Discussion Papers In Economics And Business

Pensions, Education, and Growth: A Positive Analysis

Tetsuo Ono and Yuki Uchida

Discussion Paper 14-37-Rev.2

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Pensions, Education, and Growth: A Positive Analysis

Tetsuo Ono and Yuki Uchida

Discussion Paper 14-37-Rev.2

March 2016

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Pensions, Education, and Growth: A Positive Analysis*

Tetsuo Ono
Osaka University

Yuki Uchida
Osaka University

Abstract

This study presents an overlapping generations model to capture the nature of the competition between generations regarding two redistribution policies, public education and public pensions. From a political economy viewpoint, we investigate the effects of population aging on these policies and economic growth. We show that greater longevity results in a higher pension-to-GDP ratio. However, an increase in longevity produces an initial increase followed by a decrease in the public education-to-GDP ratio. This, in turn, results in a hump-shaped pattern of the growth rate.

- Keywords: economic growth; population aging; public education; public pensions
- JEL Classification: D78, E24, H55

---

*Tetsuo Ono: Graduate School of Economics, Osaka University, 1-7, Machikaneyama, Toyonaka, Osaka 560-0043, Japan. E-mail: tono@econ.osaka-u.ac.jp. Yuki Uchida: Graduate School of Economics, Osaka University, 1-7, Machikaneyama, Toyonaka, Osaka 560-0043, Japan. E-mail: pge001uy@student.econ.osaka-u.ac.jp. Ono acknowledges financial support from Grant-in-Aid for Scientific Research (C) from the Ministry of Education, Culture, Sports, Science and Technology of Japan No. 15K03509. Uchida acknowledges financial support from Grant-in-Aid for JSPS Fellows of the Japan Society for the Promotion of Science No. 15J01375.

1 Introduction

Redistribution policy preferences are dictated largely by age. Middle-aged workers who are parents have altruistic concern for their children and are more likely to support public education because they can benefit from highly educated children. This makes expenditure on education more attractive for these parents. On the other hand, retired old people are less likely to support public education because they cannot obtain the benefits of education directly. Instead, they support public pension expenditure that compensates for loss of earnings in their retirement. The age-dependent difference in policy preferences suggests that the recent trend of aging in developed countries, which increases the political power of the old, may induce governments to shift expenditure from education to pensions.

This prediction on pensions is in line with the observation in developed countries. Figure 1 suggests that the public pension spending-to-GDP ratio is positively correlated with the share of the population aged 65 years and above. However, the aforementioned prediction on public education does not fit the observation: the public education spending-to-GDP ratio shows a hump-shaped pattern in response to the share of the aged population. In addition, the growth rate shows a hump-shaped pattern, which seems to be induced by the hump-shaped public education spending pattern. Therefore, there should be another mechanism for the non-linear relationship between aging and public education. The aim of this study to present a political economy model that provides the prediction fitting the observation in Figure 1.

[Figure 1 here.]

For analysis, this study presents a politico-economic model to characterize the nature of the competition between generations, and in addition, examines the effect of population aging on redistribution policy and economic growth. To capture generational conflict, the model economy contains an infinite sequence of overlapping generations in which each is comprised of many identical individuals who live over three periods, namely, young, middle, and old ages. The middle-aged individuals are faced with uncertain lifetimes and are endowed with altruism toward children (i.e., the young). The middle-aged and old individuals participate in voting, but policy disagreements between them arise owing to the longer planning horizon of the middle-aged individuals. In addition, the model contains physical and human capital accumulation through savings and educational investment, which enables us to demonstrate the effect of redistribution policy on economic growth.

Within this framework, we consider probabilistic voting a la Lindbeck and Weibull (1987). In each period, the middle-aged and old individuals participate in voting; the young are not enfranchised. The government in power maximizes a political objective function of the weighted sum of the utilities of the middle-aged and old population, taking account of the impact on the middle-aged population's economic decisions (for applications of the probabilistic voting for overlapping-generations models, see, e.g., Grossman and Helpman, 1998; Hassler et al., 2005; and Song, Storesletten, and Zilibotti, 2012).

In this voting environment, the redistribution preferences of the middle-aged population are state dependent. In other words, successive generations are linked through physical and human capital accumulation. The state dependence of policy preferences gives the middle-aged voters the means to influence the policy outcome when they are old. Their incentive to manipulate policy preferences stems from their desire to obtain large pension benefits in old age. We capture this forward-looking behavior of the middle-aged voters by focusing on Markov-perfect political equilibria, in which voters condition their strategies on payoff-relevant state variables (i.e., physical and human capital in the present model).

The Markov-perfect political equilibrium is affected by the three factors representing population aging: uncertain lifetimes capturing individual longevity, the political power of the old that reflects their share in voting, and the population growth rate. The political power of the old and the population growth rate have definite effects on pensions and education. With greater political power of the old and a lower population growth rate, the pension-to-GDP ratio increases, but the education-to-GDP ratio decreases. In other words, population aging stemming from a larger share of the old in voting and a lower population growth rate shifts the allocation of tax revenue from education for the young to pensions for the old.

In addition, longevity has a definite effect on pensions; greater longevity results in a higher pension-to-GDP ratio. However, the effect of longevity on public education is not straightforward. Greater longevity implies a larger weight on the utility of the old and the utility of the middle-aged population for their consumption in old age. This incentivizes the government to shift its spending from education to pensions to improve their utility. However, at the same time, greater longevity incentivizes the government to shift the allocation of spending from pensions to education because greater longevity implies a larger weight of middle-aged voters' altruistic utility from the human capital of their children. These opposing effects produce an initial increase followed by a decrease in the education-to-GDP ratio and this, in turn, results in a hump-shaped pattern of the growth rate. The model predictions described thus far are in line with the observations of developed countries demonstrated in Figure 1.

We obtain our results by assuming perfect annuity markets where individuals can purchase private pensions. However, in the real world, some countries have limited or no access to private annuity markets. To account for this possibility, we consider an alternative case without an annuity market, and show that the growth rate increases with rising longevity. This result reflects data from some Eastern European countries, such as Czech Republic, Estonia, and Hungary, with low degrees of private annuitization (OECD, 2014), as demonstrated in Panel (c) of Figure 1. Therefore, we may well conclude that the overall trend in Organisation for Economic Co-operation and Development (OECD) countries shows a non-linear relationship between longevity and growth, but some countries show a positive relationship due to limited access to private annuities.

The remainder of this paper is organized as follows. We first present a literature review in Subsection 1.1. Thereafter, Section 2 presents the model and characterizes economic equilibrium. Section 3 describes political equilibrium. Section 4 investigates how the three aging factors affect government expenditure on education and pensions. Section 5 analyzes the effects of the aging factors on economic growth. Section 6 undertakes the analysis under an alternative scenario where annuity markets are absent, and compares the result here to that in Section 5. Section 7 provides concluding remarks.

1.1 Literature Review

The present study is related to the literature on the political economy of public education and pensions by Bearse, Glomm, and Janeba (2001), Soares (2006), Iturbe-Ormaetxe and Valera (2012), Kaganovich and Meier (2012), Kaganovich and Zilcha (2012), and Naito (2012). A common feature of these studies is that the two-dimensional voting aspect is reduced to one dimension for simplicity of analysis. In other words, they consider a vote over public education for a given pension benefit, or a vote over the allocation of tax revenue for a given tax rate. Therefore, these studies do not indicate how the size of the government (i.e., the tax rate) and the allocation of government spending between education and pensions are determined jointly through voting in the presence of generational conflict.[1]

This problem is resolved by introducing two-dimensional voting (Rangel, 2003; Levy, 2005; Poutvaara, 2006; and Arawatari and Ono, 2014). However, these studies abstract from physical and/or human capital formation, and thus, show nothing about the interaction between policy and capital formation. Capital formation is introduced by Kemnitz (2000), Gradstein and Kaganovich (2004), Holz-Eakin, Lovely, and Tosun (2004), Tosun (2008), and Bernasconi and Profeta (2012). These studies assume myopic voting, in which the current voters take future policy as given. In other words, the forward-looking decisions of voters are absent in the analysis of these studies. Therefore, they abstract from the feedback mechanism between current and future redistribution policies through physical and/or human capital accumulation, which plays a crucial role in shaping redistribution

[1] An alternative to the political economy approach is the normative approach (e.g., Boldrin and Montes, 2005), which takes the two spending programs as given and focuses on their role as a means to support the complete market allocation.

policies.

The feedback mechanism is demonstrated by Beauchemin (1998), Forni (2005), Bassetto (2008), Gonzalez-Eiras and Niepelt (2008, 2012), Song (2011), Bishnu and Wang (2014), Chen and Song (2014), and Ono (2015). In particular, the present study is closely related to Lancia and Russo (2015), who analyze the politics of public education and pensions in overlapping generations models. Lancia and Russo (2015) show that greater political power of the old in voting increases pension spending but decreases education spending. However, the cross-country evidence shows that the education-to-GDP ratio shows a hump-shaped pattern in response to the share of the old in the population (see Figure 1). The present study shows that altruism toward children and uncertain lifetimes, which are absent in the model of Lancia and Russo (2015), are the keys to demonstrate such a hump-shaped pattern of education spending. In addition, this pattern induces a non-linear relationship between aging and economic growth, which is in line with the empirical evidence (e.g., An and Jeon, 2006; Kunze, 2014; and Panel (c) in Figure 1).

Ludwig, Schelkle, and Vogel (2012) and Heer and Irmen (2014) also show the potential for a non-linear relationship. An aging population decreases the share of the working-age population, which in turn increases the capital-labor ratio, lowers the rates of return on capital, and thus creates a disincentive to save. Endogenous human capital adjustment (Ludwig, Schelkle, and Vogel, 2012) or firms' incentive to invest in innovation that affect total factor productivity (Heer and Irmen, 2014) could mitigate this negative impact . The present study instead focuses on the political power of the elderly that affects growth rates through their influence on public education and pension policies.

Apart from the abovementioned studies, the present study is related to Lambrecht, Michel, and Vidal (2005) and Kunze (2014), who investigate the growth effect of redistribution policy in overlapping generations models, in which altruistic parents finance the education of their children. However, both of these studies focus on a single policy issue: public pensions in the case of Lambrecht, Michel, and Vidal (2005), and public education in the case of Kunze (2014). The present study differs from theirs in that we consider the two policy issues, investigate how they are shaped by population aging, and in turn, analyze how they affect economic growth.

2 Model

The model is based on that presented by Lambrecht, Michel, and Vidal (2005) and Kunze (2014). The economy starts at period 0 and consists of overlapping generations. Individuals are identical within a generation, live at most for three periods, namely, young, middle, and old ages. Each middle-aged individual gives birth to $1 + n$ children. The middle-aged population for the period-t is N_t, and the population grows at a constant rate $n(>-1): N_{t+1}=(1+n)N_t$.

Individuals are faced with uncertain lifetimes in the third period of life. The probability of living in old age is $\pi \in [0,1]$. This is idiosyncratic for all individuals and is constant across periods.

2.1 Individuals

The economic behavior of individuals over their life cycles is as follows. During young age, individuals make no economic decisions; they receive public education financed by the government. In middle age, individuals work, receive market wages, and make tax payments. They use after-tax income for consumption and savings. In old age, individuals are retired. They receive the returns from savings and pension benefits, and consume both.

Consider an individual born in period $t-1$. In period t, he is middle aged and is endowed with h_t units of human capital. He supplies it inelastically in the labor market, and obtains the wage $w_t h_t$, where w_t is the wage rate per efficiency unit of labor in period-t. After paying the tax $\tau_t w_t h_t$ where $\tau_t \in (0,1)$ is the period-t income tax rate, the individual distributes his after-tax income into consumption, c_t, and savings held as an annuity and invested in physical capital, s_t. Therefore, the period-t budget constraint for the middle becomes as follows:

$$c_t + s_t \leq (1-\tau_t) w_t h_t.$$

The period-$t+1$ budget constraint in old age is

$$d_{t+1} \leq \frac{R_{t+1}}{\pi} s_t + p_{t+1},$$

where d_{t+1} is consumption in old age, $R_{t+1}(>0)$ is the gross return from investment in capital, $R_{t+1}s_t/\pi$ is the return from savings, and p_{t+1} is the pension benefit. If an individual dies at the end of the middle age, his annuitized wealth is transferred to the individuals who live throughout old age via annuity markets. Therefore, the return from saving becomes R_{t+1}/π under the assumption of perfect annuity markets. The case of no annuity market will be investigated in Section 6.

A period-t middle-aged individual cares about his children's per capita human capital in period $t+1, h_{t+1}$. This is a function of the government spending on public education, x_t, and the parent's human capital, h_t. In particular, h_{t+1} is formulated by the following equation:

$$h_{t+1} = D\left(x_t\right)^{\eta}\left(h_t\right)^{1-\eta},$$

where $D(>0)$ is a scale factor, and $\eta \in (0,1)$ denotes the elasticity of education technology with respect to public education spending.

It should be noted that private investment in education may also contribute to the formation of human capital. For example, parents' time (Glomm and Ravikumar, 1995, 2001, 2003; Glomm and Kaganovich, 2008) or spending (Glomm, 2004; Lambrecht, Michel, and Vidal, 2005; Kunze, 2014) devoted to education may work as a complement to public education. In the present study, we abstract private education from the analysis to simplify the presentation of the model and to focus on the conflict between generations with respect to public spending on education and pensions. In a former version of this paper (Ono and Uchida, 2014), we assume private education spending as a complement to public education, and obtain qualitatively similar results to those in the present version of the paper.

We assume that parents are altruistic toward their children and are concerned about the disposable income of their children in middle age, $(1-\tau_{t+1})w_{t+1}h_{t+1}$. The preferences of an individual born in period $t-1$ are specified by the following expected utility function of the logarithmic form:

$$U_t = \ln c_t + \pi \ln d_{t+1} + \gamma \ln(1-\tau_{t+1})w_{t+1}h_{t+1},$$

where $\gamma(>0)$ denotes the intergenerational degree of altruism. We substitute the budget constraints and human capital production function into the utility function to write the following unconstrained maximization problem:

$$\max_{\{s_t\}} \ln\left[(1-\tau_t)w_t h_t - s_t\right] + \pi \ln\left[\frac{R_{t+1}}{\pi}s_t + p_{t+1}\right] + \gamma \ln(1-\tau_{t+1})w_{t+1}D\left(x_t\right)^{\eta}\left(h_t\right)^{1-\eta}.$$

By solving the problem, we obtain the following savings and consumption functions:

$$s_t = \frac{\pi}{1+\pi}\cdot\left[(1-\tau_t)w_t h_t - \frac{p_{t+1}}{R_{t+1}}\right], \tag{1}$$

$$c_t = \frac{1}{1+\pi}\cdot\left[(1-\tau_t)w_t h_t + \frac{\pi p_{t+1}}{R_{t+1}}\right]. \tag{2}$$

The savings function in (1) states that a higher level of after-tax wage, $(1-\tau_t)w_t$, implies higher savings, whereas a higher level of pension, p_{t+1}, implies lower savings. The consumption function in (2) states that a higher lifetime income, $(1-\tau_t)w_t h_t + \pi p_{t+1}/R_{t+1}$, results in larger spending on consumption.

2.2 Firms

In each period, there is a continuum of identical firms that are perfectly competitive profit maximizers. According to Cobb–Douglas technology, they produce a final good Y_t using two inputs, aggregate physical capital K_t and aggregate human capital $H_t \equiv N_t h_t$. The aggregate output is given by

$$Y_t = A\left(K_t\right)^{\alpha}\left(H_t\right)^{1-\alpha},$$

where $A(> 0)$ is a scale parameter and $\alpha \in (0, 1)$ denotes the capital share.

Let $k_t \equiv K_t/H_t$ denote the ratio of physical to human capital. The first-order conditions for profit maximization with respect to H_t and K_t are as follows:

$$w_t = (1 - \alpha)A (k_t)^{\alpha}, \tag{3}$$

$$\rho_t = \alpha A (k_t)^{\alpha - 1}, \tag{4}$$

where w_t and ρ_t are the wage of labor and the rental price of capital, respectively. The conditions state that firms hire human and physical capital until the marginal products are equal to the factor prices.

2.3 Government Budget Constraint

Pensions and public education are financed by the tax on labor income. The aggregate tax revenue is $\tau_t w_t h_t N_t$, while the aggregate expenditure is the spending on public pensions, $\pi N_{t-1} p_t$, plus the spending on public education, $N_{t+1} x_t$. Therefore, the government budget constraint in period t is

$$\frac{\pi}{1+n} p_t + (1 + n)x_t = \tau_t w_t h_t.$$

The left-hand side shows the expenditure on pensions, $\pi p_t/(1+n)$, and public education, $(1 + n)x_t$, and the right-hand side shows the revenue from taxing labor income.

2.4 Economic Equilibrium

The market clearing condition for capital is $K_{t+1} = N_t s_t$, which expresses the equality of total savings by the middle-aged population in period t, $N_t s_t$, to the stock of aggregate physical capital at the beginning of period $t+1$, K_{t+1}. With the use of $k_{t+1} \equiv K_{t+1}/H_{t+1}$ and $h_{t+1} = H_{t+1}/N_{t+1}$, we can rewrite the condition as $(K_{t+1}/H_{t+1}) \cdot (H_{t+1}/N_{t+1}) \cdot (N_{t+1}/N_t) = s_t$, or

$$(1 + n)k_{t+1}h_{t+1} = s_t.$$

The economic equilibrium in the present model is defined as follows.

Definition 1. Given a sequence of policies, $\{\tau_t, x_t, p_t\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations $\{c_t, d_t, s_t, k_{t+1}, h_{t+1}\}_{t=0}^{\infty}$ and prices $\{\rho_t, w_t, R_t\}_{t=0}^{\infty}$ with the initial conditions $k_0(> 0)$ and $h_0(> 0)$ such that (i) given $(w_t, R_{t+1}, \tau_t, x_t, p_{t+1})$, $\left(c_t^y, c_{t+1}^o, s_t\right)$ solves the utility maximization problem; (ii) given (w_t, ρ_t), k_t solves the profit maximization problem of a firm; (iii) given (w_t, h_t, k_t), (τ_t, x_t, p_t) satisfies the government budget constraint; (iv) $\rho_t = R_t$ holds; and (v) the capital market clears: $(1 + n)k_{t+1}h_{t+1} = s_t$.

In economic equilibrium, the indirect utility function of the middle-aged population in period t, V_t^M, and that of the old in period t, V_t^o, can be expressed as functions of government policy and physical and human capital as follows:

$$\begin{aligned} V^M\left(k_t, h_t, \tau_t, k_{t+1}, h_{t+1}, \tau_{t+1}, p_{t+1}\right) = {} & \ln \frac{1}{1+\pi}\left((1-\tau_t)(1-\alpha)A\left(k_t\right)^\alpha h_t + \frac{\pi p_{t+1}}{\alpha A\left(k_{t+1}\right)^{\alpha-1}}\right) \\ & + \pi \ln\left(\frac{\alpha A\left(k_{t+1}\right)^{\alpha-1}}{\pi}(1+n)k_{t+1}h_{t+1} + p_{t+1}\right) \\ & + \gamma \ln(1-\tau_{t+1})(1-\alpha)A\left(k_{t+1}\right)^\alpha h_{t+1}, \end{aligned} \tag{5}$$

$$\begin{aligned} V^o\left(k_t, h_t, \tau_t, p_t\right) = {} & \ln\left(\frac{\alpha A\left(k_t\right)^{\alpha-1}}{\pi}(1+n)k_t h_t + p_t\right) \\ & + \gamma \ln(1-\tau_t)(1-\alpha)A\left(k_t\right)^\alpha h_t, \end{aligned} \tag{6}$$

where some irrelevant terms are omitted from the expressions. The first and second terms in (5) correspond to the utility of consumption in middle and old ages, respectively, and the third term shows the utility from the disposable income of their children. The first term in (6) corresponds to the utility of old-age consumption and the second term shows the utility from the disposable income of their children.

3 Political Equilibrium

The present study assumes probabilistic voting developed by Lindbeck and Weibull (1987) for demonstrating the political mechanism. In each period, the government in power maximizes a political objective. Formally, the political objective function in period t is given by

$$\Omega_t = \omega\pi V^o\left(k_t, h_t, \tau_t, p_t\right) + (1+n)V^M\left(k_t, h_t, \tau_t, k_{t+1}, h_{t+1}, \tau_{t+1}, p_{t+1}\right),$$

where $\omega\pi(>0)$ and $1+n$ are the relative weights of old-age and middle-age agents, respectively. In particular, the parameter $\omega(>0)$ represents the political power of the old-age agents. An explicit microfoundation for this modeling is explained in Persson and Tabellini (2000, Chapter 3) and Acemoglu and Robinson (2005, Appendix). The government problem in period t is to maximize Ω_t subject to the government budget constraint, given the state variables, k_t and h_t.

In this study, we restrict our attention to a Markov-perfect equilibrium. In the present framework, Markov perfectness implies that outcomes depend only on the payoff-relevant state variables, that is, physical and human capital, k and h, respectively. Therefore, the expected levels of tax and public pension for the next period, τ_{t+1} and p_{t+1}, are given by functions of the next period stock of physical and human capital, $\tau_{t+1} = T(k_{t+1}, h_{t+1})$ and

$p_{t+1} = P(k_{t+1}, h_{t+1})$, respectively. By using recursive notation with z' denoting the next period z, we can define a Markov-perfect political equilibrium as follows.

Definition 2.

A Markov-perfect political equilibrium is a set of functions, $\langle T, X, P \rangle$, where $T : \Re_{++} \times \Re_{++} \to [0,1]$ is a tax rule, $\tau = T(k,h)$, $X : \Re_{++} \times \Re_{++} \to \Re_{++}$ is a public education expenditure rule, $x = X(k,h)$, and $P : \Re_{++} \times \Re_{++} \to \Re_{++}$ is a public pension expenditure rule, $p = P(k,h)$, such that the following conditions are satisfied:

(i) the capital market clears,

$$(1+n)k'h' = \frac{\pi}{1+\pi}\left[(1 - T(k,h))(1-\alpha)A(k)^{\alpha}h - \frac{P(k',h')}{\alpha A(k')^{\alpha-1}}\right], \tag{7}$$

(ii) given k and h, $\langle T(k,h), X(k,h), P(k,h) \rangle = \arg\max \Omega(k,h,\tau,x,p,p')$ subject to $p' = P(k',h')$, the capital market clearing condition in (7), the government budget constraint,

$$\frac{\pi}{1+n}P(k,h) + (1+n)X(k,h) = T(k,h)(1-\alpha)A(k)^{\alpha}h,$$

and the human capital production function, $h' = D(X(k,h))^{\eta}(h)^{1-\eta}$, where Ω is defined by $\Omega(k,h,\tau,x,p,p') \equiv \omega\pi V^{o}(k,h,\tau,p) + (1+n)V^{M}(k,h,\tau,k',h',\tau',p')$.

In order to obtain a set of functions in Definition 2, we conjecture the following functions:

$$\begin{cases} p' = \bar{P} \cdot A(k')^{\alpha} h', \\ x' = \bar{X} \cdot A(k')^{\alpha} h', \end{cases} \tag{8}$$

where $\bar{P}(>0)$ and $\bar{X}(>0)$ are constant parameters. By using this conjecture and the constraints in Definition 2(ii), we can obtain the political objective function as follows:

$$\begin{aligned} \Omega = {} & \omega\pi \ln\left(\frac{\alpha A(k)^{\alpha-1}}{\pi}(1+n)kh + p\right) \\ & + \{\omega\pi\gamma + (1+n)(1+(\pi+\gamma)\alpha)\} \ln\left((1-\alpha)A(k)^{\alpha}h - \frac{\pi}{1+n}p - (1+n)x\right) \\ & + (1+n)\eta(\pi+\gamma)(1-\alpha)\ln x, \end{aligned} \tag{9}$$

where the terms unrelated to policy are omitted from the expression. The derivation of (9) is provided in Appendix A.1.

We solve the problem of maximizing Ω. The first-order conditions with respect to p and x are

$$p : \frac{\omega\pi}{\frac{\alpha A(k)^{\alpha-1}}{\pi}(1+n)kh + p} \le \frac{\{\omega\pi\gamma + (1+n)(1+(\pi+\gamma)\alpha)\}\frac{\pi}{1+n}}{(1-\alpha)A(k)^{\alpha}h - \frac{\pi}{1+n}p - (1+n)x},$$

$$x : \frac{\{\omega\pi\gamma + (1+n)(1+(\pi+\gamma)\alpha)\}(1+n)}{(1-\alpha)A(k)^{\alpha}h - \frac{\pi}{1+n}p - (1+n)x} = \frac{(1+n)\eta(\pi+\gamma)(1-\alpha)}{x}.$$

A strict inequality holds in the first condition if $p = 0$. By using these conditions, we obtain the following result.

Proposition 1. *There is a Markov-perfect political equilibrium distinguished by $p > 0$ if $\omega\pi > \alpha\phi$, and $p = 0$ otherwise, where ϕ is defined by*

$$\phi \equiv \omega\pi + (1+n)\eta\left(\pi+\gamma\right)\left(1-\alpha\right) + \left\{\omega\pi\gamma + \left(1+n\right)\left(1+(\pi+\gamma)\alpha\right)\right\}.$$

The corresponding policy functions are as follows:

$$(P(k,h), X(k,h)) = \begin{cases} \left(\bar{P}_{p>0}A\left(k\right)^{\alpha}h, \bar{X}_{p>0}A\left(k\right)^{\alpha}h\right) & \text{if} \quad \omega\pi > \alpha\phi; \\ \left(0, \bar{X}_{p=0}A\left(k\right)^{\alpha}h\right) & \text{if} \quad \omega\pi \leq \alpha\phi, \end{cases}$$

where

$$\bar{P}_{p>0} \equiv \frac{\omega\pi - \alpha\phi}{\phi\frac{\pi}{1+n}}, \bar{X}_{p>0} \equiv \frac{\eta\left(\pi+\gamma\right)\left(1-\alpha\right)}{\phi}, \ \textit{and} \ \bar{X}_{p=0} \equiv \frac{\eta\left(\pi+\gamma\right)\left(1-\alpha\right)^{2}}{\phi-\omega\pi}.$$

Proof. See Appendix A.2.

To understand the result in Proposition 1, recall the political objective function,

$$\begin{aligned}\Omega = &\underbrace{\omega\pi}_{(*1)}\ln\left(\frac{\alpha A\left(k\right)^{\alpha-1}}{\pi}(1+n)kh+p\right) \\ &+\left\{\underbrace{\omega\pi\gamma}_{(*2)}+\underbrace{\left(1+n\right)\left(1+\left(\pi+\gamma\right)\alpha\right)}_{(*3)}\right\}\ln\left[(1-\alpha)A\left(k\right)^{\alpha}h-\frac{\pi}{1+n}p-(1+n)x\right] \\ &+\underbrace{(1+n)\eta\left(\pi+\gamma\right)\left(1-\alpha\right)}_{(*4)}\ln x. \end{aligned} \tag{10}$$

The function indicates that the political power of the old (ω), longevity (π), and the population growth rate (n) affect the provisions of public pensions and education through the four factors represented by the terms (*1), (*2), (*3), and (*4). The other terms, including π and n, have no critical impact on political decisions because the effects through these terms cancel each other out.

The term (*1) implies that greater political power of the old and greater longevity imply a larger weight of the utility of consumption for the old. This incentivizes the government to allocate tax revenue more to public pensions for the old and less to public education for the young. The term (*2) implies that greater power of the old and greater longevity signal a larger weight of the old's utility from their children's disposable income. To improve this utility, the government cuts the tax burden of the middle-aged population, thereby resulting in lower levels of public pensions and education.

The term (*3) is the weight of the utility of the middle-aged agents for their consumption. The two factors included in this term, π and n, produce opposite effects on public spending. Greater longevity implies a larger weight of the middle-aged agents' utility of their old-age consumption. This gives the government an incentive to save more for their old-age consumption by cutting current pension and education expenditures. However, a lower population growth rate works in the opposite direction because it implies a smaller weight of the middle-aged agents' utility of their consumption.

Finally, the term (*4) is the weight of the utility of the middle-aged agents from their old-age consumption, denoted by $(1+n)\eta(\pi+\gamma)\alpha$. Greater longevity provides an incentive for the government to increase pension benefits for the middle-aged agents in their old age and to increase the human capital level of their children. The government can realize these two purposes by shifting the allocation of tax revenue from pensions for the old to public education for the children. This shift expands the tax base in the future by improving human capital of the children, and thus, increases pension benefits for the middle-aged agents in their old age. A lower population growth rate works in the opposite direction because the weight $(1+n)\eta(\pi+\gamma)\alpha$ decreases as the population growth rate decreases.

Based on the argument thus far, we now consider how the condition for $p > 0$, given by $\omega\pi > \alpha\phi$, is affected by the three aging factors, ω, π, and n. The condition is rewritten as follows:

$$\frac{1-\alpha}{\alpha} \cdot \underbrace{\omega\pi}_{(*1)} > \underbrace{(1+n)\eta(\pi+\gamma)(1-\alpha)}_{(*4)} + \left\{ \underbrace{\omega\pi\gamma}_{(*2)} + \underbrace{(1+n)(1+(\pi+\gamma)\alpha)}_{(*3)} \right\},$$

where the terms (*1), (*2), (*3), and (*4) correspond to those in the political objective function. The condition states that the effect of the population growth rate is straightforward. The terms (*3) and (*4), including n, indicate that public pensions are more likely to be provided in political equilibrium if the population growth rate is lower. A lower population growth rate implies smaller weights to the utility of the middle-aged agents for their old-age consumption and to the utility of these agents for the disposable income of their children. Given a lower weight for the utility of the middle-aged agents, the government shifts its spending from education to pensions for the current old.

The effects of longevity (π) and the political power of the old (ω) on pension provision are not straightforward. First, greater longevity has a positive effect on pensions through the term (*1), while it has a negative effect on pensions through the terms (*2), (*3), and (*4). Second, greater political power of the old has a positive effect on pensions through the term (*1), while it has a negative effect on pensions through the term (*2). Greater longevity and greater political power of the old have two competing effects on pensions, but the condition implies that the public pensions are more likely to be provided in political equilibrium as longevity and the political power of the old increase.

4 Pensions and Education

The result established in Section 3 indicates that pensions and education are affected by the three aging factors, π, ω, and n. To consider their effects on pensions and education, we focus on the pension-to-GDP ratio, $\pi N_{t-1}p_t/Y_t$, and the education-to-GDP ratio, $N_{t+1}x_t/Y_t$, and analyze the effects of increases in π and ω and a decrease in n on these ratios.

The following proposition demonstrates the effects of π, ω, and n on the pension-to-GDP ratio.

Proposition 2. *Suppose that $\omega\pi > \alpha\phi$ holds: a Markov-perfect political equilibrium exists with $p > 0$. The pension-to-GDP ratio, $\pi N_{t-1}p_t/Y_t$, increases with greater longevity, greater political power of the old, and a lower population growth rate: $\partial\left(\pi N_{t-1}p_t/Y_t\right)/\partial\pi > 0, \partial\left(\pi N_{t-1}p_t/Y_t\right)/\partial\omega > 0$, and $\partial\left(\pi N_{t-1}p_t/Y_t\right)/\partial n < 0$.*

Proof. See Appendix A.3.

To confirm the statement in Proposition 2, we compute the pension-to-GDP ratio as follows:

$$\frac{\pi N_{t-1}p_t}{Y_t} = \left[1 + \underbrace{\frac{1}{\omega\pi}}_{(*1)} \cdot \left\{\underbrace{(1+n)\eta(\pi+\gamma)(1-\alpha)}_{(*4)} + \left(\underbrace{\omega\pi\gamma}_{(*2)} + \underbrace{(1+n)(1+(\pi+\gamma)\alpha)}_{(*3)}\right)\right\}\right]^{-1} - \alpha. \tag{11}$$

The derivation of Eq. (11) is given in Appendix A.3.

The terms (*1), (*2), (*3), and (*4) in the above expression correspond to those in the political objective function in Eq. (10). We can apply the interpretation for the condition of $p > 0$ to the result in Proposition 2 in the following way. First, the population growth rate has an effect on the ratio through the terms (*3) and (*4). As described in Section 3, both terms imply a positive effect on pension provision when the population growth rate decreases. Second, the political power of the old has two competing effects on the ratio through the terms (*1) and (*2); and longevity also has two competing effects on the ratio through the terms (*1), (*2), (*3), and (*4). However, when $p > 0$, the positive effect outweighs the negative effect, as demonstrated in Section 3. Therefore, greater longevity and greater political power of the old lead to a higher pension-to-GDP ratio.

Given the result in Proposition 2 and the government budgetary constraint, it is natural to conjecture that the education-to-GDP ratio, $N_{t+1}x_t/Y_t$, decreases as π and ω increase and as n decreases. The following proposition shows that the conjecture is true with regard to ω and n, but it is not necessarily true with regard to π.

Proposition 3. *The education-to-GDP ratio decreases with greater political power of the old and a lower population growth rate: $\partial\left(N_{t+1}x_t/Y_t\right)/\partial\omega < 0$ and $\partial\left(N_{t+1}x_t/Y_t\right)/\partial n >$*

0. *With greater longevity, the education-to-GDP ratio decreases if* $1+n \leq \omega(\gamma)^2$, *increases if* $\omega(1+\gamma)\gamma \leq 1+n$, *and shows a hump-shaped pattern if* $\omega(\gamma)^2 < 1+n < \omega(1+\gamma)\gamma$.

Proof. See Appendix A.4.

To confirm the statement in Proposition 3, we first compute the $N_{t+1}x_t/Y_t$ ratio when $p > 0$ and $p = 0$, as follows:

$$\left.\frac{N_{t+1}x_t}{Y_t}\right|_{p>0} = \left[1 + \frac{\overbrace{\omega\pi}^{(*1)} + \left\{\overbrace{\omega\pi\gamma}^{(*2)} + \overbrace{(1+n)\left(1+(\pi+\gamma)\alpha\right)}^{(*3)}\right\}}{\underbrace{(1+n)\eta(\pi+\gamma)(1-\alpha)}_{(*4)}}\right]^{-1},$$

$$\left.\frac{N_{t+1}x_t}{Y_t}\right|_{p=0} = (1-\alpha)\cdot\left[1 + \frac{\overbrace{\omega\pi\gamma}^{(*2)} + \overbrace{(1+n)\left(1+(\pi+\gamma)\alpha\right)}^{(*3)}}{\underbrace{(1+n)\eta(\pi+\gamma)(1-\alpha)}_{(*4)}}\right]^{-1}.$$

The terms (*1), (*2), (*3), and (*4) in the expressions correspond to those in the political objective function in Eq. (10).

The effects of the political power of the old (ω) are as follows. Greater political power of the old has a negative effect on education spending via the term (*1), representing the weight on the utility of old agents for their consumption, and the term (*2), representing the weight on the utility of old agents from the disposable income of their children. These terms give the government incentive to shift the allocation of tax revenue from education to pensions and to cut the tax burden of the children of old agents. Therefore, the education-to-GDP ratio decreases as the political power of the old increases.

A lower population growth rate has two competing effects on the education-to-GDP ratio. The positive effect comes from the term (*3), implying a smaller weight on the utility of middle-aged agents for their consumption, whereas the negative effect comes from the term (*4), implying a smaller weight on the utility of the middle-aged agents from the disposable income of their children. The result suggests that for both cases of $p > 0$ and $p = 0$, the negative effect outweighs the positive one. Therefore, a decrease in the population growth rate results in a decline in the education-to-GDP ratio.

Greater longevity affects the ratio in the following ways. The first effect involves the term (*4), representing the weight of the utility of the middle-aged agents from the disposable income of their children. When longevity increases, the government shifts the allocation of tax revenue from pensions to education in order to increase the income of the children. This positive effect is offset partly by the negative effect of longevity represented by the term (*3). This term works to decrease the tax burden of the middle-aged agents by cutting the expenditure on education.

There are two additional negative effects of longevity on public education; they are represented by the terms (*1) and (*2). These effects are enhanced as the political power of the old increases. That is, the sum of the negative effects by the terms (*1), (*2), and (*3) outweighs the positive effect by the term (*4) when the political power of the old is above a critical level. However, we should note that the effect via the term (*1) appears only when $p > 0$. Therefore, the critical values of political power that balance the two competing effects differ between the two cases, $p > 0$ and $p = 0$. This difference is a source of the hump-shaped pattern of the education-to-GDP ratio.

[Figure 2 here.]

Figure 2 illustrates a numerical example of the hump-shaped pattern. We fix the share of capital at $\alpha = 1/3$, following Song, Storesletten, and Zilibotti (2012) and Lancia and Russo (2015). Each period lasts 30 years; this assumption is standard in quantitative analyses of the two-period overlapping-generations model (see, for example, Gonzalez-Eiras and Niepelt, 2008; Lancia and Russo, 2015). The middle-age period is from 30 to 59 years old, while the old-age period is from 60 to 89 years old. Following Lancia and Russo (2015), we assume an annual gross population growth rate of 1.006, which is the OECD average rate during $1995-2009$. This assumption implies that the gross population growth rate for 30 years is $(1.006)^{30} \simeq 1.197$.

Following the literature (Gonzalez-Eiras and Niepelt, 2008; Lancia and Russo, 2015), the political power of the elderly, denoted by ω, is set equal to the per-capita influence of the middle-aged and the elderly. We set $\omega = 2.3$ to indicate that the middle-aged and the elderly have approximately the same per-capita influence when $\pi = 0.52$, that is, when individuals live for 74.6 years on average. The remaining two parameters, γ and η, are set at $\gamma = 0.61$ and $\eta = 0.259$, respectively, to satisfy the following two requirements: (i) the education-to-GDP ratio is around 5.4%, which is the OECD average for 1995-2009 (Lancia and Russo, 2015), and (ii) the condition of $\omega (\gamma)^2 < 1 + n < \omega(1+\gamma)\gamma$ has a hump-shaped pattern in the education-to-GDP ratio in the present framework.

5 Economic Growth

Based on the result established thus far, we derive the growth rate of the economy, and investigate how it is affected by population aging. For the presentation of the analysis, consider per capita output, y_t, which is defined by $y_t \equiv Y_t/N_t = A(k_t)^\alpha h_t$. Then, the growth rate of per capita output is

$$\frac{y'}{y} = \frac{A(k')^\alpha h'}{A(k)^\alpha h},$$

where x' denotes the next period $x(= k, h, y)$. In the steady state with $k' = k$, the growth rate of per capita output, y'/y, is equal to the growth rate of human capital, h'/h. Therefore, in what follows, we focus on the steady-state growth rate of human capital.

The analysis proceeds as follows. First, we consider the cases of $p > 0$ and $p = 0$, separately. We show that the physical-to-human capital ratio, $k = K/H$, stably converges to a unique steady state for each case, and that at the steady state, the growth rate of human capital remains constant across periods. Second, we undertake numerical analysis to investigate the overall effects of increases in ω and π and a decrease in n on the growth rate at the steady state.

5.1 Steady-state Growth Rate

Recall the capital market-clearing condition in Definition 2. With the use of the policy functions derived in Proposition 1, we reformulate it as follows:

$$k' = \psi_k\left(\bar{P}\right) \cdot \left[(1-\alpha)A\left(k\right)^{\alpha} h - \frac{\pi}{1+n}p - (1+n)x\right] \left(x\right)^{-\eta} \left(h\right)^{-(1-\eta)}, \tag{12}$$

where

$$\psi_k\left(\bar{P}\right) \equiv \frac{\frac{\pi}{1+\pi}}{D\left\{(1+n) + \frac{\pi}{1+\pi} \cdot \frac{\bar{P}}{\alpha}\right\}}.$$

The term $\psi_k\left(\cdot\right)$ is constant and dependent on $\bar{P}$, where $\bar{P}$ is the coefficient of the policy function, $p = \bar{P} \cdot A(k)^{\alpha}h$, and is presented in Proposition 1. The derivation of these equations and the definition of $\psi_k\left(\cdot\right)$ are provided in Appendix A.1.

Suppose that $p > 0$. The policy functions are given by

$$P(k,h) = \bar{P}_{p>0} \cdot A\left(k\right)^{\alpha} h,$$
$$X(k,h) = \bar{X}_{p>0} \cdot A\left(k\right)^{\alpha} h.$$

Substituting these functions into the physical capital formation function in (12), we obtain the law of motion of physical capital when $p > 0$, as follows:

$$k' = \psi_k\left(\bar{P}_{p>0}\right) \cdot \left[(1-\alpha) - \frac{\pi}{1+n}\bar{P}_{p>0} - (1+n)\bar{X}_{p>0}\right] \cdot \left(\bar{X}_{p>0}\right)^{-\eta} \cdot \left(A\right)^{(1-\eta)} \cdot \left(k\right)^{\alpha(1-\eta)}. \tag{13}$$

The equation implies that a unique and non-trivial steady state exists and that for any initial condition $k > 0$, the sequence of k stably converges to the unique steady state. From (13), we compute the steady-state level of k when $p > 0$, denoted by $\bar{k}_{p>0}$, as follows:

$$\bar{k}_{p>0} = \left[\psi_k\left(\bar{P}_{p>0}\right) \cdot \left\{(1-\alpha) - \frac{\pi}{1+n}\bar{P}_{p>0} - (1+n)\bar{X}_{p>0}\right\}^{1-\delta} \cdot \left(\bar{X}_{p>0}\right)^{-\eta} \cdot \left(A\right)^{(1-\eta)}\right]^{1/\{1-\alpha(1-\eta)\}}. \tag{14}$$

Using $\bar{k}_{p>0}$ in (14) and the policy functions in Proposition 1, we write the law of motion of human capital when $p > 0$ as $h'|_{p>0} = D \cdot \left(\bar{X}_{p>0} A \left(\bar{k}_{p>0}\right)^{\alpha} h\right)^{\eta} \cdot (h)^{1-\eta}$, or

$$\left.\frac{h'}{h}\right|_{p>0} = D \cdot \left(\bar{X}_{p>0} A \left(\bar{k}_{p>0}\right)^{\alpha}\right)^{\eta}. \tag{15}$$

The equation shows that the growth rate is constant across periods at the steady state. Following the same procedure, we obtain the growth rate when $p = 0$ as follows:

$$\left.\frac{h'}{h}\right|_{p=0} = D \cdot \left(\bar{X}_{p=0} A \left(\bar{k}_{p=0}\right)^{\alpha}\right)^{\eta}, \tag{16}$$

where $\bar{k}_{p=0}$ is a unique and stable steady-state level of k when $p = 0$.

5.2 Numerical Analysis

To investigate the growth effect of increases in ω and π and a decrease in n, we undertake numerical analysis. We follow the assumption introduced in Section 4: each generation lasts for 30 years; the population growth rate for 30 years is $(1.006)^{30} - 1 \simeq 0.197$; and $\alpha = 1/3$, $\gamma = 0.61$, and $\eta = 0.259$. In addition, we normalize A as $A = 1$, and set $D = 2.77$ to obtain an empirically plausible result of the growth rates.

Within these assumptions, we undertake the numerical analysis and obtain the following results, as illustrated in Figure 3. First, an increase in ω leads to a decrease in the growth rate, as depicted in Panel (a). Second, a decrease in n leads to an increase in the growth rate, as depicted in Panel (b). Third, an increase in π leads to a hump-shaped pattern when $\omega(\gamma)^2 < 1 + n < \omega(1+\gamma)\gamma$, as depicted in Panel (c). Therefore, the three parameters, ω, π, and n, representing population aging, have different effects on economic growth.

[Figure 3 here.]

To understand the mechanism behind the result, recall the growth rate of human capital in Eqs. (15) and (16). The growth rate is affected by ω, π, and n through the two factors, that is, the policy function of public education represented by $\bar{X}_{p>0}$ and $\bar{X}_{p=0}$, and the steady-state capital, represented by $\bar{k}_{p>0}$ and $\bar{k}_{p=0}$. The first factor is crucial in determining the growth rate because parameters related to population aging, ω, π, and n, directly affect the growth rate via the term $\bar{X}_{p>0}$ or $\bar{X}_{p=0}$. Therefore, hereafter, we focus on $\bar{X}_{p>0}$ and $\bar{X}_{p=0}$ to interpret the result.

Recall $\bar{X}_{p>0}$ and $\bar{X}_{p=0}$ in Proposition 1, which are reformulated as follows:

$$\bar{X}_{p>0} = \frac{\eta(\pi+\gamma)(1-\alpha)}{\phi} = \frac{\overbrace{\eta(\pi+\gamma)(1-\alpha)}^{(*4a)}}{\underbrace{\omega\pi}_{(*1)} + \underbrace{(1+n)}_{(*4b)} + \underbrace{\omega\pi\gamma}_{(*2)} + \underbrace{(1+n)(1+(\pi+\gamma)\alpha)}_{(*3)}};$$

$$\bar{X}_{p=0} = \frac{\eta(\pi+\gamma)(1-\alpha)^2}{\phi-\omega\pi} = \frac{\overbrace{\eta(\pi+\gamma)(1-\alpha)^2}^{(*4a)}}{\underbrace{(1+n)}_{(*4b)} + \underbrace{\omega\pi\gamma}_{(*2)} + \underbrace{(1+n)(1+(\pi+\gamma)\alpha)}_{(*3)}},$$

where the terms (*1), (*2), and (*3) correspond to those in the political objective function in Eq. (10), and the terms (*4a) and (*4b) correspond to the term (*4) in Eq. (10). The terms (*1) and (*2) show a negative effect of increased political power of the old on education spending, and thus, on the growth rate of human capital; the terms (*3) and (*4b) show a positive effect of a decreased population growth rate on education spending and the growth rate of human capital. Therefore, the effect on the growth rate is definite when ω and n.[2]

The effect of π on the growth rate is not straightforward. The terms (*1), (*2), and (*3) show a negative effect of longevity on education spending, and thus, on economic growth, while the term (*4a) shows a positive effect. However, the negative effect via the term (*1) appears only when the public pension is provided, because the term (*1) is irrelevant for political decision making when there is no provision of public pension. That is, the negative effect of π when $p > 0$ is larger than that when $p = 0$. Because of this difference, the negative effect outweighs the positive effect when $p > 0$, while the negative effect is outweighed by the positive effect when $p = 0$. This is the source of the hump-shaped pattern of the growth rate affected by longevity.

6 Role of Annuity Markets

The analysis thus far has assumed perfect annuity markets. However, in the real world, some countries have limited or no access to annuity markets. We expect this to influence individual economic and political decisions, which in turn affect long-run economic growth. To explore this possibility, this section modifies the model by assuming no annuity market, and investigates how the results differ in this alternative scenario.

[2]We should note that a lower population growth rate results in a lower aggregate education spending-to-GDP ratio, as demonstrated in Section 4, whereas it results in larger per capita spending on education, as demonstrated here. The difference arises because $\bar{X}_{p>0}$ (or $\bar{X}_{p=0}$) is multiplied by the gross population growth rate, $1+n$, when we compute the aggregate education spending. The factor $1+n$ works in a negative direction and outweighs the positive effect through $\bar{X}_{p>0}$ (or $\bar{X}_{p=0}$).

For this purpose, we demonstrate the no-annuity-market case in the following way. The budget constraints of the middle-aged and the elderly are now given by

$$c_t + s_t \leq (1-\tau_t) w_t h_t + b_t,$$
$$d_{t+1} \leq R_{t+1} s_t + p_{t+1},$$

where b_t is the per capita accidental bequest. If an individual dies at the end of middle age, his/her unannuitized wealth, $N_t(1-\pi)R_{t+1}s_t$, is distributed to his/her offspring as an accidental bequest: $N_{t+1}b_{t+1} = N_t(1-\pi)R_{t+1}s_t$, or

$$b_{t+1} = \frac{1-\pi}{1+n} R_{t+1} s_t. \tag{17}$$

We solve the utility maximization problem with the above budget constraints and obtain the saving and consumption functions. Then we substitute these, the government budget constraints, and (17) into the utility function of the middle-aged, and make a conjecture related to the policy functions as in Section 3 to obtain the following indirect utility function for the middle-aged:

$$\begin{aligned} V^M = &\{1 + (\pi+\gamma)\alpha\} \ln \left[(1-\alpha) A (k)^\alpha h + (1-\pi)\alpha A (k)^\alpha h - \frac{\pi}{1+n} p - (1+n)x \right] \\ &+ \eta (\pi+\gamma)(1-\alpha) \ln x, \end{aligned}$$

where the term $(1-\pi)\alpha A (k)^\alpha h$ peculiar to the modified model represents the accidental bequest. The indirect utility function of the elderly is also obtained as

$$V^o = \ln \left[(1+n)\alpha A (k)^\alpha h + p \right] + \gamma \ln \left[(1-\alpha) A (k)^\alpha h - \frac{\pi}{1+n} p - (1+n)x \right].$$

Following the same procedure as in the previous sections, we seek the policy functions that maximize the political objective, $\Omega = \omega\pi V^o + (1+n)V^M$. However, we are unable to obtain analytical solutions of the policy functions because of the presence of the additional term $(1-\pi)\alpha A (k)^\alpha h$. To resolve this difficulty, we solve the maximization problem numerically. We adopt the parameter values introduced in the previous sections, and use the numerically obtained policy functions to compute the steady-state level of capital and the rate of per capita human capital growth. Figure 4 illustrates how longevity affects the pension-to-GDP ratio and the steady-state growth rate.

[Figure 4 here.]

The figure shows that the no annuity market case differs from the case with perfect annuity markets in three respects. First, public pensions are provided for any probability of living to old age, $\pi \in [0,1]$. No annuity markets implies a lower return from savings than when there are perfect annuity markets. This strengthens the elderly population's incentives to expand public pension provision. Second, the pension-to-GDP ratio exhibits a hump-shaped pattern as longevity increases. Children receive a smaller bequest as parents' longevity increases, implying a negative income effect that gives the middle-aaged an incentive to prefer a smaller tax burden and thus a lower level of public pension provision. This negative effect on pension dominates the positive effect demonstrated in Proposition 2 when longevity is above a threshold level. Third, the negative effect on pensions incentivizes individuals to save more for the retirement period. This leads to an increase in the steady-state level of capital. This positive effect on capital dominates the negative growth effect through the policy function of public education presented in Section 5.

The result in this section indicates that the presence (or absence) of annuity markets is a key to the effect of longevity on economic growth. As demonstrated in the previous section, an increase in longevity could reduce the growth rate when individuals are able to purchase private annuity contracts. However, the result in this section shows that rising longevity definitely increases the growth rate when private annuity contracts are unavailable. To check the empirical relevance of the conflicting predictions, we again look at the observation in Panel (c) of Figure 1. In particular, we focus on four countries with a high proportion of the population aged 65 years and above: the Czech Republic, Estonia, Hungary, and the United Kingdom. The United Kingdom has GDP per capita growth below 5%, while the Czech Republic, Estonia, and Hungary have growth rates above 10%. There is a significant difference in the growth rates among these countries, although they share a similar demographic structure.

The divergence between the United Kingdom and the three Eastern European countries could be explained by the degree of private annuitization. In fact, the OECD (2014, Factbook) reports that the ratio of private expenditure on pension to GDP, which captures the degree of private annuitization, was 1.6% in 2010 for the United Kingdom. This is above the OECD average of 0.9%. On the other hand, the ratio was 0.5% for the Czech Republic, 0% for Estonia, and 0.2% for Hungary. These figures are stable for 2007-2012. Therefore, we may well conclude that the overall trend in OECD countries shows a non-linear relationship between longevity and economic growth, though some countries experience continued economic growth due to limited access to private annuities.[3]

[3]Japan also exhibits a high share of population aged 65 years and above, though is not included in the discussion above because of the lack of data related to private annuitization.

7 Summary and Conclusion

How does the conflict of interest between generations affect the two redistribution policies, namely, public education for the young and public pensions for the old? In turn, how does the conflict affect economic growth? The present study attempted to answer these questions from a political economy viewpoint.

We considered three factors representing population aging: longevity, the political power of the old, and the population growth rate. We showed that with greater political power of the old and a lower population growth rate, the pension-to-GDP ratio increases but the education-to-GDP ratio decreases. In addition, greater longevity results in a higher pension-to-GDP ratio.

We demonstrated that the effect of longevity on education spending is complex. Greater longevity increases the weight of the utility of the elderly. This incentivizes the government to shift spending allocations from education to pensions. However, greater longevity also increases the weight of middle-aged agents' utility from their children's human capital. This gives the government an incentive to increase public education spending. These opposing effects produce an initial increase followed by a decrease in the education-to-GDP ratio, which in turn results in a hump-shaped pattern in the growth rate. This model prediction generally fits the cross-country empirical evidence in developed countries. We also demonstrated that greater longevity always increases the growth rate when there is no private annuity market, and found that this result fits the data from some Eastern European countries with limited access to private pensions.

The result established in the present study have policy implications related to aging and economic growth. First, a decline in the population growth rate and an increase in longevity have different effects on economic growth. In particular, a decline in the population growth rate definitely increases per-capita growth, while an increase in longevity has a non-monotone growth effect. Therefore, policymakers should focus on increasing life expectancy and its associated costs rather than falling birth rates. Second, diminishing the political power of the elderly can increase economic growth. One way to realize this is to lower the minimum voting age. Japan, with the highest life expectancy among developed countries, has recently lowered the minimum voting age from 20 to 18. This is expected to strengthen the political power of the young, and to increase the economic growth rate in the long run.

A Proofs

A.1 Derivation of (9)

To derive (9), we first use the government budget constraint to replace τ in the indirect utility functions by x and p. Then, we substitute the conjectures in (8) into the political objective function Ω. Finally, we replace k' and h' with k, h, x and p, respectively, by using the capital market clearing condition and the human capital production function. In what follows, we provide the details of the calculation step by step.

Step 1.

Recall the government budget constraint in Definition 2(ii), which is rewritten as follows:

$$1-\tau = \frac{(1-\alpha)A\left(k\right)^{\alpha}h - \frac{\pi}{1+n}p - (1+n)x}{(1-\alpha)A\left(k\right)^{\alpha}h}.$$

Plugging this into the indirect utility functions in (5) and (6), we obtain

$$\begin{aligned}
V^M &= \ln\frac{1}{1+\pi}\left[(1-\alpha)A\left(k\right)^{\alpha}h - \frac{\pi}{1+n}p - (1+n)x + \frac{\pi p'}{\alpha A\left(k'\right)^{\alpha-1}}\right] \\
&\quad + \pi\ln\left(\frac{\alpha A\left(k'\right)^{\alpha-1}}{\pi}(1+n)k'h' + p'\right) + \gamma\ln\left[(1-\alpha)A\left(k'\right)^{\alpha}h' - \frac{\pi}{1+n}p' - (1+n)x'\right], \\
V^o &= \ln\left(\frac{\alpha A\left(k\right)^{\alpha-1}}{\pi}(1+n)kh + p\right) + \gamma\ln\left[(1-\alpha)A\left(k\right)^{\alpha}h - \frac{\pi}{1+n}p - (1+n)x\right].
\end{aligned} \tag{18}$$

Step 2.

We substitute the conjecture of the policy functions in (8) into V^M to obtain

$$\begin{aligned}
V^M &= \ln\frac{1}{1+\pi}\left[(1-\alpha)A\left(k\right)^{\alpha}h - \frac{\pi}{1+n}p - (1+n)x + \frac{\pi\bar{P}A\left(k'\right)^{\alpha}h'}{\alpha A\left(k'\right)^{\alpha-1}}\right] \\
&\quad + \pi\ln\left(\frac{\alpha A\left(k'\right)^{\alpha-1}}{\pi}(1+n)k'h' + \bar{P}A\left(k'\right)^{\alpha}h'\right) \\
&\quad + \gamma\ln\left[(1-\alpha)A\left(k'\right)^{\alpha}h' - \frac{\pi}{1+n}\bar{P}A\left(k'\right)^{\alpha}h' - (1+n)\bar{X}A\left(k'\right)^{\alpha}h'\right] \\
&= \ln\left[(1-\alpha)A\left(k\right)^{\alpha}h - \frac{\pi}{1+n}p - (1+n)x + \frac{\pi}{\alpha}\bar{P}k'h'\right] + \alpha\left(\pi+\gamma\right)\ln k' + \left(\pi+\gamma\right)\ln h',
\end{aligned} \tag{19}$$

where the terms unrelated to the political decision are omitted from the expression.

Step 3.

To replace k' and h' in (19) with k, h, p and x, we first recall the human capital production function, $h' = D(x)^{\eta}(h)^{1-\eta}$, and the capital market clearing condition,

$$\begin{aligned}(1+n)k'h' &= \frac{\pi}{1+\pi}\left[(1-\tau)\,wh - \frac{p'}{R'}\right] \\ &= \frac{\pi}{1+\pi}\left[(1-\alpha)A(k)^{\alpha}h - \frac{\pi}{1+n}p - (1+n)x - \frac{\bar{P}A(k')^{\alpha}h'}{\alpha A(k')^{\alpha-1}}\right],\end{aligned}$$

where the first line comes from the capital market clearing condition, $(1+n)k'h' = s$ with the saving function in (1), and the second line comes from the profit maximization conditions in (3) and (4) and the conjecture of the policy function p' in (8). After rearranging the terms, we rewrite the abovementioned expression as follows:

$$k'h' = \frac{\frac{\pi}{1+\pi}}{(1+n) + \frac{\pi}{1+\pi}\cdot\frac{\bar{P}}{\alpha}}\cdot\left[(1-\alpha)A(k)^{\alpha}h - \frac{\pi}{1+n}p - (1+n)x\right]. \tag{20}$$

We substitute $h' = D(x)^{\eta}(h)^{1-\eta}$ into (20) to obtain

$$k' = \psi_k(\bar{P})\cdot\left[(1-\alpha)A(k)^{\alpha}h - \frac{\pi}{1+n}p - (1+n)x\right]\cdot(x)^{-\eta}\cdot(h)^{-(1-\eta)}, \tag{21}$$

where $\psi_k(\bar{P})$ is defined by:

$$\psi_k(\bar{P}) \equiv \frac{\frac{\pi}{1+\pi}}{D\left\{(1+n) + \frac{\pi}{1+\pi}\cdot\frac{\bar{P}}{\alpha}\right\}}.$$

By using (20) and (21), we rewrite V^M in (19) as

$$\begin{aligned}V^M &= \{1 + (\pi+\gamma)\alpha\}\ln\left[(1-\alpha)A(k)^{\alpha}h - \frac{\pi}{1+n}p - (1+n)x\right] \\ &\quad + \eta(\pi+\gamma)(1-\alpha)\ln x,\end{aligned} \tag{22}$$

where constant terms are omitted from the expression. With V^o in (18) and V^M in (22), we write the political objective function $\Omega = \omega\pi V^o + (1+n)V^M$ as expressed in (9).

■

A.2 Proof of Proposition 1

Suppose that public pensions are provided in the next period, $p' > 0$. Given that preferences are specified by the logarithmic utility function, we conjecture linear policy functions of public education and public pensions for the next period, $x' = \bar{X}_{p>0}\cdot A(k')^{\alpha}h'$ and $p' = \bar{P}_{p>0}\cdot A(k')^{\alpha}h'$, respectively, where $\bar{X}_{p>0}(>0)$ and $\bar{P}_{p>0}(>0)$ are policy function parameters when $p > 0$. Under this conjecture, the solution to the problem becomes:

$$\begin{aligned}X(k,h) &= \frac{\eta}{\phi}(\pi+\gamma)(1-\alpha)A(k)^{\alpha}h, \\ P(k,h) &= \frac{\omega\pi - \alpha\phi}{\phi\frac{\pi}{1+n}}A(k)^{\alpha}h,\end{aligned}$$

where ϕ is defined in Proposition 1. The solution for $P(k,h)$ indicates that $P(k,h) > 0$ holds if and only if $\omega\pi > \alpha\phi$. When $\omega\pi > \alpha\phi$ holds, the abovementioned solution constitutes a Markov-perfect political equilibrium if

$$\begin{cases} \bar{X}_{p>0} \equiv \frac{\eta}{\phi}(\pi+\gamma)(1-\alpha), \\ \bar{P}_{p>0} \equiv \frac{\omega\pi-\alpha\phi}{\phi\frac{\pi}{1+n}}. \end{cases} \tag{23}$$

Alternatively, suppose that $p' = 0$; that is, public pensions are not provided in the next period. Consider the estimation of policy functions as $x' = \bar{X}_{p=0} \cdot A(k')^{\alpha} h'$ and $p' = \bar{P}_{p=0} \cdot A(k')^{\alpha} h'$, where $\bar{X}_{p=0}$ and $\bar{P}_{p=0}$ are policy function parameters when $p = 0$. The solution to the problem becomes:

$$\begin{aligned} X(k,h) &= \frac{\eta(\pi+\gamma)(1-\alpha)^2}{\phi-\omega\pi} A(k)^{\alpha} h, \\ P(k,h) &= 0 \text{ if } \omega\pi \le \alpha\phi. \end{aligned}$$

This solution constitutes a Markov-perfect political equilibrium if

$$\bar{X}_{p=0} \equiv \frac{\eta(\pi+\gamma)(1-\alpha)^2}{\phi-\omega\pi} \text{ and } \bar{P}_{p=0} \equiv 0. \tag{24}$$

The tax rates for $p > 0$ and $p = 0$ are obtained by substituting the corresponding policy functions X and P into the government budget constraint.

■

A.3 Proof of Proposition 2

The pension-to-GDP ratio is

$$\frac{\pi N_{t-1}p_t}{Y_t} = \frac{\pi N_{t-1}p_t}{y_t N_t} = \frac{\pi}{A(k_t)^{\alpha}h_t(1+n)} \cdot \frac{(\omega\pi-\alpha\phi)}{\phi\frac{\pi}{1+n}} \cdot A(k_t)^{\alpha}h_t = \frac{1}{\phi/\omega\pi} - \alpha,$$

where the first equality comes from $y_t = Y_t/N_t$ and the second equality comes from the policy function of p_t presented in Proposition 1. Using the definition of ϕ in Proposition 1, we rewrite this expression as

$$\frac{\pi N_{t-1}p_t}{Y_t} = \frac{1}{1+\frac{1}{\omega\pi}\cdot[(1+n)\eta(\pi+\gamma)(1-\alpha)+\{\omega\pi\gamma+(1+n)(1+(\pi+\gamma)\alpha)\}]} - \alpha,$$

that is,

$$\frac{\pi N_{t-1}p_t}{Y_t} = \frac{1}{1+(1+n)\frac{\eta}{\omega}\left(1+\frac{\gamma}{\pi}\right)(1-\alpha)+\gamma+(1+n)\frac{1}{\omega}\left(\frac{1}{\pi}+\left(1+\frac{\gamma}{\pi}\right)\alpha\right)} - \alpha.$$

This equation states that $\pi N_{t-1}p_t/Y_t$ is increasing in π and ω and is decreasing in n.

■

A.4 Proof of Proposition 3

Let $N_{t+1}x_t/Y_t|_{p>0}$ and $N_{t+1}x_t/Y_t|_{p=0}$ denote the public education-to-GDP ratio when $p > 0$ and $p = 0$, respectively. Using the policy function of x_t in Proposition 1, they are expressed as follows:

$$\left.\frac{N_{t+1}x_t}{Y_t}\right|_{p>0} = (1+n)\bar{X}_{p>0} = \frac{1}{1+\Gamma_{p>0}},$$
$$\left.\frac{N_{t+1}x_t}{Y_t}\right|_{p=0} = (1+n)\bar{X}_{p=0} = \frac{1}{1+\Gamma_{p=0}},$$

where

$$\Gamma_{p>0} \equiv \frac{\omega\pi + \{\omega\pi\gamma + (1+n)\left(1+(\pi+\gamma)\,\alpha\right)\}}{(1+n)\eta(\pi+\gamma)(1-\alpha)},$$
$$\Gamma_{p=0} \equiv \frac{\omega\pi\gamma + (1+n)\left(1+(\pi+\gamma)\,\alpha\right)}{(1+n)\eta(\pi+\gamma)(1-\alpha)}.$$

The terms $\Gamma_{p>0}$ and $\Gamma_{p=0}$ are increasing in ω and decreasing in n. Therefore, the ratio decreases as ω increases and as n decreases for both cases of $p > 0$ and $p = 0$.

To determine the effect of π on the ratio, we differentiate $\Gamma_{p>0}$ and $\Gamma_{p=0}$ with respect to π and obtain

$$\frac{\partial\Gamma_{p>0}}{\partial\pi} = \frac{1}{(1+n)\eta(\pi+\gamma)^2(1-\alpha)}\cdot\left[\omega(1+\gamma)\gamma-(1+n)\right],$$
$$\frac{\partial\Gamma_{p=0}}{\partial\pi} = \frac{1}{(1+n)\eta(\pi+\gamma)^2(1-\alpha)}\cdot\left[\omega(\gamma)^2-(1+n)\right].$$

These expressions indicate that the following holds:

$$\partial\left(\left.\frac{N_{t+1}x_t}{Y_t}\right|_{p>0}\right)/\partial\pi \geq 0 \text{ and } \partial\left(\left.\frac{N_{t+1}x_t}{Y_t}\right|_{p=0}\right)/\partial\pi > 0 \text{ if } \omega(1+\gamma)\gamma \leq (1+n),$$
$$\partial\left(\left.\frac{N_{t+1}x_t}{Y_t}\right|_{p>0}\right)/\partial\pi < 0 \text{ and } \partial\left(\left.\frac{N_{t+1}x_t}{Y_t}\right|_{p=0}\right)/\partial\pi > 0 \text{ if } \omega(\gamma)^2 < (1+n) < \omega(1+\gamma)\gamma,$$
$$\partial\left(\left.\frac{N_{t+1}x_t}{Y_t}\right|_{p>0}\right)/\partial\pi < 0 \text{ and } \partial\left(\left.\frac{N_{t+1}x_t}{Y_t}\right|_{p=0}\right)/\partial\pi \leq 0 \text{ if } (1+n) \leq \omega(\gamma)^2.$$

■

References

[1] Acemoglu, D., and Robinson, J., 2005. *Economic Origins of Dictatorship and Democracy.* Cambridge University Press, Cambridge, United Kingdom.

[2] An, C.B., and Jeon, S.H., 2006. Demographic change and economic growth: An inverted U-shape relationship. *Economics Letters* 92, 447–454.

[3] Arawatari, R., and Ono, T., 2014. Old-age social security vs. forward intergenerational public goods provision. *Japanese Economic Review* 65, 282–315.

[4] Bassetto, M., 2008. Political economy of taxation in an overlapping-generations economy. *Review of Economic Dynamics* 11, 18–43.

[5] Bearse, P., Glomm, G., and Janeba, E., 2001. Composition of government budget, non-single peakedness, and majority voting. *Journal of Public Economic Theory* 3, 471–481.

[6] Beauchemin, K.R., 1998. Intergenerational politics, fiscal policy and productivity. *Review of Economic Dynamics* 1, 835–858.

[7] Bernasconi, M., and Profeta, P., 2012. Public education and redistribution when talents are mismatched. *European Economic Review* 56, 84–96.

[8] Bishnu, M., and Wang, M., 2014. The political intergenerational welfare state. Iowa State University Department of Economics Working Paper No. 14014.

[9] Boldrin, M., and Montes, A., 2005. The intergenerational state education and pensions. *Review of Economic Studies* 72, 651–664.

[10] Chen, K., and Song, Z., 2014. Markovian social security in unequal societies. *Scandinavian Journal of Economics* 116, 982–1011.

[11] Forni, L., 2005. Social security as Markov equilibrium in OLG models. *Review of Economic Dynamics* 8, 178–194.

[12] Glomm, G., 2004. Inequality, majority voting and the redistributive effects of public education funding *Pacific Economic Review* 9, 93–101.

[13] Glomm, G., and Kaganovich, M., 2008. Social security, public education and the growth-inequality relationship. *European Economic Review* 52, 1009–1034.

[14] Glomm, G., and Ravikumar, B., 1995. Endogenous public policy and multiple equilibira. *European Journal of Political Economy* 11, 653–662.

[15] Glomm, G., and Ravikumar, B., 2001. Human capital accumulation and endogenous public expenditure. *Canadian Journal of Economics* 34, 807–826.

[16] Glomm, G., and Ravikumar, B., 2003. Public education and income inequality. *European Journal of Political Economy* 19, 289–300.

[17] Gonzalez-Eiras, M., and Niepelt, D., 2008. The future of social security. *Journal of Monetary Economics* 55, 197–218.

[18] Gonzalez-Eiras, M., and Niepelt, D., 2012. Aging, government budgets, retirement, and growth. *European Economic Review* 56, 97–115.

[19] Gradstein M., and Kaganovich, M., 2004. Aging population and education finance. *Journal of Public Economics* 88, 2469–2485.

[20] Grossman, G., and Helpman, E., 1998. Intergenerational redistribution with short-lived governments. *Economic Journal* 108, 1299–1329.

[21] Hassler, J., Krusell, P., Storesletten, K., and Zilibotti, F., 2005. The dynamics of government. *Journal of Monetary Economics* 52, 1331–1358.

[22] Heer, B., and Irmen, A., 2014. Population, pensions, and endogenous economic growth. *Journal of Economic Dynamics and Control* 46, 50-72.

[23] Holtz-Eakin, D., Lovely, M.E., and Tosun, M.S., 2004. Generational conflict, fiscal policy, and economic growth. *Journal of Macroeconomics* 26, 1–23.

[24] Iturbe-Ormaetxe, I., and Valera, G., 2012. Social security reform and the support for public education. *Journal of Population Economics* 25, 609–634.

[25] Kaganovich, M., and Meier, V., 2012. Social security systems, human capital, and growth in a small open economy. *Journal of Public Economic Theory* 14, 573–600.

[26] Kaganovich, M., Zilcha, I., 2012. Pay-as-you-go or funded social security? A general equilibrium comparison. *Journal of Economic Dynamics and Control* 36, 455–467.

[27] Kemnitz, A., 2000. Social security, public education, and growth in a representative democracy. *Journal of Population Economics* 13, 443–462.

[28] Kunze, L., 2014. Life expectancy and economic growth. *Journal of Macroeconomics* 39, 54–65.

[29] Lambrecht, S., Michel, P., and Vidal, J.P., 2005. Public pensions and growth. *European Economic Review* 49, 1261–1281.

[30] Lancia, F., and Russo, A., 2015. Public education and pensions in democracy: A political economy theory. http://www.sv.uio.no/econ/english/research/unpublished-works/working-papers/2015/memo012015.html (July 11, 2015).

[31] Levy, G., 2005. The politics of public provision of education. *Quarterly Journal of Economics* 120, 1507–1534.

[32] Lindbeck, A.J., and Weibull, W., 1987. Balanced-budget redistribution as the outcome of political competition. *Public Choice* 52, 273–297.

[33] Ludwig, A., Schelkle, T., and Vogel, E., 2012. Demographic change, human capital and welfare. *Review of Economic Dynamics* 15, 94-107.

[34] Naito, K., 2012. Two-sided intergenerational transfer policy and economic development: A politico-economic approach. *Journal of Economic Dynamics and Control* 36, 1340–1348.

[35] OECD, 2014. OECD Factbook 2014: Economic, Environmental and Social Statistics. http://www.oecd-ilibrary.org/economics/oecd-factbook-2014_factbook-2014-en (February 20, 2016).

[36] Ono, T., 2015. Public education and social security: A political economy approach. *Economics of Governance* 16, 1–25.

[37] Ono, T., and Uchida, Y., 2014. Pensions, education, and growth: A positive analysis. https://ideas.repec.org/p/osk/wpaper/1437.html (July 11, 2015).

[38] Persson, T., and Tabellini, G., 2000. *Political Economics: Explaining Economic Policy*, MIT Press, Cambridge, Massachusetts.

[39] Poutvaara, P., 2006. On the political economy of social security and public education. *Journal of Population Economics* 19, 345–365.

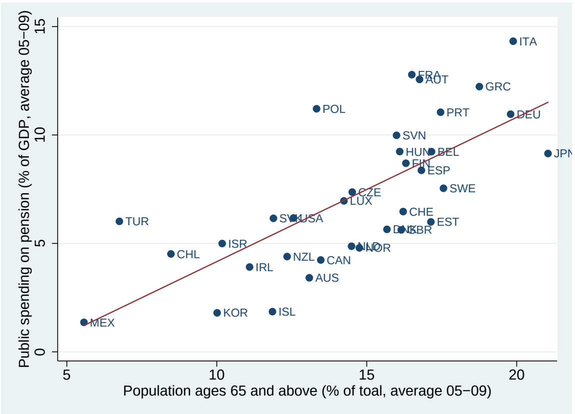
[40] Rangel, A., 2003. Forward and backward intergenerational goods: Why is social security good for the environment? *American Economic Review* 93, 813–834.

[41] Soares, J., 2006. A dynamic general equilibrium analysis of the political economy of public education. *Journal of Population Economics* 19, 367–389.

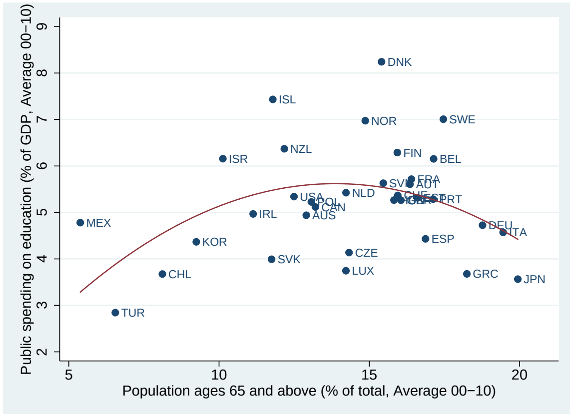
[42] Song, Z., 2011. The dynamics of inequality and social security in general equilibrium. *Review of Economic Dynamics* 14, 613–635.

[43] Song, Z., Storesletten, K., and Zilibotti, F., 2012. Rotten parents and disciplined children: A politico-economic theory of public expenditure and debt. *Econometrica* 80, 2785–2803.

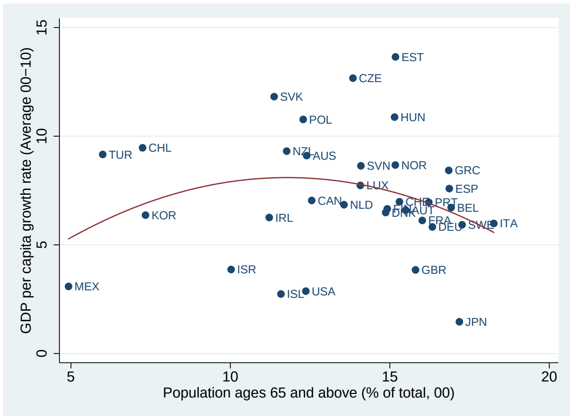
[44] Tosun, M.S., 2008. Endogenous fiscal policy and capital market transmissions in the presence of demographic shocks. *Journal of Economic Dynamics and Control* 32, 2031–2060.



(a)



(b)



(c)

Figure 1: Panel (a): public pension spending-to-GDP ratio and population aged 65 years and above, 2005–2009. Panel (b): public education spending-to-GDP ratio and population aged 65 years and above, 2000–2010. Panel (c): per capita growth rate and population aged 65 years and above. Source: World Bank, 2015, World Bank Indicators; OECD, social expenditure database (https://data.oecd.org/socialexp/pension-spending.htm, July 11, 2015).

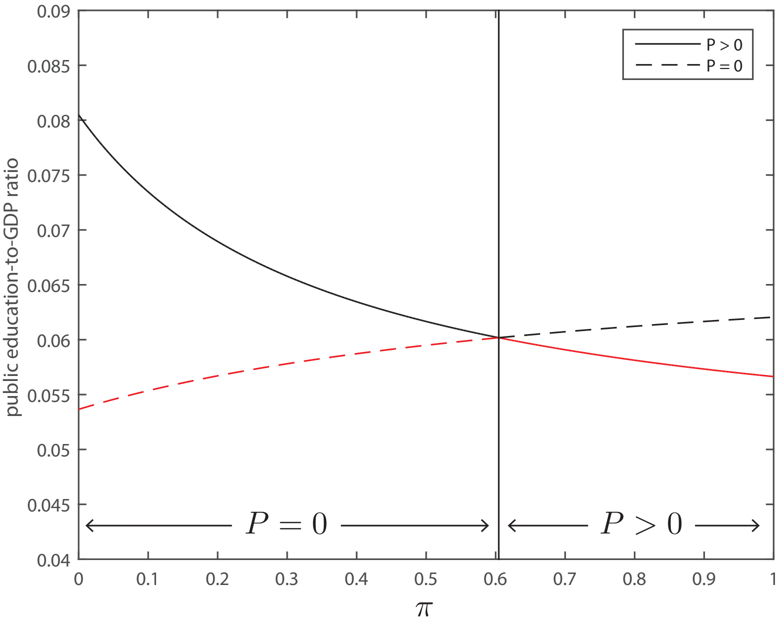


Figure 2: Education spending-to-GDP ratio and the longevity parameter π.

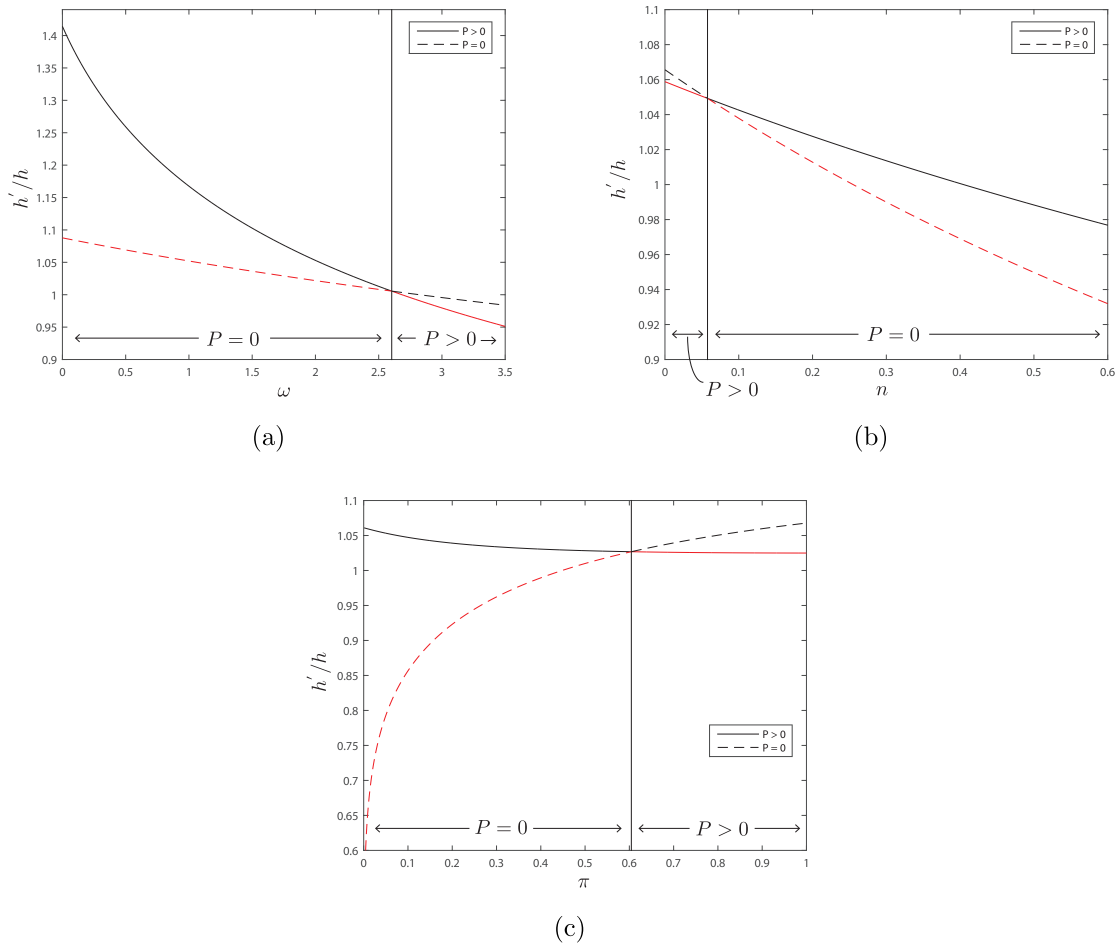
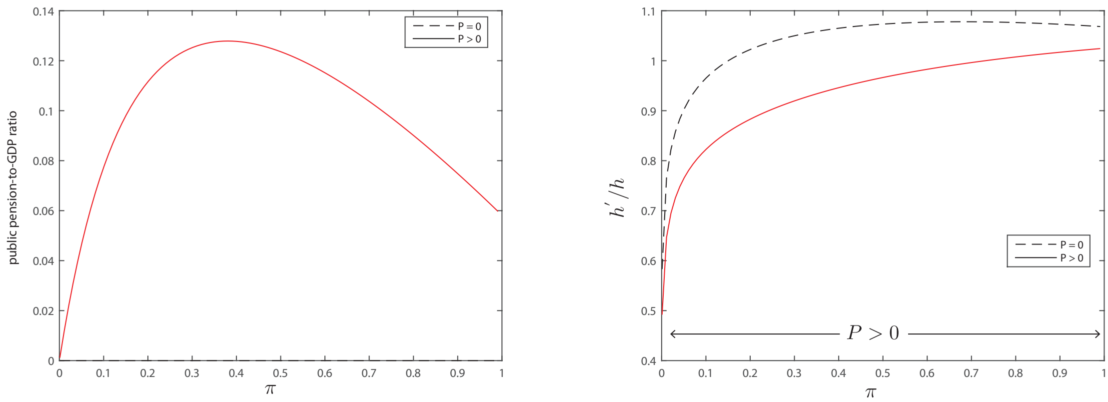


Figure 3: Panel (a): per capita human capital growth rate and ω. Panel (b): per capita human capital growth rate and n. Panel (c): per capita human capital growth rate and π.



(a) (b)

Figure 4: Panel (a): public pension-to-GDP ratio and π. Panel (b): per capita human capital growth rate and π.