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# General Equilibrium Model for an Asymmetric Information Economy without Delivery Upper Bounds

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Abstract

In this paper, we introduce production into the standard general equilibrium model with asymmetric information, which was proposed by Dubey et al. (Cowles Foundation Discussion Paper 2000; Econometrica 2005). In such an economy, there is no rational explanation for producers' delivery upper bounds while the endowments naturally limit consumers' deliveries. However, we show that the typical equilibrium allocation of the asymmetric information economy *necessarily and substantially* depends on such exogenous upper bounds (Example 1 and Theorem 1). In other words, an equilibrium existence theorem without such upper bounds, even if such exists, will typically fail to treat the asymmetric information problem, e.g., the adverse selection problem. Hence, to treat the equilibrium existence problem under the informational asymmetry appropriately, we have to extend the standard model so that the delivery upper bounds need not to be specified explicitly. For this purpose, we propose a quite natural and realistic assumption with respect to the technological condition related to the market delivery, i.e., the existence of some small standardization, commoditization, and/or transaction costs of market deliveries is shown to be sufficient (Theorem 3).

**Keywords**: general equilibrium model, asymmetric information, adverse selection, market viability problem

**JEL Classification Numbers**: C62; D51; D82

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## 1 Introduction

Informational asymmetry problems have been traditionally treated using static partial equilibrium arguments (e.g., Akerlof 1970, Rothschild and Stiglitz 1976). The approaches of Dubey et al. (2005) are groundbreaking general equilibrium treatments for asymmetric information problems.[1] In the context of asset markets, that work discussed how a certain system of *pooled* insurance may solve equilibrium-existence problems in a default economy. In their model, as a buyer, every agent obtains an average receipt (including defaults) in each asset market. As sellers, they can choose not to deliver their full obligations. If we consider a rational expectations equilibrium, the default problem does not harm the existence of a market equilibrium, i.e., the *market viability problem* is solved affirmatively under *seller–buyer informational asymmetry.*

Several researchers have investigated this seller–buyer informational asymmetry using a general competitive equilibrium model. Bisin and Gottardi (1999) considered a similar problem to Dubey et al. (2005) in a probabilistic and dynamic setting. Bisin et al. (2011), Correia-da-Silva (2012), and Meier et al. (2014) most recently considered a *static* setting.[2] Bisin et al. (2011) considered a situation where agents only have finer information (than the market) when they are sellers. Correia-da-Silva (2012) and Meier et al. (2014) investigated a situation where agents also have finer information as buyers and the information varies among agents, although their models do not have the standard general equilibrium setting. In this paper, we adopt the general equilibrium approach of Dubey et al. (2000, 2005) and Bisin et al. (2011).

Bisin et al. (2011) considered a model of an exchange economy where agents (consumers) can make a limited amount of delivery contracts, related to their endowments.[3] However, if we consider a production economy, there is no natural counterpart for these delivery upper bounds for producers and this fact causes some problematic situation for general equilibrium modeling of the economy. In this paper, we show that an equilibrium may not exist if producers have no limit on delivery contracts, even

[1] See also Dubey et al. (2000) and their earlier draft, Dubey et al. (1989).

[2] The authors investigated a static general competitive equilibrium model independently from these recent works (Urai and Yoshimachi 2005).

[3] All of the previously mentioned works also used such upper bounds for delivery contracts.

if consumers do (Example 1); that an equilibrium exists if each agent has an exogenously given upper bound for delivery contracts (Theorem 2); and that, unfortunately, the equilibria typically depend on the exogenous upper bounds if the asymmetric information does actually have an effect (Theorem 1).[4] Theorem 1 implies that we cannot treat asymmetric information problems successfully without using such delivery upper bounds. Therefore, to treat the market viability problem that determines an active market structure under the asymmetric information, appropriately, we must consider the existence problem without explicitly specified upper bounds for the delivery. Hence, to solve this problem, we extended the model by introducing some technological conditions that represent realistic costs related to delivery contracts such as standardization, commoditization, and/or transaction costs. We show that the equilibrium existence problem under asymmetric information is solved affirmatively in this extended model (Theorem 3) and that all the conditions for the market viability problem are endogenized.[5]

The paper is organized as follows. In Section 2, we establish the basic model and present some substantial results for this basic setting. In Section 3, we extend the model and state the existence theorem. All proofs are consigned to Appendix.

# 2 Basic Model and Results

## 2.1 Basic Model

The basic model and its setup are counterparts of Dubey et al. (2005) and Bisin et al. (2011), which we modified for the standard Arrow–Debreu production economy. In the following, we introduce a class of possible markets, agents' optimization problems, and an equilibrium.[6]

[4] Even in exchange economies, as long as consumers can resale their goods arbitrarily, the same problems arise. In particular, we can construct equilibrium non-existence examples of such economies (see footnote 12). Note that our solution to the non-existence problem, explicit modeling of costs that are related to delivery contracts, is relevant also in such pure exchange cases.

[5] We note that the determination problem of an active market structure is simplified here among a given *partition* of the set of real commodities $L = \{1, \ldots, \ell\}$. Extension of this point to more general subclass of $L$, which may include sets with non empty intersection, is straightforward and will be discussed in our another paper (Urai et al. 2016).

[6] Let $R$ denote the set of real numbers, let $R^n$ denote $n$-dimensional Euclidean space, let $R^n_+$ be the non-negative orthant of $R^n$, $\{x = (x_k)_{k=1}^n \in R^n | x_k \geqq 0, k = 1, \ldots, n\}$, and let $R^n_{++}$ be the strictly positive orthant of $R^n$, $\{x = (x_k)_{k=1}^n \in R^n | x_k > 0, k = 1, \ldots, n\}$. For any vectors $x, y \in R^n$, we let $x \geqq y :\Leftrightarrow x - y \in R^n_+$ and $x \gg y :\Leftrightarrow x - y \in R^n_{++}$. For any vector $x \in R^n$, we use notations $x^+ := \max\{x, 0\}$ and $x^- := \max\{-x, 0\}$. Note that for all $x \in R^n$, we have $x = x^+ - x^-$ and $x^+, x^- \in R^n_+$. For each finite set $A$, let $\sharp A$ denote the number of elements in $A$.

**Class of possible markets:** There are $\ell$ types of *real commodities (goods and services)* indexed by $k = 1, \dots, \ell$. Let $L := \{1, 2, \dots, \ell\}$ denote the set of real commodity indices and $\mathcal{M} := \{L_1, \dots, L_\lambda\}$ denote a class of non-empty subset of $L$ such that $L = \bigcup_{\kappa=1}^{\lambda} L_\kappa$. The class $\mathcal{M}$ is interpreted here as the *class of all possible markets* and, by using this, we describe a situation that in market $L_\kappa \in \mathcal{M}$ real commodities $k, k' \in L$ are not distinguished as long as $k, k' \in L_\kappa$. In our model, a *market structure* will be determined as the set of markets in $\mathcal{M}$ with non-zero amount of trade in an equilibrium. We also assume that each commodity is traded in exactly one market, i.e., $L_\kappa \cap L_{\kappa'} = \emptyset$ for all $\kappa, \kappa' \in \{1, \dots, \lambda\}$ with $\kappa \neq \kappa'$. Although we can treat a more general class of possible markets, here we adapt the simplest partitioned case to concentrate our attention on the problem concerning the exogenous delivery upper bounds. Moreover, in most standard economic settings such as *adverse selection*, we only need the partitioned possible markets framework as argued in Bisin et al. (2011). We also note that this partitioned possible markets framework contains the standard Arrow–Debreu economy as a special case with the finest partition: $\{\{1\}, \dots, \{\ell\}\}$.

Since there are only $\lambda$ types of markets indexed by $\kappa = 1, \dots, \lambda$ and the economy evaluates $\lambda$ types of *marketed contracts*, or *standardized commodities*, the price space is a subset of the $\lambda$-dimensional Euclidean space $R^\lambda$, instead of the $\ell$-dimensional one. Let $\Delta := \{(p_1, \dots, p_\lambda) \in R^\lambda_+ \mid \sum_{\kappa=1}^{\lambda} p_\kappa = 1\}$ be the price space. For each market $L_\kappa \in \mathcal{M}$, all types of real commodities $k \in L_\kappa$ are evaluated by identical price $p_\kappa$, given a price system $p = (p_1, \dots, p_\lambda) \in \Delta$.

**Agents:** There are $m$ types of *consumers* and $n$ types of *producers* in this economy and they are indexed by $i = 1, \dots, m$ and $j = 1, \dots, n$, respectively. In our model, we assume that each agent (each consumer and each producer) can buy and sell standardized commodities in all the markets. We also assume that, for all market $L_\kappa \in \mathcal{M}$ and all types of real commodities that belong to $L_\kappa$, every agents can identify each of these commodity types as long as it is possessed by themselves, and use each of them as an unit of delivery to the market $L_\kappa$. However, they can only obtain a mixture of such real commodities when they buy a standardized commodity from the market $L_\kappa$. Hence, we treat their behaviors (consumptions and productions) and their transactions (demands and supplies) as different variables. More precisely, we describe consumption plans $x_i$ $(i = 1, \dots, m)$ and production plans $y_j$ $(j = 1, \dots, n)$ as points of $R^\ell$ and demand plans $z_i^-$ $(i = 1, \dots, m+n)$ and supply plans $z_i^+$ $(i = 1, \dots, m+n)$ as points of $R^\lambda_+$.

**Agents as sellers:** For all agents $i = 1, \ldots, m+n$, we assume that the supply plan $z_i^+ = (z_{i1}^+, \ldots, z_{i\lambda}^+)$ must satisfy

$$(z_{i1}^+, \ldots, z_{i\lambda}^+) = \Big( \sum_{k \in L_1} v_{ik}, \; \ldots, \; \sum_{k \in L_\lambda} v_{ik} \Big),$$

where $(v_{i1}, \ldots, v_{ik}, \ldots, v_{i\ell}) \in R_+^\ell$ is a bundle of real commodities that agent $i$ plans to deliver to the markets. This equation describes the above mentioned situation that agents can identify each of the real commodity types in a market $L_\kappa \in \mathcal{M}$ and use each of them as an unit of delivery to the same market $L_\kappa$, although the market $L_\kappa$ does not distinguish them and hence recognizes the delivery from agent $i$ as the sum $z_{i\kappa}^+ = \sum_{k \in L_\kappa} v_{ik}$.

**Agents as buyers and expectations of real receipts:** As we described above, in this setting, a marketed contract $\kappa$ is actually a mixture of $\sharp L_\kappa$ kinds of real commodities. Therefore, we assume that agents have a certain kind of *expectation for their real receipts.* So we introduce an additional exogenous parameter, $s^\kappa$, which represents the ratio of the $\sharp L_\kappa$ types of real commodities in the total quantity of contracts supplied to market $\kappa$. Formally, for each $\kappa = 1, \ldots, \lambda$, we let $R^{L_\kappa}$ denote the subspace of $R^\ell$ constructed by elements with $k$-th coordinates equal to 0 if $k \notin L_\kappa$; that is, $R^{L_\kappa} := \{ x = (x_k)_{k=1}^\ell \in R^\ell \,|\, x_k = 0 \;\text{ if } k \notin L_\kappa \}$, and we take expectation $s = (s^1, \ldots, s^\lambda)$ as $s^\kappa \in \Delta^\kappa := \{ x = (x_1, \ldots, x_\ell) \in R_+^{L_\kappa} \,|\, \sum_{k=1}^\ell x_k = 1 \}$ for all $\kappa = 1, \ldots, \lambda$. This allows us to parametrize the uncertainty about real receipts in the same way that we treat the prices of goods in general equilibrium modelings.

Next, we describe agents' optimization problems. In this model, agents' optimization problems have two kinds of *macro* parameters: *price* $p = (p_1, \ldots, p_\lambda)$ and *expectations of real receipts* $s = (s^1, \ldots, s^\lambda)$. Note that given these two parameters, which are determined in equilibrium, agents choose *micro* variables: consumption or production plan $x_i/y_j$, demand plan $z_i^-$, and actual delivery of real commodities $v_i$ (which constitutes supply plan $z_i^+$).

**Consumers' problems:** Consumer $i = 1, \ldots, m$ has initial endowment $\omega_i \in R_{++}^\ell$ of real commodities, consumption set $X_i \subset R^\ell$, and utility function $u_i : X_i \to R$. Given price $p \in \Delta$ and the expectation of their receipts for each real commodity through the market, $s = (s^1, \ldots, s^\lambda) \in \prod_{\kappa=1}^\lambda \Delta^\kappa$, consumer $i$ chooses consumption plan $x_i$ with market transaction plans $(v_i, z_i)$, where $z_i = z_i^+ - z_i^-$, to solve the following maximization problem:

$$\max \quad u_i(x_i) \tag{1}$$

sub. to

$$(x_i, v_i, z_i) \in X_i \times R^\ell_+ \times R^\lambda \tag{2}$$

$$v_i \leqq b_i, \tag{3}$$

$$z_i^+ = (\sum_{k \in L_1} v_{ik}, \ldots, \sum_{k \in L_\lambda} v_{ik}), \tag{4}$$

$$x_i + v_i = \omega_i + z_{i1}^- s^1 + \cdots + z_{i\lambda}^- s^\lambda, \tag{5}$$

$$p \cdot z_i^- = p \cdot z_i^+ + \sum_{j=1}^n \theta_{ij} \pi_j(p, s), \tag{6}$$

where $\pi_j(p, s)$ is the profit of producer $j$ under price $p$ and expectation $s$ (in the maximization problems described below), $\theta_{ij}$ denotes $i$'s share of the profit of producer $j$ (a non-negative real number satisfying $\sum_{i=1}^m \theta_{ij} = 1$ for each $j$), and $b_i \in R^\ell_{++}$ is an arbitrary taken upper bound for the consumer's delivery. For example, it is natural to take $b_i = \omega_i$ for all $i = 1, \ldots, m$.

The following are the interpretations of the constraints. Formula (2) indicates the domains of the variables; consumption plan $x_i$ must be taken from consumption set $X_i$. Eq. (3) expresses that none of the consumer $i$ can supply arbitrarily large amounts of real commodities. Eq. (4) means that supply plan $z_i^+$ consists of the delivery of real commodity $v_i$. Eq. (5) denotes that consumption $x_i$ and actual delivery $v_i$ must be covered by the consumer's own endowments $\omega_i$ and purchase $z_{i1}^- s^1 + \cdots + z_{i\lambda}^- s^\lambda$. Note that $z_{i\kappa}^- s^\kappa \in R^\ell_+$ for $\kappa = 1, \ldots, \lambda$ because $s^\kappa \in \Delta^\kappa \subset R^\ell_+$. Finally, Eq. (6) is the budget constraint under $p \in \Delta$.

In the above, we can chose $\omega_i$ as a natural candidate of upper bound $b_i$ in the constraint (3). This is a natural assumption when we only consider the consumers in an economy, i.e., when we consider a pure exchange economy, and such an assumption was also used in all of the early works: Dubey et al. (2005), Bisin et al. (2011), Correia-da-Silva (2012), and Meier et al. (2014). For a production economy, however, since producers do not have endowments, they have no such natural upper bounds. However, Example 1 below shows that without upper bounds for the producers, an equilibrium may fail to exist even if consumers' delivery amounts are bounded, as in Formula (3) with $b_i = \omega_i$. For this reason, in the basic model, we also introduce the upper bound condition (Eq. (9)) for each producer using given upper bound parameter $b_j \in R^\ell_{++}$ although it does not have a natural interpretation.

**Producers' problems:** Producer $j = 1, \dots, n$ has production technology $Y_j \subset R^\ell$. Given two exogenous parameters, price $p$ and the expectations of their real receipts, $s = (s^1, \dots, s^\lambda)$, producer $j$ chooses production plan $y_j$ with market transaction plans $(v_j, z_j)$ to solve the following maximization problem:

$$\max \quad p \cdot z \tag{7}$$

sub. to

$$(y_j, v_j, z_j) \in Y_j \times R^\ell_+ \times R^\lambda, \tag{8}$$

$$v_j \leqq b_j, \tag{9}$$

$$z_j^+ = \Big( \sum_{k \in L_1} v_{jk}, \dots, \sum_{k \in L_\lambda} v_{jk} \Big), \tag{10}$$

$$v_j = y_j + z_{j1}^- s^1 + \cdots + z_{j\lambda}^- s^\lambda. \tag{11}$$

Constraints (8)–(11) can be interpreted in the same way as in the consumers' problems. In Formula (9), we use $b_j \in R^\ell_{++}$ as an exogenously given upper bound, as previously discussed.

Finally, we define the equilibrium.

**Equilibrium:** Let $\mathcal{E} = ((X_i, \omega_i, u_i, b_i, (\theta_{ij})_{j=1}^n)_{i=1}^m, (Y_j, b_j)_{j=1}^n, \mathcal{M})$ denote the above economy. An *equilibrium* for economy $\mathcal{E}$ is a pair $((x_i, v_i, z_i)_{i=1}^m, (y_j, v_j, z_j)_{j=1}^n) \in \prod_{i=1}^m (X_i \times R^\ell_+ \times R^\lambda) \times \prod_{j=1}^n (Y_j \times R^\ell_+ \times R^\lambda)$ and $(p, s) \in \Delta \times \prod_{\kappa=1}^\lambda \Delta^\kappa$, which satisfies Eqs. (1)–(11) and the *market clearing condition* (12) with *expectation specification* (13) for each $\kappa \in \{1, \dots, \lambda\}$:

$$\sum_{i=1}^{m+n} z_{i\kappa} = 0, \tag{12}$$

$$\frac{\sum_{i=1}^{m+n} \mathrm{pr}_{L_\kappa}(v_i)}{\sum_{i=1}^{m+n} z_{i\kappa}^+} = s^\kappa \quad \text{as long as} \quad \sum_{i=1}^{m+n} z_{i\kappa}^+ > 0, \tag{13}$$

where $\mathrm{pr}_{L_\kappa}$ denotes the projection onto subspace $R^{L_\kappa}$ of $R^\ell$ for each $\kappa = 1, \dots, \lambda$, i.e., for each $x \in R^\ell$, the $k$-th coordinate of $\mathrm{pr}_{L_\kappa}(x)$ is 0 if $k \notin L_\kappa$ and $x_k$ if $k \in L_\kappa$. Note that we only consider Eq. (13) when $\sum_{i=1}^{m+n} z_{i\kappa}^+ > 0$. Hence, if $\sum_{i=1}^{m+n} z_{i\kappa}^+ = 0$ we have no restrictions on the expectation specifications.

## 2.2 Results for the Basic Model

As previously mentioned, the upper bound condition (9) is problematic because it is difficult to interpret and/or to justify it from a natural economic point of view. However, as shown in the next theorem and the non-existence example, the condition is not only necessary but also essential to analyze asymmetric information problems. More precisely, an equilibrium allocation of the model typically depends on the upper bound parameters, $b_i$, as long as we want to analyze a situation where the asymmetric informational problem actually matters. In other words, we can prove that if an equilibrium for this standard setting does not depend on the upper bound parameters, the equilibrium allocation is typically *Pareto-optimal.*[7] To state this fact formally, we introduce some standard definitions.

We say that an *allocation* $((x_i)_{i=1}^m, (y_j)_{j=1}^n)$, where $x_i \in X_i$ $(i = 1, \ldots, m)$ and $y_j \in Y_j$ $(j = 1, \ldots, n)$, is *feasible* if it satisfies $\sum_{i=1}^m x_i = \sum_{j=1}^n y_j + \sum_{i=1}^m \omega_i$. Feasible allocation $((x_i)_{i=1}^m, (y_j)_{j=1}^n)$ is *Pareto-optimal* if there is no other feasible allocation $((x'_i)_{i=1}^m, (y'_j)_{j=1}^n)$ such that $x'_i \succsim_i x_i$ for all $i = 1, \ldots, m$ and $x'_i \succ_i x_i$ for some $i = 1, \ldots, m$, where $\succsim_i$ and $\succ_i$ are respectively agent $i$'s preference and strict preference relations.

**Theorem 1.** *Suppose that, for each $i = 1, \ldots, m$, consumer $i$'s preference $\succsim_i$ is monotone and convex.*[8] *If equilibrium state $((x_i^*, v_i^*, z_i^*)_{i=1}^m,\ (y_j^*, v_j^*, z_j^*)_{j=1}^n, p^*, s^*)$ satisfies $v_i^* \ll b_i$ for all $i = 1, \cdots, m+n$ and $s^{\kappa *} \in R_{++}^{L_\kappa}$ for all $\kappa = 1, \cdots, \lambda$, then equilibrium allocation $((x_i^*)_{i=1}^m, (y_j^*)_{j=1}^n)$ is Pareto-optimal.*

Hence, in economies with asymmetric information, where Pareto-optimality typically does not hold, there must exist some agent $i$ that delivers real commodity $k$ with its limit amount $b_{ik}$ or real commodity $k'$ that is vanishing from the market, i.e., $s_{k'}^\kappa = 0$. In this sense, to treat the asymmetric information problem such as the adverse selection problem in the standard setting, we cannot avoid the arbitrariness caused by exogenous upper bound parameters $b_i$.[9]

[7] Suppose that the asymmetric information is based on the different quality of commodities in a certain standardized market. Assuming that each agent's expectation for a standardized commodity typically satisfies the condition that for each delivery of the standardized commodity there is a non-zero possibility to obtain all real commodities that belong to this market, then an equilibrium, which does not depend on the delivery upper bound constraints, is always Pareto-optimal.

[8] Preference relation $\succsim$ is *monotone* if $x \gg y$ implies $x \succ y$. Preference relation $\succsim$ is *convex* if $x \succ y$ implies $tx + (1-t)y \succ y$ for all $t \in (0, 1]$. (For the definition of a preference relation's convexity, see also Debreu (1959).)

[9] Note that here, the situation is different from the one concerning the short selling upper bound in the incomplete

Furthermore, as we noted above, we cannot even ensure the equilibrium existence without the upper bound condition (9). In the following, we describe a non-existence example. The setting is the same as for the existence theorem (Theorem 2 below), but without the upper bound condition for the producer (Condition (9)). Hence, without the upper bounds for the producers, an equilibrium may not exist even if the consumer's delivery amount is bounded, as in Formula (3) with $b_i = \omega_i$.

**Example 1: Non-existence of equilibrium without an upper bound**

Suppose we have two real goods, one consumer, and one producer in an economy. Additionally, suppose there is only one market, and hence the two goods are traded at an identical price: $p \in R_+$. These agents expect that they will receive $s \in [0, 1]$ units of good 1 and $(1 - s)$ units of good 2 when they contract to buy one unit from the market. The producer has technology $Y = \{(y_1, -y_2) \in R^2 : y_1 \leqq 2y_2, y_2 \geqq 0\}$, and *there is no upper bound on its delivery.* (Hence this producer's problem is specified by Eqs. (7), (8), (10), and (11)). The consumer has consumption set $X = R^2_+$, utility function $u(x_1, x_2) = 2x_1 + x_2$, and endowment $(1, 1)$, and his actual delivery amounts must be bounded by his endowments. (This problem is specified by Eqs. (1)–(6) with $b_i = \omega_i = (1, 1)$. ) In this economy, the consumer prefers real good 1 to good 2, and real good 2 is the raw material for the producer. Let $(v_{p1}, v_{p2}, z_p^{\pm})$ and $(v_{c1}, v_{c2}, z_c^{\pm})$ denote the producer's and consumer's transaction plans.

First, note that price $p = 0$ never constitutes an equilibrium, because the consumer can make infinite purchases; i.e., the consumer problem has no maximum. Next, note that for any price $p > 0$, the producer's problem is reduced to $\max\{py_2 | (1 - s)z_p^- \geqq y_2 \geqq 0 \text{ and } z_p^- \geqq 0\}$, and the producer's actual deliveries are characterized by $(v_{p1}, v_{p2}) = (2y_2 + sz_p^-, -y_2 + (1 - s)z_p^-)$.[10]

If $p > 0$ and $s \neq 1$, then the producer can get his raw material (real good 2) from the market and make

market model (Hart 1975). In the incomplete market model, it is just desirable from an economic modeling point of view to remove the short selling upper bound, whereas in the model of an asymmetric information economy, the removal of the upper bound implies the removal of the asymmetric information problem itself, as shown by Theorem 1.

[10] To see that the original problem described by Eqs. (7), (8), (10), and (11) can be reduced to the stated problem with two variables ($y_2$ and $z_p^-$), we eliminate other variables as follows. First, we can eliminate $v_{p1}$ and $v_{p2}$ by substituting $v_{p1} = y_1 + sz_p^-$ and $v_{p2} = -y_2 + (1-s)z_p^-$ into all the relevant restrictions (i.e., substituting Eq. (11) into Eqs. (8) and (10)). We can also eliminate $z_p^+$ by substituting Eq. (10) into the profit function of Eq. (7). Finally, if $p > 0$, then $y_1 = 2y_2$ must hold in an optimum, so we can also eliminate $y_1$ by substituting it into all the relevant restrictions and the profit function. Then the reduced problem is actually $\max\{py_2 | (1 - s)z_p^- \geqq y_2 \geqq 0 \text{ and } z_p^- \geqq 0\}$, and this problem is equivalent to the original problem. The actual deliveries are characterized by $(v_{p1}, v_{p2}) = (2y_2 + sz_p^-, -y_2 + (1-s)z_p^-)$ by substituting the equation $y_1 = 2y_2$ to eliminate $y_1$.

twice as much product (real good 1) as raw material. Moreover, the prices of these two goods are the same, $p > 0$, and *there is no upper bound for his delivery*, so the producer can make an infinite amount of profit. Indeed, if $s \neq 1$, then the reduced problem has no maximum because $y_2$ can be made arbitrarily large by taking $z_p^-$ such that $z_p^- \geqq \frac{y_2}{1-s}$. Hence, an equilibrium only exists when $p > 0$ and $s = 1$.

If $p > 0$ and $s = 1$, then the producer expects that he cannot get raw material for production. Hence, no-production is his optimal production plan. The reduced problem has maximum solutions ($y_2 = 0$, $z_p^- =$ arbitrary) with maximum profit $\pi = 0$, and the actual deliveries are $(v_{p1}, v_{p2}) = (z_p^-, 0)$.[11] However, the consumer wants to exchange endowed good 2 with more preferable good 1 through the market, because he only expects to get good 1 when he purchases it from the market, $s = 1$. So in the optimum, $v_{c2} > 0$ must hold. Hence, there is a positive delivery of commodity 2, while the agents expect no delivery for good 2, $(1 - s) = 0$. Therefore, Eq. (13) never holds. This means that if $p > 0$ and $s = 1$, there is never an equilibrium. Consequently, no equilibrium exists in this economy.[12] □

Now we state a general equilibrium existence theorem for production economies with asymmetric information. We can ensure that an equilibrium exists as long as all agents have upper bounds for their deliveries.

**Theorem 2.** *Economy* $\mathcal{E} = ((X_i, \omega_i, u_i, b_i, (\theta_{ij})_{j=1}^n)_{i=1}^m, (Y_j, b_j)_{j=1}^n, \mathcal{M})$ *has an equilibrium,* $((x_i^*, v_i^*, z_i^*)_{i=1}^m, (y_j^*, v_j^*, z_j^*)_{j=1}^n, p^*, s^*)$, *if the following conditions are satisfied:*

> **(Consumers)** *Each consumer* $i = 1, \ldots, m$ *has* (i) *a closed convex consumption set,* $X_i \supset R_+^\ell$, *bounded from below;* (ii) *a convex preference induced by a strictly monotone and continuous utility function* $u_i : X_i \to R$; *and* (iii) *endowment* $\omega_i \in \mathrm{int} X_i$.[13]
>
> **(Producers)** *For each* $j = 1, \ldots, n$, $Y_j \subset R^\ell$ *is a closed convex set that contains* $0$.
>
> **(Attainable Set)** *The attainable sets for each agents are bounded.*[14]

[11] Here we observe that the producer's constraint correspondence does not satisfy upper semicontinuity at $s = 1$.

[12] Note that even in exchange economies, the same problem arises if consumers can resale their goods arbitrarily. Suppose two goods, one market, two consumers, and no producer in an economy. The two goods are traded at identical price $p \in R_+$. Suppose that consumers can resale their goods arbitrarily, i.e., consumers' problems are specified by Eqs. (1), (2), (4), (5) and (6). Assume that consumer 1's attributes are $X_1 = R_+^2$, $u_1(x_1, x_2) = 2x_1 + x_2$, and $\omega_1 = (1, 2)$, and that consumer 2's attributes are $X_2 = R_+^2$, $u_2(x_1, x_2) = x_1 + 2x_2$ and $\omega_2 = (1, 1)$. It is straightforward to see that no equilibrium exists in this economy because they always can get a more preferable consumption bundle through the unbounded transactions.

[13] Preference relation $\succsim$ is *strictly monotone* if $x' \geqq x$, $x' \neq x$ implies $x' \succ x$.

[14] Allocation $((x_i)_{i=1}^m, (y_j)_{j=1}^n)$ is *attainable* if $\sum_{i=1}^m x_i \leqq \sum_{j=1}^n y_j + \sum_{i=1}^m \omega_i$. The *attainable sets* for agents, $i =$

Although this theorem ensures the equilibrium existence, as shown in Theorem 1, the equilibrium typically depends on exogenous upper bounds $b_i$, and their removal induces the removal of the asymmetric information problem itself. Moreover, as shown in Example 1, we cannot even technically remove the upper bound condition because an equilibrium may not exist. In the next section, we endogenize this upper bound condition by introducing natural cost for the market deliveries and solve this problem.

## 3 Model without Exogenous Upper Bounds and Equilibrium Existence

Given the class of possible markets $\mathcal{M}= \{L_1, \dots, L_\lambda\}$, each agent must sell his real goods as $\lambda$ types of market contracts. Therefore, he should standardize or commoditize his real goods to sell them in the market depending on the class of possible markets $\mathcal{M}$. It is quite natural to assume that this procedure will cost each agent some loss of goods or services that may be delivered to the market, because it will cost each agent at least some labor to standardize, package, and/or deliver real commodities to the market, which has the class of possible markets. We describe such standardization, commoditization, and/or transaction costs by assuming that each agent has a certain kind of technology for standardizing his real goods.

Assume that each agent $i = 1, \dots, n+m$ (each of consumers and producers) has function $F_i : R^\ell_+ \to R^\ell_+$ that satisfies the following conditions:

(C1) $F_i$ is a continuous function;

(C2) for all $k = 1, \dots, \ell$, $F_{ik}$ is a concave function, where $F_{ik}$ is a $k$-th coordinate of $F_i$;

(C3) for each $\kappa = 1, \dots, \lambda$ and $v \in R^\ell_+$, $\sum_{k\in L_\kappa} F_{ik}(v) = 0$ is equivalent to $v_{L_\kappa} := (v_k)_{k\in L_\kappa} = 0$;

for each $\kappa = 1, \dots, \lambda$ and $v, v' \in R^\ell_+$, $v_{L_\kappa} \leqq v'_{L_\kappa}$ and $v_{L_\kappa} \neq v'_{L_\kappa}$ implies $\sum_{k\in L_\kappa} F_{ik}(v) < \sum_{k\in L_\kappa} F_{ik}(v')$;

(C4) $F_i(v) \leqq v$ for all $v \in R^\ell_+$; and

(C5) for each $\kappa$ such that $L_\kappa = \{k\}$ for some $k$, $F_{ik}(v) = v_k$;

for each $\kappa$ such that $\sharp L_\kappa \geqq 2$, and for each sequence $\{v^\nu\}_{\nu=1}^\infty \subset R^\ell_+$, if $\sum_{k\in L_\kappa} F_{ik}(v^\nu) \to \infty$

$1, \dots, n + m$, are defined as $\tilde{X}_i := \{x_i \in X_i | x_i \text{ constitutes an attainable allocation with some } x_{i'} \in X_{i'}\ (\forall i' \neq i), y_j \in Y_j\ (\forall j = 1, \dots, m)\}$, and $\tilde{Y}_j := \{y_j \in Y_j | y_j \text{ constitutes an attainable allocation with some } x_i \in X_i\ (\forall i = 1, \dots, n), y_{j'} \in Y_{j'}\ (\forall j' \neq j)\}$.

$(\nu \to \infty)$, then there exists some $L_{\kappa'}$ such that $\sum_{k \in L_\kappa \cup L_{\kappa'}} (v_k^\nu - F_{ik}(v^\nu)) \to \infty$ $(\nu \to \infty)$.

As we will formalize later, each agent supplies his real goods and services to the market with the class of possible markets $\mathcal{M} = \{L_1, \dots, L_\lambda\}$ as a $\lambda$-tuple $(\sum_{k \in L_1} F_{ik}(v), \dots, \sum_{k \in L_\lambda} F_{ik}(v)) \in R_+^\lambda$ (see Eqs. (16) and (20) below). We call $F_i$ agent $i$'s *standardizing technology.* Condition (C5) means that no agent can sell his real goods in the market with the class of possible markets $\mathcal{M} = \{L_1, \dots, L_\lambda\}$ at no cost, and this is the crucial assumption as we see in the following example.

**Example 1 revisited: Non-existence of equilibrium without the cost condition (C5)**

Recall that there are two goods, one market, one consumer, and one producer in Example 1. Note that in Example 1, only the consumer has an upper bound for delivery contract. Consider a model without the upper bound condition for each agents by removing consumer's upper bound in Example 1. In such a model, each agent's "standardizing technology" can be formalized as $(F_{i1}(v_1, v_2), F_{i2}(v_1, v_2)) := (v_1, v_2)$ and this formalization means that agents can standardize their goods at no cost.[15] Formally, these functions $F_i : R_+^2 \to R_+^2$ satisfy all the conditions but (C5). Indeed, for each sequence $\{v^\nu\}_{\nu=1}^\infty \subset R_+^2$, we have $\sum_{k \in \{1,2\}} (v_k^\nu - F_{ik}(v^\nu)) = \sum_{k \in \{1,2\}} (v_k^\nu - v_k^\nu) = 0$ for all $\nu = 1, 2, \dots$, and hence condition (C5) must be violated. (Note that there is only one market $L_1 := \{1, 2\}$ in this economy.) Moreover, by a similar argument in Example 1, it is straightforward to see that there is no equilibrium in this model, i.e., we cannot ensure the existence of an equilibrium without condition (C5). □

Note that the cost imposed by condition (C5) of standardizing technologies $F_i$ (or, the transaction cost) for consumers and producers $(i = 1, \dots, m + n)$ and the transformation cost represented by production technologies $Y_j$ for producers $(j = 1, \dots, n)$ are two independent concepts, and in general we specify these two independently. In particular, in Example 1, while the transformation cost represented by production set $Y = \{(y_1, -y_2) \in R^2 : y_1 \leqq 2y_2, y_2 \geqq 0\}$ is positive, the transaction cost represented by $F_i$ is zero since $\sum_{k \in \{1,2\}} (v_k^\nu - F_{ik}(v^\nu)) = \sum_{k \in \{1,2\}} (v_k^\nu - v_k^\nu) = 0$ and because of the latter fact we have no equilibrium. In the following, we show that by introducing the transaction costs expressed by (C5), we can ensure the equilibrium existence.[16]

[15] Note that with this notation and the agents' attributes given in Example 1, the model specified here is identical with a model specified by Eqs. (14)–(22) that define our extended model below.

[16] Also, note that (C5) requires such standardizing costs only if $L_\kappa$ is not a singleton set because if some $L_\kappa$ is a singleton,

Now we propose the following modifications to the model. The formalization is almost the same as the basic model in Eqs. (1)–(11), except that we remove the upper bound conditions (3) and (9) and replace (4) and (10) by "$z_i^+ = (\sum_{k\in L_1} F_{ik}(v_i),\ \dots,\ \sum_{k\in L_\lambda} F_{ik}(v_i))$".

**Agents' Problems:** The producers' problems are defined as:

$$\max \quad p\cdot z_j^+ - p\cdot z_j^- \tag{14}$$

sub. to

$$(y_j, v_j, z_j) \in Y_j \times R_+^\ell \times R^\lambda, \tag{15}$$

$$z_j^+ = \Big(\sum_{k\in L_1} F_{jk}(v_j),\ \dots,\ \sum_{k\in L_\lambda} F_{jk}(v_j)\Big), \tag{16}$$

$$v_j = y_j + z_{j1}^- s^1 + \cdots + z_{j\lambda}^- s^\lambda. \tag{17}$$

Similarly, the consumers' problems are defined as:

$$\max \quad u_i(x_i) \tag{18}$$

sub. to

$$(x_i, v_i, z_i) \in X_i \times R_+^\ell \times R^\lambda, \tag{19}$$

$$z_i^+ = \Big(\sum_{k\in L_1} F_{ik}(v_i),\ \dots,\ \sum_{k\in L_\lambda} F_{ik}(v_i)\Big), \tag{20}$$

$$x_i + v_i = \omega_i + z_{i1}^- s^1 + \cdots + z_{i\lambda}^- s^\lambda, \tag{21}$$

$$p\cdot z_i^- = p\cdot z_i^+ + \sum_{j=1}^n \theta_{ij}\pi_j(p,s). \tag{22}$$

**Equilibrium:** Let $\mathcal{E} = ((X_i, \omega_i, u_i, F_i, (\theta_{ij})_{j=1}^n)_{i=1}^m, (Y_j, F_j)_{j=1}^n, \mathcal{M})$ denote the above economy. An *equilibrium* for economy $\mathcal{E}$ is a pair $((x_i, v_i, z_i)_{i=1}^m, (y_j, v_j, z_j)_{j=1}^n)$ and $(p, s)$, satisfying (14)–(22) and the *market clearing condition* (23) with an *expectation specification* (24) for each $\kappa \in \{1, \dots, \lambda\}$ and $k \in L_\kappa$:

$$\sum_{i=1}^{m+n} z_{i\kappa} = 0, \tag{23}$$

$$\frac{\sum_{i=1}^{m+n} F_{ik}(v_i)}{\sum_{i=1}^{m+n} z_{i\kappa}^+} = s_k^\kappa \ \text{ as long as } \sum_{i=1}^{m+n} z_{i\kappa}^+ > 0. \tag{24}$$

We state the existence theorem for this modified economy.

then there is no difference between a real good and a standardized commodity in the corresponding market, and hence, in such cases, the argument about standardizing costs can be negligible. Condition (C5) expresses this point using the case $\sharp L_\kappa = 1$, i.e., $L_\kappa = \{k\}$ for some $k$.

**Theorem 3.** *Economy* $\mathcal{E} = ((X_i, \omega_i, u_i, F_i, (\theta_{ij})_{j=1}^n)_{i=1}^m, (Y_j, F_j)_{j=1}^n, \mathcal{M})$ *has an equilibrium,* $((x_i^*, z_i^*, v_i^*)_{i=1}^m, (y_j^*, z_j^*, v_j^*)_{j=1}^n, p^*, s^*)$*, if the following conditions are satisfied.*

> **(Consumers)** *Each consumer* $i = 1, \dots, m$ *has a non-empty closed convex consumption set* $X_i \supset R_+^\ell$ *that is bounded from below with a convex preference induced by a strictly monotone and continuous utility function* $u_i : X_i \to R_+$*, and initial endowment* $\omega_i \in \mathrm{int} X_i$*.*
>
> **(Producers)** *For each* $j = 1, \dots, n$*,* $Y_j \subset R^\ell$ *is a closed convex set containing* 0*.*
>
> **(Attainable Set)** *The attainable sets for all agents* ($\tilde{X}_i$ $(i = 1, \dots, m)$ *or* $\tilde{Y}_j$ $(j = 1, \dots, n)$ ) *are bounded.*[17]
>
> **(Standardizing Technologies)** *For each agent* $i = 1, \dots, m+n$ $F_i : R_+^\ell \to R_+^\ell$ *satisfies conditions (C1) – (C5).*

As shown by Theorem 3, with standardizing technologies $F_i : R_+^\ell \to R_+^\ell$ $(i = 1, \dots, n+m)$, we can remove the exogenous upper bound conditions (3) and (9). Agents choose to deliver $v_i$ so that the standardizing cost represented by $F_i : R_+^\ell \to R_+^\ell$ never harms their own total payoff. Note that the assumptions on production structure are quite standard as a general equilibrium economy of the Arrow–Debreu type. Therefore, it would be possible to interpret our existence result as follows: we can dispense with concerns over the production structure or the delivery upper bounds as long as we impose restrictions on the structure of standardization, commoditization, and/or transaction costs.

# Appendix

## Proof of Theorem 1

Suppose, on the contrary, that there is an allocation $((x_i)_{i=1}^m, (y_j)_{j=1}^n)$ which Pareto-dominates $((x_i^*)_{i=1}^m, (y_j^*)_{j=1}^n)$. Take $(v_i, z_i) \in R_+^\ell \times R^\lambda$ $(i = 1, \cdots, m+n)$ to satisfy conditions (4), (5), (10), and (11) with allocation $x_i$ or $y_i$.[18] Note, here, that (3) and/or (9) are not necessarily satisfied. Since we assume that $v_i^* \ll b_i$ for all $i = 1, \cdots, m+n$, we can construct state $((\bar{x}_i, \bar{v}_i, \bar{z}_i)_{i=1}^m, (\bar{y}_j, \bar{v}_j, \bar{z}_j)_{j=1}^n)$ such

[17] For the definitions of $\tilde{X}_i$ and $\tilde{Y}_j$, see footnote 14 of Theorem 2.

[18] Since we assume $s^{\kappa *} \in R_{++}^{L_\kappa}$ for all $\kappa = 1, \cdots, \lambda$, we can take $z_i^-$ and $v_i$ to satisfy condition (5) (or (11)) for each $i = 1, \cdots, m+n$. Then, take $z_i^+$ to satisfy (4) (or (10)).

that $((\bar{x}_i)_{i=1}^m, (\bar{y}_j)_{j=1}^n)$ is feasible and Pareto-dominates the equilibrium allocation and satisfies conditions (3), (4), (5), (9), (10), and (11). (For example, if we denote $(\bar{x}_i, \bar{v}_i, \bar{z}_i) = (1-t)(x_i^*, v_i^*, z_i^*) + t(x_i, v_i, z_i)$ and let $t \to 0$, then $\bar{v}_i \ll b_i$, in particular condition (3) (or (9)), holds for sufficiently small $t$. By taking a sufficiently small and identical $t$ for all agents $i = 1, \cdots, m+n$ we can construct a feasible state $((\bar{x}_i)_{i=1}^m, (\bar{y}_j)_{j=1}^n)$ that satisfies these conditions. Pareto-dominance is particularly preserved since each consumer's preference is convex.)

Note that the following two conditions hold:

$$\bar{x}_i \succ_i x_i^* \Longrightarrow p^* \cdot \bar{z}_i^- > p^* \cdot \bar{z}_i^+ + \sum_{j=1}^n \theta_{ij} \pi_j(p^*, s^*) \tag{25}$$

$$\bar{x}_i \succsim_i x_i^* \Longrightarrow p^* \cdot \bar{z}_i^- \geqq p^* \cdot \bar{z}_i^+ + \sum_{j=1}^n \theta_{ij} \pi_j(p^*, s^*) \tag{26}$$

(where $\pi_j(p^*, s^*)$ is the maximized profit of producer $j = 1, \cdots, n$ under $(p^*, s^*)$) since, if not, we have a contradiction to the fact that $x_i^*$ is an utility maximizing consumption plan under $(p^*, s^*)$, $s^{\kappa *} \in R_{++}^{L_\kappa}$ for all $\kappa$, and each consumer's preference is monotone. Hence, the Pareto-dominance of $((\bar{x}_i)_{i=1}^m, (\bar{y}_j)_{j=1}^n)$ with (25), (26), and the profit maximization assumption of the equilibrium state implies that $\sum_{i=1}^m p^* \cdot \bar{z}_i > \sum_{j=1}^n p^* \cdot \bar{z}_j$ holds. We can rewrite this inequality as follows:[19]

$$\sum_{i=1}^m \Big\{ \sum_{k \in L_1} p_1^*(\omega_{ik} - \bar{x}_{ik}) + \cdots + \sum_{k \in L_\lambda} p_\lambda^*(\omega_{ik} - \bar{x}_{ik}) \Big\} > \sum_{j=1}^n \Big\{ \sum_{k \in L_1} p_1^* \bar{y}_{jk} + \cdots + \sum_{k \in L_\lambda} p_\lambda^* \bar{y}_{jk} \Big\}. \tag{27}$$

However, (27) contradicts the fact that $((\bar{x}_i)_{i=1}^m, (\bar{y}_j)_{j=1}^n)$ is feasible. Indeed, if we define "extended price" $\bar{p} = (\bar{p}_1, \ldots, \bar{p}_\ell) \in R^\ell$ as $\bar{p}_k = p_\kappa^*$ if $k \in L_\kappa$ $(k = 1, \ldots, \ell,\ \ \kappa = 1, \ldots, \lambda)$ and evaluate feasibility condition $\sum_{i=1}^m (\bar{x}_i - \omega_i) = \sum_{j=1}^n \bar{y}_j$ by extended price $\bar{p}$, then (27) must hold with equality.

## Proof of Existence Theorems

Here, we make some notes regarding the proof of Theorem 2. Theorem 2 corresponds to the case $F_i(v_i) = v_i$ for all $v_i \in \{v \in R_+^\ell | v \leqq b_i\}$ in the extended standardizing technology setting of Section 3. This $F_i$ satisfies (C1)–(C4) on the compact domain $\{v \in R_+^\ell | v \leqq b_i\}$. Although it does not satisfy (C5), this assumption is not needed if the domain of $F_i$ is compact on $R^\ell$. (Footnote 22 clarifies this point.)

[19] From conditions (4), (5), (10), and (11).

Now, we proceed to the proof of Theorem 3. There are some difficulties related to the agents' problems (14)–(22). In particular, we cannot ensure that the constraint correspondences satisfy upper and lower semicontinuity and have convex values. More specifically, first, the difficulty related to the upper semi-continuity is due to non-bounded deliveries, and was observed in Example 1. Next, equality constrains (17), (21), and (22) cause some difficulties in showing the lower semi-continuity with respect to the vector valued parameters $s = (s_1, \dots, s_\lambda)$. Finally, equality constraints (16) and (20) create problems when ensuring the convexity of the values of the constraint correspondences, because $F_i$ $(i = 1, \dots, n+m)$ are generally not linear.

To avoid these difficulties, we first modify the original problems in two ways; we truncate the variables and relax the equality constraints (in **Producers' Problems** and **Consumers' Problems**). Then, we obtain a kind of an "equilibrium" for the modified problem as a fixed point of a correspondence (in **Fixed Point Argument**). Finally, we show that the fixed point is actually an equilibrium of our original model in the limit of truncation argument (in **Relationship to the Original Problem and Limit Argument**).

**Producers' Problems:** For any $t > 1$, define subset $\Omega^t \subset R^\ell \times R^\ell_+ \times R^\lambda$ as

$$\Omega^t := [-t, t]^\ell \times [0, t]^\ell \times [-t, t]^\lambda. \tag{28}$$

Now, take an arbitrarily large number $t > 1$. We consider a modified version of the original problem (14)–(17). Specifically, we replace the variable constraint (15) by the following truncated version:

$$(y_j, v_j, z_j) \in (Y_j \times R^\ell_+ \times R^\lambda) \cap \Omega^t.$$

Moreover, we relax the constraints (16) and (17) by replacing equalities "$=$" with inequalities "$\leqq$". We refer those modified constraints as (15'), (16'), and (17'), respectively. Recall that $Y_j \subset R^\ell$ is closed and convex, and contains 0, and $F_j$ satisfies (C1) – (C5) for each $j = 1, \dots, n$.

We denote by $\eta^t_j(p, s)$ the set of solutions to the modified maximization problem; (14) subject to (15'), (16'), and (17') under $(p, s)$. It is clear that $\eta^t_j(p, s)$ is non-empty, closed, and convex. (In particular, convexity is assured since we relaxed equality constraint (16) by inequality constraint (16') and $F_{ik}$ is a concave function for each $k = 1, \dots, \ell$ from condition (C2).) Also, we can prove that the correspondence

$\eta_j^t : \Delta \times \prod_{\kappa=1}^{\lambda} \Delta^\kappa \to \Omega^t$ has a closed graph. Indeed, we can see that the constraint correspondence

$$(p, s) \mapsto \{ (y_j, v_j, z_j) \in \Omega^t \mid (y_j, v_j, z_j) \text{ satisfies (15'), (16'), and (17') under } (p, s) \}$$

has a closed graph and that its value and range are compact. (The compactness of the value is assured since we truncated the variables and hence the difficulty on upper semi-continuity which we pointed out in Example 1 can be avoided. Note that the continuity of $F_j$, condition (C1), is used here.) Therefore, the constraint correspondence is upper semi-continuous. Moreover, it is clear that the constraint correspondence is lower semi-continuous because $F_j$ satisfies (C3) and constraints are inequality, and hence the standard argument is applicable. Thus, the constraint correspondence is continuous. Therefore, Berge's maximum theorem is applicable (cf. Debreu (1959, p. 19)). In this case, it is simultaneously assured that the profit function of this truncated problem, $\pi_j^t(p, s)$, is continuous.

**Consumers' Problems:** As in the producer case, we consider the modified version of the original problem (18)–(22). First, for each $t > 1$, define number $t_1(t) > 1$ as

$$t_1(t) := t \cdot \max\{1 + \sum_{j=1}^{n} \pi_j^t(p, s) | (p, s) \in \Delta \times \prod_{\kappa=1}^{\lambda} \Delta^\kappa\}. \tag{29}$$

Note that such number $t_1(t)$ exists since $\Delta \times \prod_{\kappa=1}^{\lambda} \Delta^\kappa$ is compact and each $\pi_j^t(p, s)$ is continuous. Also, note that, since $0 \in Y_j \cap [-t, t]^\ell$ implies $\pi_j^t(p, s) \geqq 0$ for all $j = 1, \ldots, n$, we have

$$t_1(t) = t \cdot \max\{1 + \sum_{j=1}^{n} \pi_j^t(p, s) | (p, s) \in \Delta \times \prod_{\kappa=1}^{\lambda} \Delta^\kappa\} \geqq t \cdot 1 = t$$

for all $t > 1$. Hence, we can take an arbitrarily large $t_1(t) > 1$ by taking sufficiently large $t > 1$.

Now, consider the modified version of the original problem. Namely, variables are truncated as $(x_i, v_i, z_i) \in (X_i \times R_+^\ell \times R^\lambda) \cap \Omega^{t_1(t)}$ where $\Omega^{t_1(t)}$ is defined as in (28), and all the equalities "$=$" in constrains (20)–(22) are replaced with inequalities "$\leqq$". Moreover, we replace each profit $\pi_j(p, s)$ in constraint (22) by $\pi_j^t(p, s)$, which is the maximized profit of each producer in the modified maximization problem we argued above. We refer those modified constraints as (19'), (20'), (21'), and (22'), respectively. Denote by $\xi_i^t(p, s)$ the set of all solutions to the modified maximization problem; (18) subject to

(19')–(22') under $(p, s)$.[20] The correspondence $\xi_i^t : \Delta \times \prod_{\kappa=1}^{\lambda} \Delta^\kappa \to \Omega^{t_1(t)}$ is non-empty closed convex valued and has a closed graph. The argument is almost the same as the producer case and hence we omit.

**Fixed Point Argument:** We have defined solution correspondences $\eta_j^t : \Delta \times \prod_{\kappa=1}^{\lambda} \Delta^\kappa \to \Omega^t$ and $\xi_i^t : \Delta \times \prod_{\kappa=1}^{\lambda} \Delta^\kappa \to \Omega^{t_1(t)}$ for agents' modified problems for any $t > 1$. Now, consider the product map $\Phi$ of these correspondences:

$$\Phi : \Delta \times \prod_{\kappa=1}^{\lambda} \Delta^\kappa \ni (p, s) \mapsto \prod_{i=1}^{m} \xi_i^t(p, s) \times \prod_{j=1}^{n} \eta_j^t(p, s) \subset (\Omega^{t_1(t)})^m \times (\Omega^t)^n. \tag{30}$$

The mapping $\Phi$ has a closed graph. Then, define a price-expectation manipulation correspondence $\Psi$ as follows:

$$\Psi : ([0, t_1(t)]^\ell \times [-t_1(t), t_1(t)]^\lambda)^m \times ([0, t]^\ell \times [-t, t]^\lambda)^n \ni (v_i, z_i)_{i=1}^{m+n} \mapsto \Theta((z_i)_{i=1}^{m+n}) \times \Xi((v_i)_{i=1}^{m+n}) \subset \Delta \times \prod_{\kappa=1}^{\lambda} \Delta^\kappa, \tag{31}$$

where $\Theta$ is the price manipulation mapping and $\Xi$ is the correspondence that assigns the real mixture ratio of the goods for each market. More precisely, we define

$$\Theta((z_i)_{i=1}^{m+n}) := \{p \in \Delta \,|\, \forall q \in \Delta, q \cdot \sum_{i=1}^{m+n} z_i \geqq p \cdot \sum_{i=1}^{m+n} z_i\}$$

for each $(z_i)_{i=1}^{n+m}$, and the $\kappa$-th coordinate of $\Xi$ by

$$\Xi_\kappa((v_i)_{i=1}^{m+n}) := \frac{\sum_{i=1}^{m+n} \mathrm{pr}_{L_\kappa}(F_i(v_i))}{\sum_{i=1}^{m+n}\big(\sum_{k \in L_\kappa} F_{ik}(v_i)\big)}, \tag{32}$$

as long as $\sum_{i=1}^{m+n}\big(\sum_{k \in L_\kappa} F_{ik}(v_i)\big) \neq 0$, and otherwise by $\Xi_\kappa((v_i)_{i=1}^{m+n}) := \Delta^\kappa$ for each $(v_i)_{i=1}^{m+n}$. (We use the notation $\mathrm{pr}_{L_\kappa}$ for the projection onto subspace $R^{L_\kappa}$ of $R^\ell$ for each $\kappa = 1, \ldots, \lambda$.) Note that the right hand side of Eq. (32) is always an element of $\Delta^\kappa$ when $\sum_{i=1}^{m+n}\big(\sum_{k \in L_\kappa} F_{ik}(v_i)\big) \neq 0$. It is routine to check that $\Theta$ and $\Xi$ are non-empty closed convex valued correspondence with a closed graph. In particular, $\Xi$ has a closed graph since the right hand side of Eq. (32) is continuous when $\sum_{i=1}^{m+n}\big(\sum_{k \in L_\kappa} F_{ik}(v_i)\big) \neq 0$.

[20] We denote the solution set by $\xi_i^t(p, s)$ instead of $\xi_i^{t_1(t)}(p, s)$ for notational simplicity.

Now, the product of the mappings $\Phi$ and $\Psi$,

$$\Phi \times \Psi : \Delta \times \big(\prod_{\kappa=1}^{\lambda} \Delta^{\kappa}\big) \times (\Omega^{t_1(t)})^m \times (\Omega^t)^n \ \to \ \Delta \times \big(\prod_{\kappa=1}^{\lambda} \Delta^{\kappa}\big) \times (\Omega^{t_1(t)})^m \times (\Omega^t)^n, \tag{33}$$

is a non-empty closed convex valued correspondence with a closed graph. By Kakutani's fixed point theorem, $\Phi \times \Psi$ has a fixed point $(\, p^t, s^t, (x_i^t, v_i^t, z_i^t)_{i=1}^m, (y_j^t, v_j^t, z_j^t)_{j=1}^n \,)$ for any $t > 1$.

**Relationship to the Original Problem and Limit Argument:** In this part, we show that the fixed point we obtained above is an equilibrium point of our original model for some number $t > 1$. Take number $t > 1$ sufficiently large so that the bounded attainable sets, $\tilde{X}_i \subset R^\ell$ and $\tilde{Y}_j \subset R^\ell$, to be subsets of the interiors of $[-t_1(t), t_1(t)]^\ell$ and $[-t, t]^\ell$, respectively. Take a fixed point $(\, p^t, s^t, (x_i^t, v_i^t, z_i^t)_{i=1}^m, (y_j^t, v_j^t, z_j^t)_{j=1}^n \,)$ we obtained above.

First, we show that (22') holds with equality, i.e., for all $i = 1, \dots, m$, we have

$$p^t \cdot z_i^{t-} = p^t \cdot z_i^{t+} + \sum_{j=1}^{n} \theta_{ij} \pi_j^t (p^t, s^t). \tag{34}$$

Indeed, if we assume $p^t \cdot z_i^{t-} < p^t \cdot z_i^{t+} + \sum_{j=1}^n \theta_{ij} \pi_j^t(p^t, s^t)$ for some $i = 1, \dots, m$, we have a contradiction to the consumer's utility maximization. More precisely, we have three cases; $z_i^{t-} \neq (t_1(t), \dots, t_1(t))$, $z_i^{t-} = (t_1(t), \dots, t_1(t))$ and $z_i^{t+} \neq (0, \dots, 0)$, and, $z_i^{t-} = (t_1(t), \dots, t_1(t))$ and $z_i^{t+} = (0, \dots, 0)$. For the first case, since consumer's utility function is strictly monotone, we can make a preferable choice $(\hat{x}_i, v_i^t, \hat{z}_i)$ by purchasing more in some market $\kappa$ and consumes more. For the second case, also, we can make a preferable choice $(\hat{x}_i, \hat{v}_i, \hat{z}_i)$ by selling less in some market $\kappa$ by using condition (C3) of the standardizing technology $F_i$ and consumes more. For the third case, we have

$$p^t \cdot (t_1(t), \dots, t_1(t)) = p^t \cdot z_i^{t-} < p^t \cdot z_i^{t+} + \sum_{j=1}^{n} \theta_{ij} \pi_j^t (p^t, s^t) = \sum_{j=1}^{n} \theta_{ij} \pi_j^t (p^t, s^t)$$

and this implies $t_1(t) < \sum_{j=1}^n \pi_j^t(p^t, s^t)$ since $p_1^t + \cdots + p_\lambda^t = 1$ and $\theta_{ij} \leqq 1$ for all $j = 1, \dots, n$. However, this inequality contradicts to the definition (29) of $t_1(t)$ and the assumption that $t > 1$.

Therefore, from condition (34) and equalities $\pi_j^t(p^t, s^t) = p^t \cdot z_j^t$ for all $j = 1, \dots, n$, it is clear that we have Walras' Law; $p^t \cdot (\sum_{i=1}^{n+m} z_i^t) = 0$. Then, by the definition of $\Theta$, we also have

$$q \cdot (\sum_{i=1}^{m+n} z_i^t) \geqq p^t \cdot (\sum_{i=1}^{m+n} z_i^t) = 0 \quad \text{for all } q \in \Delta.$$

Hnece, for each $\kappa = 1, \dots, \lambda$, the $\kappa$-th coordinates of $(\sum_{i=1}^{m+n} z_i^t)$ must satisfy that

$$\sum_{i=1}^{m+n} z_{i\kappa}^t \geqq 0 \tag{35}$$

and

$$\sum_{i=1}^{m+n} z_{i\kappa}^t > 0 \quad \text{if and only if} \quad p_\kappa^t = 0. \tag{36}$$

Here, we note that the real state $((x_i^t)_{i=1}^m, (y_j^t)_{j=1}^n)$ of the fixed point satisfies

$$\sum_{i=1}^{m} x_i^t \leqq \sum_{j=1}^{n} y_j^t + \sum_{i=1}^{m} \omega_i, \tag{37}$$

i.e., $x_i^t$ and $y_j^t$ are in the bounded attainable sets $\tilde{X}_i$ and $\tilde{Y}_j$, respectively. Indeed, for each $\kappa = 1, \dots, \lambda$ such that $\sum_{i=1}^{m+n} z_\kappa^{t+} > 0$ and each $k \in L_\kappa$, we have the following inequalities:

$$\begin{aligned}
\sum_{i=1}^{m}(\omega_{ik} - x_{ik}^t) + \sum_{j=1}^{n} y_{jk}^t &\overset{(17'),(21')}{\geqq} \sum_{i=1}^{m+n} v_{ik}^t - \sum_{i=1}^{m+n} z_\kappa^{t-} s_k^{t\kappa} \\
&\overset{(32)}{=} \sum_{i=1}^{m+n} v_{ik}^t - \sum_{i=1}^{m+n} z_\kappa^{t-} \frac{\sum_{i=1}^{m+n} F_{ik}(v_i^t)}{\sum_{i=1}^{m+n} \sum_{k' \in L_\kappa} F_{ik'}(v_i^t)} \\
&\overset{(35)}{\geqq} \sum_{i=1}^{m+n} v_{ik}^t - \sum_{i=1}^{m+n} z_\kappa^{t+} \frac{\sum_{i=1}^{m+n} F_{ik}(v_i^t)}{\sum_{i=1}^{m+n} \sum_{k' \in L_\kappa} F_{ik'}(v_i^t)} \\
&\overset{(16'),(20')}{\geqq} \sum_{i=1}^{m+n} v_{ik}^t - \sum_{i=1}^{m+n} z_\kappa^{t+} \frac{\sum_{i=1}^{m+n} F_{ik}(v_i^t)}{\sum_{i=1}^{m+n} z_\kappa^{t+}} \\
&= \sum_{i=1}^{m+n} v_{ik}^t - \sum_{i=1}^{m+n} F_{ik}(v_i^t).
\end{aligned}$$

Hence, from condition (C4), we have (37).[21] Also, note that the attainability condition (37) implies that the relaxed constraint (21') holds with equality, i.e., for all $i = 1, \dots, m$, we have

$$x_i^t + v_i^t = \omega_i + z_{i1}^{t-} s^{t1} + \cdots + z_{i\lambda}^{t-} s^{t\lambda} \tag{38}$$

because consumers' utility functions are strictly monotone and $x_i^t \neq (t_1(t), \dots, t_1(t))$ since we took number $t_1(t) > 1$ sufficiently large so that the bounded attainable set $\tilde{X}_i \subset R^\ell$ to be a subset of the interior of $[-t_1(t), t_1(t)]^\ell$.

Furthermore, we have $p^t \gg 0$. Indeed, if we assume that there is some market $\kappa$ such that $p_\kappa^t = 0$

[21] The case $\sum_{i=1}^{m+n} z_\kappa^{t+} = 0$ is obvious since (C3) implies $\sum_{i=1}^{m+n} v_{ik}^t = 0$ for all $k \in L_\kappa$ and (35) implies $\sum_{i=1}^{m+n} z_\kappa^{t-} = 0$.

then, we have a contradiction. More precisely, we have three cases; for some consumer $i = 1, \dots, m$ we have $z_{i\kappa}^{t-} < t_1(t)$, for some consumer $i = 1, \dots, m$ we have $z_{i\kappa}^{t+} > 0$, and for all consumers $i = 1, \dots, m$ we have $z_{i\kappa}^{t-} = t_1(t)$ and $z_{i\kappa}^{t+} = 0$. For the former two cases, we can make preferable choices for consumer $i$ in similar ways to the above arguments that we used to show equality (34). For the third case, because $\sum_{k \in L_\kappa} F_i(v_i^t) = z_{i\kappa}^{t+} = 0$ implies $v_{ik}^t = 0$ for all $k \in L_\kappa$ by (C3) and we have equality (38), we also have that for all $k \in L_\kappa$,

$$\sum_{i=1}^{m} x_{ik}^t = \sum_{i=1}^{m} (z_{i\kappa}^{t-} s_k^{t\kappa} + \omega_{ik}) = m t_1(t) s_k^{t\kappa} + \sum_{i=1}^{m} \omega_{ik}.$$

However, because $s^{t\kappa} \in \Delta^\kappa$ and we can take $t_1(t) > 1$ arbitrarily large, so it contradicts to the fact that, for all $i = 1, \dots, m$, $x_i^t$ belongs to attainable set $\tilde{X}_i$, which is bounded in $R^\ell$.

Therefore, because condition (36) and $p^t \gg 0$ imply $\sum_{i=1}^{m+n} z_i^t = 0$, we have the market clearing condition. Moreover, $p^t \gg 0$ implies that inequality constraints (16'), (17'), and (20') in the modified problems actually hold with equalities since if not, a contradiction to the utility or profit maximization clearly follows. Hence, these equalities, together with (34) and (38), mean that all of our modified inequality constraints actually hold with equalities for sufficiently large $t > 1$.

Thus far, we see that the state $\left(p^t, s^t, (x_i^t, v_i^t, z_i^t)_{i=1}^m, (y_j^t, v_j^t, z_j^t)_{j=1}^n\right)$ satisfies (16), (17), (20), (21), (22), (23), and (24). We call $\left(p^t, s^t, (x_i^t, v_i^t, z_i^t)_{i=1}^m,\ (y_j^t, v_j^t, z_j^t)_{j=1}^n\right)$ a $t$-equilibrium state. Since we take $t > 1$ sufficiently large so that the bounded attainable sets to be subsets of the interior of $[-t, t]^\ell$ or $[-t_1(t), t_1(t)]^\ell$, all $x_i^t$ or $y_j^t$ are interior points of $[-t, t]^\ell$ or $[-t_1(t), t_1(t)]^\ell$. Therefore, the $t$-equilibrium state $\left(p^t, s^t, (x_i^t, v_i^t, z_i^t)_{i=1}^m,\ (y_j^t, v_j^t, z_j^t)_{j=1}^n\right)$ is not an equilibrium of the original economy $\mathcal{E}$ only when $(v_i^t, z_i^t)$ is a boundary point of $[0, t]^\ell \times [-t, t]^\lambda$ or $[0, t_1(t)]^\ell \times [-t_1(t), t_1(t)]^\lambda$ for some $i = 1, \dots, m+n$.[22] Suppose that, for some $i = 1, \dots, m+n$, $(v_i^t, z_i^t)$ is a boundary point of $[0, t]^\ell \times [-t, t]^\lambda$ for all $t > 1$. If some market $\kappa$ consists of a single real commodity $k$, $L_\kappa = \{k\}$, then restriction (20) requires that $v_{ik}^t = z_{i\kappa}^{t+}$ by (C5), and $s_k^\kappa = 1$ and restriction (21) require that $x_{ik}^t + v_{ik}^t = \omega_{ik} + z_{i\kappa}^{t-}$. (The same argument is relevant for producers.) Therefore, we can also suppose that $(v_{ik}^t, z_\kappa^t)$ is bounded for such singleton markets $L_\kappa$ without loss of generality, since we can decrease the amounts of $(v_{ik}^t, z_{i\kappa}^{t-}) = (z_{i\kappa}^{t+}, z_{i\kappa}^{t-})$ without any change of utility value (and/or profit value) of $i$ or any harm in conditions we get so far. This implies

[22] If the domain of $F_i$ is compact for all $i = 1, \dots, m+n$ then $(v_i, z_i)$ $(i = 1, \dots, n+m)$ are in a fixed bounded area by the continuity of $F_i$ $(i = 1, \dots, n+m)$. Hence, in such cases, the proof is completed here. As we mentioned before, the proof of Theorem 2 corresponds to this case.

that $\| \sum_{k \in L_\kappa} F_{ik}(v_i^t)\| \to \infty$ as $t \to \infty$ for some $\kappa \in \{1, \ldots, \lambda\}$ such that $\sharp L_\kappa \geqq 2$. Note first that, for all $i = 1, \ldots, m+n$ and all $\kappa, \kappa' = 1, \ldots, \lambda$, we have

$$\sum_{i=1}^{m}\Big(\sum_{k \in L_\kappa \cup L_{\kappa'}} (\omega_{ik} - x_{ik}^t)\Big) + \sum_{j=1}^{n}\Big(\sum_{k \in L_\kappa \cup L_{\kappa'}} y_{ik}^t\Big) \geqq \sum_{k \in L_\kappa \cup L_{\kappa'}} v_{ik}^t - (z_{i\kappa}^{t+} + z_{i\kappa'}^{t+}) \tag{39}$$

by considering conditions (16), (17), (20), (21), (23), (24), and (C4). Moreover, the right hand side of Eq. (39) is $\sum_{k \in L_\kappa \cup L_{\kappa'}} v_{ik}^t - \sum_{k \in L_\kappa \cup L_{\kappa'}} F_{ik}(v_i^t)$, from conditions (16) and (20). However, if $\| \sum_{k \in L_\kappa} F_{ik}(v_i^t)\| \to \infty$ as $t \to \infty$ for some $\kappa \in \{1, \ldots, \lambda\}$ such that $\sharp L_\kappa \geqq 2$, then condition (C5) requires that there exists some $L_{\kappa'}$ such that $\sum_{k \in L_\kappa \cup L_{\kappa'}} v_{ik}^t - \sum_{k \in L_\kappa \cup L_{\kappa'}} F_{ik}(v_i^t) \to \infty$ as $t \to \infty$. This implies that the right-hand side and hence the left-hand side of Eq. (39) tends to $\infty$ as $t \to \infty$, contradicting the fact that each of $x_i^t$ and $y_j^t$ is in a bounded area in $R^\ell$. Thus, we complete the proof of our main theorem.

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