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Abstract

In this paper, we investigate a new concept of a market's *commodity-information structure* (a partition of the set of real goods that are treated as one commodity for market exchanges) and technologies relating to it, *commodity-information technologies*. Using this concept, we can always affirmatively answer the *market viability problem*, concerning the existence of general equilibrium even when information asymmetry among agents such as adverse selection prevails in the economy. Some Pareto-optimality problems and policy implications are also discussed.

Keywords: General Equilibrium Model, Asymmetric Information, Adverse Selection, Market Viability Problem, Commodity-information Structure

JEL classification: C62; D51; D82

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1 Introduction

Adverse selection problems (e.g., Akerlof (1970), Rothschild and Stiglitz (1976),) have traditionally been treated through static partial equilibrium arguments. The usual argument is that the price mechanism in a market with asymmetric information is not sufficient to assure the existence of an equilibrium if agents rationally expect their *average* receipts. From a general equilibrium theoretic viewpoint, however, partial equilibrium arguments on market unraveling or the adverse selection problem ignore several important total-market closed-income-circulation features of an economy. Even in adverse-selection cases, the owners of high-quality commodities must sell them elsewhere, so their opportunity costs in giving up trading high-quality goods must be treated endogenously.

Problems such as adverse selection assume that information asymmetry is a structure that depends not only on individuals but also on their selling and/or buying standpoints. This premise influences individuals to change their actions in a way that cannot be described under the traditional general equilibrium structure. In a recent paper (Urai et al. 2013), the new concept of the *commodity-information structure* of a market (a partition of the set of real goods that are treated as one commodity for market exchanges) is introduced, on which can be based the ordinary static general equilibrium arguments and settings for analyzing asymmetric information problems such as adverse selection.[1] We showed the existence of equilibria and examples of the non-existence of equilibria, discussed some equilibrium-optimality problems, and emphasized that the existence of an *exogenous upper bound* for market-trade amounts is essential for the existence and optimality of the general equilibrium.

The purpose of this paper is to construct a model that can treat such a market-trade upper bound *endogenously*, and to provide one of the most general answers to the *market-viability problem* by investigating conditions under which market-disappearance never happens, even with information asymmetry. The key concept used here is *commodity-information technology.* In this paper, we suppose that each commodity-information structure has an associated class of technologies that enable each agent (a possible seller) to supply in certain amounts of their outputs as a unit of market commodity, and that such a process necessarily uses a positive amount of real goods or services. We emphasize that one should not view such a technology as an ad hoc or special device for an asymmetric information economy. Even in the standard setting of general equilibrium theory, it would be interesting (from both an historical and realistic perspective) to recognize the *market* as a special place where the available *commodity* is completely different from the goods and services with which we are familiar. This paper gives a radical affirmative answer to the *market mechanism* even in such cases, that is, whether *a situation exists where all buyers obtain exactly what they expect, (given rational expectations that reflect the aggregate real features of society), and where the ordinary equilibrium conditions for all agents and the demand-supply are*

[1] We believe that the approaches of Dubey et al. (2000) and Bisin and Gottardi (1999) are groundbreaking. Our model might (technically) be classified as a complete static version of their futures-market or probabilistic-dynamics models.

satisfied.

If we view the market as a mechanism that is administrated by the government, some parts of the commodity-information technologies may be treated as policy variables. If we consider the market as an autonomous system, commodity-information technology may also be considered one of the fundamental factors behind the market commodity-information structure itself. For general equilibrium arguments, the commodity-information technology gives a normative and descriptive unified viewpoint of both symmetric and asymmetric information economies. From this perspective, it is worth noting that *market unraveling* under asymmetric information should not be seen as the disappearance or collapse of competitive markets, but as a discussion of complementary changes and replacements among several individual markets based on the strong *viability* of the market mechanism.

2 The Model

Denote the set of real numbers by R, n-dimensional Euclidean space by R^n, the non-negative orthant of R^n, $\{x = (x_k)_{k=1}^n \in R^n | \, x_k \geqq 0, k = 1, \ldots, n\}$, by R^n_+, and the strictly positive orthant of R^n, $\{x = (x_k)_{k=1}^n \in R^n | \, x_k > 0, k = 1, \ldots, n\}$, by R^n_{++}. For each finite set A, we denote by $\sharp A$ the number of elements of A.

Market Structure and Commodity Information Structure: There are m types of *consumers* and n types of *producers* indexed by $i = 1, \ldots, m$ and $j = m+1, \ldots, m+n$. In this model, ℓ types of *real goods and services* are indexed by $k = 1, \ldots, \ell$ and λ types of *market commodities* are indexed by $\kappa = 1, \ldots, \lambda$. Let L be the set of real commodity indices $\{1, 2, \ldots, \ell\}$, partitioned as $L = L_1 \cup L_2 \cup \cdots \cup L_\lambda$ where $L_\kappa \cap L_{\kappa'} = \emptyset$ for all $\kappa \neq \kappa'$. This describes a market that cannot distinguish between real commodities $k \in L$ and $k' \in L$ if and only if k and k' belong to the same L_κ. We assume that $L_\kappa \neq \emptyset$ for all $\kappa = 1, \cdots, \lambda$ and call the partition $\{L_1, \ldots, L_\lambda\}$ the *market commodity-information structure.* (In our model, agents as sellers may have finer individual information structures than the market, but as buyers, they must follow the common market commodity-information. Here, we can find the structure of the seller-buyer information asymmetry.) For each $\kappa = 1, \ldots, \lambda$, we denote by R^{L_κ} the subspace of R^ℓ containing the elements whose k-th coordinate is 0 if $k \notin L_\kappa$. Since we assume that agents can distinguish all ℓ kinds of real commodity, we treat both their consumption behavior and production behavior as points in R^ℓ and the market demand and market supply as points in R^λ.

Agents as Buyers and Expectations of Real Receipts: In our setting, commodity κ in the market is really a mixture of $\sharp L_\kappa$ kinds of real commodity. We assume that agents as buyers have an *expectation of their real receipts* for each of their trade (demand) contracts before choosing their actions. The expectation for each agent is assumed to be an identity function of the following given aggregate parameter. For each market commodity κ, let $s_\kappa > 0$ be the aggregate amount contracted to be supplied to the market, and let $\hat{s}^\kappa \in R_+^{L_\kappa} \subset R_+^\ell$ be the aggregate amount that is actually supplied. Then, for each unit of commodity κ that is demanded, an average amount $\hat{s}^\kappa/s_\kappa \in R_+^{L_\kappa}$ is delivered, and that amount will be expected by all members of the economy. (In this sense, we are considering a rational expectation equilibrium.) To simplify the model, we assume that $\|\hat{s}^\kappa/s_\kappa\| = 1$ and let $s^\kappa = \hat{s}^\kappa/s_\kappa$ for each $\kappa = 1, 2, \dots, \lambda$.[2]

Agents as Sellers and Commodity-information Technologies: Given a *market commodity-information structure* $\{L_1, \dots, L_\lambda\}$, each agent must sell their real goods as λ types of market commodity. Therefore, they should standardize real goods to sell in the market depending on the commodity-information structure. We describe such situations by assuming that each agent has a certain kind of technology for standardizing real goods. For example, the function $F : R_+^\ell \to R_+^\ell$ defined as $F(v_1, \dots, v_\ell) = (v_1, \cdots, v_\ell)$ may be a candidate for such technologies. An agent having this F can standardize real goods with no cost. However, in the real-world market economy, no one can standardize and supply their goods with no cost. Hence, we put some natural restrictions on these standardizing technologies.

Each agent $i = 1, \dots, n+m$ has a function $F_i : R_+^\ell \to R_+^\ell$ which satisfies the following conditions:

(C1) F_i is a continuous function.

(C2) F_{ik} is concave function for all $k = 1, \dots, \ell$, where F_{ik} is k-th coordinate of F_i.

(C3) F_{ik} is monotone for all $k = 1, \dots, \ell$.

(C4) $F_i(v) \leq v$ for all $v \in R_+^\ell$.

(C5) For each κ such that $\sharp L_\kappa \geqq 2$ and for each sequence $\{v^\nu\}_{\nu=1}^\infty \subset R^\ell$, if $\|\sum_{k \in L_\kappa} F_{ik}(v^\nu)\| \to \infty$ as $\nu \to \infty$ then there exists some $L_{\kappa'}$ such that $|\sum_{k \in L_\kappa \cup L_{\kappa'}} (v_k^\nu - F_{ik}(v^\nu))| \to \infty$ as $\nu \to \infty$.

We call F_i for $i = 1, \dots, n+m$ agent i's *commodity-information technology*. As we will formalize later, each agent supplies their goods and services to the market as a vector $(\sum_{k \in L_1} F_{ik}(v), \dots, \sum_{k \in L_\lambda} F_{ik}(v))$ in R_+^λ. Therefore, condition (C5) says that each agent cannot sell their real goods in the market with commodity-information structure $\{L_1, \dots, L_\lambda\}$ with no cost. Note that (C5) requires such standardizing costs to exist only if L_κ is not a singleton set. If some L_κ is a singleton, then there is no difference between a real good and a market commodity in the corresponding market. Therefore, in such cases, the argument about standardizing costs reduces to the argument about usual production costs.[3] As noted, condition (C5) allows such arguments.

[2] Note that $\|x\| = \sum_{i=1}^\ell |x_i|$ for $x = (x_1, \dots, x_\ell) \in R^\ell$. For our results (the existence of equilibrium), the boundedness of $\hat{s}^\kappa/s_\kappa$ is sufficient.

[3] This treatment of a single good market never harms existence of the equilibrium since we can make trade quantities bounded in such markets without loss of generality. We formally show this in the existence proof.

Producers' Problems: Producer $j = m+1, \ldots, m+n$ has production technology $Y_j \subset R^\ell$ such that Y_j is closed, Y_j is convex, and $0 \in Y_j$. Given a price $p \in \Delta = \{(p_1, \ldots, p_\lambda) | \, p_1 \geqq 0, \ldots, p_\lambda \geqq 0, \sum_{\kappa=1}^{\lambda} p_\kappa = 1\}$ and the expectation of their receipts for each commodity through market $s^1 \in R_+^{L_1}, \ldots, s^\lambda \in R_+^{L_\lambda}$, $\|s^1\| = 1, \ldots, \|s^\lambda\| = 1$, producer j chooses a production plan $y_j = y_j^+ - y_j^- \in Y_j$ with sales and purchasing plans $z_j \in R^\lambda$ and $v_j^+ \in R_+^\ell$, so that (y_j, z_j, v_j^+) solves the maximization problem below: [4]

$$\max \quad p \cdot z_j^+ - p \cdot z_j^- \tag{1}$$

subject to

$$z_j^+ \leqq (\sum_{k \in L_1} F_{jk}(v_j^+), \ldots, \sum_{k \in L_\lambda} F_{jk}(v_j^+)) \tag{2}$$

$$v_j^+ + y_j^- = y_j^+ + z_{j1}^- s^1 + z_{j2}^- s^2 + \cdots + z_{j\lambda}^- s^\lambda, \tag{3}$$

$$y_j = y_j^+ - y_j^- \in Y_j, \tag{4}$$

$$v_j^+ \in R_+^\ell, \quad z_j^\pm = (z_{j1}^\pm, \ldots, z_{j\lambda}^\pm) \in R_+^\lambda, \quad z_j = z_j^+ - z_j^-. \tag{5}$$

Consumers' Problems: Consumer $i = 1, \ldots, m$ has an initial endowment $\omega_i \in R_{++}^\ell$ of real commodities and a consumption set $X_i \subset R^\ell$. We assume that X_i is a non-empty closed convex subset bounded from below such that $X_i \supset R_+^\ell$ for each i. Given a price $p \in \Delta$ and the expectation of their receipts for each commodity in the market, $s = (s^1, \ldots, s^\lambda) \in \prod_{\kappa=1}^{\lambda} R_+^{L_\kappa} \subset (R_+^\ell)^\lambda$, where $\|s^\kappa\| = 1$ for $\kappa = 1, \ldots, \lambda$, consumer i chooses consumption plan x_i with market transaction plans z_i and v_i^+, so that (x_i, z_i, v_i^+) solves the following maximization problem:

$$\max \quad u_i(x_i) \tag{6}$$

subject to

$$z_i^+ \leqq (\sum_{k \in L_1} F_{ik}(v_i^+), \ldots, \sum_{k \in L_\lambda} F_{ik}(v_i^+)) \tag{7}$$

$$v_i^+ + x_i^+ = x_i^- + \omega_i + z_{i1}^- s^1 + z_{i2}^- s^2 + \cdots + z_{i\lambda}^- s^\lambda, \tag{8}$$

$$p \cdot z_i^- = p \cdot z_i^+ + \sum_{j=1}^{n} \theta_{ij} \pi_j(p, s), \tag{9}$$

$$x_i \in X_i, \tag{10}$$

$$v_i^+ \in R_+^\ell, \quad z_i^\pm = (z_{i1}^\pm, \ldots, z_{i\ell}^\pm) \in R_+^\lambda, \quad z_i = z_i^+ - z_i^-, \tag{11}$$

where u_i is a continuous concave utility function of i, $\pi_j(p, s)$ is the profit of j under price p and expectation $s = (s^\kappa)_{\kappa=1}^\lambda$ (under the maximization problem (1) – (5)) and θ_{ij} denotes consumer i's share of the profit of producer j (a non-negative real number satisfying $\sum_{i=1}^m \theta_{ij} = 1$ for each j).

[4] The variables x^+ and x^- always represent $x^+ = \sup\{x, 0\}$ and $x^- = \sup\{-x, 0\}$. We sometimes use the notation x^+ and/or x^- without referring to x to emphasize the non-negativity constraint.

Equilibrium: Denote by $\mathcal{E} = ((X_i, \omega_i, u_i, F_i, (\theta_{ij})_{j=1}^n)_{i=1}^m, (Y_j, F_j)_{j=m+1}^{m+n})$ the economy we have described above. An *equilibrium* for economy $\mathcal{E}$ is $((x_i, z_i, v_i^+)_{i=1}^m, (y_j, z_j, v_j^+)_{j=m+1}^{m+n}) \in \prod_{i=1}^m (X_i \times R^\lambda \times R_+^\ell) \times \prod_{j=m+1}^{m+n} (Y_j \times R^\lambda \times R_+^\ell)$ and $(p, s) \in \Delta \times \prod_{\kappa=1}^\lambda R_+^{L_\kappa} \subset R_+^\lambda \times (R_+^\ell)^\lambda$ satisfying (1) – (11) and the *market clearing condition* (12) with *expectation specification* (13) for each $\kappa \in \{1, \ldots, \lambda\}$ and $k \in L_\kappa$:

$$\sum_{i=1}^{m+n} z_{i\kappa} = 0, \tag{12}$$

$$\frac{\sum_{i=1}^{m+n} F_{ik}(v_i^+)}{\sum_{i=1}^{m+n} z_{i\kappa}^+} = s^{\kappa k} \quad \text{as long as} \quad \sum_{i=1}^{m+n} z_{i\kappa}^+ > 0. \tag{13}$$

We use the notation $z_i = (z_{i1}, \ldots, z_{i\lambda})$ and $s^{\kappa k}$ for the k-th coordinate of s^κ. Note that we only consider Eq. (13) when $\sum_{i=1}^{m+n} z_{i\kappa}^+ > 0$. Hence, if $\sum_{i=1}^{m+n} z_{i\kappa}^+ = 0$, we have no restriction on the expectation specifications.

3 The Existence Theorem

We now state a general-equilibrium existence theorem for economies with asymmetric information:

Theorem 1. *Economy $\mathcal{E} = ((X_i, \omega_i, u_i, F_i, (\theta_{ij})_{j=m+1}^{m+n})_{i=1}^m, (Y_j, F_j)_{j=m+1}^{m+n})$ has an equilibrium, $((x_i^*, z_i^*, v_i^{+*})_{i=1}^m, (y_j^*, z_j^*, v_j^{+*})_{j=m+1}^{m+n}, p^*, s^*)$, if the following conditions are satisfied:*

(Consumers) *Each consumer $i = 1, \ldots, m$ has a non-empty closed convex consumption set $X_i \supset R_+^\ell$ that is bounded from below with a convex preference induced by a strictly monotone (i.e., $x' \geqq x$, $x' \neq x$ implies $u_i(x') > u_i(x)$) and continuous utility function $u_i : X_i \to R_+$ and a strictly positive initial endowment $\omega_i \in R_{++}^\ell$.*

(Producers) *For each $j = m+1, \ldots, m+n$, Y_j is a closed convex set containing* 0.

(Attainable Set) *The attainable set for real state $\sum_{i=1}^m X_i \cap (\sum_{j=m+1}^{m+n} Y_j + \sum_{i=1}^m \omega_i) \subset R^\ell$ is bounded.*

(Commodity-information Technologies) *Each function $F_i : R_+^\ell \to R_+^\ell$, $i = 1, \ldots, m+n$ satisfies conditions (C1) – (C5).*

There are some difficulties with the existence proof. In our setting, we must treat demand and supply (z_i and v_i^+) as being distinguished from consumption and production (x_i and y_j), as in the case of transactions in asset markets, and treat producers or consumers whose actions are restricted not only by their technologies or standard budgets but also by their buying and selling constraints. Since transaction plans z_i and v_i^+ are not bounded, and the expectation $s = (s^\kappa)_{\kappa=1}^\lambda$ decides the estimation of real receipts, continuity of demands with respect to prices and expectations may not be warranted in some boundary cases. However, there is no appropriate reason for limiting such quantities of transactions. In this paper, we overcome such problems by (C5), which is a natural condition for commodity-information technologies. Condition (C5) is essential for the result of the theorem. For example, if we take F_i as $F_i(v) = (v_1, \ldots, v_\ell)$ for each agent $i = 1, \ldots, m+n$, then we can construct non-existence examples.[5] Indeed, this form of $F_i(v)$ satisfies conditions (C1)–(C4) but not (C5).

4 Proof of Theorem

Producers: We denote by Δ^κ the set of all real-receipt expectations, so that $\Delta^\kappa = \{s^\kappa \in R^{L_\kappa} \,|\, \|s^\kappa\| = 1\}$ for $\kappa = 1, \ldots, \lambda$. As stated in the previous section, each real technology $Y_j \subset R^\ell$, $j = m+1, \ldots, m+n$, is assumed to be closed and convex, and to contain 0, with F_j satisfying continuity (C1) and concavity (C2). Hence, we can check that the set of all solutions to the maximization problem (1)–(5) under price $p \in \Delta$ and expectation $s = (s^\kappa)_{\kappa=1}^\lambda \in \prod_{\kappa=1}^\lambda \Delta^\kappa$, $\eta_j(p, s) \subset R^\ell$ is closed and convex. Now, assume an arbitrarily large number $t > 0$ and consider maximization problem (1) subject to (2)–(5) with $(y_j, v_j^+, z_j^+, z_j^-) \in [-t, t]^\ell \times [0, t]^{\ell+2\lambda}$. The maximization problem is restricted to $[-t, t]^\ell \times [0, t]^{\ell+2\lambda}$. We denote by $\eta_j^t(p, s)$ the set of solutions to the restricted maximization problem. The non-emptiness, closedness, and convexity of $\eta_j^t(p, s)$ are clear. We can also prove that the correspondence $\eta_j^t : \Delta \times \prod_{\kappa=1}^\lambda \Delta^\kappa \to R^{2\ell+2\lambda}$ has a closed graph. Indeed, the constraint correspondence $(p, s) \mapsto \{\, (y_j, v_j^+, z_j^+, z_j^-) \,|\, (y_j, v_j^+, z_j^+, z_j^-) \in [-t, t]^\ell \times [0, t]^{\ell+2\lambda}$ satisfies (2)–(5) under $(p, s)\,\}$ has a closed graph and is lower semi-continuous, and thus also continuous. Note that the continuity (C1) and monotonicity (C3) of F_j are used to check this. Hence, Berge's maximum theorem (cf. Debreu (1959), p. 19, Theorem (4)) is applicable. In this case, it may simultaneously be confirmed that the profit function of this truncated problem, $\pi_j^t(p, s)$, is continuous.

Consumers: As in the producer case, the set of all solutions to the maximization problem (6), subject to (7)–(11) under $p \in \Delta$ and $s \in \prod_{\kappa=1}^\lambda \Delta^\kappa$, $\xi_i(p, s)$, is closed and convex. Denote by $\xi_i^t(p, s)$ the set of solution trades to maximization problem (6) subject to (7)–(11) with $(x_i, v_i^+, z_i^+, z_i^-) \in [-t, t]^\ell \times [0, t]^{\ell+2\lambda}$, and with each profit $\pi_j(p, s)$ in Eq. (9) replaced by $\pi_j^t(p, s)$, which is the maximized profit of producers in the truncated maximization problem. We assume that each consumer has a strictly positive initial endowment, $\omega_i \in R_{++}^\ell$. Then it is also possible to verify that the correspondence ξ_i^t : $\Delta \times \prod_{\kappa=1}^\lambda \Delta^\kappa \to R^{2\ell+2\lambda}$ is non-empty closed convex valued and has a closed graph. The non-emptiness, closedness, and convexity of $\xi_i^t(p, s)$ are easy to confirm. For the closed graph of $\xi_i^t : \Delta \times \prod_{\kappa=1}^\lambda \to$

[5] See Urai, Yoshimachi, and Shiozawa (2013).

$R^{2\ell+2\lambda}$, check that the constraint correspondence $(p,s) \mapsto \{\, (x_i, v_i^+, z_i^+, z_i^-) \,|\, (x_i, v_i^+, z_i^+, z_i^-) \in [-t,t]^\ell \times [0,t]^{\ell+2\lambda}$ satisfies (7)–(11) under $(p,s)\,\}$, where π_j in (9) is replaced by π_j^t, has a closed graph, and is lower semi-continuous. Then apply Berge's maximum theorem again.

Fixed Points and Limit Arguments: Take a number $t > 0$ sufficiently large for the bounded attainable set to be a subset of the interior of $[-t,t]^\ell$, and restrict the individual maximization problems (1)-(5) and (6)-(11) to the set $[-t,t]^\ell \times [0,t]^{\ell+2\lambda}$. We have defined correspondences η_j^t and ξ_i^t to be sets of solutions $(x_i, v_i^+, z_i^+, z_i^-)$ and $(y_j, v_j^+, z_j^+, z_j^-)$ for each maximization problem under (p,s). Consider the product map Φ of these correspondences:

$$\Phi : \Delta \times \prod_{\kappa=1}^{\lambda} \Delta^\kappa \ni (p,s) \mapsto \prod_{i=1}^{m} \xi_i^t(p,s) \;\times \prod_{j=m+1}^{m+n} \eta_j^t(p,s) \subset \big([-t,t]^\ell \times [0,t]^{\ell+2\lambda}\big)^{m+n}. \tag{14}$$

The mapping Φ has a closed graph. Define a price-expectation manipulation correspondence as follows:

$$\Psi : ([0,t]^{\ell+2\lambda})^{m+n} \ni (v_i^+, z_i^+, z_i^-)_{i=1}^{m+n} \mapsto \Theta((z_i)_{i=1}^{m+n}) \times \Xi((v_i^+)_{i=1}^{m+n}) \subset \Delta \times \prod_{\kappa=1}^{\lambda} \Delta^\kappa, \tag{15}$$

where Θ denotes the price manipulation mapping such that, for each $(z_i)_{i=1}^m$, $\Theta((z_i)_{i=1}^m)$ assigns a set of prices $\{p \in \Delta |\, \forall q \in \Delta, q \cdot \sum_{i=1}^{m+n} -z_i \leqq p \cdot \sum_{i=1}^{m+n} -z_i\}$, and Ξ is the correspondence that assigns the real mixture ratio of the goods for each market. More precisely, we define the κ-th coordinate of Ξ by

$$\Xi_\kappa((v_i^+)_{i=1}^{m+n}) = \frac{\sum_{i=1}^{m+n} \mathrm{pr}_{L_\kappa}(F_i(v_i^+))}{\sum_{i=1}^{m+n}\big(\sum_{k \in L_\kappa} F_{ik}(v_i^+)\big)}, \tag{16}$$

as long as $\sum_{i=1}^{m+n}\big(\sum_{k\in L_\kappa} F_{ik}(v_i^+)\big) \neq 0$, and otherwise by $\Xi_\kappa((v_i, w_i)_{i=1}^{m+n}) = \Delta^\kappa$. (We use the notation pr_{L_κ} for the projection onto subspace R^{L_κ} of R^ℓ for each $\kappa = 1, \ldots, \lambda$.) Note that the right hand side of Eq. (16) is always an element of Δ^κ when $\sum_{i=1}^{m+n}\big(\sum_{k\in L_\kappa} F_{ik}(v_i^+)\big) \neq 0$. It is routine to check that Θ is a non-empty closed convex valued correspondence with a closed graph and that the correspondence Ξ is non-empty closed convex valued. It is also easy to check that Ξ has a closed graph since the right hand side of Eq. (16) is continuous when $\sum_{i=1}^{m+n}\big(\sum_{k\in L_\kappa} F_{ik}(v_i^+)\big) \neq 0$. Now, the product of the mappings Φ and Ψ,

$$\Phi \times \Psi : \Delta \times \Big(\prod_{\kappa=1}^{\lambda} \Delta^\kappa\Big) \times \big([-t,t]^\ell \times [0,t]^{\ell+2\lambda}\big)^{m+n} \;\to\; \Delta \times \Big(\prod_{\kappa=1}^{\lambda} \Delta^\kappa\Big) \times \big([-t,t]^\ell \times [0,t]^{\ell+2\lambda}\big)^{m+n}, \tag{17}$$

is a non-empty closed convex valued correspondence with a closed graph. By Kakutani's fixed point theorem, $\Phi \times \Psi$ has a fixed point $\big(p^t, s^t, (x_i^t, v_i^t, z_i^t)_{i=1}^m, (y_j^t, v_j^t, z_j^t)_{j=m+1}^{m+n}\big) \in \Delta \times \big(\prod_{\kappa=1}^{\lambda} \Delta^\kappa\big) \times ([-t,t]^\ell \times [0,t]^{\ell+2\lambda})^{m+n}$. Equation (9) with (1) gives Walras' Law:

$$\forall (p,s) \in \Delta \times \prod_{\kappa=1}^{\lambda} \Delta^\kappa, \quad \forall z \in -\sum_{i=1}^{m} \xi_i^t(p,s) - \sum_{j=m+1}^{m+n} \eta_j^t(p,s), \quad p \cdot z = 0. \tag{18}$$

Under the standard argument, this means that, by the definition of Θ, the summation of $(z_i^t)_{i=1}^{m+n}$ must satisfy $q \cdot (-\sum_{i=1}^{m+n} z_i^t) \leqq p^t \cdot (-\sum_{i=1}^{m+n} z_i^t) = 0$ for all $q \in \Delta$, and so for each $\kappa = 1, \ldots, \lambda$, the κ-th coordinates of $(z_i^t)_{i=1}^{m+n}$, $(z_{i\kappa}^t)_{i=1}^{m+n}$, must be such that $-\sum_{i=1}^{m+n} z_{i\kappa}^t \leqq 0$, where $-\sum_{i=1}^{m+n} z_{i\kappa}^t < 0$ if and only if the price of κ-th commodity, p_κ^t, equals 0.

Considering that each individual maximizes his profit (or utility), condition (2) (or (7)) is satisfied with equality. Therefore, it follows that the state $\left(p^t, s^t, (x_i^t, v_i^t, z_i^t)_{i=1}^m, (y_j^t, v_j^t, z_j^t)_{j=m+1}^{m+n}\right)$ satisfies (12) and (13). We have that the state $\left(p^t, s^t, (x_i^t, v_i^t, z_i^t)_{i=1}^m, (y_j^t, v_j^t, z_j^t)_{j=m+1}^{m+n}\right)$ satisfies (2) with equality, (3), (7) with equality, (8), (12) and (13), and hence it follows that the real state $((x_i^t)_{i=1}^m, (y_j^t)_{j=m+1}^{m+n})$ satisfies $\sum_{i=1}^m x_i^t = \sum_{j=m+1}^{m+n} y_j^t + \sum_{i=1}^m \omega_i$, or that x_i^t and y_j^t are in the attainable set which is bounded. We call $\left(p^t, s^t, (x_i^t, v_i^t, z_i^t)_{i=1}^m, (y_j^t, v_j^t, z_j^t)_{j=m+1}^{m+n}\right)$ a t-equilibrium state.

Since we take $t > 0$ sufficiently large for the bounded attainable set to be a subset of the interior of $[-t, t]^\ell$, all x_i^t or y_j^t are interior points of $[-t, t]^\ell$. Therefore, the t-equilibrium state $\left(p^t, s^t, (x_i^t, v_i^t, z_i^t)_{i=1}^m,\right.$ $\left.(y_j^t, v_j^t, z_j^t)_{j=m+1}^{m+n}\right)$ is not an equilibrium of the original economy $\mathscr{E}$ only when $(v_i^t, z_i^t) \in [0, t]^\ell \times [-t, t]^\lambda$ is a boundary point of $[0, t]^\ell \times [-t, t]^\lambda$ for some $i = 1, \ldots, m+n$. Hence, if (v_i^t, z_i^t) is bounded for all $i = 1, \ldots, n+m$, then an equilibrium of the original economy $\mathscr{E}$ exists.

Suppose that (v_i^t, z_i^t) is not bounded for some i. If $L_\kappa = \{k\}$ then $s_k^\kappa = 1$ and restriction (3) requires that $v_{jk}^+ - z_{j\kappa}^- = y_{jk}$. (The same argument is relevant for consumers.) Therefore, we can also suppose that (v_{ik}^t, z_k^t) is bounded for such singleton markets L_κ without loss of generality. This implies that $\|\sum_{k \in L_\kappa} F_{ik}(v_i^{t+})\| \to \infty$ as $t \to \infty$ for some $\kappa \in \{1, \ldots, \lambda\}$ such that $\sharp L_\kappa \geqq 2$. Note first that

$$\sum_{i=1}^{m}\Big(\sum_{k \in L_\kappa \cup L_{\kappa'}} (\omega_{ik} - x_{ik}^t)\Big) + \sum_{j=m+1}^{m+n}\Big(\sum_{k \in L_\kappa \cup L_{\kappa'}} y_{ik}^t\Big) \geqq \sum_{k \in L_\kappa \cup L_{\kappa'}} v_{ik}^{t+} - (z_{i\kappa}^{t+} + z_{i\kappa'}^{t+}) \tag{19}$$

holds for all $i = 1, \ldots, m+n$ and all $\kappa, \kappa' = 1, \ldots, \lambda$ by considering conditions (2), (3), (7), (8), (12), and (C4). Moreover, the right hand side of Eq. (19) equals $\sum_{k \in L_\kappa \cup L_{\kappa'}} v_{ik}^{t+} - \sum_{k \in L_\kappa \cup L_{\kappa'}} F_{ik}(v_i^{t+})$, since condition (2) (or (7)) holds with equality. However, if $\|\sum_{k \in L_\kappa} F_{ik}(v_i^{t+})\| \to \infty$ as $t \to \infty$ for some $\kappa \in \{1, \ldots, \lambda\}$ such that $\sharp L_\kappa \geqq 2$, then condition (C5) requires that there exists some $L_{\kappa'}$ such that $\sum_{k \in L_\kappa \cup L_{\kappa'}} v_{ik}^{t+} - \sum_{k \in L_\kappa \cup L_{\kappa'}} F_{ik}(v_i^{t+}) \to \infty$ as $t \to \infty$. This implies that the right-hand side and hence the left-hand side of Eq. (19) tend to ∞ as $t \to \infty$, contradicting the fact that x_i^t and y_j^t are bounded.

5 Concluding Remarks

1. In this paper, we concentrate our attention on the case where the average amount of real goods and/or services is delivered to each buyer. To give an explanation of this averaging process for each commodity, we use probabilistic arguments including the law of large numbers for cases with daily perishable goods for consumers during a month, and input commodities for big buyers such as companies.

Such arguments are plausible when the cost structure (C5) for commodity-information technology is not essential for small market transaction amounts.

2. Cost condition (C5) may not necessarily mean the existence of total welfare loss under the use of commodity-information technologies. Indeed, we may assume $F_{ik}(v_i^+) = v_{ik}^+$ for each $i = 1, \ldots, m+n$ and $k = 1, \ldots, \ell$, where $F_{ik}(v_i^+)$ represents the k-th coordinate of value $F_i(v_i^+)$, as long as $|v_{ik}^+|$ is not too big (e.g., it is in the real attainable set). In this sense, the existence of the equilibrium result in our previous paper (Urai et al. 2013) may be identified as a special case of the theorem in this paper by defining F_{ik}, $i = 1, \ldots, m+n$ and $k = 1, \ldots, \ell$, as $F_{ik}(v_i^+) = v_{ik}^+$ if $v_{ik}^+ \leqq b$ and $F_{ik}(v_i^+) = b$ otherwise, for an arbitrary transaction upper bound b.

3. As we state in the introduction, some parts of the commodity-information structure and technologies may be considered to be administrated by the government. By regarding commodity-information structures and/or technologies as policy variables, we can implement some comparative statics analysis.

4. Under the equilibrium in this paper, given the structure of commodity-information technology, the transaction upper bound is treated endogenously so that we succeed in describing the situation that "bad money drives out good" as a natural equilibrium situation of the model. It would also be possible, however, to ask what defines the commodity-information technology. Specifically, while we have treated the market commodity information structure $\{L_1, \ldots, L_\lambda\}$ as given, it would be desirable for the concepts of market equilibrium, market information structure, and commodity-information technologies to be mutually related and to be treated simultaneously as an equilibrium under a more general framework.

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