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Abstract

This study theoretically investigates the effects of educational envy on schooling decisions. I develop a model where in workers enjoy utility from their levels of education and consumption. Moreover, worker's utility also depends on other worker's education levels, i.e., we assume the "keeping up with Joneses" effect in education. The main result of this study is that such envy causes workers to make decisions on education level that differ from the decision made when envy is not considered. This result can explain the United States-Japan differences in the relationship between wages and schooling decisions. Analysis from a social planner's perspective reveals that certain conditions on parameters can change the social preferences for the education level selected by individuals. Moreover, the model indicates that the economy may be overeducated in terms of the education for education's sake situation.

JEL Classification: I26; I21; I31; E24

Keywords: Education; Externality; Social preference; Multiple equilibria

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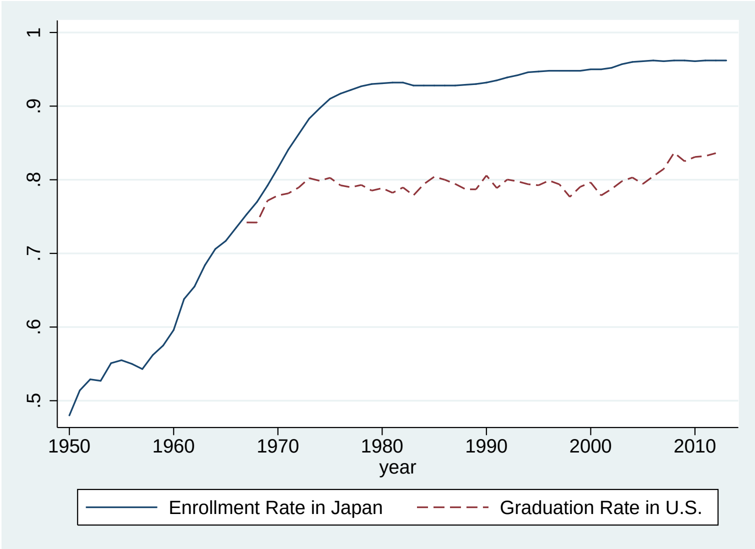
1 Introduction

Education is frequently viewed as playing a very important role in human society. For developing countries, the United Nations established the eight Millennium Development Goals (MDGs). The second target of the MDGs is to achieve a full course of primary schooling for all children throughout the world by 2015. As a result, most developing countries have attempted to facilitate elementary education. For developed countries, primary education is provided to almost of all children, and the schooling rates of more higher education have increased. Therefore, these countries have developed more sophisticated societies. Furthermore, higher education has contributed to both the growth of specific countries and overall global growth through technical innovations.

In the economic literature, Becker (1962) is famous for developing the theory of human capital formation. His study investigated the effect of human capital, which is accumulated through education and training, and affects the economy by considering the marginal benefits and marginal costs of education. Subsequently, many studies have been conducted on the relationship between human capital investment and its returns. The returns are primarily measured using wages or several other monetary measures. Therefore wages are frequently viewed as being determined by education or human capital. Mincer (1958) theorized the wage equation into the Mincerian wage equation, which is frequently used to determine the factors that affect wages. Moretti (2004) used Mincerian wage equation to empirically study the relationship among schooling year, place of habitat, and wages. The study showed that increasing the number of highly educated people in a Metropolitan Statistical Area (MSA) induces an increase in the wages in that MSA, called the spillover effect. And Calvó-Armengol et al. (2009) showed that the school performance of children in the United States is significantly and positively affected by social networks after controlling for other factors. This result suggests a positive education externality, as this study assumes. Though empirical studies are fewer in Japan because of data restrictions, Hashimoto and Heath (1995) estimated the income elasticities of educational expenditures in Japan. The average estimated elasticity of education is 1.72, a result that suggests the existence of an education externality in Japan. And there are many other empirical researches which supports the externalities. Therefore, the existence of human capital externality is undoubted.

We review actual data on education and wages to determine the relationship between wage differences and schooling decisions. Standard economic theory states that the growth of wage differences between educations induces an increasing in schooling length. However, there exist some data that are contrary to the theory. Wage differences between high school graduates and university graduates are typically introduced; however, this study introduces wage differences of high school graduates and non-high school graduates between the United States and Japan. Figure 1 depicts data from the Current Population Survey (CPS) Historical Time Series Tables on School Enrollment in the United States and time series data from School Basic Survey in Japan. In the United States, the rate of high school graduation increased slightly since 1970. In contrast, in Japan, the rate of high school enrollment significantly increased between 1950 and 1980, and increased slowly after 1980. Standard economic theory states that the growth in high school graduation rates induces a reduction in relative wages between those who graduate and those who do not. Figure 2 from the CPS data indicates that wages in the United States declined relative to the



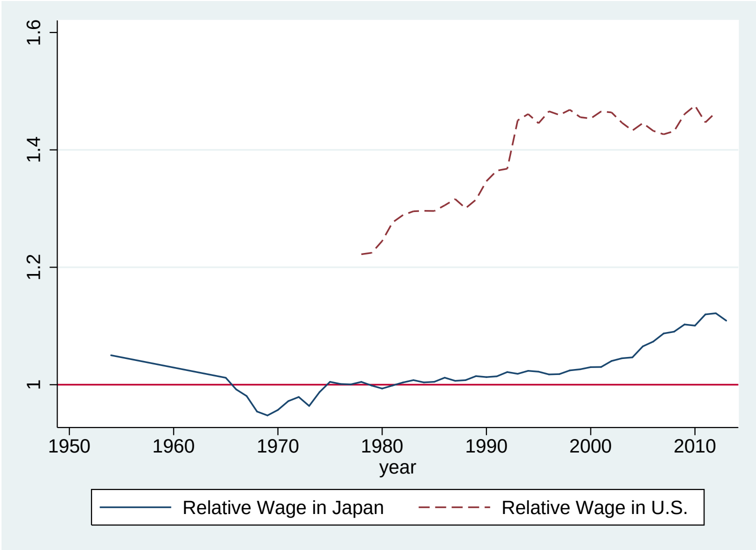
The US. rate represents the rate of high school graduates in the age group of 18-24 years old from the CPS survey. The Japanese rate shows the rate of high school enrollments in the population who completed 9 years education from the School Basic Survey. I can not obtain the graduation rate in Japan, however, the high school quitting rate in Japan is in the range of approximately 2-3% , I obtained this from the data.

Figure 1: U.S. high school graduation rates and enrollment rates in Japan

wages of high school graduates—as predicted by economic theory. In contrast, Japanese relative wages indicate two unreasonable movements and values. First, despite the growing number of high school students, wages of junior high school graduates did not decline. Typically, the labor force flows into high relative wage sectors if other conditions remain constant. However, the data show flatter movements. Second, the most unusual point is that the wage levels of high school graduates and non-graduates are almost identical, which is obvious from Figure 2. Figure 2 depicts that the relative wage of high school non-graduates is approximately 1, suggesting that graduating from high school does not affect the wages in Japan. Therefore, schooling decision of the Japanese is viewed as also being determined by non-monetary factors.[1] These puzzling motions and values are naturally caused by the “keeping up with the Joneses” thoughts of individuals getting an education. That is, a person decides to go to school not because wages of high school graduates are high but because others are attending school. In Japan, many parents want to send their children to highly ranked high schools and universities regardless of the cost. For example, they typically pay more than ¥1 million (approximately more than US$8,500) per year to cram schools (Jyuku in Japanese), in addition to paying the high school tuition to enroll in such highly ranked educational institutions. Moreover, the high cost of education cost is seen as a contributing factor to the low birth rate in Japan.

Before we begin to describe the model, we review the “keeping up with Joneses” consumption effects as the educational effect is similar. Many researchers studied the “keeping up with Joneses” consumption effects. The first study that we address is Abel (1990), who developed an asset pricing model that considered aggregate consumption per capita, investigated its effect on asset pricing, and showed that the “keeping up with the Joneses” effect is

[1] In Japan, when we consider bonuses in wages, the ratio of wages with bonus was 1.16, whereas the ratio of wages without bonus was 1.11 in 2013. Bonuses seem to increase the wage difference. However, the increase is not as large when we compared with the increase in the ratio of wages in Japan between high school graduates and university graduates, 1.33 to 1.41. Therefore, the wage difference between high school graduates and non-graduates in Japan is not as wide even we considered bonuses.



The data of U.S. relative wage is median data from weekly and hourly earnings data from the CPS. The data of Japan's relative wage is mean data from Basic Survey on Wage Structure. The difference between median and mean is due to data availability. The U.S. data is the ratio of median relative wages of males, over 25 year old that are high school graduates to those that not high school graduates. The Japanese data is the ratio of average relative wages of working males that are high school graduates to those that are not high school graduates.

Figure 2: Relative wages of high school graduates and non-graduates in the U.S. and Japan

a source of equity premiums. Similarly, Abel (1990) indicated that the "keeping up with Joneses effect" may be a source of the puzzle we mentioned in Figure 1 and 2. Ljungqvist and Uhlig (2000) explored taxation effects on "keeping up with the Joneses" consumption. Their study found that although several externalities exist, government taxation is a good way to control the economy. The implication of their paper is utilized in the discussion on policies in this study. Dupor and Liu (2003) modified Abel (1990) by measuring the utility of the "keeping up with the Joneses" effect from the marginal consumption level. Their finding is critical and is utilized to constructing the model of this study after its development. Liu and Turnovsky (2005) considered both macroeconomic dynamics with consumption and production externalities as the "keeping up with the Joneses" effect in consumption and production functions and capital stock externalities in production. In this study, the production function that determines wages is considered a human capital externality. This externality, described as the average education level of an economy, is designed to increase personal income. Mino (2008) developed an overlapping generations (OLG) model with a consumption externality and demonstrated that the consumption externality fundamentally affects both the equilibrium and the steady state characterization. Because children's education is primarily affected by parents, the OLG economy is better for considering education. However, because the OLG economy might be too complicated, this study does not consider generations. These prior articles delicately developed consumption externalities, however only considered consumption externalities and not education externalities. In addition, regarding education as consumption is incorrect because education directly and indirectly affects the entire economy.

In this study, a simple model of the education externality is developed and analyzed. The result of the analysis show that two equilibria exist that react differently to the level of productivity if an education externality is considered. Moreover, the net marginal benefit is found to determine social preferences for education.

The remainder of this paper is structured as follows. The second section is devoted to describing the model, including an education externality. The third section provides an analysis of equilibria. The fourth section develops a social planner's solution and discusses on policy implications. The final section concludes this paper.

2 Model

We develop a model of education externality. The economy comprises representative household and competitive firms.

2.1 Household

We assume that the representative household obtains utility from consumption and education. The representative household's utility function is as follows:

$$u(c, e) = c + v\left[\left(\frac{e}{E}\right)^{\gamma} - e\right], \tag{1}$$

where c represents consumption, v is a constant that represents the importance of education in utility, e denotes the education level of the household, and E indicates the reference level of education that can be interpreted as the average level of education in the society. We also assume the parameter $\gamma > 1$ to account for the "keeping up with the Joneses" effect. This effect indicates that a household increase its own education level if the education level of other households, or the reference level, is high.

The $(e/E)^{\gamma}$ term can be interpreted as the utility from the relative education level, which reflects strictly increasing returns to scale. The $-e$ term indicates the disutility from education as the effort cost. This model assumes that the peer group unilaterally affects the household and considers exogenous and full correlated effects from the classification by Manski (1993). Manski (1993) divided the social effects in a group into three types: endogenous, exogenous, and correlated effects.

Two points regarding the specification of the utility function are in order. The first is that the utility function has a quasi-linear form, for which several reasons exist. The first reason is that the function represents education externality. A non-homothetic characteristic is required for this type of externality; moreover, Alonso-Carrera et al. (2008) developed the restricted homothetic (RH) property and showed that the indeterminacy case does not hold the RH property. Second, we employ this specification to facilitate an evaluation of the education externality. Dupor and Liu (2003) suggested that the externality must be measured from the marginal utility of consumption. Therefore I added the utility function as (1) for the simple expression of the two reasons. In addition, Ljungqvist and Uhlig (2000) uses this type quasi-linear utility function in terms of consumption externality and labor disutility.

The second point is the existence of an education externality that is determined as $\gamma > 1$. No empirical study estimates the level of education externality. However, several empirical studies conjectured the presence of the externality as we noted in the introduction section.

The household earns wages from a firm by supplying one unit of labor. The household faces the following budget constraint:

$$c + pe = w(e) + \underline{w}, \tag{2}$$

where p represents the price of one unit of education, $w(e)$ represents the wage that the household earns from a firm related to one's education level, and $\underline{w}$ represents the wage that the household earns from a firm regardless of education level. Therefore, the household maximizes its utility according to the budget constraint (2).

2.2 Firm

Perfectly competitive firms pay wages to workers according to their productivity. The productivity of workers is represented in the following equation:

$$Ae^{\theta}E^{1-\theta} + \underline{w} = w(e) + \underline{w}, \tag{3}$$

where A is total factor productivity, and θ is a coefficient of the Cobb-Douglas production function that takes value in the interval between 0 and 1.

The final goods produced by firms are used in consumption or in the education system. This production function indicates that if the average level of education is low, the effect of investment in private education is weak, and vice versa. Moreover, this specification indicates that personal investment in education increases social productivity. This relationship corresponds to secondary education rather than higher education, such as lectures at a university.

The reason for this result is that, on the one hand, education from a university can be interpreted as a signal and may not necessarily improve a person's ability. On the other hand, secondary education is mainly primarily comprises learning basic skills for work, i.e., reading, writing, and calculating. Therefore, this specification is primarily considered in the relationship between secondary education and wage and is compatible with the results of the empirical studies by Trostel (2004) and Moretti (2004).

2.3 Optimization problem

The representative household maximizes its own utility when accounting for the firm's paying action. Then, by combining equations (1), (2), and (3) and imposing non-negative conditions on e and c, the maximization problem is as follows:

$$\max_{c,e} \quad u(c,e) = c + v\left[\left(\frac{e}{E}\right)^{\gamma} - e\right] \quad s.t. \quad c + pe = Ae^{\theta}E^{1-\theta} + \underline{w}, \quad c \geq 0, \quad e \geq 0. \tag{4}$$

The first-order condition of the problem is

$$v\gamma e^{\gamma-1}E^{-\gamma} - vE^{-1} = p - A\theta e^{\theta-1}E^{1-\theta}.$$

Rearranging the condition, we obtain

$$v\gamma e^{\gamma-1}E^{-\gamma} + A\theta e^{\theta-1}E^{1-\theta} = p + v. \tag{5}$$

The non-negative condition is

$$\underline{w} \geq pe - Ae^{\theta}E^{1-\theta}, \text{and} \quad e \geq 0.$$

Hereafter, we assume that $\underline{w}$ is large enough to have interior solutions and we concentrate on the interior solutions for simplicity. The left-hand side of equation (5) is the individual's marginal benefits of education and the right-hand side indicates the individual's marginal costs of education. However, this equation cannot be solved because of its non-linearity, and is analyzed in the next section.

3 Analysis of equilibria

We analyze the model by dividing it into two parts. The first part is no education externality case and the second part is the education externality case. The second part is devoted toward determining how the education externality affects the economy. Hereafter, we concentrate on variable e because it is the determinant variable of this model.

3.1 Benchmark

First, we assume that no education externality exists, $v = 0$, as the benchmark of the model. We derive the equilibrium of the model. From equation (5), we obtain:

$$A\theta e^{\theta-1}E^{1-\theta} = p.$$

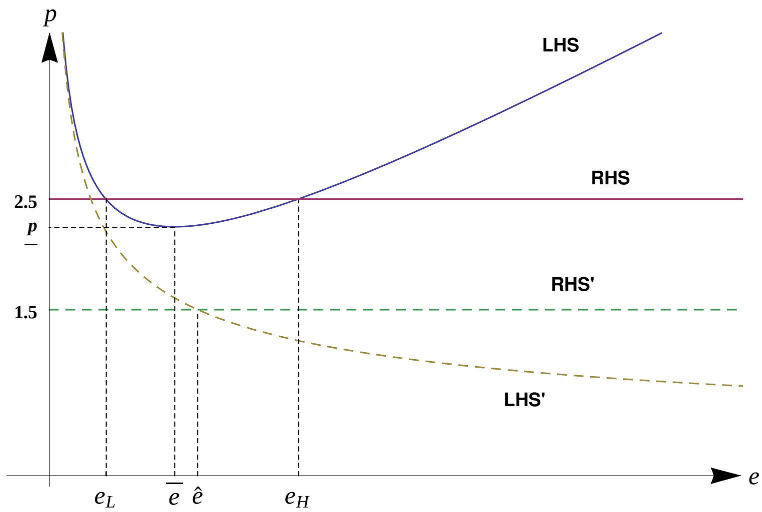
Solving the equation by e, we obtain the equilibrium:

$$e = \left(\frac{A\theta}{p}\right)^{\frac{1}{1-\theta}} E \equiv \hat{e}, \quad c = (1-\theta)A^{\frac{1}{1-\theta}} \left(\frac{\theta}{p}\right)^{\frac{\theta}{1-\theta}} E + \underline{w}. \tag{6}$$

We compare this result with the education externality case discussed in the next subsection.

3.2 Education externality

Second, we assume that a positive education externality exists, $v > 0$. We derive the optimal solution but cannot solve equation (5) in its explicit form because of its non-linearity. Therefore, we conduct a comparative statistical



LHS and RHS mean the left-hand side and the right-hand side of (5), and LHS' and RHS' are those of (6).

Figure 3: The existence of equilibria: case $A = 2,\ \theta = 0.6,\ E = 1.5,\ v = 1,\ \gamma = 2,\ p = 1.5.$

analysis on equilibria. First, we derive the condition for the two existing equilibria. If the next condition is satisfied, we have two equilibria.

$$\underline{p} \equiv \left(\frac{v\gamma}{(1-\theta)E}\right)^{\frac{1-\theta}{\gamma-\theta}} \left(\frac{A\theta}{\gamma-1}\right)^{\frac{\gamma-1}{\gamma-\theta}} (\gamma-\theta) < p + v. \tag{7}$$

See the Appendix for a derivation of the condition. Figure 3 assists in understanding the existence of equilibria. Condition (7) implies that some e exists that shows that the marginal benefits of education exceed the marginal costs of education. If condition (7) is not satisfied, the marginal benefits of education are always greater than the marginal costs of education. Then, the optimal solution for the household is yielded as $(c, e) = (+\infty, +\infty)$. Therefore, the equilibrium does not exist.

We then assume that inequality (7) is satisfied and we define the two equilibria, e_L and e_H $(e_L < e_H)$, for convenience. Hereafter, we analyze the nature of these equilibria. First, we check the stability of the equilibria. The equilibrium e_L is stable because the first-order derivative around e_L of the left-hand side of equation (5) is negative. However, the equilibrium e_H is unstable because the first-order derivative around e_H of the left-hand side of equation (5) is positive. If the equilibrium e_H suddenly increases, the marginal benefits overwhelm the marginal costs. Therefore, the level of education increases to the $c = 0$ point. If the equilibrium e_H suddenly decreases, the level of education declines until it gets to e_L. In a real economy, as Figure 1 shows the Japanese high school enrollment rate, which has increased rapidly, is approximately 100%. Therefore, because Japan seems to be in e_H equilibrium and to have deviated to increase, we should examine the e_H equilibrium and discuss whether the unstable equilibrium is common in economic literature, particularly applied economics. For example, although the saddle path in the growth theory of macroeconomics is unstable, it is frequently investigated for its economic importance. In addition, this model does not consider time. When the parameters change, we solve other problems and obtain other equilibria. For these reasons, the equilibria are worth investigating. We need to present the next lemma to clarify the range of the two equilibria.

Lemma 1. *The two equilibria e_L and e_H exist in the range $0 < e_L < \overline{e} < e_H$, where $\overline{e}$ is the minimizing value of the left-hand side of the first order condition.*

Proof. See the Appendix. □

We obtain the following proposition by comparing the benchmark case and the education externality case.

Proposition 1. *When the condition, $\gamma(A\theta/p)^{\frac{\gamma-1}{1-\theta}} < E$, holds, $e_L < \hat{e} < e_H$ is satisfied.*

Proof. See the Appendix. □

Proposition 1 implies that the condition that is primarily affected by the parameters E and γ is satisfied, and no externality equilibrium $\hat{e}$ exists between externality equilibria e_L and e_H. When the condition is satisfied and the equilibria e_L and e_H are statistically counted in the same country, the data may indicate that no externality exists in the country even though it exists. If the condition is satisfied, the equilibrium e_L represents an underinvestment in education and the equilibrium e_H represents an overinvestment in education from the no externality perspective. If the condition is not satisfied, then the case $\gamma(A\theta/p)^{\frac{\gamma-1}{1-\theta}} \geq E$ holds, and the relationship among e_L, e_H, and $\hat{e}$ is either $e_L < e_H \leq \hat{e}$ or $\hat{e} \leq e_L < e_H$. The former case implies that the education externality always decreases the investment in education, and the latter case implies that the education externality always increases the investment in education. However, we cannot distinguish among the two cases. We have the following proposition.

Proposition 2. *For each equilibrium, the following properties hold;*

$$\begin{cases} \dfrac{\mathrm{d}w_H}{\mathrm{d}A} < 0 \text{ and } \dfrac{\mathrm{d}e_H}{\mathrm{d}A} < 0, \ \dfrac{\mathrm{d}w_L}{\mathrm{d}A} > 0 \text{ and } \dfrac{\mathrm{d}e_L}{\mathrm{d}A} > 0 & \text{if} \quad \underline{p} < p + v < \tilde{p} \\ \dfrac{\mathrm{d}w_H}{\mathrm{d}A} = 0 \text{ and } \dfrac{\mathrm{d}e_H}{\mathrm{d}A} < 0, \ \dfrac{\mathrm{d}w_L}{\mathrm{d}A} > 0 \text{ and } \dfrac{\mathrm{d}e_L}{\mathrm{d}A} > 0 & \text{if} \quad \tilde{p} = p + v \\ \dfrac{\mathrm{d}w_H}{\mathrm{d}A} > 0 \text{ and } \dfrac{\mathrm{d}e_H}{\mathrm{d}A} < 0, \ \dfrac{\mathrm{d}w_L}{\mathrm{d}A} > 0 \text{ and } \dfrac{\mathrm{d}e_L}{\mathrm{d}A} > 0 & \text{if} \quad \tilde{p} < p + v, \end{cases}$$

where $\tilde{p}$ is defined as follows:

$$\tilde{p} \equiv \left(\frac{v}{E}\right)^{\frac{1-\theta}{\gamma-\theta}} \left(\frac{A\theta}{\gamma-1}\right)^{\frac{\gamma-1}{\gamma-\theta}} \gamma^{1+\frac{1-\theta}{\gamma-\theta}}. \tag{8}$$

Proof. See the Appendix. □

Proposition 2 states that, on the one hand, if productivity A increases, the wage and the level of education in equilibrium e_L also increase. On the other hand, when p is large enough, if A increases, the wage increases and the education level e_H decreases. When p is not large, but is in the range of $\underline{p} < p < \tilde{p}$, if A increases, the wage and the level of education decrease. These suspicious movements in the education level and wages are caused by the first-order condition (5). First, we consider the e_H case. When A increases, the first term on the left-hand side of

(5) decreases to sustain the first-order-condition, that is, decreases e. The wage is determined by $w_H(e_H)$ through a comparison of the increase in wages by A and the decrease in wages by e_H. Therefore, if the former is larger than the latter, the wage increases. If the latter is larger than the former, the wage decreases. Second, we consider the e_L case. When A increases, at e_L, the household increases the education level to adjust the marginal wage equally considering the effect of utility from relative education. Therefore, at e_L, an increasing A always increases education level e. Therefore, the wage $w_L(e_L)$ always increase because an increasing A itself adds to the wages and an increasing A indirectly affects wages through the positive relationship with e.

The economic theory predicts that the wage difference increases the high school graduation rate, as discussed in the introduction. And the data show the predicted movement in the United States. However, the data show the opposite movement and are not compatible with the theory in Japan. Proposition 2 theoretically explains these movements of Japan and the United States in the same model that the theory cannot explain. In Japan, productivity A declines as times passes.[2] If we ignore the education externality term, which means that $v = 0$ holds, education level e and wage level w also decrease. However, the data show the opposite movement. If we focus on the equilibrium e_H with the $\underline{p} < p + v < \tilde{p}$ condition, declining A induces an increasing education level e_H and wage $w(e_H)$. We interpret $w(e)$ as the wage of high school graduates and $\underline{w}$ as the wage of only having graduated from compulsory education. Therefore, the equilibrium with this condition is compatible with these movements in Japan.

In contrast, in the United States, the productivity A increases as times passes,[3] and the high school graduate ' s wage and graduation rate both increase. These movements are compatible with the $v = 0$ case and the e_L equilibrium case. Therefore we impose the equilibrium e_H to represent Japan and the equilibrium $e_L$to represent the United States. When the condition for Proposition 1, $\gamma(A\theta/p)^{\frac{\gamma-1}{1-\theta}} < E$, is satisfied, if we do not consider the education externality, the United States, e_L, is overeducated and Japan, e_H, is undereducated, based on the education externality case. In the next section, we examine the preferred equilibrium from the perspective of the social planner.

4 Social planner's solution and policy implications

In this section, we consider the problem from the social planner's perspective to examine how education affects the social welfare of the economy. In this section, we regard the reference level of education E as the personal level of education level e .

[2]This condition occurs because the growth rate of the multifactor productivity decreases from 1985 to 2011 according to the OECD statistics database.

[3]This occurrence is from the slight increase in the growth rate of multifactor productivity from 1985 to 2011 according to the OECD statistics database.

4.1 Social planner's solution

To examine how education affects the social welfare of the economy, we solve another problem regarding the reference level of education as the personal education decision. From the classification of social effects in a group by Manski (1993), this analysis is classified as complete correlated effects, which considers the case that the members of a group behave perfectly in the same manner. We maximize social welfare as follows:

$$\max_{c,e} \quad u(c,e) \quad = \quad c + v\left[\left(\frac{e}{e}\right)^{\gamma} - e\right] = c + v(1-e) \tag{9}$$
$$s.t. \quad c + pe = Ae^{\theta}E^{1-\theta} + \underline{w} = Ae + \underline{w}.$$

Solving problem (9), we obtain the next result, for which a is an arbitrary value:

$$(c,e) = \begin{cases} (\underline{w}, 0) & (A < p+v) \\ (\underline{w}, a) & (A = p+v) \\ (+\infty, +\infty) & (A > p+v). \end{cases}$$

The interpretation of this solution is that when $A < p + v$ holds, the social planner does not want to invest in education; when $A = p + v$ holds, the planner has no intention to pursue education; and when $A > p + v$ holds, the planner wants to invest as much as possible in education. Therefore, we have two propositions.

Proposition 3. *When $A < p + v$ holds, the education level determined by households is always higher than that determined by the social planner.*

Proof. The proof is obvious from a comparison of the social planner's and the household's solution. □

Proposition 4. *The social preferences on equilibria e_H and e_L are as follows.*

$$\begin{cases} e_H \succ e_L & A > p+v \\ e_H \sim e_L & A = p+v \\ e_H \prec e_L & A < p+v \end{cases}$$

where $\succ$ represents "strictly preferred to," $\sim$ represents "indifferent to," and $\prec$ represents " strictly worse than."

Proof. See the Appendix. □

Proposition 3 means that when $A < p + v$ holds, that is, the financial and effort costs of education are greater than the unit gain from education, the social planner believes that households should not invest in education. However, households' investment in education is greater than zero because the marginal gains from wage and education externalities pay for the marginal cost of education. Therefore, the planner always suffers from an overinvestment in education.

Proposition 4 states that social preferences are determined through a comparison of productivity A and the monetary and psychological costs of education $p+v$. When $A > p+v$ holds, a higher level of education is better for the social planner. When $A = p+v$ holds, the social planner does not consider the level of education. When $A < p+v$ holds, the social planner dislikes investments in education and, therefore, prefers a lower level of education. Normally, in a developed country, $A > p+v$ tends to hold because productivity A is relatively high and the cost is relatively low. Therefore, the social planner prefers Japan equilibrium e_H over the United States equilibrium e_L. Surely, the parameters between Japan and the United States are different. Therefore, comparing the Japan's equilibrium with the United States equilibrium using real data is useless. Proposition 4 insists that the United States can increase social welfare from the planner's perspective by moving into e_H equilibrium. However, this move may evoke reductions in large amounts of consumption, which may then reduce households' welfare because people in the United States might put significant weight on consumption. Therefore we must consider this point when considering policy implications in the next subsection.

4.2 Policy implications

Normally, in a developing country, the condition $A < p+v$ tends to hold because productivity A is relatively low compared with the costs p and v. In a developing country, the planner—the government—initially dislikes investments in education. However, although the government does not want to facilitate education, households invest in education. Finally, increasing productivity A which is caused by some exogenous factors, reverses the planner's preferences for education and the government facilitates investments in education. This chain of events represents a country's development process. Frequently, we cannot examine this theory because of a lack of accurate developing country data on education, consumption, productivity, and so on. Therefore, further research is required on this subject.

Moreover, we measure this economy using consumption level. The main motivation for individuals to pursue an education is to consume more goods by earning higher wages. By rearranging the budget constraint (2), we obtain

$$c - \underline{w} = e\left(Ae^{\theta-1}E^{1-\theta} - p\right). \tag{10}$$

Solving equation (10) to equal the zero condition gives:

$$\check{e} = \left(\frac{A}{p}\right)^{\frac{1}{1-\theta}} E. \tag{11}$$

If the equilibrium holds, $e_i > \check{e} (i = H, L)$, then at equilibrium we consume goods less than that of the zero education choice. Therefore, the following proposition is presented.

Proposition 5. *If the following condition (12) is satisfied, then we have $e_L < \check{e} < e_H$:*

$$v\left[\frac{\gamma}{E}\left(\frac{A}{p}\right)^{\frac{\gamma-1}{1-\theta}}-1\right] < (1-\theta)p. \tag{12}$$

Proof. See the Appendix. □

Proposition 5 states that, at the equilibrium e_H, if condition (12) holds, then the consumption level of the equilibrium is less than that of zero education $\underline{w}$. In this case, the level of education is relatively high and earns higher wages than the equilibrium e_L. However, the highest earned wage that is increased by education is primarily used to obtain a higher education, and the consumption level is lowered by a higher education. This result can be interpreted as education for education's sake. When $A > p + v$ holds, the planner prefers equilibrium e_H and encourages households to be more educated. Therefore, the primary motivation is not achieved. To discourage education for education's sake, increasing productivity A seems to be a good policy. Using the results of Proposition 2, increasing A induces households to decrease the equilibrium value e_H and to increase the threshold value $\check{e}$. Therefore, the condition (12) is less likely to be satisfied.

These findings show that increasing productivity A is a good policy regarding both the developing economy problem and the education for education's sake problem. To plan and execute policies that increase productivity A, a budget is required. Considering the government's budget constraints, a lump-sum tax on income and a proportional tax on consumption seem to be optimal uses of financial resources for the policy to increase productivity A. Because a proportional tax on education induces individuals to reduce education at e_L equilibrium and to increase education at e_H equilibrium by increasing relative price of education p, such a tax on education e is harmful for e_H equilibrium. Therefore, we must consider the equilibrium that we are in when we tax and subsidize economic activity.

5 Conclusion

This study investigates the effects of education externality on the economy. The two education equilibria in this model conversely react to increasing productivity. This characteristic of the model enables us to explain the puzzling movements in Japanese education, wage, and productivity. We then determined that the parameter relationship between the productivity of firms and the price of education determines the social preferences for education from the social planner's perspective.

I now look toward the prospects of extending this model. The first possibility is to treat the reference level of education E as an endogenous variable. In this study, the endogenous effects of the Manski (1993) classification are ignored. The exogenous and the correlated effects are investigated in section three and four, respectively. However, in the real world, these ideal cases are rare. Moreover, determining the reference level of education has not been well theorized. Therefore, considering E as the endogenous variable may change the results of this model.

Such future research may assist us in completely understanding education externality. The second possibility is to consider dynamics. The time series graphs are shown in the introduction; however, the model does not consider the dynamics of the economic variables. Therefore, introducing dynamics into this model may change the results. The third, estimating the parameters, particularly v and γ, is necessary. If v is not significant, the theory collapses and we must search for a new model to explain the suspicious movements of the variables for Japan. If γ satisfies the condition $0 < \gamma \leq 1$, the model does not have two equilibria. Even though equilibrium exists, which may increase the equilibrium level compared with the no externality case, productivity A affects e given that it has the same sign as the no education externality case. We may have many other extensions for this model, and research on these extensions can be fruitful.

Appendix

Existence of equilibria

First, we define the left-hand side of (5) as $f(e)$. Second, we take the first-order derivative of $f(e)$ by e and set $f'(e) = 0$ to obtain:

$$f'(e) = v\gamma(\gamma-1)e^{\gamma-2}E^{-\gamma} + A\theta(\theta-1)e^{\theta-2}E^{1-\theta} = 0.$$

Solving the equation for e gives us the solution, and we define the solution $\bar{e}$ as follows:

$$\bar{e} = \left[\frac{A\theta(1-\theta)E^{1-\theta+\gamma}}{v\gamma(\gamma-1)}\right]^{\frac{1}{\gamma-\theta}}.$$

Third, we prove that $\bar{e}$ takes the minimal value of $f(e)$. Taking the second-order derivative of $f(e)$, we obtain:

$$f''(e) = v\gamma(\gamma-1)e^{\gamma-3}E^{-\gamma} + A\theta(\theta-1)(\theta-2)e^{\theta-3}E^{1-\theta} > 0,$$

for all $e > 0$. The equation indicates that $\bar{e}$ takes the minimal value of $f(e)$. For the existence of equilibria, we must hold that the minimal value of the left-hand side of (5), $f(\bar{e})$, is smaller than the right-hand side of (5). The condition is:

$$f(\bar{e}) = \left(\frac{v\gamma}{(1-\theta)E}\right)^{\frac{1-\theta}{\gamma-\theta}}\left(\frac{A\theta}{\gamma-1}\right)^{\frac{\gamma-1}{\gamma-\theta}}(\gamma-\theta) < p + v.$$

Therefore, the condition is obtained.

Proof of lemma 1

From the previous discussion, $\bar{e}$ takes the minimal value of $f(e)$. From the intermediate value theorem and the limits of the first-order condition:

$$\lim_{e \to +0} v\gamma e^{\gamma-1}E^{-\gamma} + A\theta e^{\theta-1}E^{1-\theta} = +\infty$$

and

$$\lim_{e \to \infty} v\gamma e^{\gamma-1}E^{-\gamma} + A\theta e^{\theta-1}E^{1-\theta} = +\infty,$$

there exist equilibria in the open interval $(0, \bar{e})$ and $(\bar{e}, +\infty)$, respectively. Because $f(e)$ is monotonically decreasing in interval $(0, \bar{e})$ and its lower bound is $f(\bar{e})$ in interval $(\bar{e}, +\infty)$, $f(e)$ is monotonically increasing and its lower bound is $f(\bar{e})$. Therefore, $0 < e_L < \bar{e} < e_H$ holds.

Proof of proposition 1

Use (5) and (6). Arranging (5), we obtain:

$$A\theta e^{\theta-1}E^{1-\theta} + v\gamma e^{\gamma-1}E^{-\gamma} - p - v = 0. \tag{13}$$

To evaluate the left-hand side of (13), substitute $\hat{e}$ into the left-hand side of (13), then to obtain the value at $\hat{e}$:

$$v\left[\frac{\gamma}{E}\left(\frac{p}{A\theta}\right)^{\frac{\gamma-1}{\theta-1}} - 1\right]. \tag{14}$$

If (14) is negative, then $e_L < \hat{e} < e_H$, which is obvious from the form of $f(e)$ previously shown.

Proof of proposition 2

First, from the proof of proposition 1, use (13) as follows:

$$A\theta e^{\theta-1}E^{1-\theta} + v\gamma e^{\gamma-1}E^{-\gamma} - p - v = 0.$$

To determine the relationship between e and A, differentiate the left-hand side of (13) by e and A to obtain:

$$\frac{\mathrm{d}e}{\mathrm{d}A} = -\frac{\theta e^{\theta-1}E^{1-\theta}}{A\theta(\theta-1)e^{\theta-2}E^{1-\theta} + v\gamma(\gamma-1)e^{\gamma-2}E^{-\gamma}}. \tag{15}$$

The numerator always takes a positive value; therefore, the sign of $\mathrm{d}e/\mathrm{d}A$ depends on the denominator. We know that the denominator is equivalent to $f'(e)$. Therefore, at the equilibrium e_L, the denominator has a negative value and $\mathrm{d}e/\mathrm{d}A > 0$ holds. At e_H, the denominator has a positive value and $\mathrm{d}e/\mathrm{d}A < 0$ holds.

Second, because the wage is determined by $w = Ae^{\theta}E^{1-\theta} + \underline{w}$, we differentiate between A and e, to obtain:

$$\frac{\mathrm{d}w}{\mathrm{d}A} = e^{\theta} E^{1-\theta} + A\theta e^{\theta-1} E^{1-\theta} \frac{\mathrm{d}e}{\mathrm{d}A}.$$

Then, we substitute (15) into this equation to obtain:

$$\frac{\mathrm{d}w}{\mathrm{d}A} = \frac{e^{\theta-1} E^{1-\theta} \left[v\gamma(\gamma-1) e^{\gamma-1} E^{-\gamma} - A\theta e^{\theta-1} E^{1-\theta} \right]}{A\theta(\theta-1) e^{\theta-2} E^{1-\theta} + v\gamma(\gamma-1) e^{\gamma-2} E^{-\gamma}}.$$

The sign of the denominator as a function of e is known because it is the same as $f'(e)$. If an equilibrium exists between 0 and $\bar{e}$, $f'(e)$ is negative at the equilibrium. If an equilibrium is greater than $\bar{e}$, $f'(e)$ is positive at the equilibrium. Therefore, the sign of the numerator reveals the sign of $\mathrm{d}w/\mathrm{d}A$. The threshold value $\tilde{e}$ of the numerator is determined by the following equation:

$$v\gamma(\gamma-1) e^{\gamma-1} E^{-\gamma} - A\theta e^{\theta-1} E^{1-\theta} = 0,$$

for $e^{\theta-1} E^{1-\theta}$ is positive. We solve the equation by e to obtain:

$$\tilde{e} = \left[\frac{A\theta E^{1-\theta+\gamma}}{v\gamma(\gamma-1)} \right]^{\frac{1}{\gamma-\theta}}.$$

We easily check that $\tilde{e} > \bar{e}$ using $(1-\theta)^{\frac{1}{\gamma-\theta}} < 1$. If an equilibrium is between 0 and $\tilde{e}$, the numerator is negative at the equilibrium. If an equilibrium is greater than $\tilde{e}$, the numerator is positive at the equilibrium. Then, to obtain the parametrized condition on $\tilde{e}$, substitute $\tilde{e}$ into (5) to obtain the threshold parameter as follows:

$$\tilde{p} \equiv \left(\frac{A\theta}{\gamma-1} \right)^{\frac{\gamma-1}{\gamma-\theta}} \left(\frac{E}{v} \right)^{\frac{\theta-1}{\gamma-\theta}} \gamma^{1+\frac{1-\theta}{\gamma-\theta}} = p + v.$$

From the above, we summarize the result for $\mathrm{d}w/\mathrm{d}A$ as follows. At e_L, the denominator and numerator are both negative because $e_L < \bar{e} < \tilde{e}$. Then, we obtain $\mathrm{d}w/\mathrm{d}A > 0$. At $e_H < \tilde{e}$, the denominator is positive and the numerator is negative because $\bar{e} < e_H < \tilde{e}$. Further, we obtain $\mathrm{d}w/\mathrm{d}A < 0$. At $e_H = \tilde{e}$, the denominator is 0 and we obtain $\mathrm{d}w/\mathrm{d}A = 0$. At $e_H > \tilde{e}$, both the denominator and the numerator are positive because $\bar{e} < \tilde{e} < e_H$ and we obtain $\mathrm{d}w/\mathrm{d}A > 0$. Therefore, the proposition is proved.

Proof of proposition 4

From budget constraint (2), we obtain:

$$c = (A-p)e + \underline{w}.$$

By substituting the equation into the utility function of the social planner, the utility level is measured by $(A-p-v)e+v+\underline{w}$. From the proof of lemma 1, $e_L < e_H$ is satisfied. Therefore, when $A > p+v$, the social planner prefers e_H to e_L. When $A < p+v$, the planner prefers e_L to e_H, and when $A = p+v$, education choice e_H is indifferent to e_L from the social planner's perspective.

Proof of proposition 5

Write the left-hand side of (13) as follows:

$$A\theta e^{\theta-1}E^{1-\theta} + v\gamma e^{\gamma-1}E^{-\gamma} - p - v.$$

Substitute (11) into the equation to obtain:

$$v\left[\frac{\gamma}{E}\left(\frac{A}{p}\right)^{\frac{\gamma-1}{1-\theta}} - 1\right] - (1-\theta)p.$$

References

ABEL, A. B. (1990): “Asset Prices under Habit Formation and Catching Up with the Joneses,” *American Economic Review*, 80, 38–42.

ALONSO-CARRERA, J., J. CABALLÉ, AND X. RAURICHC (2008): “Can Consumption Spillovers be a Source of Equilibrium Indeterminacy?” *Journal of Economics of Dynamics and Control*, 32, 2883–2902.

BECKER, G. S. (1962): “Investment in Human Capital: A Theoretical Analysis,” *Journal of Political Economy*, 70, 9–49.

CALVÓ-ARMENGOL, A., E. PATACCHINI, AND Y. ZENOU (2009): “Peer Effects and Social Networks in Education,” *Review of Economic Studies*, 76, 1239–1267.

DUPOR, B. AND W.-F. LIU (2003): “Jealousy and Equilibrium Overconsumption,” *The American Economic Review*, 93, 423–428.

HASHIMOTO, K. AND J. A. HEATH (1995): “Income Elasticity of Educational Expenditure by Income Class: The Case of Japanese Households,” *Economics of Education Review*, 14, 63–71.

LIU, W.-F. AND S. J. TURNOVSKY (2005): “Consumption Externalities, Production Externalities, and Long-run Macroeconomic Efficiency,” *Journal of Public Economics*, 89, 1097–1129.

LJUNGQVIST, L. AND H. UHLIG (2000): “Tax Policy and Aggregate Demand Management Under Catching Up with the Joneses,” *The American Economic Review*, 90, 356–366.

MANSKI, C. F. (1993): "Identification of Endogenous Social Effects: The Reflection Problem," *The Review of Economic Studies*, 60, 531–542.

MINCER, J. (1958): "Investment in Human Capital and Personal Income Distribution," *Journal of Political Economy*, 66, 281–302.

MINO, K. (2008): "Growth And Bubbles With Consumption Externalities," *The Japanese Economic Review*, 59, 33–53.

MORETTI, E. (2004): "Estimating the Social Return to Higher Education: Evidence from Longitudinal and Repeated Cross-sectional Data," *Journal of Econometrics*, 121, 175–212.

TROSTEL, P. A. (2004): "Returns to Scale in Producing Human Capital from Schooling," *Oxford Economic Papers*, 56, 461–484.