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Abstract

This paper empirically studied the model-free implied volatility indices constructed from options prices of the Nikkei 225 index during 2005-2010. The concept of corridor volatility index is compared and contrasted with the methodology of the famous VIX index developed by the Chicago Board Options Exchange (CBOE). The relative corridor widths are found to be able to explain relative variations between volatility indices with different corridors' widths. Also, the corridor volatility index is found to be the better predictor of the future realized volatility of the underlying stock index.

JEL Classification Code: G01, G100, G130

Key words: Volatility Index, Crisis, Financial Crisis, Option, Option-implied

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1. Introduction

A stock market volatility index is an index of stock market returns volatility. Returns volatility of the stock market is a measure of its riskiness and uncertainty. In the simplest form, the volatility of stock is a standard deviation of stock's returns during the period of interest. Data of stock's volatility is used in portfolio construction and optimization and is one of the earliest notions learnt by any persons taking an introduction to finance course. It is also one of the important parameters that financial risk managers must pay careful attention to in managing their portfolio. Volatility calculated from historical data of stock's returns is called realized volatility. As most financial market participants may have learnt, history might not repeat itself. It is more important to be able to calculate or predict volatility of stock's returns in the future.

Chicago Board Options Exchange's VIX Index

The most referred stock market volatility index is the Chicago Board Options Exchange's VIX index (CBOE's VIX), which was first introduced in 1993. It is designed to measure the market's expectation of the next 30-days' volatility as implied by the at-the-money S&P100 Index option price.[1] The index is also known as "the Investor Fear Gauge" because high levels of VIX are coincident with a high degree of market turmoil.[2] However, because the returns' volatilities used in the calculation of the index are derived from the Black-Scholes model, the index is model-dependent and is subject to all the underlying assumptions of the Black-Scholes model. Though the VIX index became well-known and widely accepted as a benchmark for U.S. stock market volatility, it has been criticized for its practicality from both academia and industry practitioners.[3]

In 2003, the Chicago Board Options Exchange and Goldman Sachs revised the calculation method of the index. The old VIX became VXO and the new one is currently the world's most referenced stock market volatility index. The VIX index is now calculated based on S&P500

[1] See The CBOE Volatility Index – VIX or VIX White Paper for more details.
[2] Whaley (2000) explained in details regarding the origin of this name.
[3] See more comparison of the old VIX and new VIX in Carr and Wu (2006).

Index option data. The interpretation of the index is unchanged; it measures the market's expectation of 30-day volatility implicit in the S&P500 index option.

The major improvement in the new VIX index is that it no longer depends on the Black-Scholes model; in other words, it is now a model-free implied volatility index. Also, because the calculation method of the index is applied from variance swap contracts, the VIX index is replicable.[4] With the possibility of replication, the Chicago Board Options Exchange has introduced both VIX Futures and VIX Options enabling not only institutional investors but also retail investors to easily assess volatility exposure.

The new calculation method is so effective that it does not calculate only the volatility index of the S&P500; volatility indexes of Dow Jones, NASDAQ100, and Russell 2000 are calculated as well. Moreover, the Chicago Board Options Exchange calculates a volatility index of crude oil, gold, and Euro currency. In addition, exchanges around the world adopt CBOE's VIX calculation method in constructing their own volatility indices. Andersen et al. (2011) provides a summary and comparison of the calculation methods adopted by international exchanges.

Volatility Index Japan

Despite being one of the world's leading capital markets, Japan lags behind the U.S. in introducing a stock market volatility index. It was not until recent years that the Center for the Study of Finance and Insurance of Osaka University (CSFI), with the cooperation of the Osaka Securities Exchange, introduced the Volatility Index Japan (VXJ).

VXJ index is calculated from Nikkei 225 options prices. It reflects the market's expectation of volatility of the Nikkei 225 over the coming 30 days as implicit in the option prices. The calculation method is similar to the method of VIX index with differences in underlying options data.

In addition, CSFI developed its own version of volatility index, called the CSFI-VXJ. The methodology of CSFI-VXJ uses a spline function to interpolate the implied variances from

[4] For more details on volatility swaps contract, refer to Demeterfi et al. (1999).

observed options prices. The new methodology aims to reduce the approximation error associated with the methodology of CBOE's VIX, detailed in Fukusawa et al. (2011). VXJ and CSFI-VXJ share many characteristics with its counterpart, the VIX index, such as the statistical relationship between the volatility index and its underlying stock market index.[5]

However, unlike the VIX index, the VXJ and CSFI-VXJ indices are far from being the benchmark volatility index of Japan's stock market. They are constructed with academic purposes in mind. Therefore, there are no derivative securities on VXJ or CSFI-VXJ indices. It is possible that trading and hedging volatility in Japan is not well developed as the U.S. market or industry practitioners still feel more comfortable undertaking volatility transactions in an over-the-counter style.

Corridor Implied Volatility Index

Corridor implied volatility is a kind of model free implied volatility based on the industry-practiced corridor variance swap concept. A corridor variance swap contract is a conditional variance swap contract that accumulates variance only when the underlying asset's prices fall within the predetermined range.[6]

Andersen and Bondarenko (2007) proposed that a volatility index constructed from a corridor implied volatility concept is more accurate in reflecting the true realized variance, compared to the current method employed by CBOE. The main argument against the VIX index is that the random truncation of options used in calculation creates inaccuracy. The calculation concept of VIX index demands the use of options with strike prices ranging from zero to infinity to support the whole range of return distribution, but in practice, only limited and finite strikes of options are traded. CBOE uses random truncation in screening the option data to be used in the calculation of the VIX index, which is a source of inaccuracy.

[5] The Center for the Study of Finance and Insurance , Osaka University provided details of VXJ on its website, http://www-csfi.sigmath.es.osaka-u.ac.jp/en/activity/vxj/vxj.pdf

[6] For more documents regarding Corridor Volatility, see Allen et al. (2006), Carr & Madan (1998), Lieberman & Omprakash (2007)

Instead of random truncation, Andersen and Bondarenko (2007) constructed corridor implied volatility indexes based on options on S&P500 futures. The corridor widths were determined according to percentile of estimated risk-neutral density. They found that the volatility indexes created under this method added more information in a realized volatility prediction and produced lower forecasting root-mean-square errors when compared to other benchmark volatility measures: realized volatility itself, Black-Scholes implied volatility, and VIX index.

Nevertheless, Andersen et al. (2011) revised the Andersen and Bondarenko (2007) method, and pointed out potential drawbacks in determining corridor widths based on percentile of risk-neutral density (RND), which may distort the estimated volatility index. The revised method based on the findings of Andersen and Bondarenko (2010) that option prices will reflect tail moments of RND. The cut off strike price is a function of RND's tail moments which can be calculated directly from option prices.

In this study, corridor volatility indices for the Japanese market estimated under the revised method of Andersen et al. (2011) are compared to the volatility index calculated under CBOE's VIX method, the VXJ, and the CSFI-VXJ. In particular, the forecast performance of estimated volatility indices is compared against the realized volatility.

2. CBOE's VIX methodology

In this section, the method of CBOE's VIX methodology is discussed in detail. It will be applied to options on the Nikkei 225 index to construct a volatility index for the Japanese market. Though the CSFI constructs the VXJ based on the CBOE's VIX methodology, the VXJ's method is slightly different from the CBOE's VIX in various aspects. The exact method of CBOE's VIX will be used to construct a benchmark volatility index for the Japanese stock market.

The CBOE's VIX index is essentially a square-root of a model-free implied variance based on out-of-the-money (OTM) options prices. A model-free implied variance is an integrated quadratic return variation under a risk-neutral expectation. Assuming that the log price of the underlying asset, $log(S_t)$, follows the continuous Brownian semi-martingale diffusion process;

$$dlog(S_t) = \mu_t dt + \sqrt{v_t} dW_t \quad , \tag{1}$$

where W_t denotes a standard Brownian motion. μ_t is the drift parameter and v_t is the variance parameter of (1) process without specific parametric assumption. Demeterfi et al.(1999) and Andersen et al. (2010) showed that the theoretical realized variance under (1) process during a time interval $[0, T]$ is an integrated variance or a total quadratic return variation during $[0, T]$, which can be expressed by:

$$IVar = \int_0^T v_u du \quad . \tag{2}$$

The expectation of realized variance in (2) under the risk-neutral measure can be expressed as;

$$\hat{\sigma}_T^2 = \frac{1}{T} E_q \left[\int_0^T v_u du \right] = \frac{1}{T} E_q [IVar] \quad , \tag{3}$$

where $E_q[\cdot]$ denotes the expectation under risk-neutral measure.

Demeterfi et al. (1999) and Andersen et al. (2011) showed that the model-free implied variance in (3) from the current time t for the next τ period can be derived from option prices by:

$$\hat{\sigma}_{t,\tau}^2 = \frac{2e^{r_{t,\tau}\tau}}{\tau} \left[\int_0^{F_{t,\tau}} \frac{P(K,\tau)}{K^2} dK + \int_{F_{t,\tau}}^{\infty} \frac{C(K,\tau)}{K^2} dK \right] = \frac{2e^{r_{t,\tau}\tau}}{\tau} \left[\int_0^{\infty} \frac{Q(K,\tau)}{K^2} dK \right] \quad , \tag{4}$$

where $r_{t,\tau}$ is a risk-free interest rate and $F_{t,\tau}$ is the forward price of an underlying asset at current time t for the next τ period. τ is the remaining time-to-maturity of options measured in years. $P(K,\tau)$ and $C(K,\tau)$ are the prices of European put and call options with strike K and the remaining time-to-maturity τ. $Q(K,\tau)$ denotes the price of an OTM option at strike K. For $K < F_{t,\tau}$, $Q(K,\tau)$ is the price of an OTM put option and vice versa for an OTM call option. For $K = F_{t,\tau}$, both put and call options are said to be at-the-money (ATM).

Nevertheless, in reality, the number of option strikes is limited and discrete. Consequently, CBOE approximates the implied variance $\hat{\sigma}^2_{t,\tau}$ under (4) by the following formula;

$$\hat{\sigma}^2_{t,\tau} = \frac{2e^{r_{t,\tau}\tau}}{\tau}\left[\sum_{i=1}^{n}\frac{\Delta K_i}{K_i^2}Q(K_i,\tau)\right] - \frac{1}{\tau}\left[\frac{F_{t,\tau}}{K_f} - 1\right]^2 \quad , \tag{5}$$

where $0 < K_1 < \cdots < K_f \leq F_{t,\tau} < K_{f+1} < \cdots < K_n$ represents observed strikes and K_f is the first strike directly below the forward underlying asset price $F_{t,\tau}$. ΔK_i is an interval between observed strikes calculated as $\Delta K_1 = K_2 - K_1$, $\Delta K_n = K_n - K_{n-1}$, and for $1 < i < n, \Delta K_i = (K_{i+1} - K_{i-1})/2$. $Q(K_i,\tau)$ is a mid-quote price of OTM options with a K_i strike. For $K_i = K_f$ or ATM level, $Q(K_i,\tau)$ is an average of put mid-quote and call mid-quote.

The CBOE's VIX is a square root of a linear combination of (5) calculated from options with two nearest maturities to a 30-day horizon, excluding options with a time-to-maturity less than seven days.

$$VIX_t = 100 \times \sqrt{\left[w_1\tau_1\hat{\sigma}^2_{t,\tau_1} + w_2\tau_2\hat{\sigma}^2_{t,\tau_2}\right]\frac{365}{30}} \quad , \tag{6}$$

where $w_1 = \frac{\tau_2 - \tau_{30days}}{\tau_2 - \tau_1}$ and $w_2 = \frac{\tau_{30days} - \tau_1}{\tau_2 - \tau_1}$ so that $w_1 + w_2 = 1$. CBOE measures τ_i in minutes as a fraction of years. For more details regarding the methodology of CBOE's VIX, refer to CBOE's VIX whitepaper, which is publicly available for download from CBOE's website[7].

3. The corridor volatility index

A corridor volatility index is based on the concept of corridor variance swaps and can be regarded as a restricted version of the CBOE's VIX. As Andersen et al. (2011) pointed out, the

[7] http://www.cboe.com/micro/VIX/vixintro.aspx

need for a corridor volatility index arises from the problematic random cut-off rule to exclude options from the VIX calculation. The random cut-off rule is the source of spurious breaks and artificial jumps in the VIX index unrelated to the variation of the underlying return series. Due to the proliferation of derivative securities with the VIX index as an underlying asset, Andersen et al. (2011) stressed the need for a less erroneous benchmark volatility index.

The current cut-off rule of CBOE indicates that all OTM options with positive bid prices are eligible to be included in the calculation of the VIX. Additional rules state that, starting from the K_f toward the lowest strike K_1, when two-consecutive zero bid prices of OTM puts are observed, the OTM puts beyond that point will be excluded from the calculation, even if they have positive bid prices. Similarly, starting from K_f toward the maximum strike K_n, OTM calls beyond observed two-consecutive zero bid prices will be excluded. This random truncation induces a time-varying effective strike range for the VIX index.

A corridor volatility index alleviates us of the spurious break arising from random truncation by pre-determining and controlling the effective strike range in a time-consistent manner. Define an indicator function $I_t(B_1, B_2)$ for a given pair of barriers B_1 and B_2such that $0 < B_1 < B_2$, specifically;

$$I_t(B_1, B_2) = 1[B_1 \leq S_t \leq B_2] \quad ,$$

in other words, $I_t(B_1, B_2)$ takes the value of one only when the underlying asset price at t is within $[B_1, B_2]$. Carr and Madan (1998) show that an expected integrated variance under risk-neutral measure during a time interval $[0, T]$ derived from a corridor variance swaps with barriers $[K_1, K_n]$ can be expressed as;

$$\frac{1}{T} E_q[CIVar(K_1, K_n)] = \frac{1}{T} E_q\left[\int_0^T v_u I_u(K_1, K_n) du\right] = \frac{2e^{rT}}{T}\left[\int_{K_1}^{K_n} \frac{Q(K,\tau)}{K^2} dK\right] \quad , \quad (7)$$

where $E_q[\cdot]$ denotes the expectation under risk-neutral measures and $Q(K,\tau)$ is the price of the OTM option at strike K as previously defined. It is easy to recognize that (7) is simply a restricted version of (4). The implied corridor variance can be estimated using (5). Then, in the same manner with the CBOE's VIX, the corridor volatility index for the exact 30-day horizon can be derived using (6).

4. Defining corridors

Andersen et al. (2011) pointed out that a corridor of volatility index should provide broad coverage of strikes but should not include excessive extrapolation of noisy or non-existing quotes for deep OTM options. A desired corridor should be automatically or endogenously adjusted over a time horizon, provide a direct and transparent linkage to the option prices, and center on the forward price of the underlying asset $F_{t,\tau}$.

Andersen and Bondarenko (2007) proposed defining a corridor based on a percentile of RND. However, Andersen et al. (2011) pointed out that the corridors defined by a percentile of RND are not centered on the forward price $F_{t,\tau}$ and that the estimation of RND requires additional assumptions regarding the density model, which complicates the calculation and make it less transparent. Instead, Andersen et al. (2011) proposed a new method based on the work of Andersen and Bondarenko (2010), which stated that option prices reflect tail moments of RND. The left and right tail moments of a positive random variable, x, with strictly positive density $f(x)$ for all x are given by;

$$LT(K) = \int_0^K (K-x)f(x)dx \quad \text{and} \quad RT(K) = \int_K^\infty (x-K)f(x)dx \quad ,$$

where $LT(\cdot)$ and $RT(\cdot)$ stand for left tail and right tail moments at a given point K . In the next step, Andersen et al. (2011) defined a ratio statistic, $R(K)$, as an indicator of the location of K within the support of x,

$$R(K) = \frac{LT(K)}{LT(K) + RT(K)} \quad .$$

$R(K)$ can be regarded as a cumulative density function (CDF) because it is strictly increasing on $(0, \infty)$, with $R(0) = 0$ and $R(\infty) = 1$. In addition, $R(K)$ is centered on the mean of x, which equals to $\overline{K} = E[x] = \int_0^\infty xf(x)dx$. For any given percentile q in the range of $R(K)$ function, K_q is defined as;

$$K_q = R^{-1}(q),\ q \in [0,1] \quad . \tag{8}$$

In the context of RND of forward prices, $R(K)$ can be expressed in terms of OTM put and call options prices as;

$$R(K) = \frac{P(K)}{P(K) + C(K)} \quad . \tag{9}$$

Andersen et al. (2011) showed that the $R(K)$ ratio provides direct linkage to the option prices, is independent of RND functional form, and is centered on the forward price. Using (8) and (9), strikes associated with left and right truncation points at the required percentile of RND can be easily obtained.

5. Data

Option data on the Nikkei 225 index from December 2005 to December 2010, totaling 1,227 observation dates, obtained from NEEDS-FinancialQUEST2.0, are used to estimate volatility indices as defined in section 2 and 3. Nikkei 225 options are European-style options that can be exercised on the second Friday of the expiration month. The settlement price is based on special quotation (SQ) calculated from the total opening prices of each component stock of the Nikkei 225 on the business day following the last trading day. Strike prices are multiples of ¥500 intervals based on the Nikkei 225, but for the nearest three expiration months the strike prices are multiples of ¥250 intervals. On any trading day, strike prices were set such that 17 strike prices are available for any maturity month, eight below and eight above the at-the-money (ATM) strike price.

The screening process began by eliminating options with maturities less than seven days. Then, for each trading day, the pair of ATM call and put options with the least difference in the closing mid-quotes was identified for each maturity. If a pair could not be identified, the options in that maturity were entirely disregarded. During this step, an implied forward index level, $F_{t,\tau}$, was calculated based on the closing mid-quotes of ATM call and put options from the relationship,

$$F_{t,\tau} = K_{ATM} + e^{r_{f,t}\tau}[C_t(K_{ATM},\tau) - P_t(K_{ATM},\tau)] \quad , \tag{10}$$

where $r_{f,t}$ is the risk-free rate, which is an average of both buying and selling rates of new issues of three-month certificate of deposits (CDs). τ is the remaining time-to-maturity of the options, and C_t and P_t are the prices of ATM call and put options, respectively. K_{ATM} is the ATM strike price.

For maturities where ATM options were identified, only options with non-zero bid prices were retained. Next, mid-quotes were calculated for all options. The estimated volatility indices were based on OTM options that have more liquidity than in-the-money options and, hence, are less subject to pricing error.

Table 1 shows summary statistics of the options data for 1,227 trading days, from the end of December 2005 to the end of December 2010. The total number of options available during the sampling period is 618,686 options for all maturities, or an average of 504 options per observation day. For the near-term calculation, 66,188 put and call options were available before the screening process. Only 30% or 19,982 OTM put and call options passed the screening process and were used in index calculation. For the next term calculation, there were 60,254 options in total before the screening process. Around 42% or 25,145 OTM put and call options were included in the index calculation. A sum of OTM puts and calls, minus one, is the number of available strikes used in the estimation of volatility indices. On average, 17 and 21 strikes were available, respectively, for the near-term and the next-term calculation of indices.

[Table 1: Summary statistic of Nikkei 225 options]

It should be noted that an additional filtering rule was applied to the CBOE's methodology. At least three OTM options must exist after the screening process; otherwise, the integrated variance of the previous trading day, calculated from (5) and (7), will be used instead.

6. Empirical results

Table 2 shows summary statistic of volatility indices calculated using methodology described in section 2 and 3, as well as, the VXJ and CSFI-VXJ obtained from the CSFI's website. CBVX denotes the volatility index calculated under CBOE's methodology described in section 2. CX99, CX97, and CX95 denote the corridor volatility indices calculated under Andersen et al. (2011)'s method described in section 3. The number behind 'CX' indicates the upper percentile range of $R(K)$ function in (9). CX99 corresponds to the widest corridor volatility index calculated from the options within [1, 99] percentile range of $R(K)$, while CX95 corresponds to the narrowest corridor volatility index calculated from options within [5,95] percentile range of $R(K)$.

MFIV is a special case of corridor implied volatility index, in which the corridor width is set to $(-\infty, \infty)$. In other words, MFIV represents a full-blown version of the CBOE's VIX, which uses all the available options data residing deep in both tails of RND, or deep-out-of-the-money options.

RV denotes realized volatility for the next 30-day period from the observation date of a volatility index. In particular, RV is calculated from daily closing spot prices of the Nikkei 225 index from t to $t + 30$:

$$RV_t = 100 \times \sqrt{\sum_{t=1}^{\tau-1}\left[log\left(\frac{S_{t+1}}{S_t}\right)\right]^2} \quad , \tag{11}$$

where τ is the target time-to-maturity of options, in this case, equal to 30 days. S_t is the daily closing price of Nikkei 225 index. RV is normalized into a percentage for ease of comparison to the volatility indices.

During the sampling period from December 2005 to December 2010, there were a total of 1,277 observation dates. However, there existed times when the integrated variance under (5) or (7) could not be obtained due to the unavailability of valid options data. When the implied forward index level in (10) cannot be obtained, ATM strike cannot be determined and all the options data are disregarded. Instead, the integrated variance of the near-term or the next-term was carried over from the previous trading day. CSFI and CBOE have similar procedures to ensure the availability of volatility indices. However, the carry-over procedure may cause artificial correlations or characteristics of the volatility indices that are unrelated to differences in the methodology. Consequently, the observation dates, in which the integrated variance of any volatility indices could not be calculated by normal procedures, were excluded from further analysis. Under this policy, the final observations were reduced to 1,071 days, a loss of around 16% of total observations.

[Table 2: Summary Statistics of 30-day Implied Volatility Indices]

According to Table 2, all the volatility indices, including RV, are positively skewed as expected. The standard deviation of volatility indices are in compliance with the width of corridors. The smallest corridor index shows the smallest standard deviation. CSFI-VXJ, which is calculated using different approach from the CBOE's methodology, shows the largest standard deviation among the implied volatility indices, even larger than the full-blown MFIV index.

The statistics of MFIV and CBVX are nearly identical. It is possible that the deep OTM options data may hardly exist so that the CBOE's truncation rule is enough to cover all the options quoted within reasonable strikes range.

7. Volatility indices over time

Figure 1 shows the movement of volatility indices as well as RV over time. As seen from Table 2, volatility indices normally fluctuate between 20-30 levels; however, volatility indices spiked in response to major financial events. The first spike observed in August 2007 may be a response to major financial assets sell-off in the U.S. as a result of the subprime loan crisis. The volatility indices peaked in October 2008 when the subprime loan crisis hit the Japanese market the hardest. In Japan, the financial crisis during this time is called the Lehman shock. After the bankruptcy of Lehman Brothers Holding Inc. and a series of attempts of the U.S. Government to bail out distressed major financial institutions in September 2008, the Japanese market finally could not hold on. The Daiwa Life Assurance declared bankruptcy and the Nikkei 225 index fell 10% during the day on October 10th, 2008. The spike during May 2010 may be a response to the Standard & Poor's reduction of Greece's sovereign debt rating to a junk status, a part of European debt crisis that has been lingering until now.

[Figure 1: Daily volatility indices and realized volatility during December 2005 to December 2010.]

The shaded areas in Figure 1 indicate the periods that are magnified and depicted in Figure 2. The left shaded area is the second half of 2006, which is a period just before the breakout of the subprime loan crisis in the U.S.. The shaded area in the middle represents the second half of 2008 when the subprime loan crisis in the U.S. became Global financial crisis and severely affected the Japanese market. The right shaded area denotes the second half of 2010 period, a post-crisis period when the Japanese market already recovered from the financial slump.

During the pre-crisis and the post-crisis period depicted in Panel a) and c) of Figure 2, the RV seemed to fluctuate below all the implied volatility indices, in line with the negative variance risk premium found in recent financial literatures, for example, Carr and Wu (2009) and Bollerslev et al. (2009). The relationship between implied volatility indices and RV reversed during August to October 2008.

The MFIV is expected to cap all other volatility indices as it uses all the available options data. This relationship is true for CBVX, CX99, CX97, and CX95 which are all based on the filtering rules of the CBOE. The relationship can be confirmed from statistics presented in Table 1, in which means and standard deviations of the narrower corridors volatility indices are less than those of the wider corridors volatility indices. However, the VXJ and CSFI-VXJ occasionally exceeded the MFIV, especially during the crisis period in October 2008.

According to the VXJ documentation[8], VXJ adopted the same methodology as CBOE's VIX but the determination of ATM options were based on transaction prices rather than mid-quotes. In addition, CSFI applies carry-over procedures or uses previous trading day data, when the integrated variance in (5) cannot be estimated due to the unavailability of options data. These discrepancies may cause the break in the relationship between the MFIV and the VXJ. On the contrary, CSFI-VXJ uses a different approach in estimation of integrated variance. Consequently, the relationship between MFIV and other narrower corridors volatility indices does not apply to CSFI-VXJ.

[Figure 2: Closed-up of implied volatility indices and realized volatility by observation periods.]

[8] http://www-csfi.sigmath.es.osaka-u.ac.jp/en/activity/vxj_download.php?

8. Effective strike range of corridor volatility indices

According to Andersen et al. (2011), an effective range, or ER, measures the coverage of options data used in calculation of implied volatility indices, and is defined as;

$$ER_{t,\tau} = \left[\frac{log\left(\frac{K_1}{F_{t,\tau}}\right)}{\hat{\sigma}_{BS}\sqrt{\tau}}, \frac{log\left(\frac{K_n}{F_{t,\tau}}\right)}{\hat{\sigma}_{BS}\sqrt{\tau}} \right] , \quad (12)$$

where $F_{t,\tau}$ is the implied forward index level in (10), τ is the remaining maturity of options, and $\hat{\sigma}_{BS}$ is the Black-Scholes implied volatility estimated from ATM options. $K_1$and $K_n$denotes the lowest and the highest strikes used in calculation of the integrated variance in (5) or (7). On each observation date, effective ranges were calculated for both the near-term and next-term integrated variance estimation.

Figure 3 shows effective strike ranges for both the near-term and the next-term options data on each observation date. Only effective ranges of MFIV, the widest corridor index, and CX95, the narrowest corridor index, were presented for ease of comparison. Effective ranges of CBVX are similar to those of MFIV. Effective ranges of CX99 and CX97 fluctuates between those of MFIV and CX95. Ranges of MFIV were plotted as line graphs, while ranges of CX95 were shown as area. The coverage above the zero line is the coverage of OTM call options, while the coverage below the zero line represents coverage of OTM put options.

As pointed out by Andersen et al. (2011), shifts in ER can cause artificial jumps in the implied volatility index unrelated to the fluctuation in the underlying asset. From Figure 3, it is obvious that strike ranges of CX95 are more stable than the ranges of MFIV whether in the near-term or the next-term options. The width of MFIV fluctuated between 2.43 and 10.50, the difference over 4 times as much between the widest and narrowest range observed. The width of CX95 varied from 0.89 to 2.44. MFIV is likely to produce artificial jumps in the volatility index more than the CX95. In particular, during October 2008, the Lehman Shock crisis period, the coverage of MFIV exhibited a sudden shrinkage and expansion on day-to-day basis. The asymmetry in the coverage of OTM puts and calls found in Andersen et al. (2011), which reflects the relative importance of OTM puts over OTM calls for pricing volatility, can also be observed from Figure 3.

[Figure 3: Daily effective strike range of MFIV and CX95.]

To systematically investigate the effect of the differences in strike ranges between MFIV and CX95, the following simple regression test as used by Andersen et al. (2011) is applied;

$$Z_t = \alpha + \beta\, DR_t \quad , \tag{13}$$

where $Z_t = MFIV_t/CX95_t - 1$. α and β are regression coefficients to be estimated. On each observation date t, the corridor widths of the near-term and the next-term options were linearly combined, separately for MFIV and CX95, using a similar weighting method to the index calculation defined in (6). DR_t is then calculated by taking differences between the widths of corridors of MFIV and CX95. The test in (13) essentially aims to measure how much variations in relative values of MFIV and CX95 can be explained by differences in effective strike ranges between the two indices.

[Table 3: MFIV/CX95 relative strikes ranges regression results.]

Using OLS estimation, the regression results show that both coefficients in (13) are statistically different from zero. The differences in strike ranges between MFIV and CX95 contribute to relative variations between the two indices. Though the adjusted R-squared found in this study is less than what documented in Andersen et al. (2011), the adjusted R-squared nearly reaches the 50% level. The sample standard deviation of DR_t equals 1.17, so the regression results imply that an increase in the effective range of MFIV by one standard deviation will inflate a relative value of MFIV to CX95 by an additional 1.67% (1.17 x 1.43%).

The finding that effective ranges has significant explanatory power for the relative size of implied volatility indices is similar to the results of Andersen et al. (2011).

9. Correlations of volatility indices

The summary statistics presented in Table 2 suggested that volatility indices are very similar in properties. Figure 1 and Figure 2 also showed that indices fluctuate closely to each other. Table 4 shows a correlations matrix of volatility indices as well as RV, Nikkei 225 returns for the next 30-days, and Nikkei 225 returns for the next day. The future-period returns of Nikkei 225 were input to see if the volatility indices have potentials to predict the future returns of their underlying asset. RV is the 30-days-ahead realized volatility calculated from (11). Since implied volatility indices are the expectation of future realized volatility under risk-neutral measure, it is interesting to test the correlation between the predictive measures and the realized measure.

[Table 4: Correlations of volatility indices and volatility index returns.]

The correlations between levels of the implied volatility indices are high and positive. They are all statistically different from zero at 5% significant level. The correlation results confirm the properties found in summary statistics as well as Figure 1 and Figure 2. The correlations of volatility index returns share the same characteristics and are all statistically different from zero. The correlations between volatility indices and future realized volatility, RV, are statistically different from zero and are all nearly reaching the 70% level. The high correlations between volatility indices and the realized volatility for the next 30 days period are expected, because volatility indices are expectations of future volatility under a risk-neutral measure.

The correlations between volatility indices and future returns of Nikkei 225 are not statistically different from zero at 5% significant level, implying that volatility indices may not be a good predictor of future-period returns. The correlations between daily returns of volatility indices, defined as $(V_t/V_{t-1}) - 1$, and the future-period returns of the Nikkei 225 index are neither statistically different from zero.

Nevertheless, on separate tests not presented here, correlations between implied volatility indices and daily return of the Nikkei 225 index on the same trading day are negative and significantly different from zero. Similarly, correlations between daily returns of implied volatility indices and daily returns of Nikkei 225 index are all above 60% and are statistically different from zero at the 5% significant level. This fact is in accordance with recent financial literatures that found negative correlations between stock returns and volatility.

To investigate if the width of corridors and different observation periods affect the negative relationship between stock returns and implied volatility index returns, Figure 4 shows scatter plots between Nikkei 225 returns and MFIV returns, as well as, Nikkei 225 returns and CX95 returns. MFIV is considered a special case as corridor volatility index with the widest corridor, which uses all the available options strikes. CX95, the narrowest corridor volatility index, is calculated using only options strikes which fell within [5, 95] percentile of $R(K)$ defined in (9). The Nikkei 225 returns and implied volatility index returns were standardized by de-meaning and dividing by respective standard deviations. Observation periods are divided into the pre-crisis, crisis, and post-crisis periods, corresponding to the periods between 2005/12-2007/06, 2007/07-2009/03, and 2009/04-2010/12, respectively.

[Figure 4: Volatility index returns vs. Nikkei 225 index returns.]

The negative relationship between Nikkei 225 returns and returns of both volatility indices held regardless of observation periods and the width of corridors as evidenced from the downward sloping regression lines in Figure 4. To confirm the finding, a simple regression test using OLS estimation with $r_t = \alpha + \beta\, IV_t$, was performed for each observation period. r_t is the daily returns of the Nikkei 225 index and IV_t is the daily returns or percentage change of MFIV or CX95. The estimated coefficients β were all negative and significant at 5% level.

It should be noted that Andersen et al. (2011) found that the use of a corridor implied volatility index assists in preserving the relationship between volatility index returns and the underlying stock market index returns when there is a sudden shrinkage or expansion of the effective strike ranges. Though the effect of corridor widths was found to be important in explaining variations in implied volatility indices as discussed in section 8, the corridor widths did not affect the relationship shown in Figure 4.

[Figure 5: Volatility index returns vs. Nikkei 225 index returns in October 2008 and January 2009.]

To confirm if the corridor width does not affect the relationship between stock index returns and volatility index returns, scatter plots similar to Figure 4 were reproduced using data in October 2008 and January 2009. In October 2008, the effective strike ranges of MFIV shrink dramatically, while in January 2009, the opposite occurred. Figure 5 shows that different corridor widths may affect the relationship between the underlying stock index returns and the volatility index returns but the impact was minimal, unlike the findings documented in Andersen et al. (2011). The drastic impact of corridor differences may become apparent under intraday observations.

10. Implied volatility index as predictor of realized volatility

Predictive performance of volatility indices toward the realized volatility for the next 30-days is compared in this section. By construction, an implied volatility index estimated from options data with a 30-day remaining maturity is an expectation of future realized volatility for the same horizon under risk-neutral measure. In section 8, it is evidenced that the choice of corridors affects the characteristic of the volatility index. This section tests wheter variations in corridors affect predictive performance of the volatility index.

The parameters of following a simple regression between the future-realized volatility and the volatility index are obtained using an OLS estimation.

$$RV_t = \alpha + \beta \, IV_t \quad , \qquad (14)$$

where IV_t denotes the implied volatility indices, MFIV, CBVX, CX99, CX97, CX95, VXJ, and CSFI-VXJ. Though RV_t and IV_t has the same subscript t, RV_t denotes the annualized realized volatility calculated from the current time t to $t + 30$ using (11). It should be noted that the regression was estimated on a monthly basis using non-overlapping series of RV_t to prevent autocorrelation.

The estimation window for regression in (14) was set between December 2005 to December 2009, a total of 49 observations. The data during 2010 will be used in testing the forecast performance of the volatility indices against the realized volatility. All the coefficients of regression results in Table 5 are statistically significant at 5% level. The estimated coefficients and adjusted R-squared are similar across alternative implied volatility measures. However, the narrow corridor volatility indices CX97 and CX95 performed slightly better than others based on adjusted R-squared statistics.

[Table 5: Regression results of realized volatility]

As a robustness test, the same regression was performed using all the observations from December 2005 to December 2010, a total of 61 observations. The adjusted R-squared measure still indicates that CX97 performed the best in explaining variations in the future-realized volatility, followed by CX95. However, if the estimation window were set such that 21 sampling months during the subprime crisis period, between July 2007 to March 2009, were excluded from the total observations, the CSFI-VXJ performs the best with adjusted R-squared around 42%, followed by VXJ with adjusted R-squared around 41%. However, the intercept of regression for all the volatility measures became statistically insignificant.

In the next step, the estimated coefficients in Table 5 are used to calculate a root mean squared error (RMSE) of out-of-sample forecast performance during 2010. In particular, the RMSE is calculated from;

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}\left(\widetilde{RV}_t - RV_t\right)^2} \quad , \tag{15}$$

where $\widetilde{RV}_t$ is the forecasted future realized volatility based on the estimated coefficients of (14). N is the total number of forecast periods. It should be noted here that, though the estimation of coefficients used non-overlapping monthly data, the forecast process used all the available daily data. Each forecast is regarded as an independent forecast for the next 30-days' realized volatility.

Table 6 shows RMSE results of 238 daily forecasts from January to December 2010. The forecast performance is in line with the adjusted R-squared criteria presented in Table 5. The CX97 corridor volatility index has the best forecast performance with the lowest RMSE of 4.24, followed by CX95 index with RMSE equaling 4.26. When the non-overlapping data of all 61 observation months were used to estimate coefficients of (14), the RMSE criteria of in-sample daily forecasts during 2010 confirm that CX97 has the best forecast performance with the lowest RMSE of 4.12. Nevertheless, when the forecast is conducted using estimated parameters from the estimation window that excluded the 21 months during the crisis period, the MFIV and CBVX indices gave the lowest RMSE of 4.05.

With exception to the third estimation window showed in Table 6, CX97, the corridor volatility index with a moderate corridor width, seems to outperform other volatility measures as a predictor of realized volatility over the next 30 days.

[Table 6: RMSE of realized volatility forecast.]

11. Conclusions

In this paper, various implied volatility measures were introduced, compared, and contrasted. The methodology of the well-known CBOE's VIX index was applied to construct a volatility index of the Japanese stock market using Nikkei 225 options data. A new approach proposed by Andersen et al. (2011) to overcome the random truncation of the CBOE's VIX method was applied and tested against the CBOE's method.

Volatility indices calculated under CBOE's method and Andersen et al. (2011) shared very similar properties. The corridor volatility indices fluctuated to a lesser degree than the CBOE's VIX and the index that used all options strikes, MFIV, during the observation period. The negative volatility risk premium, which dictated that a realized volatility should be less than its expectation under a risk-neutral measure, can be observed when the realized volatility with a matching horizon was plotted with volatility indices. However, a large positive premium can be observed during the Lehman shock crisis in 2008.

The corridor widths have explanatory power against the relative variations between two volatility indices with different strikes coverage and corridor volatility indices seemed to perform better in predicting future realized volatility. However, the merit of using corridor volatility index was not fully realized as the differences in performance were minimal. Further study using high-frequency intraday data may help convey the strengths of corridor volatility indices as documented by Andersen et al. (2011).

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Table 1: Summary statistic of Nikkei 225 options.

Out-of-the-money is abbreviated as OTM. Total denotes total number of options observed under each category during the whole 1,227 observation dates. Average refers to average number of options per day across all observation dates. Min and Max are the minimum and maximum number of options observed during the sampling period.

Options before screening process (all maturities)				618,686
Average options before screening per day				504
	Total	Average	Min	Max
Near term options before screening	66,188	54	28	98
OTM put after screening	10,706	9	3	22
OTM call after screening	9,276	8	3	29
Next term options before screening	60,254	52	22	96
OTM put after screening	12,947	11	3	21
OTM call after screening	12,198	11	4	34

Table 2: Summary Statistics of 30-day Implied Volatility Indices.

From left to right, the corridor widths were sorted from the widest to the narrowest, with MFIV representing the widest corridor and CX95 representing the narrowest corridor implied volatility indices. VXJ, CSFI-VXJ, and RV are presented for comparison. Q25 to Q75 denotes the 25th to 75th percentile of the total observations. There were a total of 1,227 observation dates during December 2005 to December 2010. However, after the elimination of incalculable sampling dates, 1,071 observations remain.

	MFIV	CBVX	CX99	CX97	CX95	VXJ	CSFI VXJ	RV
Mean	29.93	29.93	29.17	28.20	27.43	29.55	29.35	24.88
Std Dev	12.66	12.65	12.48	12.15	11.84	12.49	13.03	14.76
Skewness	2.10	2.10	2.19	2.25	2.28	2.09	2.23	3.33
Kurtosis	8.07	8.04	8.54	8.92	9.09	7.98	8.87	17.01
min	14.32	14.32	14.12	13.77	13.77	14.12	13.47	8.50
max	93.53	93.53	93.05	92.12	89.12	91.45	97.27	113.90
Q25	22.31	22.31	21.64	20.91	20.40	21.99	21.65	17.33
Q50	26.60	26.60	25.85	24.97	24.44	26.34	25.88	22.25
Q75	32.51	32.51	31.54	30.27	29.33	32.08	31.89	27.12

Table 3: MFIV/CX95 relative strikes ranges regression results.

The simple regression test specified in (13) was conducted to test if relative differences in corridor widths contribute to the relative variations between MFIV and CX95. After elimination of outliers and invalid data, a total of 1,067 observations were used in the regression under OLS estimation. Standard errors of coefficients were Newey-West HAC standard errors calculated using 20 lags. P-values for both coefficients were less than 1e-6.

	$\hat{\alpha}$	$\hat{\beta}$
Coefficient	0.0336	0.0143
Std. Error	0.0071	0.0014
t-Statistic	4.74	9.88
p-value	0	0
$\bar{R}^2$= 0.466		

Table 4: Correlations of volatility indices and volatility index returns.

RV denotes 30-days-ahead realized volatility calculated using (11). RET_{30} is the 30-days-ahead log-returns of the Nikkei 225 index. RET_d is the daily return of Nikkei 225 index in the following trading day. The volatility index return is (V_t/V_{t-1})-1, where V_t denotes each volatility index.

Panel a) Correlations of volatility indices

	MFIV	CBVX	CX99	CX97	CX95	VXJ	CSFI VXJ	RV	RET_{30}	RET_d
MFIV	1									
CBVX	1.0000	1								
CX99	0.9995	0.9994	1							
CX97	0.9985	0.9985	0.9995	1						
CX95	0.9977	0.9977	0.9990	0.9994	1					
VXJ	0.9985	0.9985	0.9981	0.9971	0.9963	1				
CSFI VXJ	0.9965	0.9964	0.9968	0.9968	0.9965	0.9978	1			
RV	0.6581	0.6582	0.6654	0.6723	0.6749	0.6646	0.6763	1		
RET_{30}	0.0380	0.0379	0.0356	0.0332	0.0336	0.0370	0.0357	-0.4218	1	
RET_d	-0.0116	-0.0115	-0.0120	-0.0113	-0.0092	-0.0110	-0.0048	-0.1597	0.2425	1

Panel b) Correlations of volatility index returns

	MFIV	CBVX	CX99	CX97	CX95	VXJ	CSFI VXJ	RV	RET_{30}	RET_d
MFIV	1									
CBVX	0.9994	1								
CX99	0.9958	0.9951	1							
CX97	0.9821	0.9815	0.9812	1						
CX95	0.9644	0.9639	0.9638	0.9471	1					
VXJ	0.9403	0.9397	0.9372	0.9185	0.9008	1				
CSFI VXJ	0.9422	0.9412	0.9381	0.9241	0.9035	0.9577	1			
RV	-0.0293	-0.0284	-0.0293	-0.0270	-0.0268	-0.0255	-0.0262	1		
RET_{30}	-0.0297	-0.0297	-0.0280	-0.0272	-0.0246	-0.0255	-0.0264	-0.2332	1	
RET_d	0.0174	0.0189	0.0182	0.0151	0.0144	0.0509	0.0425	-0.0667	0.2423	1

Table 5: Regression results of realized volatility

Regression equation (14) is estimated using an OLS estimation with non-overlapping 30-days-ahead realized volatility, RV_t , as the dependent variable and each series of implied volatility index, IV_t as the predictor variable. The estimation window is from the end of December 2005 to the end of December 2009, totaling of 49 observations. Standard errors of coefficients are Newey-West HAC standard errors with one lag, automatically determined by AIC criteria.

IV_t		$\hat{\alpha}$	$\hat{\beta}$	$\bar{R}^2$
MFIV	Coefficient	4.869	0.668	0.360
	Std. Error	(2.164)	(0.086)	
CBVX	Coefficient	4.869	0.668	0.360
	Std. Error	(2.164)	(0.086)	
CX99	Coefficient	4.847	0.685	0.368
	Std. Error	(2.171)	(0.082)	
CX97	Coefficient	4.886	0.706	0.373
	Std. Error	(2.169)	(0.081)	
CX95	Coefficient	4.853	0.728	0.371
	Std. Error	(2.259)	(0.066)	
VXJ	Coefficient	4.670	0.684	0.364
	Std. Error	(2.168)	(0.091)	
CSFI-VXJ	Coefficient	5.603	0.656	0.368
	Std. Error	(2.105)	(0.079)	

Table 6: RMSE of realized volatility forecast.

RMSEs of 238 daily forecast of realized volatility over the next 30-days period during January to December 2010 are calculated using (15). The coefficients estimated from regression in (14) with different estimation windows were used to derive the RMSE. Lower RMSE indicates better forecast performance.

	Estimation window 1 (2005/12 – 2009/12) 49 observations	Estimation window 2 (2005/12 – 2010/12) 61 observations	Estimation window 3 (2005/12 –2007/06 : 2009/04-2010/12) 39 observations
MFIV	4.347	4.172	4.050
CBVX	4.347	4.172	4.050
CX99	4.283	4.134	4.066
CX97	4.248	4.115	4.085
CX95	4.260	4.134	4.125
VXJ	4.383	4.211	4.096
CSFI VXJ	4.360	4.182	4.069

Figure 1: Daily volatility indices and realized volatility during December 2005 to December 2010.

The shaded areas are magnified in Figure 2. RV plotted on each observation date is the realized volatility for the next 30-day period.

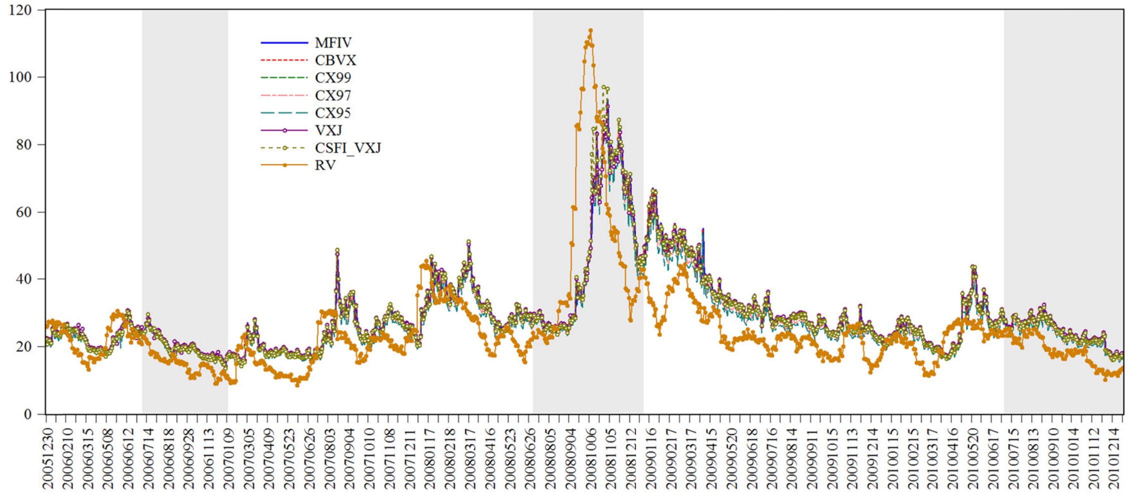
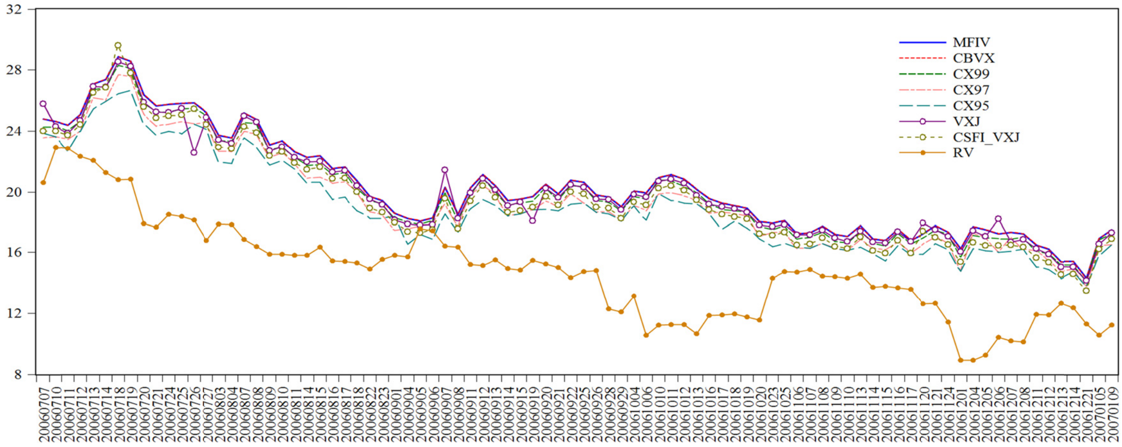


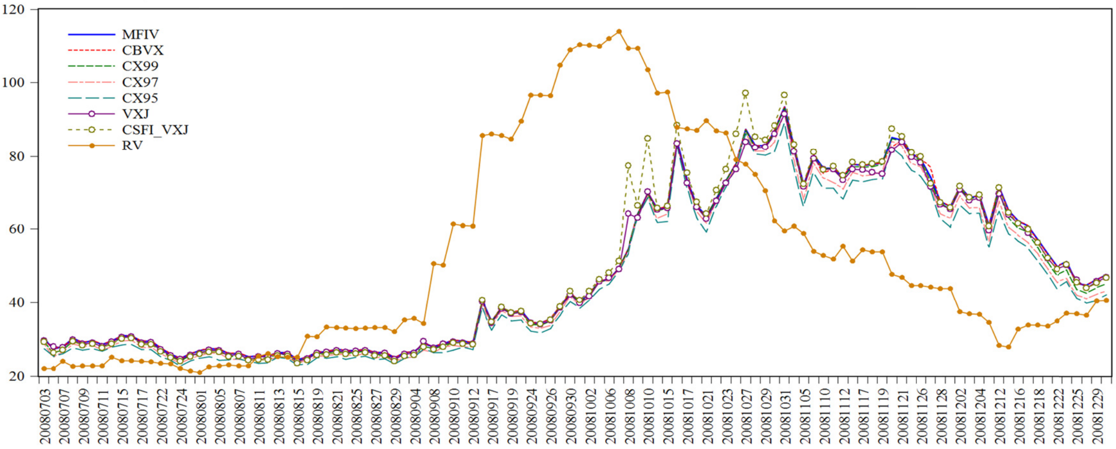
Figure 2: Closed-up of implied volatility indices and realized volatiliy by observation periods.

The volatility indices plotted in Figure 1 for each sub-sampling periods are magnified. Panel a) shows indices before the subprimeloan crisis began to emerge in the U.S. Panel b) shows indices during Lheman Shock crisis hit Japanese market. Panel c) shows the indices during the last half of 2010 when the Nikkei 225 index already recovered from the slump.

a) Pre-crisis: July – December 2006



b) Crisis: July – December 2008



c) Post-crisis: July – December 2010

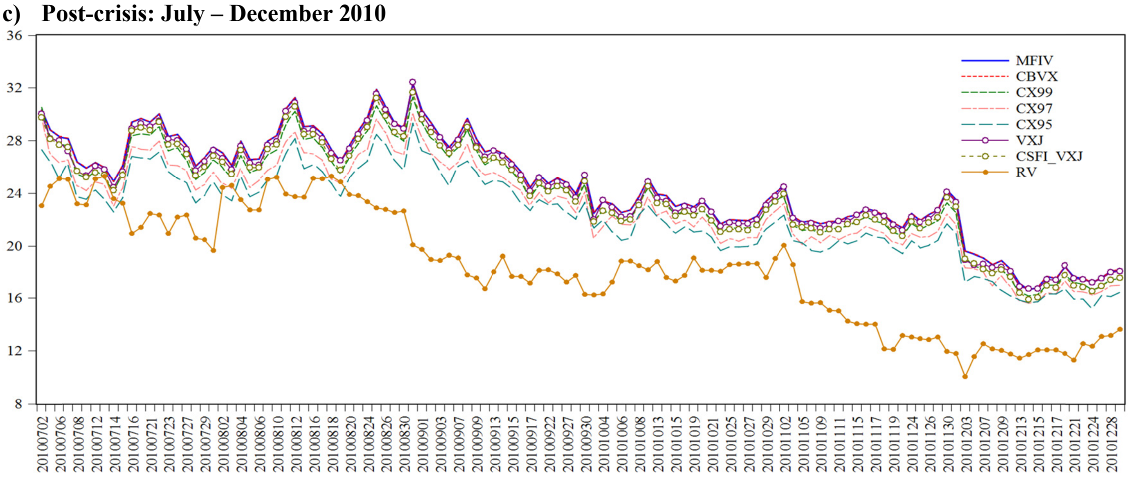
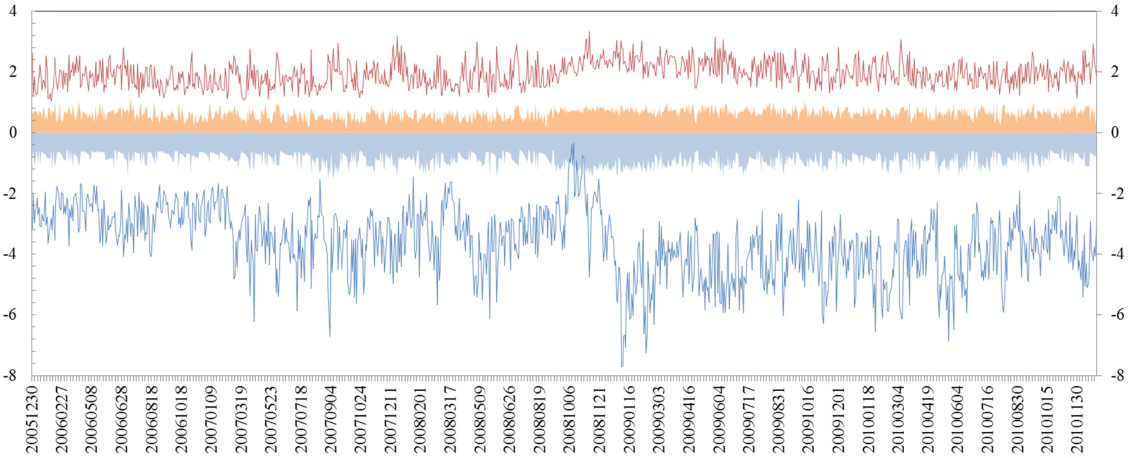


Figure 3: Daily effective strike range of MFIV and CX95.

The positive region shows the upper bound coverage for OTM call options, while the negative region shows the lower bound coverage for OTM put options. The ranges of MFIV were plotted as line, while the ranges of CX95 were plotted as shaded area. Panel a) and Panel b) show, respectively, ER for the near-term and the next-term options data.

Panel a) Near-term maturity



Panel b) Next-term maturity

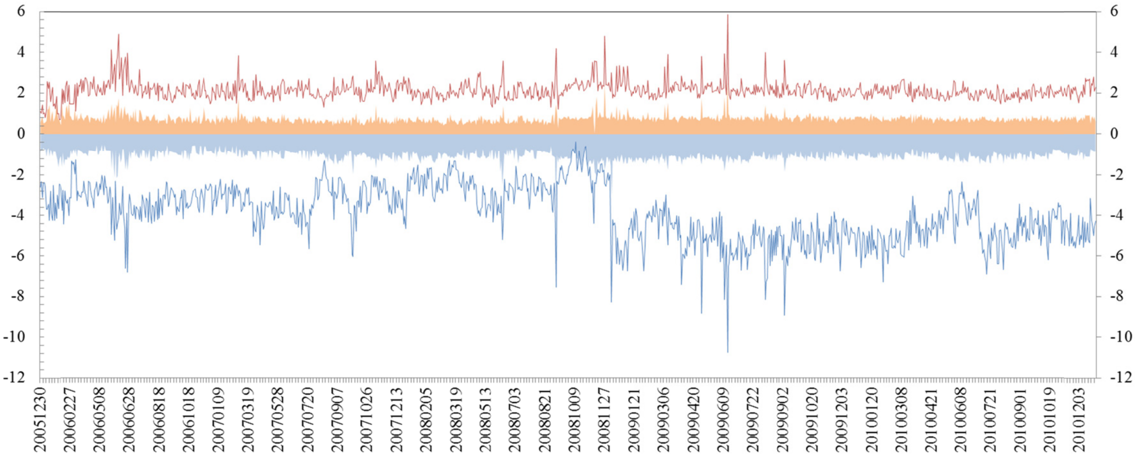


Figure 4: Volatility index returns vs. Nikkei 225 index returns.

Pre-crisis, crisis, and post-crisis refer to 2005/12–2007/06, 2007/07–2009/03, and 2009/04– 2010/12, respectively. Y-axis shows daily returns of volatility indices and X-axis is daily returns of the Nikkei 225 index. Both returns of volatility indices and the Nikkei 225 index were de-meaned and standardized by their standard deviations.

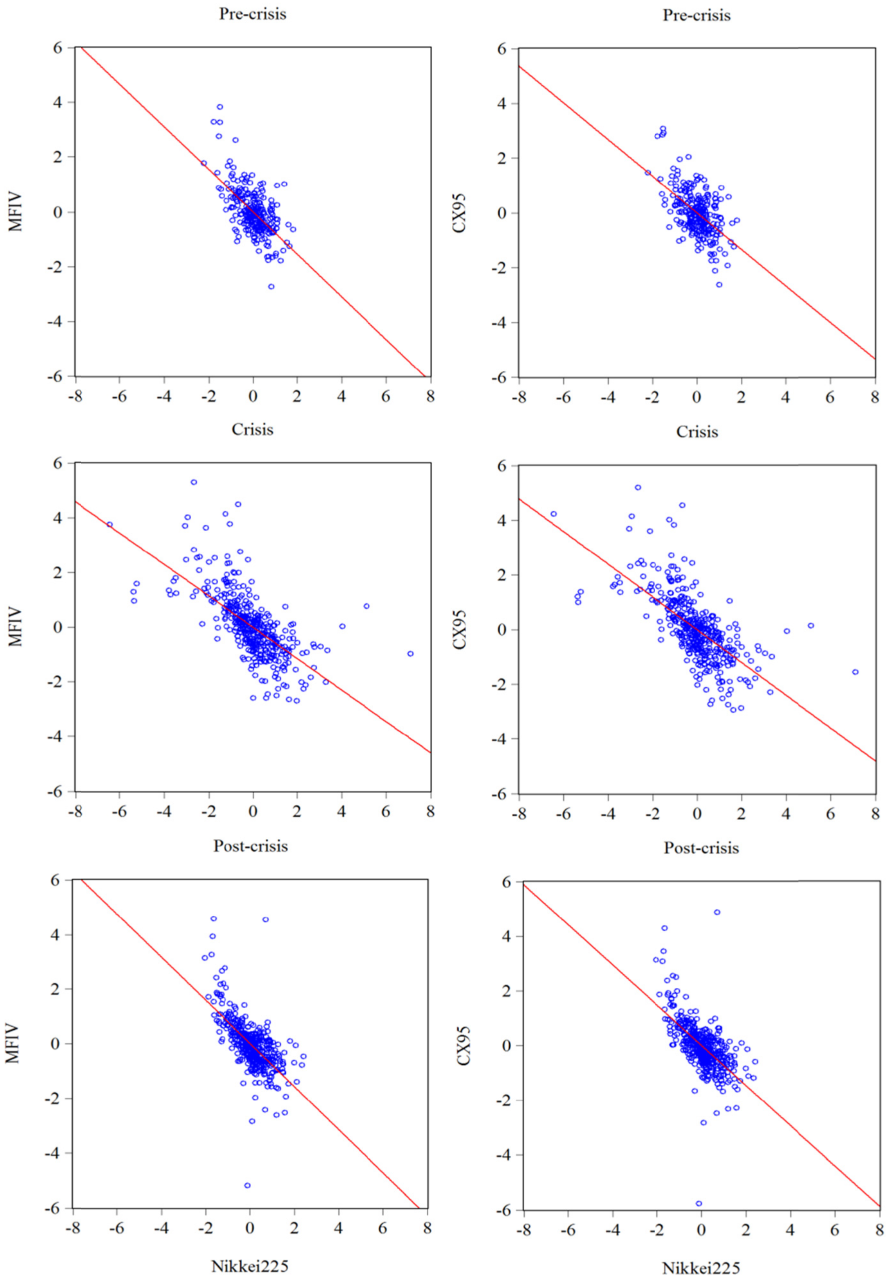


Figure 5: Volatility index returns vs. Nikkei 225 index returns in October 2008 and January 2009.

MFIV is the volatility index with the widest corridor, and CX95 is the volatility index with the narrowest corridor. The Y-axis shows daily returns of volatility indices and the X-axis is daily returns of the Nikkei 225 index. Both returns of volatility indices and the Nikkei 225 index were de-meaned and standardized by their standard deviations.