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Abstract

In this study, we reconsider the optimal non-linear tax problem with the public goods from the perspective of the commitment issue and examine how it affects the condition of the public goods provision. We show that the Samuelson rule should be modified when the government cannot commit and the skill types of taxpayers are revealed. Even if taxpayers have the same preference, which is separable and additive with respect to consumption and leisure, the Samuelson rule breaks down. Our analysis focuses on the effect of commitment issue on the marginal cost of public funds and the level of public goods provision. Our findings imply that the level of investment in public goods may be excessive in comparison to the case where the commitment issue is not considered.

Keywords: Public goods provision, Optimal Taxation, Time consistency

Journal of Economic Literature Classification Numbers: D82, H21

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1 Introduction

An important role in the public sector is to manage several kinds of services; for instance, providing health care, police protection, or education. In order to examine the optimal level of these activities, many researchers analyze the provision of public goods. As a first-best optimum, the public goods should satisfy the condition that the sum of the marginal utility for those who benefit from the goods is equal to the marginal cost. This is known as the Samuelson rule. Boadway and Keen (1993) consider another situation; that is, when the government cannot observe the marginal productivity of taxpayers and it is assumed that the public goods are financed by non-linear income taxation. They found that if the utility function of taxpayers is of the weakly separable form, then the Samuelson rule should be applied in the second-best environment.

Although they use the static model, public goods have the property of durable goods. Therefore, a dynamic setting is necessary. The traditional dynamic Samuelson rule implies that the marginal social benefit of public goods should be equal to the marginal cost plus the static Pigou effect[1] and the dynamic efficiency effect. The extension leads to topics that we need to include in our analysis. Batina (1990) considered a time inconsistency. If the government can use public debt policy and maintain the optimal expansion path, that is, the government can commit to its expenditure policy in the future, then the dynamic efficiency effect disappears. He pointed out that such a preliminary finding is unrealistic and showed that if the government cannot control its allocation over all periods, then the dynamic efficiency effect may decrease the Pigou effect. As for the non-linear income tax model, Aronsson and Grandlund (2011) focused on the situation where consumers have a time inconsistent preference for public goods. Assuming a paternalistic government, they showed that the social optimal condition of public goods provision should deviate from the Samuelson rule.

This paper differs from these related studies as it focuses on the commitment issue. Previous studies assume full commitment by the government. Then, once taxpayers reveal their productivities, the government can reconsider its tax policy in the next period on the basis of information gained from the previous period. However, anticipating this action by the government, taxpayers adjust their decision making in the first period. This is also known as the ratchet effect. Although previous studies of the optimal tax problem without commitment are discussed in the literature: for instance, Roberts (1984), Apps and Rees (2006), Bisin and Rampini (2006), Brett and Weymark (2008), Krause (2009), Guo and Krause (2011, 2013), and Berliant and Ledyard (2014), no previous studies examined how the commitment issue affects the provision of public goods. In this study, we utilize two periods and the partial equilibrium version of the model discussed in Pirttilä and Tuomala (2001). We show that even if the utility function is satisfied by the condition of separability, then the Samuelson rule should not be applied when considering the commitment issue. Moreover, we analyze

[1] Pigou (1928) pointed out that when the public good is financed by the distortionary tax, the social marginal cost of public goods is relatively higher than financed by the non-distortionary tax. This is because the dead weight loss is generated by distortionary taxes. The effect is called "Pigou effect."

how the commitment issue affects the marginal cost of public funds (hereafter MCPF).

The study is organized as follows. Section 2 describes the model and considers the benchmark case. Section 3 analyzes the time-consistent public goods provision. As an additional analysis, we examine how the commitment issue affects the level of public goods provision and social welfare in the economy. Thereafter, we provide a numerical example in Section 4. Conclusion is given in Section 5. Proofs of our main results are presented in the Appendix.

2 The Model

2.1 Environment

Herein, we introduce a model with two time period denoted by $t = 1, 2$[2] and a production technology with constant returns to scale in terms of labor and capital, where wage rates and rates of return correspond to productivity. For simplicity, the interest rate is assumed to be unity. The economy consists of consumers who have two kinds of productivity, θ^H and θ^L, where $\theta^H > \theta^L$. A consumer who has a productivity θ^H is called a high-type agent, while another agent is called a low-type agent. For simplicity, it is assumed that the number of each type of consumer is unity[3] and their productivities are constant over time. Heterogeneity of productivity between consumers means that income differentials exist among consumers. In order to redistribute consumer incomes, the government designs a tax policy and a program for the provision of public goods. However, the government cannot observe the productivity of consumers. The purpose of the government is to achieve an allocation that maximizes the social welfare through a redistribution policy that offsets the incentive for consumers to choose a mimicking strategy. The tax problem discussed in this paper can be regarded as a direct mechanism, whereby consumers report their own productivities to the government. In this situation, the revelation principle is applied in order to determine the optimal allocation. We adopt the classical setting of the optimal tax theory proposed by Stiglitz (1982, 1987).

2.2 Consumers

Consumers choose the amount of labor supply and consumption in both periods, where the consumption of type i agent in period t is defined as c_t^i and the labor supply of type i agent in period t is defined as l_t^i. Then, the labor income is $y_t^i \equiv \theta^i l_t^i$. Since consumers live in a two-periods economy, they can only save in the first period. The amount of savings by the type i agent can be denoted by s^i. As with the production technology, the savings technology

[2]Even if we extend this model to an economy with n periods, the result does not change. But, under the infinite-period model, the complete pooling case introduced in Section 3 cannot be defined. We therefore consider the two-period model, for simplicity.

[3]The same result remains in an environment where there is a continuum of consumers. However, we ought to take into account the partial-pooling case discussed by Apps and Rees (2006) and Krause (2009). Even if this case is included in our analysis, the result does not change.

is linear in terms of the amount of savings. Consumers choose these variables in order to maximize their utility. It is assumed that all consumers have the same preference with respect to consumption, the pubic goods, and the labor supply. Moreover, it is also assumed that the preference for consumption and the pubic goods is additive and separable for the labor supply.[4] Then, the utility function can be given by

$$U^i(c_1^i, G_1, l_1^i, c_2^i, G_2, l_2^i) = \sum_t \beta^{t-1}\{u(c_t^i, G_t) - v(l_t^i)\} \tag{1}$$

where $\beta \in (0, 1)$ is a discount factor. The sub-utility function $u(\cdot)$ is strictly increasing and a concave function, and $v(\cdot)$ is strictly increasing and a convex function.

2.3 The government

The purpose of the government is to implement an allocation that maximizes the social welfare through its redistributive policy. Herein, the social welfare function is defined as the sum of consumer payoffs. It is assumed that the government cannot observe individual consumer productivity θ^i, while the amount of labor income and savings are assumed to be observable. This allows the government to levy non-linear taxes on these variables. The tax revenue is divided between the public goods provision and savings by the government, which is denoted by s^G. As for the public goods, it has the property of durable goods. Then, it evolves according to the following difference equation:

$$G_2 = \delta G_1 + g_2 \tag{2}$$

where $\delta \in (0, 1)$ is the depreciation factor and g_2 is the amount of investment in public goods in period 2. For simplicity, public goods contribute to the welfare of consumers but not to the production sector. However, the government can save when consumers use the same technology. As a result of this saving, the government compensates for the lack of redistribution between the first and second periods. Thus, the budget constraint in each period can be formulated as follows:

$$\sum_i y_1^i - \sum_i c_1^i - (\sum_i s^i + s^G) - pG_1 \geq 0 \tag{3}$$

$$\sum_i y_2^i + (\sum_i s^i + s^G) - \sum_i c_2^i - pg_2 \geq 0 \tag{4}$$

where p is the marginal cost of investment in the public goods. In sum, the government collects tax revenues through labor and capital income taxes, and uses these taxes to provide public goods and incur saving in accordance with its redistribution policy.

[4] In Boadway and Keen (1993), they found that the Samuelson rule is applied in the second-best environment if the utility function is $u^i(v(c_t^i, G_t), l_t^i)$, which is weakly separable. The utility function in this paper is a more restrictive formulation. It is natural that their result remains the same when the utility function is assumed to be that shown in equation (1). The details are discussed in our benchmark analysis.

2.4 Benchmark analysis

In order to examine the effect of commitment, we first consider the case where we assume full commitment by the government. Under this assumption, the government can control the allocation in both periods through its tax policy in the beginning of period 1, and all consumers believe that the commitment by the government is credible. Then, the government chooses c_1^H, y_1^H, c_1^L, y_1^L, s^H, s^L, s_G, G_1, c_2^H, y_2^H, c_2^L, y_2^L and g_2, which maximizes

$$\sum_t \beta^{t-1} \sum_i \{u(c_t^i, G_t) - v(\frac{y_t^i}{\theta^i})\} \tag{5}$$

subject to equation (2), (3), (4), and

$$\sum_t \beta^{t-1}\{u(c_t^H, G_t) - v(\frac{y_t^H}{\theta^H})\} \geq \sum_t \beta^{t-1}\{u(c_t^L, G_t) - v(\frac{y_t^L}{\theta^H})\} \tag{6}$$

Equation (5) is a utilitarian social welfare function, which is the objective function of the government. Equation (6) is the incentive compatibility constraint for a high-type agent. It means that the government faces the constraint that induces a high-type agent to choose his/her own labor supply and consumption, rather than the labor supply and consumption assigned to a low-type agent. Suppose that $u_{c_t}^i$ is the marginal utility with respect to consumption in period t for a type i agent and $u_{G_t}^i$ is the marginal utility with respect to the public good in the period t for a type i agent. In this case, the social optimal condition with respect to the public good is given by:

$$\sum_i MRS^i_{G_1,c_1^i} + \delta\beta \sum_i MRS^i_{G_2,c_2^i} = p \tag{7}$$

where $MRS^i_{G_t,c_t^i} \equiv \frac{u_{G_t}^i}{u_{c_t}^i}$ is the marginal rate of substitution of the public good for private consumption for a type i agent in period t. Equation (7) implies that the Samuelson rule should be applied; that is, the social marginal benefit of increasing a unit of the public good over the two periods should be equal to the producer prices of the public goods related to the aggregate amount of private goods. This is because the utility function is of the separable form and all agents have the same preference; this assumption means that agents evaluate the marginal benefit in the same manner. In other words, the relation between the agent's evaluations is

$$\hat{MRS}_{G_t,c_t^L} = MRS^L_{G_t,c_t^L} \tag{8}$$

where the superscript hat denotes the choice by a high-type agent who chooses a mimicking strategy. For a high-type agent, the marginal benefit of choosing the truth-telling strategy is equal to that of choosing the mimicking strategy. When equation (8) holds, an increase in the level of the public goods never affects the incentive compatibility constraint. Moreover, in the benchmark case, the rule for the social optimal allocation should be constant over periods, since the government can commit to the second-period policy in the first period. Therefore, the government does not need to deviate from the Samuelson rule in both periods. [5]

3 The time consistent public goods rule

In the beginning of the second period, the government can try to exploit the efficiency gain in the second period by using information gathered in the first period. This implies that the optimal tax problem with multiple-periods involves time inconsistency. Being aware of this relation, agents (in particular, a high-type agent) also adjust their own decision-making in the first period. This is known as a commitment issue. When the assumption of full commitment is relaxed, we can consider two cases with respect to how to adjust agent behavior. These cases are candidates for solving the non-commitment case and they called the complete pooling case and complete separation case. [6] Consumers adjusting their behavior affects both tax revenues and provision of public goods because the tax revenues finance the public goods. The main purpose of this section is to examine how the commitment issue affects the optimal condition of public goods provision.

3.1 The complete pooling case

Herein, a high-type agent decreases his/her own labor supply in order to benefit from the information rent that he/she receives in the second period. Consequently, the government cannot observe which agent is a high-type agent at the beginning of the second period. This means that the tax contract assigned in the first period does not depend on the type of agents: $(\bar{c}_1, \bar{y}_1, \bar{s})$. In the second period, given as $\bar{\mathbf{v}} = (\bar{G}_1, \bar{s}, s^G)$, the government chooses c_2^H, y_2^H, c_2^L, y_2^L, and g_2 , which maximizes

$$\sum_i \{u(c_2^i, G_2) - v(\frac{y_2^i}{\theta^i})\} \tag{9}$$

subjet to the budget constraint in the second period that corresponds to equation (4) and

$$u(c_2^H, G_2) - v(\frac{y_2^H}{\theta^H}) \geq u(c_2^L, G_2) - v(\frac{y_2^L}{\theta^H}) \tag{10}$$

Equation (9) is the social welfare function in the second period and equation (10) is the incentive compatibility constraint in the same period. On the basis of the results in the second

[5] It is a rule issue; that is, the government should follow the Samuelson rule in the second-best environment, is obtained in the first-best environment. Therefore, the amount of public goods in the second-best environment is not necessarily equal to the level of public goods in the first-best environment.

[6] Which case is the solution to the non-commitment case depends on the shape of the function and the range of the exogenous variables. The details are analyzed using numerical examples in Section 4.

period, the government chooses $\bar{c}_1$, $\bar{y}_1$, $\bar{s}$, s^G, and $\bar{G}_1$, in order to maximize

$$\sum_i \{u(\bar{c}_1, \bar{G}_1) - v(\frac{\bar{y}_1}{\theta^i})\} + \beta V_2^{Pool}(\bar{\mathbf{v}}) \tag{11}$$

subject to

$$\sum_i \{\bar{y}_1 - \bar{c}_1 - \bar{s}\} - s^G \geq p\bar{G}_1 \tag{12}$$

where $V_2^{Pool}(\bar{\mathbf{v}})$ is the value function in the second period.

Herein, it is easy to show that the dynamic formulation of the Samuelson rule in the benchmark case should be applied as the social optimal condition of the public goods provision in the complete pooling case. In the first period, the incentive compatibility constraints are binding for both type agents. Therefore, the government faces no incentive compatibility constraints. In the second period, the planning problem corresponds to the standard non-linear income tax problem. In terms of the evaluation of public goods between a high-type agent and a low-type agent, the outcome in this period conforms with the benchmark case. Therefore, the social optimal condition of public goods provision in the benchmark case remains the same as that in the complete pooling case.

3.2 The complete separation case

We consider the case in which the tax system separates a high-type agent from a low-type agent in the first period; that is, the asymmetric information can be resolved in the second period. At an optimum, the government can utilize the lump-sum tax policy in order to redistribute income between agents. We turn to the planning problem in this case. Given as $\mathbf{v} = (G_1, s^H, s^L, s^G)$, the government chooses c_2^H, c_2^L, y_2^H, y_2^L, and g_2, in order to maximize

$$\sum_i \{u(c_2^i, G_2) - v(\frac{y_2^i}{\theta^i})\} \tag{13}$$

subject to equation (4). Let $V_2^{Sep}(\mathbf{v})$ be the value function in the second period of the complete separation case. With this in mind, the government chooses c_1^H, c_1^L, y_1^H, y_1^L, and G_1 to maximize

$$\sum_i \{u(c_1^i, G_1) - v(\frac{y_1^i}{\theta^i})\} + \beta V_2^{Sep}(\mathbf{v}) \tag{14}$$

subject to equation (3) and

$$\begin{aligned} &u(c_1^H, G_1) - v(\frac{y_1^H}{\theta^H}) + \beta\{u(c_2^H(\mathbf{v}), G_2(\mathbf{v})) - v(\frac{y_2^H(\mathbf{v})}{\theta^H})\} \\ &\geq u(c_1^L, G_1) - v(\frac{y_1^L}{\theta^H}) + \beta\{u(c_2^L(\mathbf{v}), G_2(\mathbf{v})) - v(\frac{y_2^L(\mathbf{v})}{\theta^H})\} \end{aligned} \tag{15}$$

where $c_2^H(\mathbf{v})$ and $y_2^H(\mathbf{v})$ are the first-best contracts for a high type agent, and $c_2^L(\mathbf{v})$ and $y_2^L(\mathbf{v})$ are the first-best contracts for a low-type agent. Equation (15) is the incentive compatibility constraint for a high-type agents. The left-hand side is the utility for a high-type agent that chooses the truth-telling strategy and the right-hand side is the utility received when he/she chooses the mimicking strategy; that is, the second period payoff received by a high-type agent in the complete pooling case. Suppose that ϕ is the Lagrange multiplier for the incentive compatibility constraint, λ_t is the Lagrange multiplier for the resource constraint in period t, and $\hat{v}_{y_2}$ is the marginal dis-utility with respect to labor income in period 2 for an agent who mimics. Combining these factors with the optimal conditions allows us to obtain the social optimal condition with respect to the public good:

$$[\sum_i MRS^i_{G_1,c_1^i} + \delta\beta\frac{\lambda_2}{\lambda_1}\sum_i MRS^i_{G_2,c_2^i}] + \Psi_G = p \tag{16}$$

where

$$\Psi_G \equiv \frac{\phi}{\lambda_1}[\underbrace{\{u^H_{G_2} - u^L_{G_2}\}\{\delta + \frac{dg_2}{dG_1}\}}_{\text{The direct effect}} + \underbrace{\{u^H_{c_2}\frac{dc_2^H}{dG_1} - v^H_{y_2}\frac{dy_2^H}{dG_1}\} - \{u^L_c\frac{dc_2^L}{dG_1} - \hat{v}_y\frac{dy_2^L}{dG_1}\}}_{\text{The indirect effect}}]$$

The derivation of equation (16) is presented in Appendix A. This equation contains two additional effects.

Firstly, it relates to the durability of public goods. Since the government takes into account the outcome in the second period and public goods have the property of durable goods, increasing a unit of the public goods in the first period influences the incentive compatibility constraint. The effect is represented by Ψ_G, and is named "the durable effect." Furthermore, we can classify this effect into two terms; the direct effect and indirect effect. The first term, Ψ_G, indicates the direct effect. Since the public goods provision contributes to the welfare of agents, an additional public goods provision directly affects payoffs both agents, the agent who chooses the true-telling strategy and the agent who chooses the mimicking strategy. The indirect effect appears since the endogenous variables chosen in the second period depend on the public good provision in the first period. This also affects the incentive compatibility constraint. These are illustrated within the second and third brackets of Ψ_G. A critical question is whether the direct effect exceeds or falls the indirect effect. To see the difference between these effects, suppose that the preference is additive and separable between private consumption and the public good. Under this assumption, the direct effect disappears because the marginal utility of the public good between agents is identical. In contrast, the indirect effect is negative, as shown in Appendix B. The sign of the indirect effect implies that an incremental unit of the public good restricts the incentive compatibility constraint. Specifically, the additional public good provision decreases the payoff of agent who chooses the truth-telling strategy, but increases the payoff of agent who chooses the mimicking strategy.

The second novel effect is a consequence of production inefficiency. The engine of this effect is the ratio of the shadow price of public fund across time periods. The first-order condition with respect to the government's saving reveals the following:

$$-\lambda_1 + \Psi_{s_G} + \beta\lambda_2 = 0$$

where

$$\Psi_{s_G} \equiv \beta\phi[\{u^H_{G_2} - u^L_{G_2}\}\{\delta + \frac{dg_2}{ds_G}\} + \{u^H_{c_2}\frac{dc^H_2}{ds_G} - v^H_{y_2}\frac{dy^H_2}{ds_G}\} - \{u^L_c\frac{dc^L_2}{ds_G} - \hat{v}_y\frac{dy^L_2}{ds_G}\}]$$

The variable Ψ_{s_G} indicates how an additional saving by the government affects the incentive compatibility constraint. When the disutility function of effort is iso-elastic, the sign of Ψ_{s_G} is negative. The proof is shown in Appendix C. The negative sign implies that the production efficiency theorem by Diamond and Mirrlees (1971) breaks down, since the marginal social value of public funds is not equal to the marginal rate of transformation of commodities between the first and second periods. Therefore, as the shadow price of pubic funds is higher in the second period than that in the first period, the discount rate of the social marginal benefit is greater than unity. Hereinafter, we call this effect "the public saving effect" and this feature allow us to propose the following.

Proposition 1. *Suppose that the government cannot commit to the second period tax policy. When all agents are completely separate in the first period (even if the preference satisfies the separability condition and is constant for all agents), then the dynamic formulation of the Samuelson rule should be modified.*

It is more interesting for us to examine the interaction with MCPF. Rearranging equation (16) yields the following:

$$SMB_1 + \underbrace{\Psi_G}_{\text{the durable effect}} + \underbrace{(\frac{\beta\lambda_2}{\beta\lambda_2 + \Psi_{s_G}} - 1)\delta \sum_i MRS^i_{G_2,c^i_2}}_{\text{the public saving effect}} = p$$

where $SMB_1 \equiv \sum_i MRS^i_{G_1,c^i_1} + \delta \sum_i MRS^i_{G_2,c^i_2}$ is the social marginal benefit of an increase in the level of public goods in the first period. As Ψ_G is negative, the durable effect means that MCPF is greater than unity. To begin with, an increase in a unit of the public goods provision requires additional tax revenues. Then, if the government utilizes distortionary taxes, MCPF is greater than unity because of the Pigou effect. In contrast, an additional public goods provision affects MCPF through the demand for taxed goods. The durable effect is different from this response, because increasing a unit of the public goods provision influences the welfare level through the incentive compatibility constraint. As for the public saving effect, it implies that MCPF is less than unity because $\Psi_{s_G} < 0$. As λ_1 is greater than zero, the sign of this term is positive. The weight for the marginal rate of the substitution of public goods for private consumption in the second period is higher than that in the first period. This means it has the opposite effect on MCPF compared with the durable effect. Therefore, if the durable effect dominates the public saving effect at the social optimum, then MCPF is greater than unity and vice versa.

4 Numerical examples

Herein, we provide numerical examples in order to answer the following questions:, (i) Which case of social welfare is greater, the complete separation case or the complete pooling case; and (ii) How different is the social optimal public goods provision through the social optimal condition with respect to the pubic goods discussed in the previous sections?

4.1 Specification and Parameterization

It is assumed that utility function is represented by the the constant relative risk aversion (CRRA) function that is satisfied by separability,

$$\sum_t \beta^{t-1}\{\frac{1}{1-\sigma}(c_t^i \cdot G^i)^{1-\sigma} - (\frac{y_t^i}{\theta^i})^\gamma\} \tag{17}$$

where σ and γ are non-negative. It is also assumed that $\sigma = 1.0$ and $\gamma = 2.0$. The other preference parameter (a discount factor) is $\beta = 0.38$, the wage rate of a high-type agent is $\theta^H = 2.0$, and θ^L is equal to unity. Concerning a setting of the public goods, the marginal cost of investment in the public goods is unity and the depreciation factor is 0.5. [7] Table 1 summarizes the results of our simulation.

4.2 The level of social welfare

The level of social welfare in each case is shown in the last line of Table 1. The highest level of social welfare can be achieved in the benchmark case. This outcome corresponds to previous studies by Bisin and Rampini (2006) and Guo and Krause (2013). The complete separation case is ranked second and the complete pooling case is ranked third. The asymmetric information remains in the second period of the complete pooling case, while it resolves in the complete separation case. The difference of information structures between the two cases can enhance the social welfare level in the complete separation case.

4.3 The level of public goods provisions

In order to analyze the level of public goods determined by the social optimal condition discussed in the previous sections, we focus on the value of public goods provided in the first period. In fact, the level of public goods is highest in the benchmark case and, lowest in the complete pooling case. Since the Samuelson rule should be applied in the benchmark and

[7] This specification and parameterization, as outlined below, refer to Golosov et al. (2006) and Guo and Krause (2013). Their simulations are based on the optimal non-linear income tax problem without the public goods provision. They examine the two-period case and the infinite-horizon case. In the former case, the level of social welfare in the complete separation case is larger than that in the complete pooling case; the same result as we find in this paper.

Table 1: Results of simulation

	Benchmark case		Pooling case		Separation case	
	High	*Low*	*High*	*Low*	*High*	*Low*
c_1^i	0.893466	0.443849	0.590996	0.590996	1.01529	0.2694
y_1^i	1.88829	1.20472	1.99574	1.99574	1.88046	1.39809
s^i	0.553725	0.104287	0.533473	0.533473	0.0397036	0.105052
c_2^i	0.948375	0.787473	1.62404	1.26582	0.970125	0.970125
y_2^i	1.65153	1.02771	1.69052	1.21287	1.66288	1.36307
Payoff	-0.846492	-0.945162	-1.53149	-2.13712	-0.836572	-1.90097
G_1	1.97365		1.31917		1.54457	
g_2	0.725424		1.50384		1.53499	
s_G	-0.875972		0.423365		0.304534	
SMB_1	1.00		1.00		1.25221	
$\frac{\lambda_2}{\lambda_1}$	1.00		1.00		1.69018	
Ψ_G	-		-		-1.51776	
Social welfare	-1.79164		-3.66859		-2.73754	

complete pooling cases, the difference in the public goods levels between the cases depends on the values of the endogenous variables. On the other hand, the level of public goods in the complete separation case is ranked second. A key variable to discuss the difference between the benchmark case and the complete separation case is Ψ_G. Since the value of this variable in the complete separation case is negative, the sum of the marginal rates of substitution should be greater than the producer price of the public good in terms of the private good. By comparing the Samuelson rule in this case with the benchmark case, we find that the level of public goods in the complete separation case is lower than in the benchmark case.

5 Conclusion

In this study, we reconsider the optimal non-linear tax problem with the public goods provision from the perspective of the commitment issue. If taxpayers have the same preferences and the preferences are separable consumption and leisure, the Samuelson rule does not need to be modified, as discussed in previous studies: Christiansen (1981) and Boadway and Keen (1993). However, the outcome is altered dramatically when the commitment issue is considered. We show that the Samuelson rule should be modified when the taxpayers are completely separated by tax policy, even if the utility function satisfies the separability condition. Moreover, we point out through numerical examples that the effects of the commitment issue also affect the level of public goods.

Appendices

A. Proof of equation (16)

Consider the second-period planning problem in the complete separation case. The Lagrangian can be written as follows:

$$\mathcal{L}_2 = \sum_i \{u(c_2^i, \delta G_1 + g_2) - v(\frac{y_2^i}{\theta^i})\} + \lambda_2[\sum_i \{y_2^i + s^i - c_2^i\} + s_G - pg_2] \tag{18}$$

The first order conditions are

$$\frac{\partial \mathcal{L}_2}{\partial c_2^H} = u_{c_2}^H - \lambda_2 = 0 \tag{19}$$

$$\frac{\partial \mathcal{L}_2}{\partial c_2^L} = u_{c_2}^L - \lambda_2 = 0 \tag{20}$$

$$\frac{\partial \mathcal{L}_2}{\partial y_2^H} = -v_{y_2}^H + \lambda_2 = 0 \tag{21}$$

$$\frac{\partial \mathcal{L}_2}{\partial y_2^L} = -v_{y_2}^L + \lambda_2 = 0 \tag{22}$$

$$\frac{\partial \mathcal{L}_2}{\partial g_2} = \sum_i u_{G_2}^i - \lambda_2 p = 0 \tag{23}$$

$$\frac{\partial \mathcal{L}_2}{\partial \lambda_2} = \sum_i \{y_2^i + s^i - c_2^i\} + s_G - pg_2 = 0 \tag{24}$$

where the notations on derivatives are denoted by $u_{c_t}^i = \frac{\partial u}{\partial c_t^i}(c_t^i, G_t)$, $v_{y_t}^i = \frac{dv}{dl_t^i}\frac{1}{\theta^i}$ and $u_{G_t}^i = \frac{\partial u}{\partial G_t}(c_t^i, G_t)$. From the envelope theorem, we find

$$\frac{dV_2^{Sep}}{dG_1} = \delta \sum_i u_{G_2}^i \tag{25}$$

$$\frac{dV_2^{Sep}}{ds_G} = \lambda_2 \tag{26}$$

Next, we turn to the first period planning problem. The Lagrangian in the first period is

$$\begin{aligned}\mathcal{L}_1 = \sum_i \{u(c_1^i, G_1) - v(\frac{y_1^i}{\theta^i})\} + \beta V_2^{Sep}(\mathbf{v}) + \lambda_1[\sum_i \{y_1^i - c_1^i - s^i\} - s_G - pG_1] \\ +\phi[u(c_1^H, G_1) - v(\frac{y_1^H}{\theta^H}) + \beta\{u(c_2^H(\mathbf{v}), G_2(\mathbf{v})) - v(\frac{y_2^H(\mathbf{v})}{\theta^H})\} \\ -u(c_1^L, G_1) + v(\frac{y_1^L}{\theta^H}) - \beta\{u(c_2^L(\mathbf{v}), G_2(\mathbf{v})) - v(\frac{y_2^L(\mathbf{v})}{\theta^H})\}]\end{aligned} \tag{27}$$

The optimality conditions with respect to the private consumption of each consumer, the public goods and saving are as follows:

$$\frac{\partial \mathcal{L}_1}{\partial c_1^H} = (1+\phi)u_{c_2}^H - \lambda_1 = 0 \tag{28}$$

$$\frac{\partial \mathcal{L}_1}{\partial c_1^L} = (1-\phi)u_{c_2}^L - \lambda_1 = 0 \tag{29}$$

$$\frac{\partial \mathcal{L}}{\partial G_1} = \sum_i u_G^i + \beta \frac{dV_2^{Sep}}{dG_1} - \lambda_1 p \tag{30}$$

$$+\beta\phi[\{u_{c_2}^H \frac{dc_2^H}{dG_1} - v_{y_2}^H \frac{dy_2^H}{dG_1} + u_{G_2}^H \frac{dG_2}{dG_1}\} - \{u_c^L \frac{dc_2^L}{dG_1} - \hat{v}_y \frac{dy_2^L}{dG_1} + u_G^L \frac{dG_2}{dG_1}\}] = 0$$

$$\frac{\partial \mathcal{L}_1}{\partial s_G} = \beta \frac{dV_2^{sep}}{ds_G} - \lambda_1 \tag{31}$$

$$+\beta\phi[\{u_{c_2}^H \frac{dc_2^H}{ds_G} - v_{y_2}^H \frac{dy_2^H}{ds_G} + u_{G_2}^H \frac{dG_2}{ds_G}\} - \{u_c^L \frac{dc_2^L}{ds_G} - \hat{v}_y \frac{dy_2^L}{ds_G} + u_G^L \frac{dG_2}{ds_G}\}] = 0$$

By the definition of public goods between the first period and the second period (equation (2)), it is easy to show that the marginal effect of public goods in the first period on in the second period is $\frac{dG_2}{dG_1} = \delta + \frac{dg_2}{dG_1}$. Using equations (19), (20), (24), (26), (28), and (29), equation (30) can be simplified to equation (16). □

B. Proof of the sign of Ψ_G

We consider the sign of Ψ_G. It is assumed that $\frac{\partial^2 u}{\partial c \partial G}$ is zero. Let A be the Hessian with respect to equation (19) to (24). We can rewrite these equations as a matrix formation,

$$A \begin{pmatrix} dc_2^H \\ dc_2^L \\ dy_2^H \\ dy_2^L \\ dg_2 \\ d\lambda_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \delta \sum_i u_{g,g}^i \\ 0 \end{pmatrix} dG_1$$

where

$$A \equiv \begin{pmatrix} u_{c,c}^H & 0 & 0 & 0 & 0 & -1 \\ 0 & u_{c,c}^L & 0 & 0 & 0 & -1 \\ 0 & 0 & -v_{y,y}^H & 0 & 0 & 1 \\ 0 & 0 & 0 & -v_{y,y}^L & 0 & 1 \\ 0 & 0 & 0 & 0 & \sum_i u_{g,g}^i & -p \\ -1 & -1 & 1 & 1 & -p & 0 \end{pmatrix}$$

The determinant of A is

$$|A| = -u_{c,c}^H u_{c,c}^L v_{y,y}^H v_{y,y}^L p^2 + \sum_{i \neq j} u_{c,c}^i u_{c,c}^j v_{y,y}^i \sum_i u_{g,g}^i - \sum_{i \neq j} u_{c,c}^i v_{y,y}^i v_{y,y}^j \sum_i u_{g,g}^i < 0 \tag{32}$$

It implies that the inverse matrix of $|A|$ exist and Cramer's theorem is available. Then, we obtain

$$\frac{dc_2^H}{dG_1} = \frac{\delta \sum_i u^i_{g,g}}{|A|} p u^L_{c,c} v^H_{y,y} v^L_{y,y} < 0 \tag{33}$$

$$\frac{dy_2^H}{dG_1} = \frac{\delta \sum_i u^i_{g,g}}{|A|} p u^H_{c,c} u^L_{c,c} v^L_{y,y} > 0 \tag{34}$$

$$\frac{dc_2^L}{dG_1} = -\frac{\delta \sum_i u^i_{g,g}}{|A|} p u^H_{c,c} v^H_{y,y} v^L_{y,y} > 0 \tag{35}$$

$$\frac{dy_2^L}{dG_1} = -\frac{\delta \sum_i u^i_{g,g}}{|A|} p u^H_{c,c} u^L_{c,c} v^H_{y,y} < 0 \tag{36}$$

$$\frac{dg_2}{dG_1} = \frac{\delta \sum_i u^i_{g,g}}{|A|} v^H_{y,y}\{u^H_{c,c}u^L_{c,c} - u^H_{c,c}v^L_{y,y} - u^L_{c,c}v^L_{y,y}\} > 0 \tag{37}$$

These equations imply that Ψ_G is negative.

C. Proof of the sign of Ψ_{s_G}

We show that Ψ_{s_G} is negative. Suppose that the dis-utility is an iso-elastic utility function of work effort, that is, $v(\frac{y^i_t}{\theta^i}) = \frac{\kappa}{\nu}(\frac{y^i_t}{\theta^i})^\nu, \kappa > 0, \nu \geq 1$. We can rewrite equations (19) to (24) as a matrix formation,

$$A \begin{pmatrix} dc_2^H \\ dc_2^L \\ dy_2^H \\ dy_2^L \\ dg_2 \\ d\lambda_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} ds_G$$

Applicaition of Cramer's rule yields,

$$\frac{dc_2^H}{ds_G} = \frac{\sum_i u^i_{g,g}}{|A|} u^L_{c,c} v^H_{y,y} v^L_{y,y} < 0 \tag{38}$$

$$\frac{dy_2^H}{ds_G} = \frac{\sum_i u^i_{g,g}}{|A|} u^H_{c,c} u^L_{c,c} v^L_{y,y} > 0 \tag{39}$$

$$\frac{dc_2^L}{ds_G} = \frac{\sum_i u^i_{g,g}}{|A|} u^H_{c,c} u^H_{c,c} v^L_{y,y} < 0 \tag{40}$$

$$\frac{dy_2^H}{ds_G} = \frac{\sum_i u^i_{g,g}}{|A|} u^H_{c,c} u^L_{c,c} v^H_{y,y} > 0 \tag{41}$$

$$\frac{dg_2}{ds_G} = \frac{1}{|A|} u^H_{c,c} u^L_{c,c} v^H_{y,y} v^L_{y,y} p < 0 \tag{42}$$

From equations (19) and (20), it is easy to show that $u^H_{c_2}$ is equal to $u^L_{c_2}$. Moreover, it implies that c^H_2 is also equal to c^L_2, if $\frac{\partial^2 u}{\partial c \partial G} = 0$. Then, we can find that $\frac{\partial c^H_2}{\partial s_G} - \frac{\partial c^L_2}{\partial s_G}$ is zero. These results give the following equation:

$$\Psi_{s_G} = (\frac{1}{\theta^H})\frac{1}{|A|}u^H_{c,c}u^L_{c,c}\sum_i u^i_{g,g}[\frac{\hat{v}_y v^H_{y,y}(\theta^L)^2}{v^H_y v^L_{y,y}(\theta^H)^2} - 1] \tag{43}$$

Since the dis-utility function is iso-elastic, equation (41) can be rewritten as

$$\Psi_{s_G} = (\frac{1}{\theta^H})\frac{1}{|A|}u^H_{c,c}u^L_{c,c}\sum_i u^i_{g,g}[(\frac{y^L_2}{y^H_2})(\frac{\theta^L}{\theta^H})^{\nu} - 1] \tag{44}$$

By subtracting equation (21) from equation (22), we find that $y^H_2 > y^L_2$. Then, equation (42) has a negative sign and Ψ_{s_G} is also negative. □

References

[1] Aronsson, Thomas and Grandlund, David. 2011, Public goods and optimal paternalism under present-biased preferences, Economic Letters, 113, 54-57

[2] Apps, P and Rees, R, 2006, Repeated Optimal Nonlinear Income Taxation, mimeo.

[3] Batina, Raymond G., 1990, Public goods and dynamic efficiency The Modified Samuelson Rule, Journal of Public Economics, 41, 389-400

[4] Berliant, Marcus and Ledyard, John O, 2014, Optimal dynamic nonlinear income taxes with no commitment, Journal of Public Economic Theory, 16, 196-221

[5] Bisin, Alberto, and Rampini, Adriano, 2006, Markets as Beneficial Constraints on the Government, Journal of Public Economics, 90, 601-629

[6] Boadway, Robin, and Keen, Michael, 1993, Public goods, self-selection and optimal income taxation, International Economic Review, 34, 463-478

[7] Brett, Craig, and John Weymark, 2008, Optimal nonlinear taxation of income and savings without commitment, Working paper 08-W05. Vanderbilt University

[8] Christiansen, Vidar., 1981, Evaluation of public projects under optimal taxation, Review of Economic Studies, 48, 447-457

[9] Diamond, Peter and Mirrlees, James, 1971, Optimal taxation and public production, American Economic Review, 61, nos.1 and 3, 8-27, 261-278

[10] Golosov, Mikhail, Tsyvinski, Aleh and Werning, Iv́an, 2006, New dynamic public finance: a user's guide, NBER Macroeconomics Annual, 317-387

[11] Guo, Jang-Ting, and Krause, Alan, 2011, Optimal nonlinear income taxation with habit formation, Journal of Public Economic Theory, 13, 463-480

[12] Guo, Jang-Ting, and Krause, Alan, 2013, Optimal nonlinear taxation of income and education expenditure, Oxford Economic Paper, 65, 74-95

[13] Krause, Alan, 2009, Optimal nonlinear income taxation with learning-by-doing, Journal of Public Economics, 93, 1098-1110

[14] Pigou, Arthur C., 1928, A Study in Public Finance, London: Macmillan.

[15] Pirttilä, Jukka, and Tuomala, Matti, 2001, On optimal non-linear taxation and public good provision in an overlapping generation economy, Journal of Public Economics, 79, 485-501

[16] Roberts, Kevin, 1982, The Theoretical Limits to Redistribution, Review of Economic Studies, 51, 177-195

[17] Stiglitz, Joseph E., 1982, "Self-selection and Pareto efficient taxation" Journal of Public Economics 17, 213-240

[18] Stiglitz, Joseph E., 1987, "Pareto efficient and optimal taxation in the new new welfare economics", In: Auerbach, A. J. and Feldstein, M. (Eds.), Handbook of Public Economics. Vol.2. North-Holland, Amsterdam, 991-1042