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Michi NISHIHARA[†‡] Takashi SHIBATA[§]

Abstract

This paper considers a dynamic model in which shareholders of a firm in distress have a choice of whether to proceed to debt restructuring or direct liquidation at an arbitrary time. In the model, we show the following results. Fewer asset sales, lower financing, debt renegotiation, and running costs, a lower premium to the debt holders, a lower cash flow volatility, and a higher initial coupon increase the shareholders' incentive to choose debt restructuring to avoid full liquidation. In the debt renegotiation process, the shareholders arrange the coupon reduction and use equity financing to retire a part of the debt value to the debt holders. The timing of debt restructuring always coincides with that of liquidation without debt renegotiation. Most notably, the shareholders do not prefer asset sale in debt restructuring even if they face high financing costs. The possibility of debt renegotiation in the future increases the initial leverage ratio in the optimal capital structure.

JEL Classifications Code: C73; G31; G33.

Keywords: real options; asset sale; debt renegotiation; liquidation; capital structure.

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[†]Corresponding Author. Graduate School of Economics, Osaka University, 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, Japan, E-mail: nishihara@econ.osaka-u.ac.jp, Phone: 81-6-6850-5242, Fax: 81-6-6850-5277

[‡]Swiss Finance Institute, École polytechnique fédérale de Lausanne, 1015 Lausanne, Switzerland

[§]Graduate School of Social Sciences, Tokyo Metropolitan University, 1-1 Minami-Osawa, Hachioji, Tokyo 192-0397, Japan, E-mail: tshibata@tmu.ac.jp, Phone: 81-42-677-2310, Fax: 81-42-677-2298

1 Introduction

There has been a growing trend to investigate dynamic investment and financing models in corporate finance. One of the advantages of analyzing dynamic models over static models is that we can deal with optimal timing problems more clearly. For instance, Mauer and Sarkar (2005) and Sundaresan and Wang (2007) derived the optimal investment timing with the optimal capital structure.[1] Compared to an increasing number of papers regarding dynamic investment and financing problems, there are not so many papers that analyze dynamic models of divestment and deleveraging. Although a number of the papers, including seminal works by Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997) and Fan and Sundaresan (2000), focus on liquidation and debt renegotiation problems of a firm in financial distress, they usually ignore problems related to downsizing and continuing to operate a firm.[2] In the real world, it is not unusual to downsize and/or deleverage a firm's operations during an economic downturn. For example, in the Japanese electronics industry, Panasonic, Sony, and Sharp corporations have experienced large-scale downsizing in recent years.

We reveal interactions of downsizing, debt restructuring, and liquidation in the following dynamic model. The equity holders of a levered firm in distress face a problem of choosing either debt restructuring or direct liquidation. They can also choose the optimal timing. If they choose debt restructuring, they can reduce the coupon payments but are forced to sell a fraction of assets, where following the standard assumption (e.g., Mella-Barral (1999) and Gryglewicz (2009)), we assume that partial liquidation is less efficient than full liquidation in terms of asset price (economies of scale). We do not focus on the debt holders' strategic behaviour and suppose that the debt holders accept the shareholders' proposal with a sufficient premium compared to the value without debt renegotiation.[3] Naturally, the equity holders need to pay back more to the debt holders when they wish to reduce more coupon payments and continue the firm's operation longer. The equity holders also pay debt renegotiation costs, such as transaction costs, as well as equity financing costs that arise when the proceeds from selling assets cannot cover all costs associated with debt renegotiation. Considering the trade-off, the equity holders optimize the coupon reduction and its timing. The equity holders can choose direct liquidation when it is more beneficial than debt restructuring.

Our analysis of the model yields several results about when, how, and whether the firm proceeds to debt restructuring or direct liquidation. First, we show that the timing of debt restructuring is always equal to that of liquidation without debt renegotiation if it occurs.

[1] Recently, Shibata and Nishihara (2012), Shibata and Nishihara (2015a), and Shibata and Nishihara (2015b) extended their analysis to the cases involving debt financing constraints.

[2] A notable exception is Reindl (2013), who clarified conditions under which a firm decreases its leverage ratio along with selling assets by analyzing a dynamic game between equity and debt holders.

[3] This paper also explores the impact of the debt holders' bargaining power by varying the premium.

This is because of the timing when the shareholders can most efficiently reduce the value which they retire to the debt holders. This result is in line with Lambrecht (2001) and Moraux and Silaghi (2014), but they do not consider asset sale and payback to the debt holders in exchange for the coupon reduction. We show that, unlike the previous results, the shareholders greatly decrease the coupon so that they need to pay back the partial debt value to the debt holders. The shareholders use costly equity financing in addition to the proceeds from selling assets to repay the partial debt value in debt restructuring. This is because operating the firm longer through deleveraging is more beneficial to the equity holders even though they pay temporarily higher costs in debt restructuring.

A most notable result is that the shareholders do not prefer asset sale in debt restructuring. In other words, if asset sale is not forced, they adjust the coupon reduction and equity financing to avoid asset sale. This result is different from the following intuition: the equity holders may wish to sell more assets to cover the costs of debt renegotiation even if asset sale is less advantageous from the viewpoint of asset price. It is not optimal for the shareholders to sell assets to finance the repayment value even when equity financing is very costly. Instead of avoiding asset sale, they mitigate the coupon reduction and decrease the repayment value to the debt holders.

Our result stems not only from the assumption of economies of scale. A more important motivation is the timing of debt restructuring. Because the debt restructuring time is earlier than the final liquidation time, more losses are associated with asset sale at the debt restructuring time. Indeed, the shareholders can reduce the loss by deferring asset sale as long as possible, i.e., until the final liquidation time. This result is consistent with the following empirical evidence. Maksimovic and Phillips (1998) and Maksimovic and Phillips (2001) showed that the timing of asset sale is not related to debt renegotiation but that it is motivated by improvement of the resource allocation. Because asset sale does not improve the resource allocation in our model, the shareholders' unwillingness to sell assets is in line with their result. Arnold, Hackbarth, and Puhan (2013) also showed that asset sales and investments are significantly and positively correlated. Relatedly, Weiss and Wruck (1998) illustrated a real-world example of inefficient asset sales during the debt restructuring process.

Another notable result is the impact of the initial coupon of debt on the shareholders' choice of whether to proceed to debt restructuring or direct liquidation. As the initial coupon is higher, the shareholders are more likely to proceed to debt renegotiation to avoid direct liquidation. This is because for a higher initial coupon the equity holders can possibly reduce more coupon payments via earlier debt renegotiation. Then, they can greatly extend the firm's survival time, which leads to more surplus from debt renegotiation. This result predicts that larger/older firms, which tend to have more debt, are more likely to avoid direct liquidation. This prediction is consistent with the empirical evidence in Bris, Welch, and Zhu (2006). Although we do not consider multiple debt renegotiations, our finding is also consistent with that of Moraux and Silaghi (2014), who showed that due to renegotiation costs equity holders give up any further renegotiation after the coupon is reduced to a sufficiently low level.

We also show that, in addition to fewer asset sales and a higher initial coupon, lower financing, debt renegotiation, and running costs, as well as a lower premium to the debt holders and a lower volatility, increase the shareholders' incentive to proceed to debt restructuring. These results are consistent with the stylized fact that larger/older/higher-productivity firms are more likely to avoid direct liquidation (e.g., Bris, Welch, and Zhu (2006), Maksimovic and Phillips (1998)).

In addition, we examine how the optimal capital structure, where the initial coupon of debt is chosen to maximize the firm value, differs in cases with and without debt renegotiation in the future. In the case of taking account of debt renegotiation in the future, the firm takes a higher coupon, leverage, and credit spread at the initial time. This is because, by increasing the coupon, leverage, and credit spread at the initial time, the firm can gain more benefits of debt renegotiation. Similar results are also documented in Moraux and Silaghi (2014) and Christensen, Flor, Lando, and Miltersen (2014).

The remainder of this paper is organized as follows. As a benchmark, Section 2 examines the direct and partial liquidation options of an unlevered firm and shows that the unlevered firm always prefers direct liquidation to downsizing under the assumption of scales of economies. In Section 3.1, we examine the liquidation option of the levered firm. In Section 3.2, we explore the downsizing and debt restructuring option of the levered firm. In Section 4, we present a wide range of numerical examples and explain key results. Section 5 concludes the paper.

2 Unlevered firm

2.1 Direct liquidation

Throughout this paper, we consider a risk-neutral firm that is receiving EBIT (Earnings before interests and taxes) $X(t) - w$ at time t, where $X(t)$ is a stochastic component and $w(\geq 0)$ is a constant running cost. Following the standard real options literature, we assume that $X(t)$ follows a geometric Brownian motion:

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}B(t) \ (t > 0), \quad X(0) = x,$$

where $B(t)$ denotes the standard Brownian motion defined in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and $\mu, \sigma(> 0)$ and $x(> 0)$ are constants. Positive constants r and $\tau \in (0, 1)$ denote the interest rate and the corporate tax rate, respectively. For convergence, we assume that $r > \mu$. For the economic rationale behind these standard assumptions, refer to Dixit and Pindyck (1994).

Consider an all-equity firm that has an option to sell whole assets (denoted by "direct liquidation") and gain $P_U(X(T_U), 1)$ after taxes at an arbitrary time T_U . The subscript

U stands for the unlevered case. We assume that by selling a fraction $a \in (0,1)$ of assets (denoted by "partial liquidation" or "asset sale") at time t, the shareholders receive the proceeds $P_U(X(t),a)$ after taxes. Assume that $P_U(x,a) = F_U(a)x + G_U(a)$, where the functions $F_U(\cdot)$ and $G_U(\cdot)$ are non-decreasing and convex functions with $F_U(0) = G_U(0) = 0$.[4] The convexity means that partial liquidation destroys existing economies of scale, and hence selling assets sequentially is less profitable than selling assets simultaneously. Following most papers, including Mella-Barral (1999) and Gryglewicz (2009), we assume the economies of scale. Also, note that in most cases, assets sold piecemeal are not as valuable as the same assets sold as a going concern. By this assumption, we can also differentiate our study from Reindl (2013), who do not directly relate a partial liquidation value with a full liquidation value. We presume that the initial value x is sufficiently high to exclude (partial) liquidation at the initial time.

Now, consider the problem of the equity holders who optimize the direct liquidation time T_U. The equity value, denoted by $E_U(x)$, which is the same as the firm value in the unlevered case, is equal to

$$E_U(x) = \sup_{T_U} \mathbb{E}[\int_0^{T_U} \mathrm{e}^{-rt}(1-\tau)(X(t)-w)\mathrm{d}t + \mathrm{e}^{-rT_U} P_U(X(T_U),1)], \tag{1}$$

where the liquidation time T_U is optimized over all stopping times. In the standard manner (e.g., Dixit and Pindyck (1994)), we can explicitly solve problem (1) as follows. For $(1-\tau)/(r-\mu) - F_U(1) > 0$, we have

$$\begin{aligned} E_U(x) &= \frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)w}{r} + \sup_{T_U} \mathbb{E}[\mathrm{e}^{-rT_U}\left(-\left(\frac{1-\tau}{r-\mu} - F_U(1)\right)X(T_U) + \frac{(1-\tau)w}{r} + G_U(1)\right)] \\ &= \frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)w}{r} + \left(-\left(\frac{1-\tau}{r-\mu} - F_U(1)\right)x_U^* + \frac{(1-\tau)w}{r} + G_U(1)\right)\left(\frac{x}{x_U^*}\right)^{\gamma} \end{aligned} \tag{2}$$

for $x \geq x_U^*$, where $\gamma = 1/2 - \mu/\sigma^2 - \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2} (< 0)$ and the liquidation trigger x_U^* is given by

$$x_U^* = \frac{\gamma}{\gamma - 1}\left(\frac{1-\tau}{r-\mu} - F_U(1)\right)^{-1}\left(\frac{(1-\tau)w}{r} + G_U(1)\right). \tag{3}$$

The optimal liquidation time is expressed as $T_U^* = \{t \geq 0 \mid X(t) \leq x_U^*\}$. Throughout the paper, following the standard literature (e.g., Leland (1994) and Mella-Barral (1999)), we presume that $(1-\tau)/(r-\mu) - F_U(1) > 0$. Note that only when w and $G_U(1)$ are equal to zero, the equity holders perpetually operate the firm without liquidation. In that case, we have $x_U^* = 0$ and $E_U(x) = (1-\tau)x/(r-\mu)$, which correspond to the unlevered case in Goldstein, Ju, and Leland (2001). In the next subsection, we will derive the equity value in the case of partial liquidation and compare it with (2) to clarify whether the shareholders prefer direct liquidation or asset sale.

[4]Following the standard literature, such as Leland (1994), Mella-Barral and Perraudin (1997), Mella-Barral (1999), and Goldstein, Ju, and Leland (2001), we assume that the liquidation value is linear with respect to the state variable.

2.2 Partial liquidation

We consider a problem of whether the firm should sell its assets at once or piecemeal. In order to focus the problem, in the piecemeal case, we limit our attention to a case in which the firm sell assets only twice. We assume that the shareholders first sell a fraction $a \in (0, 1)$ of assets and after that liquidate the firm by selling the remaining fraction $1-a$. We now suppose that a is a given constant.

Consider an all-equity firm that has an option to sell assets and gain $P_U(X(T_{U1}), a)$ at an arbitrary time T_{U1}. Assume that the firm's downsizing reduces EBIT from $X(t) - w$ to $(1-a)(X(t) - w)$. The equity value, denoted by $E_{U1}(x)$, is equal to

$$E_{U1}(x) = \sup_{T_{U1}} \mathbb{E}[\int_0^{T_{U1}} \mathrm{e}^{-rt}(1-\tau)(X(t)-w)\mathrm{d}t + \mathrm{e}^{-rT_{U1}} P_U(X(T_{U1}), a)) + \mathrm{e}^{-rT_{U1}} E_{U2}(X(T_{U1}))], \tag{4}$$

where the downsizing time T_{U1} is optimized over all stopping time, and the equity value after downsizing, denoted by $E_{U2}(X(T_{U1}))$, is given by

$$\begin{aligned} E_{U2}(X(T_{U1})) = \sup_{T_{U2}} \mathbb{E}^{X(T_{U1})}[\int_{T_{U1}}^{T_{U2}} & \mathrm{e}^{-r(t-T_{U1})}(1-\tau)(1-a)(X(t)-w)\mathrm{d}t \\ & + \mathrm{e}^{-r(T_{U2}-T_{U1})} P_U(X(T_{U1}), 1-a))], \end{aligned} \tag{5}$$

where the liquidation time T_{U2} is optimized over all stopping time later than T_{U1}. The notation $\mathbb{E}^{X(T_{U1})}[\cdot]$ denotes the expectation conditional to $t = T_{U1}$ and $X(t) = X(T_{U1})$. As in (2), we can explicitly solve problem (5) as follows:

$$\begin{aligned} E_{U2}(X(T_{U1})) &= \frac{(1-\tau)(1-a)X(T_{U1})}{r-\mu} - \frac{(1-\tau)(1-a)w}{r} \\ &+ \left(- \left(\frac{(1-\tau)(1-a)}{r-\mu} - F_U(1-a) \right) x^*_{U2} + \frac{(1-\tau)(1-a)w}{r} + G_U(1-a) \right) \left(\frac{X(T_{U1})}{x^*_{U2}} \right)^\gamma \end{aligned}$$

for $X(T_{U1}) \geq x^*_{U2}$, where the optimal liquidation trigger x^*_{U2} is

$$x^*_{U2} = \frac{\gamma}{\gamma-1} \left(\frac{1-\tau}{r-\mu} - \frac{F_U(1-a)}{1-a} \right)^{-1} \left(\frac{(1-\tau)w}{r} + \frac{G_U(1-a)}{1-a} \right). \tag{6}$$

Note that $E_{U2}(X(T_{U1})) = F_U(1-a)X(T_{U1}) + G_U(1-a)$ for $X(T_{U1}) < x^*_{U2}$.

Then, we can explicitly solve problem (4) as follows:

$$\begin{aligned} E_{U1}(x) = \frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)w}{r} &+ \left\{ - \left(\frac{(1-\tau)a}{r-\mu} - F_U(a) \right) x^*_{U1} + \frac{(1-\tau)aw}{r} + G_U(a) \right\} \left(\frac{x}{x^*_{U1}} \right)^\gamma \\ + \left\{ - \left(\frac{(1-\tau)(1-a)}{r-\mu} - F_U(1-a) \right) x^*_{U2} \right. &+ \left. \frac{(1-\tau)(1-a)w}{r} + G_U(1-a) \right\} \left(\frac{x}{x^*_{U2}} \right)^\gamma \end{aligned} \tag{7}$$

for $x \geq x^*_{U1}$, where the downsizing trigger x^*_{U1} is

$$x^*_{U1} = \frac{\gamma}{\gamma-1} \left(\frac{1-\tau}{r-\mu} - \frac{F_U(a)}{a} \right)^{-1} \left(\frac{(1-\tau)w}{r} + \frac{G_U(a)}{a} \right) \tag{8}$$

when $x^*_{U1} > x^*_{U2}$ is satisfied. Otherwise,

$$E_{U1}(x) = \frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)w}{r} + \Bigg\{ - \left(\frac{(1-\tau)}{r-\mu} - F_U(a) - F_U(1-a) \right) x^{**}_{U1} + \frac{(1-\tau)w}{r} + G_U(a) + G_U(1-a) \Bigg\} \left(\frac{x}{x^{**}_{U1}} \right)^{\gamma} \quad (9)$$

for $x \geq x^{**}_{U1}$, where liquidation occurs immediately after asset sale and the trigger x^{**}_{U1} is defined by

$$x^{**}_{U1} = \frac{\gamma}{\gamma - 1} \left(\frac{1-\tau}{r-\mu} - F_U(a) - F_U(1-a) \right)^{-1} \left(\frac{(1-\tau)w}{r} + G_U(a) + G_U(1-a) \right).$$

In this "piecemeal" case, we assume that the shareholders receive $P_U(X(T_{U1}), a) + P_U(X(T_{U1}), 1 - a)$ at the trigger x^{**}_{U1}.

We can easily prove that $E_{U1}(x) \leq E_U(x)$ for an arbitrary a as follows. We consider (4) replaced $P_U(X(T_{U1}), a)$ and $P_U(X(T_{U1}), 1-a)$ with $aP_U(X(T_{U1}), 1)$ and $(1-a)P_U(X(T_{U1}), 1)$, respectively. We denote this value function by $\tilde{E}_{U1}(x)$. Because of the convexity of $F_U(\cdot)$ and $G_U(\cdot)$, we have $P_U(X(T_{U1}), a) \leq aP_U(X(T_{U1}), 1)$ and $P_U(X(T_{U1}), 1 - a) \leq (1-a)P_U(X(T_{U1}), 1)$. Then, we have $E_{U1}(x) \leq \tilde{E}_{U1}(x)$. Because we can calculate $\tilde{E}_{U1}(x)$ following (9) and check that $\tilde{E}_{U1}(x) = E_U(x)$. Thus, we have $E_{U1}(x) \leq \tilde{E}_{U1}(x) = E_U(x)$.

This result means that the all-equity firm always prefers direct liquidation to partial liquidation. As we can see from the proof above that the reason lies in the price advantage of selling whole assets over piecemeal assets (economies of scale). In the absence of economies of scale (e.g., Reindl (2013)), a firm may prefer partial liquidation. One example is a case in which fire sales are accompanied only by final liquidation. In the following sections, we will explore the optimal decisions of the equity holders of the levered firm. It is a key question to be answered whether the levered firm always chooses direct liquidation under the assumption of economies of scale.

3 Levered firm

3.1 Direct liquidation

In this subsection, we derive the equity, debt, and firm values of the firm that chooses direct liquidation. Consider the firm that issued console debt with coupon c and is operating with the asset size a. We denote the equity, debt, and firm values of the firm by $E(x, a, c), D(x, a, c)$, and $V(x, a, c)$, respectively. As in Section 2, we consider the asset price function $P_L(x, a) = F_L(a)x + G_L(a)$ for asset sale, where the functions $F_L(\cdot)$ and $G_L(\cdot)$ are non-decreasing and convex functions with $F_L(0) = G_L(0) = 0$. In the levered case, the price function $P_L(x, a)$ can be different from $P_U(x, a)$, although we do not specify the relation.

Following the standard manner (e.g., Leland (1994) and Goldstein, Ju, and Leland (2001)), we have

$$
\begin{aligned}
E(x,a,c) =& \mathbb{E}[\int_0^{T_L^*} \mathrm{e}^{-rt}(1-\tau)(aX(t)-aw-c)\mathrm{d}t] \\
=& \frac{(1-\tau)ax}{r-\mu} - \frac{(1-\tau)(aw+c)}{r} - \left(\frac{(1-\tau)ax_L^*(a,c)}{r-\mu} - \frac{(1-\tau)(aw+c)}{r}\right)\left(\frac{x}{x_L^*(a,c)}\right)^{\gamma}
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
D(x,a,c) =& \mathbb{E}[\int_0^{T_L^*} \mathrm{e}^{-rt}c\mathrm{d}t + \mathrm{e}^{-rT_L^*}P_L(X(T_L^*),a)] \\
=& \frac{c}{r} - \left(\frac{c}{r} - P_L(x_L^*(a,c),a)\right)\left(\frac{x}{x_L^*(a,c)}\right)^{\gamma}
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
V(x,a,c) =& E(x,a,c) + D(x,a,c) \\
=& \frac{(1-\tau)ax}{r-\mu} - \frac{(1-\tau)aw}{r} + \frac{\tau c}{r} \\
& - \left(\frac{(1-\tau)ax_L^*(a,c)}{r-\mu} - \frac{(1-\tau)aw}{r} + \frac{\tau c}{r} - P_L(x_L^*(a,c),a)\right)\left(\frac{x}{x_L^*(a,c)}\right)^{\gamma}
\end{aligned} \tag{12}
$$

for $x \geq x_L^*(a,c)$, where $x_L^*(a,c)$ is the liquidation trigger determined by the shareholders who maximize (10). Actually, it is equal to

$$
x_L^*(a,c) = \frac{\gamma(r-\mu)(aw+c)}{(\gamma-1)ra}, \tag{13}
$$

and $T_L^* = \inf\{t \geq 0 \mid X(t) \leq x_L^*(a,c)\}$ (we omit arguments (a,c) of T_L^*). The equations above presume that debt is risky, i.e., $P_L(x_L^*(a,c),a) < c/r$ because we are interested only in risky debt. This condition is satisfied in all numerical examples in Section 4.

When the firm with an initial coupon c_0 chooses direct liquidation without debt renegotiation, its equity, debt, and firm values become $E(x,1,c_0), D(x,1,c_0)$, and $V(x,1,c_0)$, respectively. In this case, we denote by $E(x) = E(x,1,c_0)$ and $x_L^* = x_L^*(1,c)$ to simplify the notations. Comparing $E(x)$ with the equity value, denoted by $E_{L1}(x)$, in the case of debt restructuring, the equity holders decide whether they proceed to direct liquidation or debt restructuring. In the next subsection, we will examine the case of debt restructuring.

3.2 Debt restructuring with asset sale

In this section, we derive the equity, debt, and firm values, denoted by $E_{L1}(x), D_{L1}(x)$, and $V_{L1}(x)$, respectively, of the firm that chooses debt restructuring along with partial liquidation. As in Section 2, we assume that the shareholders first liquidates a fraction $a \in (0,1)$ of assets and gain $P_L(X(T_{L1}),a)$ at time T_{L1}, and after that they liquidate the remaining fraction $1-a$ and gain $P_L(X(T_{L1}),1-a)$ at time T_{L2}. The shareholders optimize the partial liquidation and debt restructuring time T_{L1} and the final liquidation time T_{L2}. We now assume a constant a, and we will also examine an optimal a in Section 4.2.1. In the real world, the debt holders or the third party sometimes force the equity holders to sell assets used as the collateral to repay the debt value in default. As documented in Djankov, Hart, McLiesh, and Shleifer (2008), the forced asset sale is one of the major debt enforcement frictions.

As in Lambrecht (2001), Moraux and Silaghi (2014), and Christensen, Flor, Lando, and Miltersen (2014), we assume that the shareholders demand a lump-sum and permanent reduction in the coupon of debt. Actually, it might be difficult for a firm to continuously and marginally (e.g., Mella-Barral and Perraudin (1997) and Fan and Sundaresan (2000)) adjust the coupon because of costs associated with the negotiation, adjustment, and transaction. For instance, Djankov, Hart, McLiesh, and Shleifer (2008) reported high costs of debt enforcement procedures. At time T_{L1}, the shareholders can reduce the coupon c_0 into a new level c_{L1} by debt renegotiation.[5] At T_{L1}, the equity, debt, and firm values change to $E(X(T_{L1}), 1-a, c_{L1}), D(X(T_{L1}), 1-a, c_{L1})$, and $V(X(T_{L1}), 1-a, c_{L1})$, respectively. The final liquidation time is equal to $T^*_{L2} = \inf\{t \geq 0 \mid X(t) \leq x^*_{L2} = x^*_L(1-a, c_{L1})\}$ under the assumption that $T_{L1} \leq T^*_{L2}$. The debt holders would refuse the coupon reduction proposed by the shareholders unless it is beneficial to them. For simplicity, we do not model a dynamic game between the equity and debt holders. Instead, we assume that the shareholders need to pay back $(1+k_D)\underbrace{D(X(T_{L1}), 1, c_0)}_{\text{original debt value}} - \underbrace{D(X(T_{L1}), 1-a, c_{L1})}_{\text{new debt value}}$ to satisfy the debt holders, where a parameter $k_D(\geq 0)$ measures the debt holders' premium, if it is positive. Although we do not model a bargaining game between equity and debt holders, k_D can be also regarded as proxy for the debt holders' bargaining power in debt renegotiation.[6]

In addition, we assume that external costs $k_R D(X(T_{L1}), 1, c_0)$, such as transaction costs, are accompanied by debt restructuring, where k_R is a non-negative parameter. Note that k_D (k_R) becomes higher (lower) as debt holdings are more concentrated. In total, at time T_{L1}, the shareholders pay the costs $(1+k_D+k_R)D(X(T_{L1}), 1, c_0) - D(X(T_{L1}), 1-a, c_{L1})$ in exchange for the coupon reduction from c_0 to c_{L1}.

Now, we solve the shareholders' problem of optimizing $T_{L1} = \inf\{t \geq 0 \mid X(t) \leq$

[5]This paper focuses on a financially distressed situation, and hence, the model does not allow the shareholders to increase the coupon when EBIT goes up. Christensen, Flor, Lando, and Miltersen (2014) assumed the callable debt to consider the possibility of increasing debt.

[6]In other words, we assume a sort of debt enforcement procedure that protects the creditors from suffering loss by the shareholders' strategic default. Although we take k_D as a given parameter, it should satisfy that $P_L(X(T_{L1}), 1) \leq D(X(T_{L1}), 1, c_0) \leq (1+k_D)D(X(T_{L1}), 1, c_0) \leq c/r$. $(1+k_D)D(X(T_{L1}), 1, c_0)$ is not necessarily equal to c/r (face value) because violations of absolutely priority rule frequently occur in debt restructuring (e.g., Bris, Welch, and Zhu (2006)).

$x_{L1}\}(\le T^*_{L2})$ and $c_{L1}(\le c_0)$ as follows:

$$
\begin{aligned}
E_{L1}(x) = \sup_{T_{L1},c_{L1}} \mathbb{E}[&\underbrace{\int_0^{T_{L1}} \mathrm{e}^{-rt}(1-\tau)(X(t)-w-c_0)\mathrm{d}t}_{\text{value before asset sale}} + \mathrm{e}^{-rT_{L1}}\{\underbrace{P_L(X(T_{L1}),a)}_{\text{asset price}} \\
&- \underbrace{\{(1+k_D+k_R)D(X(T_{L1}),1,c_0) - D(X(T_{L1}),1-a,c_{L1})\}}_{\text{total costs}} + \underbrace{E(X(T_{L1}),1-a,c_{L1})}_{\text{value after asset sale}}\}] \\
&= \frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)(w+c_0)}{r} + \sup_{x_{L1},c_{L1}} \Bigg\{ V(x_{L1},1-a,c_{L1}) + P_L(x_{L1},a) \\
&- (1+k_D+k_R)D(x_{L1},1,c_0) - \frac{(1-\tau)x_{L1}}{r-\mu} + \frac{(1-\tau)(w+c_0)}{r} \Bigg\} \left(\frac{x}{x_{L1}}\right)^{\gamma}, \qquad (14)
\end{aligned}
$$

where we reduce the original problem to the problem (14) of optimizing the debt restructuring trigger, denoted by $x_{L1} \in [x^*_{L2}, x]$, and the coupon $c_{L1} \in [0, c_0]$.

Next, we consider the shareholders' behavior at the debt renegotiation trigger x_{L1} more carefully. At x_{L1}, they need the funds

$$
EF = \underbrace{(1+k_D+k_R)D(x_{L1},1,c_0) - D(x_{L1},1-a,c_{L1})}_{\text{total costs}} - \underbrace{P_L(x_{L1},a)}_{\text{asset price}} \qquad (15)
$$

if EF is positive. Formally, they use equity financing to raise the funds. In fact, however, it is difficult and costly for especially smaller/younger firms during an economic downturn to use external equity financing (e.g., Greenwald, Stiglitz, and Weiss (1984), Bernanke and Gertler (1989), and Bernanke, Gertler, and Gilchrist (1996)). Taking the stylized fact into account, we assume the following equity financing cost k_F. The shareholders need to pay the proportional financing cost

$$
k_F \max\{EF, 0\} \qquad (16)
$$

when using equity financing, where $k_F(\ge 0)$ is a constant. The parameter k_F is higher for smaller/younger firms in a worse economy. If the proceeds from selling assets cover the total costs of the debt renegotiations, the shareholders do not raise the funds. Unlike in the unlevered case, the financing costs may be a firm's great motive for selling assets even though partial liquidation is less advantageous in terms of asset price. This point will be closely explored in Section 4.2.1. In the presence of the equity financing cost, we modify the objective function of the problem (14) with the subtractive term (16) $\times (x/x_{L1})^{\gamma}$. Note that if the financing cost k_F goes to infinity, the problem is subject to the constraint

$$
(1+k_D+k_R)D(x_{L1},1,c_0) - D(x_{L1},1-a,c_{L1}) \le P_L(x_{L1},a). \qquad (17)
$$

When $a = 0, k_D = 0$, and the financing constraint (17) are assumed, our model corresponds to the debt renegotiation model of Lambrecht (2001) and the single debt renegotiation model of Moraux and Silaghi (2014) (the case where the shareholders have full bargaining power).

The equity holders obtain the higher value of $E(x)$ and $E_{L1}(x)$. Note that the inequality between $E(x)$ and $E_{L1}(x)$ does not depend on x, and hence, the shareholders have no incentive to change their decision on the way to liquidation. As will be shown in Section 4, unlike in the unlevered case in Section 2, the decision whether to proceed to debt restructuring or direct liquidation depends on the parameter values.

We derive also the debt and firm values, denoted by $D_{L1}(x)$ and $V_{L1}(x)$ respectively, in the debt restructuring case. We denote the optimal debt restructuring time, debt restructuring trigger, final liquidation trigger and coupon by $T^*_{L1}, x^*_{L1}, x^*_{L2}$, and c^*_{L1}. Using T^*_{L1}, x^*_{L1}, and c^*_{L1}, we have

$$\begin{aligned}
D_{L1}(x) &= \mathbb{E}[\int_0^{T^*_{L1}} \mathrm{e}^{-rt} c_0 \mathrm{d}t + \mathrm{e}^{-rT^*_{L1}}\{(1+k_D)D(X(T^*_{L1}),1,c_0) - D(X(T^*_{L1}),1-a,c^*_{L1}) \\
&\quad + D(X(T^*_{L1}),1-a,c^*_{L1})\}] \\
&= \frac{c}{r} - \left(\frac{c}{r} - (1+k_D)D(x^*_{L1},1,c_0)\right)\left(\frac{x}{x^*_{L1}}\right)^{\gamma} \qquad (18)\\
V_{L1}(x) &= E_{L1}(x) + D_{L1}(x) \\
&= \frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)w}{r} + \frac{\tau c_0}{r} - \left\{\frac{(1-\tau)x^*_{L1}}{r-\mu} - \frac{(1-\tau)w}{r} + \frac{\tau c_0}{r}\right. \\
&\quad \left. - V(x^*_{L1},1-a,c^*_{L1}) + P_L(x^*_{L1},a) - k_R D(x^*_{L1},1,c_0)\right\}\left(\frac{x}{x^*_{L1}}\right)^{\gamma} \qquad (19)
\end{aligned}$$

for $x \geq x^*_{L1}$. We presume that $c/r \geq (1+k_D)D(x^*_{L1},1,c_0)$, which is satisfied in all numerical examples in Section 4. Also note that in the debt restructuring case, the final liquidation occurs at time $T^*_{L2} = \inf\{t \geq 0 \mid X(t) \leq x^*_{L2} = x^*_L(1-a,c^*_{L1})\}$.

In the remainder of this section, we add an explanation why this paper does not assume the shareholders who maximize the firm value and share the surplus with the debt holders through a bargaining game. In the case of firm value maximization, the firm does not need to proceed to debt renegotiation but just continues to operate permanently or liquidate the firm at a very low liquidation trigger x^*_V. In other words, the firm can choose the "renegotiation" trigger x_{L1} at 0 or x^*_V with setting the "new" coupon c_{L1} at c_0. Indeed, when we consider the firm value maximization problem, the firm value becomes

$$\begin{aligned}
&\frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)w}{r} + \frac{\tau c_0}{r} + \sup_{x_V}\left\{-\frac{(1-\tau)x_V}{r-\mu} + \frac{(1-\tau)w}{r} - \frac{\tau c_0}{r} + P_L(x_V,1)\right\}\left(\frac{x}{x_V}\right)^{\gamma} \\
&= \begin{cases} \dfrac{(1-\tau)x}{r-\mu} - \dfrac{(1-\tau)w}{r} + \dfrac{\tau c_0}{r} + \left\{-\dfrac{(1-\tau)x^*_V}{r-\mu} + \dfrac{(1-\tau)w}{r} - \dfrac{\tau c_0}{r} + P_L(x^*_V,1)\right\}\left(\dfrac{x}{x^*_V}\right)^{\gamma} \\ (\text{if } (1-\tau)w/r - \tau c_0/r + G_L(1) > 0) \\ \dfrac{(1-\tau)x}{r-\mu} - \dfrac{(1-\tau)w}{r} + \dfrac{\tau c_0}{r} \ (\text{otherwise}), \end{cases} \qquad (20)
\end{aligned}$$

where the liquidation trigger x^*_V is defined by

$$x^*_V = \frac{\gamma}{\gamma-1}\left(\frac{1-\tau}{r-\mu} - F_L(1)\right)^{-1}\left(\frac{(1-\tau)w}{r} - \frac{\tau c_0}{r} + G_L(1)\right), \qquad (21)$$

if $(1-\tau)w/r - \tau c_0/r + G_L(1) > 0$. Note that (21) is of the same form as (3) except for the tax advantage $\tau c_0/r$. When $(1-\tau)w/r - \tau c_0/r + G_L(1) \leq 0$, the firm continues to operate perpetually.

In Mella-Barral and Perraudin (1997) and Mella-Barral (1999), due to $\tau = 0$, the firm liquidates at the liquidation trigger like (3) through debt renegotiation, which maximizes the firm value. On the other hand, in the presence of tax advantages of debt, most of the literature restricts benefits of tax savings when the equity and debt holders cooperate to maximize the firm value. For example, Fan and Sundaresan (2000) considered the debt renegotiation process assuming no tax advantage of debt while the equity and debt holders cooperate to adjust the coupon. Without this sort of restriction, in the basic models, such as those provided by Leland (1994) and Goldstein, Ju, and Leland (2001) (note that $w = 0$), the equity and debt holders could continue to operate the firm perpetually and enjoy tax savings without potential default costs. Thus, in our model, it is not meaningful to consider the firm value maximization problem (20), allowing tax advantage of debt and a bargaining problem between the equity and debt holders. As in Lambrecht (2001), Reindl (2013), Christensen, Flor, Lando, and Miltersen (2014), Moraux and Silaghi (2014), we consider the equity holders' value maximization problem (14).

4 Numerical analysis and implications

4.1 Basic results

In this section, we compute problem (14) with the financing costs (16) for the base parameter values as follows:[7]

$$r = 0.06, \mu = 0.01, \sigma = 0.2, \tau = 0.15, x = 2, w = 0.5, a = 0.1, c_0 = 2, k_D = k_R = 0.05, k_F = 0.1. \tag{22}$$

We also define the asset price by $P_L(x, a) = 0.6a^{1.01}x/(r - \mu) + 2a^{1.01}$ in the base case. Note that for $x = 1$ the full liquidation value $P_L(x, 1) = 0.6x/(r - \mu) + 2$ is the same as $0.7x/(r - \mu)$, which is often used as the liquidation value of the basic literature.[8]

For the base parameter (22), we have Table 1, where $LV = D(x^*_L, 1-a, c^*_{L1})/V(x^*_L, 1-a, c^*_{L1})$ (leverage ratio), $CS = c^*_{L1}/D(x^*_L, 1-a, c^*_{L1}) - r$ (credit spread), and $EF = (1 + k_D + k_R)D(x^*_{L1}, 1, c_0) - D(x^*_{L1}, 1-a, c^*_{L1}) - P_L(x^*_{L1}, a)$ (newly issued equity value). Because $E_{L1}(x) = 5.91 > E(x) = 5.58$, the equity holders choose debt renegotiation. At the debt renegotiation trigger $x^*_{L1} = 1.25$, they reduce the coupon from $c_0 = 2$ to $c^*_{L1} = 1.07$; instead, they need to pay the total costs $(1 + k_D + k_R)D(x^*_{L1}, 1, c_0) - D(x^*_{L1}, 1-a, c_{L1}) = 3.02$. The proceeds from asset sale, $P_L(x^*_{L1}, a) = 1.44$, are not sufficient to cover the costs, and hence they raise new equity financing $EF = 1.58$. Then, the new leverage ratio and

[7]The growth rate μ is set at a small value because we focus on the firm approaching liquidation in an economic downturn. Following the standard literature, we set r and τ. The initial state x is not substantial because it can be normalized. In Section 4.2, we will show the comparative statics with respect to the other parameter values.

[8]In the parameter values, the liquidation trigger is around 1. We also computed many results, changing the parameter values of $0.6, 1.01$, and 2. The results are straightforward and are omitted from the paper.

credit spread become $LV = 0.846$ and $CS = 0.0207$, respectively. The firm continues to operate until the state variable $X(t)$ hits the final liquidation trigger $x^*_{L2} = 0.842$. Although this is a typical outcome, several values depend on the parameter values. In the next subsection, we will explain the comparative statics results. Now, we explain several robust findings which hold true regardless of parameter values.

In the base case, because of $EF = 1.58 > 0$, the shareholders need equity financing to pay the costs associated with debt renegotiation. According to our computation for a wide range of parameter values, this result remains true even when the equity financing cost k_F is very high. Of course, the shareholders sufficiently mitigate the coupon reduction to decrease EF as k_F increases. However, EF is always positive unless k_F is infinite. To our knowledge, there has been no paper that shows the efficiency of equity financing in debt restructuring. Most of the literature about debt renegotiation, including Lambrecht (2001) and Moraux and Silaghi (2014), considered neither the partial retirement of debt nor equity financing. Reindl (2013), who considered the problem of buying back debt and deleveraging along with selling asset, denied the possibility of equity financing.

(Insert Figures 1 and 2 around here.)

Next, in Table 1 we find that the debt restructuring trigger x^*_{L1} is exactly equal to the original liquidation trigger x^*_L without debt renegotiation. This means that the leverage ratio without debt renegotiation increases up to 1 right before debt renegotiation. We made numerous computations in addition to presented examples and verified that $x^*_{L1} = x^*_L$ in all cases. For instance, Figure 1 shows that $x^*_{L1} = x^*_L = 1.25$ without regard to the fraction of asset sale a, although, as will be checked in Figure 3 of Section 4.2.1, the new coupon c^*_{L1} varies with a. While this result does not hold in debt renegotiation models with a temporary reduction in the coupon (e.g., Mella-Barral and Perraudin (1997) and Fan and Sundaresan (2000)), it is in line with the results in previous models with a lump-sum and permanent reduction in the coupon. Actually, Lambrecht (2001) and Moraux and Silaghi (2014) showed the same result, although they did not consider either asset sale or debt retirement. In Reindl (2013), the debt restructuring timing can be earlier depending on the asset price. This is because his assumption, unlike our assumption of $P_L(x, a)$, does not directly relate partial liquidation value and the bankruptcy value, and hence partial liquidation can be more profitable than full liquidation. In our view, debt restructuring always takes place at the original liquidation trigger as long as we assume economies of scale in production relating asset sale with the full liquidation value.

Using Figure 2, we explain the rationale behind the result that $x^*_{L1} = x^*_L$. Figure 2 depicts the debt value $D(x_{L1}, 1, c_0)$ as a function of x_{L1}. For $x_{L1} \geq x^*_L$, $D(x_{L1}, 1, c_0)$ is concave and increasing. Because of the concavity, the shareholders can efficiently decrease the repayment value by waiting until $X(t)$ reaches x^*_L. They have no incentive to strategically default before the original liquidation time because of the high repayment value. On the other hand, the shareholders do not delay the debt renegotiation timing after the original liquidation time because they do not wish to continue to pay the initial coupon

c_0 any longer. The debt value $D(x_{L1}, 1, c_0)$ coincides with a linear function $P_L(x_{L1}, 1)$ for $x_{L1} \leq x_L^* = 1.25$, and hence there is no incentive for the shareholders to delay debt renegotiation any longer. Accordingly, the equity holders optimally choose the same debt renegotiation timing as the liquidation timing, i.e., $x_{L1}^* = x_L^*$.

Based on the result above, from now on we suppose that $x_{L1}^* = x_L^*$ and examine the equity holders' decision in more details. Under the condition, we can reduce (14) to the following:

$$\begin{aligned} E_{L1}(x) = \frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)(w+c_0)}{r} + \sup_{c_{L1}} V(x_L^*, 1-a, c_{L1}) \left(\frac{x}{x_L^*}\right)^{\gamma} + \Bigg\{ P_L(x_L^*, a) \\ - (1+k_D+k_R)P_L(x_L^*, 1) - \frac{(1-\tau)x_L^*}{r-\mu} + \frac{(1-\tau)(w+c_0)}{r} \Bigg\} \left(\frac{x}{x_L^*}\right)^{\gamma}, \end{aligned} \tag{23}$$

where we used $D(x_L^*, 1, c_0) = P_L(x_L^*, 1)$. In the absence of financing costs, the condition under which debt renegotiation is preferred is as follows:

$$\begin{aligned} & E_{L1}(x) > E(x) \\ \Leftrightarrow & V(x_L^*, 1-a, c_{L1}^*) + P_L(x_L^*, a) - (1+k_D+k_R)P_L(x_L^*, 1) > 0. \end{aligned} \tag{24}$$

For $a = w = k_D = k_R = 0$, we can find $c < c_0$ such that $D(x_L^*, 1, c) = P_L(x_L^*, 1)$ because $D(x_L^*, 1, c)$ is hump-shaped with respect to $c \leq c_0$ (e.g., Leland (1994), Lambrecht (2001), and Figure 1 of Moraux and Silaghi (2014)). Using this c, we have

$$\begin{aligned} V(x_L^*, 1, c_{L1}^*) - P_L(x_L^*, 1) \geq & V(x_L^*, 1, c) - P_L(x_L^*, 1) \\ = & E(x_L^*, 1, c) \\ > & 0. \end{aligned}$$

That is, without asset sale, operating costs, and costs associated with debt restructuring, the shareholders always prefer debt renegotiation to liquidation. This result remains unchanged even in the presence of a financing cost because we have $EF = 0$ for the same c. The similar results were found in many papers (e.g., Lambrecht (2001), Reindl (2013), and Moraux and Silaghi (2014)). On the other hand, if any of a, w, k_D, and k_R are positive, the shareholders may proceed to direct liquidation. In the next subsection, we will examine how the parameter values affect the equity holders' decision of whether to proceed to debt restructuring or direct liquidation.

4.2 Comparative statics

This section analyzes comparative statics with respect to parameters a, c_0, σ, k_F, k_D, and w, and it reveals how these parameters affect the shareholders' optimal decision. Above all, the impacts of the fraction of asset sale a and the initial coupon c_0 are novel and discussed in detail. In each figure, the other parameter values are fixed at the base case (22). Each figure contains six panels which show equity values ($E_{L1}(x)$ and $E_L(x)$), new coupons (c^*_{L1}), new and original liquidation (renegotiation) triggers (x^*_{L2} and $x^*_L(= x^*_{L1})$), new leverage ratios ($LV = D(x^*_L, 1-a, c^*_{L1})/V(x^*_L, 1-a, c^*_{L1})$), new credit spread ($CS = c^*_{L1}/D(x^*_L, 1-a, c^*_{L1}) - r$), and the newly issued equity value ($EF = (1 + k_D + k_R)D(x^*_{L1}, 1, c_0) - D(x^*_{L1}, 1-a, c^*_{L1}) - P_L(x^*_{L1}, a)$). In some figures, we show other values to explore the results more closely.

4.2.1 Fraction of asset sale a

(Insert Figure 3 around here.)

Figure 3 shows the panels with varying levels of the fraction of asset sale a. We can see from the top left panel that $E_{L1}(x)$ monotonically decreases with a. For $a \geq 0.34$, the shareholders prefer to liquidate the firm because of the inefficiency in the partial asset sale in debt renegotiation. According to a number of computations, we find that partial liquidation always destroys the equity value. Partial liquidation is less efficient not only because $P_L(x, \cdot)$ is convex but also because for $x = x^*_L$, which is larger than x^*_{L2}, the equity holders pay the fundamental costs of partial liquidation, i.e., $(a - 0.6a^{1.01})x/(r - \mu)$. In other words, they wish to defer all liquidation costs until $X(t)$ hits the final liquidation trigger x^*_{L2}. If the shareholders can optimize a in debt renegotiation, they always choose $a = 0$ (debt reorganization with no asset sale). Instead, they optimally adjust the coupon reduction and use equity financing. Actually, as checked in the bottom right panel of Figure 3, selling assets adversely increases the newly issued equity value because it reduces the new debt value $D(x^*_L, 1-a, C^*_{L1})$ more than the increase in $P_L(x^*_L, a)$.

(Insert Figure 4 around here.)

Even if a financing cost k_F is higher (in an extreme case infinite) or $P_L(x, \cdot)$ is linear, they do not prefer a positive a. Figure 4 shows $E_{L1}(x)$ and $E(x)$ with varying levels of a, where k_F is set at $0.5, 1$, and 1.5. The left and right panels show the results for $P_L(x, a) = 0.6a^{1.01}x/(r - \mu) + 2a^{1.01}$ (base case) and $P_L(x, a) = 0.6ax/(r - \mu) + 2a$ (linear case), respectively. Although the regions of direct liquidation increase with higher k_F, the equity holders do not voluntarily sell assets in order to repay the partial debt value.

This result is different from the finding of Reindl (2013), who argued that the possibility of selling assets causes deleveraging. Again, in our view, this is because partial liquidation can be more profitable than full liquidation in the setup of Reindl (2013). Our result can potentially account for several empirical findings. Actually, Maksimovic and Phillips (1998) and Maksimovic and Phillips (2001) showed that the frequency of asset sales does not increase during the debt reorganization process. They argued that lower-productivity firms are more likely to sell assets to higher-productivity firms when industry output increases, which leads to improvement in the resource allocation. Note that our model focuses only on a firm's behaviour in declining economies and assumes that some costs are associated with liquidation. Arnold, Hackbarth, and Puhan (2013) also showed that asset sales are related to an increase in investment and resource allocation.

Relatedly, Weiss and Wruck (1998) documented the case of Eastern Airlines, in which inefficient asset sales caused Chapter 11's failure.

The values x^*_{L2}, x^*_L, LV, and CS are almost constants over a (note the scale of the vertical axis in the figure). This is because the debt service ratio $(1-a)/c^*_{L1}$ is almost constant while c^*_{L1} decreases with a (see the top right panel of Figure 3). That is, the fraction of asset sale a does not greatly influence the capital structure after debt restructuring.

4.2.2 Initial coupon c_0

(Insert Figure 5 around here.)

Figure 5 shows the panels with varying levels of initial coupons c_0. Naturally, $E_{L1}(x)$ and $E(x)$ decrease with a higher c_0, while x^*_{L2} and x^*_L increase. Interestingly, in the top left panel, for $c_0 \leq 1.65$ the shareholders prefer liquidation to debt renegotiation. This is because, for a higher c_0, the equity holders can reduce the coupon more (see the difference between the two lines in the top right panel), which leads to more surplus from debt renegotiation. Accordingly, a higher c_0 increases the incentive to proceed to debt restructuring rather than direct liquidation.

This result may align with the "too big to fail" theory. Indeed, our model suggests that larger/older firms with more debt gain more surplus from succeeding in debt renegotiation and avoiding full liquidation. Note that due to information asymmetry problems, smaller/younger firms tend to have limited access to debt (e.g., Bernanke, Gertler, and Gilchrist (1996)). Our prediction is also consistent with the stylized fact that larger/older firms are more likely to proceed to debt renegotiation rather than direct liquidation (e.g., Bris, Welch, and Zhu (2006)). Although we do not consider multiple debt renegotiations, our result is also similar to the result of Moraux and Silaghi (2014). They showed that due to renegotiation costs equity holders give up any more renegotiation round after the coupon is reduced to a sufficiently low level. We will also return to this point while examining another aspect in Section 4.3.

The sensitivities of c_0 on LV and CS are counter-intuitive (see the middle right and bottom left panels of Figure 5). The firm with a higher c_0 will be less risky after debt restructuring. The reason can be explained by the top right and middle left panels. Indeed, we find from the panels that, for a higher c_0, the coupon reduction $c_0 - c^*_{L1}$ is larger and the firm's survival time after debt restructuring is longer. Because of this efficient capital restructuring on the early timing, the firm with a higher c_0 can be less risky after debt renegotiation. On the other hand, due to the large coupon reduction, the firm needs more equity financing to pay back to the debt holders (see the bottom right panel).

4.2.3 Volatility σ

(Insert Figure 6 around here.)

Figure 6 shows the panels with varying levels of cash flow uncertainty σ. In the top and middle left panels, $E_{L1}(x)$ and $E(x)$ (x_L^* and x_{L2}^*) increase (decrease) with a higher σ. This corresponds to the standard volatility effect (e.g., Dixit and Pindyck (1994)) that a higher σ increases the option value of waiting and delays the liquidation timing. More notably, we find that the increase in $E(x)$ (decrease in x_L^*) dominates that of $E_{L1}(x)$ (x_{L2}^*). As a result, $E(x)$ exceeds $E_{L1}(x)$ for $\sigma \geq 0.35$. The reason is that the convexity of shareholders' option to liquidate is stronger in the direct liquidation case than in the piecemeal liquidation case. Kort, Murto, and Pawlina (2010) also documented a similar logic in the context of stepwise investment. Our result is consistent with the empirical evidence in Favara, Schroth, and Valta (2012) and Favara, Morellec, Schroth, and Valta (2014). They showed that debt renegotiation decreases the convexity of shareholders' claim and their incentives for risk-taking.

The finding that a firm with a high σ is more likely to liquidate without debt renegotiation is also consistent with the following empirical evidence. Smaller/younger firms tend to have a higher σ, and such firms are more likely to proceed to direct liquidation (e.g., Bris, Welch, and Zhu (2006)). We can see from LV and CS that the firm with a higher σ is more risky, despite that the newly arranged coupon c_{L1}^* is lower. The graphs of LV and CS are similar to those presented by Shibata and Nishihara (2015b). In the bottom right panel, the result on EF is not monotonic but unimodal.

4.2.4 Financing cost k_F

(Insert Figure 7 around here.)

Figure 7 shows the panels with varying levels of equity financing costs k_F. $E_{L1}(x)$ monotonically decreases with k_F. In this example, once k_F increases beyond 0.8, the shareholders prefer direct liquidation to debt restructuring. In the figure, we find that a higher k_F increases c_{L1}^* and decreases EF; nevertheless, the financing cost (16) increases (see Cost in the bottom right panel). That is, with a higher equity financing cost, the shareholders give up a larger coupon reduction. Then, the time interval between debt renegotiation and final liquidation becomes shorter with a higher k_F. Indeed, the middle left panel shows that a higher k_F increases the final liquidation trigger x_{L2}^* while it does not change the renegotiation trigger x_L^*. As well as the liquidation probability, LV and CS increase with a higher k_F. Especially smaller/younger firms tend to face more difficulty in accessing external financing during a recession (e.g., Greenwald, Stiglitz, and Weiss (1984), Bernanke and Gertler (1989), and Bernanke, Gertler, and Gilchrist (1996)). Taking this into account, our result is consistent with the empirical evidence that, during a recession, weaker firms are likely to proceed to direct liquidation rather than debt renegotiation (e.g., Bris, Welch, and Zhu (2006)).

4.2.5 Premium to the debt holders k_D

(Insert Figure 8 around here.)

Figure 8 shows the panels with varying levels of premiums k_D, which the debt holders receive. Although this paper does not model any bargaining game, k_D can be proxy for the debt holders' bargaining power in debt renegotiation. As is shown clearly, $E_{L1}(x)$ monotonically decreases with k_D. In this example, for $k_D \geq 0.091$, the shareholders prefer liquidation to debt renegotiation. This negative impact of k_D on debt renegotiation is straightforward and consistent with the previous findings regarding the debt holders' bargaining power (e.g., Moraux and Silaghi (2014) and Favara, Morellec, Schroth, and Valta (2014)). If the debt holders have full bargaining power, they optimally choose the critical premium $k_D = 0.091$, which is the cross point of $E_{L1}(x)$ and $E(x)$. Note that the debt holders gain more in debt renegotiation than in direct liquidation as long as k_D is larger than 0.

In Figure 8, $c^*_{L1}, x^*_{L2}, x^*_L, LV$, and CS do not depend on k_D. These are obvious from the equations (23) and (16). Indeed, the shareholders choose $c^*_{L1} = \arg\max_{c_{L1}} V(x^*_L, 1-a, c_{L1}) - k_F D(x^*_L, 1-a, c_{L1})$ regardless of the values of k_D. This means that the premium to the debt holders does not influence the capital structure after debt renegotiation. Lastly, as is easily checked by (15), EF increases with k_D (see the bottom right panel) because the shareholders need to pay back more to the debt holders. We omit the comparative statics with respect to the renegotiation cost k_R because its effects are quite similar to those of k_D.

4.2.6 Running cost w

(Insert Figure 9 around here.)

Figure 9 shows the panels with varying levels of running costs w. In the top left panel, a higher w decreases both $E_{L1}(x)$ and $E(x)$. The effect on $E_{L1}(x)$ is larger because the firm that chooses debt renegotiation will operate and suffer from the running cost after the original liquidation time. In this example, for $w \geq 0.62$ the shareholders prefer liquidation to debt renegotiation. The impact of w is opposite from that of c_0 (cf. Figure 5), although both w and c_0 are costs to the equity holders. This difference stems from the fact that w, unlike c_0, cannot be reduced in debt renegotiation.

In the middle left panel, we find that both x^*_{L2} and x^*_L increase with w. Furthermore, the distance between the two triggers monotonically decreases with w. This means that the firm with a higher w is more risky even when it succeeds in debt renegotiation. LV and CS in the middle right and the bottom left panels also support this result. Our result is consistent with the stylized fact that lower-productivity firms tend to fail in debt renegotiation and proceed to liquidation (e.g., Maksimovic and Phillips (1998)).

4.3 Optimal capital structure

So far, we have changed parameter values with the initial coupon c_0 fixed at 2 in the base case except for Figure 5. We also check the comparative statics with c_0 maximizing the firm value at the initial time. The firm chooses an initial coupon c_0 for which the shareholders can choose whether to proceed to debt restructuring or direct liquidation. Taking account of the shareholders' future behavior contingent on c_0, we need to compute an optimal c_0.

For the base parameter (22) where c_0 replaced the optimal coupon, we have Table 2, where the first and second rows present the optimal capital structure in the case allowing debt restructuring (DR) and the case with direct liquidation only (L), respectively. LV_0 and CS_0 denote the leverage ratio and credit spread at the initial time. In the DR case, we have the optimal coupon $c_0 = 1.95$, which will lead to debt renegotiation rather than direct liquidation in the future.[9] Table 2 shows that the firm that will proceed to debt renegotiation has a higher c_0, LV_0, and CS_0 than in the direct liquidation case. This is because, as shown in Figure 5, the firm can gain more surplus from debt renegotiation when the initial coupon is high. This may be related to a moral hazard problem caused by prospective debt renegotiation. Our result is similar to the previous findings. For instance, Moraux and Silaghi (2014) showed that a firm increases the initial coupon, taking account of debt renegotiation in the future. Christensen, Flor, Lando, and Miltersen (2014) also showed that the possibility of debt renegotiation increases the initial leverage ratio because it reduces the default costs, which is the negative impact of debt financing.

(Insert Figure 10 around here.)

The comparative statics results, except for σ in the previous subsection, remain similar even when we take the optimal c_0 at the initial time, and thus they are omitted here. Figure 10 shows the panels with varying levels of σ. Differently from Figure 6 in Section 4.2.5, the firm proceeds to debt renegotiation regardless of σ. We see from the top right panel that the optimal c_0 increases with σ. Because of the increased c_0, the equity holders can make enough of a coupon reduction (see the gap between c_0 and c_{L1} in the top right panel) to gain more profits from debt renegotiation than from direct liquidation. Note that the new coupon c_{L1}, unlike in Figure 6, increases with σ, but the gap between c_0 and c_{L1} increases more than in Figure 6. The efficiency of debt restructuring is also seen in the gap between x_L^* and x_{L2}^* in the middle left panel of Figure 10, compared to that of Figure 6. In the middle right panel, both LV and LV_0 decrease with σ in the optimal capital structure case, differently from Figure 6. CS, CS_0 and EF have similar shapes to those of Figure 6.

(Insert Figure 11 around here.)

When we optimally adjust the capital structure, the timing of adjustment also influences the results. To see the impact, we present the comparative statics with respect to the initial state variable x. It is worth noting that in the previous subsection with c_0 fixed, the results do not depend on x. Figure 11 shows the panels with varying levels of x.

[9] As will be seen in Figure 11 later, an optimal coupon, which maximizes the firm value, may lead to direct liquidation depending on the parameter values.

For $x \leq 1.52$, the firm is better off leaving the equity holders to choose liquidation rather than debt renegotiation.[10] Because of this, $E_{L1}(x), c_0^*, x_L^*, LV_0$ and CS_0 jump at $x = 1.52$. All graphs, especially LV_0 and CS_0, have quite different shapes for $x \in (1.52, 1.75)$ from those for larger x. This is because in the region the firm dares to choose a higher c_0 to make the equity holders proceed to debt renegotiation rather than liquidation. Recall that with a higher c_0 the equity holders are more likely to proceed to debt restructuring (see Figure 5). The firm wishes to choose a lower c_0, but then the equity holders would liquidate the firm. Because of this, the firm chooses high c_0 (hence, LV_0 and CS_0) when x is close to 1.52. This motivation greatly distorts the result for $x \in (1.52, 1.75)$.

Once x decreases below 1.52, the firm is better off setting a low c_0 and leaving the shareholders to choose direct liquidation, rather than a sufficiently high c_0 to lead to debt renegotiation. Then, the optimal c_0^* jumps down when x decreases below the threshold. At the same threshold, the (ex-post) equity value adversely jumps up due to the downward jump in c_0^*. On the other hand, the firm value $V_{L1}(x)$, which can be regarded as the ex-ante equity value, continuously increases with x. As in Moraux and Silaghi (2014), our result predicts that the firm is more likely to proceed to direct liquidation when the latest capital adjustment occurs for a lower state variable.

5 Conclusion

In this paper, we investigated whether and how the shareholders of a firm in distress proceed to direct liquidation or debt restructuring along with partial liquidation. We showed the following results.

In debt restructuring, the shareholders arrange the coupon reduction and use equity financing to retire the partial debt value. The optimal debt restructuring time is always equal to the original liquidation time without debt renegotiation because the shareholders can most efficiently decrease the debt repayment value on this timing. Most notably, even if they face high financing costs, the shareholders do not prefer partial liquidation in debt restructuring because the costs arise earlier. Instead, they prefer to adjust the coupon reduction at a small level so that they do not need to pay back a significant amount to the debt holders. Our result about the inefficiency of asset sale is consistent with empirical findings.

Fewer forced asset sales, lower financing, renegotiation, and running costs, a lower premium to the debt holders, a lower volatility, and more debt increase the shareholders' incentive to choose debt renegotiation to avoid full liquidation. These results suggest that larger/older/higher-productivity firms are more likely to proceed to debt renegotiation, which is supported by the empirical evidence. When the firm optimizes its capital structure, taking account of debt renegotiation in the future, it chooses a higher coupon,

[10]For $x \leq 1.52$ (liquidation region), the figure does not depict $c_{L1}^*, x_{L2}^*, LV, CS$, and EF.

leverage, and credit spread than in the case without debt renegotiation. The firm tends to proceed to direct liquidation rather than another debt restructuring shortly after the capital adjustment.

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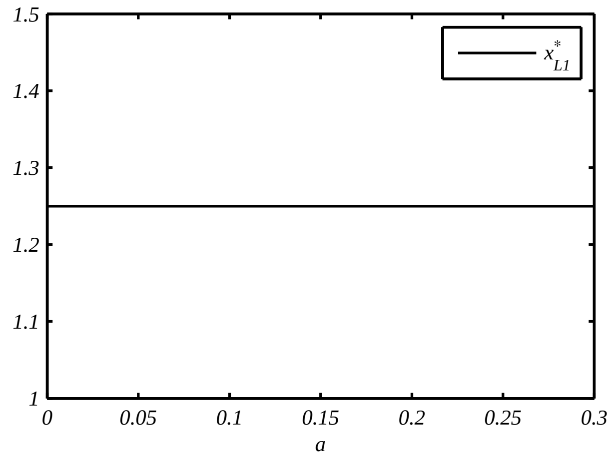

Figure 1: The debt renegotiation trigger x^*_{L1} with varying levels of the fraction of asset sale a. The other parameter values are set at the base case (22). Note that debt renegotiation, rather than direct liquidation, is chosen for these parameter values.

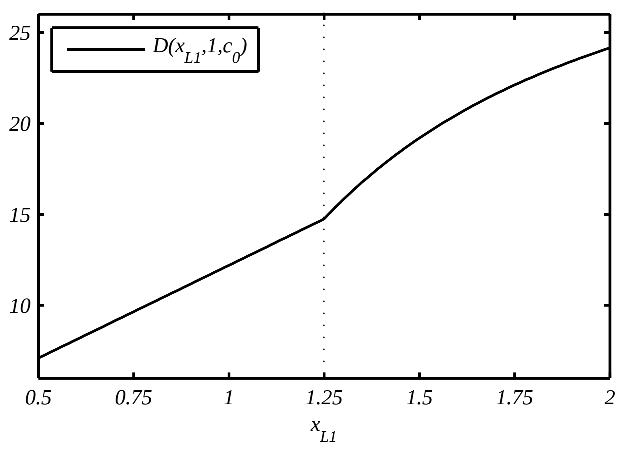

Figure 2: The debt value $D(x_L, 1, c_0)$ with varying levels of x_L. The parameter values are set at the base case (22).

Table 1: Base case.

$E_{L1}(x)$	$E(x)$	c^*_{L1}	x^*_L	x^*_{L1}	x^*_{L2}	LV	CS	EF
5.91	5.58	1.07	1.25	1.25	0.842	0.846	0.0207	1.58

Table 2: Optimal capital structure.

	V	E	c_0, c^*_{L1}	LV_0, LV	CS_0, CS	x^*_L, x^*_{L2}	EF
DR	30.4	6.21	$1.95, 1.05$	$0.796, 0.852$	$0.0205, 0.0209$	$1.23, 0.833$	1.55
L	30.1	9.58	$1.51, N/A$	$0.682, N/A$	$0.0134, N/A$	$1.01, N/A$	N/A

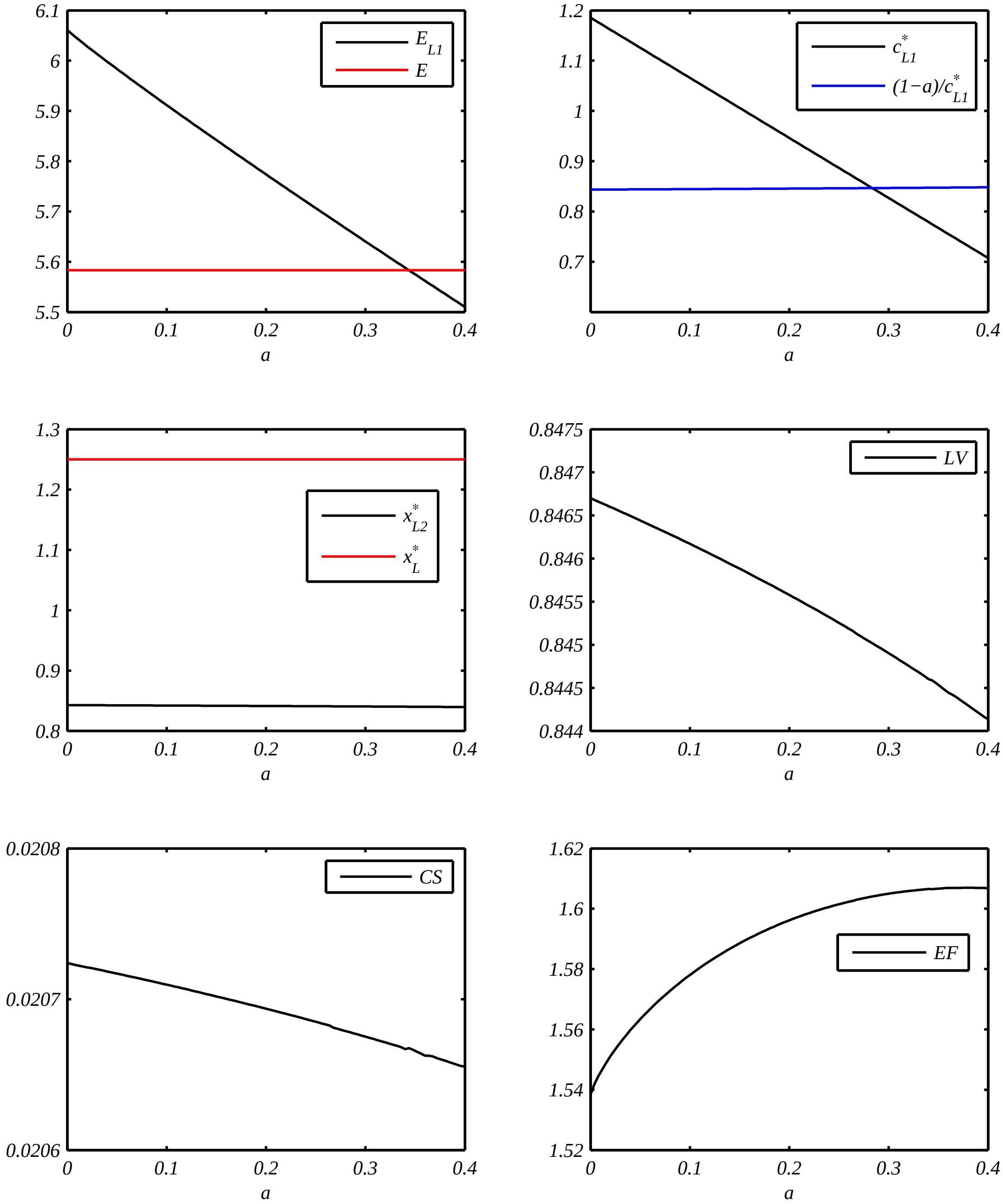

Figure 3: Comparative statics with respect to the fraction of asset sale, a. The other parameter values are set at the base case (22).

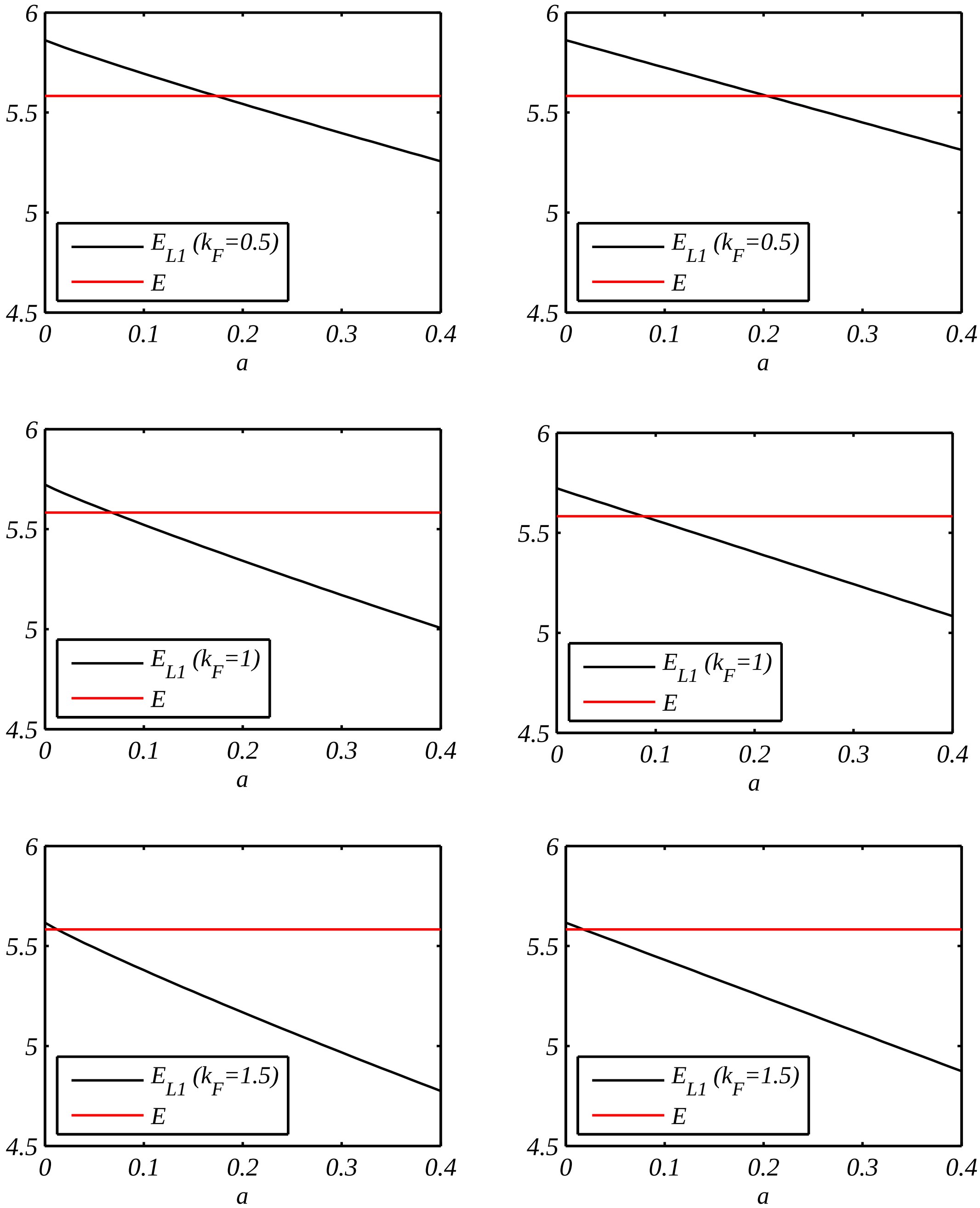

Figure 4: $E_{L1}(x)$ and $E(x)$ with respect to the fraction a and the financing cost k_F. The left and right panels show the values for $P_L(x, a) = 0.6a^{1.01}x/(r - \mu) + 2a^{1.01}$ (base case) and $P_L(x, a) = 0.6ax/(r - \mu) + 2a$ (linear case). The other parameter values are set at the base case (22).

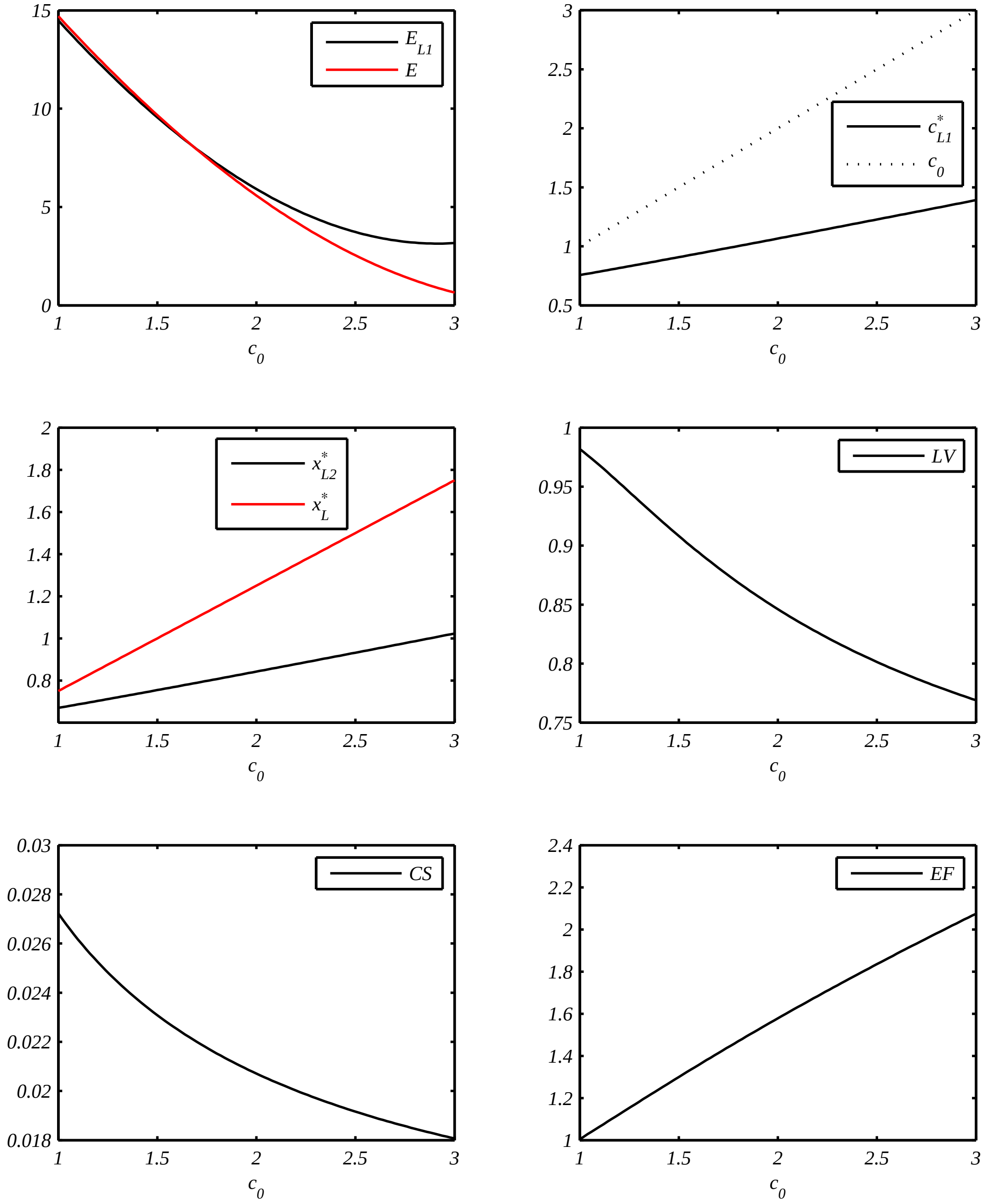

Figure 5: Comparative statics with respect to the initial coupon c_0. The other parameter values are set at the base case (22).

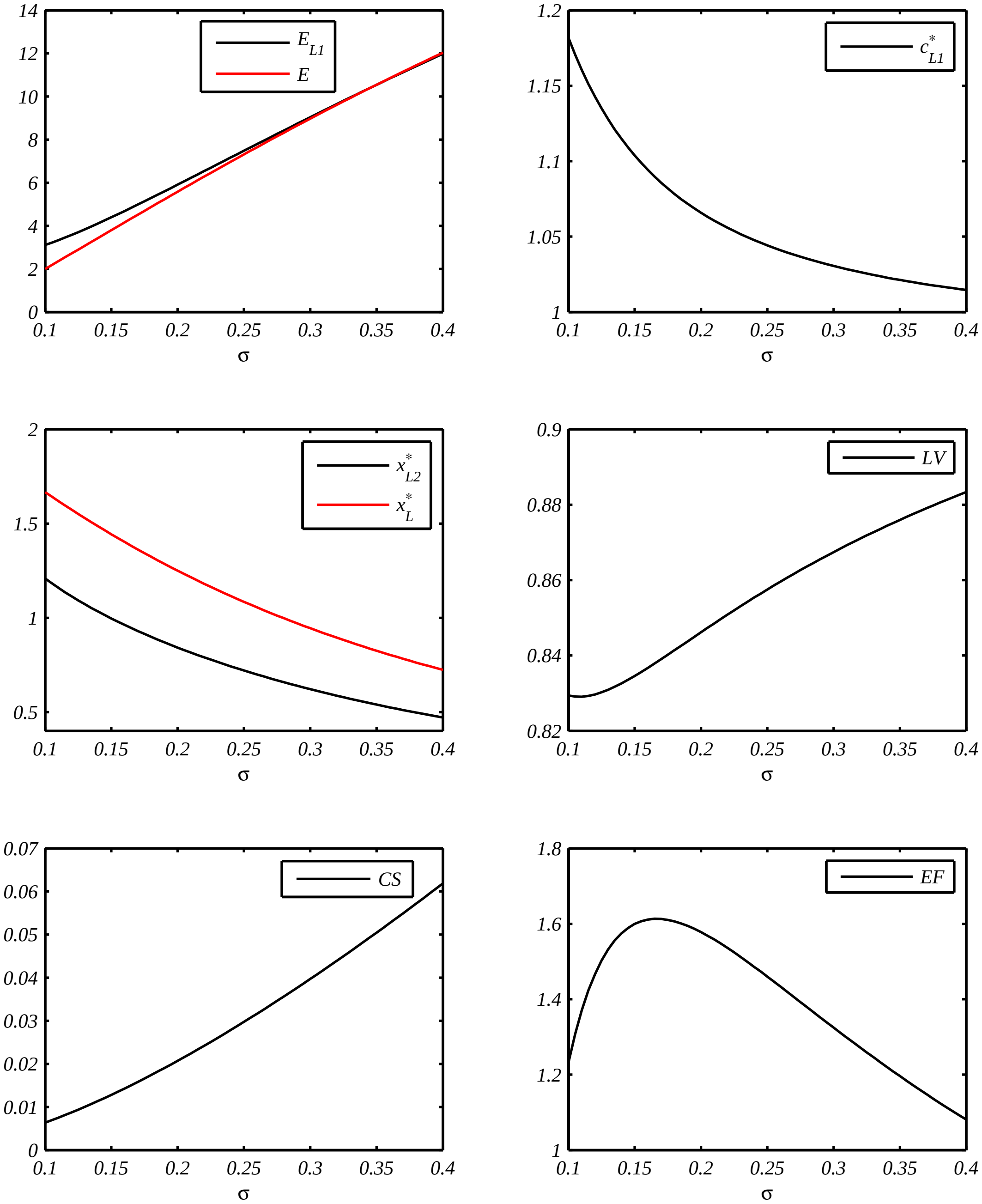

Figure 6: Comparative statics with respect to the cash flow volatility σ. The other parameter values are set at the base case (22).

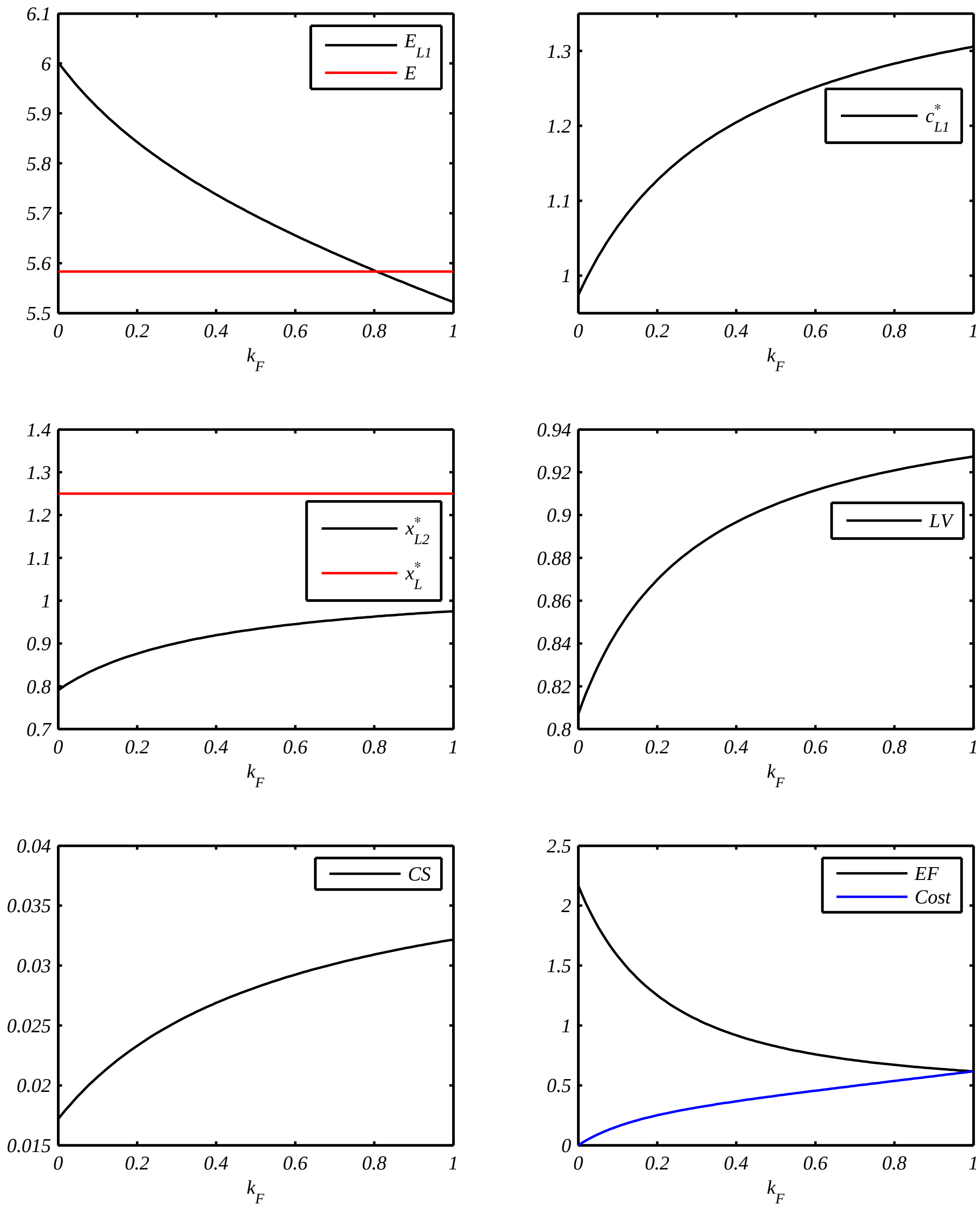

Figure 7: Comparative statics with respect to the financing cost k_F. The other parameter values are set at the base case (22).

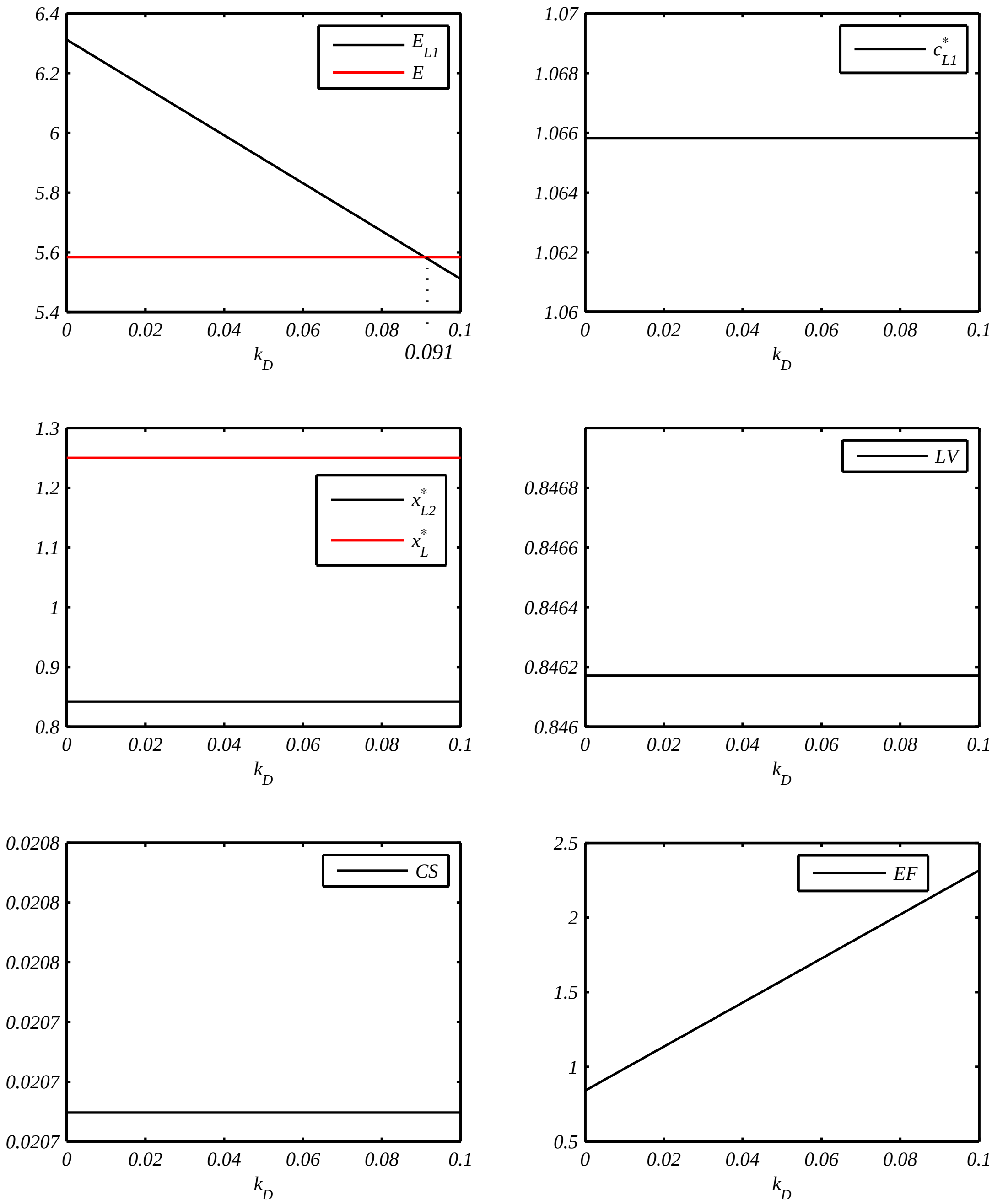

Figure 8: Comparative statics with respect to the premium to the debt holders k_D. The other parameter values are set at the base case (22).

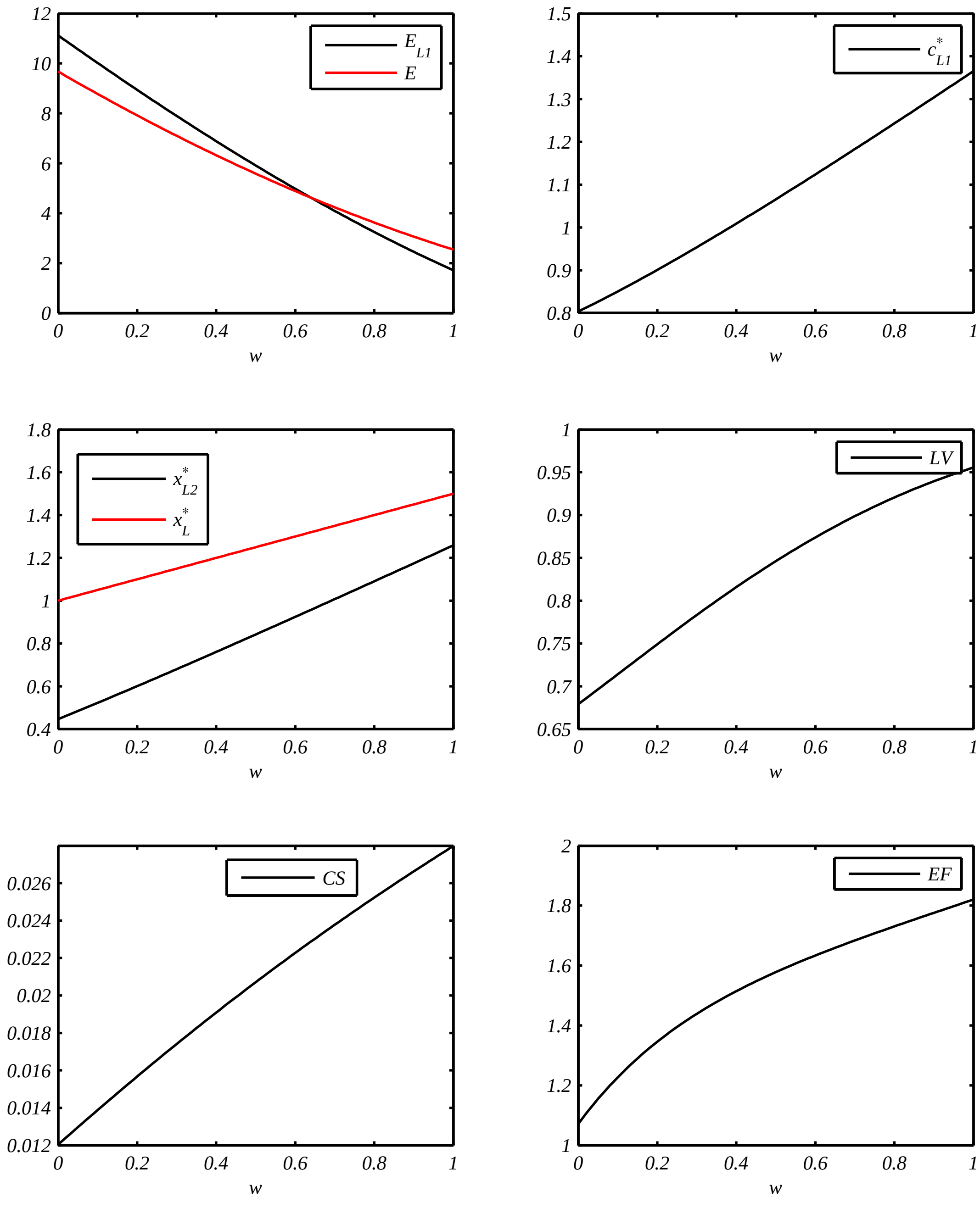

Figure 9: Comparative statics with respect to the running cost w. The other parameter values are set at the base case (22).

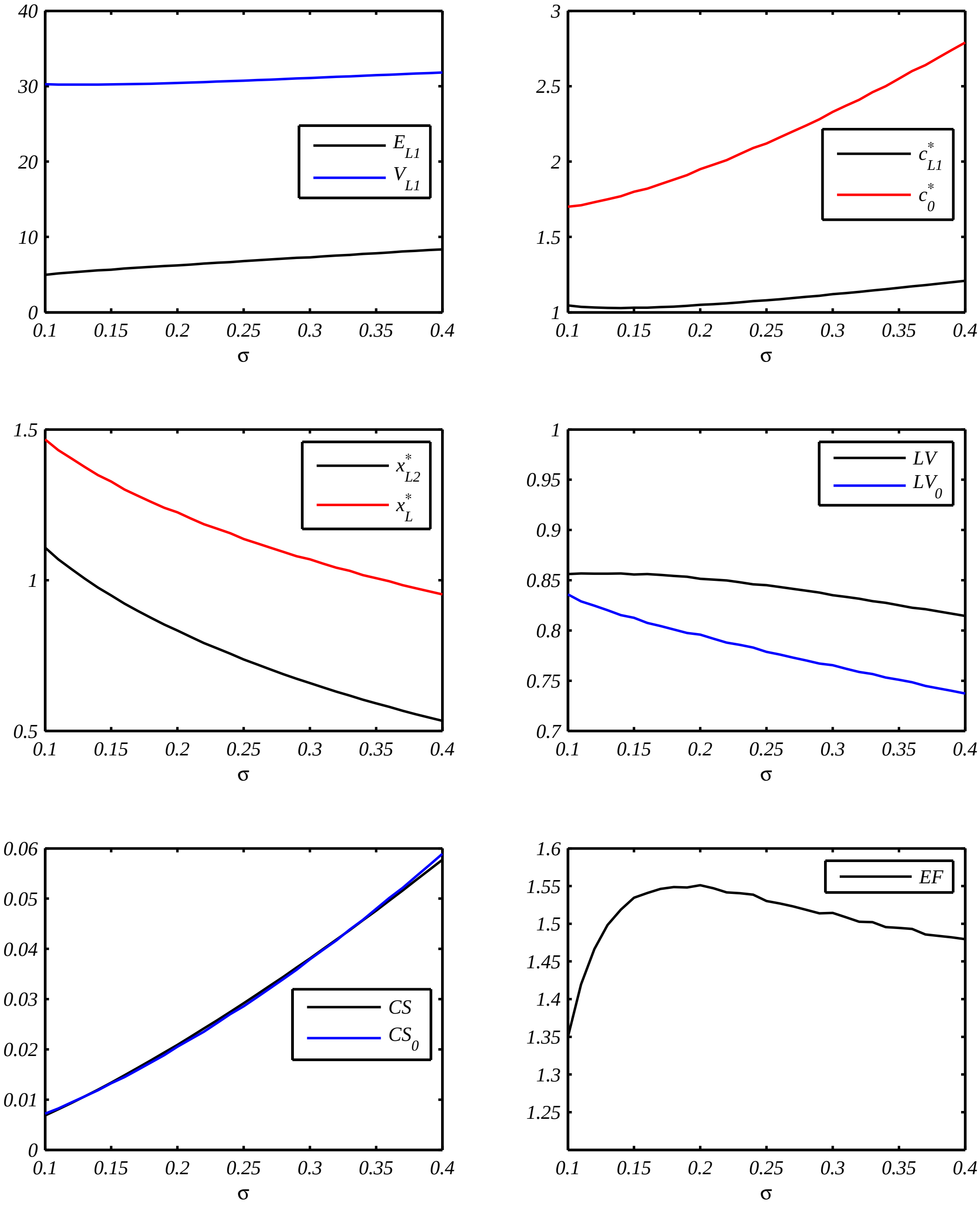

Figure 10: Comparative statics with respect to volatility σ in the optimal capital structure case. The other parameter values are set at the base case (22).

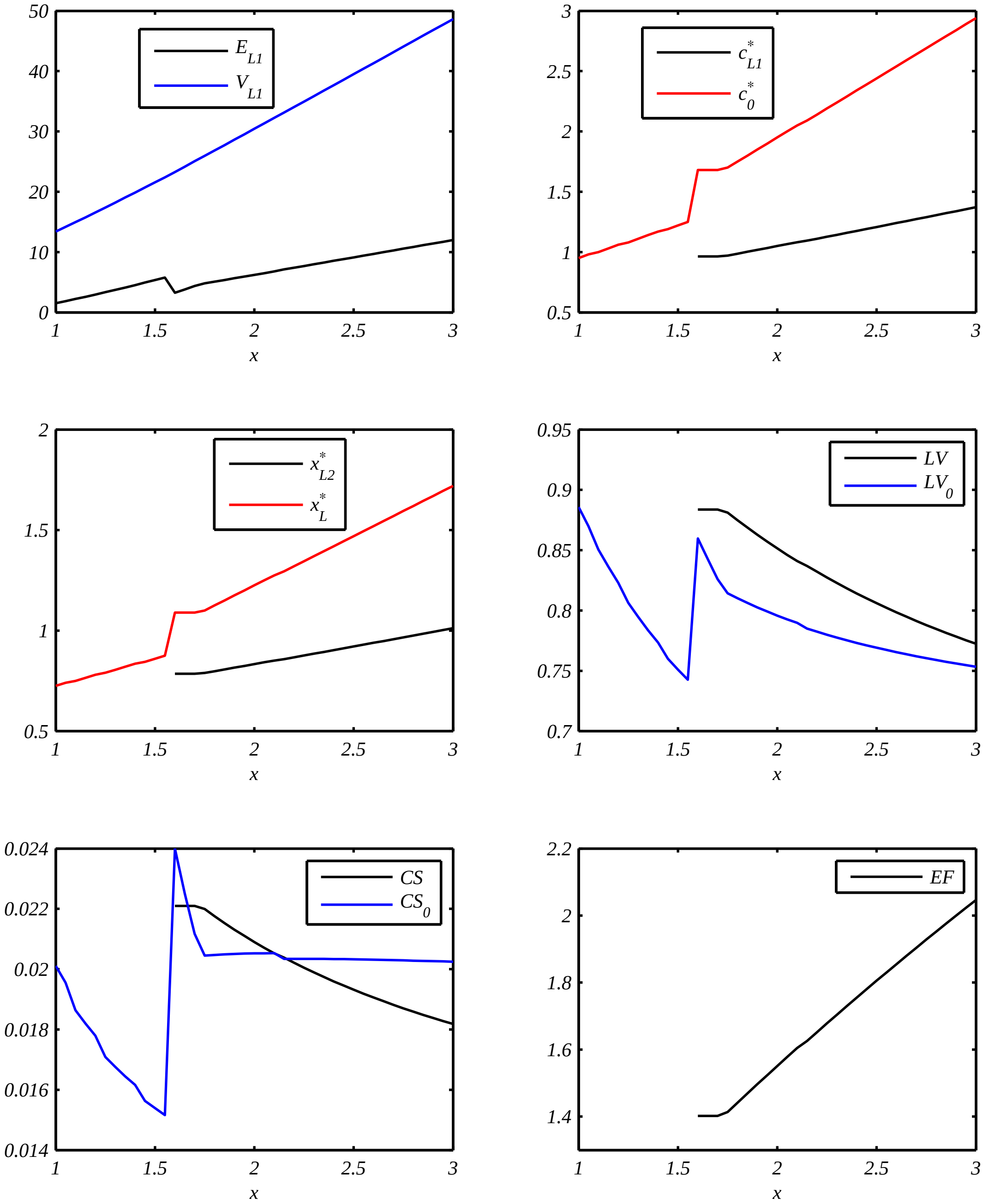

Figure 11: Comparative statics with respect to the initial state variable x in the optimal capital structure case. The other parameter values are set at the base case (22).