Discussion Papers In Economics And Business

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April 2018

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Tatsuro Iwaisako†‡and Kazuyoshi Ohki§

Abstract

We develop a Schumpeterian growth model in which both leaders and followers conduct R&D activities and in which leaders have different quality leads over their followers, determined by a random draw, and thus have different profit flows. We show that leaders with larger quality leads make smaller R&D investments; this result is consistent with the actual behaviors of some previous leader firms such as Sony and Eastman–Kodak. Moreover, we show that subsidizing followers' R&D can promote leaders' aggregate R&D, because promotion of followers' R&D decreases (increases) the number of leaders with larger (smaller) quality leads and smaller (larger) R&D investments.

Keywords: Schumpeterian growth; Heterogeneous leaders; R&D subsidies

JEL classification: L16, O31, O38

*The authors would like to thank Paul Klein (co-editor) and two anonymous referees for their helpful comments and suggestions, which have improved this paper significantly, and Daishoku Kanehara, Akihisa Shibata, Miho Sunaga, Hitoshi Tanaka and participants at the Japanese Economic Association Annual Meeting 2013 and Kyoto Uinversity for their helpful comments on an earlier version. The authors also gratefully acknowledge the financial support of a Japan Society for the Promotion of Science (JSPS) Grant-in-Aid for Young Scientists (B) #25780134, JSPS Grant-in-Aid for Specially Promoted Research #23000001, and JSPS Grant-in-Aid for Scientific Research (No. 16H02026, No. 17K03623). The usual disclaimer applies.

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1 Introduction

In developed economies, total factor productivity (hereafter, TFP) growth from research and development (hereafter, R&D) activities is the most important source of sustained economic growth. Since the seminal contributions by Grossman and Helpman (1991) and Aghion and Howitt (1992), many studies have developed R&D-based growth models and examined how aggregate R&D activities and the growth rate of output are determined. However, most of the earlier studies have two features that are not consistent with the evidence. First, in most of the earlier studies, innovation is conducted by new entrants or follower firms, not by the leader that has the state-of-the-art technology in each industry. In reality, in many industries, the leader firm conducts R&D activities and contributes to innovation. For instance, Bartelsman and Doms (2000) reported that 75 percent of TFP growth results from R&D activities by the incumbent firms. Second, in most of the earlier studies, profits from innovation are symmetric among firms. However, in reality, the profits from innovation differ among firms. Indeed, commencing with Scherer (1965), many empirical studies have examined the distribution of profits among firms.

To construct an R&D-based growth model that is consistent with this evidence, we assume that (i) R&D technology of leader firms exhibits decreasing returns and (ii) the size of the quality increment is determined by a random draw from a given distribution. As a result, we develop a Schumpeterian growth model where leader firms have different profits and some of them conduct R&D activities. Examining this heterogeneous-leaders model, first, we find the following interesting tendency in the leader firms' behaviors. The leader that has higher quality compared with its followers and earns larger profits makes smaller R&D investments.[1] The reason is as follows. The leaders lose the value of their current position when they succeed in the next innovation. Therefore, the net benefit of R&D for the leader that has a larger quality lead over their followers is lower, and thus, a higher-quality leader has lower R&D expenditure. This theoretical result is consistent with actual firm behavior. Sony succeeded in developing the high-quality flat cathode-ray tube TV and did not invest in the development of a liquid crystal TV. Consequently, Sony did not obtain a large share of the market for liquid crystal TVs. Eastman–Kodak, which was the earliest developer of film cameras, went bankrupt because of the diffusion of digital

[1] Akcigit and Kerr (2018) showed similar result with ours. They examined the situation where multi-product incumbents conduct both internal R&D and external R&D. Internal R&D is undertaken by leaders to improve their product, whereas external R&D is undertaken by both leaders and followers to obtain leadership over products that they do not currently possess. They showed that larger firms have a smaller ratio of R&D to their size, because internal R&D is proportional to the firm size while external R&D is constant. They mentioned that the reason why large firms have proportionately less external R&D relates to difficulty to protect the advanced position with too many project, limitation of managerial capabilities, and organizational rigidities etc.

cameras.[2] Second, we examine the effects of subsidizing followers' R&D and obtain a counter-intuitive result. R&D subsidies for followers promote R&D by followers naturally. However, this promotion of followers' R&D can also promote aggregate R&D by leaders. The reason is as follows. The promotion of followers' R&D impedes R&D by leaders. However, the promotion of followers' R&D also shifts the distribution of leaders; it reduces the number of leaders with larger leads and smaller R&D investment, while it increases that of leaders with smaller leads and larger R&D investment. If this positive effect associated with the changing distribution outweighs the negative effects on individual firms' R&D, the promotion of followers' R&D increases leaders' aggregate R&D. This result implies that R&D subsidies for new entry firms are effective in raising aggregate productivity growth, and provides a rationale for actual promotion policies for R&D activities by new entry firms.[3]

Some earlier studies, including Thompson and Waldo (1994), Segerstrom and Zornierek (1999), and Segerstrom (2007), developed R&D-based growth models where the leader firms conduct R&D activities and reexamined how aggregate R&D is determined. However, in the models, there is no factor that brings about heterogeneity among the leaders, and thus the leader firms are symmetric among industries. In contrast, Minniti, Parello, and Segerstrom (2013) incorporated uncertainty of innovation size (quality increment) into Segerstrom (1998) and constructed a Schumpeterian growth model where the innovation size and profits are different among industry leaders. However, this study assumes that R&D technology has constant returns, and thus the leader firms do not conduct R&D activities in equilibrium.[4] Therefore, constructing a realistic model where the R&D activities of leaders are asymmetric is one of the contributions of this study.[5] The finding that subsidies for followers' R&D increase the aggregate R&D is not

[2]Carroll and Mui (2008) mentioned that Polaroid as well as Eastman–Kodak did not spend enough on digital photography because it earned larger gross margins from its existing business: that is, film photography.

[3] In this paper, we focus our analysis mainly on the effect of subsidies for followers' R&D. There are two reasons for this. First, in reality, subsidy policies for entrants (followers) are used commonly. For instance, one of the innovation-promoting policies that the European Commission decided to implement in the "Innovation Union" was to promote entry and R&D activities by small and medium-sized enterprises (European Commission, 2010). Second, some empirical studies imply that competition can enhance R&D. We can regard subsidies for followers' R&D as one type of competition policy, and thus our result can support the above empirical result partly.

[4]Chu (2011) developed the quality-ladder model where there are heterogeneous industries and thus there are heterogeneous R&D firms. However, leader firms do not conduct R&D activities because of the constant returns of R&D technology, as in Minniti, Parello, and Segerstrom (2013).

[5]Acemoglu and Cao (2015) developed the quality-ladder model where leaders can innovate several times, and consequently the R&D activities of the leaders are asymmetric also in their model. However, in their model, R&D by leaders and R&D by followers are traded off against each other, and they do not necessarily adopt the position of advocating R&D subsidies for followers. In contrast with the result of the study, we show that promotion of followers' R&D can increase aggregate R&D by

new, as a similar result was obtained by Denicolo (2001) and Denicolo and Zanchettin (2012). However, as mentioned above, the mechanism in the present paper occurs through changing the distribution of heterogeneous leaders, which is different from the mechanism in these other papers.

The rest of the paper is structured as follows. Section 2 develops a heterogeneous-leaders Schumpeterian growth model and derives the equilibrium of the model. Section 3 presents the comparative statics. Section 4 discusses several extensions of the model. Section 5 concludes.

2 Model

As stated in the Introduction, we extend the quality-ladder model of Grossman and Helpman (1991) in the following two ways.[6]

First, to obtain the equilibrium where leaders conduct R&D activities as we observe in the real world, we assume that leaders have a diminishing-returns R&D technology, while followers have a constant-return R&D technology. In the quality-ladder model, where followers can leapfrog leaders, leaders lose the value of their current position when they succeed in the next innovation, while followers lose nothing. As long as leaders and followers have the same constant-returns R&D technology, leaders do not conduct R&D activity.[7] Second, to consider heterogeneity of leaders, we assume that innovation size—namely, the quality increment—is stochastic.

2.1 Consumers and workers

The economy has a fixed measure L of consumers. Intertemporal utility is given by:

$$U = \int_0^\infty e^{-\rho t} \left[\ln u(t) + (\bar{\ell} - \ell(t)) \right] dt, \tag{1}$$

leaders.

[6] Our model is based on a first-generation R&D-based growth model that exhibits a strong scale effect. That is, in the model, an increase in the total amount of resources devoted to R&D raises the rate of economic growth. Jones (1995a) showed that a strong scale effect is inconsistent with the empirical evidence, and Jones (1995b) and Segerstrom (1998) constructed R&D-based growth models that do not exhibit a scale effect.

[7] By assuming that followers cannot leapfrog leaders and can only improve their quality step-by-step, we can obtain positive R&D activities by leaders, as in Aghion, Harris, Howitt, and Vickers (2001) and Acemoglu and Akcigit (2012). In this setting, leaders can make the quality-gap from followers larger by conducting R&D activities and improving their quality, and keep their position as leaders longer.

Alternatively, by assuming that leaders can decide their amounts of R&D before followers decide, we can obtain positive R&D activities by leaders, as in Etro (2004), Ledezma (2013), and Kiedaisch (2015). In this setting, leaders conduct more R&D activities than followers in equilibrium.

where ρ is a subjective discount rate, $\ell(t)$ denotes supplied labor at time t, $\bar{\ell}$ denotes the maximum labor supply, and $u(t)$ represents utility from consumption at time t.[8] We specify utility per capita from consumption as follows:

$$\ln u(t) = \int_0^1 \ln \sum_j q(j,\omega,t) d(j,\omega,t) d\omega, \tag{2}$$

where $q(j,\omega,t)$ and $d(j,\omega,t)$ denote the quality j in industry ω and the consumption volume of the good, respectively.

Solving the utility maximization problem of the household, we now derive the demand for each differentiated good $\omega \in [0,1]$. First, in each differentiated goods industry, households consume only the good with the lowest quality-adjusted price, which is provided by the firm having the highest quality in equilibrium. Letting $p(\omega,t)$ denote the price of the highest quality in industry ω, the demand for the good with the highest quality in industry ω is given by:

$$d(\omega,t) = \frac{E(t)}{p(\omega,t)}, \tag{3}$$

where $E(t)$ denotes expenditures. Substituting the demand function into the utility function, we can derive the indirect utility as:

$$\ln u(t) = \ln E(t) - \left\{ \int_0^1 \ln \left[\frac{p(\omega,t)}{q(\omega,t)} \right] dj \right\}.$$

Second, we must solve the dynamic optimization problem as follows:

$$\max \int_0^\infty e^{-\rho t} \left[\ln E(t) - \left\{ \int_0^1 \ln \left[\frac{p(\omega,t)}{q(\omega,t)} \right] dj \right\} + (\bar{\ell} - \ell(t)) \right] dt,$$

subject to

$$\dot{a}(t) = r(t)a(t) + w(t)\ell(t) - E(t) - T(t),$$

where $a(t)$ denotes the asset holding per capita and $w(t)$ and $T(t)$ denote the wage rate and the lump-sum tax. We take labor rate as a numeraire, therefore $w(t) = 1$. As long as $\ell(t) < \bar{\ell}$, $1 = \frac{w(t)}{E(t)}$ must hold. This optimality condition can be reduced to $E(t) = 1$, and thus from the Euler equation, $\frac{\dot{E}(t)}{E(t)} = r(t) - \rho$, we obtain $r(t) = \rho$; that is, the interest rate is constant over time. Hereafter, we let r denote the constant interest rate.

[8]Labor supply is assumed to be perfectly elastic as in Aghion, Harris, and Vickers (1997). As a result, utility is quasilinear.

2.2 Firms

In each industry, there are different qualities that consumers can buy. However, as mentioned before, consumers buy only the good with the lowest quality-adjusted price.[9]

Hereafter, we refer to the firm producing the highest quality in each industry as the leader and refer to the other firms conducting R&D activities to leapfrog the leader as followers. If a firm succeeds in achieving quality improvement, the quality increases by λ, which we assume is determined by a random draw from a given distribution. If the highest quality in industry ω is $q(\omega)$ before innovation, the quality of the new leader is given by $\lambda q(\omega)$ while the quality of its followers is $q(\omega)$. Regardless of the quality, every leader must hire one unit of labor to produce one unit of a good. As we take labor as a numeraire, the marginal cost is equal to one. In each industry, the leader charges a price so that the followers cannot earn a positive profit. Therefore, the leader charges a price equal to the quality lead λ. Thus, the profit per consumer is given by $\pi(t) = (p(\omega,t) - 1)d(\omega,t) = 1 - \frac{1}{\lambda}$.

Originally, we identify leaders with quality increments λ that are obtained by their innovations. However, for analytical simplicity, we assume that profit per consumer π is among $[0, \bar{\pi}]$, and hereafter we identify leaders with profit sizes of $\pi \in [0, \bar{\pi}]$ and refer to the leader with profit π as π-leader.

We let $V(\pi)$ and $I(\pi)$ denote the value and R&D intensity of π-leader respectively. The R&D technology of leaders is assumed to exhibit diminishing returns. By devoting $\frac{C}{2}I(\pi)^2 dt$ units of labor in infinitesimal time interval dt, the leaders will succeed in the invention of new quality with probability $I(\pi)dt$. In addition, for analytical simplicity, we assume that once new quality is invented, the previous quality becomes public domain; that is, any other firm can produce the previous quality. On this assumption, the value of the leader who obtains quality lead λ is simply $V(\pi)$ irrespective of the previous quality of the leader. Therefore, the Hamilton–Jacobi–Bellman equation (hereafter, HJB equation) of the π-leader is given by:

$$rV(\pi) = \max_{I(\pi)} \left\{ \pi L - \frac{C}{2}[I(\pi)]^2 + I(\pi)\left[EV - V(\pi)\right] - I_F V(\pi) \right\}, \tag{4}$$

where EV is the expected value of the firm that succeeds in innovating and I_F is the probability of innovation by followers. Letting $g(\pi)$ denote the distribution of π, we can express EV as follows:

$$EV \equiv E[V(\tilde{\pi})] = \int_0^{\bar{\pi}} V(\pi)g(\pi)d\pi.$$

From (4), the first-order condition for leaders is:

$$EV - V(\pi) = CI(\pi), \tag{5}$$

[9] Some studies have examined the situation where more than two firms operate in each industry. For example, Denicolo and Zanchettin (2010) examined the situation where several asymmetric firms can be simultaneously active in an industry.

if $EV \geq V(\pi)$, otherwise $I(\pi) = 0$.

The firms other than the leaders—that is, the followers—also conduct R&D activities. The R&D technology of followers is constant returns to scale in contrast with the R&D technology of leaders. We let V_F denote the value of a follower. The HJB equation of the follower is:

$$rV_F = \max_{I_F} \left\{0 - (1 - s_F)\bar{C}_F I_F + I_F \left[EV - 0\right]\right\}$$

where $\bar{C}_F$ denotes the R&D cost for followers and s_F denotes the R&D subsidy rate for followers. [10] The R&D equilibrium condition for followers is:

$$EV = (1 - s_F)\bar{C}_F \equiv C_F \tag{6}$$

and thus we obtain $V_F = 0$.

3 Equilibrium

We derive the equilibrium in the present model and the effects of subsidies for followers' R&D on followers' R&D, the π-leader's R&D, and aggregate R&D by leaders, and the sum of leaders' R&D and followers' R&D.

3.1 R&D by leaders

First, we derive the equilibrium R&D of π-leader $I(\pi)$ given I_F.

From the first-order condition of R&D for leaders (5) and the equilibrium condition of R&D for followers (6), we obtain:

$$CI(\pi) = \begin{cases} C_F - V(\pi) & \text{if} \quad 0 \leq V(\pi) \leq C_F, \\ \\ 0 & \text{if} \quad V(\pi) > C_F. \end{cases} \tag{7}$$

[10] Like Segerstrom (2007), we assume that R&D by leaders exhibits decreasing returns while R&D by followers exhibits constant returns to obtain the equilibrium where both leaders and followers engage in R&D activities; if we assume that R&D by leaders exhibits constant returns, either leaders or followers do all the R&D in the economy. This assumption means that industry leaders have cost advantages over followers, and the related literature provides several plausible interpretations of the assumption; only leaders have state-of-the-art technology in their industries, and have better experience, ability or knowledge on R&D activities in their industries. Alternatively, we can assume that leaders have first-mover advantage, such as Gilbert and Newbery (1982), Etro (2004), and Kiedaisch (2015). The first-mover-advantage setting is interesting and intuitive. However, to simplify the equilibrium of the model as much as possible, the present paper assumes the difference in R&D technologies between leaders and followers, as in Segerstrom (2007).

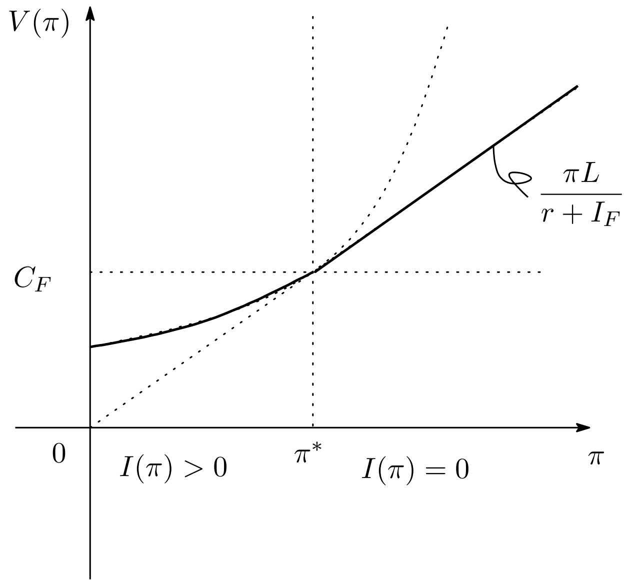

Figure 1: The value of π-leader

For the leaders with a stock value that satisfies $0 \leq V(\pi) \leq C_F$, substituting the first-order condition into the HJB equation (4) yields the quadratic equation with respect to $V(\pi)$, as follows:

$$V(\pi) = \frac{\pi L + \frac{1}{2C}\left[C_F - V(\pi)\right]^2}{r + I_F}. \tag{8}$$

For the leaders with a stock value that satisfies $V(\pi) > C_F$, they conduct no R&D activity and the stock value consists of only profit flow, as follows:

$$V(\pi) = \frac{\pi L}{r + I_F}.$$

We define the critical level of profit in terms of whether the leader conducts R&D activities or not as π^*. π^* satisfies $V(\pi^*) = C_F$ and π^* is given by:

$$\pi^* = (r + I_F)\frac{C_F}{L}. \tag{9}$$

As a result, if the profit flow π exceeds the critical level, the leader with profit flow π conducts no R&D activity, and the stock value is given by $\pi L/(r + I_F)$, which satisfies $V(\pi) > C_F$. If the profit flow π is lower than the critical level, the leader conducts R&D activities, and the stock value is given by the smaller solution of the quadratic equation (8), which satisfies $V(\pi) < C_F$, as shown in Figure 1. The stock value is given by:

$$V(\pi) = \begin{cases} C_F + (r+I_F)C - C\sqrt{(2\dfrac{C_F}{C} + r + I_F)(r+I_F) - 2\pi\dfrac{L}{C}} & \text{if} \quad 0 \le \pi \le \pi^*, \\ \\ \dfrac{\pi L}{r+I_F} & \text{if} \quad \pi^* < \pi \le \bar{\pi}. \end{cases} \tag{10}$$

Substituting the stock value into the optimality condition for R&D (7), given I_F, we obtain the π-leader's R&D as follows:

$$I(\pi) = \begin{cases} \sqrt{(2\dfrac{C_F}{C} + r + I_F)(r+I_F) - 2\dfrac{L}{C}\pi} - (r+I_F), & \text{if} \quad 0 \le \pi \le \pi^*, \\ \\ 0 & \text{if} \quad \pi^* < \pi \le \bar{\pi}. \end{cases} \tag{11}$$

(11) shows that the innovation intensity of the leader is a decreasing function of π. Therefore, we can summarize the result in the following proposition.

Proposition 1 *The leader that has higher quality compared with followers and earns higher profit makes a smaller R&D investment.*

As mentioned in the Introduction, the intuition of this result is as follows. Leaders lose the value of their current position when they succeed in the next innovation. Therefore, the return of R&D for the leader that has a larger quality lead over their followers is lower, and thus, a higher-quality leader makes a smaller R&D investment.[11]

In the quality-ladder model of Grossman and Helpman (1991), leaders and followers have the same constant-returns R&D technology, and the return on R&D for leaders is necessarily lower than that of followers, and thus, leaders never conduct R&D activities. This is called the Arrow effect. In this paper, where R&D technologies are different between leaders and followers, some leaders conduct R&D activities. However, leaders with a larger quality lead lose the larger value when they succeed in the next innovation, and thus, leaders with a larger quality lead conduct smaller R&D activities. Therefore, we can call this a weak form of the Arrow effect.[12]

The theoretical result in Proposition 1 is consistent with some actual firms' behaviors. For instance, as mentioned in the Introduction, Sony succeeded in developing the high-quality flat cathode-ray tube TV

[11] We assume that leaders cannot withdraw the innovation when they draw a quality increment that is smaller than that which they own currently. However, even if leaders can withdraw the smaller quality increment, we obtain the same result as Proposition 1, as we show in Section 4.4.

[12] See Cozzi (2007) for a discussion of the Arrow effect.

and did not invest in the development of a liquid crystal TV. Consequently, Sony did not obtain a large share of the market for liquid crystal TVs. In addition, some empirical studies have showed that R&D investment by smaller firms accounts for a large proportion of important innovations.[13] This paper's result is consistent with this tendency.

3.2 R&D by followers

Next, we derive the equilibrium R&D of followers I_F.

In the rest of the analysis, we assume that the distribution of profit is a uniform distribution over $[0, \bar{\pi}]$ for analytical simplicity.[14] Hereafter, we use $R \equiv r + I_F$ instead of I_F, and we use $\tilde{C}_F \equiv C_F/C$ and $A \equiv L/C$ for notational simplicity. We can calculate the expected value of $V(\pi)$ as follows:

$$\begin{aligned} E[V(\tilde{\pi})] &= \int_0^{\pi^*} V(\pi)g(\pi)d\pi + \int_{\pi^*}^{\bar{\pi}} V(\pi)g(\pi)d\pi \\ &= \frac{C}{A\bar{\pi}}\left((\tilde{C}_F + R)\tilde{C}_F R + \frac{1}{3}\left\{R^3 - \left[(2\tilde{C}_F + R)R\right]^{\frac{3}{2}}\right\}\right) + \frac{C}{2A\bar{\pi}}\left(\frac{(A\bar{\pi})^2}{R} - \tilde{C}_F^2 R\right). \end{aligned}$$

We can rewrite this as follows:

$$E[V(\tilde{\pi})] = \frac{C}{A\bar{\pi}} G(R, \tilde{C}_F)$$

where

$$G(R, \tilde{C}_F) \equiv \left((\tilde{C}_F + R)\tilde{C}_F R + \frac{1}{3}\left\{R^3 - \left[(2\tilde{C}_F + R)R\right]^{\frac{3}{2}}\right\}\right) + \frac{1}{2}\left(\frac{(A\bar{\pi})^2}{R} - \tilde{C}_F^2 R\right).$$

This expected value must satisfy the R&D equilibrium condition for followers, (6), and thus, we obtain:

$$G(R, \tilde{C}_F) = A\bar{\pi}\, \tilde{C}_F. \tag{12}$$

This equation determines the equilibrium value of R.

As shown in Appendix A, we can show that $\frac{dR}{d\tilde{C}_F} < 0$. Thus, we can summarize the result as follows.

Proposition 2 *A decrease in R&D costs for followers from, for example, subsidies promotes followers' R&D.*

[13] According to Scherer (1984), companies with fewer than 1,000 employees were responsible for 47.3 percent of important innovations, while large companies with over 10,000 employees were responsible for only 34.5 percent of important innovations.

[14] In Section 4.3, we consider the realistic case where the distribution of quality lead is a Pareto distribution.

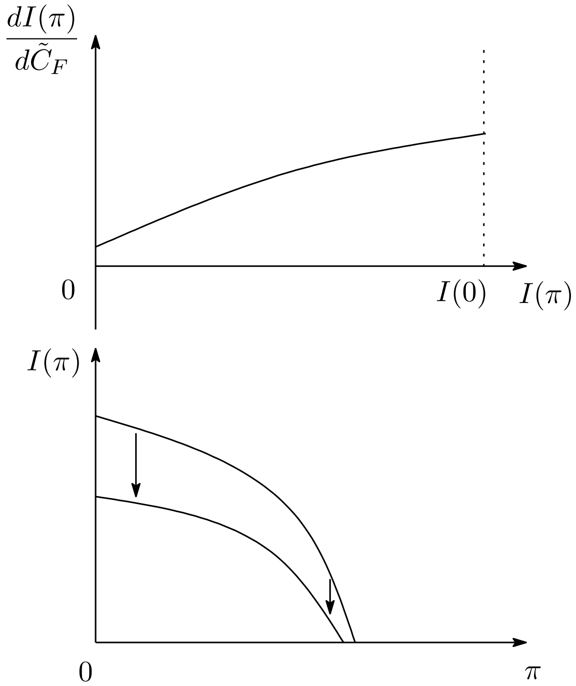

Figure 2: The effects of subsidies for followers' R&D on leaders' R&D

3.3 Effects of R&D subsidies for followers on leaders' R&D

We have shown that subsidies for followers' R&D promote followers' R&D. Now we examine the effects of subsidies for followers' R&D on leaders' R&D.

Differentiating (11) when $\pi < \pi^*$, we obtain:

$$\frac{dI(\pi)}{d\tilde{C}_F} = \frac{1}{I(\pi) + R} \left\{ \left[\tilde{C}_F - I(\pi) \right] \frac{dR}{d\tilde{C}_F} + R \right\}. \tag{13}$$

As shown in Appendix B, if the elasticity of followers' R&D with respect to R&D costs is sufficiently high to satisfy the condition:

$$\text{Condition (R)} \qquad -\frac{\tilde{C}_F}{R}\frac{dR}{d\tilde{C}_F} > \frac{\tilde{C}_F}{R + \tilde{C}_F},$$

the effect of an increase in $\tilde{C}_F$ on $I(\pi)$ is an increasing function of $I(\pi)$, as depicted in Figure 2. We can summarize the result as follows.

Proposition 3 *If the elasticity of followers' R&D with respect to R&D subsidies is sufficiently high to satisfy Condition (R), the effect of subsidies for followers' R&D on the π-leader's R&D decreases as π increases.*

Next, we examine the effect of an increase in $\tilde{C}_F$ on the R&D of the π^*-leader, the leader with the minimum profit, who conducts no R&D activity. When $I(\pi^*) = 0$, we can rewrite (13) as follows:

$$\frac{dI(\pi^*)}{d\tilde{C}_F} = \frac{\tilde{C}_F}{R}\frac{dR}{dC_F} + 1. \tag{14}$$

Therefore, if the elasticity of followers' R&D is sufficiently high to satisfy the following condition,

$$\text{Condition (R')} \qquad -\frac{\tilde{C}_F}{R}\frac{dR}{d\tilde{C}_F} > 1,$$

then R&D subsidies for followers force the leader who would otherwise conduct no R&D investment to undertake investment.[15]

3.4 Effects on aggregate R&D by leaders

In this section, we examine how aggregate R&D by leaders is determined and examine the effects of subsidies for followers' R&D on aggregate R&D by leaders.

We let $\mu(\pi, t)$ denote the distribution of π-leaders, and thus, we can express the aggregate innovation of leaders as follows:

$$I_L(t) = \int_0^{\bar{\pi}} I(\pi)\mu(\pi, t)d\pi. \tag{15}$$

We derive the stationary distribution of π-leaders. The inflow of π-leaders in time interval dt is a proportion $g(\pi)$ of the number of leaders and followers that succeed in innovation at the interval; that is, $[I_L(t)dt + I_F dt]\, g(\pi)$. The outflow of π-leaders is the sum of the number of π-leaders that succeed in innovation in the interval and that of followers that target the industries of present π-leaders and succeed in innovation in the interval; that is, $[I(\pi)dt + I_F dt]\, \mu(\pi, t)$. Therefore, the change in the distribution of π-leaders is given by:

$$\frac{\partial \mu(\pi, t)}{\partial t} = [I_L(t) + I_F]\, g(\pi) - [I(\pi) + I_F]\, \mu(\pi, t). \tag{16}$$

We focus our analysis on the stationary distribution for simplicity. Then, from (16), we obtain the stationary distribution of the leaders as follows:

$$\mu(\pi) = \frac{I_L + I_F}{I(\pi) + I_F}\frac{1}{\bar{\pi}}. \tag{17}$$

Substituting (17) into (15) yields the equation that determines the equilibrium leaders' aggregate R&D I_L as follows:

$$I_L = \int_0^{\bar{\pi}} \frac{I_L + I_F}{I(\pi) + I_F}\frac{1}{\bar{\pi}} I(\pi)d\pi. \tag{18}$$

[15] On the assumption that profit is uniformly distributed, this case does not hold.

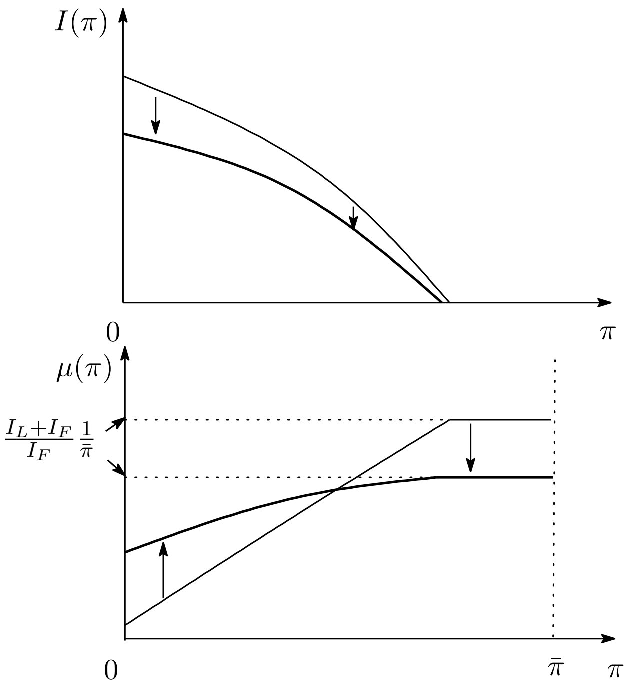

Figure 3: Change in the stationary distribution of leaders

(17) shows that higher π results in smaller $I(\pi)$, and thus higher $\mu(\pi)$, as depicted in Figure 3. This means that the distribution of the leader with higher π, which conducts a lower level of R&D $I(\pi)$, is larger. Moreover, if $I(\pi) > (<) I_L$, a rise in I_F increases (decreases) the distribution of the π-leader, as depicted in the lower panel of Figure 3. This means that an increase in followers' R&D increases (decreases) the distribution of the π-leader that conducts higher (lower) levels of R&D than the average level of R&D by leaders. The intuitions for these results are as follows. The stationary distribution of the π-leader satisfies:

$$[I(\pi) + I_F]\,\mu(\pi) = [I_L + I_F]\,g(\pi). \tag{19}$$

First, the leaders with higher π conduct lower levels of R&D $I(\pi)$, and thus the outflow of leaders with higher π is smaller. Hence the distribution of the leaders with higher π is larger. Second, if $I(\pi) > (<) I_L$, the outflow is relatively larger (smaller) than the inflow. An increase in followers' R&D raises both flows but raises the inflow (outflow) proportionately more, and thus increases (decreases) the distribution of the π-leader.

Now, we examine the effects of subsidies for followers' R&D on aggregate R&D by leaders I_L. As mentioned above, if the elasticity of followers' R&D is lower, subsidies for followers' R&D tend to decrease leaders' R&D irrespective of the profit level of the leader. Regardless, subsidies for followers' R&D may increase aggregate R&D by leaders. Subsidies for followers' R&D affect the aggregate R&D

by leaders through the following two channels. First, subsidies for followers' R&D affect the R&D level of each leader. We refer to this effect as the *individual effect*. Through this effect, subsidies necessarily reduce the aggregate R&D by leaders, as depicted in the upper panel of Figure 3. Second, they change the stationary distribution of the π-leader. Subsidies for followers' R&D promote followers' R&D I_F. As we showed before, an increase in I_F increases (decreases) the distribution of the π-leader that conducts higher (lower) levels of R&D more than the average level of R&D by leaders; that is, $I(\pi) > (<) I_L$, as depicted in the lower panel of Figure 3. From Proposition 1, leaders with lower profits conduct higher levels of R&D. Therefore, through the change in the distribution, followers' R&D increased by subsidies raises the aggregate R&D by leaders. We refer to this effect as the *distribution effect*. If the distribution effect overwhelms the individual effect, subsidies for followers' R&D raise the aggregate R&D by leaders.

By providing numerical examples, we show that subsidies for followers' R&D raise the aggregate R&D by leaders under certain parameter values.[16] In the numerical example, we use the following parameters: $\rho = 0.04, \bar{\pi} = 0.4, A(\equiv L/C) = 1$, and $\tilde{C}_F \in [0.5, 3]$. First, following Jones and Williams (2000), we set the interest rate to 0.04. In this model, $r(t) = \rho$, and we set the subjective discount rate at 0.04. Second, empirical studies such as Norrbin (1993) and Basu (1996) estimate the range of markup of price over marginal cost as [1.05, 1.4]. Thus, we set the upper value of the profit $\bar{\pi}$ at 0.4 so that the expected profit equals 0.2, which is obtained when the markup is 1.25: that is, in the middle of the estimated range. Finally, we choose the value of A and the range of $\tilde{C}_F$ so that the range of the growth rate of utility equals the range of the GDP per capita growth rate.[17]

As shown in the left panel of Figure 4, under larger followers' R&D costs, a decrease in followers' R&D costs increases the aggregate R&D by leaders, while under smaller followers' R&D costs, it decreases the aggregate R&D by leaders. We can surmise the reason for this result as follows. When followers' R&D costs are smaller, followers' R&D is larger, as shown in the middle panel of Figure 4. This makes the distribution of leaders flatter and the increase in aggregate R&D by leaders through the distribution effect tends to be smaller, and thus, overwhelmed by the decrease in aggregate R&D by leaders through the individual effect.

[16] On the assumption that profit is uniformly distributed, we can derive aggregate R&D by leaders, I_L, as a function of followers' R&D I_F, as shown in Appendix C, and we can provide numerical examples easily.

[17] We derive the growth rate of utility in this model later.

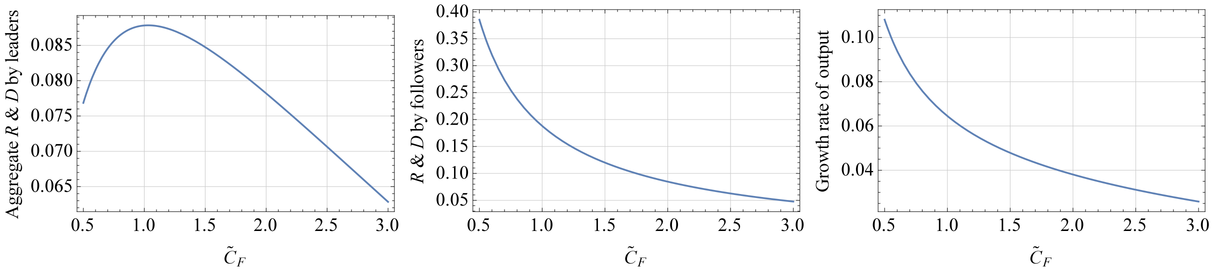

Figure 4: Aggregate R&D by leaders I_L, R&D by followers I_F, and growth rate of output

The horizontal axis represents the relative cost of followers' R&D, $\tilde{C}_F \equiv (1 - s_F)\bar{C}_F/C$.

3.5 Effects on aggregate R&D

Next, we examine how the aggregate R&D—that is, the sum of R&D by followers and R&D by leaders—is determined. Rewriting the definition of I_L (18), we obtain the equation of the aggregate R&D $I(\equiv I_L + I_F)$ as follows: [18]

$$\frac{1}{I_L + I_F} = \int_0^{\pi^*} \frac{1}{I(\pi) + I_F} \frac{1}{\bar{\pi}} d\pi + \int_{\pi^*}^{\bar{\pi}} \frac{1}{I_F} \frac{1}{\bar{\pi}} d\pi. \tag{20}$$

This equation determines the equilibrium aggregate R&D I. Differentiating (20) with respect to $\tilde{C}_F$ yields:

$$\frac{1}{I^2} \frac{dI}{d\tilde{C}_F} = \int_0^{\pi^*} \frac{1}{[I(\pi) + I_F]^2} \left[\frac{dI(\pi)}{d\tilde{C}_F} + \frac{dI_F}{d\tilde{C}_F} \right] \frac{1}{\bar{\pi}} d\pi + \int_{\pi^*}^{\bar{\pi}} \frac{1}{I_F{}^2} \frac{dI_F}{d\tilde{C}_F} \frac{1}{\bar{\pi}} d\pi. \tag{21}$$

By using (13), we can rewrite $\frac{dI(\pi)}{d\tilde{C}_F} + \frac{dI_F}{d\tilde{C}_F}$ as follows:

$$\begin{aligned} \frac{dI(\pi)}{d\tilde{C}_F} + \frac{dI_F}{d\tilde{C}_F} &= \frac{1}{I(\pi) + R} \left\{ \left[\tilde{C}_F - I(\pi) \right] \frac{dR}{d\tilde{C}_F} + R \right\} + \frac{dR}{d\tilde{C}_F} \\ &= \frac{1}{I(\pi) + R} \left\{ \left(\tilde{C}_F + R \right) \frac{dR}{d\tilde{C}_F} + R \right\}. \end{aligned}$$

From this equation, if Condition (R) holds, subsidies for followers' R&D increase the sum of leaders' R&D and followers' R&D in each industry where leaders' R&D is positive irrespective of the profit of the industry leader. Then, the first term of the RHS of (21) is negative. Moreover, from Proposition 2, the

[18]Dividing both sides of (18) by $I_L + I_F$ yields:

$$\frac{I_L}{I_L + I_F} = \int_0^{\bar{\pi}} \frac{I(\pi)}{I(\pi) + I_F} \frac{1}{\bar{\pi}} d\pi.$$

Subtracting both sides of this equation from one, we obtain:

$$\frac{I_F}{I_L + I_F} = \int_0^{\bar{\pi}} \frac{I_F}{I(\pi) + I_F} \frac{1}{\bar{\pi}} d\pi.$$

Dividing both sides of this equation by I_F, we obtain (20).

second term of the RHS of (21) is negative. Thus, the effects on the aggregate R&D are in the following proposition.

Proposition 4 *If the elasticity of followers' R&D with respect to R&D costs is sufficiently high to satisfy Condition (R), subsidies for followers' R&D increase the sum of leaders' R&D and followers' R&D in each industry, and consequently increase the aggregate R&D.*

This result implies that R&D subsidies for new entry firms are effective in raising aggregate productivity growth. [19]

3.6 Effects on the growth rate

We derive the growth rate of output. In the quality-ladder model, $u(t)$ in (2) is the volume of final output. We can calculate $\ln u(t)$ as follows:

$$\begin{aligned} \ln u(t) &= \int_0^1 \ln \sum_j q(j,\omega,t) d(j,\omega,t) d\omega \\ &= \int_0^1 \ln q(\omega,t) d\omega + \int_0^1 \ln d(\omega,t) d\omega, \end{aligned}$$

where $q(\omega,t)$ and $d(\omega,t)$ denote the state-of-the-art quality and its consumption volume in industry ω at time t, respectively. The above equation shows that the utility can be decomposed into the utility from quality $\ln Q(t)(\equiv \int_0^1 \ln q(\omega,t)d\omega)$ and the utility from quantity $\ln D(t) \equiv \int_0^1 \ln d(\omega,t)d\omega$. On the balanced growth path, the utility from quantity is constant, and the growth rate of $u(t)$ equals that of $Q(t)$, which is given by:

$$\frac{d \ln Q(t)}{dt} = E[\ln \lambda] \left(I_L + I_F \right). \tag{22}$$

We assume that the distribution of λ is a uniform distribution, and we can calculate $E[\ln \lambda]$ as follows:

$$E[\ln \lambda] = \frac{1 - [1 - \ln(1-\bar{\pi})]\,(1-\bar{\pi})}{\bar{\pi}}.$$

The growth rate is proportional to aggregate R&D, $(I_L + I_F)$. Under the parameter values that we use in the numerical example, aggregate R&D increases as the cost for followers' R&D decreases, and thus, the growth rate rises as shown in the right panel of Figure 4.

[19] We cannot provide a numerical example where subsidies for followers' R&D reduce aggregate R&D, even when Condition (R) does not hold. Therefore, we guess that subsidies for followers' R&D always increase aggregate R&D in our model.

3.7 Effects on welfare

Finally, we examine whether subsidies for followers' R&D improve welfare. As in Appendix D, we can show that the market equilibrium level of R&D by leaders is lower than the socially optimal level for all industries. This result is due to the Arrow effect; the leaders lose the value of their current position when they succeed in the next innovation in the market equilibrium, as shown in (4), and thus, the leaders' incentive is weaker than that of the social optimum level. If Condition (R) holds, subsidies for followers' R&D enhance R&D by leaders and thus, subsidies for followers' R&D are likely to improve welfare. In this section, we examine whether subsidies for followers' R&D raise welfare.

First, we derive the welfare function. We focus our analysis on the balanced growth path as before. From (1), we can derive the welfare on the balanced growth path as follows:

$$\begin{aligned} U &= \int_0^\infty e^{-\rho t}\left[\ln Q(t) + \ln D + (\bar{\ell}-\ell)\right] dt \\ &= \int_0^\infty e^{-\rho t}\left[E[\ln\lambda](I_L+I_F)t + \ln Q(0) + \ln D + (\bar{\ell}-\ell)\right] dt \\ &= \frac{1}{\rho^2}E[\ln\lambda]\underbrace{(I_L+I_F)}_{\text{aggregate R\&D (+)}} + \frac{1}{\rho}\left[\ln Q(0) + \underbrace{\ln D}_{\text{production (+)}} + \underbrace{(\bar{\ell}-\ell)}_{\text{disutility (-)}}\right]. \end{aligned} \tag{23}$$

We can calculate the utility from quantity $\ln D$ and the labor supply per capita ℓ on the balanced growth path.[20]

Promoting followers' R&D using subsidies affects welfare in the following three ways. First, according to Proposition 4, as long as the elasticity of followers' R&D with respect to R&D cost is sufficiently

[20]First, we can calculate the utility from quantity as follows:

$$\ln D = \int_0^1 \ln d(\omega,t)d\omega = \int_0^{\bar{\pi}} \ln(1-\pi)\mu(\pi)d\pi = \int_0^{\bar{\pi}} \ln(1-\pi)\frac{I_L+I_F}{I(\pi)+I_F}\frac{1}{\bar{\pi}}d\pi,$$

where we use (17). Second, we can also derive the total labor supply per capita ℓ. The total labor supply is the sum of the production labor and the R&D labor. The labor devoted to production is:

$$\int_0^1 d(\omega,t)d\omega = \int_0^{\bar{\pi}} (1-\pi)\mu(\pi)d\pi = \int_0^{\bar{\pi}} (1-\pi)\frac{I_L+I_F}{I(\pi)+I_F}\frac{1}{\bar{\pi}}d\pi,$$

where we also use (17). The labor devoted to R&D activities by leaders is:

$$\int_0^1 \frac{C}{2}[I(\pi)]^2\mu(\pi)d\pi = \int_0^{\bar{\pi}} \frac{C}{2}[I(\pi)]^2\frac{I_L+I_F}{I(\pi)+I_F}\frac{1}{\bar{\pi}}d\pi.$$

The labor devoted to R&D activities by followers is $C_F I_F$. Therefore, the labor supply per capita ℓ is given by:

$$\ell = \int_0^{\bar{\pi}} (1-\pi)\frac{I_L+I_F}{I(\pi)+I_F}\frac{1}{\bar{\pi}}d\pi + \int_0^{\bar{\pi}} \frac{C}{2}[I(\pi)]^2\frac{I_L+I_F}{I(\pi)+I_F}\frac{1}{\bar{\pi}}d\pi + C_F I_F.$$

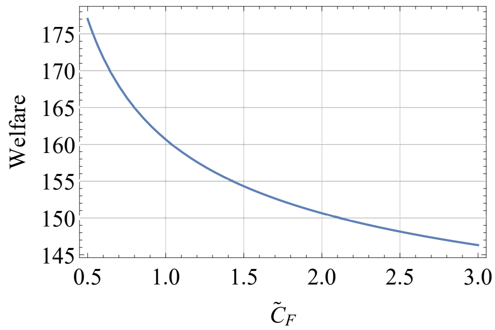

Figure 5: Welfare

The horizontal and vertical axes represent $\tilde{C}_F \equiv (1 - s_F)\bar{C}_F/C$ and welfare U, respectively.

We set $\bar{\ell} = 6$ so that the labor supply is positive, and we choose $C = 1$, which is a relatively low R&D cost, here.

high to satisfy Condition (R), promoting followers' R&D increases the aggregate R&D $I_L + I_F$, and thus, raises the growth rate. This increases the welfare. We refer to this effect as the *aggregate R&D-promoting effect*, which is indicated by the first term of the RHS of (23). Second, promoting followers' R&D changes the distribution of leaders. The increase in followers' R&D increases the distribution of leaders with lower profit. This means an increase in the distribution of leaders with larger production volumes, which reduces the welfare loss, because of monopoly power, and raises the utility from quantity $\ln D$. We refer to this effect as the *competition promoting effect*, which is indicated by the third term of the RHS of (23). Third, promoting followers' R&D affects the labor devoted to production and R&D activities. As mentioned above, promoting followers' R&D increases the distribution of leaders with larger production volumes, and thus, increases the labor devoted to production. As also mentioned above, promoting followers' R&D also increases the aggregate R&D, and thus, is likely to increase the labor devoted to R&D activities. These increases in the labor supply increase the disutility of labor and lowers the welfare. We refer to this effect as the *disutility effect*, which is indicated by the fourth term of the RHS of (23).

We examine how subsidies for followers' R&D affect the welfare numerically. As shown in Figure 5, the numerical results show that as long as the R&D cost is not extremely large, the aggregate R&D-promoting effect and the production-increasing effect outweigh the disutility effect, and thus, promoting followers' R&D raises the welfare.[21]

[21] If we choose a high R&D cost, such as $C = 20$, the disutility effect is larger than the two positive welfare effects; a decrease in R&D cost for followers relative to that for leaders $\tilde{C}_F$ can reduce the welfare.

4 Extensions

4.1 Effects of R&D subsidies for leaders

In the previous sections, we focus on R&D subsidies for followers.

In this section, we examine the effects of R&D subsidies for leaders. Only in this section, we let C denote the parameter of R&D cost for leaders after subsidies. That is, $C \equiv (1 - s_L)\bar{C}$ where $\bar{C}$ denotes the R&D-cost parameter for leaders and s_L denotes the R&D subsidy rate for leaders. We examine the effects of reducing C in this section.

First, we examine the effect of R&D subsidies for leaders on followers' R&D. To examine the effect of R&D subsidies for leaders, we differentiate (10) with respect to C. For $\pi \in [0, \pi^*]$, we obtain:

$$\frac{\partial V(\pi)}{\partial C} = -\left(\frac{2}{C}(C_F R - L\pi) + R^2\right)^{-\frac{1}{2}} \left\{\frac{1}{C}(C_F R - L\pi) + R^2 - R\left(\frac{2}{C}(C_F R - L\pi) + R^2\right)^{\frac{1}{2}}\right\} < 0. \quad (24)$$

This inequality shows that R&D subsidies for leaders increase the value of leaders that conduct R&D as long as followers' R&D is constant.[22] For $\pi \in (\pi^*, \bar{\pi}]$, $V(\pi)$ is independent of C as long as followers' R&D is constant. Therefore, followers' R&D must increase to satisfy (6), and we obtain:

$$\frac{dR}{dC} < 0. \quad (25)$$

The intuition of this result is that R&D subsidies for leaders increase the value of leaders that conduct R&D, and thus, stimulates entrants' incentives to be leaders. Then followers' R&D increases, and this mechanism continues until the expected gain for followers reaches zero.[23]

Next, we examine the effect of R&D subsidies for leaders on leaders' R&D. Differentiating (11) with respect to C when leaders' R&D is positive, we obtain:

$$\frac{dI(\pi)}{dC} = \frac{1}{I(\pi) + R}\left(\left(\tilde{C}_F - I(\pi)\right)\frac{dR}{dC} - \frac{R\tilde{C}_F - \pi A}{C}\right) < 0, \quad (26)$$

where we use $\pi < \pi^* \equiv \frac{R\tilde{C}_F}{A}$. Then we can confirm that the individual effect is positive for all leaders that conduct R&D. Moreover, we can confirm that the critical level of profit, π^*, also increases with R&D subsidies for leaders. Differentiating (26) with respect to $I(\pi)$, we obtain:

$$\frac{d}{dI(\pi)}\frac{dI(\pi)}{dC} = \frac{1}{[I(\pi) + R]^2}\left(-\underbrace{\frac{dR}{dC}}_{-}\left[\tilde{C}_F + R\right] + \frac{R\tilde{C}_F - \pi A}{C} - \frac{A}{C}\underbrace{\frac{d\pi}{dI(\pi)}}_{-}[I(\pi) + R]\right) > 0. \quad (27)$$

[22] In Appendix E, we show $\frac{1}{C}(C_F R - L\pi) + R^2 - R\left(\frac{2}{C}(C_F R - L\pi) + R^2\right)^{\frac{1}{2}} > 0$.

[23] This result means that R&D subsidies for leaders also benefit followers. Acemoglu and Akcigit (2012) found a similar result in their model and referred to this effect as the "trickle-down effect".

This inequality shows that the effect of subsidies for leaders' R&D on π- leader's R&D increases as π increases. The intuition is as follows: subsidies for leaders' R&D have two effects on leaders' R&D and the value of leaders. The first is the direct effect; the decrease in R&D cost stimulates leaders' R&D and increases the value of leaders that conduct R&D. The second is the effect through the increase in followers' R&D. This decreases the value of leaders, and the size of the effect increases as π increases. Through these two effects, subsidies for leaders' R&D increase the value of low π- leaders more than they increase the value of high π- leaders. This weakens (strengthens) the Arrow effect of high (low) π- leaders. As the direct effect outweighs the effect of strengthening the Arrow effect even for low π- leaders, the effect of subsidies for leaders' R&D on the individual effect is always positive. As π increases, the effect of strengthening the weak form of the Arrow effect decreases. Therefore, subsidies for leaders' R&D tend to stimulate high π- leaders' R&D relative to low π- leaders' R&D.[24]

Finally, we examine the effect of R&D subsidies for leaders on aggregate R&D. Differentiating (20) with respect to C yields:

$$\frac{1}{I^2}\frac{dI}{dC} = \int_0^{\pi^*} \frac{1}{\left[I(\pi)+I_F\right]^2}\left[\frac{dI(\pi)}{dC}+\frac{dI_F}{dC}\right]\frac{1}{\bar{\pi}}d\pi + \int_{\pi^*}^{\bar{\pi}} \frac{1}{I_F^2}\frac{dI_F}{dC}\frac{1}{\bar{\pi}}d\pi. \tag{28}$$

In the same procedure as Section 3.5, we can rewrite $\frac{dI(\pi)}{dC}+\frac{dI_F}{dC}$ as follows:

$$\frac{dI(\pi)}{dC}+\frac{dI_F}{dC} = \frac{1}{I(\pi)+R}\left(\left(\tilde{C}_F+R\right)\frac{dR}{dC} - \frac{R\tilde{C}_F-\pi A}{C}\right) < 0. \tag{29}$$

Therefore, we can confirm

$$\frac{dI}{dC} < 0. \tag{30}$$

From (22), we can confirm that the effect of subsidies for leaders' R&D on the growth rate is also positive. We can summarize these results in the following proposition.

Proposition 5 *Subsidies for leaders' R&D increase followers' R&D, individual leaders' R&D, aggregate R&D, and the growth rate.*

4.2 The case where leaders' R&D capability increases as lead increases

In our basic model, we assume that R&D capability between leaders is homogenous. This assumption makes the calculations easy; however, it does not reflect a real economy. It is natural to think that leaders

[24]The effect of R&D subsidies for leaders on the distribution effect is ambiguous because it increases both followers' R&D and individual leaders' R&D (see equation (17)).

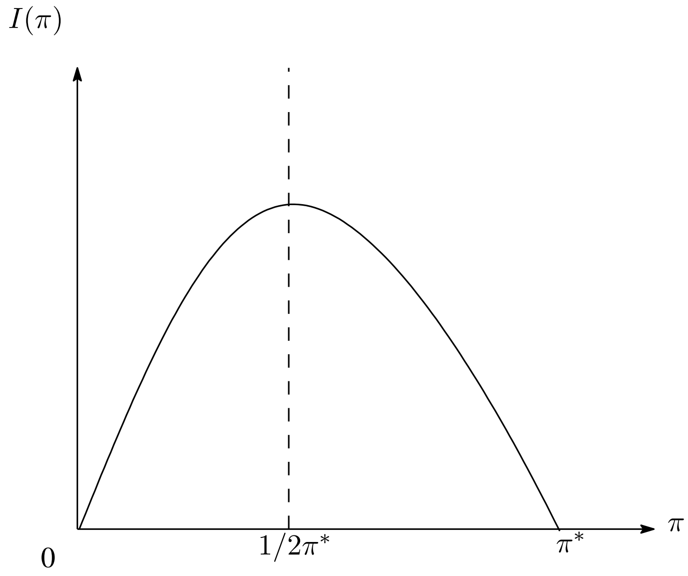

Figure 6: Leaders' R&D considering heterogeneity of leaders' R&D capability.

with a large lead might have superior R&D capability than leaders with a small lead. Then, in this section, we assume that by devoting $\frac{C}{2\pi}I^2(\pi)\,dt$ units of labor in an infinitesimal time interval dt, the leaders will succeed in the invention of new quality with probability $I(\pi)dt$. This extension changes (10) and (11) as follows:

$$I(\pi) = \sqrt{\left(2\tilde{C}_F\pi + R\right)R - 2A\pi^2} - R \quad if \quad 0 \leq \pi \leq \pi^*, \tag{31}$$

$$V(\pi) = C_F - C\left[\sqrt{\left(2\tilde{C}_F\pi + \frac{R}{\pi}\right)\frac{R}{\pi} - 2A} - \frac{R}{\pi}\right] \quad if \quad 0 \leq \pi \leq \pi^* \ . \tag{32}$$

Differentiating (31) with respect to π, we obtain:

$$\frac{dI(\pi)}{d\pi} = \frac{\tilde{C}_F R - 2A\pi}{I(\pi) + R}.$$

Therefore, we can confirm:

$$\begin{array}{ll} \frac{dI(\pi)}{d\pi} > 0 \quad if & 0 \leq \pi < \frac{\tilde{C}_F R}{2A} \\ \frac{dI(\pi)}{d\pi} \leq 0 \quad if & \frac{\tilde{C}_F R}{2A} \leq \pi \leq \frac{\tilde{C}_F R}{A} = \pi^*. \end{array}$$

This inverse U-shape relationship is not obtained in our basic model. The intuition is as follows: this extension adds a new effect where the R&D cost decreases as π increases. Using the same logic as the basic model, the weak form of the Arrow effect increases as π increases. The former effect outweighs when π is small, whereas the latter effect outweighs when π is large. We can derive R analytically:[25]

$$R = \frac{-\bar{\pi}\tilde{C}_F + \bar{\pi}\sqrt{\tilde{C}_F^2 - 2A\left[\ln\frac{2A}{2A+\tilde{C}_F^2} + \frac{2\tilde{C}_F}{\sqrt{2A}}\arctan\frac{\tilde{C}_F}{\sqrt{2A}} - \frac{\tilde{C}_F^2}{2A}\right]}}{2\left[\ln\frac{2A}{2A+\tilde{C}_F^2} + \frac{2\tilde{C}_F}{\sqrt{2A}}\arctan\frac{\tilde{C}_F}{\sqrt{2A}} - \frac{\tilde{C}_F^2}{2A}\right]}. \quad (33)$$

In the same way as in Section 3.5, we can derive the aggregate R&D and the growth rate numerically.

4.3 The case where quality increment is drawn from a Pareto distribution

In the previous sections, we assumed that profit is drawn from a uniform distribution so that we can examine the effects of subsidies for followers' R&D analytically. However, as Minniti et al. (2013) mentioned, empirical research has shown that patent citations and profit returns from innovations are highly skewed toward the low-value side, with a very long tail on the high-value side. Therefore, in this section, we assume that the quality increment λ is drawn from the Pareto distribution instead, and conduct the previous analyses again.

As in Minniti et al. (2013), we assume that λ is drawn from the following Pareto distribution:

$$f(\lambda) = \frac{a}{\lambda^{a+1}}, \qquad \lambda \in [1, \infty). \quad (34)$$

Parameter a characterizes the distribution; a higher value of a corresponds to a thinner upper tail of the distribution of λ, and the expected value of λ is given by $a/(a-1)$, which decreases in a. We assume that $a > 1$ so that the expected value is finite. In this extension, we cannot examine the effects of subsidies for followers' R&D analytically, and thus, we examine them numerically.[26] Figure 7 shows the numerical results of the extension. We use the same parameter values as in the uniform distribution case, other than the distribution parameter. They show that a decrease in followers' R&D cost, such as an increase in R&D subsidies, enhances aggregate R&D by leaders, R&D by followers, and growth rate of output. In particular, we obtain a monotone decreasing relation between followers' R&D cost and aggregate R&D by leaders, as shown in Figure 7, while in the uniform distribution case, the relation is not monotone, and we obtain an increasing relation when followers' R&D cost is very low as in the left panel of Figure 4. The reason is as follows. An increase in followers' R&D decreases the R&D of each leader. However, it increases the distribution of leaders having lower λ, and thus it may increase aggregate R&D by leaders. In the previous sections, we call the former negative effect and the latter positive effect, the *individual effect* and the *distribution effect*, respectively. If the distribution effect overwhelms the individual effect, an increase in followers' R&D raises aggregate R&D by leaders.

[25] Details of the derivation of R are given in Appendix F.

[26] Details of derivation of the equilibrium are given in Appendix G.

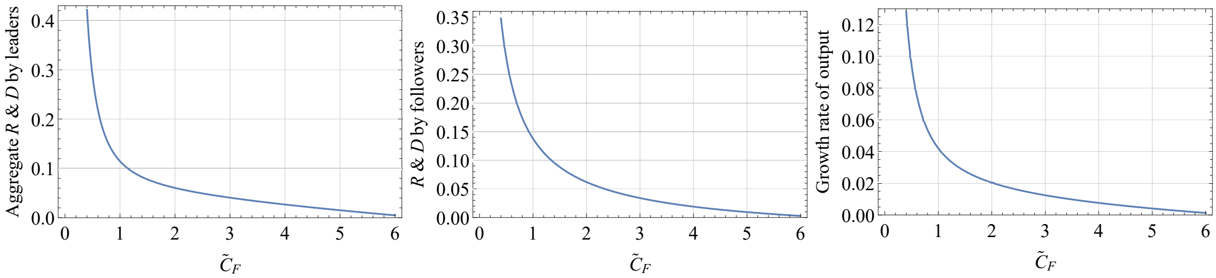

Figure 7: Aggregate R&D by leaders I_L, R&D by followers I_F, and growth rate of output in the case where quality increment is drawn from a Pareto distribution

The horizontal axis represents the relative cost of followers' R&D, $\tilde{C}_F \equiv (1 - s_F)\bar{C}_F/C$.

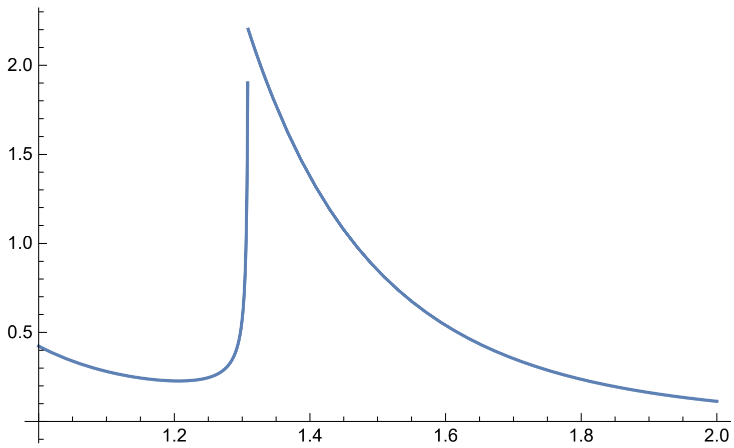

Figure 8: Stationary distribution of leaders in the case where quality increment is drawn from a Pareto distribution

The horizontal and vertical axes represent λ and $\mu(\lambda)$, respectively. We set $\tilde{C}_F = 4$.

When the quality increment is drawn from a Pareto distribution, new leaders have a low λ with higher probability compared with the uniform distribution case. Therefore, the Pareto distribution strengthens the distribution effect, and thus, a decrease in followers' R&D cost necessarily raises aggregate R&D by leaders.

The stationary distribution of λ-leader is quite different from that in the uniform distribution case; the distribution is almost an inverted U-shape as in Figure 8. As for leader firms with an intermediate lead smaller than λ^*, the number of larger firms is larger, and this tendency is the same as before. However, for leader firms with a much smaller lead or larger lead than λ^*, the number of larger firms is smaller. In reality, the number of larger firms is smaller, and thus, the results of the extension to a Pareto distribution is consistent with actual tendency.

4.4 The case where leaders can withdraw any quality increment that is smaller than that which they own currently

In the basic model, we assume that leaders cannot withdraw any innovation with the quality increment that is smaller than that which they own currently. In this section, we relax this somewhat strong assumption and assume that leaders can withdraw any smaller quality increment than that which they own currently.[27] We can show that the main result that leaders with a larger lead make smaller R&D investment still holds even under the more realistic assumption.

Under this assumption, the π-leader withdraws π' lower than π. We let $E[V|\pi]$ denote the expected value of the π-leader that succeeds in innovating. $E[V|\pi]$ depends on π and thus we define that $\widehat{V}(\pi) = E[V|\pi]$. In the present setting, the value of the π-leader depends not only on π but also on $\widehat{V}(\pi)$, and we let $V(\pi, \widehat{V}(\pi))$ denote the value of the π-leader. Then the HJB equation of the π-leader is given by:

$$rV(\pi, \widehat{V}(\pi)) = \max_{I(\pi)} \left\{ \pi L - \frac{C}{2}[I(\pi)]^2 + I(\pi) \left[\widehat{V}(\pi) - V(\pi, \widehat{V}(\pi)) \right] - I_F V(\pi, \widehat{V}(\pi)) \right\}. \qquad (35)$$

The π-leaders can withdraw π' that is lower than π, and thus, the expected value of the π-leader that succeeds in innovating is no less than the value of the π-leader; $\widehat{V}(\pi) \geq V(\pi, \widehat{V}(\pi))$, and the equality holds only if $\pi = \bar{\pi}$. From the first-order condition of R&D for leaders, therefore, we obtain:

$$CI(\pi) = \begin{cases} \widehat{V}(\pi) - V(\pi, \widehat{V}(\pi)) & \text{if } \ 0 \leq V(\pi, \widehat{V}(\pi)) < \widehat{V}(\pi), \\ 0 & \text{if } \ V(\pi, \widehat{V}(\pi)) = \widehat{V}(\pi). \end{cases} \qquad (36)$$

In a similar way as before, we can also derive the stock value of the π-leader as follows:

$$V(\pi, \widehat{V}(\pi)) = \begin{cases} \widehat{V}(\pi) + RC - C\sqrt{(2\frac{\widehat{V}(\pi)}{C} + R)R - 2\pi\frac{L}{C}} & \text{if } \ 0 \leq \pi < \bar{\pi}, \\ \frac{\bar{\pi}L}{R} & \text{if } \ \pi = \bar{\pi}. \end{cases} \qquad (37)$$

where $R \equiv r + I_F$. If π' is not higher than π, π-leaders withdraw the quality increment and the value remains $V(\pi, \widehat{V}(\pi))$. The probability is $G(\pi) \equiv \int_0^{\pi} g(\pi')d\pi'$. If π' is higher than π, π-leaders adopt the quality increment and the value becomes $V(\pi', \widehat{V}(\pi'))$. Therefore, $\widehat{V}(\pi)$, which is the expected value of the π-leader that succeeds in innovating, satisfies:

$$\widehat{V}(\pi) \ = \ V(\pi, \widehat{V}(\pi))G(\pi) + \int_{\pi}^{\bar{\pi}} V(\pi', \widehat{V}(\pi'))g(\pi')d\pi'. \qquad (38)$$

Differentiating both sides of (38) with respect to π yields:

$$\frac{d\widehat{V}(\pi)}{d\pi} \ = \ \left[\frac{\partial V(\pi, \widehat{V}(\pi))}{\partial \pi} + \frac{\partial V(\pi, \widehat{V}(\pi))}{\partial \widehat{V}} \frac{d\widehat{V}(\pi)}{d\pi} \right] G(\pi).$$

[27] In addition, we assume that if the leader withdraws the new quality, the previous quality does not become public domain.

Solving this equation for $\frac{d\widehat{V}(\pi)}{d\pi}$, we obtain:

$$\left[1 - \frac{\partial V(\pi, \widehat{V}(\pi))}{\partial \widehat{V}} G(\pi)\right] \frac{d\widehat{V}(\pi)}{d\pi} = \frac{\partial V(\pi, \widehat{V}(\pi))}{\partial \pi} G(\pi). \tag{39}$$

From (37), $\frac{\partial V(\pi, \widehat{V}(\pi))}{\partial \widehat{V}} < 1$ and $\frac{\partial V(\pi, \widehat{V}(\pi))}{\partial \pi} > 0$. Therefore, the LHS of the above equation is positive, and thus we can show $\frac{d\widehat{V}(\pi)}{d\pi} > 0$. Differentiating both sides of (36) with respect to π, we obtain:

$$C\frac{dI(\pi)}{d\pi} = \left[1 - \frac{\partial V(\pi, \widehat{V}(\pi))}{\partial \widehat{V}}\right] \frac{d\widehat{V}(\pi)}{d\pi} - \frac{\partial V(\pi, \widehat{V}(\pi))}{\partial \pi}.$$

Substituting (39) into this equation, we obtain:

$$\begin{aligned} & C\frac{dI(\pi)}{d\pi} \\ = & \left[1 - \frac{\partial V(\pi, \widehat{V}(\pi))}{\partial \widehat{V}} G(\pi)\right]^{-1} \left\{\left[1 - \frac{\partial V(\pi, \widehat{V}(\pi))}{\partial \widehat{V}}\right] G(\pi) - \left[1 - \frac{\partial V(\pi, \widehat{V}(\pi))}{\partial \widehat{V}} G(\pi)\right]\right\} \frac{\partial V(\pi, \widehat{V}(\pi))}{\partial \pi} \\ = & \left[1 - \frac{\partial V(\pi, \widehat{V}(\pi))}{\partial \widehat{V}} G(\pi)\right]^{-1} \{-[1 - G(\pi)]\} \frac{\partial V(\pi, \widehat{V}(\pi))}{\partial \pi}. \end{aligned}$$

This is negative as long as $\pi < \bar{\pi}$, because $G(\pi) < 1$ for $\pi < \bar{\pi}$.

Hence, we can show that $\frac{dI(\pi)}{d\pi} < 0$ even on the more realistic assumption; that is, even if leaders can withdraw any smaller quality increment than that which they own currently, leaders with a larger lead make smaller R&D investments.

5 Conclusion

Incorporating stochastic quality increments and diminishing returns of R&D of leaders into the standard quality-ladder model, we constructed a new Schumpeterian growth model where leader firms' quality leads over their followers are different among industries and leaders' R&D investment depends on the size of the quality lead. In this model, we obtained two main results. First, leaders with larger quality leads make smaller R&D investments. This tendency is consistent with actual behaviors of previous leader firms such as Sony and Eastman–Kodak. Second, subsidies for followers' R&D can increase the aggregate R&D of leaders. This theoretical result provides a rationale for the actual promotion policies for R&D activities of new entry firms, such as the Innovation Union, which is the innovation policy of the EU.

In the present paper, we focused our attention on only the R&D activities of leader firms; however, in reality, leader firms also conduct rent-protecting activities such as patent blocking and litigation.[28] Incorporating rent-protecting activities into the present model where the leaders are heterogeneous would change the R&D activities of the leaders and the determination of aggregate R&D. This extension is beyond the scope of the present paper, but it would be worth conducting in the future.

Although we examined mainly the effects of subsidies for followers' R&D in this paper, our model is relatively tractable and future work should examine the effects of other policies. For example, considering the "minimum inventive step requirement", which is examined by Chor and Lai (2014), in our setting can give rise to interesting results. In our model, the "minimum inventive step requirement" removes the leaders with lower leads that conduct larger R&D investment. Therefore, there is a possibility that aggregate R&D decreases with the "minimum inventive step requirement", which is clearly different from Chor and Lai (2014).

A Appendix

In this appendix, we show that $\frac{dR}{d\tilde{C}_F} < 0$.

Totally differentiating (12) yields:

$$\frac{\partial G}{\partial R}\frac{dR}{d\tilde{C}_F} = A\bar{\pi} - \frac{\partial G}{\partial \tilde{C}_F}. \tag{40}$$

First, from (12) we obtain $\frac{\partial G(R)}{\partial R}$ as follows:

$$\frac{\partial G(R)}{\partial R} = \frac{1}{2}\left(\tilde{C}_F^2 - \left(\frac{A\bar{\pi}}{R}\right)^2\right) + \left[\left(2\tilde{C}_F + R\right)R\right]^{\frac{1}{2}}\left\{\left[\left(2\tilde{C}_F + R\right)R\right]^{\frac{1}{2}} - \left(\tilde{C}_F + R\right)\right\}.$$

The first term of the RHS is negative because $\pi^* = \frac{R\tilde{C}_F}{A} < \bar{\pi}$ is satisfied at the equilibrium and the second term of the RHS is also negative because

$$\begin{aligned}
&\left[\left(2\tilde{C}_F + R\right)R\right]^{\frac{1}{2}} - \left(\tilde{C}_F + R\right) < 0 \\
&\Leftrightarrow 2\tilde{C}_F R + R^2 - \tilde{C}_F^2 - 2\tilde{C}_F R - R^2 < 0 \\
&\Leftrightarrow -\tilde{C}_F^2 < 0.
\end{aligned}$$

Thus, we can show that $\frac{\partial G(R)}{\partial R} < 0$.

[28]Recent studies examined how aggregate R&D is determined incorporating rent-protecting activities (Dinopoulos and Syropoulos, 2007; Grieben and Sener, 2009; Davis and Sener, 2012; Furukawa, 2013). Grossmann and Steger (2008) also incorporated the creation of entry barriers by incumbent firms into an R&D-based growth model and examined the effects of the activity on growth and welfare.

Second, we examine the sign of the term $(A\bar{\pi} - \frac{\partial G(R)}{\partial R})$. From (12) we obtain $\frac{\partial G(R)}{\partial \tilde{C}_F}$ as follows:

$$\frac{\partial G(R)}{\partial \tilde{C}_F} = R\left(\tilde{C}_F + R\right) - R\left[\left(2\tilde{C}_F + R\right)R\right]^{\frac{1}{2}}.$$

Substituting this into $(A\bar{\pi} - \frac{\partial G(R)}{\partial R})$, we obtain:

$$\begin{aligned} A\bar{\pi} - \frac{\partial G(R)}{\partial R} &= A\bar{\pi} - R\left(\tilde{C}_F + R\right) + R\left[\left(2\tilde{C}_F + R\right)R\right]^{\frac{1}{2}} \\ &= A\left\{\bar{\pi} - \frac{R\tilde{C}_F}{A}\right\} + R\left\{\left[\left(2\tilde{C}_F + R\right)R\right]^{\frac{1}{2}} - R\right\}. \end{aligned}$$

The first term of the RHS is positive because $\pi^* = \frac{R\tilde{C}_F}{A} < \bar{\pi}$ is satisfied in the equilibrium. The second term is also positive. Thus, we obtain $(A\bar{\pi} - \frac{\partial G(R)}{\partial R}) > 0$. As a result, we can show that $\frac{dR}{d\tilde{C}_F} < 0$.

B Appendix

We show that $\frac{dI(\pi)}{d\tilde{C}_F}$ is an increasing function of $I(\pi)$ if Condition (R) holds.

$$\begin{aligned} \frac{\partial}{\partial I(\pi)} \frac{dI(\pi)}{d\tilde{C}_F} &= \frac{1}{[I(\pi) + R]^2}\left(-[I(\pi) + R]\frac{dR}{d\tilde{C}_F} - \left\{\left[\tilde{C}_F - I(\pi)\right]\frac{dR}{d\tilde{C}_F} + R\right\}\right) \\ &= \frac{1}{[I(\pi) + R]^2}\left[\left(-\frac{dR}{d\tilde{C}_F}\right)\left(\tilde{C}_F + R\right) - R\right]. \end{aligned}$$

Therefore, we obtain $\frac{\partial}{\partial I(\pi)} \frac{dI(\pi)}{d\tilde{C}_F} > 0$ if $\frac{\tilde{C}_F}{R}\left(-\frac{dR}{d\tilde{C}_F}\right) > \frac{\tilde{C}_F}{\tilde{C}_F + R}$.

C Appendix

In this appendix, we derive aggregate R&D by leaders I_L.

Using $R = r + I_F$, $\tilde{C}_F = C_F/C$ and $A = L/C$, we rewrite (11) as:

$$I(\pi) = \begin{cases} \sqrt{(2\tilde{C}_F + R)R - 2A\pi} - R, & \text{if } \ 0 \le \pi \le \pi^*, \\ 0 & \text{if } \ \pi^* < \pi \le \bar{\pi}. \end{cases}$$

Substituting this into (20), we obtain:

$$\frac{1}{I_L + I_F} = \int_0^{\pi^*} \frac{1}{\sqrt{(2\tilde{C}_F + R)R - 2A\pi} - r}\frac{1}{\bar{\pi}}d\pi + \frac{\bar{\pi} - \pi^*}{\bar{\pi}}\frac{1}{I_F}. \tag{41}$$

By using integration by substitution, we can solve the integration in the first term of the RHS of (41). We define that $x \equiv \sqrt{(2\tilde{C}_F + R)R - 2A\pi} - r$ and replace π with x in the integration. First, we derive the interval of the integration of x. If $\pi = 0$, $x = \sqrt{(2\tilde{C}_F + R)R} - r$ $(\equiv x_0)$. From the definition of x, $x = I(\pi) + I_F$. Thus $x = I(\pi^*) + I_F = I_F$ if $\pi = \pi^*$. Second, we derive the relation between dx and $d\pi$. Solving the equation of x for π, we obtain:

$$\pi = \frac{1}{2A}\left[(2\tilde{C}_F + R)R - (x+r)^2\right].$$

The total differentiation of the equation yields:

$$d\pi = -\frac{x+r}{A}dx.$$

Using them, we can rewrite the first term of the RHS of (41) as follows.

$$\begin{aligned}
\int_0^{\pi^*} \frac{1}{\sqrt{(2\tilde{C}_F + R)R - 2A\pi} - r}\frac{1}{\bar{\pi}}d\pi &= \int_{x_0}^{I_F} \frac{1}{x}\frac{1}{\bar{\pi}}\left(-\frac{x+r}{A}\right)dx \\
&= \frac{1}{A\bar{\pi}}\int_{x_0}^{I_F}\left(-\frac{x+r}{x}\right)dx \\
&= \frac{1}{A\bar{\pi}}\left[-x - r\ln x\right]_{x_0}^{I_F} \\
&= \frac{1}{A\bar{\pi}}\left[x_0 - I_F + r\left(\ln x_0 - \ln I_F\right)\right].
\end{aligned}$$

Substituting this result into (41), we obtain aggregate R&D by leaders I_L as a function of I_F, as follows:

$$\frac{1}{I_L + I_F} = \frac{1}{A\bar{\pi}}\left[x_0 - I_F + r\left(\ln x_0 - \ln I_F\right)\right] + \frac{\bar{\pi} - \pi^*}{\bar{\pi}}\frac{1}{I_F}.$$

Using this, we can obtain the values of I_L given I_F easily.

D Appendix

In this appendix, we solve the social optimum problem and show that the market equilibrium level of innovation by leaders is necessarily smaller than the social optimum level.

We maximize

$$U = \int_0^\infty e^{-\rho t}\left[\int_0^1 \ln\sum_j q\left(j,\omega,t\right)d\left(j,\omega,t\right)d\omega + \left(\bar{l} - l\left(t\right)\right)\right]Ldt,$$

subject to resource constraints. We solve this optimization problem by dividing it into the following three steps.

We let $X_P(t)$ and $X_R(t)$ denote the labor devoted to the production activities and R&D activities, respectively. First, given $X_P(t)$, we solve the following production allocation problem:

$$\begin{aligned}
\max_{d(j,\omega,t)} \quad & \textstyle\int_0^1 \ln\sum_j q\left(j,\omega,t\right)d\left(j,\omega,t\right)d\omega \\
s.t. \quad & \textstyle\int_0^1 \sum_j d\left(j,\omega,t\right)d\omega \le X_P\left(t\right).
\end{aligned}$$

Solving this problem yields $d(\omega, t) = X_P(t), \forall\omega$ and $\int_0^1 d(\omega, t)\, d\omega = X_P(t)$.

Second, given $X_R(t)$, we solve the following problem of allocation between the leaders' innovation and the followers' innovation:

$$\begin{array}{cc} \max\limits_{I_L(\omega,t), I_F(t)} & I(t) = \int_0^1 I_L(\omega, t)\, d\omega + I_F(t) \\ s.t. & \int_0^1 \frac{C}{2}(I_L(\omega, t))^2 d\omega + C_F I_F(t) \le X_R(t). \end{array}$$

Solving this problem, we obtain $I_F(t) = \frac{1}{C_F} X_R(t) - \frac{1}{2}\frac{\bar{C}_F}{C}$ and $I(t) = \frac{1}{C_F} X_R(t) + \frac{1}{2}\frac{\bar{C}_F}{C}$, and in particular we obtain the social optimum level of innovation by leaders I_L^{SO} as follows:

$$I_L^{SO} = I_L(\omega, t) = \frac{\bar{C}_F}{C} \quad \text{for} \quad \forall\omega. \tag{42}$$

Finally, we solve the problem of dynamic allocation. From the solution of the above static optimization problems, $X_P(t) + X_R(t) + X_L(t) = \bar{L}$ and the discussion of Section 3.6, the dynamic problem can be expressed as follows:

$$\begin{array}{cc} \max\limits_{X_P(t), X_L(t), \ln Q(t)} & U = \int_0^\infty e^{-\rho t}[\ln Q(t) + \ln X_P(t) + X_L(t)]dt \\ s.t. & \frac{d \ln Q(t)}{dt} = E[\ln\lambda]\left\{\frac{1}{\bar{C}_F}\left[\bar{L} - X_P(t) - X_L(t)\right] + \frac{1}{2}\frac{\bar{C}_F}{C}\right\}, \end{array}$$

where $X_L(t)$ is labor neither distributed in production nor innovation activity, and $\bar{L} \equiv \bar{l}L(t)$ is the maximum labor supply in the economy. We solve this problem by using a Hamiltonian function as follows:

$$H = \ln Q(t) + \ln X_P(t) + X_L(t) + \theta(t) E[\ln\lambda]\left\{\frac{1}{\bar{C}_F}\left[\bar{L} - X_P(t) - X_L(t)\right] + \frac{1}{2}\frac{\bar{C}_F}{C}\right\},$$

where $\theta(t)$ is the current-value shadow price. We obtain the following first-order condition:

$$\frac{1}{X_P(t)} = \theta(t)\frac{E[\ln\lambda]}{\bar{C}_F},$$

$$\dot{\theta}(t) - \rho\theta(t) = -1.$$

In the equilibrium, $\theta = \frac{1}{\rho}$ is satisfied from $\dot{\theta}(t) = 0$. Then $X_P^{SO} = \frac{\bar{C}_F \rho}{E[\ln\lambda]}$ is satisfied.

We limit our attention to the case where the expected value of $\ln\lambda$ is so high that $E[\ln\lambda]/\rho > 1$. Because of the assumption that the utility function is quasilinear and the R&D technology of the followers is linear, we obtain the corner solution of leisure: that is, $X_L^{SO} = 0$.[29] Consequently, we obtain the

[29]Otherwise, there are the following two cases. When $\frac{E[\ln\lambda]}{\rho} < 1$, $I_F^{SO} = 0$, $X_L^{SO} = \bar{L} - \frac{\bar{C}_F\rho}{E[\ln\lambda]}$ and $I^{SO} = \frac{\bar{C}_F}{C}$ are satisfied. When $\frac{E[\ln\lambda]}{\rho} = 1$, $I_F^{SO} \in \left[0, \frac{\bar{L}}{\bar{C}_F} - \frac{\rho}{E[\ln\lambda]} - \frac{1}{2}\frac{\bar{C}_F}{C}\right]$, $X_L^{SO} \in \left[0, \bar{L} - \frac{\bar{C}_F\rho}{E[\ln\lambda]}\right]$ and $I^{SO} \in \left[\frac{\bar{C}_F}{C}, \frac{\bar{L}}{\bar{C}_F} - \frac{\rho}{E[\ln\lambda]} + \frac{1}{2}\frac{\bar{C}_F}{C}\right]$ are satisfied.

socially optimal innovation by followers and aggregate innovation, I_F^{SO} and I^{SO}, as follows.

$$I_F^{SO} = \frac{\bar{L}}{\bar{C}_F} - \frac{\rho}{E[\ln\lambda]} - \frac{1}{2}\frac{\bar{C}_F}{C}, \quad (43)$$

$$I^{SO} = \frac{\bar{L}}{\bar{C}_F} - \frac{\rho}{E[\ln\lambda]} + \frac{1}{2}\frac{\bar{C}_F}{C}. \quad (44)$$

From (7) and (42), we can show that the market equilibrium level of innovation by leaders is lower than the socially optimal level for all industries. This result is due to the Arrow effect; as shown in (7), the higher the value of current status, the weaker the incentive to innovate for the leader. For innovation by followers, the result is ambiguous; whether the market equilibrium level is lower or higher than the socially optimal level depends on the parameters.

E Appendix

In this appendix, we show $\frac{1}{C}(C_F R - L\pi) + R^2 - R\left(\frac{2}{C}(C_F R - L\pi) + R^2\right)^{\frac{1}{2}} > 0$.

$$\begin{aligned} &\tfrac{1}{C}(C_F R - L\pi) + R^2 > R\left(\tfrac{2}{C}(C_F R - L\pi) + R^2\right)^{\frac{1}{2}} \\ &\Leftrightarrow \left(\tfrac{1}{C}(C_F R - L\pi)\right)^2 + \tfrac{2}{C}(C_F R - L\pi)R^2 + R^4 > R^2\left(\tfrac{2}{C}(C_F R - L\pi) + R^2\right) \\ &\Leftrightarrow \left(\tfrac{1}{C}(C_F R - L\pi)\right)^2 > 0 \end{aligned} \quad (45)$$

As $\pi \leq \frac{C_F R}{L} = \pi^*$ is satisfied for active R&D leaders, we obtain $\left(\frac{1}{C}(C_F R - L\pi)\right)^2 > 0$.

F Appendix

In this appendix, we derive (33). From (32), the expected value of leaders is:

$$E[V(\tilde{\pi})] = \frac{C}{\bar{\pi}}\int_0^{\pi^*}\left\{\tilde{C}_F - \left[\sqrt{\left(2\tilde{C}_F\pi + \frac{R}{\pi}\right)\frac{R}{\pi} - 2A} - \frac{R}{\pi}\right]\right\}d\pi + \frac{L}{R\bar{\pi}}\int_{\pi^*}^{\bar{\pi}}\pi d\pi.$$

We can easily confirm $\frac{L}{R\bar{\pi}}\int_{\pi^*}^{\bar{\pi}}\pi d\pi = \frac{1}{2}\frac{C}{\bar{\pi}}\frac{A}{R}\left(\bar{\pi}^2 - \left(\frac{\tilde{C}_F R}{A}\right)^2\right)$, however, calculating

$$Z \equiv \int_0^{\pi^*}\left\{\tilde{C}_F - \left[\sqrt{\left(2\tilde{C}_F\pi + \frac{R}{\pi}\right)\frac{R}{\pi} - 2A} - \frac{R}{\pi}\right]\right\}d\pi$$

is troublesome.

Let $\nu \equiv \tilde{C}_F - \left[\sqrt{\left(2\tilde{C}_F\pi + \frac{R}{\pi}\right)\frac{R}{\pi} - 2A} - \frac{R}{\pi}\right]$, then $\nu = 0$ when $\pi = 0$ and $\nu = \tilde{C}_F$ when $\pi = \pi^*$ are satisfied respectively. Furthermore, we can calculate $d\pi = 2R\frac{2A + \tilde{C}_F^2 - \nu^2}{\left[2A + \left(\tilde{C}_F - \nu\right)^2\right]^2}d\nu$. Then we can rewrite Z as:

$$Z = 2R\int_0^{\tilde{C}_F} \nu \frac{2A + \tilde{C}_F^2 - \nu^2}{\left[2A + \left(\tilde{C}_F - \nu\right)^2\right]^2} d\nu.$$

Let $F(\nu) \equiv \frac{\nu}{2A+\left(\tilde{C}_F-\nu\right)^2}$, then $F'(\nu) = \frac{2A+\tilde{C}_F^2-\nu^2}{\left[2A+\left(\tilde{C}_F-\nu\right)^2\right]^2}$ is satisfied. Using $F(\nu)$ and $F'(\nu)$, Z can be rewritten as:

$$\begin{aligned} Z &= 2R\int_0^{\tilde{C}_F} \nu F'(\nu)\, d\nu \\ &= 2R\left\{\left[\nu F(\nu)\right]_0^{\tilde{C}_F} - \int_0^{\tilde{C}_F} F(\nu)\, d\nu\right\} \\ &= 2R\left\{\frac{\tilde{C}_F^2}{2A} - \int_0^{\tilde{C}_F} \frac{\nu}{2A+\left(\tilde{C}_F-\nu\right)^2} d\nu\right\} \\ &= 2R\left\{\frac{\tilde{C}_F^2}{2A} - \int_0^{\tilde{C}_F} \frac{1}{2}\frac{2\left(\nu-\tilde{C}_F\right)}{2A+\left(\tilde{C}_F-\nu\right)^2} d\nu - \int_0^{\tilde{C}_F} \frac{\tilde{C}_F}{2A+\left(\tilde{C}_F-\nu\right)^2} d\nu\right\} \end{aligned}.$$

We can confirm

$$\int_0^{\tilde{C}_F} \frac{1}{2}\frac{2\left(\nu-\tilde{C}_F\right)}{2A+\left(\tilde{C}_F-\nu\right)^2} d\nu = \frac{1}{2}\left[\ln\left(2A+\left(\tilde{C}_F-\nu\right)^2\right)\right]_0^{\tilde{C}_F} = \frac{1}{2}\ln\frac{2A}{2A+\tilde{C}_F^2}.$$

Furthermore, let $\xi \equiv \frac{\tilde{C}_F-\nu}{\sqrt{2A}}$, then $\xi = \frac{\tilde{C}_F}{\sqrt{2A}}$ when $\nu = 0$ and $\xi = 0$ when $\nu = \tilde{C}_F$ are satisfied respectively. Then we can calculate $d\nu = -\sqrt{2A}d\xi$ and rewrite $\int_0^{\tilde{C}_F} \frac{\tilde{C}_F}{2A+\left(\tilde{C}_F-\nu\right)^2} d\nu$ as follows:

$$\begin{aligned} \int_0^{\tilde{C}_F} \frac{\tilde{C}_F}{2A+\left(\tilde{C}_F-\nu\right)^2} d\nu &= \int_{\frac{\tilde{C}_F}{\sqrt{2A}}}^0 -\frac{\tilde{C}_F}{2A+\left(\tilde{C}_F-\nu\right)^2}\sqrt{2A}d\xi \\ &= \frac{\tilde{C}_F}{\sqrt{2A}}\int_0^{\frac{\tilde{C}_F}{\sqrt{2A}}} \frac{1}{1+\xi^2} d\xi \\ &= \frac{\tilde{C}_F}{\sqrt{2A}}\left[\arctan\xi\right]_0^{\frac{\tilde{C}_F}{\sqrt{2A}}} \\ &= \frac{\tilde{C}_F}{\sqrt{2A}}\arctan\frac{\tilde{C}_F}{\sqrt{2A}} \end{aligned}.$$

Therefore, Z can be rewritten as:

$$Z = 2R\left\{\frac{\tilde{C}_F^2}{2A} - \frac{1}{2}\ln\frac{2A}{2A+\tilde{C}_F^2} - \frac{\tilde{C}_F}{\sqrt{2A}}\arctan\frac{\tilde{C}_F}{\sqrt{2A}}\right\},$$

and $E\left[V\left(\tilde{\pi}\right)\right]$ can be rewritten as:

$$E\left[V\left(\tilde{\pi}\right)\right] = \frac{C}{\bar{\pi}}\left\{-R\left[\ln\frac{2A}{2A+\tilde{C}_F^2} + \frac{2\tilde{C}_F}{\sqrt{2A}}\arctan\frac{\tilde{C}_F}{\sqrt{2A}} - \frac{\tilde{C}_F^2}{2A}\right] + \frac{1}{2}\frac{A}{R}\bar{\pi}^2\right\}.$$

Substituting this into (6), and calculating the quadratic equation yields (33).

G Appendix: Analysis in the case where quality increment is drawn from a Pareto distribution

In this appendix, we show how the equilibrium is determined in the extension with a Pareto distribution.

First, we derive the equation that determines R&D by followers. We can express the expected value of $V(\lambda)$ as follows:

$$E[V(\tilde{\lambda})] = \int_1^{\lambda^*} V(\lambda)f(\lambda)d\lambda + \int_{\lambda^*}^{\infty} V(\lambda)f(\lambda)d\lambda,$$

where $\lambda^* = (1 - R\tilde{C}_F/A)^{-1}$ from (9). From (10), we can derive the value of the leader with any λ, $V(\lambda)$. Using $V(\lambda)$, we can calculate the first term of the RHS as follows:

$$\begin{aligned}\int_1^{\lambda^*} V(\lambda)f(\lambda)d\lambda &= \int_1^{\lambda^*} C\left[\tilde{C}_F + R - \sqrt{(2\tilde{C}_F + R)R - 2A + 2A/\lambda}\right]\frac{a}{\lambda^{a+1}}d\lambda \\ &= C\left[(\tilde{C}_F + R)\int_1^{\lambda^*}\frac{a}{\lambda^{a+1}}d\lambda - \int_1^{\lambda^*}\sqrt{[(2\tilde{C}_F + R)R - 2A]\lambda + 2A}\frac{a}{\lambda^{a+3/2}}d\lambda\right] \\ &= C\left[(\tilde{C}_F + R)(1 - \lambda^{*-a}) - \int_1^{\lambda^*}\sqrt{[(2\tilde{C}_F + R)R - 2A]\lambda + 2A}\frac{a}{\lambda^{a+3/2}}d\lambda\right].\end{aligned}$$

For larger values than λ^*, we can analytically derive the integral of $V(\lambda)$. We can calculate the second term of the RHS as follows:

$$\begin{aligned}\int_{\lambda^*}^{\infty} V(\lambda)f(\lambda)d\lambda &= \int_{\lambda^*}^{\infty} C\frac{A}{R}\left(1 - \frac{1}{\lambda}\right)\frac{a}{\lambda^{a+1}}d\lambda \\ &= C\frac{A}{R}\left[\int_{\lambda^*}^{\infty}\frac{a}{\lambda^{a+1}}d\lambda - \int_{\lambda^*}^{\infty}\frac{a}{\lambda^{a+2}}d\lambda\right] \\ &= C\frac{A}{R}\left[\lambda^{*-a} - \frac{a}{a+1}\lambda^{*-(a+1)}\right].\end{aligned}$$

The equilibrium condition for followers' R&D is given by $E[V(\tilde{\lambda})] = C_F$, and then we can derive the equilibrium condition for followers' R&D as follows:

$$(\tilde{C}_F + R)(1 - \lambda^{*-a}) - \int_1^{\lambda^*}\sqrt{[(2\tilde{C}_F + R)R - 2A]\lambda + 2A}\frac{a}{\lambda^{a+3/2}}d\lambda + \frac{A}{R}\left[\lambda^{*-a} - \frac{a}{a+1}\lambda^{*-(a+1)}\right] = \tilde{C}_F. \quad (46)$$

We cannot calculate the integral in the second term of the LHS of the above equation analytically. Now we specify the value of the parameter of the Pareto distribution a. The expected value of the markup is determined by a as follows:

$$E[\lambda] = \int_1^{\infty}\lambda f(\lambda)d\lambda = \frac{a}{a-1}.$$

According to empirical studies such as Basu (1996) and Norrbin (1993), the range of the average markup has been estimated as $[1.05, 1.4]$. We adopt $a = 6$, and then the average markup becomes $E[\lambda] = 1.2$, which is in the middle of this range. By specifying $a = 6$, we can calculate the integral. Equation (46) determines the equilibrium value of R in the same way as in the uniform distribution case.

Next, we derive the equation that determines aggregate R&D by leaders. The change in the distribution of λ-leaders is given by: $\partial\mu(\lambda,t)/(\partial t) = (I_L(t) + I_F)f(\lambda) - [I(\lambda) + I_F]\,\mu(\lambda,t)$. Then, we derive the stationary distribution of λ-leader $\mu(\lambda)$ as follows:

$$\mu(\lambda) = \frac{I_L + I_F}{I(\lambda) + I_F}\frac{a}{\lambda^{a+1}} \tag{47}$$

In the same way as in the uniform distribution case, aggregate R&D $I(= I_L + I_F)$ is determined by the following equation:

$$\frac{1}{I_L + I_F} = \int_1^{\lambda^*} \frac{1}{I(\lambda) + I_F} f(\lambda)d\lambda + \int_{\lambda^*}^{\infty} \frac{1}{I_F} f(\lambda)d\lambda.$$

From (11), we can derive the R&D of the leader with quality increment λ, $I(\lambda)$. Using $I(\lambda)$, we can rewrite the above equation as follows:

$$\begin{aligned}
\frac{1}{I_L + I_F} &= \int_1^{\lambda^*} \frac{1}{\sqrt{(2\tilde{C}_F + R)R - 2A + 2A/\lambda} - r} \frac{a}{\lambda^{a+1}} d\lambda + \int_{\lambda^*}^{\infty} \frac{1}{I_F} \frac{a}{\lambda^{a+1}} d\lambda \\
&= \int_1^{\lambda^*} \frac{1}{\sqrt{(2\tilde{C}_F + R)R - 2A + 2A/\lambda} - r} \frac{a}{\lambda^{a+1}} d\lambda + \frac{1}{I_F}\lambda^{*-a}.
\end{aligned}$$

By specifying $a = 6$, we can calculate the integral analytically, and by using this we provide the aggregate R&D I numerically, as in Figure 7.

Finally, we derive the growth rate of output. The growth rate of output is given by (22). When λ is drawn from the Pareto distribution of parameter a, we obtain $E[\ln[\lambda]] = [-\lambda^{-a}(1/a + \ln\lambda)]_1^{\infty} = 1/a$. We specify $a = 6$, and then the growth rate of output is given by: $\frac{d\ln Q(t)}{dt} = 1/6\,(I_L + I_F)$.

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