Discussion Papers In Economics And Business

Growth effect of bubbles in a non-scale endogenous growth model with in-house R&D

Kizuku Takao

Discussion Paper 14-11

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Growth effect of bubbles in a non-scale endogenous growth model with in-house R&D

Kizuku Takao

Discussion Paper 14-11

February 2014

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Growth effect of bubbles in a non-scale endogenous growth model with in-house R&D*

Kizuku Takao†

Abstract

This paper provides a theoretical explanation for why the presence of asset bubbles can lead to higher economic growth in concurrence with high consumption by using a simple endogenous growth model. In the model economy, long-lived value-maximizing firms continuously improve the quality of their specific products through in-house R&D, while at the same time new firms also enter into the market. Due to an absence of intergenerational altruism, asset bubbles can exist as pyramid schemes whose value is not backed by fundamental value. The presence of asset bubbles then leads to higher interest rates. This requires product proliferation to be impeded, which would result in an increase in the demand for differentiated goods at the level of an individual firm. A larger scale of production at the level of an individual firm can encourage in-house R&D of firms and promote economic growth.

Keywords: Bubbles, R&D, Overlapping generations

JEL classification: E44, O32, O41

*I would like to express my sincere gratitude to Koichi Futagami, Ching-Chong Lai, and Been-Lon Chen for the beneficial discussions and suggestions. I would also like to thank Katsunori Yamada and the participants at the 2013 Autumn Annual Meeting of the Japanese Economic Association at the University of Kanagawa. This paper was written while visiting the Institute of Economics, Academia Sinica. The financial support from Strategic Young Researcher Overseas Visits Program for Accelerating Brain Circulation (Japan Society for the Promotion of Science) and Grants for Excellent Graduate Schools (Ministry of Education, Culture, Sports, Science and Technology, Japan) are gratefully acknowledged. All remaining errors are mine.

†Graduate School of Economics, Osaka University, 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, Japan. E-mail address: kge006tk@mail2.econ.osaka-u.ac.jp

1 Introduction

Asset bubbles sometimes emerge, and they are accompanied by higher economic growth and a consumption boom.[1][2] A seminal study by Tirole (1985) shows the existence of a rational deterministic bubble on an asset in an economy with overlapping generations. It is found that the presence of asset bubbles increases consumption but retards economic growth (e.g.. Saint-Paul (1992), Grossman and Yanagawa (1993), King and Ferguson (1993), and Futagami and Shibata (2000)). The reason is that the higher consumption absorbs resources and crowds out investment. As a result, the theoretical prediction runs contrary to the mentioned above fact. The purpose of this paper is to overcome this conflict and provide a theoretical explanation for why the presence of asset bubbles can lead to higher economic growth in concurrence with high consumption by using a simple endogenous growth model.

The present analysis is based on a recent endogenous growth model developed by Peretto (2007, 2011). Specifically, the model considers the economy where long-lived value-maximizing firms continuously improve upon the quality of their specific product through in-house R&D, while simultaneously new firms also enter into the market. That is, in the model economy, there are two dimensional investment opportunities: in-house R&D (quality improving) and the creation of a new firm (product proliferation). The main source of economic growth is obtained from technological progress, endogenously derived from the in-house R&D of firms. The advantage of the model is elimination of the well-known undesirable scale effect (e.g. Jones (1995)), while keeping the policy effect property supported by recent growing empirical literature.[3] Increases in the scale of the aggregate economy are perfectly fragmented by the endogenous product proliferation.

We introduce the above-mentioned structure into a continuous-time overlapping generations model developed by Weil (1989). Specifically, we assume that economic agents live

[1]Asset bubbles are defined as the difference between the fundamental value of an asset and its market value. For example, if an intrinsically useless asset whose fundamental value is zero has a positive value, we say there exist asset bubbles.

[2]The introduction of Martin and Ventura (2012) provides excellent surveys about the fact about asset bubbles.

[3]See, for example, Laincz and Peretto (2006), Ha and Howitt (2007), and Ang and Madsen (2011)).

forever but have no intergenerational altruism. We also assume that new generations are born at a constant rate.[4] Additionally, we assume that there initially exists an intrinsically useless asset (fiat money).[5] The reason there exist asset bubbles in equilibrium is the same as the reason spelled out in Tirole (1985). Due to an absence of intergenerational altruism, one generation can transfer the intrinsically useless asset to the other generation as a pyramid scheme only if the price appreciation of the intrinsically useless asset is equal to the return on a real asset (equity) whose value is backed by the fundamental value.

Our analysis stipulates the theoretical mechanism by which the presence of asset bubbles can lead to higher economic growth with high consumption. Here the endogeneity of the market structure plays a key role. If asset bubbles emerge, households think that they are wealthier and thus want to consume more. This leads to a higher reservation interest rate of households in the asset market. To satisfy the higher reservation interest rate, the return on a real asset (equity) must increase. This requires product proliferation to be impeded so that demand for differentiated goods, at the level of an individual firm, increases. Consequently, if the positive effect on in-house R&D of firms due to the larger scale of production at the level of an individual firm exceeds the negative effect derived from the higher interest rate, the intensity of in-house R&D increases, thus enhancing economic growth. In addition, the lower product proliferation provides available economic resources for a higher consumption by households.

This paper is related to several recent studies that examine the relationship between asset bubbles and economic growth (e.g.. Hirano and Yanagawa (2010) and Martin and Ventura (2012)). Their analyses consider an environment where the credit market is imperfect and the efficiency of investment among firms is heterogeneous. In this environment, the presence of asset bubbles can reallocate resources from inefficient to efficient investments. As a result, the reallocation may raise the productivity of aggregate outputs and

[4]If we employ the alternative setup that there exists a positive death rate following Blanchard (1985), our main claims do not change.

[5]For simplicity, as a theoretical devise, we distinguish the pure bubbles from real assets whose values are backed by the fundamental values along with many other studies. As one notable exception, Olivier (2000) analyzes the case where bubbles attach directly on real assets such as the equity of firms, and Olivier (2000) also demonstrates that bubbles on equity may enhance economic growth in the framework of the endogenous growth model developed by Romer (1990).

promote economic growth. By contrast, our model economy is characterized by an environment, where the credit market is perfect, and the efficiency of investment among firms is homogeneous. The key feature of our analysis is that the endogeneity of market structure affects the scale of production at the level of an individual firm, which in turn determines the intensity of in-house R&D.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 analyzes the dynamic system of the market equilibrium. Section 4 analyzes the steady state equilibrium as well as compares the steady state equilibrium with and without asset bubbles.

2 The model

In this section, we establish a continuous-time overlapping-generations version of the model developed by Peretto (2011).[6] We also assume that there exists an initial and intrinsically useless asset, that is, fiat money. The economy is closed and consists of a final goods sector, an intermediate goods sector, and households.

2.1 The final goods sector

Perfect competition prevails in the final goods sector. The (real) price of final goods is set to be numeraire. Final goods are consumed by households and used as only one factor of the production and investment of intermediate goods sector. The final goods, Y_t is produced by the following production function:

$$Y_t = \int_0^{N_t} X_{it}^{\theta} \left(Z_{it}^{\alpha} \bar{Z}_t^{1-\alpha} L_{it} \right)^{1-\theta} di, \quad 0 < \alpha, \theta < 1, \tag{1}$$

where N_t is the variety of intermediate goods (the number of intermediate goods firms), X_{it} is the input of intermediate good $i \in [0, N_t]$ (produced by firm i), and L_{it} is the input of labor which uses intermediate goods i. The productivity of L_{it} depends not only

[6]The model of Peretto (2011) is a lab-equipment style versions of the model developed by Peretto (2007).

on the quality of intermediate good i, Z_{it}, but also on the average level of the quality across intermediate goods, $\bar{Z}_t \equiv \int_0^{N_t} \frac{1}{N_t} Z_{jt} dj$. Therefore, we obtain the following optimal conditions:

$$X_{it} = \left(\frac{\theta}{p_{it}^I} \right)^{\frac{1}{1-\theta}} \left(Z_{it}^{\alpha} \bar{Z}_t^{1-\alpha} L_{it} \right), \tag{2}$$

$$L_{it} = \left(\frac{1-\theta}{w_t} \right)^{\frac{1}{\theta}} X_{it} \left(Z_{it}^{\alpha} \bar{Z}_t^{1-\alpha} \right)^{\frac{1-\theta}{\theta}}, \tag{3}$$

where p_{it}^I and w_t represent the price of intermediate good i and the wage rate, respectively.

2.2 The intermediate goods sector

Monopolistic competition prevails in the intermediate goods sector. Firm i exclusively produces its differentiated good with its quality Z_{it}. The monopoly of each firm is permanently protected by perfect patent protection. Producing one unit of intermediate goods requires one unit of final goods. Firms improve upon the quality of their specific product through their in-house R&D.[7] The law of motion of the firm-specific quality is as follows:

$$\dot{Z}_{it} = R_{it}, \tag{4}$$

where R_{it} is the inputs for in-house R&D. Fixed operating costs, $\phi \bar{Z}_t$ ($\phi > 0$), are also required at each point in time. Then, the profit flow of firm i is $\Pi_{it} = \left(p_{it}^I - 1 \right) X_{it} - \phi \bar{Z}_t - R_{it}$. Without loss of generality, the number of equity per firm is normalized to one. The return on equity of firm i is given by

$$r_t = \frac{\Pi_{it}}{V_{it}} + \frac{\dot{V}_{it}}{V_{it}}. \tag{5}$$

Integrating (5) forward yields the value of firm i as $V_{it} = \int_t^{\infty} \Pi_{is} \left(e^{-\int_t^s r_v dv} \right) ds$. Throughout this exercise we consider a symmetric equilibrium in which $Z_{it} = Z_t$ by assuming that any new entry firm starts with the same level of technologies as incumbents so that the subscript

[7]Since here we assume that perfect capital market prevails and that there are no distortionary taxes, it does not matter whether firms finance their in-house R&D by issuing equity or by using retained earnings.

i can be dropped. In equilibrium, $Z_t = \bar{Z}_t$ holds. Each firm maximizes its value, subject to (2) and (4), given $\bar{Z}$. Solving the inter-temporal optimization problem, we obtain the following optimal conditions:

$$p_t^I = \frac{1}{\theta}, \tag{6}$$

$$q_t = 1 \;\Leftrightarrow\; R_t > 0, \tag{7}$$

$$r_t = \alpha \frac{1-\theta}{\theta} \theta^{\frac{2}{1-\theta}} Z_t^{\alpha-1} \bar{Z}_t^{1-\alpha} L_t + \frac{\dot{q}_t}{q_t}, \tag{8}$$

and the transversality condition is given by $\lim_{s\to\infty} q_s Z_s e^{-\int_t^s r_v dv} = 0$, where q_t is a co-state variable (a shadow value of in-house R&D) associated with the current-value Hamiltonian for this optimization problem, where (6) represents pricing rule with constant mark-up, and where (8) represents the no-arbitrage relationship between the return on equity and the return on in-house R&D.

2.3 Firm entry

Development of new product requires βZ_t ($\beta > 1$).[8] A new firm is set up by issuing equities. Free-entry condition yields

$$V_t = \beta Z_t \;\Leftrightarrow\; \dot{N}_t > 0. \tag{9}$$

This equation implies entry is positive until the value of a firm is equal to the set up cost. In our model economy, as we will see later, population grows perpetually and then the aggregate demand for intermediate goods also continues to grow. Therefore, at each point in time, new entry of firms occurs.

[8] Since any new entrant firm starts with the same level of technologies of incumbents, it is natural to assume that the cost of creating a new product with the initial quality level Z_t is proportional to the quality level of incumbents. Moreover, it is natural to assume that the new entrant firm has to pay an additional cost more than the cost already paid by the incumbents (see Peretto (2011)).

2.4 Households

The specification of household behavior is based on that in Weil (1989).[9] At each point in time, a new generation is born at a constant rate $\lambda > 0$. Since the probability of death is assumed to be zero, each generation lives forever. And there is no intergenerational altruism. Without loss of generality, the total population at time 0 is normalized to 1 so that the total population at time t is $e^{\lambda t}$.

At the beginning of the time 0, there is an initial generation endowed with initial wealth. Hereafter we call this generation as generation 0^-. The initial wealth consists of a real asset (equity) and an intrinsically useless asset (fiat money). To decide upon a fixed terminology, we label the intrinsically useless asset a “bubbly asset”. Specifically, the generation 0^- initially possesses the equity of firms (the initial number of firms is N_0), whose value is given by N_0V_0 in real terms and B_0 pieces of the bubbly asset in nominal terms.[10] By contrast, the subsequent generation does not have any initial wealth. Hereafter we label the generation born at time $t \geq 0$ as “generation t”.

Each household supplies one inelastic unit of labor at each point in time. The representative household in generation $s(\leq t)$ maximizes the following utility function:

$$U(s,t) = \int_t^{\infty} \log\left[c(s,x)\right] e^{-\rho(x-t)} dx, \quad \rho > 0, \tag{10}$$

subject to the following budget constraint of generation s:

$$\frac{db(s,t)}{dt}\frac{1}{p_t^M} + \frac{dk(s,t)}{dt} = r_t k(s,t) + w_t - c(s,t),$$

where $b(s,t)$ is holdings of pieces of a bubbly asset (in nominal terms), $1/p_t^M$ is the price of the bubbly asset, $k(s,t)$ is holdings of a real asset, and $c(s,t)$ is consumption.

Since the bubbly asset has no intrinsic or fundamental value, households possess both the real asset and the bubbly asset only if the price appreciation of the bubbly asset is

[9] Futagami and Shibata (2000) also uses the same specification.

[10] That is, the total value of the initial asset is equal to $N_0V_0 + B_0/p_0^M$ in real terms, where p_0^M is the price of the bubbly asset at time 0.

equal to the return on the real asset, that is,

$$-\frac{\dot{p}_t^M}{p_t^M} = r_t. \tag{11}$$

Let $m(s,t) \equiv b(s,t)/p_t^M$ and $a(s,t) \equiv k(s,t) + m(s,t)$. Using (11) and the definitions, the flow budget constraint of generation s can be rewritten as

$$\frac{da(s,t)}{dt} = r_t a(s,t) + w_t - c(s,t). \tag{12}$$

Maximizing (10) subject to (12) yields the following Euler equation of generation s:

$$\frac{dc(s,t)}{dt} = (r_t - \rho)\, c(s,t). \tag{13}$$

And the following No Ponzi game condition must be satisfied: $\lim_{x\to\infty} a(s,x)e^{\left(-\int_t^x r_v dv\right)} = 0$.

In what follows, we will aggregate the consumption and wealth of each generation. To do so, we sequentially define aggregate consumption, aggregate wealth, and aggregate wage income:

$$\begin{aligned}
C_t &\equiv c(0^-,t) + \lambda \int_0^t c(s,t)e^{\lambda s} ds, \\
A_t &\equiv a(0^-,t) + \lambda \int_0^t a(s,t)e^{\lambda s} ds, \\
W_t &\equiv w_t + \lambda \int_0^t w_s e^{\lambda s} ds.
\end{aligned}$$

In Appendix 1, we show that the dynamic path of aggregate consumption and aggregate wealth is respectively given by

$$\dot{C}_t = (r_t - \rho + \lambda)\, C_t - \rho\lambda A_t, \tag{14}$$

$$\dot{A}_t = r_t A_t + W_t - C_t. \tag{15}$$

3 Market equilibrium and Dynamics

In this section, we derive the dynamics system of the market equilibrium based on the model in the preceding section. The market equilibrium condition of final goods is given by $Y_t = C_t + N_t [X_t + \phi Z_t + R_t] + \beta Z_t \dot{N}_t$. The market equilibrium condition of assets is given by $A_t = M_t + N_t V_t$, where M_t is defined as the aggregate value of the bubbly asset, that is B_0/p_t^M. This condition holds that in equilibrium the aggregate wealth must be equal to the aggregate value of the bubbly asset and the aggregate value of equity ($N_t V_t$). Here we remark that the real asset in the economy is only the equity of firms. The market equilibrium condition of labor is given by $e^{\lambda t} = N_t L_t$.

Define the number of firms per capita as $n_t \equiv N_t/e^{\lambda t}$. From (7) and the market equilibrium condition of labor, if $R_t > 0$, (8) is rewritten by

$$r_t = \alpha\theta(1-\theta)\frac{\Omega}{n_t}, \tag{16}$$

where $\Omega \equiv \theta^{\frac{2\theta}{1-\theta}}$. From (9) and the market equilibrium condition of labor, if $R_t > 0$, (5) is rewritten by

$$r_t = \frac{1}{\beta}\left[\theta(1-\theta)\frac{\Omega}{n_t} - \phi\right] + \frac{\beta-1}{\beta}\frac{\dot{Z}_t}{Z_t}. \tag{17}$$

From (16) and (17), the rate of quality growth is given by

$$\hat{z}_t(n_t) \equiv \frac{\dot{Z}_t}{Z_t} = \begin{cases} \dfrac{\alpha\beta-1}{\beta-1}\theta(1-\theta)\dfrac{\Omega}{n_t} + \dfrac{\phi}{\beta-1}, & \text{if} \quad n_t > \tilde{n}, \\ 0, \quad \text{if} \quad \alpha\beta < 1 \ \text{ and } \ 0 < n_t \le \tilde{n}, \end{cases} \tag{18}$$

and the interest rate (the return on equity) is given by

$$r_t = \begin{cases} \alpha\theta(1-\theta)\dfrac{\Omega}{n_t}, \quad \text{if} \quad n_t > \tilde{n}, \\ \dfrac{1}{\beta}\left[\theta(1-\theta)\dfrac{\Omega}{n_t} - \phi\right], \quad \text{if} \quad \alpha\beta < 1 \ \text{ and } \ 0 < n_t \le \tilde{n}, \end{cases} \tag{19}$$

where $\tilde{n} = \max\left\{0, \frac{(1-\alpha\beta)\theta(1-\theta)\Omega}{\phi}\right\}$. More specifically, the intensity of in-house R&D is determined at the point where the return on in-house R&D (the LHS of (16)) is equal to the return on equity (the LHS of (17)). (18) indicates if $\alpha\beta > 1$ (< 1), the lower number of firms per capita leads to the higher (lower) rate of quality growth. If α and β are sufficiently high (low), the return on in-house R&D is more (less) sensitive to changes in the number of firms per capita. Hence, the lower number of firms per capita raises the return on in-house R&D than the return on equity, resulting in the higher (lower) rate of quality growth. On the other hand, (19) shows that the interest rate is unambiguously decreasing in the number of firms per capita. The lower number of firms per capita leads to higher demand for intermediate goods at the level of an individual firm.

More intuitively, whether the rate of quality growth is increasing or decreasing in the number of firms per capita is dependent on the extent of the following two contradictory forces. Other things being equal, the lower number of firms per capita increases demand for intermediate goods at the level of an individual firm. The larger scale of production at the level of an individual firm allow in-house R&D expenditures to be spread over more units of goods, thus having a positive effect on incentives to in-house R&D (the cost spreading effect). On the other hand, a higher interest rate associated with a lower number of firms, per set of firms, lowers incentives to in-house R&D (the interest effect). If $\alpha\beta > 1$, the positive cost spreading effect exceeds the negative interest effect. The same logic applies to the case where $\alpha\beta < 1$.[11]

The perfect distribution in the final goods sector implies $N_t X_t = \theta^2 Y_t$ and $w_t N_t L_t = (1-\theta)Y_t$. Applying these relationships to (1), we obtain the following reduced-form aggregate production function of final goods, which is given as

$$Y_t = \Omega e^{\lambda t} Z_t. \tag{20}$$

The growth rate of outputs per capita is given as $\hat{z}_t(n_t)$.

Define the ratio of aggregate consumption to outputs as $c_t \equiv C_t/Y_t$ and the ratio of aggregate value of the bubbly asset to outputs as $m_t \equiv M_t/Y_t$. In Appendix 2, we show

[11] If $\phi = 0$, $\alpha\beta$ is restricted to be higher than 1 so that the steady state equilibrium with a positive rate of quality growth exists.

that the following three equations constitute the dynamic system of the economy:

$$\dot{n}_t = \frac{\Omega}{\beta}\left[1-\theta^2-c_t\right] - \frac{n_t}{\beta}\left[\phi+\hat{z}_t(n_t)+\beta\lambda\right], \tag{21}$$

$$\dot{c}_t = \left[\varphi(n_t)-\rho\right]c_t - \lambda\rho\left[m_t+\beta\frac{n_t}{\Omega}\right], \tag{22}$$

$$\dot{m}_t = \left[\varphi(n_t)-\lambda\right]m_t, \tag{23}$$

where the difference between the interest rate and the rate of quality growth is given by

$$\varphi(n_t) \equiv r_t - \hat{z}_t = \begin{cases} \dfrac{(1-\alpha)\theta(1-\theta)}{\beta-1}\dfrac{\Omega}{n_t} - \dfrac{\phi}{\beta-1}, & \text{if} \quad n_t > \tilde{n}, \\ \dfrac{1}{\beta}\left[\theta(1-\theta)\dfrac{\Omega}{n_t} - \phi\right], & \text{if} \quad \alpha\beta < 1 \text{ and } 0 < n_t \le \tilde{n}. \end{cases} \tag{24}$$

4 The steady state equilibrium

This section characterizes the steady state equilibrium. As we will see later, there may exist multiple steady state equilibrium: the steady state equilibrium without asset bubbles and the steady state equilibrium with asset bubbles. In Appendix 2, we examine the local stability of the steady state equilibrium. We prove that the equilibrium path toward the steady state equilibrium with asset bubbles is locally saddle-point stable, whereas the equilibrium path toward the steady state equilibrium without asset bubbles is locally indeterminate. Thus, a global indeterminacy arises along with previous studies, such as in Grossman and Yanagawa (1993) and Futagami and Shibata (2000).

From (21), $\dot{n}_t = 0$-locus is given by

$$c = \begin{cases} (1-\theta)\left[1+\dfrac{\beta\theta(1-\alpha)}{\beta-1}\right] - \beta\dfrac{n}{\Omega}\left[\dfrac{\phi}{\beta-1}+\lambda\right], & \text{if} \quad n > \tilde{n}, \\ (1-\theta^2) - \beta\dfrac{n}{\Omega}\left[\dfrac{\phi}{\beta}+\lambda\right], & \text{if} \quad \alpha\beta < 1 \text{ and } 0 < n \le \tilde{n}. \end{cases} \tag{25}$$

(25) shows that the $\dot{n}_t = 0$-locus is independent of the value of m and on the $\dot{n}_t = 0$-locus

c is negatively related to n. This locus represents the resource constraint of the economy. The downward-sloping shape of this locus implies that the higher number of firms per capita absorbs available resources for consumption of households. From (22), $\dot{c}_t = 0$-locus is given by

$$c = \begin{cases} \dfrac{\lambda\rho}{\left[\dfrac{(1-\alpha)\theta(1-\theta)}{\beta-1}\dfrac{\Omega}{n} - \dfrac{\phi}{\beta-1} - \rho\right]}\left[m + \beta\dfrac{n}{\Omega}\right], & \text{if} \quad n > \tilde{n}, \\ \dfrac{\lambda\rho}{\left[\dfrac{\theta(1-\theta)}{\beta}\dfrac{\Omega}{n} - \dfrac{\phi}{\beta} - \rho\right]}\left[m + \beta\dfrac{n}{\Omega}\right], & \text{if} \quad \alpha\beta < 1 \text{ and } 0 < n \leq \tilde{n}. \end{cases} \tag{26}$$

(26) shows that on the $\dot{c}_t = 0$-locus, given the value of m, c is positively related to n starting from the origin in the region of $0 < n < \bar{n}$ where $\bar{n} \equiv \text{argsolve}\{\varphi(n) = \rho\}$.[12] This locus is derived from the aggregation of the Euler equation of households. The upward-sloping shape of this locus implies that the higher number of firms per capita leads to a lower interest rate, and thus households want to consume more.[13] First, we consider a steady state equilibrium without asset bubbles, $\{n^*, c^*, m^*\}$ where $m^* = 0$. In Figure 1 and Figure 2, we plot the $\dot{n}_t = 0$-locus and $\dot{c}_t = 0$-locus with $m^* = 0$ by solid lines.[14] These figures show that $\{n^*, c^*\}$ is uniquely determined.

Second, we consider the possibility of the existence for the steady state equilibrium with asset bubbles, $\{n^{**}, c^{**}, m^{**}\}$, where $m^{**} > 0$. From (23), if $m^{**} > 0$, $\dot{m}_t = 0$ implies

[12]If the steady state number of firms per capita is higher than $\bar{n}$, the steady state consumption ratio becomes negative.

[13]The higher number of firms per capita also yields a higher ratio of the aggregate value of equity to output, that is, NV/Y. Hence, the higher wealth ratio induces households to consume more, given an interest rate. This reinforces the extent of the upward-sloping shape of this locus. However, this is dependent on the specification of the entry cost. For example, if we consider the alternative assumption in which the entry cost is related to the production volumes of incumbents, that is, X_t along with Peretto (2007), then the ratio of the aggregate value of equity to output becomes constant.

[14]Specifically, Figure 1 shows the configuration of $\dot{n}_t = 0$-locus and $\dot{c}_t = 0$-locus when $\alpha\beta > 1$, while Figure 2 shows that when $\alpha\beta < 1$ and $\phi > \tilde{\phi}$ (see the definition of $\tilde{\phi}$ for the latter main text). If $\alpha\beta < 1$ and $\phi > \tilde{\phi}$ is satisfied, the steady state quality growth is positive both in the steady state equilibrium without asset bubbles and in that with asset bubbles.

$\varphi(n^{**}) = \lambda$. Therefore, from (18) and (24), we obtain

$$n^{**} = \begin{cases} \dfrac{(1-\alpha)\theta(1-\theta)\Omega}{(\beta-1)\lambda+\phi}, & \text{if} \quad \alpha\beta > 1 \quad \text{or} \quad \alpha\beta < 1 \ \textit{and} \ \phi > \tilde{\phi}, \\ \dfrac{\theta(1-\theta)\Omega}{\beta\lambda+\phi}, & \text{if} \quad \alpha\beta < 1 \ \textit{and} \ 0 < \phi \leq \tilde{\phi}, \end{cases} \tag{27}$$

and

$$\hat{z}(n^{**}) = \begin{cases} \dfrac{(\alpha\beta-1)\lambda+\alpha\phi}{1-\alpha}, & \text{if} \quad \alpha\beta > 1 \quad \text{or} \quad \alpha\beta < 1 \ \textit{and} \ \phi > \tilde{\phi}, \\ 0, \quad \text{if} \quad \alpha\beta < 1 \ \textit{and} \ 0 < \phi \leq \tilde{\phi}, \end{cases} \tag{28}$$

where $\tilde{\phi} \equiv \max\left\{0, \frac{(1-\alpha\beta)\lambda}{\alpha}\right\}$. From (25), (27), and (28), c^{**} is given by

$$c^{**} = (1-\theta). \tag{29}$$

From (26), (27) and (29), m^{**} is given by

$$m^{**} = \begin{cases} \dfrac{(1-\theta)}{\lambda\rho}\left[\lambda - \rho - \dfrac{(1-\alpha)\beta\theta\lambda\rho}{(\beta-1)\lambda+\phi}\right], & \text{if} \quad \alpha\beta > 1 \quad \text{or} \quad \alpha\beta < 1 \ \textit{and} \ \phi > \tilde{\phi}, \\ \dfrac{(1-\theta)}{\lambda\rho}\left[\lambda - \rho - \dfrac{\beta\theta\lambda\rho}{\beta\lambda+\phi}\right], & \text{if} \quad \alpha\beta < 1 \ \textit{and} \ 0 < \phi \leq \tilde{\phi}. \end{cases} \tag{30}$$

Therefore, the necessary and sufficient condition for the existence of the steady state equilibrium with an asset bubble is given by

$$\begin{cases} \lambda - \rho > \frac{(1-\alpha)\beta\theta\lambda\rho}{(\beta-1)\lambda+\phi} > 0, & \text{if} \quad \alpha\beta > 1 \quad \text{or} \quad \alpha\beta < 1 \ \textit{and} \ \phi > \tilde{\phi} \\ \lambda - \rho > \frac{\beta\theta\lambda\rho}{\beta\lambda+\phi} > 0, & \text{if} \quad \alpha\beta < 1 \ \textit{and} \ 0 < \phi \leq \tilde{\phi}. \end{cases} \tag{31}$$

If this condition is satisfied, we find that in the steady state equilibrium without asset bubbles the growth rate of final output exceeds the interest rate.[15] That is, the necessary condition for the presence of asset bubbles is same as previous studies such as Grossman and Yanagawa (1993) and Futagami and Shibata (2000).

[15] See Appendix A.4 for the proof.

If there exists the steady state equilibrium with asset bubbles, $\dot{c}_t = 0$-locus moves up in a counterclockwise direction as opposed to the case in which no asset bubbles exist. In Figure 1 and Figure 2, we plot $\dot{c}_t = 0$-locus with $m^{**} > 0$ by a dotted line. These figures show that the number of firms per capita in the steady state with asset bubbles is unambiguously lower than that in the steady state equilibrium without asset bubbles. Moreover, it is shown that the consumption ratio in the steady state equilibrium with asset bubbles is unambiguously higher than that in the steady state equilibrium without asset bubbles.

The key mechanism is as follows: If asset bubbles emerge, households think that they are wealthier, and thus want to consume more. This leads to higher reservation interest rate of households in the asset market. In order to satisfy the higher reservation interest rate, the return on equity must increases. This requires the number of firms per capita to be lower given the market size of the economy because the return on equity is positively related to demand for intermediate goods at the level of an individual firm. As a result, the presence of asset bubbles yields a larger scale of production at the level of an individual firm.

Consequently, if the cost spreading effect exceeds the interest effect (if $\alpha\beta > 1$), as previously noted, the presence of asset bubbles encourages in-house R&D of firms and promotes economic growth. The same logic applies to the case in which the interest effect surmounts the cost spreading effect (if $\alpha\beta < 1$). In addition, the lower product proliferation provides available economic resources for a higher household consumption, regardless of the effect on economic growth.

A Appendix

A.1 Appendix 1

Integrating (12) yields

$$\lim_{x\to\infty} a(s,x)e^{\left(-\int_t^x r_v dv\right)} - a(s,t) + \int_t^\infty c(s,x)e^{\left(-\int_t^x r_v dv\right)}dx = \int_t^\infty (w_x)\, e^{\left(-\int_t^x r_v dv\right)}dx \equiv h_t.$$

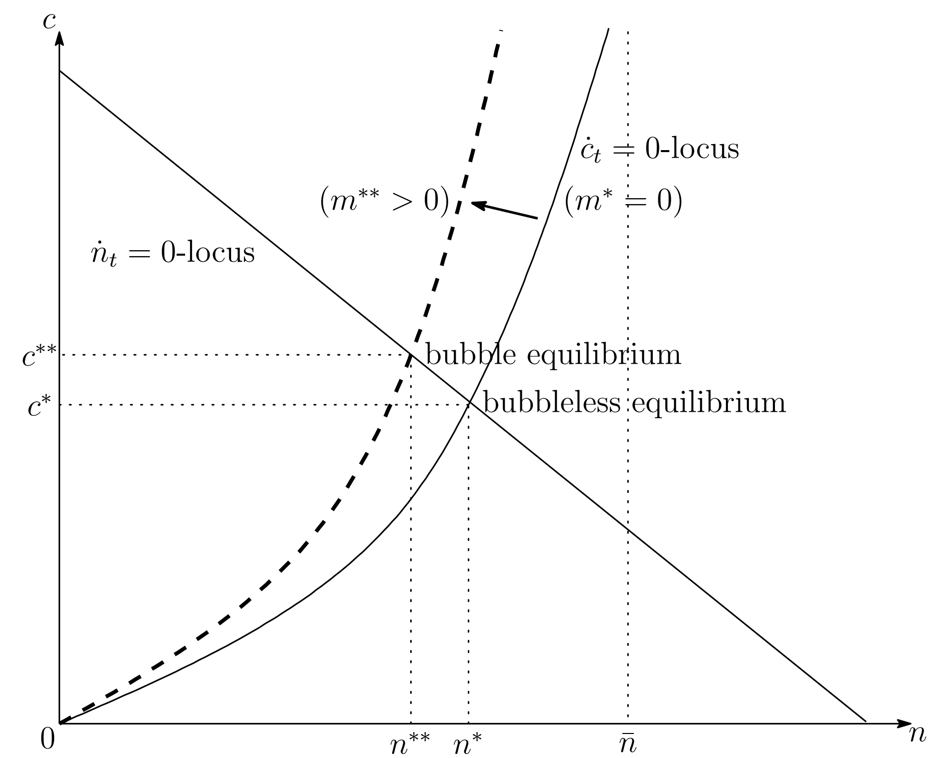

Figure 1: Steady state equilibrium (in the case where $\alpha\beta > 1$)

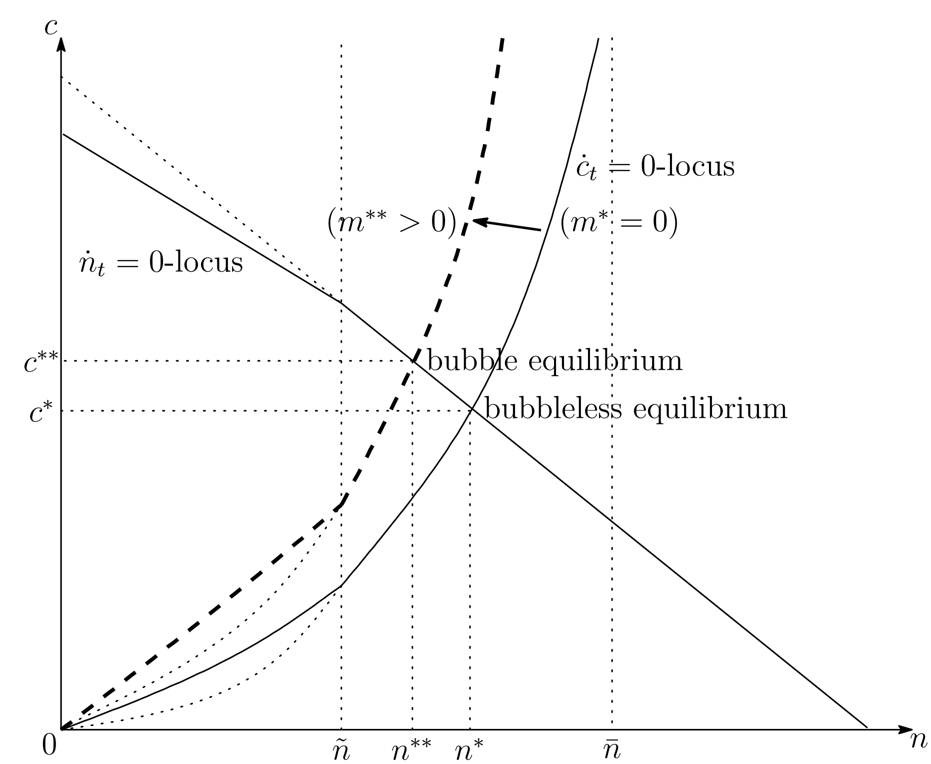

Figure 2: Steady state equilibrium (in the case where $\alpha\beta < 1$ and $\phi > \tilde{\phi}$)

Applying (13) and the No Ponzi game condition to the above equation, we obtain $c(s,t) = \rho\left[a(s,t)+h_t\right]$. Aggregating this yields

$$C_t = \rho\left[a(0^-,t)+h_t\right] + \rho\lambda\int_0^t a(s,t)e^{\lambda s}ds + \rho\lambda\int_0^t h_t e^{\lambda s}ds,$$

$$\Leftrightarrow \; C_t = \rho\left(A_t + H_t\right), \quad \text{where } \; H_t \equiv h_t + \lambda\int_0^t h_t e^{\lambda s}ds. \tag{A-1}$$

From (13) and (A-1), differentiating the aggregate consumption with respect to time yields

$$\dot{C}_t = \left(r_t-\rho\right)c(0^-,t) + c(t,t)\lambda e^{\lambda t} + \left(r_t-\rho\right)\lambda\int_0^t c(s,t)e^{\lambda s}ds,$$

$$\Leftrightarrow \; \dot{C}_t = \rho\lambda e^{\lambda t}\left[a(t,t)+h_t\right] + \left(r_t-\rho\right)C_t,$$

$$\Leftrightarrow \; \dot{C}_t = \rho\lambda H_t + \left(r_t-\rho\right)C_t, \quad (\because \;\; a(t,t)=0\,),$$

$$\Leftrightarrow \; \dot{C}_t = \left(r_t-\rho+\lambda\right)C_t - \rho\lambda A_t.$$

And aggregating (12) yields (15).

A.2 Appendix 2

Let $a_t \equiv A_t/Y_t$. From (14) and (20),

$$\frac{\dot{c}_t}{c_t} = \frac{\dot{C}_t}{C_t} - \frac{\dot{Y}_t}{Y_t} = \frac{\dot{C}_t}{C_t} - \hat{z}_t - \lambda \;\Leftrightarrow\; \dot{c}_t = \left[\varphi(n_t)-\rho\right]c_t - \lambda\rho a_t. \tag{A-2}$$

From (20), the market equilibrium condition of assets is rewritten by

$$a_t = m_t + \beta\frac{n_t}{\Omega}.$$

Then, substituting this condition into (A-2) yields (22).

From (20) and $N_tX_t = \theta^2 Y_t$, the market equilibrium condition of final goods is rewritten by

$$1 = c_t + \theta^2 + \frac{n_t}{\Omega}\left[\phi + \hat{z}_t + \beta\frac{\dot{N}_t}{N_t}\right],$$

Then, from this condition and the definition of n_t, we obtain (21).

From the market equilibrium condition of labor and $N_t w_t L_t = (1-\theta)Y_t$, (15) is rewritten by

$$\frac{\dot{A}_t}{A_t} = r_t + (1-\theta)\frac{1}{a_t} - \frac{c_t}{a_t}.$$

Using the above equation and (20), we obtain

$$\dot{a}_t = [\varphi(n_t) - \lambda]\, a_t + (1-\theta) - c_t.$$

Differentiating the market equilibrium condition of assets with respect to time and using (21) and the law of motion of the asset ratio, we obtain (23).

A.3 Appendix 3

From (21), (22), and (23), the system of the linearized differential equation around the steady state is given by

$$\begin{pmatrix} \dot{n}_t \\ \dot{c}_t \\ \dot{m}_t \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix} \begin{pmatrix} n_t - n^{st} \\ c_t - c^{st} \\ m_t - m^{st} \end{pmatrix}, \tag{A-3}$$

where $\{n^{st}, c^{st}, m^{st}\}$ represents $\{n^*, c^*, m^*\}$ or $\{n^{**}, c^{**}, m^{**}\}$ and

$$\begin{aligned}
&J_{11} = -\frac{1}{\beta}\left[\phi + \hat{z}(n^{st}) + \beta\lambda\right] - \frac{n^{st}}{\beta}\frac{\partial \hat{z}(n)}{\partial n}\bigg|_{n=n^{st}} < 0, \\
&J_{12} = -\frac{\Omega}{\beta} < 0, \\
&J_{13} = 0, \\
&J_{21} = -c^{st}\frac{\partial \varphi(n)}{\partial n}\bigg|_{n=n^{st}} - \lambda\rho\frac{\beta}{\Omega} < 0, \\
&J_{22} = \varphi(n^*) - \rho > 0 \quad \text{if} \;\; m^{st} = m^* = 0 \quad \left(\because \;\; n^* > n^{**}\right), \\
&\left\{ J_{22} = \lambda - \rho > 0 \quad \text{if} \;\; m^{st} = m^{**} > 0 \right\}, \\
&J_{23} = -\lambda\rho < 0, \\
&J_{31} = 0 \quad \text{if} \;\; m^{st} = m^* = 0,
\end{aligned}$$

$$\left\{ J_{31} = m^{**} \frac{\partial \varphi(n)}{\partial n}\bigg|_{n=n^{**}} < 0 \quad \text{if} \;\; m^{st} = m^{**} > 0 \right\},$$

$$J_{32} = 0,$$

$$J_{33} = \varphi(n^*) - \lambda < 0 \quad \text{if} \;\; m^{st} = m^* = 0 \quad (\because \;\; n^* > n^{**}) \, ,$$

$$\left\{ J_{33} = 0 \quad \text{if} \;\; m^{st} = m^{**} > 0 \right\}.$$

We define matrix J as $J \equiv \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix}$. We obtain the following the characteristic equation of the coefficient matrix on the right-hand side of (A-3):

$$0 = |J - qI| = -q^3 + \mathrm{Tr}Jq^2 - BJq + \mathrm{Det}J \equiv f(q),$$

where I is the identity matrix, TrJ is the trace of J, Det$J$ is the determinant of J, and $BJ \equiv \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix} + \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix}$.

The dynamic system has one state variable, n_t, and two jump variables, c_t, and m_t. Hence, if the number of negative eigenvalues around the steady state is one, there exists a unique equilibrium saddle path toward the steady state. If the number of negative eigenvalues around the steady state is more than one, there exists a continuum of equilibrium path toward the steady state.

Following Benhabib and Perli (1994), we now check the number of negative eigenvalues around the steady state by applying the Routh-Hurwitz theorem:

Theorem. *The number of roots of the polynomial in (A-3) with positive real parts is equal to the number of variations of sign in the scheme*

$$-1 \qquad \mathrm{Tr}J \qquad -\mathrm{B}J + \frac{\mathrm{Det}J}{\mathrm{Tr}J} \qquad \mathrm{Det}J. \tag{A-4}$$

Around the steady state without asset bubbles, we obtain $\mathrm{Det}J = J_{12}J_{23}J_{31} - J_{12}J_{21}J_{33} > 0$ and $\mathrm{Tr}J = J_{11} + J_{22} + J_{33} < 0 \, (\because \varphi(n^*) < \lambda)$. Then, we can conclude that around the steady state without asset bubbles there are one positive eigenvalue and two eigenvalues with negative real part because there is only one change of the sign in the scheme (A-4) regardless of the value of BJ. Therefore, there exists a continuum of equilibrium path toward the steady state without asset bubbles.

Around the steady state with asset bubbles, we obtain Det$J = J_{12}J_{23}J_{31} < 0$, Tr$J = J_{11} + J_{22} < 0$, and B$J = J_{11}J_{22} - J_{12}J_{21} < 0$. Hence, we also obtain $-BJ + \text{Det}J/\text{B}J > 0$. Then, we can conclude that around the steady state with asset bubbles there are two eigenvalues with one positive real part and one negative eigenvalue, because there are two changes of the sign in the scheme (A-4). Therefore, the equilibrium path toward the steady state with asset bubbles is locally determinate.

A.4 Appendix 4

From (24), we find that the difference between the interest rate and the growth rate of final output, $\varphi(n) - \lambda$, is decreasing in n. In the steady state equilibrium with asset bubbles, we obtain $\varphi(n^{**}) - \lambda = 0$. Since it is shown that n^* is higher than n^{**}, we find that $\varphi(n^*) - \lambda < 0$.

References

[1] Ang, J. B. and J.B. Madsen (2011) "Can Second-Generations Endogenous Growth Models explain the Productivity and Knowledge Production in the Asian Miracle Economies?", *The Review of Economics and Statistics*, Vol. 93, No. 4, pp. 1360-1373.

[2] Benhabib, J. and R. Peril (1994), "Uniqueness and indeterminacy: on the dynamics of endogenous growth", *Journal of Economic Theory*, 63, pp. 113-142.

[3] Blanchard, O.J. (1985), "Debts, deficits, and finite horizons", *Journal of Political Economy*, 93, pp. 223-247.

[4] Futagami, K. and Shibata, A. (2000), "Growth effects of bubbles in an endogenous growth model", *The Japanese Economic Reviews*, 2, pp. 221-235.

[5] Grossman, G. M. and Yanagawa, N. (1993), "Asset bubbles and endogenous growth", *Journal of Monetary Economics*, 31, pp. 3-19.

[6] Ha, J. and Howitt, P. (2007) "Accounting for Trends in Productivity and R&D: A Schumpeterian Critique of Semi-Endogenous Growth Theory", *Journal of Money, Credit and Banking*, Vol. 39, No. 4, pp. 733-744.

[7] Hirano, T. and N. Yanagawa (2010) "Asset bubbles, endogenous growth, and financial frictions", mimeo.

[8] Jones, C. I. (1995) "Time Series Tests of Endogenous Growth Models", *The Quarterly Journal of Economics*, 110, pp. 495-525.

[9] King, I. and D. Ferguson (1993) "Dynamic Inefficiency, Endogenous Growth, and Ponzi Games", *Journal of Monetary Economics*, 32, pp. 79-104.

[10] Laincz, C. A. and Peretto, P. F. (2006) "Scale Effects in endogenous growth theory: an error of aggregation not specification", *Journal of Economic Growth*, Vol. 11, No. 3, pp. 263-288.

[11] Martin, A. and Ventura, J. (2012), "Economic growth with bubbles", *American Economic Review*, 102, pp. 3033-3058.

[12] Olivier, J. (2000), "Growth-enhancing bubbles", *International Economic Review*, 41, pp. 133-151.

[13] Peretto, P.F. (2007), "The manhattan metaphor", *Journal of Economic Growth*, 12, pp. 329-350.

[14] Peretto, P.F. (2011), "The growth and welfare effects of deficit-financed dividend tax cuts", *Journal of Money, Credit and Banking*, 43, pp. 835-870.

[15] Romer, P. M. (1990), "Endogenous Technological Change", *Journal of Political Economy*, 98, pp. S71-S102.

[16] Saint-Paul, G. (1992), "Fiscal policy in an Endogenous Growth Model", *Quarterly Journal of Economics*, 107, pp. 1234-1259.

[17] Tirole, J. (1985), "Asset bubbles and overlapping generation", *Econometrica*, 53, pp. 1499-1528.

[18] Weil, P. (1989), "Overlapping families of infinitely lived agents", *Journal of Public Economics*, 38, pp. 183-198.