\ln\left(\rho_t^L + \lambda_t\tau_t\right) + \theta\ln\left(1-\lambda_t\right)\tau_t.$$

The second term corresponds to the utility of consumption, which is dependent on ρ_t^L; and the third term corresponds to the utility of public education via the human capital production function, which is independent of ρ_t^L. As observed in the second term, the marginal utility of lump-sum transfer becomes larger as ρ_t^L becomes lower. This incentivizes the type-L agents to choose lump-sum transfer. Therefore, the type-L agents choose $e_t > 0$ and $b_t > 0$ in voting for this case.

Notice that the two sorts of equilibria, demonstrated in the present model, come from the external effect of parent's human capital associated with private education. This effect is observed by the term $\left(\rho_t^L\right)^\mu$ in the type-L's human capital production function, $h_{t+1}^L = A^L \left(\bar{h}_t\right)^\mu \cdot \left[e_t + \left(\rho_t^L\right)^\mu z_t^L\right]$. If the effect is removed from the model, we fail to demonstrate the two types of empirically relevant equilibria. This point is discussed more in detail in Section 5.

4 Steady-state Equilibrium

The characterization of the period-t political equilibrium enables us to clarify the role of inequality in the determination of redistribution policy and to provide empirically relevant results. However, it does not deal with the mutual interaction between inequality and redistribution policy because inequality is considered given. To illustrate the interaction and to explore its implication, we here demonstrate a motion of inequality across periods, and characterize a steady-state equilibrium in which $\rho_t^L = \rho_{t+1}^L$ holds along the equilibrium path. Based on this characterization, we examine how the structural parameters α and μ, representing family background effects, affect the determination of the steady-state inequality and redistribution policy. We then discuss the implication of the results.

We substitute private educational investment and equilibrium policy into the human capital production function, $h_{t+1}^i = A^i \left(\bar{h}_t\right)^\mu \cdot \left[e_t + (\rho_t^i)^\mu z_t^i\right]$, and obtain the period-$t+1$ human capital, h_{t+1}^i, as a function of $\bar{h}_t$ and h_t^i, as follows:

$$h_{t+1}^L = h^L\left(\bar{h}_t, h_t^L\right); h_{t+1}^H = h^H\left(\bar{h}_t, h_t^H\right).$$

Given the definition of the average human capital, $\bar{h}_{t+1} = \phi^L h_{t+1}^L + \phi^H h_{t+1}^H$, we can write $\bar{h}_{t+1}$ as a function of $\bar{h}_t$, h_t^L, and h_t^H as follows:

$$\bar{h}_{t+1} = \phi^L h^L\left(\bar{h}_t, h_t^L\right) + \phi^H h^H\left(\bar{h}_t, h_t^H\right).$$

Therefore, the period-$t+1$ measure of inequality, ρ_{t+1}^L, becomes

$$\rho_{t+1}^L \equiv \frac{h_{t+1}^L}{\bar{h}_{t+1}} = \frac{h^L\left(\bar{h}_t, h_t^L\right)}{\phi^L h^L\left(\bar{h}_t, h_t^L\right) + \phi^H h^H\left(\bar{h}_t, h_t^H\right)}.$$

After some calculations, we obtain the following law of motion of ρ_t^L (see Appendix A.2 for the derivation):

$$\rho_{t+1}^L = \tilde{\Omega}_i\left(\rho_t^L, \rho_t^H\right) \equiv \frac{\alpha}{\phi^L\alpha + \phi^H \cdot \omega_i\left(\rho_t^L, \rho_t^H\right)}, i = 1, 2, 3, \tag{7}$$

where $\omega_i\left(\rho_t^L, \rho_t^H\right) = \alpha h_{t+1}^H / h_{t+1}^L$. The function $\omega_i(\cdot, \cdot)$ is defined by

$$\omega_i\left(\rho_t^L, \rho_t^H\right) = \begin{cases} \omega_1\left(\rho_t^L, \rho_t^H\right) \equiv \theta + \frac{(1-\theta)-(1+\theta)\rho_t^L + 2\rho_t^H}{1+\rho_t^L} \cdot \left(\rho_t^H\right)^\mu & \text{if} \quad 0 < \rho_t^L < \frac{1-\theta}{1+\theta}, \\ \omega_2\left(\rho_t^L, \rho_t^H\right) \equiv \theta + (1+\theta) \cdot \left(\rho_t^H\right)^\mu & \text{if} \quad \frac{1-\theta}{1+\theta} \leq \rho_t^L \leq \left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)}, \\ \omega_3\left(\rho_t^L, \rho_t^H\right) \equiv \frac{1-\left(\rho_t^L\right)^{1+\mu} + 2\left(\rho_t^H\right)^{1+\mu}}{1+\left(\rho_t^L\right)^{1+\mu}} & \text{if} \quad \left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)} < \rho_t^L < 1, \end{cases}$$

where $\phi^L \rho_t^L + \phi^H \rho_t^H = 1$, that is,

$$\rho_t^H = \rho^H\left(\rho_t^L\right) \equiv \frac{1}{\phi^H}(1 - \phi^L \rho_t^L).$$

Let us denote $\tilde{\Omega}_i\left(\rho_t^L, \rho_t^H\right) = \tilde{\Omega}_i\left(\rho_t^L, \rho^H\left(\rho_t^L\right)\right) \equiv \Omega_i\left(\rho_t^L\right)$. Then, given an initial condition, $\rho_0^L(>0)$, the equilibrium sequence $\left\{\rho_t^L\right\}$ is represented by the first-order difference equation, $\rho_{t+1}^L = \Omega\left(\rho_t^L\right)$, where

$$\Omega\left(\rho_t^L\right) = \begin{cases} \Omega_1\left(\rho_t^L\right) & \text{if} \quad 0 < \rho_t^L < \frac{1-\theta}{1+\theta}, \\ \Omega_2\left(\rho_t^L\right) & \text{if} \quad \frac{1-\theta}{1+\theta} \leq \rho_t^L \leq \left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)}, \\ \Omega_3\left(\rho_t^L\right) & \text{if} \quad \left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)} < \rho_t^L < 1. \end{cases}$$

The function $\Omega(\cdot)$ has the following property: $\Omega_1(0) \in (0,1), \Omega_3(0) \in (0,1), \partial\Omega_i(\cdot)/\partial\rho_t^L > 0\ (i = 1,2,3), \Omega_1\left(\frac{1-\theta}{1+\theta}\right) = \Omega_2\left(\frac{1-\theta}{1+\theta}\right)$ and $\Omega_2\left(\left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)}\right) = \Omega_3\left(\left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)}\right)$. Therefore, we obtain the following result.

Proposition 2. *There exists a unique and stable steady-state equilibrium with $\rho^L \in (0,1)$.*

Having established the existence of a unique and stable steady-state equilibrium, we now investigate the effect of family background, represented by the parameters α and μ, on the determination of steady-state inequality and policy. Figure 3 illustrates numerical examples of the equation. In Panel (a), we set $\theta = 0.5, \phi^L = 0.6$ and $\mu = 0.25$, and illustrate three cases, $\alpha = 0.6, 0.75$ and 0.9. The figure shows that the economy attains a high-inequality steady state with government expenditure favoring lump-sum transfer when $\alpha = 0.6$, while it attains a low-inequality steady state with government expenditure favoring public education when $\alpha = 0.75$ and 0.9.

[Figure 4 here.]

To understand the role of α more precisely, consider first the case of a low α, that is, $\alpha = 0.6$. A low α implies that the type-L agents suffer from low productivity of human capital investment. Given this, the type-L agents expect a low rate of return of private and public education, which gives them a disincentive to pay for private education, and to choose lump-sum transfer rather than public education. Then, the economy realizes a high-inequality equilibrium with government expenditure favoring lump-sum transfer. The opposite result holds when α is high such that $\alpha = 0.75$ and 0.9.

In Panel (b), we fix α at 0.75 and focus on the parameter μ for three cases: $\mu = 0.05, 0.3$ and 0.55. The figure shows that the economy attains a high-inequality steady state with government expenditure favoring lump-sum transfer when $\mu = 0.55$, while it attains a low-inequality steady state with government expenditure favoring public education when $\mu = 0.3$ and 0.05. A lower μ implies less external effect of parental human capital on the child's human capital formation and thus narrows the gap between the two classes via human capital accumulation. Therefore, the economy is more likely to attain a less unequal state as μ becomes lower.

In summary, the present numerical investigation suggests that the economy with the weaker family background effect attains a more equal state with government expenditure favoring public education, and the economy with the stronger family background effect attains a less equal state with government expenditure favoring lump-sum transfer. This model prediction should be viewed with caution because it depends on the present model specification. However, it provides a hypothesis about the cross-country difference in inequality and redistribution policies, which should be tested in future research.

5 Discussion and Extension

The result established so far depends on the assumption of the external effect of parents' human capital on the productivity of private education. The result also depends on the assumption of the type-L majority and on the specification of the logarithmic utility function. In this section, we briefly consider the role of each assumption, and investigate how the result would change if either of them is relaxed or modified.

5.1 External Effect of Human Capital

The present model assumes that the average human capital $\bar{h}_t$ affects the marginal productivity of public education. The model also assumes that the parents' human capital h_t^i affects the marginal productivity of private education. When μ is set to be zero, these effects vanish in the human capital formation; the human capital production function is reduced to

$$h_{t+1}^i = A^i \cdot \left[e_t + z_t^i\right],$$

which is similar to that in Gradstein and Justman (1996).

Given this reduced form of the function, the preferences of a type-L agents are represented by the following indirect utility function:

$$V_t^L = \begin{cases} V_{t,z>0}^L \equiv \ln\left(1-\tau_t\right)\left[\rho_t^L + \lambda_t \tau_t + \left(1-\lambda_t\right)\tau_t\right] \\ \qquad \text{if } \lambda_t > \left(\frac{1-\theta}{\theta} - \frac{\rho_t^L}{\tau_t}\right)\cdot\left(1+\frac{1-\theta}{\theta}\right)^{-1}; \\ V_{t,z=0}^i \equiv \ln(1-\tau_t) + (1-\theta)\ln\left(\rho_t^L + \lambda_t\tau_t\right) + \theta \ln\left(1-\lambda_t\right)\tau_t \\ \qquad \text{if } \lambda_t \le \left(\frac{1-\theta}{\theta} - \frac{\rho_t^L}{\tau_t}\right)\cdot\left(1+\frac{1-\theta}{\theta}\right)^{-1}. \end{cases}$$

By comparing Eq. (3) with the above equation, we find that when $z^L = 0$, the preferences are not affected by the presence or the absence of the external effects of human capital. The result in Lemma 2 is applicable to the present case. However, the external effects of human capital matter when $z^L > 0$. Lump-sum transfer and public education are perfect substitutes in the absence of the external effects, as we can see in the above expression of $V_{t,z>0}^L$. In other words, they are indifferent for the type-L agents from the viewpoint of utility maximization. Therefore, the allocation of tax revenue, λ_t, becomes indeterminate, which makes sharp prediction difficult. The assumption of the external effects is one of the ways to resolve the problem of indeterminacy.

5.2 Type-H Majority

In the main body of the paper, we have conducted the analysis by assuming type-L majority. This assumption reflects the right-skewed income distribution in the real economy. However, readers may wonder how the result would change when the type-H agents are the majority. Following the same procedure as in the case of the type-L majority, we can characterize the type-H majority political equilibrium and obtain the following result: there is no provision of public education and lump-sum transfer when the type-H agents are the majority (see Appendix A.3 for the proof).

To understand the statement, recall the type-H's human capital production function given by

$$h_{t+1}^H = A^H \left(\bar{h}_t\right)^\mu \cdot \left[e_t + \left(\rho_t^H\right)^\mu z_t^H\right].$$

The return from investment is higher in private education than in public education because $\left(\rho_t^H\right)^\mu > 1$ holds. Because of this property, the type-H agents have no incentive to allocate the tax revenue to public education. In addition, the type-H agents obtain no benefit from the lump-sum transfer because it is a redistribution from the type-H agents to the type-L agents. Therefore, the type-H agents prefer no provision of both public education and lump-sum transfer, and choose no taxation as a decisive voter.

5.3 A Constant Elasticity-of-Substitution Utility Function

At this point, we have conducted the analysis by assuming a logarithmic utility function. This specification makes the analysis tractable, but results in a private investment function that is independent of the productivity of human capital, A^i, the average human capital, $\bar{h}_t$, and other parameters. This subsection introduces a constant elasticity-of-substitution utility function to resolve this problem. This subsection demonstrates that the result is qualitatively unchanged even if we generalize the utility function.

Consider the following utility function:

$$U_t^i = (1-\theta)\frac{\left(c_t^i\right)^{1-\sigma} - 1}{1-\sigma} + \theta\frac{\left(h_{t+1}^i\right)^{1-\sigma} - 1}{1-\sigma}, \tag{8}$$

where $\sigma > 1$ and $\sigma \neq 1$ hold.[4] When agents' preferences are specified by this utility

[4] The function becomes the logarithmic utility function $U_t^i = (1-\theta)\ln c_t^i + \theta \ln h_{t+1}^i$ if $\sigma \to 1$.

function, the period-t political equilibrium policy is given by

$$(\tau_t, \lambda_t) = \begin{cases} \left(\frac{1}{2}\left(1-\rho_t^L\right), \frac{1-\left\{\frac{\theta}{1-\theta}\left(A^L\right)^{1-\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)}\right\}^{1/\sigma}\frac{2\rho_t^L}{1-\rho_t^L}}{1+\left\{\frac{\theta}{1-\theta}\left(A^L\right)^{1-\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)}\right\}^{1/\sigma}} \right) & \text{if} \quad \rho_t^L < \tilde{\rho}_t^L\left(\bar{h}_t, A^L\right) \\ \left(\hat{\tau}\left(\rho_t^L, \bar{h}_t; A^L\right), 0\right) & \text{if} \quad \tilde{\rho}_t^L\left(\bar{h}_t, A^L\right) \le \rho_t^L \le \hat{\rho}_t^L\left(\bar{h}_t, A^L\right) \\ \left(\frac{1}{2}\left(1-\left(\rho_t^L\right)^{1+\mu}\right), 0\right) & \text{if} \quad \hat{\rho}_t^L\left(\bar{h}_t, A^L\right) < \rho_t^L, \end{cases}$$

where $\hat{\tau}\left(\rho_t^L, \bar{h}_t; A^L\right)$ satisfies $(1-\theta)\left(\rho_t^L\right)^{1-\sigma} = \theta\left(A^L\right)^{1-\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)}(\tau_t)^{-\sigma}(1-2\tau_t)$, and $\tilde{\rho}_t^L\left(\bar{h}_t, A^L\right)$ and $\hat{\rho}_t^L\left(\bar{h}_t, A^L\right)$ denote the critical values of ρ_t^L that satisfy

$$\rho_t^L = \left[1 + 2\left\{\frac{\theta}{1-\theta}\left(A^L\right)^{1-\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)}\right\}^{1/\sigma}\right]^{-1},$$

and

$$1 = \left(\rho_t^L\right)^{1+\mu} + 2\left(\frac{\theta}{1-\theta}\right)^{1/\sigma}\left(A^L\right)^{(1-\sigma)/\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)/\sigma}\left(\rho_t^L\right)^{\mu/\sigma+1},$$

respectively. The proof of the statement here is provided in Appendix A.4.

The result described thus far indicates that the critical value of ρ_t^L depends on the average human capital, $\bar{h}_t$, and the type-L's human capital productivity, A^L. This property is not observed in the logarithmic utility function case as demonstrated in Proposition 1. However, the equilibrium redistributive policy established here is qualitatively equivalent to that in Proposition 1. There is a provision of both public education and lump-sum transfer when inequality is high, while there is no provision of lump-sum transfer when inequality is low.

The reason for this equivalence lies in the fact that there is no intertemporal decision making on savings in the present model. In other words, there is no substitution effect through the interest rate, which may dominate the income effect, depending on the elasticity of substitution. Given this lack of the substitution effect, we have found that the generalization of the utility function does not qualitatively affect the choice of redistribution policy through voting.

6 Conclusion

In this paper, we have developed a theoretical framework that studies the voting on public education and lump-sum transfer in an overlapping generations model with two types of family dynasties classified according to their endowed level of human capital. In doing this, we have argued that the marginal productivity of private education increases with the parent's human capital level, and this family background effect influences agents' preferences for the two redistribution policies.

Our model predicts that a higher (lower) inequality is associated with the government expenditure favoring lump-sum transfer (public education). In other words, there is a positive correlation between inequality and the ratio of lump-sum transfer to public education. This model prediction is in line with the empirical evidence observed in OECD countries. To illustrate the source of this correlation, we focus on the family background effect. We show that the economy with stronger (weaker) family background effect attains a less (more) equal state with the government expenditure favoring lump-sum transfer (public education).

A Appendix

A.1 Proof of Lemma 2.

As demonstrated in Figure 2, two cases should be considered: (a) the case of $\frac{\theta}{1+\theta} \leq \frac{\theta}{1-\theta}\rho_t^L$, that is, $(1-\theta)/(1+\theta) \leq \rho_t^L$ and (b) the case of $\frac{\theta}{1+\theta} > \frac{\theta}{1-\theta}\rho_t^L$, that is, $(1-\theta)/(1+\theta) > \rho_t^L$.

In case (a), there are two possible solutions: one characterized by no redistribution with $\lambda_t = 0$, and the other characterized by some redistribution with $\lambda_t > 0$. They are given by

$$(\tau_t, \lambda_t) = \left(\frac{\theta}{1+\theta}, 0\right) \text{ and } \left(\hat{\tau}_t, \hat{\lambda}_t\right) \text{ where } \hat{\lambda}_t > 0.$$

By direct calculation, we find that the former solution, $(\theta/(1+\theta), 0)$, satisfies the $z_t^L = 0$ condition if $\rho_t^L \leq ((1-\theta)/(1+\theta))^{1/(1+\mu)}$.

The latter solution, $\left(\hat{\tau}_t, \hat{\lambda}_t\right)$, does not satisfy the $z_t^L = 0$ condition. To prove this argument, suppose that $\left(\hat{\tau}_t, \hat{\lambda}_t\right)$ is available in equilibrium in the case of $(1-\theta)/(1+\theta) \leq \rho_t^L$. The curve representing the reaction function $\tau_t = \tau_{z=0}^L(\lambda_t)$ crosses the curve representing the reaction function $\lambda_t = \lambda_{z=0}^L(\tau_t)$ twice within the range $\tau_t \in \left(\frac{\theta}{1-\theta}\rho_t^L, \frac{1}{2}\right)$. That is, the following condition holds for some $\tau_t \in \left(\frac{\theta}{1-\theta}\rho_t^L, \frac{1}{2}\right)$:

$$\frac{(1+\theta) - \theta/\tau_t}{1-2\tau_t}\rho_t^L < (1-\theta) - \frac{\theta\rho_t^L}{\tau_t}.$$

The condition is rewritten as follows:

$$\rho_t^L < 1 - 2\tau_t \text{ for some } \tau_t \in \left(\frac{\theta}{1-\theta}\rho_t^L, \frac{1}{2}\right),$$

or

$$\rho_t^L < \frac{1-\theta}{1+\theta}.$$

This contradicts the presumption of $(1-\theta)/(1+\theta) \leq \rho_t^L$. Therefore, there is no solution with $\lambda_t > 0$ when $(1-\theta)/(1+\theta) \leq \rho_t^L$ holds.

The analysis thus far indicates that the solution is limited to $(\tau_t, \lambda_t) = (\theta/(1+\theta), 0)$ when $(1-\theta)/(1+\theta) \leq \rho_t^L$. We substitute this solution into the condition $z_t^L = 0$ in (6) and find that $z_t^L = 0$ holds if and only if $\rho_t^L \leq ((1-\theta)/(1+\theta))^{1/(1+\mu)}$. That is, the solution $(\tau_t, \lambda_t) = (\theta/(1+\theta), 0)$ is realized as an equilibrium if $(1-\theta)/(1+\theta) \leq \rho_t^L \leq ((1-\theta)/(1+\theta))^{1/(1+\mu)}$.

In case (b), the solution is interior as illustrated in Panel (b) of Figure 2. By direct calculation, we obtain

$$(\tau_t, \lambda_t) = \left(\frac{1}{2}(1-\rho_t^L), (1+\theta) - \frac{2\theta}{1-\rho_t^L}\right).$$

We substitute this solution into the condition $z_t^L = 0$ in (6) and find that $z_t^L = 0$ holds if $\rho_t^L \leq 1$.

Finally, we show that $z_t^H > 0$ holds for both cases. With the use of (2), we have

$$z_t^H > 0 \Leftrightarrow \lambda_t > \frac{1}{1 + \frac{1-\theta}{\theta} \cdot \frac{1}{\left(\rho_t^H\right)^\mu}} \cdot \left(\frac{1-\theta}{\theta} \cdot \frac{1}{\left(\rho_t^H\right)^\mu} - \frac{\rho_t^H}{\tau_t} \right).$$

Suppose that $(1-\theta)/(1+\theta) \leq \rho_t^L$ holds. We substitute the solution $(\tau_t, \lambda_t) = (\theta/(1+\theta), 0)$ into the above condition and obtain

$$\left(\frac{1-\theta}{1+\theta} \right)^{1/(1+\mu)} < \rho_t^H,$$

which holds for any ρ_t^H because the left-hand side is less than one and the right-hand side is greater than one by the definition of ρ_t^H.

Alternatively, suppose that $\rho_t^L < (1-\theta)/(1+\theta)$ holds. We substitute the solution $(\tau_t, \lambda_t) = \left(\frac{1}{2}(1-\rho_t^L), (1+\theta) - 2\theta/(1-\rho_t^L)\right)$ into the $z_t^H > 0$ condition. After some calculation, we obtain

$$z_t^H > 0 \Leftrightarrow (1-\theta) \left(1 - \frac{1}{\left(\rho_t^H\right)^\mu} \right) \left(1 + \rho_t^L\right) + 2 \left(\rho_t^H - \rho_t^L\right) > 0,$$

which holds for any $\rho_t^H (> 1)$ and $\rho_t^L (< 1)$.

A.2 Derivation of (7)

To derive Eq. (7), recall that the average human capital in period $t+1$, $\bar{h}_{t+1}$, is defined by

$$\bar{h}_{t+1} \equiv \phi^L h_{t+1}^L + \phi^H h_{t+1}^H.$$

With the definition of $\rho_{t+1}^L \equiv h_{t+1}^L / \bar{h}_{t+1}$, we have

$$\rho_{t+1}^L = \frac{h_{t+1}^L}{\phi^L h_{t+1}^L + \phi^H h_{t+1}^H} = \frac{\alpha}{\phi^L \alpha + \phi^H \alpha \frac{h_{t+1}^H}{h_{t+1}^L}}.$$

The next task is to compute $\alpha \cdot \left(h_{t+1}^H / h_{t+1}^L \right)$. With the use of the results in Lemmas 1 and 2, we can write h_{t+1}^H and h_{t+1}^L as follows:

$$h_{t+1}^H = \begin{cases} \frac{1}{4} A\theta \cdot \left(1 + \rho_t^L\right) \cdot \left(\bar{h}_t\right)^{1+\mu} \\ \times \left[\left\{ (1-\theta) - (1+\theta)\rho_t^L \right\} \left(\rho_t^H\right)^\mu + \theta \left(1 + \rho_t^L\right) + 2 \left(\rho_t^H\right)^{1+\mu} \right] & \text{if } \rho_t^L < \frac{1-\theta}{1+\theta}, \\ A \frac{\theta}{1+\theta} \cdot \left(\bar{h}_t\right)^{1+\mu} \cdot \left[\frac{\theta}{1+\theta} + \left(\rho_t^H\right)^{1+\mu} \right] & \text{if } \frac{1-\theta}{1+\theta} \leq \rho_t^L \leq \left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)}, \\ \frac{1}{4} A\theta \cdot \left(\bar{h}_t\right)^{1+\mu} \cdot \left(1 + \left(\rho_t^L\right)^{1+\mu}\right) \cdot \left(1 - \left(\rho_t^L\right)^{1+\mu} + 2\left(\rho_t^H\right)^{1+\mu}\right) & \text{if } \left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)} < \rho_t^L, \end{cases}$$

and

$$h_{t+1}^{L}=\begin{cases}\frac{1}{4}\alpha A\theta\cdot\left(1+\rho_t^L\right)^2\cdot\left(\bar{h}_t\right)^{1+\mu} & \text{if } \rho_t^L<\frac{1-\theta}{1+\theta},\\ \alpha A\cdot\frac{\theta}{(1+\theta)^2}\cdot\left(\bar{h}_t\right)^{1+\mu} & \text{if } \frac{1-\theta}{1+\theta}\leq\rho_t^L\leq\left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)},\\ \frac{1}{4}\alpha A\theta\cdot\left(\bar{h}_t\right)^{1+\mu}\cdot\left(1+\left(\rho_t^L\right)^{1+\mu}\right)^2 & \text{if } \left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)}<\rho_t^L.\end{cases}$$

Therefore, $\alpha\cdot\left(h_{t+1}^H/h_{t+1}^L\right)$ is given by

$$\alpha\frac{h_{t+1}^H}{h_{t+1}^L}=\begin{cases}\omega_1\left(\rho_t^L,\rho_t^H\right)\equiv\theta+\frac{(1-\theta)-(1+\theta)\rho_t^L+2\rho_t^H}{1+\rho_t^L}\left(\rho_t^H\right)^\mu & \text{if } \rho_t^L<\frac{1-\theta}{1+\theta},\\ \omega_2\left(\rho_t^L,\rho_t^H\right)\equiv\theta+(1+\theta)\left(\rho_t^H\right)^{1+\mu} & \text{if } \frac{1-\theta}{1+\theta}\leq\rho_t^L\leq\left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)},\\ \omega_3\left(\rho_t^L,\rho_t^H\right)\equiv\frac{1-\left(\rho_t^L\right)^{1+\mu}+2\left(\rho_t^H\right)^{1+\mu}}{1+\left(\rho_t^L\right)^{1+\mu}} & \text{if } \left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)}<\rho_t^L.\end{cases}$$

A.3 The Type-H Majority

First, suppose that $z_t^H>0$ holds. The type-H's indirect utility function when $z_t^H>0$ is

$$V_{t,z>0}^H=\ln(1-\tau_t)\cdot\left[\rho_t^H+\lambda_t\tau_t+\frac{1}{\left(\rho_t^H\right)^\mu}\left(1-\lambda_t\right)\tau_t\right]+X_{z>0}\left(\bar{h}_t,\rho_t^H\right).$$

The first-order conditions with respect to τ_t and λ_t are as follows:

$$\left.\frac{\partial V_{t,z>0}^H}{\partial\tau_t}\right|_{\tau_t=0}=-1+\frac{1}{\rho_t^H}\cdot\left[\lambda_t+\frac{1-\lambda_t}{\left(\rho_t^H\right)^\mu}\right]<0,$$
$$\frac{\partial V_{t,z>0}^H}{\partial\lambda_t}=\frac{\tau_t\left(1-1/\left(\rho_t^H\right)^\mu\right)}{\rho_t^H+\lambda_t\tau_t+\left(1-\lambda_t\right)\tau_t/\left(\rho_t^H\right)^\mu}>0.$$

These conditions imply that $(\tau_t,\lambda_t)=(0,1)$ holds for any $\rho_t^H>0$. Plugging this pair of policy into the condition of $z_t^H>0$, we obtain

$$z_t^H>0\Leftrightarrow 1>\frac{\frac{1-\theta}{\theta}\cdot\frac{1}{\left(\rho_t^H\right)^\mu}-\frac{\rho_t^H}{\tau_t}}{1+\frac{1-\theta}{\theta}\cdot\frac{1}{\left(\rho_t^H\right)^\mu}}=-\infty,$$

which holds for any $\rho_t^H>0$. Given that $\tau_t=0$, there is no tax revenue and thus no provision of public education and lump-sum transfer.

Next, suppose that $z_t^H=0$ holds. The type-H's indirect utility function when $z_t^H=0$ is

$$V_{t,z=0}^H=\ln(1-\tau_t)+(1-\theta)\ln\left(\rho_t^H+\lambda_t\tau_t\right)+\theta\ln\left(1-\lambda_t\right)\tau_t.$$

We solve the problem of maximizing $V_{t,z=0}^H$ to obtain the following reaction functions:

$$\tau_t=\tau_{z=0}^H\left(\lambda_t\right)\Leftrightarrow\lambda_t=\frac{1+\theta-\theta/\tau_t}{1-2\tau_t}\rho_t^H,$$
$$\lambda_t=\lambda_{z=0}^H\left(\tau_t\right)\equiv\max\left\{0,(1-\theta)-\frac{\theta\rho_t^H}{\tau_t}\right\}.$$

These reaction functions coincide with those in the case of type-L majority if we replace ρ_t^H with ρ_t^L. We can apply the analysis in Section 3.2. The possible solutions are

$$(\tau_t, \lambda_t) = \begin{cases} \left(\frac{\theta}{1+\theta}, 0\right) & \text{if} \quad \rho_t^H \in \left[\frac{1-\theta}{1+\theta}, \left(\frac{1-\theta}{1+\theta}\right)^{1/(1+\mu)}\right] \\ \left(\frac{1}{2}\left(1-\rho_t^H\right), (1+\theta) - \frac{2\theta}{1-\rho_t^H}\right) & \text{if} \quad \rho_t^H < \frac{1-\theta}{1+\theta}. \end{cases}$$

However, the solution is not feasible because it requires $\rho_t^H < 1$, which contradicts the presumption of $\rho_t^H > 1$. Therefore, there is no feasible policy when $z_t^H = 0$.

A.4 A Constant Elasticity-of-Substitution Utility Function

Consider the following utility function:

$$U_t^i = (1-\theta)\frac{\left(c_t^i\right)^{1-\sigma} - 1}{1-\sigma} + \theta\frac{\left(h_{t+1}^i\right)^{1-\sigma} - 1}{1-\sigma},$$

where $\sigma > 1$ and $\sigma \neq 1$ hold. We compute the type-L's indirect utility function as follows:

$$V_t^L = \begin{cases} V_{t,z>0}^L \equiv \frac{1}{1-\sigma}\phi_{z>0}(\rho_t^L, \bar{h}_t)\left(1-\tau_t\right)^{1-\sigma}\left[\left(\rho_t^L + \lambda_t\tau_t\right)\left(\rho_t^L\right)^{\mu} + \left(1-\lambda_t\right)\tau_t\right]^{1-\sigma} - \frac{1}{1-\sigma} \\ \qquad\qquad \text{if } \lambda_t > \Phi\left(\tau_t; \rho_t^L\right), \\ V_{t,z=0}^L \equiv \frac{1}{1-\sigma}\left(\bar{h}_t\right)^{1-\sigma}\left(1-\tau_t\right)^{1-\sigma} \\ \quad \times \left[(1-\theta)\left(\rho_t^L + \lambda_t\tau_t\right)^{1-\sigma} + \theta\left(A^L\right)^{1-\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)}\left(\left(1-\lambda_t\right)\tau_t\right)^{1-\sigma}\right] - \frac{1}{1-\sigma} \\ \qquad\qquad \text{if } \lambda_t \leq \Phi\left(\tau_t; \rho_t^L\right), \end{cases}$$

where $\Phi\left(\tau_t; \rho_t^L\right)$ is defined as follows:

$$\Phi\left(\tau_t; \rho_t^L\right) \equiv \frac{1 - \left(\frac{\theta}{1-\theta}\right)^{1/\sigma}\left(A^L\right)^{(1-\sigma)/\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)/\sigma}\left(\rho_t^L\right)^{\mu/\sigma+1}\left(1/\tau_t\right)}{1 + \left(\frac{\theta}{1-\theta}\right)^{1/\sigma}\left(A^L\right)^{(1-\sigma)/\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)/\sigma}\left(\rho_t^L\right)^{\mu/\sigma}},$$

and the term $\phi_{z>0}(\rho_t^L, \bar{h}_t)$ includes ρ_t^L and $\bar{h}_t$ and other unrelated terms.[5]

Suppose that $z_t^L > 0$ holds, that is, $\lambda_t > \Phi\left(\tau_t; \rho_t^L\right)$ holds. The first-order condition with respect to τ_t leads to the following reaction function of τ_t for a given λ_t:

$$\tau_t = \tau_{z>0}^L\left(\lambda_t\right) \equiv \frac{1}{2}\left[1 - \frac{\left(\rho_t^L\right)^{1+\mu}}{\lambda_t\left(\rho_t^L\right)^{\mu} + \left(1-\lambda_t\right)}\right].$$

[5] The term $\phi_{z>0}(\rho_t^L, \bar{h}_t)$ is defined as follows:

$$\phi_{z>0}(\rho_t^L, \bar{h}_t) \equiv \left[\frac{\bar{h}_t}{\left(\rho_t^L\right)^{\mu} + \left(\frac{\theta}{1-\theta}\right)^{1/\sigma}\left(A^L\right)^{(1-\sigma)/\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)/\sigma}\left(\rho_t^L\right)^{\mu/\sigma}}\right]^{1-\sigma} \cdot \left(\bar{h}_t\right)^{1-\sigma}$$
$$\times \left[(1-\theta) + \theta\left(A^L\right)^{1-\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)}\left\{\left(\frac{\theta}{1-\theta}\right)^{1/\sigma}\left(A^L\right)^{(1-\sigma)/\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)/\sigma}\left(\rho_t^L\right)^{\mu/\sigma}\right\}^{1-\sigma}\right].$$

The differentiation of $V_{t,z>0}^L$ with respect to λ_t is

$$\frac{1}{\phi_{z>0}(\rho_t^L,\bar{h}_t)\left(1-\tau_t\right)^{1-\sigma}}\cdot\frac{\partial V_{t,z>0}^L}{\partial\lambda_t}=\left[\left(\rho_t^L+\lambda_t\tau_t\right)\left(\rho_t^L\right)^{\mu}+\left(1-\lambda_t\right)\tau_t\right]^{-\sigma}\tau_t\left(\left(\rho_t^L\right)^{\mu}-1\right)<0.$$

This condition implies that $\lambda_t=0\forall\rho_t^L$. We substitute $\lambda_t=0$ into $\tau_t=\tau_{z>0}^L\left(\lambda_t\right)$ and obtain $\tau_t=\left(1-\left(\rho_t^L\right)^{1+\mu}\right)/2$.

The solution $\left(\tau_t,\lambda_t\right)=\left(\left(1-\left(\rho_t^L\right)^{1+\mu}\right)/2,0\right)$ is feasible if it satisfies the condition of $z_t^L>0$, that is, $\lambda_t>\Phi\left(\tau_t;\rho_t^L\right)$. We substitute the solution into the condition $\lambda_t>\Phi\left(\tau_t;\rho_t^L\right)$ and rearrange the terms to obtain

$$1<\left(\rho_t^L\right)^{1+\mu}+2\left(\frac{\theta}{1-\theta}\right)^{1/\sigma}\left(A^L\right)^{(1-\sigma)/\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)/\sigma}\left(\rho_t^L\right)^{\mu/\sigma+1},$$

where the right-hand side is increasing in ρ_t^L. Therefore, there exists a critical value of ρ_t^L, denoted by $\hat{\rho}_t^L\left(\bar{h}_t,A^L\right)$, such that $z_t^L>0$ holds at $\left(\tau_t,\lambda_t\right)=\left(\left(1-\left(\rho_t^L\right)^{1+\mu}\right)/2,0\right)$ if $\hat{\rho}_t^L\left(\bar{h}_t,A^L\right)<\rho_t^L$ holds.

Next, suppose that $z_t^L=0$ holds, that is, $\lambda_t\le\Phi\left(\tau_t;\rho_t^L\right)$ holds. The first-order condition with respect to τ_t is

$$\begin{aligned}&(1-\theta)\left(\rho_t^L+\lambda_t\tau_t\right)^{-\sigma}\left[\rho_t^L-\lambda_t(1-2\tau_t)\right]\\&=\theta\left(A^L\right)^{1-\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)}\left(\left(1-\lambda_t\right)\tau_t\right)^{-\sigma}\left(1-\lambda_t\right)\left(1-2\tau_t\right).\end{aligned}\tag{9}$$

The first-order condition with respect to λ_t is

$$(1-\theta)\left(\rho_t^L+\lambda_t\tau_t\right)^{-\sigma}=\theta\left(A^L\right)^{1-\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)}\left(\left(1-\lambda_t\right)\tau_t\right)^{-\sigma}.\tag{10}$$

Taking account of a corner solution, we obtain the following reaction function of λ_t:

$$\lambda_t=\lambda_{z=0}^L\left(\tau_t\right)\equiv\max\left\{0,\frac{1-\left\{\frac{\theta}{1-\theta}\left(A^L\right)^{1-\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)}\right\}^{1/\sigma}\frac{\rho_t^L}{\tau_t}}{1+\left\{\frac{\theta}{1-\theta}\left(A^L\right)^{1-\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)}\right\}^{1/\sigma}}\right\}.\tag{11}$$

Suppose that $\lambda_t>0$ holds in (11). With the use of (9) and (10), we obtain

$$\left(\tau_t,\lambda_t\right)=\left(\frac{1}{2}\left(1-\rho_t^L\right),\frac{1-\left\{\frac{\theta}{1-\theta}\left(A^L\right)^{1-\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)}\right\}^{1/\sigma}\frac{2\rho_t^L}{1-\rho_t^L}}{1+\left\{\frac{\theta}{1-\theta}\left(A^L\right)^{1-\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)}\right\}^{1/\sigma}}\right).$$

Therefore, $\lambda_t>0$ holds if and only if $1-\left\{\frac{\theta}{1-\theta}\left(A^L\right)^{1-\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)}\right\}^{1/\sigma}\frac{2\rho_t^L}{1-\rho_t^L}>0$, that is,

$$\rho_t^L<\tilde{\rho}_t^L\left(\bar{h}_t,A^L\right)\equiv\left[1+2\left\{\frac{\theta}{1-\theta}\left(A^L\right)^{1-\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)}\right\}^{1/\sigma}\right]^{-1}.$$

Alternatively, suppose that $\lambda_t=0$ holds. This case holds for $\rho_t^L\in\left[\hat{\rho}_t^L\left(\bar{h}_t,A^L\right),\tilde{\rho}_t^L\left(\bar{h}_t,A^L\right)\right]$. Substituting $\lambda_t=0$ into (9), we find that the solution of τ_t, denoted by $\hat{\tau}\left(\rho_t^L,\bar{h}_t;A^L\right)$, satisfies $(1-\theta)\left(\rho_t^L\right)^{1-\sigma}=\theta\left(A^L\right)^{1-\sigma}\left(\bar{h}_t\right)^{\mu(1-\sigma)}\left(\tau_t\right)^{-\sigma}\left(1-2\tau_t\right)$.

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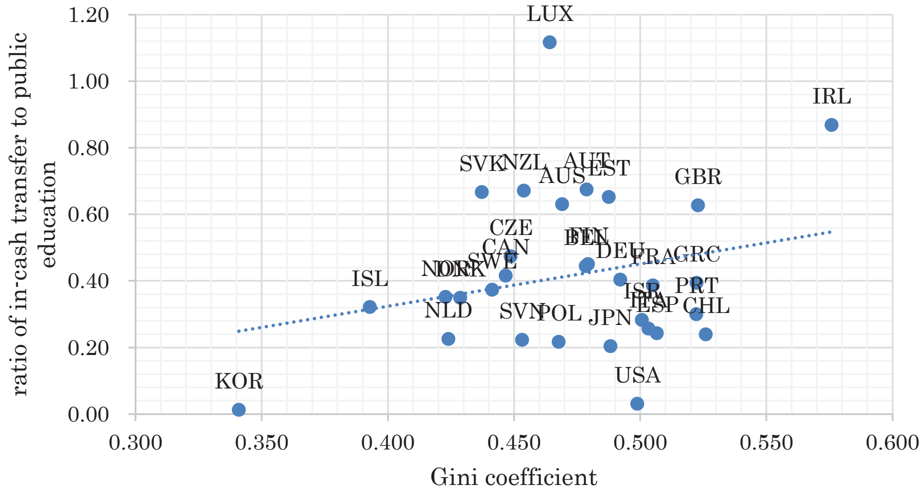
Panel (a)



Panel (b)

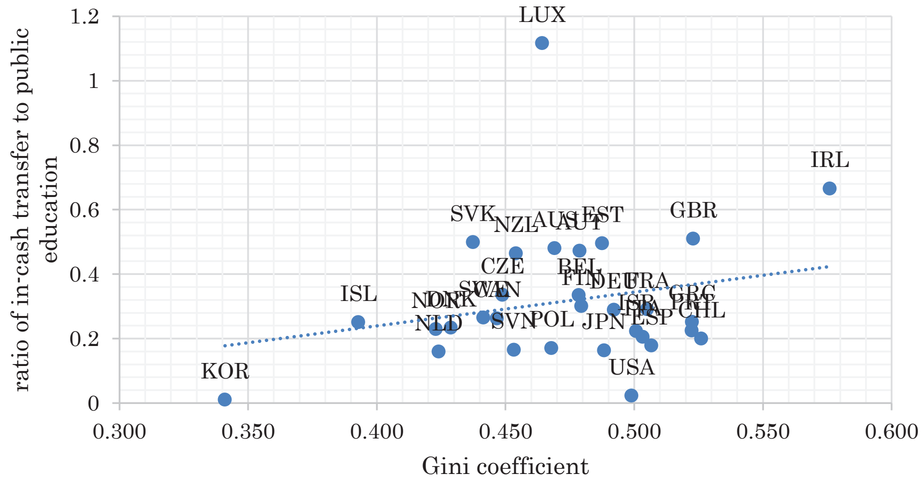


Figure 1: The figure illustrates the scatter plot of Gini coefficients in 2010 and the ratio of in-cash transfer to public education expenditure in 2009 for OECD countries. Public education expenditure includes primary and secondary education in Panel (a), and primary, secondary and tertiary education in Panel (b).

Source: Social Expenditure Database (www.oecd.org/els/social/expenditure), December 2013; OECD Education Database, 2013, and Eurostat Education Database, 2013; OECD Income Distribution Database (via www.oecd.org/social/income-distribution-database.htm)

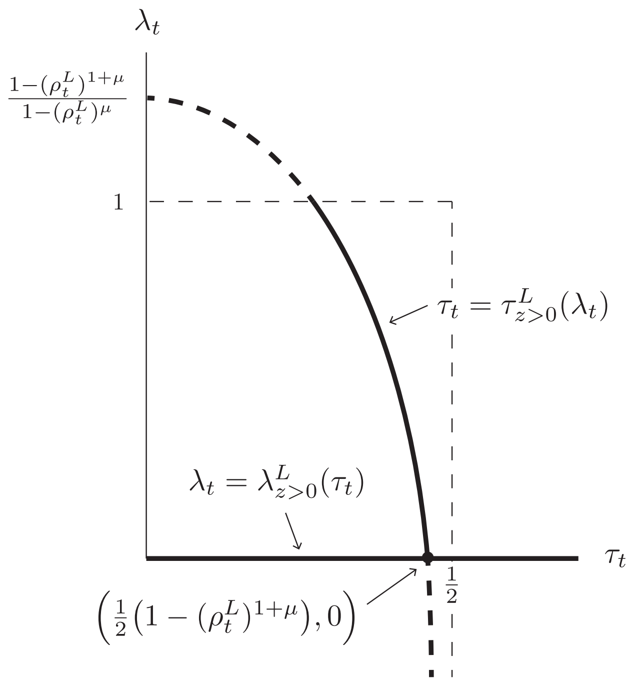
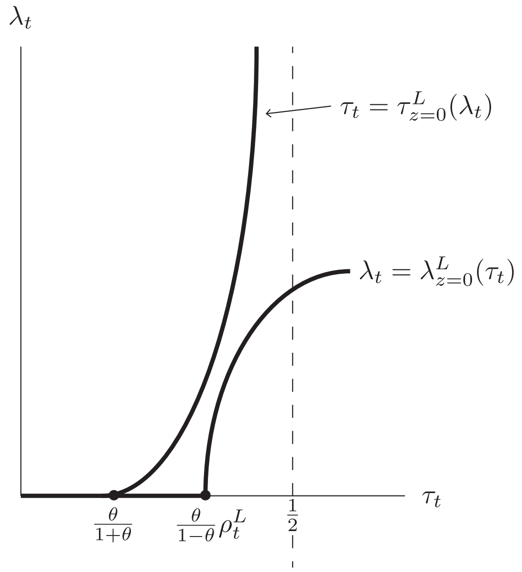


Figure 2: Type-L's reaction functions when $z_t^L > 0$.

Panel (a)



Panel (b)

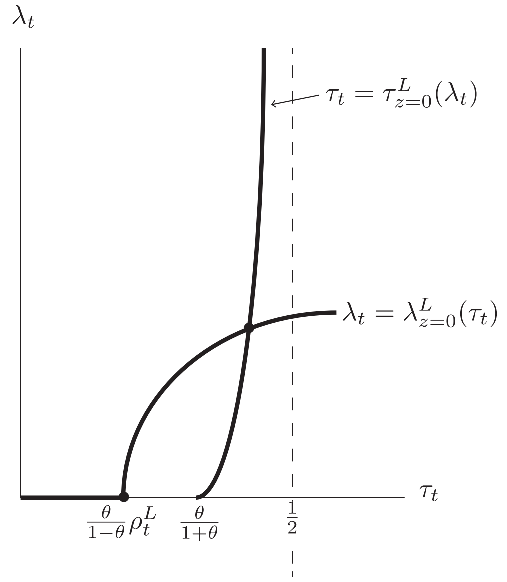
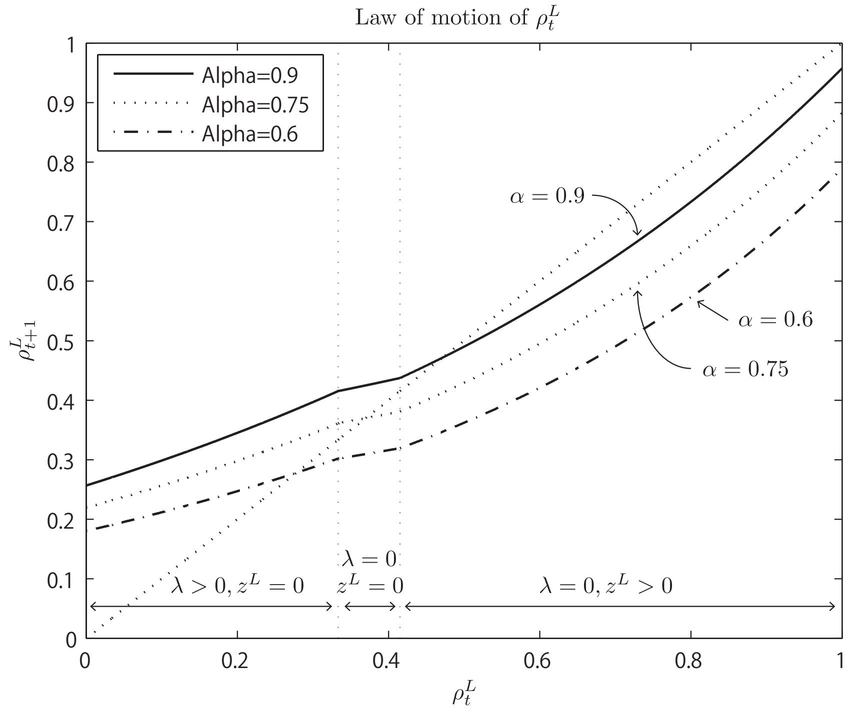


Figure 3: Type-L's reaction functions when $z^L_t = 0$. Panel (a) illustrates the case of $\rho^L_t \geq \frac{1-\theta}{1+\theta}$; Panel (b) illustrates the case of $\rho^L_t < \frac{1-\theta}{1+\theta}$.

Panel (a)



Panel (b)

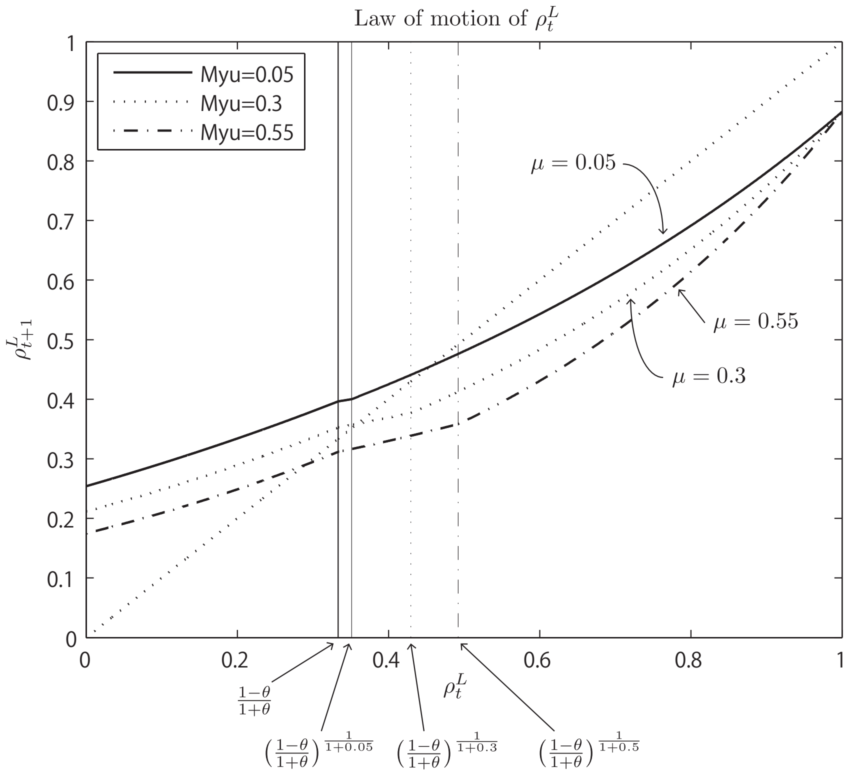


Figure 4: The figure illustrates the graph of $\rho_{t+1}^L = \Omega\left(\rho_t^L\right)$. We set $\theta = 0.5$ and $\phi^L = 0.6$. In Panel (a), we fix μ at 0.25, and illustrate three cases, $\alpha = 0.6, 0.75$, and 0.9. In Panel (b), we fix α at 0.75 and illustrate three cases, $\mu = 0.05, 0.3$, and 0.55.