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Abstract

This study considers how individuals determine the ratio of two kinds of educational investment. One kind contributes to labor skills and the other does not. We refer to the former as human capital investment and the latter as unproductive investment which improves test scores, but has no beneficial effect on students' human capital. We formulate an overlapping generations economy in which the rich and poor invest in both types of education. We argue that the ratio of human capital investment to unproductive investment is a U-shaped function of the wage differentials between the rich and poor. Moreover, we identify three patterns of stable steady states for these wage differentials, namely, no-inequality, high-inequality, and multiple steady states. Using these results, we conclude that a rapid increase in the level of skill-biased technology may switch the steady state from no inequality to high inequality. Further, it causes a temporary or permanent increase in the ratio of unproductive investment during the transition to the new steady state.

Keywords: human capital investment, unproductive education, wage differentials, overlapping generations, and multiple steady states

JEL classification: O11, D81, I24

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1 Introduction

For several decades, many countries have experienced expanded access to higher education. One reason is an increase in demand for high skilled worker from skill-biased technological progress, such as the IT revolution. In addition to the fact, it is known that higher educational attainment serves as a signal and has a big impact on students' postgraduation earnings. Using US data, Jaeger and Page (1996) and Park (1999) find evidence of diploma effects: earning gains of 9–10% associated with high school graduation, 11% with associate's degrees, and 20–30% with bachelor's degrees. Thus, higher education plays a dual role of "educating" and "credentialing".

The main purpose of this paper is to investigate whether higher education is socially productive, or whether it is partly a social waste. From the perspective of credentialing, students are concerned not with learning, but with being certified as having learned. In other words, their main objective is to obtain higher test scores and better grades. We see that these credentialing activities are individually valuable but may contain a lot of socially wasteful expenditure. Ronald P. Dore, who is widely regarded as the leading authority on sociology, sates in his book that:

> *The effect of schooling, the way it alters a man's capacity and will to do things, depends not only on what he learns, or the way he learns it, but also on* ***why he learns it****. That is at the basis of the distinction between schooling which is education, and schooling which is only qualification, a mere process of certificating - or 'credentialing'. (Dore (1976) [pp. 8])*

According to the argument, we can see that whether schooling contributes to build one's capacity depends on what his/her purpose of schooling is.

In this study, we separate educational investment into schooling that is education, or human capital investment, and schooling that is only qualification, or unproductive investment. Unproductive investment in our model implies investment undertaken only to obtain an observable credential, which does not contribute to labor skills. Conversely, human capital investment not only contributes to labor skills but is also helpful for obtaining the credential. Thus, the ratio of human capital investment to unproductive investment captures the degree of credentialing: a lower ratio means that credentialing is households' major purpose of receiving higher education.

We introduce an overlapping generations economy, in which parents are heterogeneous in their wages, being either rich or poor. Each parent invests in the above-mentioned two types of education for their children. Moreover, uncertainty exists in the acquisition of the credential, which is determined endogenously by the differences in each type of educational investment by the rich and poor. That is, as the gap in the education level between rich and poor increases, the probability that the child of rich parent obtains the credential increases, whereas the probability that the child of poor parent obtains the credential decreases. In other words, this model considers competition to obtain the credential between the two young, and this competition becomes an important factor that affects the ratio of human capital to unproductive investment.

The results of our study are divided into two parts, static and dynamic analysis. In static analysis, we examine how the ratio of human capital to unproductive investment changes with the wage differentials. We find that the ratio of human capital to unproductive investment for both rich and poor becomes a U-shaped function of the wage differentials. That is to say, the ratio is high when the wage differential is small or large, while it is low with medium levels of wage differentials. The ratio of the two types of investment well describes the degree of credentialing in higher education; a lower ratio of human capital to unproductive investment implies that a larger amount of socially wasteful activities is involved in educational investment.

Our model also contributes to explaining the dynamics of the wage differentials of middle-aged generation. In dynamic analysis, we find that three patterns of stable steady states of wage differentials exist: a unique steady state with no inequality, a unique steady state with high inequality, and multiple steady states. These alternative steady states are distinguished by the parameter representing the level of skill-biased technology, and we can show that an increase in the level of skill-biased technology tends to lead to high inequality steady state. Thus, this result well explains the expansion of wage differentials experienced in the period of skill-biased technical change (SBTC) in many advanced countries. Further, and more importantly, we show that rapid progress of skill-biased technology switches the dynamics of the economy from no-inequality to high-inequality system, and the ratio of human capital to unproductive investment decreases at least temporarily during the transition to the high-inequality steady state. Thus, we can say that SBTC, such as the IT revolution from the 1970s to the 1990s, would enhance socially wasteful unproductive investment.

To understand credentialing activities, signaling theory developed by Spence (1973, 1974) have often been used in economic models. Many empirical works, including those of Lang and Kropp (1986), Hungerford and Solon (1987), Belman and Heywood (1991), and Bedard (2001), argue that higher education serves as a signal of higher productivity in addition to increasing individual human capital, and that this signal is rewarded in the labor market. Following these results, theoretical studies focusing on signaling as private educational investment have been developed. Futagami and Ishiguro (2004), Hendel et al. (2005), Yuki (2009), D'Amato and Mookherjee (2012), and Balart (2016) study educational signaling and wage inequality in a dynamic model (using an overlapping generations model). Futagami and Ishiguro (2004) address macroeconomic issues by considering how signaling behavior of individuals affects physical capital accumulation of the economy. Hendel et al. (2005) and Balart (2016) examine the dynamics of education and wage inequality using a signaling framework that incorporates walth heterogeneity and imperfect credit markets. Although our model is not signaling model, our study have a certain similarity to the studies in that we consider unproductive education under asymmetric information.[1]

Our model is also related to Blankenau and Camera (2006, 2009), in which investment for obtaining qualifications is separated from skill-enhancing investment. Blankenau and Camera (2009) constructs a model in which homogeneous students choose their educational investment as follows: skilled (with qualification and skill), schooled (with qualification but no skill), and unschooled (no qualification and no skill). They suggest that increased tuition subsidies may not be beneficial because they increase enrollment but they may lower the incentives of education or making achievement.

In addition, our results in dynamic analysis are consistent with the arguments in the literature on the relation between wage inequality and SBTC. They argue that SBTC can be offered as explanations for an increase in US wage inequality between college and high school graduates from the 1980s (for seminal empirical and theoretical works, see Katz and Murphy, 1992; Acemoglu, 1998; Autor et al., 1998; and Galor and Moav, 2000). Some subsequent theoretical studies analyze the relationship between the emergence of skill-biased technologies and educational attainment of workers. They show that higher skill premiums driven by SBTC increase educational attainment of individuals with higher ability or income (see Caselli, 1999; Miyake et al., 2009; and Restuccia and Vandenbroucke, 2013), while the

[1] Aside from studies of dynamics, several studies describe individual incentives of investment to acquire skills by applying the basic model of signaling theory. Hopkins (2012) and Bidner (2014) show that by assuming positive assortative matching of worker–firm or worker–worker pair production, a higher level of investment not only serves as a signal and leads to matches with higher productive partners but also increases productivity and leads to higher wages. Ordine and Rose (2011) introduce both innate and schooling ability in determining the individuals' educational choice, and analyze within wage inequality in terms of educational mismatch and inefficient self-selection into education. They shows that wage inequality rise along with ability-complementary technological progress whenever education equality is low.

attainments of individuals with lower ability or income decrease because of an increased cost of educational investment (see Crifo, 2008; Nakamura, 2013). These results, like our study, explain the widening wage inequality between college and high school graduates and the contemporary increase in college enrollment in many advanced countries in the decades after SBTC.

The remainder of the paper is structured as follows. Section 2 describes the construction of the model. Section 3 provides the results of static analysis. We illustrates the mechanism underlying the U-shaped relationship between the ratio of human capital to unproductive investment and the wage differentials. In Section 4, we analyze the dynamics of the wage differentials of middle-aged and old workers. Section 5 provides micro-foundation theory for the main assumption of this model which is about the description of the credential-gaining process defined in section 2.1. Section 6 concludes.

2 The Model

Consider a discrete-time overlapping generations economy in which each agent lives for three periods: young, middle, and old. Each generation is composed of two agents, rich and poor. We refer to a young agent born to a rich (poor) middle-aged parent as “rich (poor) young”. We assume that there is no population growth and, for simplicity, that all agents have no access to an intertemporal storage technology.[2]

In the first period of life, young agents receive education from their parents. We assume both young agents are born with the same ability. At the end of the young period, either the rich young agent or the poor young agent obtains a credential, and the probability is determined as a result of their educational investments. In the second period of life, middle-aged agents work and obtain wage incomes. They give birth to one child and allocate income between their own consumptions and investments in education for the child. Further, we assume that firms can not observe the individual output of each middle-aged worker. In the final period of life, old agents work and consume, but they neither bear children nor invest in education. We assume that, unlike the middle-aged worker, firms can observe the individual output of each old worker.

Thus, our model divides individuals’ working period into two terms in order to introduce

[2]Thus, we assume that individuals are credit constrained due to financial market imperfections. This assumption is made in order to ensure that middle-aged agents remains in the two types regardless of their income levels in old age. This constraint can be relaxed without altering any of the qualitative features of the results presented in this paper, if we assume a small open economy.

both symmetric and asymmetric information cases. That is, having the credential works as a signaling device for middle-aged workers and positively affects their wages. By contrast, the wages of the second working period are paid according to the individuals' actual working skills, regardless of whether they have the credential.[3]

2.1 Educational investment and uncertainty of the credential

Parents invest in two types of education for their children: one is productive and the other is not. Here, we refer to the former as human capital investment and and the latter as unproductive investment. Assume that the human capital production function is given by the following expression:

$$h_{i,t} = {e_{i,t}}^{\mu}, \tag{1}$$

where $h_{i,t}$ is the human capital of individual i born in period t, $e_{i,t}$ is the amount of human capital investment put in by his/her parent, and $\mu \in (0, 1)$ measures efficiency of human capital investment. In addition, parents can invest in unproductive activities aimed only at credentialing. We denote by $s_{i,t}$ the amount of the unproductive investment. We assume that the levels of e and s are unobservable to firms.

Now we define the probability that describes the uncertainty of the credential. We denote by $\theta_{i,t}$ the probability that young agent i born in period t obtains the credential, and assume that $\theta_{i,t}$ increases in $h_{i,t}$ and $s_{i,t}$. Since one of the two young agents can obtain the credential, $\theta_{i,t}$ has to be equal to the probability that young agent j fails to obtain the credential, that is, $\theta_{i,t} = 1 - \theta_{j,t}$. In particular, we define $\theta_{i,t}$ as the following function:

$$\theta_{i,t} = \Theta\left(\widetilde{h}_{i,t}, \widetilde{s}_{i,t}\right) = 1 - \Theta\left(\widetilde{h}_{j,t}, \widetilde{s}_{j,t}\right) = 1 - \theta_{j,t}, \tag{2}$$

where $\widetilde{h}_{i,t} = h_{i,t}/h_{j,t} = 1/\widetilde{h}_{j,t}$ and $\widetilde{s}_{i,t} = s_{i,t}/s_{j,t} = 1/\widetilde{s}_{j,t}$. Further, we assume $\Theta : R^+ \times R^+ \to (0, 1)$, $\Theta_{\widetilde{h}} > 0$, $\Theta_{\widetilde{h}\widetilde{h}} < 0$, $\Theta_{\widetilde{s}} > 0$, $\Theta_{\widetilde{s}\widetilde{s}} < 0$, $\Theta_{\widetilde{h}\widetilde{s}} = 0$, $\lim_{\widetilde{h}\to 0, \widetilde{s}\to 0} \Theta = 0$, $\Theta(1, 1) = \frac{1}{2}$, and $\lim_{\widetilde{h}\to\infty, \widetilde{S}\to\infty} \Theta = 1$. In addition, we assume that $\theta_{i,t} = \overline{\Theta}\big(\widetilde{h}_{i,t}\big)$ when $s_{i,t} = s_{j,t} = 0$, $\theta_{i,t} = \underline{\Theta}\big(\widetilde{s}_{i,t}\big)$ when $e_{i,t} = e_{j,t} = 0$, and $\theta_{i,t} = 1/2$ when $e_{i,t} = e_{j,t} = s_{i,t} = s_{j,t} = 0$.

In this model, the two young agents compete for obtaining one credential. The assumptions $\Theta_{\widetilde{h}} > 0$ and $\Theta_{\widetilde{s}} > 0$ mean that Θ is an increasing function of the educational gaps

[3]For a justification behind this assumption, see Altonji and Pierret (2001). They argue that firms in general have only limited information about the quality of workers in the early stages of their careers, and distinguish among workers on the basis of easily observable variables, such as years of education or degree. On the other hand, with each successive period, firms can observe more of workers' performance.

between the two young agents. That is, as $e_{i,t}$ and $s_{i,t}$ increase, young agent i can obtain the credential more easily compared to young agent j. Likewise, the increase in $e_{j,t}$ and $s_{j,t}$ makes young agent j easier to obtain the credential compared to young agent i. See Section 5 for a more detailed discussion of these assumptions.

The reason for calling s_{it} unproductive investment is apparent now. Equation (2) indicates that, a young agent with higher productive skill is likely to get the credential on one hand, but on the other hand, a young agent with larger amount of the unproductive investment is also likely to obtain the credential. As will become apparent shortly, the rich young agent receives larger amount of investments on both education in a unique equilibrium in this model. Further, the equilibrium wage of the worker with the credential is higher than that of the worker without the credential in middle-age. Therefore, not only human capital investment but also unproductive investment are privately worthwhile for expected middle-aged wages, but the latter is socially wasteful. Thus we interpret e_t/s_t as the degree of credentialing: a lower ratio indicates households invest more in socially wasteful education.

2.2 Production

Firms produce final goods using only labor. We assume that, for agent i and j in each generation, if $h_{i,t} > h_{j,t}$, the individual outputs of i and j are $\phi h_{i,t}$ and $h_{j,t}$, where $\phi > 1$. That is, only the higher skilled worker can work with the superior technology, and thus ϕ can be interpreted as the level of skill-biased technology.

We assume that more than two risk-neutral firms simultaneously make wage offers to workers. If all firms make the same offers, one randomly selected firm acquires all workers and produce.[4] For each middle-aged worker, firms can observe whether the worker has the credential, but cannot observe the individual output and the education level. Hence, according to the expected outputs of the worker with the credential and the worker without the credential, firms make wage offers to the workers. Let $w_{t+1}^m \in \{w_{t+1}^c, w_{t+1}^n\}$ denote the wages of the middle-aged worker with the credential ($m = c$) and the worker with no credential ($m = n$) in period $t + 1$. On the other hand, firms can observe the individual output of each old-age worker. Therefore, old age workers are paid their individual outputs, that is, the wages of old workers are paid regardless of the credential.

[4] Thus, in equilibrium, the competition between firms drives their profits down to zero.

2.3 Household

The preferences of middle-aged agents in period t (born in period $t-1$) are defined over consumption and the wage of their offspring. They are represented by a log-linear utility function:

$$U(c_t^m, c_{t+1}^o, w_{t+1}^m, w_{t+2}^o) = \alpha \ln c_t^m + \beta\Big[\alpha \ln c_{t+1}^o + (1-\alpha)E\big(\ln w_{t+1}^m\big)\Big] + \beta^2(1-\alpha)\ln w_{t+2}^o, \quad (3)$$

where c_t^m is own middle-age consumption in period t, c_{t+1}^o is own old-age consumption in period $t+1$, $w_{t+1}^m \in \{w_{t+1}^c,\, w_{t+1}^n\}$ is the wages that their offspring earn in middle age, w_{t+2}^o is the wage that their offspring get in old age, $\alpha \in (0,1)$ is an inverse measure of parental altruism, $\beta \in (0,1)$ is the time discount rate, and E is the expectation operator. The budget constraint of middle age and old age are expressed as follows:

$$c_t^m + e_t + s_t \leq w_t^m, \quad (4)$$

and

$$c_{t+1}^o \leq w_{t+1}^o. \quad (5)$$

2.4 Optimization and equilibrium

We now define a competitive equilibrium. Let us denote by $h_{H,t}$ and $h_{L,t}$ the human capital levels of higher skilled and lower skilled worker in generation t. That is, $h_{i,t} = h_{H,t}$ and $h_{j,t} = h_{L,t}$ if $h_{i,t} > h_{j,t}$. Further, let ϑ_t denotes the value of firms' expectation about the probability that the worker with the credential has higher skill $h_{H,t}$. A Competitive Equilibrium consists of sequence of wages $\{w_{t+1}^c, w_{t+1}^n, w_{t+2}^o\}$, individuals' allocation $\{c_t^m, c_{t+1}^o, e_t, s_t\}$, and the probabilities $\{\theta_{i,t}, \theta_{j,t}, \vartheta_t\}$, such that

- taking as given w_{t+1}^c, w_{t+1}^n, $e_{j,t}$ and $s_{j,t}$, middle-aged parent i maximizes her utility subject to the budget constraints and the wages that her offspring earns in old age (8):

$$\begin{cases} \max\limits_{c^m,e,s} \;\; \alpha \ln c_{i,t}^m + \beta(1-\alpha)\left[\Theta\left(\widetilde{h}_{i,t}, \widetilde{s}_{i,t}\right)\ln w_{t+1}^c + \left(1-\Theta\left(\widetilde{h}_{i,t}, \widetilde{s}_{i,t}\right)\right)\ln w_{t+1}^n\right] \\ \qquad\qquad\qquad\qquad\qquad + \beta\left[\alpha \ln c_{i,t+1}^o + \beta(1-\alpha)\ln w_{t+2}^o\right], \\ \text{s.t.} \quad c_{i,t}^m + e_{i,t} + s_{i,t} = w_t^m, \;\; c_{i,t+1}^o = w_{t+1}^o, \text{ and (8)}; \end{cases} \quad (6)$$

- taking as given $e_{i,t}$, $s_{i,t}$, $e_{j,t}$ and $s_{j,t}$, and formulating the expectation about the probability ϑ_t, firms make wage offers to the middle-aged workers as:

$$\begin{cases} w_{t+1}^c = \vartheta_t \phi h_{H,t} + (1-\vartheta_t)h_{L,t}, \\ w_{t+1}^n = \vartheta_t h_{L,t} + (1-\vartheta_t)\phi h_{H,t}; \end{cases} \quad (7)$$

- observing the individual outputs of each old-age worker, firms offer wages that equal to the actual outputs of each old-age worker:

$$w_{t+2}^{o} = \begin{cases} \phi h_{H,t} & \text{if the worker has higher skill,} \\ h_{L,t} & \text{otherwise;} \end{cases} \tag{8}$$

- firms' expectations about ϑ_t coincide with the actual value:

$$\vartheta_t = \begin{cases} \theta_{i,t} & \text{if } h_{i,t} > h_{j,t}, \\ \theta_{j,t} & \text{otherwise.} \end{cases} \tag{9}$$

We now solve the equilibrium. Without loss of generality, we assume the middle-aged agent i has the credential, and thus, her wage in period t is w_t^c. Note that $w^c \leq w^n$ is never realized for any period in equilibrium. Suppose, by contradiction, that $w_{t+1}^c \leq w_{t+1}^n$. From (7), it implies firms expect that the worker with the credential is more likely of being low skilled, that is, $\vartheta \leq 1/2$. Then, households' maximization decision with respect to the unproductive investment becomes $s_{i,t} = s_{j,t} = 0$. Given that $\theta_{i,t} = \overline{\Theta}\big(\widetilde{h}_{i,t}\big)$ when $s_{i,t} = s_{j,t} = 0$, then the worker with the credential is more likely of being high-skilled. However, it contradicts the equilibrium condition (9), we thus have $w^c > w^n$ in equilibrium.

The lifetime events of young agents born in period t are illustrated in Figure 1 below. The solid lines represent the case where rich young (i) obtains the credential, and the dashed lines represent the case where poor young (j) obtains the credential.

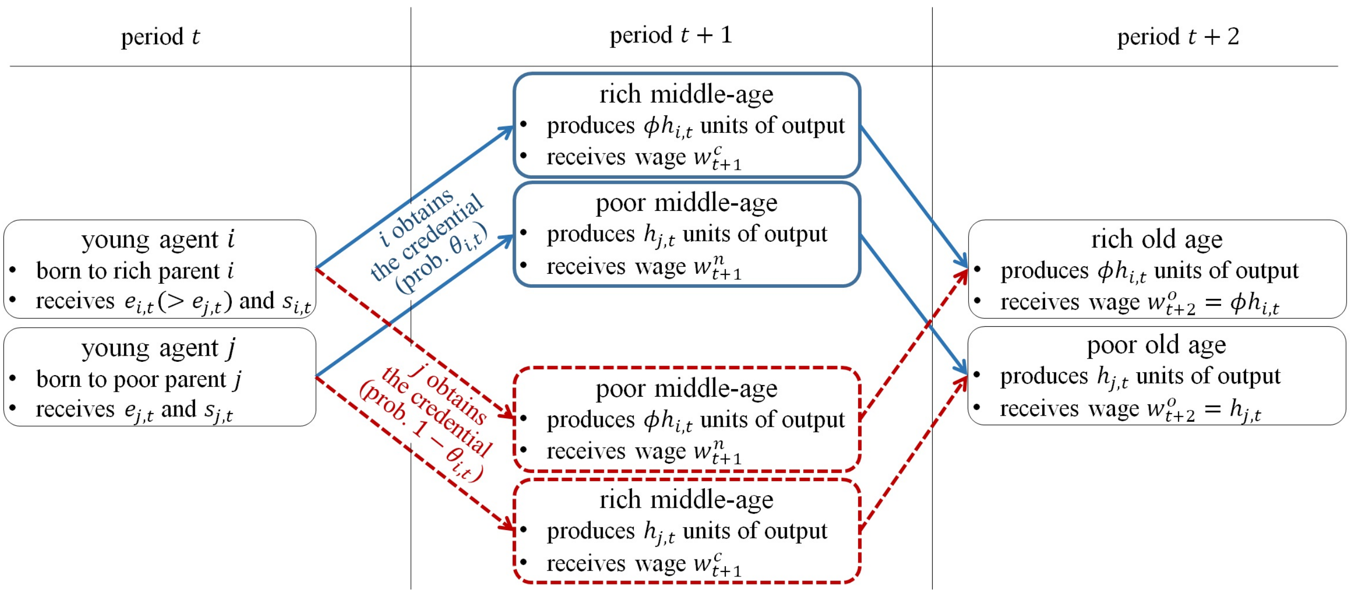


Figure 1 : Lifetime events of young agents born in period t

First, we solve the household's problem (6). We have the first order conditions for $e_{i,t}$ and $s_{i,t}$ as follows:

$$e_{i,t} \ : \ \frac{e_{i,t}}{w_t^c - e_{i,t} - s_{i,t}} = \frac{\beta(1-\alpha)\mu}{\alpha}\left[\beta + \frac{\partial\Theta(\widetilde{h}_{i,t}, \widetilde{s}_{i,t})}{\partial \widetilde{h}_{i,t}}\ \widetilde{h}_{i,t} \ln \frac{w_{t+1}^c}{w_{t+1}^n}\right], \tag{10}$$

$$s_{i,t} \ : \ \frac{s_{i,t}}{w_t^c - e_{i,t} - s_{i,t}} = \frac{\beta(1-\alpha)}{\alpha}\frac{\partial\Theta(\widetilde{h}_{i,t}, \widetilde{s}_{i,t})}{\partial \widetilde{s}_{i,t}}\ \widetilde{s}_{i,t} \ln \frac{w_{t+1}^c}{w_{t+1}^n}, \tag{11}$$

Likewise, by using $\Theta\left(\widetilde{h}_{j,t}, \widetilde{s}_{j,t}\right) = 1 - \Theta\left(\widetilde{h}_{i,t}, \widetilde{s}_{i,t}\right)$, the optimal condition of the middle-aged agent j can be expressed as follows:

$$e_{j,t} \ : \ \frac{e_{j,t}}{w_t^n - e_{j,t} - s_{j,t}} = \frac{\beta(1-\alpha)\mu}{\alpha}\left[\beta + \frac{\partial\Theta(\widetilde{h}_{i,t}, \widetilde{s}_{i,t})}{\partial \widetilde{h}_{i,t}}\ \widetilde{h}_{i,t} \ln \frac{w_{t+1}^c}{w_{t+1}^n}\right], \tag{12}$$

$$s_{j,t} \ : \ \frac{s_{j,t}}{w_t^n - e_{j,t} - s_{j,t}} = \frac{\beta(1-\alpha)}{\alpha}\frac{\partial\Theta(\widetilde{h}_{i,t}, \widetilde{s}_{i,t})}{\partial \widetilde{s}_{i,t}}\ \widetilde{s}_{i,t} \ln \frac{w_{t+1}^c}{w_{t+1}^n}. \tag{13}$$

A derivation of these equations can be found in Appendix. We can see from (10) and (12) that $e_{i,t}/c_{i,t} = e_{j,t}/c_{j,t}$, and from (11) and (13) that $s_{i,t}/c_{i,t} = s_{j,t}/c_{j,t}$. Using them, we have

$$\frac{e_{i,t}}{e_{j,t}} = \frac{s_{i,t}}{s_{j,t}} = \frac{w_t^c}{w_t^n}.$$

The above expression shows an important equilibrium property in this model. First, given that $w_t^c/w_t^n > 1$, the rich parent who has the credential invests more in both education. Let $e_{H,t}$ ($e_{L,t}$) and $s_{H,t}$ ($s_{L,t}$) denote the equilibrium investment levels made by the rich (poor) and $\widetilde{w}_t \equiv w_t^c/w_t^n$ denote the equilibrium wage differentials of the middle-aged generation. Rewriting the above expression in terms of the equilibrium values, we have

$$\frac{e_{H,t}}{e_{L,t}} = \frac{s_{H,t}}{s_{L,t}} = \widetilde{w}_t, \tag{14}$$

which indicates that the the levels of the educational gaps between rich and poor young are consistent with the levels of parental wage differentials. (14) also indicates that the rich and poor always choose the same level of e/s, that is,

$$\frac{e_{H,t}}{s_{H,t}} = \frac{e_{L,t}}{s_{L,t}}. \tag{15}$$

We define $\widetilde{h}_t = h_{H,t}/h_{L,t}$ and $\widetilde{s}_t = s_{H,t}/s_{L,t}$ as the gap of education levels between rich and poor in equilibrium, then we have

$$\widetilde{h}_t = \left(\frac{e_{H,t}}{e_{L,t}}\right)^{\mu} = \widetilde{w}_t^{\mu} \quad \text{and} \quad \widetilde{s}_t = \frac{s_{H,t}}{s_{L,t}} = \widetilde{w}_t. \tag{16}$$

Moreover, from (9) and (16), the equilibrium value of θ is expressed as a function of $\widetilde{w}_t$:

$$\vartheta_t = \Theta\big(\widetilde{h}_t, \widetilde{s}_t\big) = \Theta\big(\widetilde{w}_t^{\mu}, \widetilde{w}_t\big). \tag{17}$$

Let us define $\Theta\big(\widetilde{w}_t^{\mu}, \widetilde{w}_t\big) \equiv \Gamma(\widetilde{w}_t)$ where Γ satisfies $\Gamma(\widetilde{w}_t) : [1, \infty) \to \big[\frac{1}{2}, 1\big)$, $\Gamma'(\widetilde{w}_t) > 0$, $\Gamma''(\widetilde{w}_t) < 0$, $\Gamma(1) = \frac{1}{2}$ and $\lim_{\widetilde{w}_t \to \infty} \Gamma(\widetilde{w}_t) = 1$.[5] Finally, we can express the equilibrium dynamics of $\widetilde{w}_t$, by using (7), (16) and (17), as follows:

$$\widetilde{w}_{t+1} = \frac{w^c_{t+1}}{w^n_{t+1}} = \frac{\Gamma(\widetilde{w}_t)\phi\widetilde{w}_t^{\mu} + (1 - \Gamma(\widetilde{w}_t))}{\Gamma(\widetilde{w}_t) + (1 - \Gamma(\widetilde{w}_t))\phi\widetilde{w}_t^{\mu}}. \tag{18}$$

Thus, we can express the right-hand sides of (10)–(13) as a function of $\widetilde{w}_t$, so we can derive the equilibrium value of each endogenous variable, $e_{H,t}$, $s_{H,t}$, $e_{L,t}$ and $s_{L,t}$, when we specify the functional form of Θ. However, we omit this work because it is not necessary for our subsequent analysis.

3 Ratio of Human Capital to Credentialing Investment

This section proposes a static analysis to examine how the ratio of human capital to unproductive investment changes with $\widetilde{w}_t$. That is, here we explore the relationship between e_t/s_t and $\widetilde{w}_t$ by regarding e_t/s_t as a function of $\widetilde{w}_t$. As shown in (15), rich and poor parents divide educational expenditure into human capital and unproductive investment in the same ratio. Let us define η_t as the ratio of human capital to unproductive investment, that is, $\eta_t \equiv e_{H,t}/s_{H,t} = e_{L,t}/s_{L,t}$. Then, by using (10) and (11), we can express η_t in terms of the equilibrium values, $\widetilde{h}_t$, $\widetilde{s}_t$ and $\widetilde{w}_{t+1}$, as follows:

$$\begin{aligned}\eta_t &= \mu \frac{\frac{\partial\Theta(\widetilde{h}_t, \widetilde{s}_t)}{\partial\widetilde{h}_t}\,\widetilde{h}_t \ln \widetilde{w}_{t+1} + \beta}{\frac{\partial\Theta(\widetilde{h}_t, \widetilde{s}_t)}{\partial\widetilde{s}_t}\,\widetilde{s}_t \ln \widetilde{w}_{t+1}} \\ &= \mu\left[\frac{\varepsilon_h}{\varepsilon_s} + \frac{\beta}{\frac{\partial\Theta(\widetilde{h}_t, \widetilde{s}_t)}{\partial\widetilde{s}_t}\,\widetilde{s}_t \ln \widetilde{w}_{t+1}}\right],\end{aligned} \tag{19}$$

where $\quad \varepsilon_h = \frac{\partial\Theta(\widetilde{h}_t, \widetilde{s}_t)}{\partial\widetilde{h}_t}\Big/\frac{\Theta(\widetilde{h}_t, \widetilde{s}_t)}{\widetilde{h}_t} \quad$ and $\quad \varepsilon_s = \frac{\partial\Theta(\widetilde{h}_t, \widetilde{s}_t)}{\partial\widetilde{s}_t}\Big/\frac{\Theta(\widetilde{h}_t, \widetilde{s}_t)}{\widetilde{s}_t}$.

In the following analysis, we focus on the denominator of the second term in (19) and show that, regardless of the value of $\varepsilon_h/\varepsilon_s$, η_t is a U-shaped function of $\widetilde{w}_t$. First, we examine how $\partial\Theta(\widetilde{h}_t, \widetilde{s}_t)/\partial\widetilde{s}_t$ varies with $\widetilde{w}_t$. From (16) and the assumptions about the Θ function, we have

$$\frac{d}{d\widetilde{w}_t}\left(\frac{\partial\Theta(\widetilde{h}_t, \widetilde{s}_t)}{\partial\widetilde{s}_t}\right) = \underbrace{\frac{\partial^2\Theta(\widetilde{h}_t, \widetilde{s}_t)}{\partial\widetilde{s}_t\partial\widetilde{h}_t}}_{=0}\underbrace{\frac{d\widetilde{h}_t}{d\widetilde{w}_t}}_{>0} + \underbrace{\frac{\partial^2\Theta(\widetilde{h}_t, \widetilde{s}_t)}{\partial\widetilde{s}_t^2}}_{<0}\underbrace{\frac{d\widetilde{s}_t}{d\widetilde{w}_t}}_{>0} < 0, \tag{20}$$

[5] Thus, we obtain a negative relationship between income inequality and intergenerational social mobility since an increase in θ decreases the mobility. This relationship is supported by both empirical and theoretical studies (see Andrews and Leigh, 2009; Ferrer, 2005; Solon, 2004).

$\lim_{\widetilde{w}_t \to \infty} \frac{\partial \Theta(\widetilde{h}_t, \widetilde{s}_t)}{\partial \widetilde{s}_t} = 0$, and $\lim_{\widetilde{w}_t \to 1} \frac{\partial \Theta(\widetilde{h}_t, \widetilde{s}_t)}{\partial \widetilde{s}_t} > 0$. Next, we examine $\widetilde{s}_t \ln \widetilde{w}_{t+1}$. We obtain from (16) and (18) that this term is increasing in $\widetilde{w}_t$ and that $\lim_{\widetilde{w}_t \to \infty}(\widetilde{s}_t \ln \widetilde{w}_{t+1}) = \infty$. Further, as it is clear from (18) that $\widetilde{w}_{t+1} = 1$ when $\widetilde{w}_t = 1$, we have $\lim_{\widetilde{w}_t \to 1}(\widetilde{s}_t \ln \widetilde{w}_{t+1}) = 0$.

From the above analysis, we see that the denominator of the second term in (19) converges to zero when $\widetilde{w}_t \to 1$ and that it takes positive and finite values for finite $\widetilde{w}_t > 1$. Further, we have the following lemma for the case where $\widetilde{w}_t \to \infty$.

Lemma 1. $\frac{\partial \Theta(\widetilde{h}_t, \widetilde{s}_t)}{\partial \widetilde{s}_t} \widetilde{s}_t \ln \widetilde{w}_{t+1} \to 0$ when $\widetilde{w}_t \to \infty$ (see Appendix for details).

It is clear from the above discussion and Lemma 1 that η_t in (19) approaches infinity when $\widetilde{w}_t \to 1$ and $\widetilde{w}_t \to \infty$. Conversely, for finite $\widetilde{w}_t > 1$, since $\varepsilon_h/\varepsilon_s$ is positive and finite, η_t is also positive and finite. Therefore, we have the following proposition.

Proposition 1. Finite value $\widetilde{w}' > 1$ exists such that η_t decreases in $\widetilde{w}_t$ for all $\widetilde{w}_t < \widetilde{w}'$, and finite value $\widetilde{w}''(> \widetilde{w}')$ exists such that η_t increases in $\widetilde{w}_t$ for all $\widetilde{w}_t > \widetilde{w}''$.

From Proposition 1, we can say that η_t is a U-shaped function of $\widetilde{w}_t$, because regardless of the value of $\varepsilon_h/\varepsilon_s$, we have $\eta_t = \infty$ at the boundaries. Strictly speaking, single-peakedness of the objective does not hold for every possible Θ function. However, if we restrict Θ to some classes of functions, we obtain single-peaked function as shown in the next figure. Figure 2 shows a numerical example of η_t. A numerical example in Figure 2 shows a case of $\widetilde{w}' = \widetilde{w}''$. That is, in this example, η_t is a single-peaked.

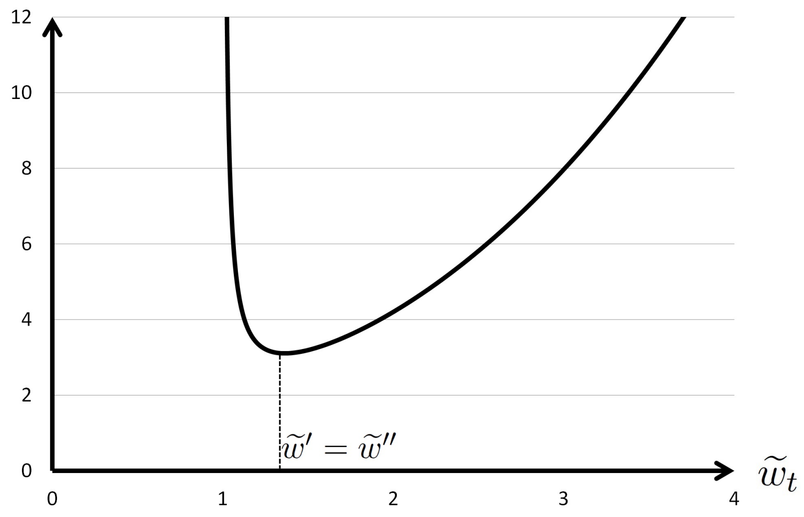


Figure 2

Notes : Figure 2 visualizes equation (19) by using $\Theta(\widetilde{h}_t, \widetilde{s}_t) = 1 - \frac{1}{4}\left(\frac{1}{\widetilde{h}_t}\right)^{a_1} - \frac{1}{4}\left(\frac{1}{\widetilde{s}_t}\right)^{a_2}$, $a_1 = 3$, $a_2 = 3$, $\mu = 0.85$, $\phi = 4$, and $\beta = 0.2$. In this case we have $\widetilde{w}' = \widetilde{w}'' = 1.36$, that is, η_t is a single-peaked.

A mathematical interpretation of Proposition 1 is that the U-shapedness of η_t is derived mainly from $\frac{d}{d\widetilde{w}_t}\left(\frac{\partial\Theta(\widetilde{h}_t,\widetilde{s}_t)}{\partial\widetilde{s}_t}\right) < 0$ and $\frac{d\ln\widetilde{w}_{t+1}}{d\widetilde{w}_t} > 0$. Hence, to see the intuition of Proposition 1, here we focus on the two opposite effects of increasing the wage differentials. First, we consider the intuition of $\frac{d}{d\widetilde{w}_t}\left(\frac{\partial\Theta(\widetilde{h}_t,\widetilde{s}_t)}{\partial\widetilde{s}_t}\right) < 0$. Given that Θ is increasing in $\widetilde{s}_t$ but with an upper bound of 1, the marginal change in Θ becomes progressively smaller as $\widetilde{w}_t$ increases. That is, when $\widetilde{w}_t$ is large, marginal increases in e and s lead to very little change in the probability of obtaining the credential for both rich and poor households. Therefore, as $\widetilde{w}_t$ increases, households reduce their unproductive investment ratio because it does not contribute to promoting their productivities. Thus, from the perspective of the probability of obtaining the credential, an increase in $\widetilde{w}_t$ decreases households' incentives to make additional investment in unproductive education.

On the other hand, from $\frac{d\ln\widetilde{w}_{t+1}}{d\widetilde{w}_t} > 0$, we can see that it becomes more valuable for households to obtain the credential as $\widetilde{w}_t$ increases. Clearly, larger wage differentials of the parental generation lead to larger wage differentials of the offspring generation. Further, note that a higher value of $\widetilde{w}_{t+1}$ implies a larger benefit from obtaining the credential because $\widetilde{w}_{t+1}$ represents the wage gap between the worker with the credential and the worker with no credential in the offspring generation. Therefore, from the perspective of the benefit from obtaining the credential, a larger $\widetilde{w}_t$ leads households to have more incentive to invest in unproductive education. Thus, these two opposite effects of wage differentials creates the U-shape of η_t.

Proposition 1 implies that private investment in higher education may contain much wasteful investment in economies with medium-sized wage differentials. That is, at medium wage differentials, because obtaining the credential is of great concern to households, they may spend a lot on unproductive training that contributes only to test scores. The same logic can be applied to some publicly-funded educational investments. If households have strong incentive to obtain the credential, part of public expenditure (such as education vouchers) may be used wastefully as unproductive investment by households. As is shown theoretically by Blankenau and Camera (2009), our result in Proposition 1 also shows the mechanism of weak connection between education expenditure and accumulation of human capital.

Furthermore, note that an increase in $\Theta_{\widetilde{h}}$ or a decrease in $\Theta_{\widetilde{s}}$ increases η. $\Theta_{\widetilde{h}}$ and $\Theta_{\widetilde{s}}$ here are interpreted as the sensitivity of the credential-gaining process to human capital and unproductive investment respectively. Therefore, we can say that as test scores that lead students to better earnings become more sensitive to human capital, households' private educational investments in human capital increase relatively.

4 The Dynamics of Wage Differentials

4.1 The dynamics of $\widetilde{w}_t$

In this section, we investigate the dynamics of middle-aged wage differentials $\widetilde{w}_t$ by using the dynamic equation (18). In order to clarify the analysis, here we specify the functional form of $\Gamma(\widetilde{w}_t)$. Note that $\Gamma(\widetilde{w}_t)$ derived in (17) has to satisfy $\Gamma(\widetilde{w}_t) : [1, \infty) \to \left[\frac{1}{2}, 1\right)$, $\Gamma'(\widetilde{w}_t) > 0$, $\Gamma''(\widetilde{w}_t) < 0$, $\Gamma(1) = \frac{1}{2}$, and $\lim_{\widetilde{w}_t \to \infty} \Gamma(\widetilde{w}_t) = 1$. Further, we obtain from (17) that $\Gamma(\widetilde{w}_t)$ is increasing in μ. For simplicity, we specify $\Gamma(\widetilde{w}_t)$ as

$$\Gamma(\widetilde{w}_t) = 1 - \frac{1}{2\widetilde{w}_t^{\mu}}, \tag{21}$$

which satisfies the above conditions. Using (21), we can rewrite (18) as follows:

$$\widetilde{w}_{t+1} = \frac{2\phi\widetilde{w}_t^{2\mu} - \phi\widetilde{w}_t^{\mu} + 1}{(2+\phi)\widetilde{w}_t^{\mu} - 1}. \tag{22}$$

Our purpose here is to examine how the dynamics of middle-aged wage differentials varies with the parameters in (22), namely ϕ and μ. We show in Appendix that the dynamics of $\widetilde{w}_t$ has three patterns of steady states depending on the values of ϕ and μ: a unique steady state with no inequality, a unique steady state with high inequality, and multiple steady states. For the following proposition, we define $\bar{\phi}$ and $\widehat{\phi}$ as follows:

$$\bar{\phi} = \frac{1+2\mu}{2\mu - 1} \text{ and } \widehat{\phi} = \frac{1+\mu}{3\mu - 2}. \tag{23}$$

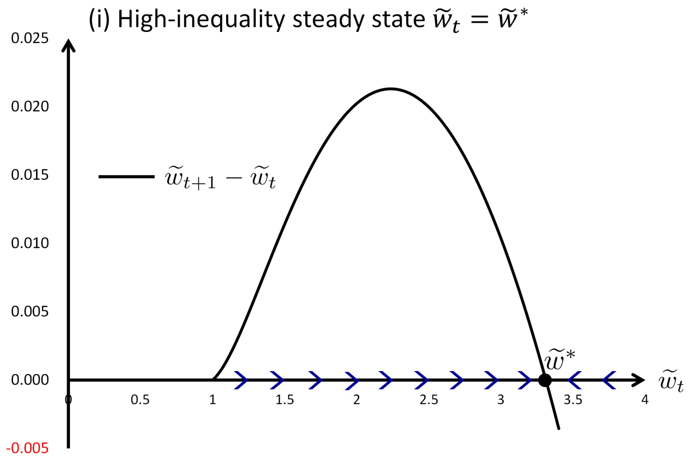
Proposition 2. Assume that $\mu > 2/3$. Further, we specify $\Gamma(\widetilde{w}_t)$ as in (21), and define $\bar{\phi}$ and $\widehat{\phi}$ as in (23). Then,

(i) When $\phi > \bar{\phi}$, there is a unique stable steady state in which $\widetilde{w} > 1$.

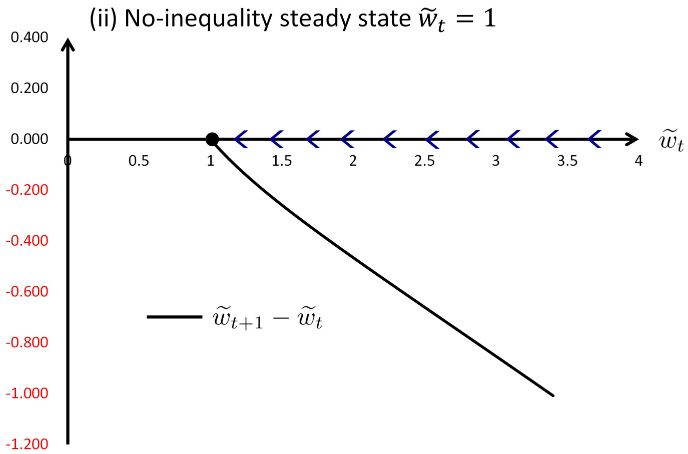
(ii) When $\phi \leq \min\{\bar{\phi}, \widehat{\phi}\}$, there is a unique stable steady state in which $\widetilde{w} = 1$.

(iii) When $\mu > (1+\sqrt{5})/4$ and $\widehat{\phi} < \phi \leq \bar{\phi}$, multiple steady states may exist.

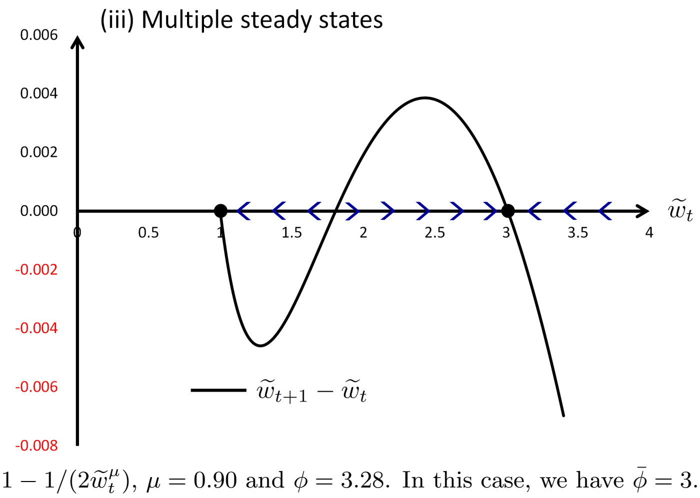
Proof: See Appendix for the technical proof of the statements (i) and (ii). Figure 3 illustrates the existence of multiple steady states mentioned in (iii).



For $\Gamma(\widetilde{w}_t) = 1 - 1/(2\widetilde{w}_t^{\mu})$, $\mu = 0.87$ and $\phi = 3.80$. In this case, we have $\bar{\phi} = 3.70$ and $\widehat{\phi} = 3.07$.



For $\Gamma(\widetilde{w}_t) = 1 - 1/(2\widetilde{w}_t^{\mu})$, $\mu = 0.87$ and $\phi = 1.50$. In this case, we have $\bar{\phi} = 3.70$ and $\widehat{\phi} = 3.07$.



For $\Gamma(\widetilde{w}_t) = 1 - 1/(2\widetilde{w}_t^{\mu})$, $\mu = 0.90$ and $\phi = 3.28$. In this case, we have $\bar{\phi} = 3.50$ and $\widehat{\phi} = 2.71$.

Notes : In Figure 3, we provides numerical examples of Proposition 2. It plots $\widetilde{w}_{t+1} - \widetilde{w}_t$. As shown in (iii) in Figure 3, we verify the existence of multiple steady states. In the following, we denote by $\widetilde{w}^*$ the steady state value of (i) High-inequality steady state.

Figure 3

The most notable parameter here is ϕ, which represents the level of skill-biased technology. From Proposition 2, we can say that a high-inequality steady state tends to be realized for higher values of ϕ. Therefore, it can be said that if an economy with an initial steady state of no inequality experiences rapid increases of skill-biased technologies, the dynamics of the economy may switch from (ii) to (i) in Figure 3. As a result, its steady state changes from no-inequality to high-inequality, and the economy suffers a large increase in wage differentials during the transition to the high-inequality steady state. This is consistent with a commonly asserted notion that SBTC from the 1970s to the 1990s caused large increases of wage inequality between highly educated and poorly educated workers. In turn, we see from (23) that $\bar{\phi}$ and $\widehat{\phi}$ are decreasing in μ. That is, it implies that a high-inequality steady state tends to be realized for higher values of μ. The effects of μ are also intuitive. This parameter represents the productivity of human capital investment. Therefore, in addition, Proposition 2 states that more productive education leads to a high-inequality steady state.

4.2 The dynamics of η_t

Now that we have solved for the dynamics of $\widetilde{w}_t$, we turn to the dynamics of η_t. We showed in section 3 that the ratio of human capital to unproductive investment, η_t, is a U-shaped function of $\widetilde{w}_t$. Our purpose of this subsection is to examine how the educational investment ratio varies with SBTC.

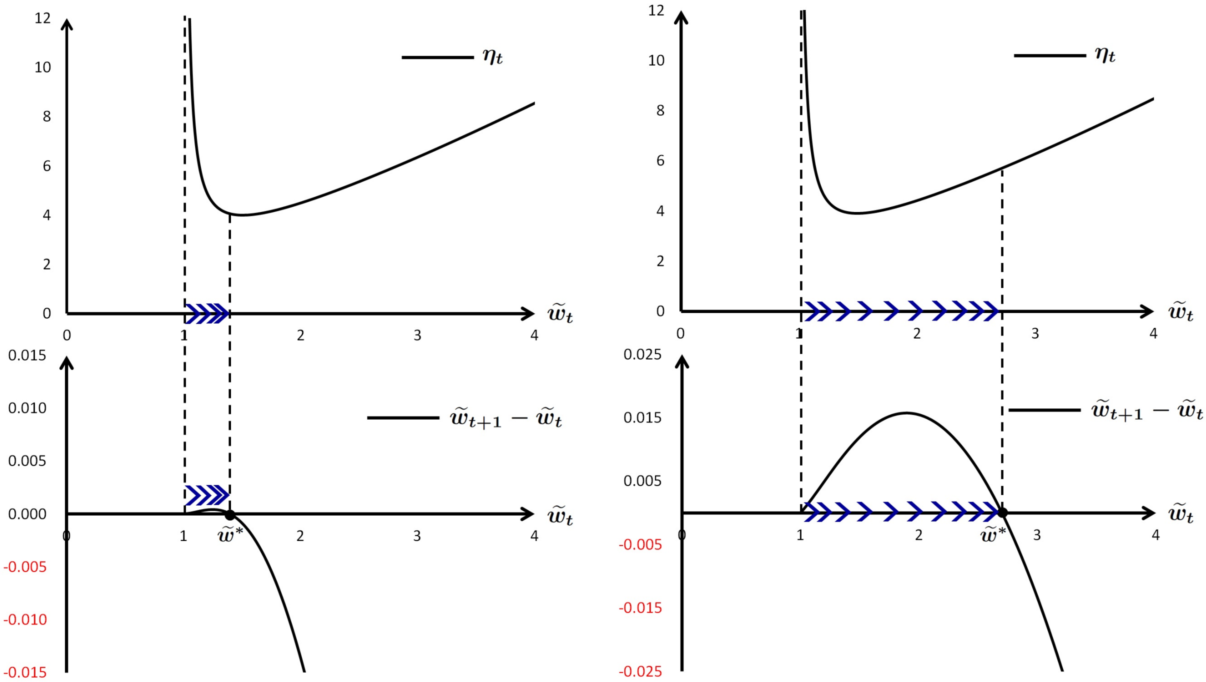
We assume here that ϕ initially satisfies $\phi \leq \min\{\bar{\phi}, \widehat{\phi}\}$. We also assume that the economy is initially in the neighborhood of the no-inequality steady state.[6] Further, let us consider that, in period $t_0 (> 0)$, the economy faces a rapid increase in the level of skill-biased technology and ϕ rises to a level greater than $\bar{\phi}$. It means that the steady state of the economy switches from no-inequality to high-inequality in period t_0, and then, the wage differentials increase toward the high-inequality steady state $\widetilde{w}^*$. What is important here is that η decreases for at least a couple of periods from period t_0 because $\eta_{t_0} = \infty$. Then we have the following proposition.

Proposition 3. Suppose that, at $t = 0$, $\phi \leq \min\{\bar{\phi}, \widehat{\phi}\}$ holds and $\widetilde{w}_0$ is in the neighborhood of $\widetilde{w} = 1$. Further, suppose that ϕ increases to a level higher than $\bar{\phi}$ in $t = t_0$. Then, there exists $t_1 \geq t_0$ such that $\eta_{t+1} < \eta_t$ holds for all $t \in [t_0, t_1]$.

[6] This assumption can be relaxed by considering a sufficiently long but finite time period. This is because, when $\phi \leq \min\{\bar{\phi}, \widehat{\phi}\}$, for any initial condition $\widetilde{w}_0 > 1$, $\widetilde{w}_t$ converges to no-inequality steady state $\widetilde{w} = 1$.

We can interpret Proposition 3 to mean that the rapid technological progress from the 1980s may have enhanced educational investment for the purpose of credentialing. That is, the recent increase in higher educational attainment experienced in many countries may stem from an increase in socially wasteful educational expenditure caused by SBTC. If this is the case, the increase in educational attainment would make a slight contribution to human capital accumulation.

Further, Proposition 3 indicates that the decrease in η after period t_0 may be temporary or permanent. That is, t_1 in Proposition 3 may be finite or infinite. When the high-inequality steady state value $\widetilde{w}^*$ is less than $\widetilde{w}'$ (which was defined as the smallest value of $\widetilde{w}_t$ satisfying $d\eta_t/d\widetilde{w}_t = 0$ in Proposition 1), η decreases for all $t \in [t_0, \infty)$. On the other hand, if $\widetilde{w}^* > \widetilde{w}'$, the decrease in η is temporary and t_1 is finite. Instead of giving an analytical proof, we provide numerical examples of the existence of the above two cases in the following Figure 4. Note that, in both cases, the ratio of unproductive investment increases for some period after the switch of steady state, as shown in Proposition 3.



Notes : We draw η function and transition path of $\widetilde{w}_t$ which converges to the high-inequality steady state $\widetilde{w}^*$. Here, we assume that the paths start from around the $\widetilde{w} = 1$. The figure on the left is drawn by using $\Gamma(\widetilde{w}_t) = 1 - 1/(2\widetilde{w}_t^{\mu})$, $\mu = 0.83$, $\phi = 4.03$ and $\beta = 0.1$. In this case, we have $\widetilde{w}^* = 1.40$ and $\widehat{w}'(= \widehat{w}'') = 1.50$. On the other hand, the figure on the right is drawn by using $\Gamma(\widetilde{w}_t) = 1 - 1/(2\widetilde{w}_t^{\mu})$, $\mu = 0.85$, $\phi = 4.00$ and $\beta = 0.2$. In this case, we have $\widetilde{w}^* = 2.71$ and $\widehat{w}'(= \widehat{w}'') = 1.36$.

Figure 4

4.3 The dynamics of the wage differentials of old workers

In this paper, we have focused on the wage differentials of middle-aged workers because it determines the educational investment ratio η. However, it is also worthwhile to examine the wage differentials of old workers. Let us denote by $\widetilde{W}$ the wage differentials of old workers. Then, from (8), we obtain $\widetilde{W}_t = \phi\widetilde{h}_{t-2}$. Further, from (16), its equilibrium value is as follows:

$$\widetilde{W}_t = \phi\widetilde{w}^{\mu}_{t-2}. \tag{24}$$

(24) indicates that the wage differentials of old workers in period t, $\widetilde{W}_t$, is expressed as an increasing function of the wage differentials of middle-aged workers in period $t-2$, $\widetilde{w}_{t-2}$. Thus, without yielding a dynamic equation of $\widetilde{W}_t$, we can see that $\widetilde{W}_t$ varies in the same direction as $\widetilde{w}_t$ but with a two-period lag.[7]

5 Discussion

In this section, we discuss the two main features of this study: first, only one of two students obtains the credential, the probability of which is determined by the education gaps between the two students; second, we treat productive and unproductive educational investment separately.

5.1 Competition for the credential

The model describes the competition between the students for the credential. It involves a broad concept that includes competition in high school or college admissions, in high-level academic transcripts, and in certification exams. In this subsection, we first offer a simple interpretation to the competition for the credential and provide a micro-foundation for the credential-gaining process between two students, which is expressed in equation (2). Then, we argue that the credential-gaining process in equation (2) can be supported by the aforementioned broad concept.

The most characteristic aspect of this model is that only one of two students can obtain the credential. Simply stated, we can interpret the credential in this model as a diploma of

[7]Using (22) and (24), we have the dynamics of $\widetilde{W}_t$ as follows:

$$\widetilde{W}_{t+1} = \phi\left[\frac{2\widetilde{W}_t^2 - \phi\widetilde{W}_t + \phi}{(2+\phi)\widetilde{W}_t - \phi}\right]^{\mu}.$$

higher education. In the case where the enrollment limit of the educational institution is fixed at one, only one student can obtain the diploma at the institution. Then, we can say that the probability that the highly (poorly) educated student gets the diploma increases (decreases) as the gap of education levels between two students increases, as we supposed in equation (2). However, this situation might be seen as ad hoc because the enrollment limit is always fixed regardless of the education levels of the two students. Generally, governments control the pupil strength in higher education in accordance with the education level of the population. In order to tackle this problem, we next show a micro-foundation model for the credential-gaining process in equation (2) and then discuss a broader concept of the competition for the credential by using the micro-foundation model.

Here, we suppose there is an educational institution that can provide comprehensive education from primary to post-secondary curriculum. The households invest in two types of education by paying e and s to the educational institution. After receiving education, the two students, i and j, with the education levels, (h_i, s_i) and (h_j, s_j), take a test, the score of which is given by

$$\text{Score} = \varepsilon_i \Phi(h_i, s_i)$$

where Φ is an increasing function of both h_i and s_i, and ε_i is an i.i.d. individual level shock with $E(\varepsilon_i) = 1$. Then, the winner and the loser of the test receive credentials H and L, respectively, from the educational institution. Then, the probability that a young agent born to parent i obtains the credential H can be written as

$$\begin{aligned}\theta_i &= \text{prob}\left[\varepsilon_i \Phi(h_i, s_i) > \varepsilon_j \Phi(h_j, s_j)\right] \\ &= \text{prob}\left[\frac{\Phi(h_i, s_i)}{\Phi(h_j, s_j)} > \xi\right],\end{aligned}$$

where $\xi \equiv \varepsilon_j/\varepsilon_i$ and $E(\xi) = 1$ because ε_i and ε_j are independent. Now, we assume that $\Phi(h_i, s_i)/\Phi(h_j, s_j)$ can be expressed as a function of the educational gaps, $\varphi(\widetilde{h}_i, \widetilde{s}_i)$, where $\widetilde{h}_i = h_i/h_j$ and $\widetilde{s}_i = s_i/s_j$. Then we have $\text{prob}[\varphi(\widetilde{h}_i, \widetilde{s}_i) > \xi] : R^2 \to (0, 1)$, which is an increasing function of $\widetilde{h}_i$ and $\widetilde{s}_i$, and satisfies $\text{prob}[\varphi(1, 1) > \xi] = 1/2$. Further, note that since $\theta_i = 1 - \theta_j$, that is

$$\text{prob}\left[\varepsilon_i \Phi(h_i, s_i) > \varepsilon_j \Phi(h_j, s_j)\right] = 1 - \text{prob}\left[\varepsilon_j \Phi(h_j, s_j) > \varepsilon_i \Phi(h_i, s_i)\right],$$

then we have

$$\text{prob}[\varphi(\widetilde{h}_i, \widetilde{s}_i) > \xi] = 1 - \text{prob}[\varphi(\widetilde{h}_j, \widetilde{s}_j) > \xi].$$

Therefore, we can derive the definition of Θ in equation (2) by applying the second-order derivatives, $\varphi_{\widetilde{h},\widetilde{h}} < 0$, $\varphi_{\widetilde{s},\widetilde{s}} < 0$, and $\varphi_{\widetilde{h},\widetilde{s}} = 0$.

We can interpret from the above micro-foundation that the credential in this model can be seen as a certificate for distinguishing the winner of the test from the loser. Generally, educational institutions make relative evaluations on students' achievements across various subjects or classes, and thus, certificates offer substantial information about each student. However, for simplicity, this model describes an economy with only two students, a one-dimensional productive skill, and one educational institution that certifies the winner and the loser on the test. Therefore, one credential is enough to distinguish between the two students. This is why we assume, in section 2, that only one of two students can obtain the credential. More importantly, now, we do not have to rely on the limited interpretation mentioned above. The two students freely progress toward a higher stage of educational curriculum without being bound to any enrollment limits, but their success or failure are finally evaluated by noisy test scores. In reality, workers could have many certificates such as high school or college admission acceptances, academic transcripts, and diplomas, and the final judgment of success or failure is determined by comprehensive assessments of these. Therefore, the competitive educational investments in this model capture broad private expenditures on education.

In economics, education has frequently not been interpreted as competition. However, especially in East Asia including South Korea, Japan and China, students traditionally have been in tough competition for high-stake tests.[8] Becker (2013) argues that such competition in East Asian countries has spread to other nations owing the worldwide boom in higher education. For example, he indicates that, in the US, competition to get into top colleges has become much tougher, and families are spending a lot more on private tutoring partly for direct preparation for the STA exams.[9] Further, Becker (2013) states that "These results do not answer the basic question of whether the extra effort by high school students is socially productive, or whether it is partly a social waste (an "arms race")". Our study can provide one answer to the question.

5.2 Productive and unproductive educational investment

The model assumes that there exists an unproductive education that can be invested separately from a productive education. To our knowledge, few studies address these two kinds of education, while some models of multi-task principal agent theory have a concept close

[8]College Scholastic Ability Test in South Korea, National Center Test for University Admissions in Japan and National Higher Education Entrance Examination in China can be seen as representative examples of high-stake tests.

[9]Scholastic Assessment Test

to our study. For example, in Acemoglu et al. (2007), teachers with career concerns chose two types of effort separately, one that influences the human capital of their students and one that improves the test scores but has no beneficial effect on students' human capital.[10] They interpret that the latter involves rote learning, where a teacher forces students to cram certain essential facts or methods, without explaining the concepts or phenomena behind them.

Further, the existence of productive and unproductive education at schools is accepted among experts in education. Popham (2001) identifies two kinds of instruction methods of teachers: "curriculum teaching" and "item teaching". Curriculum teaching involves the body of knowledge and skills regarding a subject in accordance with the curricular content, while item teaching narrows the instruction and uses clone items of the particular questions most likely to appear in the test. Critics argue that students who receive item teaching lack a comprehensive and lasting understanding of the subject matter, because item teachers exclude the strengthening of creative skills and abstract-thinking ability.

Our study differs from the perspective of Popham (2001) in that recipients of education decide the educational choice between productive and unproductive investments. However, when we focus on private educational investments, there is no substantial difference between our study and Popham (2001). When we consider that private education is competitive, we can see that private teachers provide exactly what households want, and thus, the households can buy the two types of educational services by paying tuition fees. Examples of these private educations include after-class tutoring and coaching schools. In another respect, especially in higher education, students themselves, not teachers, may be able to decide learning methods among productive and unproductive efforts. For example, college students can either exert effort to achieve a comprehensive understanding of a particular topic or for achieving higher test scores. Thus, the unproductive investment can exist both in secondary and post-secondary private education.

6 Conclusions and Remarks

In this study, we presented a simple dynamic model that contains both human capital investment and unproductive investment. By introducing two opposing types of education, human

[10] Acemoglu et al. (2007) show that high-powered incentives can induce more unproductive effort of teachers. Therefore, when principals have a commitment problem because of their intention of providing high-powered incentives to teachers, government provision may be useful as a credible commitment to low-powered incentives.

capital investment and unproductive investment, into the model, we described the degree of credentialing from the investment ratio. Furthermore, we focused on the wage differentials between the rich and poor. Because parents care not only about the productivity of their offspring but also about the credential, the rich and poor compete with one another to obtain the credential. Therefore, the investment ratio of both the rich and poor depends on the wage differentials.

Our static analysis proved that the ratio of human capital investment to unproductive investment becomes a U-shaped function of the wage differentials. On the other hand, from the dynamic analysis, we showed that three patterns of stable steady states exist with respect to the wage differentials: no-inequality, high-inequality, and multiple steady states. Under a higher level of skill-biased technology, an economy is more likely to have a high-inequality steady state. Furthermore, we showed that if the economy were initially around the no-inequality steady state, rapid progress of skill-biased technology would switch the steady state from no inequality to high inequality, and the ratio of unproductive investment would increase, at least for some periods. From this result, we see that the substantial observed increase in educational attainment experienced after SBTC in many countries may have resulted not only in an increase in human capital, but also in a large increase in wasteful investment aimed at obtaining credentials.

Clearly, it may be hard to distinguish between productive and unproductive investment in existing educational investment. In reality, no educational investment would contribute nothing to human capital accumulation. However, the ratio of human capital investment to unproductive investment in our model well describes whether households' primary objectives for educational attainment are learning or credentialing. Because private decisions between learning and credentialing are less observable, our theoretical analysis makes a significant contribution to explaining households' decisions.

Moreover, our model is related to the analysis of the negative relationship between income inequality and intergenerational social mobility, known as "The Great Gatsby Curve". In our model, social mobility is driven by the probability of obtaining the credential, then parents invest in dissipative expenses in order to improve these chance to their offspring. Introducing heterogeneity in innate ability into the model may lead to a richer analysis of social mobility or occupational mobility. Although these extensions are beyond the scope of the present paper, they are worth examining.

Appendix

Derivation of the first order conditions (10)–(13)

First, we solve the utility maximization problem of middle-aged parent i represented in (6). Given that the optimal allocations in old age are simply $c^o_{i,t+1} = w^o_{t+1}$, we exclude it from the optimization. Further, because ϕ in w^o_{t+2} does not affect the optimization, the objective function in (6) can be rewritten as follows:

$$\alpha \ln(w^c_t - e_{i,t} - s_{i,t}) + \beta(1-\alpha)\left[\Theta\left(\widetilde{h}_{i,t}, \widetilde{s}_{i,t}\right)\ln\frac{w^c_{t+1}}{w^n_{t+1}} + \ln w^n_{t+1}\right] + \beta^2(1-\alpha)\ln h_{i,t}.$$

The first order condition with respect to e_{it} is

$$-\frac{\alpha}{w^c_t - e_{i,t} - s_{i,t}} + \beta(1-\alpha)\frac{\partial\Theta\left(\widetilde{h}_{i,t}, \widetilde{s}_{i,t}\right)}{\partial\widetilde{h}_{i,t}}\frac{\mu e^{\mu-1}_{i,t}}{h_{j,t}}\ln\frac{w^c_{t+1}}{w^n_{t+1}} + \beta^2(1-\alpha)\frac{\mu e^{\mu-1}_{i,t}}{h_{i,t}} = 0$$

$$\Leftrightarrow \qquad \frac{e_{i,t}}{w^c_t - e_{i,t} - s_{i,t}} = \frac{\beta(1-\alpha)\mu}{\alpha}\left[\beta + \frac{\partial\Theta(\widetilde{h}_{i,t}, \widetilde{s}_{i,t})}{\partial\widetilde{h}_{i,t}}\,\widetilde{h}_{i,t}\ln\frac{w^c_{t+1}}{w^n_{t+1}}\right].$$

Thus we have (10). In the same manner, we have the first order condition with respect to s_{it} represented in (11). Next, we solve the problem of j. Using (2), the objective function of middle-aged parent j can be written as follow:

$$\begin{aligned}
&\alpha \ln(w^n_t - e_{j,t} - s_{j,t}) + \beta(1-\alpha)\left[\Theta\left(\widetilde{h}_{j,t}, \widetilde{s}_{j,t}\right)\ln\frac{w^c_{t+1}}{w^n_{t+1}} + \ln w^n_{t+1}\right] + \beta^2(1-\alpha)\ln h_{j,t} \\
=&\alpha \ln(w^n_t - e_{j,t} - s_{j,t}) + \beta(1-\alpha)\left[\Theta\left(\widetilde{h}_{i,t}, \widetilde{s}_{i,t}\right)\ln\frac{w^n_{t+1}}{w^c_{t+1}} + \ln w^c_{t+1}\right] + \beta^2(1-\alpha)\ln h_{j,t}.
\end{aligned}$$

The first order condition with respect to e_{jt} is

$$-\frac{\alpha}{w^n_t - e_{j,t} - s_{j,t}} + \beta(1-\alpha)\frac{\partial\Theta\left(\widetilde{h}_{i,t}, \widetilde{s}_{i,t}\right)}{\partial\widetilde{h}_{i,t}}\left(-\frac{h_{i,t}}{h^2_{j,t}}\mu e^{\mu-1}_{j,t}\right)\ln\frac{w^n_{t+1}}{w^c_{t+1}} + \beta^2(1-\alpha)\frac{\mu e^{\mu-1}_{j,t}}{h_{j,t}} = 0$$

$$\Leftrightarrow \qquad \frac{e_{j,t}}{w^n_t - e_{j,t} - s_{j,t}} = \frac{\beta(1-\alpha)\mu}{\alpha}\left[\beta + \frac{\partial\Theta(\widetilde{h}_{i,t}, \widetilde{s}_{i,t})}{\partial\widetilde{h}_{i,t}}\,\widetilde{h}_{i,t}\ln\frac{w^c_{t+1}}{w^n_{t+1}}\right].$$

Thus we have (12). In the same manner, we have the first order condition with respect to s_{jt} represented in (13).

$\square$

Proof of Lemma 1

To simplify the exposition, we use Θ instead of $\Theta(\widetilde{h}_t, \widetilde{s}_t)$. Since we have $\ln \widetilde{w}_{t+1} < \ln \phi \widetilde{h}_t < \phi \ln \widetilde{h}_t$ when $\widetilde{w}_t \to \infty$, using the denominator of the second term in (19) we have:

$$\frac{\partial \Theta}{\partial \widetilde{s}_t} \widetilde{s}_t \ln \widetilde{w}_{t+1} \ < \ \phi \frac{\partial \Theta}{\partial \widetilde{s}_t} \widetilde{s}_t \ln \widetilde{h}_t.$$

In addition, we have $\widetilde{h}_t = \left(\widetilde{s}_t\right)^{\frac{\mu}{\nu}}$, and thus, from the right-hand side of the above equation, we have

$$\phi \frac{\partial \Theta}{\partial \widetilde{s}_t} \widetilde{s}_t \ln \widetilde{h}_t \ = \ \frac{\mu \phi}{\nu} \frac{\partial \Theta}{\partial \ln \left(\ln \widetilde{s}_t\right)} \to 0 \ \ (\mathit{when} \ \widetilde{w}_t \to \infty).$$

The last term of limit to zero is obtained by properties that $\Theta \to 1$ when $\widetilde{w}_t \to \infty$, while $\ln(\ln \widetilde{s}_t) \to \infty$ when $\widetilde{w}_t \to \infty$. Then, we obtain:

$$\lim_{\widetilde{w}_t \to \infty} \frac{\partial \Theta(\widetilde{h}_t, \widetilde{s}_t)}{\partial \widetilde{s}_t} \widetilde{s}_t \ln \widetilde{w}_{t+1} = 0.$$

$\square$

Proof of Proposition 2

Rewriting (22), we have

$$\widetilde{w}_{t+1} - \widetilde{w}_t = \frac{1}{(2+\phi)\widetilde{w}_t^{\mu} - 1} \big(\Phi(\widetilde{w}_t) - \Psi(\widetilde{w}_t)\big) \tag{25}$$

where

$$\Phi(\widetilde{w}_t) = 2\phi \widetilde{w}_t^{2\mu} - \phi \widetilde{w}_t^{\mu},$$

$$\Psi(\widetilde{w}_t) = (2+\phi)\widetilde{w}_t^{1+\mu} - \widetilde{w}_t - 1.$$

We can see from (25) that the sign of $\widetilde{w}_{t+1} - \widetilde{w}_t$ depends on $\Phi(\widetilde{w}_t) - \Psi(\widetilde{w}_t)$, and thus we focus on the relative sizes of $\Phi(\widetilde{w}_t)$ and $\Psi(\widetilde{w}_t)$ in the following analysis.

It is immediately seen that $\Phi(1) = \Psi(1) = \phi$. It means that $\widetilde{w} = 1$ is a steady state of the dynamics of $\widetilde{w}_t$. Note that the stability of the steady state at $\widetilde{w} = 1$ is unclear at present. Further, we have $\lim_{\widetilde{w}_t \to \infty} (\widetilde{w}_{t+1} - \widetilde{w}_t) < 0$, and thus we see that $\widetilde{w}_t$ does not diverge to infinity.

Next, let us examine the shapes of functions $\Phi(\widetilde{w}_t)$ and $\Psi(\widetilde{w}_t)$. The first and second derivatives of $\Phi(\widetilde{w}_t)$ are

$$\Phi'(\widetilde{w}_t) = 4\phi\mu \widetilde{w}_t^{2\mu-1} - \phi\mu \widetilde{w}_t^{\mu-1},$$

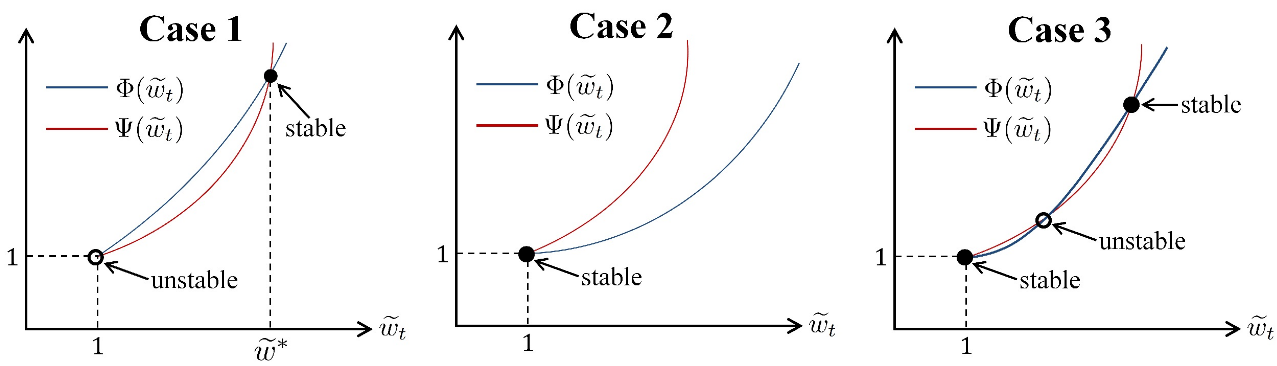
$$\Phi''(\widetilde{w}_t) = 4\phi\mu(2\mu-1)\widetilde{w}_t^{2\mu-2} - \phi\mu(\mu-1)\widetilde{w}_t^{\mu-2},$$

which satisfy $\Phi'(\widetilde{w}_t) > 0$ and $\Phi''(\widetilde{w}_t) > 0$ for all $\widetilde{w}_t \geq 1$ when $\mu > 1/2$. Further we get

$$\Psi'(\widetilde{w}_t) = (2+\phi)(1+\mu)\widetilde{w}_t^{\mu} - 1,$$

$$\Psi''(\widetilde{w}_t) = (2+\phi)(1+\mu)\mu\widetilde{w}_t^{\mu-1},$$

which satisfy $\Psi'(\widetilde{w}_t) > 0$ and $\Psi''(\widetilde{w}_t) > 0$ for all $\widetilde{w}_t > 1$. Therefore, when $\mu > 1/2$, $\Phi(\widetilde{w}_t)$ and $\Psi(\widetilde{w}_t)$ are both convex and increasing in $\widetilde{w}_t$. In the following analysis, we show that three types of steady states may arise depending on the intersections of $\Phi(\widetilde{w}_t)$ and $\Psi(\widetilde{w}_t)$.



To begin with, we can see that $\Phi''(\widetilde{w}_t) - \Psi''(\widetilde{w}_t)$ is decreasing with $\widetilde{w}_t$ when $\mu > 1/2$. Therefore, if $\Phi'(1) > \Psi'(1)$ holds, $\Phi(\widetilde{w}_t)$ intersects with $\Psi(\widetilde{w}_t)$ at $\widetilde{w}_t = 1$ and $\widetilde{w}_t > 1$, as depicted in Case 1 in the above figure. Let us denote by $\widetilde{w}^*$ the steady state value at $\widetilde{w}_t > 1$. From (25), we see that $\widetilde{w}_{t+1} > \widetilde{w}_t$ holds for $\widetilde{w}_t \in (1, \widetilde{w}^*)$, and $\widetilde{w}_{t+1} < \widetilde{w}_t$ holds for $\widetilde{w}_t \in (\widetilde{w}^*, \infty)$. Thus, we see that a steady state at $\widetilde{w}_t = \widetilde{w}^*$ is stable, but that at $\widetilde{w}_t = 1$ is unstable. Using the above first-order derivatives and rewriting $\Phi'(1) > \Psi'(1)$, we have

$$\phi > \frac{1+2\mu}{2\mu-1} \equiv \bar{\phi}.$$

Then we can say that, when $\mu > 1/2$ and $\phi > \bar{\phi}$, there exists a unique stable steady state with positive wage differentials.

Next, we consider $\phi \leq \bar{\phi}$. That is, $\Phi(\widetilde{w}_t)$ is lower than $\Psi(\widetilde{w}_t)$ at least in the neighborhood of $\widetilde{w}_t = 1$. Since $\Phi''(\widetilde{w}_t) - \Psi''(\widetilde{w}_t)$ is decreasing with $\widetilde{w}_t$, if $\Phi''(1) \leq \Psi''(1)$ holds, we have $\Phi(\widetilde{w}_t) < \Psi(\widetilde{w}_t)$ for all $\widetilde{w}_t \in (1, \infty)$. That is, $\Phi(\widetilde{w}_t)$ intersects with $\Psi(\widetilde{w}_t)$ only at $\widetilde{w}_t = 1$, as depicted in Case 2. Assuming that $\mu > 2/3$ and rewriting $\Phi''(1) \leq \Psi''(1)$, we have

$$\phi \leq \frac{1+\mu}{3\mu-2} \equiv \widehat{\phi}.$$

Hence, when $\mu > 2/3$ and $\phi \leq \min\{\bar{\phi}, \widehat{\phi}\}$, $\widetilde{w} = 1$ is a unique stable steady state.

Finally, let us consider $\widehat{\phi} < \phi \leq \bar{\phi}$. Here, we have to suppose that $\mu > (1+\sqrt{5})/4$ in order to satisfy $\widehat{\phi} < \bar{\phi}$. In this case, $\Phi(\widetilde{w}_t)$ may have three intersections with $\Psi(\widetilde{w}_t)$, as depicted in

Case 3, that is, multiple steady states may exist (but not always). Otherwise, $\Phi(\widetilde{w}_t)$ intersects with $\Psi(\widetilde{w}_t)$ only at $\widetilde{w}_t = 1$ (that is, we have Case 2). Instead of giving sufficient conditions for the existence of multiple steady states, we provide a numerical example of this case. See Figure 3 for numerical examples of the three types of steady states. $\square$

References

[1] Acemoglu, D. (1998). "Why do new technologies complement skills? Directed technical change and wage inequality." Quarterly Journal of Economics, vol. 113, issue 4, pp. 1055–1089.

[2] Altonji, J. and Pierret, C. (2001). "Employer learning and statistical discrimination." Quarterly Journal of Economics, vol. 116, issue 1, pp. 313–350.

[3] Andrews, D. and Leigh, A. (2009). "More inequality, less social mobility." Applied Economics Letters, vol. 16, issue 15, pp. 1489–1492.

[4] Autor, D., Katz, L., and Krueger, A. (1998). "Computing inequality: Have computers changed the labor market?" Quarterly Journal of Economics, vol. 113, issue 4, pp. 1169–1213.

[5] Balart, P. (2016). "The increase in college premium and the decline in low-skill wages: A signaling story." Journal of Public Economic Theory, vol. 18, issue 3, pp. 363–384.

[6] Becker, G. (2013). "Why has competition to get into top colleges become much tougher?." Blog Entry (Becker-Posner Blog: http://www.becker-posner-blog.com/2013/01/why-has-competition-to-get-into-top-colleges-become-much-tougher-becker.html).

[7] Bedard, K. (2001). "Human capital vs. signaling models: University access and high school dropouts." Journal of Political Economy, vol. 109, pp. 749–75.

[8] Belman, D. and Heywood, J. (1991). "Sheepskin effects in the returns to education: An examination of women and minorities." Review of Economics and Statistics, vol. 73, issue 4, pp. 720–724.

[9] Bidner, C. (2014). "A spillover-based theory of credentialism." Canadian Journal of Economics, forthcoming.

[10] Blankenau, W. and Camera, G. (2006). "A simple economic theory of skill accumulation and schooling decisions." Review of Economic Dynamics, vol. 9, issue 1, pp. 93–115.

[11] Blankenau, W. and Camera, G. (2009). "Public spending on education and the incentives for student achievement." Economica, vol. 76, issue 303, pp. 505–527.

[12] Caselli, F. (1999). "Technological revolutions." American Economic Review, vol. 89, issue 1, pp. 78–102.

[13] Crifo, P. (2008). "Skill supply and biased technical change." Labour Economics, vol. 15, issue 5, pp. 812–830.

[14] D'Amato, M. and Mookherjee, D. (2012). "Educational signaling, credit constraints and inequality dynamics." CSEF Working Paper No. 311.

[15] Dore, R. (1976). The Diploma Disease. London: George Allen & Unwin.

[16] Ferrer, M. (2005). "Signaling, inequality and the social structure." Economica, vol. 72, issue 287, pp. 515–529.

[17] Futagami, K. and Ishiguro, S. (2004). "Signal-extracting education in an overlapping generations model." Economic Theory, vol. 24, issue 1, pp. 129–146.

[18] Galor, O. and Moav, O. (2000). "Ability-biased technological transition, wage inequality, and economic growth." Quarterly Journal of Economics, vol. 115, issue 2, pp. 469–497.

[19] Hendel, I., Shapiro, J., and Willen, P. (2005). "Educational opportunity and income inequality." Journal of Public Economics, vol. 89, issue 5-6, pp. 841–870.

[20] Hopkins, E. (2012). "Job market signaling of relative position, or Becker married to Spence." Journal of European Economic Association, vol. 10, issue 2, pp. 290–322.

[21] Hungerford, T. and Solon, G. (1987). "Sheepskin effects in the returns to education." Review of Economics and Statistics, vol. 69, issue 1, pp. 175–177.

[22] Jaeger, D. and Page, M. (1996). "Degrees matter: New evidence on sheepskin effects in the returns to education." Review of Economics and Statistics, vol. 78, issue 4, pp. 733–740.

[23] Katz, L. and Murphy, K. (1992) "Changes in relative wages, 1963–1987: Supply and demand factors." Quarterly Journal of Economics, vol. 107, issue 1, pp. 35–78.

[24] Lang, K. and Kropp, D. (1986). “Human capital versus sorting: The effects of compulsory attendance laws.” Quarterly Journal of Economics, vol. 101, issue 3, pp. 609–624.

[25] Miyake, A., Muro, K., Nakamura, T., and Yasuoka, M. (2009) “Between- and within-group wage inequalities, and the advent of new technology.” Journal of Economic Inequality, vol. 7, issue 4, pp. 387–394.

[26] Nakamura, H. (2013) “Wages of regular and irregular workers, the price of education, and income inequality.” Journal of Economic Inequality, vol. 11, issue 4, pp. 517–533.

[27] Ordine, P. and Rose, G. (2011) “Inefficient self-selection into education and wage inequality.” Economics of Education Review, vol. 30, issue 4, pp. 582–597.

[28] Park, J. (1999). “Estimation of sheepskin effects using the old and the new measures of educational attainment in the Current Population Survey.” Economics Letters, vol. 62, issue 2, pp. 237–240.

[29] Popham, W. J. (2001). “Teaching to the test.” Educational Leadership, vol. 58 issue 6, pp. 16–20.

[30] Restuccia, D. and Vandenbroucke, G. (2013). “The evolution of education: A macroeconomic analysis.” International Economic Review, vol. 54, issue 3, pp. 915–936.

[31] Solon, G. (2004). “A model of intergenerational mobility variation over time and place.” In Corak, M. (ed.) Generational Income Mobility in North America and Europe, Cambridge: Cambridge University Press, Ch. 2, pp. 38–47.

[32] Spence, M. (1973). “Job market signaling.” Quarterly Journal of Economics, vol. 87, pp. 355–374.

[33] Spence, M. (1974). “Competitive and optimal responses to signals: An analysis of efficiency and distribution.” Journal of Economic Theory, vol. 7, issue 3, pp. 296–332.

[34] Yuki, K. (2009). “Education, signaling, and wage inequality in a dynamic economy.” MPRA Working Paper No. 16982.