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# Indeterminacy and utility-generating government spending under balanced-budget fiscal policies*

Takeo Hori† Noritaka Maebayashi‡

**Abstract**

We reexamine indeterminacy and utility-generating public spending under balanced-budget rules in a simple one-sector growth model. The introduction of consumption tax (subsidy) as well as subsidies for savings and labor modify indeterminacy conditions in the existing studies. We show that if consumption subsidies and income taxes exist, indeterminacy occurs even when private and public consumption are Edgeworth substitutes and public spending and leisure are separable in the utility function. Indeterminacy also occurs even when they are weak Edgeworth complements if consumption tax and subsidies for savings and labor are present. Surprisingly, when they are strong Edgeworth complements, the stronger external effects of public consumption tend to lower the possibility of equilibrium indeterminacy in the presence of consumption tax.

JEL classification: E32, E62

Keywords: utility-generating public spending, indeterminacy, balanced-budget rule

*We are responsible for any remaining errors.

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# 1 Introduction

It is well known that in the presence of certain of externalities or market incompleteness, a wide variety of growth models exhibit equilibrium indeterminacy. It has been shown that production and/or consumption externalities may be sources of indeterminacy.[1] Other possible sources of indeterminacy are public goods or services because they are typical goods that yield external effects. In a model where public spending financed by income taxes exerts positive external effects on the productivity of firms, Guo and Harrison (2008) show that indeterminacy may arise if the external effects of public spending are sufficiently strong.[2,3]

The external effects of public spending are not limited to productivity and can extend to the utility of households, which may be a source of indeterminacy. Fernández et al. (2004) and Guo and Harrison (2008) introduce utility-generating public spending financed by income tax with constant tax rates into otherwise standard one-sector real business cycle (RBC) models and examine the conditions for indeterminacy under balanced-budget rules. Assuming that private and public consumption are Edgeworth substitutes, Fernández et al. (2004) show that if public consumption and leisure are non-separable in the utility function, indeterminacy might arise. On the other hand, in a model where public consumption and leisure are separable, Guo and Harrison (2008) show that if private and public consumption are Edgeworth substitutes, indeterminacy never arises, whereas the model economy exhibits equilibrium indeterminacy only when private and public consumption are Edgeworth complements and the external effects of public consumption are sufficiently strong.[4] These studies show that under balanced-budget rules,

[1]See Benhabib and Farmer (1994), Benhabib and Perli (1994), Benhabib and Nishimura (1998) and Mino (2001), for studies on production externalities. Alonso-Carrera et al. (2008), Chen and Hsu (2007) and others study indeterminacy and consumption externalities.

[2]Extending the model of Guo and Harrison (2008), Kamiguchi and Tamai (2001) show that if productive public spending is financed by consumption tax, indeterminacy does not arise.

[3]Cazzavillan (1996), Raurich (2003) and Palivos, Yip and Zhang (2003) examine conditions for equilibrium indeterminacy in endogenous growth models with productive government spending.

[4]Lloyd-Braga et al. (2008) study the same issue in a segmented asset market economy model of the Woodford (1986) type, rather than standard one-sector RBC models. Under Edgeworth complementarity between private and public consumption, they also show that with constant income tax rates, indeterminacy arises if the external effects of public consumption are sufficiently strong. In addition, they show that if the consumption tax rate responds negatively to the tax base, indeterminacy is possible in a segmented asset markets framework.

preference structure is crucial for indeterminacy in standard one-sector RBC models with utility-generating public spending.

We also introduce utility-generating public spending into a standard one-sector RBC model where public consumption and leisure are separable and then reexamine conditions for indeterminacy under balanced-budget rules. In contrast to Fernández et al. (2004) and Guo and Harrison (2008), we allow the government to finance its spending by consumption tax as well as income taxes. Subsidies for savings, labor and consumption are also considered. All of these tax (subsidy) rates are assumed to be constant. We show that even when private and public consumption are Edgeworth substitutes and public consumption and leisure are separable in the utility function, indeterminacy arises in the presence of consumption subsidies as well as income taxes. The indeterminacy conditions under Edgeworth complementarity between private and public consumption are as follows. Even when the external effect of public spending is not strong, indeterminacy occurs if consumption tax and subsidies for savings and labor are present. When the external effects of public spending are strong, the financing of public spending is crucial. If public spending is mainly financed by income taxes, indeterminacy arises whereas it does not arise if the revenue from consumption tax is a main source of public spending. As the consumption tax rate increases, the occurrence of indeterminacy becomes unlikely. In addition, unlike most existing studies on indeterminacy and externalities, we show that as the external effects of public spending become stronger, the model economy tends to be more *unlikely* to exhibit indeterminacy under Edgeworth complementarity with strong externalities. Our results suggest that in addition to preference structure, the designs of fiscal policy are crucial for the stability of the economy.

The rest of the paper is organized as follows. Section 2 presents the model. Conditions for indeterminacy are examined in Section 3. Concluding remarks are in Section 4.

# 2 The Model

We follow the model presented in Subsection 3.2 of Guo and Harrison (2008). Departing from Guo and Harrison (2008), we consider consumption tax (or subsidy) and subsidies for savings and labor as well as income taxation. Guo and Harrison (2008) focus only on the case where private and government consumption are Edgeworth complements. We examine the case where they are Edgeworth substitutes as well as the case where they are Edgeworth complements.

Consider a competitive economy that consists of firms, households and the government. Time is continuous and is denoted as $t \geq 0$. The representative firm produces a single final good by using the following technology:

$$Y_t = A{K_t}^{\alpha}{L_t}^{1-\alpha}, \quad A > 0 \text{ and } \alpha \in (0,1), \tag{1}$$

where $Y_t$ is the output, $K_t$ is capital input and $L_t$ is labor input at time $t$. Through profit maximization, the rental rate of capital, $r_t$, and the wage rate, $w_t$, are equal to the marginal products:

$$r_t = \alpha \frac{Y_t}{K_t} = \alpha A \left(\frac{K_t}{L_t}\right)^{\alpha-1}, \tag{2a}$$

$$w_t = (1-\alpha)\frac{Y_t}{L_t} = (1-\alpha)A\left(\frac{K_t}{L_t}\right)^{\alpha}. \tag{2b}$$

The utility function of the representative household is given by

$$U = \int_0^{\infty} \left\{ \frac{({C_t}^{\theta_C}{G_t}^{\theta_G})^{1-\sigma}}{1-\sigma} - BL_t \right\} e^{-\rho t}dt, \quad B,\ \theta_C,\ \theta_G,\ \sigma > 0 \text{ and } \sigma \neq 1, \tag{3}$$

where $\rho > 0$ is subjective discount rate, $C_t$ is private consumption, $L_t$ is the labor supply, and $G_t$ is the public spending of the government. The assumption $\theta_G > 0$ implies the presence of external effects of public spending. $\sigma$ is the inverse of the intertemporal elasticity of substitution in effective consumption, ${C_t}^{\theta_C}{G_t}^{\theta_G}$. To ensure strict concavity with respect to $C_t$, we assume $\theta_C(1-\sigma) - 1 < 0$. If $\sigma = 1$, then $C_t$ and $G_t$ are separable and thus, the presence of $G_t$ in (3) does not affect the dynamic properties of the model. Hence, we restrict our attention to $\sigma \neq 1$. Assuming $\sigma < 1$, Guo and Harrison (2008) consider only the case where $C_t$ and $G_t$ are Edgeworth complements. We examine the case where $C_t$ and $G_t$ are Edgeworth substitutes ($\sigma > 1$), and the case where they are

Edgeworth complementarity ($\sigma < 1$).[5] In contrast to Fernández et al. (2004), $G_t$ and $L_t$ are separable in (3).

The budget constraint of the household is given by

$$\dot{K}_t = (1 - \tau_r) r_t K_t - \delta K_t + (1 - \tau_w) w_t L_t - (1 + \tau_c) C_t, \tag{4}$$

where $\tau_r$, $\tau_w$ and $\tau_c$ ($\in (-1, 1)$) are the tax (or subsidy) rates on capital income, labor income and consumption, respectively. When $\tau_x > (<) 0$ holds ($x = r$, $w$ and $c$), $\tau_x$ denotes the tax (subsidy) rate. The capital depreciation rate is denoted as $\delta > 0$.

The household maximizes (3) subject to (4) and the no-Ponzi game condition, which yields the following first order conditions:

$$\theta_C {C_t}^{\theta_C(1-\sigma)-1} {G_t}^{\theta_G(1-\sigma)} = \Lambda_t (1 + \tau_c), \tag{5a}$$

$$B = \Lambda_t (1 - \tau_w) w_t, \tag{5b}$$

$$\dot{\Lambda}_t = \{\rho + \delta - (1 - \tau_r) r_t\} \Lambda_t, \tag{5c}$$

where $\Lambda_t$ is the co-state variable associated with (4).

Under the balanced-budget rule, the budget constraint of the government is given by $G_t = \tau_r r_t K_t + \tau_w w_t L_t + \tau_c C_t$. Public spending, $G_t$, must be strictly positive in equilibrium. Using (2a) and (2b), we rewrite the budget constraint of the government as

$$G_t = \tau Y_t + \tau_c C_t, \tag{6}$$

where $\tau \equiv \alpha \tau_r + (1 - \alpha) \tau_w$. The good market equilibrium condition is given by

$$\dot{K}_t = Y_t - \delta K_t - C_t - G_t. \tag{7}$$

[5] Many researchers have tested whether private and government consumption are complements or substitutes. However, empirical evidence is mixed. Aschauer (1985), Bean (1986), and Kormendi and Meguire (1995) find evidence for substitutability. Karras (1994) finds evidence for complementarity, and Ni (1995) finds that results are sensitive to model specifications. More recently, Hamori and Asako (1999) and Chiu (2001) find substitutability for Japan and Taiwan, respectively. Fleissig and Rossana (2007) find Morishima substitutes between government and private consumption from US data. In contrast, Ohkubo (2003) finds that the two types of consumption may be complements or may be unrelated from Japanese quarterly data. Fiorito and Kollintzas (2004) find that they are complements from data for European countries.

Using (1), (2a), (2b), (5b), (5c), (6) and (7), we can reduce the equilibrium conditions to (5a) and the following four equations:

$$\dot{\Lambda}_t = \left\{\rho + \delta - (1-\tau_r)\alpha A \left(K_t/L_t\right)^{\alpha-1}\right\} \Lambda_t, \tag{8a}$$

$$\dot{K}_t = (1-\tau)A{K_t}^{\alpha}{L_t}^{1-\alpha} - \delta K_t - (1+\tau_c)C_t, \tag{8b}$$

$$G_t = \tau A{K_t}^{\alpha}{L_t}^{1-\alpha} + \tau_c C_t, \tag{8c}$$

$$B = \Lambda_t(1-\tau_w)(1-\alpha)A\left(K_t/L_t\right)^{\alpha}. \tag{8d}$$

When $\tau_r = \tau_w > 0$, $\tau_c = 0$ and $\sigma < 1$ hold, our model reduces to the one examined in Guo and Harrison (2008).

To derive the steady state equilibrium in which $\Lambda_t$, $K_t$, $C_t$, $L_t$, and $G_t$ are constant over time, let us denote $\hat{K} \equiv K^*/L^*$, $\hat{C} \equiv C^*/L^*$, and $\hat{G} \equiv G^*/L^*$.[6] From (8a), (8b), (8c), $\dot{\Lambda}_t = 0$ and $\dot{K}_t = 0$, $\hat{K}$, $\hat{C}$ and $\hat{G}$ are given by

$$\hat{K} = \left\{\frac{(1-\tau_r)\alpha A}{\rho+\delta}\right\}^{\frac{1}{1-\alpha}}, \tag{9a}$$

$$\hat{C} = \frac{\{(1-\tau)A\hat{K}^{\alpha-1} - \delta\}\hat{K}}{1+\tau_c}, \tag{9b}$$

$$\hat{G} = \tau A\hat{K}^{\alpha} + \tau_c \hat{C}. \tag{9c}$$

To ensure $\hat{G} > 0$ (or equivalently, $G^* > 0$), at least one of $\tau$ and $\tau_c$ must be positive. From (8d), we have $\Lambda^* = B\{(1-\tau_w)(1-\alpha)A\hat{K}^{\alpha}\}^{-1}$. Apparently, $\hat{K}$, $\hat{C}$, $\hat{G}$ and $\Lambda^*$ are uniquely determined. We rearrange (5a) as

$$\theta_C \hat{C}^{\theta_C(1-\sigma)-1}\hat{G}^{\theta_G(1-\sigma)}L^{*(1-\sigma)(\theta_C+\theta_G)-1} = (1+\tau_c)\Lambda^*.$$

When $(1-\sigma)(\theta_C+\theta_G) - 1 < (>) 0$ holds, the left-hand side (LHS) of the above equation monotonically decreases (increases) from $\infty$ (zero) to zero ($\infty$) as $L^*$ increases from zero to $\infty$. Irrespective of whether $\sigma < 1$ or $\sigma > 1$ holds, there exists a unique $L^*$. Then, $K^*$, $C^*$ and $G^*$ are also unique and there exists a unique steady state.

[6] We use asterisks to indicate steady state variables.

# 3 Local Stability and (In)determinacy

To examine the local stability of the steady state, we consider a log-linear approximation around the steady state. Let us define $\lambda_t \equiv \ln(\Lambda_t/\Lambda^*)$, $k_t \equiv \ln(K_t/K^*)$, $l_t \equiv \ln(L_t/L^*)$, $c_t \equiv \ln(C_t/C^*)$, and $g_t \equiv \ln(G_t/G^*)$. The log-linearized equilibrium conditions are

$$[\theta_C(1-\sigma)-1]c_t + \theta_G(1-\sigma)g_t = \lambda_t, \tag{10a}$$

$$\dot{\lambda}_t = (1-\alpha)(\rho+\delta)(k_t - l_t), \tag{10b}$$

$$\dot{k}_t = \left\{\frac{1-\tau}{1-\tau_r}(\rho+\delta)-\delta\right\}k_t + \frac{1-\tau}{1-\tau_r}\frac{1-\alpha}{\alpha}(\rho+\delta)l_t - \left\{\frac{1-\tau}{1-\tau_r}\frac{\rho+\delta}{\alpha}-\delta\right\}c_t, \tag{10c}$$

$$g_t = \tau\frac{\alpha Y^*}{G^*}k_t + \tau\frac{(1-\alpha)Y^*}{G^*}l_t + \tau_c\frac{C^*}{G^*}c_t, \tag{10d}$$

$$0 = \lambda_t + \alpha k_t - \alpha l_t, \tag{10e}$$

where $Y^* \equiv AK^{*\alpha}L^{*1-\alpha}$ is the output level in the steady state equilibrium. The derivations of (10a)–(10e) are in Appendix. Using (9c), (10a), (10d) and (10e), we eliminate $l_t$ and $c_t$ from (10b) and (10c), which yields the following dynamic system:

$$\begin{bmatrix} \dot{\lambda}_t \\ \dot{k}_t \end{bmatrix} = \begin{bmatrix} -\frac{(1-\alpha)(\rho+\delta)}{\alpha} & 0 \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} \lambda_t \\ k_t \end{bmatrix}, \tag{11}$$

where

$$m_{21} = \frac{1-\tau}{1-\tau_r}\frac{(1-\alpha)(\rho+\delta)}{\alpha^2} - \left[\frac{1-\tau}{1-\tau_r}\frac{\rho+\delta}{\alpha}-\delta\right]\frac{1-\theta_G(1-\sigma)\frac{1-\alpha}{\alpha}\frac{\tau Y^*}{G^*}}{(\theta_C+\theta_G\frac{\tau_C C^*}{G^*})(1-\sigma)-1},$$

$$m_{22} = \left[\frac{1-\tau}{1-\tau_r}\frac{\rho+\delta}{\alpha}-\delta\right]\frac{(\theta_C+\theta_G)(1-\sigma)-1}{(\theta_C+\theta_G\frac{\tau_C C^*}{G^*})(1-\sigma)-1}.$$

The system, (11), includes only one predetermined variable, $k_t$. One of the eigenvalues of the coefficient matrix is apparently negative, $-\frac{(1-\alpha)(\rho+\delta)}{\alpha} < 0$. If the other eigenvalue, $m_{22}$, is positive, the steady state is saddle stable and locally determinate. If $m_{22}$ is negative, the steady state exhibits local indeterminacy. If $\theta_G = 0$ holds, $m_{22}$ is positive.[7] Without the external effects of $G_t$ in (3), equilibrium indeterminacy never arises.

[7] If $\theta_G = 0$, we have $m_{22} = \frac{1-\tau}{1-\tau_r}\frac{\rho+\delta}{\alpha} - \delta$. Because of $\tau_r < 1$, $\tau_w < 1$, we have $\frac{1-\tau}{1-\tau_r} = \frac{(1-\alpha)(1-\tau_w)}{1-\tau_r} + \alpha > \alpha$, which ensures $m_{22} > 0$.

## 3.1 The Case of Edgeworth Complements

This subsection considers the local stability under $\sigma < 1$.

### 3.1.1 Stability under Strong Externalities

We first consider the case where $\theta_G$ is large enough to satisfy $(\theta_C + \theta_G)(1-\sigma) - 1 > 0$. Equilibrium indeterminacy arises only when $\left(\theta_C + \theta_G \frac{\tau_c C^*}{G^*}\right)(1-\sigma) - 1 < 0$ or, equivalently,

$$\frac{(\theta_C + \theta_G)(1-\sigma) - 1}{1 - \theta_C(1-\sigma)} \tau_c C^* < \tau Y^* \tag{13}$$

holds. If the reverse inequality holds in (13), the steady state is locally determinate. If public spending and consumption subsidy are financed by income taxes ($\tau > 0$ and $\tau_c \leq 0$), (13) is satisfied and hence the steady state is locally indeterminate. However, if public spending and subsidies for saving and labor are financed by consumption tax ($\tau \leq 0$ and $\tau_c > 0$), (13) is violated and the steady state is locally determinate. The source of public spending is crucial for the stability of the economy. When public spending is financed mainly by consumption tax, equilibrium indeterminacy does not arise. If the main source of public spending is income taxation, indeterminacy tends to occur.

To obtain further results, assuming $\tau > 0$ and $\tau_c > 0$, we rearrange (13) to obtain

$$\theta_G < \frac{1 - \theta_C(1-\sigma)}{1-\sigma} \left\{ 1 + \frac{1+\tau_c}{\tau_c} \frac{(\rho+\delta)\tau}{\rho + (1-\alpha)\delta - \alpha\rho\tau_r - (\rho+\delta)(1-\alpha)\tau_w} \right\} \equiv \Gamma. \tag{14}$$

In deriving the above inequality, we use (9a), (9b) and (9c). It is shown that $\rho + (1 - \alpha)\delta - \alpha\rho\tau_r - (\rho+\delta)(1-\alpha)\tau_w > \rho + (1-\alpha)\delta - \alpha\rho - (\rho+\delta)(1-\alpha) = 0$ because of $\tau_r < 1$ and $\tau_w < 1$. As shown in Figure 1, as $\tau_c$ increases from zero to $+\infty$, $\Gamma$ monotonically decreases from $+\infty$ to $\hat{\Gamma} > 0$, where $\hat{\Gamma}$ is

$$\hat{\Gamma} \equiv \frac{1 - \theta_C(1-\sigma)}{1-\sigma} \left\{ 1 + \frac{(\rho+\delta)\tau}{\rho + (1-\alpha)\delta - \alpha\rho\tau_r - (\rho+\delta)(1-\alpha)\tau_w} \right\}.$$

Apparently, $\hat{\Gamma}$ monotonically increases with $\tau_r$ and $\tau_w$. When $\tau_r = \tau_w = 0$, we have $\theta_G > \hat{\Gamma}$ because $(\theta_C + \theta_G)(1-\sigma) - 1 > 0$. When both $\tau_r$ and $\tau_w$ tend to one, $\hat{\Gamma}$ tends to $+\infty(> \theta_G)$. Then, when $\tau_r > 0$ and $\tau_w > 0$ are sufficiently small, $\theta_G > \hat{\Gamma}$ holds. There exists a unique $\hat{\tau}_c > 0$ such that if $\tau_c < (>)\hat{\tau}_c$, we have $\theta_G < (>)\Gamma$. If the government sets $\tau_c$ larger than $\hat{\tau}_c$, it can eliminate equilibrium indeterminacy. When $\tau_r$ and $\tau_w(> 0)$ are sufficiently large, $\theta_G < \hat{\Gamma}$ holds and we have $\theta_G < \Gamma$ for any $\tau_c > 0$. Then, the government cannot stabilize the economy by setting a high consumption tax rate. We obtain the next proposition.

**Proposition 1** *Suppose* $0 < \sigma < 1$ *and* $(\theta_C + \theta_G)(1 - \sigma) - 1 > 0$.

1. *If* $\tau \leq 0$ *and* $\tau_c > 0$, *the steady state is locally determinate.*

2. *If* $\tau > 0$ *and* $\tau_c \leq 0$, *the steady state is locally indeterminate.*

3. *Assume* $\tau > 0$ *and* $\tau_c > 0$. *(a) Given* $\tau_r$ *and* $\tau_w$, *if* $\theta_G > \hat{\Gamma}$, *there exists a unique* $\hat{\tau}_c > 0$ *such that if* $\tau_c < \hat{\tau}_c$, *the steady state is locally indeterminate, whereas it is locally determinate if* $\tau_c > \hat{\tau}_c$. *(b) Given* $\tau_r$ *and* $\tau_w$, *if* $\theta_G < \hat{\Gamma}$, *the steady state is locally indeterminate for any* $\tau_c > 0$.

[Figures 1 and 2]

Apparently, $\hat{\tau}_c$ decreases with $\theta_G$ (see Figure 1). As the external effects of $G_t$ become stronger, the range of $\tau_c$ where indeterminacy arises becomes narrower and hence indeterminacy is more unlikely to occur. Assuming $\tau_r = \tau_w = \tau$, Figure 2 shows this result graphically on the $(\tau, \tau_c)$ plane.[8] Equilibrium indeterminacy occurs in the shaded areas while the steady state is locally determinate in the other regions. Apparently, as the external effects increase, the shaded area becomes narrower and hence the steady state tends to exhibit determinacy for wider ranges of tax rates.

The last result may be somewhat counterintuitive because most existing studies on indeterminacy and externalities show that as external effects become stronger, indeterminacy tends to occur. The intuition behind Proposition 1 helps our understanding. We focus on the case where $\tau > 0$ and $\tau_c > 0$ hold. Suppose that agents expect future public spending to increase. Because of Edgeworth complementarity, the marginal utility of future private consumption is expected to increase. The household has an incentive to save

[8]The parameter values in Figure 2 are $A = 1$, $\alpha = 0.36$, $\delta = 0.1$, $\sigma = 0.3$, $\rho = 0.05$, $B = 1$, and $\theta_c = 1.3$.

more, which stimulates capital accumulation. Future output will be larger, which will have positive effects on future public spending. At the same time, capital accumulation will raise the future wage rate. From (5b), future $\Lambda_t$ will decrease. Because (5a) must hold in equilibrium, future private consumption will increase, causing external effects and increasing future public spending further. If $\tau_c$ is large enough to satisfy $\tau_c > \hat{\tau}_c$, the external effects of future private consumption on future public consumption are strong, and hence, future public spending increases more than expected. As a result, (5a) is violated, which cannot happen in equilibrium. The expectation is not self-fulfilling, and hence, the steady state is locally determinate. As $\theta_G$ increases, the LHS of (5a) responds more to increases in future public spending, and hence, (5a) tends to be violated. Then, as the external effects increase, the steady state tends to exhibit determinacy for wider ranges of tax rates. When $\tau_c$ is not too large ($\tau_c < \hat{\tau}_c$), increases in future private consumption have only negligible effects on future public spending. Thus, future public spending increases as much as expected. The expectation is self-fulfilling, and hence, indeterminacy arises.

### 3.1.2 Stability under Weak Externalities

Next, we observe that in contrast to Guo and Harrison (2008), even when $\theta_G$ is sufficiently small such that it satisfies $(\theta_C+\theta_G)(1-\sigma)-1 < 0$, indeterminacy arises if consumption tax is present. Indeterminacy arises only when $\left(\theta_C + \theta_G \frac{\tau_c C^*}{G^*}\right)(1-\sigma)-1 > 0$ or, equivalently,

$$\frac{\theta_G(1-\sigma)}{1-\theta_C(1-\sigma)}\tau_c C^* > G^* \tag{15}$$

holds. Because of $(\theta_C+\theta_G)(1-\sigma)-1 < 0$, we have $0 < \theta_G(1-\sigma)/\{1-\theta_C(1-\sigma)\} < 1$. The above inequality holds only when $\tau_c C^* > G^*$ holds. The last inequality implies $\tau_c > 0$ and $\tau < 0$. In other cases, indeterminacy never arises. To obtain further results, we assume $\tau_c > 0$ and $\tau < 0$ and rearrange (15) as $\theta_G > \Gamma$, where $\Gamma$ is defined as in (14). Because $\Gamma$ is equal to $\frac{1-\theta_C(1-\sigma)}{1-\sigma}\frac{G^*}{\tau_c C^*}$, $\Gamma$ must be positive. When $\tau < 0$, $\Gamma$ increases from $-\infty$ to $\hat{\Gamma}$ as $\tau_c$ increases from zero to $+\infty$ (see Figure 3). We assume $\alpha < 1/2$ to ensure $\hat{\Gamma} > 0$.[9] If $\theta_G < \hat{\Gamma}$ holds, there exist $\underline{\tau}_c(> 0)$ and $\overline{\tau}_c(> \underline{\tau}_c)$ such that $\theta_G > \Gamma > 0$ holds for $\tau_c \in (\underline{\tau}_c, \overline{\tau}_c)$

[9] $\hat{\Gamma}$ has the same sign as the following equation: $\rho + (1-\alpha)\delta - \alpha\rho\tau_r - (\rho+\delta)(1-\alpha)\tau_w + (\rho+\delta)\tau = \rho + (1-\alpha)\delta + \alpha\delta\tau_r > \rho + (1-2\alpha)\delta > 0$. The assumption of $\alpha < 1/2$ ensures the last inequality.

and $\theta_G < \Gamma$ holds for $\tau_c > \overline{\tau}_c$. If $\theta_G > \hat{\Gamma}$ holds, we have $\theta_G > \Gamma > 0$ for $\tau_c > \underline{\tau}_c$. We obtain the following proposition.

**Proposition 2** *Suppose $\alpha < 1/2$, $0 < \sigma < 1$ and $(\theta_C + \theta_G)(1-\sigma) - 1 > 0$.*

1. *If $\tau_c \leq 0$ and $\tau > 0$, or if $\tau \geq 0$ and $\tau_c \geq 0$ hold and at least one of them holds with strict inequality, the steady state is locally determinate.*

2. *Assume $\tau < 0$ and $\tau_c > 0$. (a) When $\theta_G < \hat{\Gamma}$ holds, there exist unique $\underline{\tau}_c$ and $\overline{\tau}_c$ $(0 < \underline{\tau}_c < \overline{\tau}_c)$, given $\tau_r$ and $\tau_w$. The steady state is locally indeterminate if $\tau_c \in (\underline{\tau}_c, \overline{\tau}_c)$ $(0 < \underline{\tau}_c < \overline{\tau}_c)$, whereas it is locally determinate if $\tau_c > \overline{\tau}_c$. (b) When $\theta_G > \hat{\Gamma}$ holds, there exists a unique $\underline{\tau}_c$, given $\tau_r$ and $\tau_w$. If $\tau_c > \underline{\tau}_c$, the steady state is locally indeterminate. In both cases, if $\tau_c < \underline{\tau}_c$, $G^* < 0$ holds.*

[Figures 3 and 4]

Apparently, $\overline{\tau}_c$ increases with $\theta_G$, while $\underline{\tau}_c$ is independent of $\theta_G$. Unlike the case where $\theta_G$ satisfies $(\theta_C + \theta_G)(1-\sigma) - 1 > 0$, as the external effects of $G$ increase, indeterminacy is more likely to occur. Figure 4 presents this result.[10] Indeterminacy occurs in the shaded areas.[11] As $\theta_G$ increases, indeterminacy occurs for wider ranges of tax rates.

The intuition behind Proposition 2 is as follows. Assume $\tau < 0$ and $\tau_c > 0$. Again, suppose that agents expect future public spending to increase. As discussed following Proposition 1, the expectation stimulates capital accumulation, thus increasing future output, and bearing has negative effects on future public spending. At the same time, because of increases in the future wage rate, future private consumption will increase, thus bearing positive external effects on future public spending. When $\tau_c$ is larger than $\underline{\tau}_c$, the positive effect on future public spending dominates the negative one. However, if $\tau_c$ is too large $(\tau_c > \overline{\tau}_c)$, future public spending increases more than expected, which cannot be equilibrium. If $\tau_c < \overline{\tau}_c$, future public spending increases as much as expected. The expectation is self-fulfilling, and indeterminacy arises.

[10]The parameter values in Figure 4 are $A = 1$, $\alpha = 0.36$, $\delta = 0.1$, $\sigma = 0.3$, $\rho = 0.05$, $B = 1$, and $\theta_c = 1.1$.

[11]In the areas to the left of the shaded areas, $G^* < 0$ holds. In the other regions, the steady state is locally determinate.

## 3.2 The Case of Edgeworth Substitutes

This subsection will demonstrate that in contrast to Fernández et al. (2004) and Guo and Harrison (2008), even when $C_t$ and $G_t$ are Edgeworth substitutes ($\sigma > 1$) and $G_t$ and $L_t$ are separable in (3), indeterminacy arises if consumption subsidy is present. When $\sigma > 1$, indeterminacy arises only when $\left(\theta_C + \theta_G \frac{\tau_c C^*}{G^*}\right)(1-\sigma) - 1 > 0$ or, equivalently, (15) holds. Because of $\theta_G(1-\sigma)/\{1-\theta_C(1-\sigma)\} < 0$, (15) holds only when $\tau_c < 0$ and $\tau > 0$. In other cases, indeterminacy never arises. To obtain further results, we assume $\tau_c < 0$ and $\tau > 0$ and then rearrange (15) as $\theta_G > \Gamma$. Again, $\Gamma$ must be positive in equilibrium because of $\Gamma = \frac{1-\theta_C(1-\sigma)}{1-\sigma}\frac{G^*}{\tau_c C^*}$. When $\tau > 0$, $\Gamma$ increases from $\frac{1-\theta_C(1-\sigma)}{1-\sigma} < 0$ to $+\infty$ as $\tau_c(< 0)$ increases from -1 to zero (see Figure 5). Then, there exist $\underline{\underline{\tau}}_c$ and $\overline{\overline{\tau}}_c$ $(-1 < \underline{\underline{\tau}}_c < \overline{\overline{\tau}}_c < 0)$ such that $\theta_G > \Gamma > 0$ holds for $\tau_c \in (\underline{\underline{\tau}}_c, \overline{\overline{\tau}}_c)$ and $\theta_G < \Gamma$ holds for $\tau_c > \overline{\overline{\tau}}_c$. The next proposition is obtained.

**Proposition 3** *Suppose* $\sigma > 1$.

1. *If* $\tau < 0$ *and* $\tau_c > 0$, *or if* $\tau \geq 0$ *and* $\tau_c \geq 0$ *hold and at least one of them holds with strict inequality, the steady state is locally determinate.*

2. *Assume* $\tau > 0$ *and* $\tau_c < 0$. *Given* $\tau_r$ *and* $\tau_w$, *there exist unique* $\underline{\underline{\tau}}_c$ *and* $\overline{\overline{\tau}}_c$ $(-1 < \underline{\underline{\tau}}_c < \overline{\overline{\tau}}_c < 0)$. *The steady state is locally indeterminate if* $\tau_c \in (\underline{\underline{\tau}}_c, \overline{\overline{\tau}}_c)$, *whereas it is locally determinate if* $\tau_c > \overline{\overline{\tau}}_c$. *If* $\tau_c < \underline{\underline{\tau}}_c$, $G^* < 0$ *holds.*

Apparently, $\overline{\overline{\tau}}_c$ increases with $\theta_G$ while $\theta_G$ does not affect $\underline{\underline{\tau}}_c$. As the external effects of $G$ increase, indeterminacy is more likely to occur. In the shaded areas of Figure 6, indeterminacy occurs.[12] As $\theta_G$ increases, the steady state tends to exhibit indeterminacy for wider ranges of tax rates.

[Figures 5 and 6]

The intuition of the above results is as follows. Assume $\tau > 0$ and $\underline{\underline{\tau}}_c < \tau_c < 0$. Suppose that agents expect future public spending to increase. Because of Edgeworth

[12]The parameter values in Figure 6 are $A = 1$, $\alpha = 0.36$, $\delta = 0.1$, $\sigma = 2$, $\rho = 0.05$, $B = 1$, and $\theta_c = 1$. We have $G^* < 0$ in the areas to the left of the shaded areas. In the other regions, the steady state is locally determinate.

substitutability, the future marginal utility of private consumption will decrease. Capital accumulation is discouraged and future output will decrease, which will have negative effects on future public spending. At the same time, the future wage rate decreases and thus, future $\Lambda_t$ increases (see (5b)). Because (5a) must hold in equilibrium, future private consumption will decrease, which will have positive effects on future public spending. When $\tau_c$ is smaller than $\overline{\overline{\tau}}_c(< 0)$, the positive effect on future public spending dominates the negative one.[13] Hence, the expectation is self-fulfilling, and indeterminacy arises.

# 4 Concluding Remarks

We reexamine indeterminacy and utility-generating public spending in a simple one-sector growth model. We allow the government to finance its spending by consumption tax as well as income taxes. Subsidies for savings, labor supply, and consumption are also considered. We show that even when private and public consumption are Edgeworth substitutes and public consumption and leisure are separable in the utility function, indeterminacy can arise with consumption subsidies. Indeterminacy conditions under Edgeworth complementarity between private and public consumption are as follows. (i) Even when the external effect of public spending is not strong, indeterminacy occurs if public spending and subsidies for income are financed by consumption tax. (ii) When the external effect of public spending is strong, indeterminacy does not arise if the revenue from consumption tax is a main source of public spending, whereas it arises if public spending is financed mainly by income taxation. (iii) When the external effect of public spending is strong, as the external effects of public spending become stronger, the economy tends to be more unlikely to exhibit indeterminacy. These results suggest that not only the preference structure but also the designs of fiscal policy are crucial for indeterminacy conditions.

# Appendix

After dividing both sides of (5a) by $\theta_C C^{*\theta_C(1-\sigma)-1} G^{*\theta_G(1-\sigma)} = \Lambda^*(1+\tau_c)$, we take the logarithm to obtain (10a). We can derive (10e) by using (8d) in a similar way. From the

[13]Note $\tau_c < 0$. Then, as $\tau_c$ is smaller, the positive effect tends to dominate the negative one.

definition of $\lambda_t$, we have $\Lambda_t = \Lambda^* e^{\lambda_t}$. Note $\lambda_t = 0$ in the steady state. Around the steady state, $\dot{\Lambda}$ is approximated as $\dot{\Lambda} = \Lambda^* \dot{\lambda}_t$. In the steady state, we have $\partial \Lambda_t / \partial \lambda_t |_{\lambda_t = 0} = \Lambda^*$. These relationships hold for $K_t$, $C_t$, $L_t$, and $G_t$. Then, (8a) is approximated as follows:

$$\Lambda^* \dot{\lambda}_t = \Lambda^* (1-\alpha)(\rho+\delta) \left[ \frac{1}{K^*} \frac{\partial K_t}{\partial k_t}\bigg|_{k_t=0} k_t - \frac{1}{L^*} \frac{\partial L_t}{\partial l_t}\bigg|_{l_t=0} l_t \right] = \Lambda^* (1-\alpha)(\rho+\delta)\left(k_t - l_t\right).$$

In the first equality, we use (9a). Dividing the left- and right-hand sides by $\Lambda^*$ yields (10b). Similarly, (8b) is approximated as

$$K^* \dot{k}_t = \left[ (1-\tau)\alpha A \hat{K}^{\alpha-1} - \delta \right] K^* k_t + (1-\tau)(1-\alpha) A \hat{K}^{\alpha} L^* l_t - (1+\tau_c) C^* c_t.$$

After dividing both sides by $K^*$, we rearrange the resulting equation by using (9a) and (9b) to obtain (10c). Finally, (8c) is approximated as $G^* g_t = \tau \alpha Y^* k_t + \tau(1-\alpha) Y^* l_t + \tau_c C^* c_t$. Dividing both sides by $G^*$ results in (10d).

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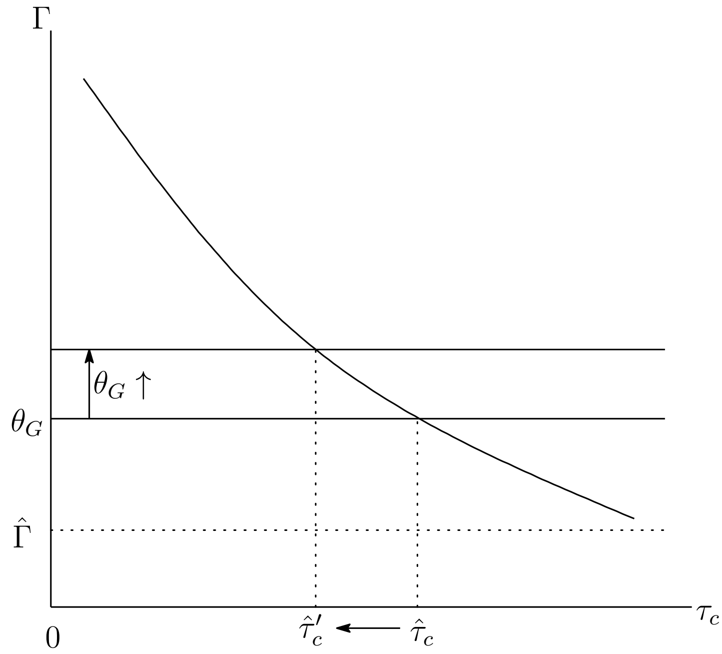


Figure 1: The Case of Edgeworth Complements with Strong Externalities

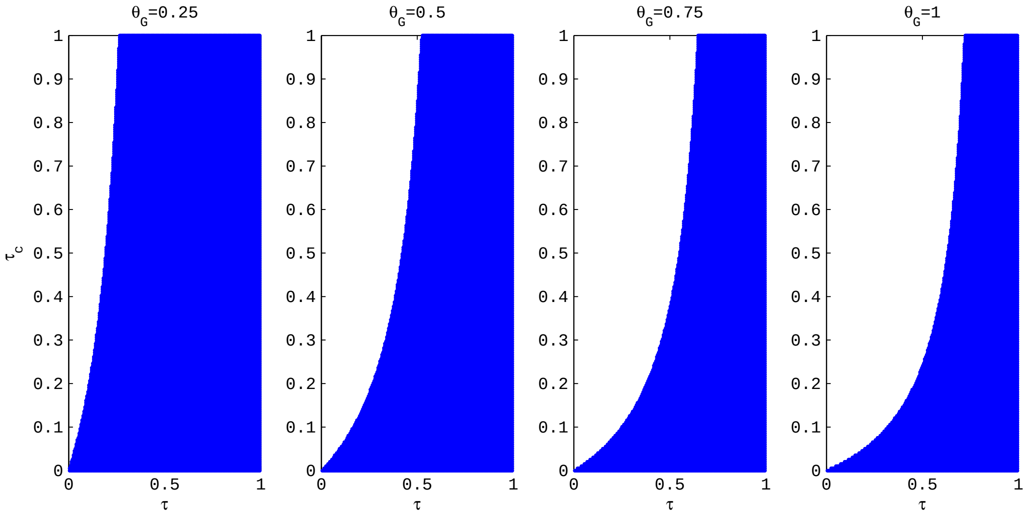


Figure 2: Indeterminacy under Edgeworth Complements with Strong Externalities

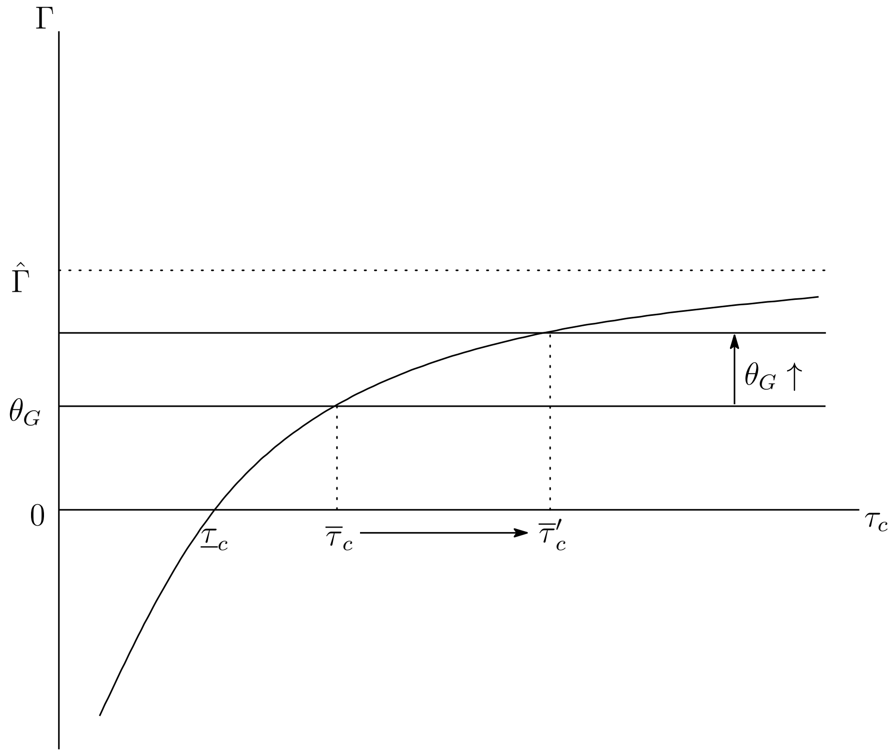


Figure 3: The Case of Edgeworth Complements with Weak Externalities

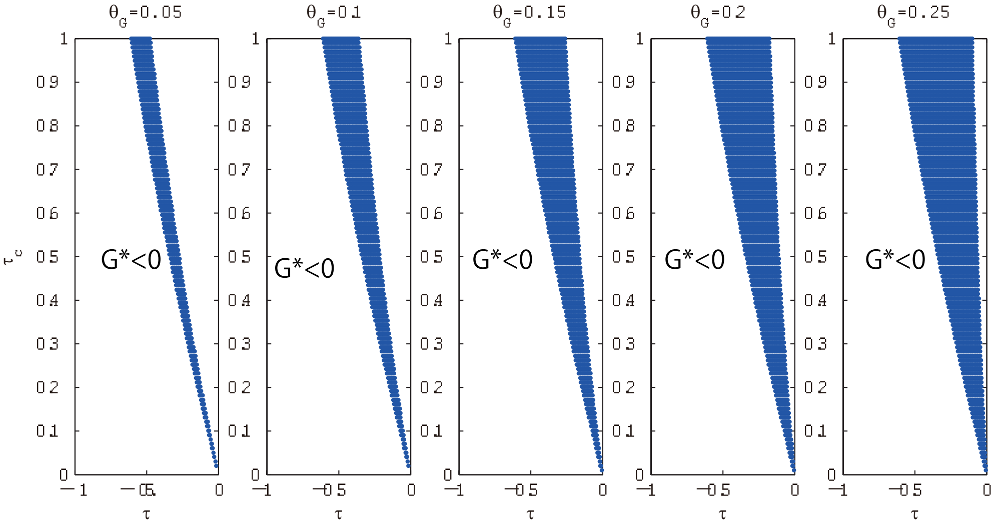


Figure 4: Indeterminacy under Edgeworth Complements with Weak Externalities

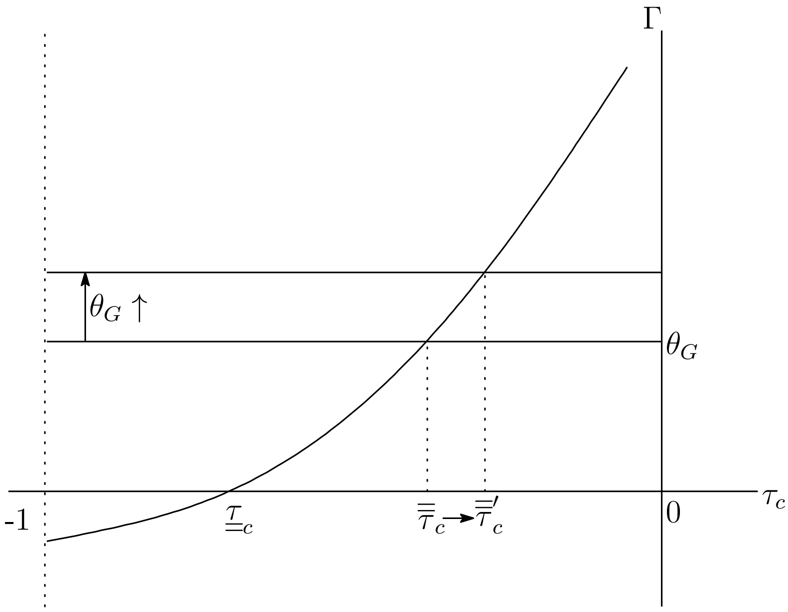


Figure 5: The Case of Edgeworth Substitutes

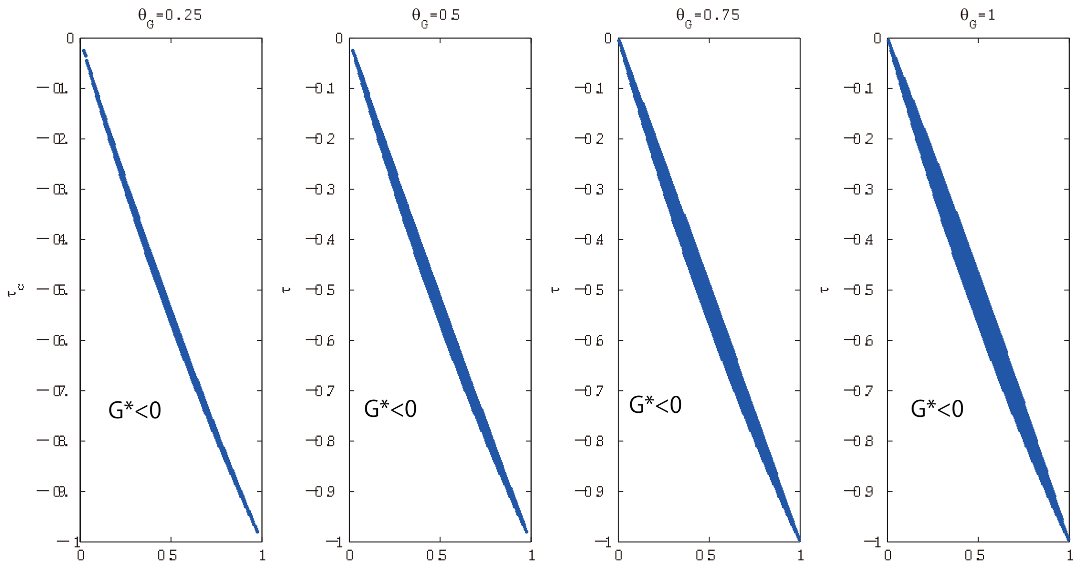


Figure 6: Indeterminacy under Edgeworth Substitutes