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Abstract

It is very important for service industries to decentralize consumers at peak time, and thereby to increase sales not at peak time. This study discusses an optimal number of business hours for a service industry when the service provider offers a price discount immediately after the opening time and just before the closing time. For a specific ideal service time distribution of consumers, the optimal opening and closing time are explored. Clarified are the conditions under which an optimal number of business hours exists to maximize the social welfare. Numerical examples are also presented to illustrate the theoretical underpinnings of the proposed model.

KEY WORDS: Price discount, Service hours, Ideal service time distribution, Social welfare

JEL Classification: L80, M20

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1. Introduction

The regulations of business hours have traditionally generated a central issue in many European countries (see, e.g., De Meza(1984), Ferris(1990, 1991), Clemenz(1990, 1994), Inderst and Irmen (2005)). In the real world, however, it is very important for service providers to decentralize consumers at peak time and increase the sales not at peak time to increase the sales and/or the profit. Shy and Stenbacka(2006) have proposed a model to discuss an optimal number of business hours for a service provider against a specific consumers' ideal service time distribution.

In the real circumstance, it can observed that a special offer such as time discount has commonly implemented by service providers as an effective strategy, e.g., morning perm at a beauty salon, happy hour at a hotel, midnight discount of a telecommunications industry, special time discount in business logistics and so forth. From this point of view, Kim and Sandoh(2014) have introduced a special offer of discount of price immediately after the opening time and just before the closing time to discuss an optimal number of business hours, where the service provider is interested in maximizing his profit. This type of price promotion is an effective management tool since a service provider can attract extra consumers whose ideal or convenient service times are before the opening time or after the closing time.

In this study, we focus on the social welfare as an objective function to be maximized and clarify the conditions where an optimal number of business hours exists. Numerical examples are also presented to illustrate the insights of our analysis.

2. Model Formulation

2.1 Assumptions and notations

The assumptions along with their relevant notations in this study are as follows:

(1) Each individual consumer has her own ideal time to visit the provider to receive service from the service provider.

(2) Each consumer obtains utility, u_0, by purchasing a service product.

(3) The regular selling price of a service product is p.

(4) During the price discount period, the provider sells his service product at price αp as his special offer, where $0 < \alpha < 1$.

(5) The length of the price discount period is denoted by $\tau(>0)$.

(6) A consumer owes ω per unit of time to shift her actual service time from her own ideal service time to purchase a service product.

(7) The opening and closing times are, respectively, denoted by t_o and t_c, where we have $0 \le t_o \le t_c \le 1$.

(8) The raw price per service product is given by c_1, while the operation cost of the service provider per unit of time is c_2.

2.2 Ideal service time distribution

In this study, we assume that a customer distribution q_t in which the number of consumers with an ideal service time $t(0 \le t \le 1)$ is

$$q_t = \begin{cases} n[\mu + 4(1-\mu)t], & 0 \le t < \dfrac{1}{2} \\ n[4 - 3\mu - 4(1-\mu)t], & \dfrac{1}{2} \le t \le 1 \end{cases}, \tag{2.1}$$

where n represents the population size and μ $(0 \le \mu \le 1)$ measures the degree of uniformity. Figure 1 shows the ideal time distribution given by Eq. (2.1) for n =1 against various values of μ.

Shy and Stenbacka[6] have assumed the above ideal time distribution on the unit circle with the view to formalizing the idea that there are spillovers between time periods. In this study, however, we assume the same structure of the ideal time distribution on the unit time interval [0, 1]. This is because spillovers are an important factor only when the service provider sells his products for almost whole unit time period, and in such a situation the strategic determination of service hours might not be necessary.

We here introduce and additional assumption as follows:

(9) When the selling price is discounted to αp at t, demand quantity q_t increases $\beta(\alpha)q_t$ to $[1+\beta(\alpha)]q_t$ for $\beta(\alpha)>0$ and $0<\alpha<1$.

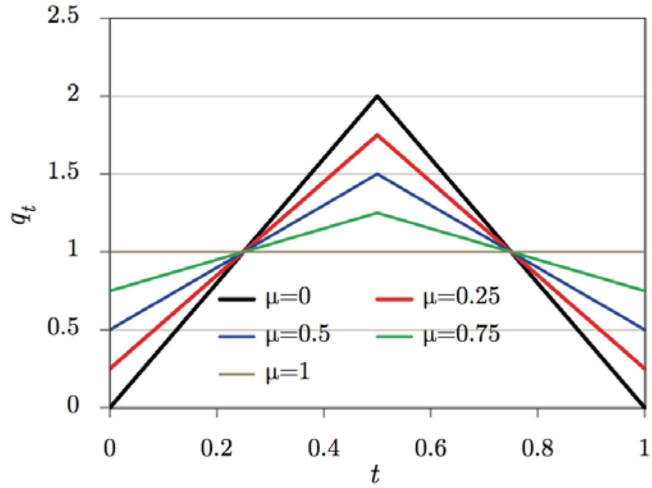

Figure 1: Ideal time distribution(n=1).

In the above, Assumption (9) signifies the price elasticity η of demand is given by

$$\eta = -\frac{\dfrac{\beta(\alpha)q_t}{q_t}}{\dfrac{(\alpha-1)p}{p}} = \frac{\beta(\alpha)}{1-\alpha}, \quad 0<\alpha<1, \tag{2.2}$$

where $\lim_{\alpha\to 1-0}\beta(\alpha)=0$.

In the following, consumers involved in and represented by the demand quantity q_t are called type $\mathcal{A}$, while those expressed by $\beta(\alpha)q_t$ are referred to type $\mathcal{B}$. Moreover, we concentrate upon the case where values of α and $\beta(\alpha)$ are both specified to specific values, and therefore $\beta(\alpha)$ is written as β for simplicity.

3. Consumers' Behavior

3.1 Best response

Since the ideal time distribution by Eq. (2.1) reveals a symmetrical shape, the opening time,

t_o, and the closing time, t_c, are also symmetrical with respect to $t = \frac{1}{2}$, accordingly we have

$$t_c = 1 - t_o .$$

Hence, we focus on the former half $\left[0,\ \frac{1}{2}\right)$ of period to discuss the opening time, t_o, hereafter.

(1) Type $\mathcal{A}$ consumers' response
When the provider offers early birds specials and/or closing time discount/sale, the best response of type $\mathcal{A}$ consumers with ideal time t becomes as follows:

i) If $t \in \left[0,\ t_o^{(1a)}\right]$, type $\mathcal{A}$ consumers are reluctant to wait until t_o, and purchase no service product, where

$$t_o^{(1a)} = t_o - \frac{u_0 - \alpha p}{\omega}. \tag{3.1}$$

Consequently, their net utility, U_t, becomes

$$U_t = 0 .$$

ii) If $t \in \left(t_o^{(1a)},\ t_o\right]$, type $\mathcal{A}$ consumers purchase a service product at the discounted price αp by waiting until t_o, and hence their net utility is given by

$$U_t = u_0 - \alpha p - \omega(t_o - t) .$$

iii) Type $\mathcal{A}$ consumers with their ideal time $t \in \left(t_o,\ t_o + \tau\right]$ purchase a service product at their own ideal time t, at the discounted price αp. In this case, their net utility becomes

$$U_t = u_0 - \alpha p .$$

iv) In the case of $t \in \left(t_o + \tau,\ t_o^{(2)}\right]$, the consumers purchase a service product earlier than their own ideal time t, at the special price αp, yielding

$$U_t = u_0 - \alpha p - \omega\left[t - (t_o + \tau)\right],$$

where

$$t_o^{(2)} = t_o + \tau + \frac{(1-\alpha)p}{\omega}. \tag{3.2}$$

It should be noted in Eq. (3.2) that $t_o^{(2)} \neq t_o + \tau + \frac{u_0 - \alpha p}{\omega}$ since consumers with their own ideal time t, can obtain positive utility, $u_0 - p$, even at t, and $t_o^{(2)}$ should be derived from the condition in reference to t;

$$u_0 - \alpha p - \omega\left[t - \left(t_o + \tau\right)\right] \geq u_0 - p.$$

v) When $t \in \left(t_o^{(2)},\ \frac{1}{2}\right]$, type $\mathcal{A}$ consumers will purchase a service product at the regular price, p, at their ideal time t, and hence

$$U_t = u_0 - p.$$

(2) Type $\mathcal{B}$ consumers' response.

The best response of type $\mathcal{B}$ consumers with their ideal time t is described as follows:

i) If $t \in \left[0,\ t_o^{(1b)}\right]$, type $\mathcal{B}$ consumers would not wait until t_o because they purchase no service, where

$$t_o^{(1b)} = t_o - \frac{(1-\alpha)p}{\omega}. \tag{3.3}$$

Consequently, their net utility becomes

$$U_t = 0.$$

ii) If $t \in \left(t_o^{(1b)},\ t_o\right]$, type $\mathcal{B}$ consumers purchases a service product at αp, by shifting their actual service time from their own ideal time to t_o. In this case, the maximum value of their net utility can be represented by

$$U_t = (1-\alpha)p - \omega(t_o - t).$$

iii) Consumers with $t \in \left(t_o,\ t_o + \tau\right]$ purchase a service product at the discounted price αp, at their own ideal service time, and hence their maximum net utility can be expressed as

$$U_t = (1-\alpha)p.$$

iv) In the case of $t \in \left(t_o + \tau,\ t_o^{(2)}\right]$, type $\mathcal{B}$ consumers purchase a product earlier than their ideal time at αp, and their maximum net utility becomes

$$U_t = (1-\alpha)p - \omega\left[t - (t_o + \tau)\right].$$

v) $t \in \left(t_o^{(2)},\ \frac{1}{2} \right]$, type $\mathcal{B}$ consumers would purchase no service product yielding

$$U_t = 0.$$

3.2 Domain of opening time

It is neither reasonable nor proper for a consumer with ideal time $t < 0$ to shift her actual service time to t_o, and thereby we assume

$$\min\left(t_o^{(1a)},\ t_o^{(1b)}\right) = t_o^{(1a)} = t_o - \frac{u_0 - \alpha p}{\omega} \geq 0,$$

which constrains the opening time to satisfy

$$t_o \geq \frac{u_0 - (1-\alpha)p}{\omega}. \tag{3.4}$$

The right-hand-side of Eq. (3.4) is denoted by t_L in the following.

Likewise, it is reasonable to assume

$$t_o^{(2)} \leq \frac{1}{2},$$

which is equivalent to

$$t_o \leq \frac{1}{2} - \tau - \frac{(1-\alpha)p}{\omega}. \qquad (3.5).$$

The right-hand-side of Eq. (3.5) is denoted by t_U hereafter.

It should be noted here that Eqs. (3.4) and (3.5) yield,

$$\frac{u_0 - \alpha p}{\omega} \leq \frac{1}{2} - \tau - \frac{(1-\alpha)p}{\omega},$$

which signifies, at the same time, that ω should satisfy

$$\omega \geq \frac{2[u_0 + (1-2\alpha)p]}{1-2\tau}. \tag{3.6}$$

From Eqs. (3.4) and (3.5), the domain of t_o is, as a result, given by

$$t_L \equiv \frac{u_0 - \alpha p}{\omega} \leq t_o \leq \frac{1}{2} - \tau - \frac{(1-\alpha)p}{\omega} \equiv t_U. \qquad (3.\ 7)$$

4. Social Welfare
4.1 Provider's profit

Let $Q_{1A}(t_o)$ express the number of Type $\mathcal{A}$ consumers who purchase a service product at the discounted price αp, then we have

$$Q_{1A}(t_o) = 2\int_{t_o^{(1a)}}^{t_o^{(2)}} q_t dt$$
$$= 2n\left[\tau + \frac{u_0 + (1-2\alpha)p}{\omega}\right] \times \left[\mu + 2(1-\mu)\left(2t_o + \tau - \frac{u_0 - p}{\omega}\right)\right] \qquad (4.1)$$

By letting $Q_{1B}(t_o)$ signfy the number of type $\mathcal{B}$ consumers who purchase a service product at αp, we have

$$Q_{1B}(t_o) = 2\int_{t_o^{(1b)}}^{t_o^{(2)}} \beta q_t dt$$
$$= 2n\beta\mu\left[\tau + \frac{2(1-\alpha)p}{\omega}\right] + 4n\beta(1-\mu)\left[\tau + \frac{2(1-\alpha)p}{\omega}\right](2t_o + \tau). \qquad (4.2)$$

On the other hand, let us denote, by $Q_2(t_o)$, the number of consumers who purchase a service product at the regular price p, then we have

$$Q_2(t_o) = 2\int_{t_o^{(2)}}^{\frac{1}{2}} q_t dt$$
$$= 2n\left(\frac{1}{2} - t_o - \tau - \frac{(1-\alpha)p}{\omega}\right) \times \left[\mu + 2(1-\mu)\left(\frac{1}{2} + t_o + \tau + \frac{(1-\alpha)p}{\omega}\right)\right]. \qquad (4.3)$$

Hence, the provider's profit is given by

$$\Pi(t_o) = (\alpha p - c_1)\left[Q_{1A}(t_o) + Q_{1B}(t_o)\right] + (p - c_1)Q_2(t_o) - c_2(1 - 2t_o). \qquad (4.4)$$

We here introduce the following additional constraints so that the provider's profit can take on a positive value at its demand peak and a negative value at its demand off-peak;

$$n(2-\mu)(p - c_1) > c_2, \qquad (4.5)$$

$$n\mu(p - c_1) < c_2. \qquad (4.6)$$

Further, we also assume

$$c_1 \le \alpha p,$$

not to lose profit by the special offer. This provides a lower bound for α and consequently the domain of α is given by

$$\frac{c_1}{p} < \alpha < 1. \tag{4.7}$$

4.2 Consumers' surplus

Let us denote by $C_i(t_o)$ the total surplus of Type i (i =A, B) consumers, then we have

$$C_A(t_o) = (u_0 - \alpha p) Q_{1A}(t_o) + (u_0 - p) Q_2(t_o) - 2\omega \int_{t_o^{(1a)}}^{t_o} (t_o - t) q_t dt$$
$$- 2\omega \int_{t_o+\tau}^{t_o^{(2)}} [t - (t_o + \tau)] q_t dt, \tag{4.8}$$
$$C_B(t_o) = (1-\alpha) p Q_{1B}(t_o) - 2\omega \int_{t_o^{(1b)}}^{t_o} (t_o - t) \beta q_t dt - 2\omega \int_{t_o+\tau}^{t_o^{(2)}} [t - (t_o + \tau)] \beta q_t dt. \tag{4.9}$$

Hence, the whole consumers' surplus, which is denoted by $C(t_o)$, is given by

$$C(t_o) = C_A(t_o) + C_B(t_o). \tag{4.10}$$

4.3 Social welfare

The social welfare is defined by the sum of the total consumers' surplus and the provider's profit, i.e., let $\Psi(t_o)$ denote the social welfare, then we have

$$\begin{aligned}\Psi(t_o) &= \Pi(t_o) + C(t_o) \\ &= (u_0 - c_1)\left[Q_{1A}(t_o) + Q_2(t_o)\right] + (p - c_1) Q_{1B}(t_o) \\ &\quad - \left[D_1(t_o) + D_2(t_o)\right] - c_2(1 - 2t_o),\end{aligned} \tag{4.11}$$

where

$$D_1(t_o) = 2\omega \left[\int_{t_o^{(1a)}}^{t_o} (t_o - t) q_t dt + \beta \int_{t_o^{(1b)}}^{t_o} (t_o - t) q_t dt \right],$$
$$D_2(t_o) = 2(1+\beta)\omega \int_{t_o+\tau}^{t_o^{(2)}} \left[t - (t_o + \tau)\right] q_t dt.$$

5. Optimal Strategy

This section seeks for the socially optimal opening time t_o^*, which can provide an optimal closing time t_c^* by the symmetric structure of the ideal time distribution. Numerical examples are also presented to illustrate the proposed model formulation.

5.1 Analysis

From Eq. (4.11), we have

$$\frac{d\Psi(t_o)}{dt_o} = (u_0 - c_1)\left(\frac{dQ_{1A}(t_o)}{dt_o} + \frac{dQ_2(t_o)}{dt_o}\right) + (p - c_1)\frac{dQ_{1B}(t_o)}{dt_o} - \left[\frac{dD_1(t_o)}{dt_o} + \frac{dD_2(t_o)}{dt_o}\right] + 2c_2. \tag{5.1}$$

Let us denote, by $\varphi(t_o)$, the right-hand-side of Eq. (5.1), and let a and b be defined by

$$a \equiv \frac{u_0 - \alpha p}{\omega},$$
$$b \equiv \frac{(1-\alpha)p}{\omega}.$$

Then, we have

$$\varphi(t_o) = (u_0 - c_1)\left[8n(1-\mu)(a - t_o) - 2n\mu\right] + (p - c_1)8n\beta(1-\mu)(\tau + 2b) - 2n\omega\left[2(1-\mu)(a^2 + b^2) + 4\beta(1-\mu)b^2\right] + 2c_2, \tag{5.2}$$

which indicates $\varphi(t_o)$ is strictly decreasing in t_o.

In addition, we have

$$\varphi\left(\frac{u_0 - \alpha p}{\omega}\right) = -2n\mu(u_0 - c_1) + (p - c_1)8n\beta(1-\mu)(\tau + 2b) - 2n\omega\left[2(1-\mu)(a^2 + b^2) + 4\beta(1-\mu)b^2\right] + 2c_2, \tag{5.3}$$

$$\varphi\left(\frac{1}{2} - \tau - \frac{(1-\alpha)p}{\omega}\right) = 2n(u_0 - c_1)\left[2(1-\mu)(2\tau + 2b + 2a - 1) - \mu\right] + (p - c_1)8n\beta(1-\mu)(\tau + 2b) - 2n\omega\left[2(1-\mu)(a^2 + b^2) + 4\beta(1-\mu)b^2\right] + 2c_2. \tag{5.4}$$

Now, let A and B be defined by

$$A \equiv -n\mu(u_0 - c_1) + (p - c_1)4n\beta(1-\mu)(\tau + 2b) - n\omega\left[2(1-\mu)(a^2 + b^2) + 4\beta(1-\mu)b^2\right] + c_2, \tag{5.5}$$

$$B \equiv n(u_0 - c_1)\left[2(1-\mu)(2\tau + 2b + 2a - 1) - \mu\right] + (p - c_1)4n\beta(1-\mu)(\tau + 2b) - n\omega\left[2(1-\mu)(a^2 + b^2) + 4\beta(1-\mu)b^2\right] + c_2, \tag{5.6}$$

and then the optimal opening time, t_o^*, can be discussed under the following classification:

(a) If we have $A > 0$, further classification is necessary.

i) In the case of $B \geq 0$, t_o^* is given by

$$t_o^* = \frac{1}{2} - \tau - \frac{(1-\alpha)p}{\omega} = t_{U.}$$

ii) On the contrary, in case we have $B < 0$, t_o^* is given by

$$t_o^* = a - \frac{\mu}{4(1-\mu)(u_0 - c_1)} + \frac{\beta\left(p - c_1\right)(\tau + 2b)}{u_0 - c_1} - \frac{\omega\left(a^2 + b^2\right)}{2(u_0 - c_1)} - \frac{\beta\omega b^2}{u_0 - c_1} + \frac{c_2}{4n(1-\mu)(u_0 - c_1)}.$$

(b) If we have $A \leq 0$, then $\pi(t_o) \leq 0$ and hence

$$t_o^* = \frac{u_0 - \alpha p}{\omega} = t_{L.}$$

As for the optimal opening time, t_o^*, we have the following proposition:

Proposition 1 *For the ideal time distribution with* $\mu = 1$, *if* $u_0 - c_1 \geq \frac{c_2}{n}$, *the opening time becomes*

$$t_o^* = \frac{u_0 - \alpha p}{\omega} = t_L,$$

otherwise we have

$$t_o^* = \frac{1}{2} - \tau - \frac{(1-\alpha)p}{\omega} = t_U.$$

Proof. In the case of $\mu = 1$, the relationship $u_0 - c_1 \geq \frac{c_2}{n}$ reveals $A \leq 0$ from Eq. (5.5) along with $\varphi(t_o) \leq 0$. On the contrary, $u_0 - c_1 < \frac{c_2}{n}$ agrees with $B > 0$, accordingly we have $\varphi(t_o) > 0$.

5.2 Numerical examples

This subsection presents numerical examples to illustrate the proposed model. Table 1 shows the optimal opening time, t_o^*, and its corresponding welfare, $\Psi(t_o^*)$, together with t_L and t_U against various values of μ and α when we set the para-meters involved in the model as $(n,\ \tau, u_0,\ p,\ \omega,\ c_1, c_2, \beta) =$ (1, 0.05, 10, 9.49, 80, 4.99, 2.29, 0.35). It is observed in Table 1

that the optimal opening time, t_o^*, satisfies $t_L < t_o^* < t_U$ in the case of $\mu = 0.25$. In the oth er
cases, we have $t_o^* = t_{L.}$ Table 1 indicates that as the distribution becomes closer to uniform, service hours maximizing the social welfare would increase towards "open 24 hours".

Table 1: Optimal strategies.

μ	0.25			0.5			0.75		
α	0.75	0.8	0.85	0.75	0.8	0.85	0.75	0.8	0.85
t_L	0.036	0.0301	0.0242	0.036	0.0301	0.0242	0.036	0.0301	0.0242
t_U	0.4203	0.4263	0.4322	0.4203	0.4263	0.4322	0.4203	0.4263	0.4322
t_o^*	0.1171	0.1149	0.1111	0.036	0.0301	0.0242	0.036	0.0301	0.0242
$\Psi(t_o^*)$	3.042	3.0358	3.0188	2.9641	2.957	2.9403	2.985	2.9864	2.9754

Figure 2 shows the shape of the social welfare, $\Psi(t_o)$, for $\mu = 0.25, 0.50$ and 0.75 against $\alpha = 0.8$ with the other parameter values set to the same values in Table 1. It is also observed in Fig.1 that $\Psi(t_o)$ has apparently its maximum when $\mu = 0.25$, while it is decreasing in t_o against $\mu = 0.50,\ 0.75$ taking its maximum at $t_o^* = 0.0301 = t_L$.

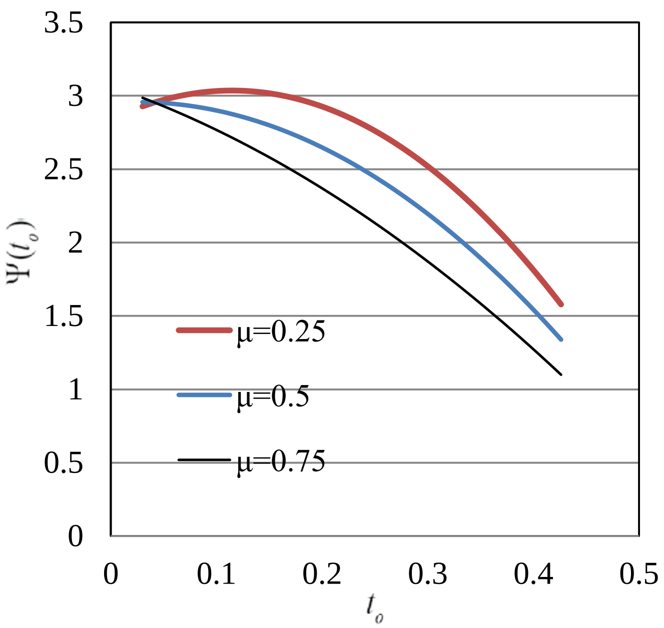

Figure 2: Behavior of social welfare.

6. Concluding Remarks

In this paper, we discussed an optimal number of business hours for a service provider, where he offers a special discounted price immediately after opening the store and just before closing it. Under a specific ideal service time distribution of consumers, derived was the social welfare which is an objective function to be maximized. We clarified the conditions under which there exists an optimal opening time along with an optimal closing one. It was also shown that the optimal opening time decreases with increasing uniformity of the ideal time distribution. Numerical examples were also presented to illustrate the theoretical underpinnings of the proposed mathematical model.

The authors are to make a comparison between the results of this study and those in Kim and Sandoh(2015), which will appear in the forthcoming paper.

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