Discussion Papers In Economics And Business

Human Capital, Public Debt, and Economic Growth:
A Political Economy Analysis

Tetsuo Ono and Yuki Uchida

Discussion Paper 16-01-Rev.3

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Human Capital, Public Debt, and Economic Growth: A Political Economy Analysis

Tetsuo Ono and Yuki Uchida

Discussion Paper 16-01-Rev.3

October 2017

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Human Capital, Public Debt, and Economic Growth: A Political Economy Analysis*

Tetsuo Ono† Yuki Uchida‡
Osaka University Seikei University

Abstract

This study considers the politics of public education policy in an overlapping-generations model with physical and human capital accumulation. In particular, this study examines how debt and tax financing differ in terms of growth and welfare across generations, as well as which fiscal stance voters support. The analysis shows that the growth rate in debt financing is lower than that in tax financing, and that debt financing creates a tradeoff between the present and future generations. The analysis also shows that debt financing attains slower economic growth than that realized by the choice of a social planner who cares about the welfare of all generations.

- Keywords: Economic growth, Human capital, Public debt, Political equilibrium
- JEL Classification: D70, E24, H63,

*We are grateful for the comments from participants at the 2016 Japanese Economic Association Spring Meeting and 2016 Asian Meeting of the Econometric Society. Tetsuo Ono acknowledges financial support from Grant-in-Aid for Scientific Research (C) from the Ministry of Education, Culture, Sports, Science and Technology of Japan No. 15K03509. Yuki Uchida acknowledges financial support from Grant-in-Aid for JSPS Fellows of the Japan Society for the Promotion of Science No. 15J01375.

†Tetsuo Ono: Graduate School of Economics, Osaka University, 1-7, Machikaneyama, Toyonaka, Osaka 560-0043, Japan. E-mail: tono@econ.osaka-u.ac.jp.

‡Yuki Uchida: Faculty of Economics, Seikei University, 3-3-1 Kichijoji-Kitamachi, Musashino, Tokyo 180-8633, Japan. E-mail: yuchida@econ.seikei-u.ac.jp.

1 Introduction

This paper considers the political determinants of fiscal policy and its impact on growth and welfare across generations, with a focus on public education expenditures, which work as both an inter-generational transfer from parents to children and an engine of economic growth. Parents with altruistic concern for their children are likely to support public education and bear the tax burden because they can benefit from highly educated children. However, when the government has access to debt financing, parents may prefer debt to taxes because the former enables them to shift the fiscal burden to future generations.

The discussion above leads to the following questions: how do debt and tax financing differ in terms of growth and welfare across generations and which fiscal stance do voters adopt? To consider these questions, this study presents a three-period overlapping-generations model with physical and human capital accumulation (see, e.g., Lambrecht, Michel, and Vidal, 2005; Kunze, 2014). Parents care about their children's wage income. Public education spending and parental human capital are inputs in the human capital formation process. Governments finance spending through tax on labor income and issue bonds if they have access to debt financing.

Within this framework, we investigate the politics of fiscal policy formation. In each period, three successive generations, the young, middle-aged, and elderly, can participate in voting. However, the elderly are indifferent between any two policies because they have no tax burden and obtain no benefit from public education expenditure. The young may benefit from public education in the future, but they are below the voting age. Therefore, the government's aim is to choose a fiscal policy that maximizes the utility of the middle-aged. Given this process, we characterize the political equilibrium in both the tax- and debt-financing cases, which yields the following findings.

First, the government's choice of fiscal policy depends on the parents' degree of altruism toward their children. Greater altruism encourages parents to leave higher wages to their children, and they will do so by decreasing public bond issues and, thus, weakening the crowding-out effect on capital accumulation. In particular, there is a threshold level of altruism such that the government finds it optimal (not) to issue public bonds when altruism is below (above) the threshold level. This result suggests that the degree of altruism is a key explanation for the choice of fiscal policy.

Second, the growth rate in the debt-financing case is lower than that in the tax-financing case. The debt overrides capital accumulation and adds the cost of debt repayment, reducing the government's available resources and, thus, decreasing public education expenditure as an engine of economic growth. The peculiar negative impact on the expenditure in the debt-financing case explains this result.

Third, debt financing has a tradeoff between the present and future generations. When altruism is below the threshold level, the government in each period chooses debt financing for the given physical and human capital levels. However, debt financing discourages human capital accumulation and, thus, bequeaths less physical and human capital to future generations. This implies that future governments must choose a fiscal policy subject to lower levels of capital than those expected from tax financing. In other words, the choice of the future governments is itself optimal for the given capital levels, but they obtain less utility from debt financing than from tax financing.

Fourth, the political equilibrium generally attains under-accumulation of capital and underinvestment of education, relative to the choice of the social planner who is assumed to have the ability to commit to all of his or her choices at the beginning of a period, subject to the resource constraint. The discrepancy between the choices of the planner and politician arises because the planner values the welfare of all generations, whereas the politician cares only about existing generations. This shortsightedness of the politician causes less physical and human capital formation and, thus, slower economic growth than do those realized by the planner's choice.

The rest of this paper is organized as follows. We, first, present a literature survey in Subsection 1.1. Thereafter, Section 2 presents the model. Section 3 describes the political equilibrium and Section 4 compares the debt- and tax-financing political equilibria. Section 5 characterizes the planner's allocation and compares it to the political equilibrium. Section 6 offers some extensions to the basic model. Finally, Section 7 presents concluding remarks.

1.1 Literature Review

Our study is related to research on public education and economic growth in terms of parental altruism toward children. Examples include studies from Glomm and Kaganovich (2003, 2008) and Glomm and Ravikumar (1992, 2001, 2003), based in the competitive equilibrium context and assuming that the central government can control policy processes and outcomes across periods. Under this assumption, these researchers investigate how changes in educational policy affect growth and welfare across generations as well as the optimal policy in terms of long-term growth and/or welfare.

Several studies have attempted to relax this assumption by introducing voting into the policy-making process. These studies tend to focus on factors that affect policy, such as aging (Zhang, Zhang, and Lee, 2003; Gradstein and Kaganovich, 2004; Kunze, 2014), inequality (Saint-Paul and Verdier, 1993), expectations of future policy (Glomm and Ravikumar, 1995), private education as an alternative to public education (Gradstein and Justman, 1996), social cohesion (Gradstein and Justman, 2002), and social security (Kemnitz, 2000; Poutvaara, 2006; Soares, 2006; Gonzalez-Eiras and Niepelt, 2012; Iturbe-

Ormaetxe and Valera, 2012; Kaganovich and Meier, 2012; Naito, 2012; Ono, 2015; Lancia and Russo, 2016; Ono and Uchida, 2016). However, all of these studies assume financing only from taxes and that the government budget constraint is balanced each period. In other words, debt financing of public education is abstracted away from their analyses.

One exception is Zhang's (2003) study that demonstrates the optimal policy for public education when the government can issue public bonds. Thus, Zhang (2003) is based in the competitive equilibrium context, and mentions nothing about the political determinants of debt-financing policy. As such, our study demonstrates the endogenous determinants of debt-financed public education expenditures and their impact on growth and welfare across generations. With this analysis, we can evaluate the relative performance of the two fiscal stances.

Our study is also related to the literature on the golden rule of public finance (see, e.g., Greiner and Semmler, 2000; Ghosh and Mourmouras, 2004a, 2004b; Yakita, 2008; Minea and Villieu, 2009; Greiner, 2012). The rule allows the government to run a budget deficit as long as it uses the deficit to finance productive public capital expenditures. In the present framework, public education expenditures represent productive public capital expenditures, as in Greiner (2008), who investigates the effect of different budgetary regimes on growth and welfare. Yet, the present study differs from Greiner (2008) in that it incorporates the effect of voting on fiscal policy, and considers its welfare effects across generations.

2 Model

The discrete time economy starts at period 0 and consists of overlapping generations. Individuals are identical within a generation, and live for three periods: youth, middle, and elderly ages. Each middle-aged individual gives birth to $1+n$ children. The middle-aged population for the period t is N_t, and the population grows at a constant rate of $n(>-1) : N_{t+1} = (1+n)N_t$.

2.1 Individuals

Individuals display the following economic behavior over their life cycles. During youth, they make no economic decisions and receive public education financed by the government. In middle age, individuals work, receive market wages, and make tax payments. They use after-tax income for consumption and savings. Individuals retire in their elderly years, and receive and consume returns from savings.

Consider an individual born in period $t-1$. In period t, the individual is middle-aged and endowed with h_t units of human capital. The individual supplies them inelastically in the labor market, and obtains labor income $w_t h_t$, where w_t is the wage rate per efficient unit of labor in period t. After paying tax $\tau_t w_t h_t$, where $\tau_t \in (0,1)$ is the period t income tax rate, the individual distributes the after-tax income between consumption c_t and savings invested in physical capital s_t. Therefore, the period t budget constraint for the middle age becomes:

$$c_t + s_t \leq (1-\tau_t) w_t h_t.$$

The period $t+1$ budget constraint in the elderly age is:

$$d_{t+1} \leq R_{t+1} s_t,$$

where d_{t+1} is consumption, $R_{t+1}(>0)$ is the gross return from investment in capital, and $R_{t+1} s_t$ is the return from savings.

Period t middle-aged individuals care about their children's income , $w_{t+1} h_{t+1}$. Children's human capital in period $t+1$, h_{t+1}, is a function of government spending on public education, x_t, and the parents' human capital, h_t. In particular, h_{t+1} is formulated using the following equation:

$$h_{t+1} = D \left(x_t\right)^{\eta} \left(h_t\right)^{1-\eta},$$

where $D(>0)$ is a scale factor and $\eta \in (0,1)$ denotes the elasticity of education technology with respect to education spending.

We note that private investment in education may also contribute to human capital formation. For example, parents' time (Glomm and Ravikumar, 1995, 2001, 2003; Glomm and Kaganovich, 2008) or spending (Glomm, 2004; Lambrecht, Michel, and Vidal, 2005; Kunze, 2014) devoted to education may complement public education. In the present study, we abstract private education from the main analysis to simplify the presentation of the model and focus on the effect of public education on growth and utility. Section 6.1 presents how the analysis and result would change when the model includes private education.

We assume that parents are altruistic toward their children and concerned about their income in middle age, $w_{t+1} h_{t+1}$. The preferences of an individual born in period $t-1$ are specified by the following expected utility function of the logarithmic form:

$$U_t = \ln c_t + \beta \ln d_{t+1} + \gamma \ln w_{t+1} h_{t+1},$$

where $\beta \in (0,1)$ is a discount factor and $\gamma(>0)$ denotes the intergenerational degree of altruism. We substitute the budget constraints and human capital production function into the utility function to form the following unconstrained maximization problem:

$$\max_{\{s_t\}} \ln\left[(1-\tau_t) w_t h_t - s_t\right] + \beta \ln R_{t+1} s_t + \gamma \ln w_{t+1} D \left(x_t\right)^{\eta} \left(h_t\right)^{1-\eta}.$$

By solving this problem, we obtain the following savings and consumption functions:

$$s_t = \frac{\beta}{1+\beta} \cdot (1-\tau_t) w_t h_t, \tag{1}$$

$$c_t = \frac{1}{1+\beta} \cdot (1-\tau_t) w_t h_t \text{ and } d_{t+1} = \frac{\beta R_{t+1}}{1+\beta} \cdot (1-\tau_t) w_t h_t. \tag{2}$$

2.2 Firms

Each period contains a continuum of identical firms that are perfectly competitive profit maximizers. According to Cobb–Douglas technology, they produce a final good Y_t using two inputs, aggregate physical capital K_t and aggregate human capital $H_t \equiv N_t h_t$. The aggregate output is given by:

$$Y_t = A\left(K_t\right)^{\alpha}\left(H_t\right)^{1-\alpha},$$

where $A(>0)$ is a scale parameter and $\alpha \in (0,1)$ denotes the capital share.

Let $k_t \equiv K_t/H_t$ denote the ratio of physical to human capital. The first-order conditions for profit maximization with respect to H_t and K_t are:

$$w_t = (1-\alpha) A\left(k_t\right)^{\alpha}, \text{ and } \rho_t = \alpha A\left(k_t\right)^{\alpha-1}, \tag{3}$$

where w_t and ρ_t are labor wages and the rental price of capital, respectively. The conditions state that firms hire human and physical capital until the marginal products are equal to the factor prices.

2.3 Government Budget Constraint

Public education expenditures are financed by both tax on labor income and public bond issues. Let B_t denote the aggregate inherited debt. A government budget constraint in period t is:

$$B_{t+1} + \tau_t w_t h_t N_t = N_{t+1} x_t + R_t B_t,$$

where B_{t+1} is newly issued public bonds, $\tau_t w_t h_t N_t$ is the aggregate labor income tax revenue, $N_{t+1} x_t$ is the aggregate expenditure for public education, and $R_t B_t$ is debt repayment. We assume a one-period debt structure to derive analytical solutions from the model, and assume that the government in each period is committed to not repudiating the debt.

By dividing both sides of the above expression, we obtain a per-capita form of the constraint:

$$(1+n)\hat{b}_{t+1} + \tau_t w_t h_t = (1+n) x_t + R_t \hat{b}_t, \tag{4}$$

where $\hat{b}_t \equiv B_t/N_t$ is the per-capita public debt. We use the notation $\hat{b}_t$, rather than b_t, to distinguish the per-capita public debt, $\hat{b}_t \equiv B_t/N_t$, from the public debt per human capital, $b_t \equiv B_t/H_t$, which we introduce in the next section.

2.4 Economic Equilibrium

Public bonds are traded in a domestic capital market. The market clearing condition for capital is $B_{t+1} + K_{t+1} = N_t s_t$, which expresses the equality of total savings by the middle-aged population in period t, $N_t s_t$, to the sum of the stocks of aggregate public debt and aggregate physical capital at the beginning of period $t+1$, $B_{t+1} + K_{t+1}$. Using $k_{t+1} \equiv K_{t+1}/H_{t+1}$, $h_{t+1} = H_{t+1}/N_{t+1}$, and the savings function in (1), we can rewrite the condition as:

$$(1+n) \cdot \left(k_{t+1}h_{t+1} + \hat{b}_{t+1}\right) = \frac{\beta}{1+\beta} \cdot (1-\tau_t)w_t h_t. \tag{5}$$

The following defines the economic equilibrium in the present model.

Definition 1. Given a sequence of policies, $\{\tau_t, x_t\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations $\left\{c_t, d_t, s_t, k_{t+1}, \hat{b}_{t+1}, h_{t+1}\right\}_{t=0}^{\infty}$ and prices $\{\rho_t, w_t, R_t\}_{t=0}^{\infty}$ with the initial conditions $k_0(>0), \hat{b}_0(\geq 0)$ and $h_0(>0)$, such that (i) given $(w_t, R_{t+1}, \tau_t, x_t)$, $\left(c_t^y, c_{t+1}^o, s_t\right)$ solves the utility maximization problem; (ii) given (w_t, ρ_t), k_t solves a firm's profit maximization problem; (iii) given $\left(w_t, h_t, k_t, \hat{b}_t\right)$, $\left(\tau_t, x_t, \hat{b}_{t+1}\right)$ satisfies the government budget constraint; (iv) $\rho_t = R_t$ holds; and (v) the capital market clears: $(1+n) \cdot \left(k_{t+1}h_{t+1} + \hat{b}_{t+1}\right) = s_t$.

3 Political Equilibrium

In this section, we consider voting on fiscal policy. In the present framework, the young, middle-aged, and elderly can participate in voting. However, the elderly are indifferent between any two policies because they bear no tax burden and obtain no benefits. The young may benefit from the public education expenditure in the future, but we assume that they are unable to participate in voting because they are below the voting age.

Under the assumption described above, the government's aim in each period is to maximize the indirect utility of the middle-aged, given by:

$$V_t = (1+\beta)\ln\left\{(1-\alpha)A\left(k_t\right)^{\alpha}h_t - \tau_t w_t h_t\right\} + \left\{\beta(\alpha-1)+\gamma\alpha\right\}\ln k_{t+1} + \gamma\eta\ln x_t + \gamma(1-\eta)\ln h_t + C_0, \tag{6}$$

where C_0 includes constant terms and is defined by:

$$C_0 \equiv \ln\frac{1}{1+\beta} + \beta\ln\frac{\beta}{1+\beta} + \gamma\ln D + \beta\ln\alpha A + \gamma\ln(1-\alpha)A.$$

The term $\left\{(1-\alpha)A\left(k_t\right)^{\alpha}h_t - \tau_t w_t h_t\right\}$ shows the after-tax income and, thus, represents utility from consumption. The second term includes the utility from the return on savings, $\beta(\alpha-1)\ln k_{t+1}$, and that from the next generation's wage income, $\gamma\alpha\ln k_{t+1}$. The third and fourth terms correspond to the utility from the next generation's human capital, h_{t+1}.

Given the state variables k_t, h_t and $\hat{b}_t$, the period$-t$ government chooses fiscal policy to maximize V_t subject to the capital-market-clearing condition and government budget constraint. In particular, we assume away government lending to the private sector, and focus on the government borrowing case, $\hat{b}_{t+1} \geq 0$. Formally, Definition 2 describes the political equilibrium in this framework.

Definition 2. A *political equilibrium* is a sequence of policies $\{\tau_t, x_t\}_{t=0}^{\infty}$, allocations $\left\{c_t, d_t, s_t, k_{t+1}, \hat{b}_{t+1}, h_{t+1}\right\}_{t=0}^{\infty}$, and prices $\{\rho_t, w_t, R_t\}_{t=0}^{\infty}$ with the initial conditions $k_0(>0), h_0(>0)$ and $\hat{b}_0(\geq 0)$, such that (i) the conditions in Definition 1 (Economic Equilibrium) are satisfied, and (ii) in period $t(\geq 0)$, the government chooses $x_t\,(\geq 0)$ and $\hat{b}_{t+1}\,(\geq 0)$ to maximize V_t, subject to the capital market clearing condition and the government budget constraint, given by k_t, h_t, and $\hat{b}_t$.

To formulate the government problem, we substitute the government budget constraint in (4) into V_t in (6) and obtain:

$$V_t = (1+\beta)\ln\left(i_t - (1+n)x_t + (1+n)\hat{b}_{t+1}\right) + \{\beta(\alpha-1)+\gamma\alpha\}\ln k_{t+1} + \gamma\eta\ln x_t + \gamma(1-\eta)\ln h_t + C_0, \tag{7}$$

where i_t is defined by:

$$i_t \equiv (1-\alpha)A\,(k_t)^{\alpha}\,h_t - \alpha A\,(k_t)^{\alpha-1}\,\hat{b}_t.$$

The term $(1-\alpha)A\,(k_t)^{\alpha}\,h_t$ denotes labor income, and $\alpha A\,(k_t)^{\alpha-1}\,\hat{b}_t$ denotes debt repayment. Therefore, i_t presents the government's available resources for its expenditures.

The government's problem is to maximize V_t in (7) subject to the government budget constraint in (4) and capital-market-clearing condition in (5). To more conveniently reformulate the problem, we substitute the government budget constraint and human capital formation function into the capital-market-clearing condition, and obtain:

$$k_{t+1} = \frac{1}{(1+n)D(x_t)^{\eta}(h_t)^{1-\eta}} \cdot \frac{\beta}{1+\beta} \cdot \left[i_t - (1+n)x_t - \frac{(1+n)\hat{b}_{t+1}}{\beta}\right]. \tag{8}$$

We substitute this into V_t in (7) and rearrange the terms to obtain:

$$\begin{aligned} V_t &= (1+\beta)\ln\left[i_t - (1+n)x_t + (1+n)\hat{b}_{t+1}\right] \\ &\quad + \{\beta(\alpha-1)+\gamma\alpha\}\ln\left[i_t - (1+n)x_t - \frac{(1+n)\hat{b}_{t+1}}{\beta}\right] \\ &\quad + \eta(\beta+\gamma)(1-\alpha)\ln x_t + (1-\gamma)(\beta+\gamma)(1-\alpha)\ln h_t + C_1, \end{aligned} \tag{9}$$

where C_1 includes the following constant terms:

$$C_1 \equiv C_0 + \{\beta(\alpha-1)+\gamma\alpha\}\ln\frac{\beta}{(1+\beta)\,(1+n)D}.$$

The first three terms in (9) relate to policy-making. The first term represents the utility of lifetime consumption; the second term shows the utility of the interest income and next generation's wage income affected by physical capital accumulation; and the third term includes the three factors affected by public education: the utility of the return on savings, utility of the next generation's wage income, and utility of the next generation's human capital. The government's problem is to choose a pair of $(x_t, \hat{b}_{t+1})$ that maximizes V_t in (9) and the corresponding tax rate that satisfies the government budget constraint. Solving the problem yields the following policy functions.

Lemma 1. *The policy functions of x_t, $\hat{b}_{t+1}$, and τ_t in the political equilibrium are:*

$$\begin{aligned} x_t &= X_0 \cdot i_t, \\ \hat{b}_{t+1} &= B_0 \cdot i_t, \\ \tau_t &= (1+n)(X_0 - B_0) + \frac{\alpha}{1-\alpha} \cdot \{1 - (1+n)(X_0 - B_0)\} \cdot \frac{b_t}{k_t}, \end{aligned}$$

where:

$$\begin{aligned} X_0 &\equiv (1+n)^{-1} \cdot \left[\frac{(1+\beta) + \{\beta(\alpha-1) + \gamma\alpha\}}{(\beta+\gamma)(1-\alpha)\eta} + 1 \right]^{-1}, \\ B_0 &\equiv \max \left\{ 0, \frac{(1+\beta)\beta - \{\beta(\alpha-1) + \gamma\alpha\}}{(1+n)[1 + (\beta+\gamma)\{\alpha + (1-\alpha)\eta\}]} \right\}. \end{aligned}$$

Proof. See Appendix A.1.

To understand the property of the policy function of x_t, let us focus on the three terms of X_0, $1+\beta$, $\beta(\alpha-1)+\gamma\alpha$, and $(\beta+\gamma)(1-\alpha)\eta$; they correspond to the coefficients of the first, second, and third terms of V_t in (9), respectively. The term $1+\beta$ is the weight on the utility of consumption; a greater weight on it incentivizes the government to reduce public education expenditures in order to increase the disposable income of the individuals and, thus, to raise consumption levels in their middle and elderly ages. The term $\beta(\alpha-1)+\gamma\alpha$ implies that an increase in public education expenditures reduces the physical-to-human-capital ratio, and this, in turn, creates two opposing effects on the utility: an increase in the interest rate income, presented by $\beta(\alpha-1)$, and a decrease in the children's wage income, presented by $\gamma\alpha$. Finally, the term $(\beta+\gamma)(1-\alpha)\eta$ is the weight on the public education expenditures; a greater weight enhances the children's human capital formation and, thus, improves parent's utility. In summary, the terms $\beta(\alpha-1)$ and $(\beta+\gamma)(1-\alpha)\eta$ work to increase public education expenditures, while the terms $1+\beta$ and $\gamma\alpha$ work to decrease them.

The policy function of $\hat{b}_{t+1}$ indicates that $(1+\beta)\beta$ and $\{\beta(\alpha-1)+\gamma\alpha\}$ are crucial to the government's financial state. They correspond to the coefficients of the first and second terms of V_t in (9), respectively, so we can apply the abovementioned interpretation for the policy function of x_t. The term $(1+\beta)\beta$ implies that a greater weight on the utility of consumption incentivizes the government to issue more public bonds in order to raise consumption levels in the middle and elderly ages. The term $\{\beta(\alpha-1)+\gamma\alpha\}$ implies that an increase in public bond issues crowds out physical capital accumulation and, thus, increases the interest rate income, but decreases the children's wage income. The government chooses $\hat{b}_{t+1} > 0$ and, therefore, borrows in the capital market if the effect of the first term, $(1+\beta)\beta$, outweighs the effect of the second term, $\{\beta(\alpha-1)+\gamma\alpha\}$; that is, if $B_0 > 0 \Leftrightarrow \gamma < \beta\{(1+\beta)+(1-\alpha)\}/\alpha$. However, if $B_0 = 0$, that is, $\gamma \geq \beta\{(1+\beta)+(1-\alpha)\}/\alpha$, the government finds it optimal to issue no public bonds.

The policy function of τ_t suggests an influence from both public education expenditures and public bond issues. The first term $(1+n)(X_0 - B_0)$ indicates that the government attempts to increase the tax rate to finance public education expenditures, but can cut the tax rate and finance a part of the expenditure by issuing public bonds. The second term shows the effect of debt repayment on the tax rate. In particular, the term "1" of $\{1-(1+n)(X_0-B_0)\}$ indicates that the government raises the tax rate to finance debt repayment. However, debt repayment pressure incentivizes the government to cut public education expenditures and issue less public bonds. The former effect, represented by $(1+n)X_0$, works to decrease the tax rate, while the latter, represented by $(1+n)B_0$, works in the opposite direction.

Having established the policy functions, we are now ready to demonstrate the accumulation of physical and human capital as well as public debt. We substitute the policy functions in Lemma 1 into the capital accumulation equation in (8), government budget constraint in (4), and human capital formation function given by $h_{t+1} = D(x_t)^{\eta}(h_t)^{1-\eta}$, and obtain:

$$\begin{aligned}
k_{t+1} &= \Psi_K \cdot \left[(1-\alpha)A(k_t)^{\alpha} - \alpha A(k_t)^{\alpha-1} b_t\right]^{1-\eta}, \\
b_{t+1} &= \Psi_B \cdot \left[(1-\alpha)A(k_t)^{\alpha} - \alpha A(k_t)^{\alpha-1} b_t\right]^{1-\eta}, \\
\frac{h_{t+1}}{h_t} &= D(X_0)^{\eta} \cdot \left[(1-\alpha)A(k_t)^{\alpha} - \alpha A(k_t)^{\alpha-1} b_t\right]^{\eta},
\end{aligned}$$

where Ψ_K and Ψ_B are defined by:

$$\Psi_K \equiv \frac{\beta}{1+\beta} \cdot \frac{1-(1+n)X_0 - \frac{1+n}{\beta}B_0}{(1+n)D(X_0)^{\eta}} \text{ and } \Psi_B \equiv \frac{B_0}{D(X_0)^{\eta}},$$

respectively.

Given $\{k_0, b_0, h_0\}$, the sequence $\{k_t, b_t, h_t\}$ is distinguished by the above three equations, which we use to obtain Proposition 1.

Proposition 1. *Suppose that the government can issue public bonds to finance its expenditures. Given* $b_0/k_0 \in (0, (1-\alpha)/\alpha)$,

(i) *there is a unique, stable, steady-state equilibrium with* $k_{t+1} > 0, b_{t+1} = 0$, *and* $\tau_t \in (0,1)$ *if* $\gamma \geq \beta\left\{(1+\beta)+(1-\alpha)\right\}/\alpha$*;*

(ii) *there is a unique, stable, steady-state equilibrium with* $k_{t+1} > 0, b_{t+1} > 0$, *and* $\tau_t \in (0,1)$ *if*

$$\max\left\{\frac{\beta\left\{(1+\beta)+(1-\alpha)(1-\eta)\right\}}{\alpha+(1-\alpha)\eta}, \frac{\beta(1+\alpha\beta)}{\alpha}\right\} < \gamma < \frac{\beta\left\{(1+\beta)+(1-\alpha)\right\}}{\alpha}.$$

Proof. See Appendix A.2.

The result in Proposition 1 suggests that the degree of altruism, denoted by γ, plays a role in shaping fiscal policy. To gain the intuition for the γ assumption, let us, first, consider the assumption in Proposition 1(i);

$$\gamma \geq \frac{\beta}{\alpha}\left\{(1+\beta)+(1-\alpha)\right\} \Rightarrow B_0 = 0.$$

As mentioned above, the government finds it optimal to issue no public bond if the above condition holds. However, if the condition is reversed, as expressed in the second inequality condition in Proposition 1(ii), part of the expenditures is financed by borrowing in the capital market.

Next, consider the role of the lower bound of γ. When γ is below the first lower bound given by $\beta\left\{(1+\beta)+(1-\alpha)(1-\eta)\right\}/\left\{\alpha+(1-\alpha)\eta\right\}$, the degree of altruism is too low to incentivize adults to provide higher levels of education and, thus, they would rather support more public bond issues to finance public education expenditures. In this case, the government gains a surplus in revenue and, therefore, can refund it by subsidizing households. We can rule out this possibility using the first lower bound of γ. In addition, more public bond issues increases the debt repayment cost and, as such, results in a tax rate higher than 100%. The second lower bound of γ guarantees that the tax rate is below 100% for period $t \geq 1$, while the upper bound of b_0/k_0 guarantees that the period 0 tax rate is below 100%.

We should note that the present framework does not involve time inconsistency. In period t, the government representing the period-t middle announces and implements period-t policy. Such individuals are concerned about the period-t policy because it creates costs and benefits in terms of their utility. However, they are indifferent between any two sets of fiscal policies in period $t+1$ (i.e., in their old age) because they owe no costs and receive no benefits from the period-$t+1$ policy. Thus, they have no incentive to act against any policy chosen by the next-period government.

4 Tax and Debt Financing

Public bond issues enable the current generation to pass costs on to future generations. In fact, the public education expenditure, $(1+n)x_t$, plus the interest payment, $(R_t-1)\hat{b}_t$, is greater than the tax revenue, $\tau_t w_t h_t$, along the steady-state debt-financing political equilibrium path with $h'/h > 1$, where h' denotes the next-period h. To confirm this statement, note that $h'/h > 1$ implies $(1+n)h'/h > 1$. We reformulate this as $(1+n)h'b > hb$, or $(1+n)\hat{b}' > \hat{b}$ in the steady state. The government budget constraint in (4) implies that $(1+n)\hat{b}' > \hat{b}$ leads to $(1+n)x < \tau wh + (R-1)\hat{b}$. Therefore, part of the fiscal burden is passed on to future generations via public bond issues when adults have low altruism toward children, $\gamma < \beta\left\{(1+\beta)+(1-\alpha)\right\}/\alpha$.

To consider the impact of debt financing in more detail, here, we assume the following to focus on a situation in which the government chooses debt financing from the viewpoint of maximizing its objective.

Assumption 1. $\gamma < \frac{\beta}{\alpha}\left\{(1+\beta)+(1-\alpha)\right\} \Leftrightarrow B_0 > 0.$

To check the plausibility of this assumption, we calibrate the parameter γ to match the Organisation for Economic Co-operation and Development (OECD) data during 2000–2010 (see Appendix A.6). For this aim, we fix the share of capital at $\alpha = 1/3$ following Song, Storesletten, and Zilibotti (2012) and Lancia and Russo (2016). We set it at $\beta = (0.98)^{30}$, implying that each period lasts 30 years, and that the period discount factor is 0.98. In this setting, we obtain $\gamma = 3.47 < \beta\left\{(1+\beta)+(1-\alpha)\right\}/\alpha = 3.62$, suggesting that Assumption 1 is empirically plausible.

Under Assumption 1, we consider a fiscal rule that prohibits debt financing and requires tax financing as an alternative. Here, we denote $z|_{\rm tax}$ and $z|_{\rm debt}$ the steady-state level of $z(= k, b, \tau$, and $h'/h)$ for the tax- and debt-financing cases, respectively. We, first, compare the aggregate expenditure-GDP ratio and the tax rate in the debt-financing case with those in the tax-financing case, and obtain Proposition 2.

Proposition 2. *Consider the aggregate expenditure-GDP ratio, $N_{t+1}x_t/Y_t$ and the tax rate, τ_t.* (i) *For $t=0$, $N_1x_0/Y_0|_{\rm tax} = N_1x_0/Y_0|_{\rm debt}$. For $t \geq 1$, $N_{t+1}x_t/Y_t|_{\rm tax} > N_{t+1}x_t/Y_t|_{\rm debt}$.* (ii) *For $t=0$, $\tau_0|_{\rm tax} > \tau_0|_{\rm debt}$. For $t \geq 1$, $\tau_t|_{\rm tax} \lesseqgtr \tau_t|_{\rm debt}$ if and only if $\gamma \lesseqgtr \beta(1-\alpha)/\alpha + (1+\beta)^2/(1-\alpha)$.*

Proof. See Appendix A.3.

The first part of Proposition 2 states that both cases have equal aggregate expenditure-GDP ratios in the initial period, but they differ from period 1. To understand this result, recall the policy function x_t, which indicates that the government uses a fraction, X_0, of its available resources, i_t, for public education expenditures in both cases. The government has the same resources available in period 0, given by $i_0 = \left\{(1-\alpha)A\left(k_0\right)^{\alpha} - \alpha A\left(k_0\right)^{\alpha-1} b_0\right\} h_0$ for both cases. Therefore, the expenditure-GDP ratios are equal in period 0. However, they differ from period 1. In the debt-financing case, the government must manage debt repayment, which reduces its available resources. Therefore, the government attains a lower expenditure-GDP ratio in the debt-financing case than in the tax-financing case.

The second part of Proposition 2 states that the tax rate in the tax-financing case is higher than that in the debt-financing case in the initial period, but under a certain condition, this relationship reverses in the next period. In the debt-financing case, the government can implement a tax cut financed by public bond issues. Because of this tax cut, the period 0 tax rate is lower in the debt-financing case than in the tax-financing case. However, from period 1, the government has debt repayment costs in the debt-financing case, and must then raise the tax rate. The second part of Proposition 2 shows that this tax-hike effect is greater (less) than the tax-cut effect if γ is below (above) the critical value given by $\beta(1-\alpha)/\alpha + (1+\beta)^2/(1-\alpha)$. Our calibration in Appendix A.6 suggests that γ is below the critical value. This indicates that the tax rate decreases from period 1 onward when debt financing is prohibited.

Next, we compare the steady-state growth rates for the two cases.

Proposition 3. *The steady-state growth rate in the tax-financing political equilibrium is higher than in the debt-financing political equilibrium;* $h'/h|_{\text{tax}} > h'/h|_{\text{debt}}$.

Proof. See Appendix A.4.

The steady-state growth rates in the tax- and debt-financing cases are given by:

$$\left.\frac{h'}{h}\right|_{\text{tax}} = D \cdot \left[X_0 \left(\frac{\beta}{1+\beta} \cdot \frac{1-(1+n)X_0}{(1+n)D\left(X_0\right)^{\eta}}\right)^{\frac{\alpha}{1-\alpha(1-\eta)}} \left\{(1-\alpha)A\right\}^{\frac{1}{1-\alpha(1-\eta)}}\right]^{\eta},$$

$$\left.\frac{h'}{h}\right|_{\text{debt}} = D \cdot \left[X_0 \left(\frac{\beta}{1+\beta} \cdot \frac{1-(1+n)X_0 - \frac{1+n}{\beta}B_0}{(1+n)D\left(X_0\right)^{\eta}}\right)^{\frac{\alpha}{1-\alpha(1-\eta)}} \left((1-\alpha)A - \alpha A\frac{\Psi_B}{\Psi_K}\right)^{\frac{1}{1-\alpha(1-\eta)}}\right]^{\eta},$$

respectively. The equation $h'/h|_{\text{debt}}$ suggests that there are two negative effects on the growth rate peculiar to the debt-financing case. First, the cost of debt repayment reduces the government's resources, which, in turn, decreases public education expenditures as an engine of economic growth. The term $-(1+n)B_0/\beta$ demonstrates this effect. Second, the cost of debt repayment overwhelms capital accumulation and, thus, lowers the steady-state level of capital, resulting in a higher interest rate, which then further increases the debt repayment cost. The term $\alpha A\Psi_B/\Psi_K$ illustrates this effect. These two negative effects from public debt result in a lower steady-state growth rate in the debt-financing case than in the tax-financing case.

Debt financing enables the government to cut the tax rate (Proposition 2(ii)). The tax cut increases households' disposable income and, thus, lifetime consumption for the period-0 middle-aged agents, while the public education expenditures remain equal in both cases (Proposition 2(i)). The period-0 middle-aged agents find it optimal to choose debt rather than tax financing to maximize their utility when $\gamma < \beta(1-\alpha)/\alpha + (1+\beta)^2/(1-\alpha)$. However, future generations would be worse off due to debt repayment and low growth rates. This suggests the possibility of intergenerational utility tradeoffs.

To investigate the welfare consequences of debt financing across generations, here, we compare the tax- and debt-financing cases in terms of utility, and obtain Proposition 4.

Proposition 4. *Along the equilibrium path with $h'/h > 1$, the steady-state utility of the middle-aged is higher in the tax-financing case than in the debt-financing case: $V_{tax} > V_{debt}$.*

Proof. See Appendix A.5.

In each period, the government, which represents the middle-aged agents, finds it optimal to choose debt financing for the state variables k, b, and h. The choice of debt financing discourages human capital accumulation (Proposition 3), which, in turn, bequeaths less physical and human capital to future generations. This implies that future governments are forced to choose a fiscal policy subject to lower levels of capital. If the government introduces tax financing and continues this for long periods, then future governments can enjoy higher capital and realize a higher utility for the middle-aged, although they are constrained by the choice of fiscal policy. In other words, the choice of future governments is itself optimal for given capital levels, but they obtain less utility for the middle-aged from debt financing than from tax financing.

5 Planner's Allocation

This section demonstrates an allocation chosen by a benevolent planner. The planner has the ability to commit to all of his or her choices at the beginning of a period, subject to the resource constraint. We compare the planner's allocation with the political equilibrium allocation in terms of long-run capital, growth rates, and education-GDP ratios.

The planner is assumed to value the welfare of all generations. In particular, the objective of the planner is to maximize a discounted sum of the life-cycle utility of all current and future generations:

$$SW = \sum_{t=-1}^{\infty} \theta^t U_t,$$

under the resource constraint:

$$N_t c_t + N_{t-1} d_t + K_{t+1} + N_{t+1} x_t = A \left(K_t\right)^{\alpha} \left(H_t\right)^{1-\alpha},$$

or

$$c_t + \frac{1}{1+n}d_t + (1+n)k_{t+1}h_{t+1} + (1+n)x_t = A\left(k_t\right)^{\alpha} h_t,$$

where k_0 and h_0 are given. The parameter $\theta \in (0,1)$ is a planner's discount factor.

In the present framework, the state variable h_t does not lie in a compact set because it continues to grow along an optimal path. To reformulate the problem into one in which the state variable lies in a compact set, here, we undertake the following normalization:

$$\tilde{c}_t \equiv c_t/h_t, \tilde{d}_t \equiv d_t/h_t, \text{ and } \tilde{x}_t \equiv x_t/h_t.$$

Then, the above resource constraint is rewritten as:

$$\tilde{c}_t + \frac{1}{1+n}\tilde{d}_t + (1+n)k_{t+1}D\left(\tilde{x}_t\right)^{\eta} + (1+n)\tilde{x}_t = A\left(k_t\right)^{\alpha}, \tag{10}$$

and the utility function becomes:

$$\begin{aligned}
U_{-1} &= \beta \ln \tilde{d}_0 + (\beta+\gamma)\ln h_0 + \gamma\alpha \ln k_0 + \gamma \ln(1-\alpha)A, \\
U_0 &= \ln \tilde{c}_0 + \beta \ln \tilde{d}_1 + \gamma\alpha \ln k_1 + (\beta+\gamma)\,\eta \ln \tilde{x}_0 + (1+\beta+\gamma)\ln h_0 \\
&\quad + \gamma \ln(1-\alpha)A + (\beta+\gamma)\ln D, \\
U_t &= \ln \tilde{c}_t + \beta \ln \tilde{d}_{t+1} + \gamma\alpha \ln k_{t+1} + (\beta+\gamma)\,\eta \ln \tilde{x}_t + (1+\beta+\gamma)\,\eta \sum_{j=0}^{t-1} \ln \tilde{x}_j \\
&\quad + (1+\beta+\gamma)\ln h_0 + \gamma \ln(1-\alpha)A + \{(\beta+\gamma) + t\cdot(1+\beta+\gamma)\}\ln D, \ t \geq 1.
\end{aligned}$$

The planner's objective function is now given by:

$$SW\left(k_0\right) \simeq \frac{\alpha\gamma}{\theta}\ln k_0 + \sum_{t=0}^{\infty}\theta^t\left[\ln \tilde{c}_t + \frac{\beta}{\theta}\ln \tilde{d}_t + \gamma\alpha \ln k_{t+1} + \left\{(\beta+\gamma) + (1+\beta+\gamma)\frac{\theta}{1-\theta}\right\}\eta \ln \tilde{x}_t\right],$$

where constant terms are omitted from the expression. Thus, we can express the Bellman equation for the problem as follows:

$$SW(k) = \max_{\left\{\tilde{c},\tilde{d},k',\tilde{x}\right\}}\left\{\ln \tilde{c} + \frac{\beta}{\theta}\ln \tilde{d} + \gamma\alpha \ln k' + \left\{(\beta+\gamma) + (1+\beta+\gamma)\frac{\theta}{1-\theta}\right\}\eta \ln \tilde{x} + \theta SW(k')\right\}, \tag{11}$$

subject to (10), where k' denotes the next-period stock of capital.

Solving the problem in (11) leads to the policy functions of k' and x (see Appendix A.7). We compute the corresponding steady-state capital level, growth rate, and education-GDP ratio based on the calibration in Appendix A.6, and compare them to those in the political equilibrium, as depicted in Figure 1. The numerical result shows that the concerned variables in the planner's allocation realize larger values than those in the political equilibrium unless the planner's discount factor is below 0.2. This result suggests that the political equilibrium generally attains under-accumulation of capital and underinvestment of education and, thus, suffers from a lower growth rate due to the shortsightedness of politicians.

[Figure 1 here.]

6 Extensions and Further Analysis

Up to this point, the analysis has assumed (i) no private education; (ii) no alternative public spending, such as old-age benefits; (iii) no fertility choice; and (iv) no international lending and borrowing. This section briefly considers how the analysis and results would change if we relax each of these assumptions. A supplementary explanation for this section is provided in Appendix B.

6.1 Private Education

This subsection introduces private education, denoted by e_t. The budget constraint of the young is $c_t + s_t + (1+n)e_t \leq (1-\tau_t)w_t h_t$, and human capital formation is $h_{t+1} = D\left(x_t\right)^{\eta}\left(e_t\right)^{\delta}\left(h_t\right)^{1-\eta-\delta}$, where δ is the elasticity of human capital with respect to e_t. Solving the utility maximization problem leads to the following indirect utility of the middle-aged:

$$V_t = \left\{1 + \beta + \underbrace{\delta\left(\gamma + \beta\left(1-\alpha\right) - \gamma\alpha\right)}_{e.1}\right\} \ln\left(i_t - (1+n)x_t + (1+n)\hat{b}_{t+1}\right)$$
$$+ \left\{\beta(\alpha-1) + \gamma\alpha\right\} \ln\left(i_t - (1+n)x_t - \underbrace{\frac{1+\gamma\delta}{\beta}}_{e.2}\cdot(1+n)\hat{b}_{t+1}\right) + (\gamma+\beta)(1-\alpha)\eta \ln x_t,$$

where the terms (e.1) and (e.2) are relevant to private education. When $\delta = 0$, this expression matches the indirect utility function in the absence of private education.

The term (e.1) includes the following effects of private education. First, an increase in private education improves the human capital of the next generation, represented by the term γ in parentheses. However, an improvement in the human capital decreases the physical-to-human-capital ratio and, thus, lowers the next generation's wage, but raises the interest rate. These effects are represented by the terms $\beta\left(1-\alpha\right)$ and $-\gamma\alpha$ in parentheses, respectively. The net effect of these three effects on utility is positive since $\gamma + \beta\left(1-\alpha\right) - \gamma\alpha > 0$. Therefore, the term (e.1) implies that the middle-aged attach larger weights to the cost of public education as well as to the benefit of public bond issues when private education is available than in its absence.

The term (e.2) shows that private education increases the cost of public bond issues because it lowers savings, which, in turn, strengthens the crowding-out effect of public bond issues. Overall, private education definitely decreases public education spending, while it has two opposing effects on public bond issues. Direct calculation suggests that the net effect is negative: the availability of private education lowers public bond issues.

To examine the long-run effect of private education, here, we undertake the numerical analysis based on the calibration of the model to OECD countries, as presented in

Appendix A.6. For analysis, we focus on the elasticity of private education, δ, and investigate its impact on steady-state debt, public and private education-GDP ratios, capital, and growth rate, as illustrated in Figure 2. To see this, we consider an increase in δ, accompanied by a one-to-one decrease in η, so that $\delta + \eta$ does not change and is fixed at 0.12.

[Figure 2 here.]

Panel (a) shows that steady-state debt decreases as δ increases. A higher δ implies a higher elasticity of private education and, thus, incentivizes individuals to invest more in private education as a substitute for public education. In turn, this reduces government expenditure on public education, so the government finds it optimal to reduce public bond issues. In particular, there is a threshold value of δ, such that the government issues no public bonds and finances its expenditure solely by taxation when δ is above the threshold value.

Panel (b) shows that public education expenditure decreases, while the private education expenditure increases as δ increases. This result is straightforward because a higher elasticity of private education is associated with a lower elasticity of public education in the present assumption. The results in Panels (a) and (b) suggest that there are two opposing effects on capital accumulation: a positive effect by the reduction of the crowding-out effect of public debt, and a negative effect by the increase in private education associated with a decrease in savings. Below the threshold value of δ, the positive effect outweighs the negative effect; beyond the threshold, the positive effect disappears. Thus, we can observe a hump-shaped pattern of the steady-state capital, as depicted in Panel (c).

Panel (d) shows an initial decrease in the growth rate, followed by an increase, in response to an increase in the elasticity, δ. An increase in δ has the following three effects on the growth rate: (i) a decrease in public education expenditure; (ii) an increase in private education expenditure; and (iii) an increase in capital in the presence of public bond issues, and a decrease in capital in its absence. When δ is below the threshold level, such that the government issues public bonds, the positive effects created by the last two are outweighed by the first negative effect. However, when δ is above the threshold level, such that the government issues no public bonds, the positive effect of the second outweighs the negative effects of the first and third. Therefore, the U-shaped growth pattern is observed in the presence of private education.

6.2 Old-Age Benefits

This subsection introduces old-age benefits as an alternative mode of government spending, and investigates how they affect public education expenditures. For this purpose, we modify the government budget constraint in (4) as:

$$(1+n)\hat{b}_{t+1} + \tau_t w_t h_t = (1+n)x_t + \frac{p_t}{1+n} + R_t \hat{b}_t,$$

where p_t denotes the per-capita benefits to the elderly living in period t. The budget constraint of the elderly is $d_t \leq R_t s_{t-1} + p_t$. The elderly now have an incentive to control the benefits through voting. We keep the assumption that the young are below voting age.

To reflect the preferences of the elderly and middle-aged in voting, here, we employ the probabilistic voting mechanism according to Lindbeck and Weibull (1987). In particular, the period-t political objective function is a weighted average of the elderly and middle-aged:

$$\Omega_t = \omega V_t^o + (1+n)(1-\omega)V_t^m,$$

where $\omega(>0)$ is the relative political weight of the elderly, and V_t^o and V_t^m are the indirect utility functions of the elderly and middle-aged, respectively.

The problem of the government in period t is to choose a fiscal policy to maximize Ω_t subject to the government budget constraint and capital-market-clearing condition. The first-order conditions with respect to p_t and x_t are summarized as follows:

$$\frac{\omega\beta}{j_t + \frac{p_t}{1+n}} = \frac{(1-\omega)(\beta+\gamma)(1-\alpha)}{x_t},$$

where the left-hand side shows the marginal benefit of p and the right-hand side shows the marginal benefit of x. The government chooses x and p to equate these benefits. This expression suggests that the government allocates its spending from public education to old-age benefits as the elderly's political power, ω, increases.

6.3 Endogenous Fertility

This subsection introduces the fertility decisions of the individuals, and investigates how this extension would affect the result. Let n_{t+1} denote the population growth rate from period t to period $t+1$. Following de la Croix and Doepke (2004), we assume that parents care about both child quantity, $1+n_{t+1}$, and quality, h_{t+1}. In particular, the problem of the period-t middle is:

$$\begin{aligned}
&\max \ln c_t + \beta \ln d_{t+1} + \gamma \ln(1+n_{t+1}) w_{t+1} h_{t+1} \\
&\text{s.t. } \; c_t + s_t \leq (1-\tau_t)\, w_t h_t \,(1-\phi(1+n_{t+1})) \\
&d_{t+1} \leq R_{t+1} s_t,
\end{aligned}$$

where $\phi \in (0,1)$ means that raising one child takes fraction ϕ of an adult's time.

Solving the problem leads to:

$$1+n_{t+1}=\frac{1}{\phi}\cdot\frac{\gamma}{1+\beta+\gamma},$$

showing that the fertility rate is constant across periods as in de la Croix and Deopke (2004). Thus, the introduction of endogenous fertility does not qualitatively affect the result, as long as we keep the assumption of a logarithmic utility function.

6.4 Small Open Economy

In this subsection, we modify the model by assuming a small open economy, which allows for international lending and borrowing for a given world interest rate, R. The profit maximization conditions imply that for a given R, per-capita capital k and wage w are uniquely determined and constant over time. This implies that there is no inter-temporal effect of current fiscal policy choice through capital accumulation. Within this specification, we compare the tax- and debt-financing cases in terms of growth and utility.

We, first, consider the tax-financing case. We assume $\hat{b}_{t+1}=0$ and maximize V_t in (7), with respect to x_t, to obtain:

$$x_t|_{tax}=\begin{cases}\frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)}\left(wh_0-R\hat{b}_0\right) & \\ \frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)}w\ h_t|_{tax} & \text{for } t\geq 1.\end{cases}$$

This is the public education expenditure in the tax-financing case.

In the debt-financing case, the government's objective function in (7) suggests that since k is constant, there is no crowding-out effect of public bond issues. This incentivizes the government to borrow as much as possible in the capital market. The borrowing reaches its ceiling, that is, the natural debt limit, $\bar{b}_t$, which is defined by setting $\tau_t=1$ and $x_t=0$ in the government budget constraint. Thus, $\bar{b}_t$ satisfies $(1+n)\bar{b}_t+wh_t=0+R\bar{b}_t$, or:

$$\bar{b}_t=\frac{w}{R-(1+n)}h_t=\bar{b}\cdot h_t,$$

where $\bar{b}\equiv w/(R-(1+n))$. We assume $R>1+n$ in the following.

Given $\hat{b}_{t+1}=\bar{b}\cdot h_t$, the solution maximizing V_t in (7) is:

$$x_t|=\begin{cases}\frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)}\left(wh_0-R\hat{b}_0+(1+n)\bar{b}_0\right) & \\ 0 & \text{for } t\geq 1.\end{cases}$$

In period 0, the government can finance its public education expenditure by borrowing in the capital market. However, from period 1 onward, the government is unable to spend its revenue for public education because it must use the revenue to repay debt. Therefore, the choice of fiscal policy in the debt-financing case results in no human capital growth,

$h_{t+1}/h_t|_{debt} = 0$. This, in turn, leads to $V_t|_{debt} = -\infty$ from period 1 onward because of the specification of the logarithmic utility function.

The result, thus far, is summarized as:

$$x_0|_{tax} < x_0|_{debt}\,,$$
$$x_t|_{tax} > x_t|_{debt} = 0 \text{ for } t \geq 1.$$

Given the property that growth and utility increase in response to an increase in public education expenditure, we can conclude that in period 0, growth and utility are higher in the debt-financing case than in the tax-financing case. However, the result is reversed from period 1 onward because there is no growth as well as no available resources for consumption and public education in the debt-financing case. This result is qualitatively similar to that shown in the closed economy analysis of the main text.

7 Conclusion

This paper developed an overlapping-generations model with physical and human capital accumulation using public education and parental human capital as inputs in the process of human capital formation. Public education spending is financed through tax on labor income in the tax-financing case, and by both tax and public bond issues in the debt-financing case. Within this framework, we demonstrated the endogenous policy formation and showed that the current generation will prefer debt financing to tax financing, since the former enables the current generation to pass the debt repayment costs on to future generations. However, the debt repayment costs induce successive generations to cut the expenditure, making them worse off than in the tax-financing case. Our results provide one possible explanation for why debt financing continues to dominate policy in many developed countries, despite being expected to perform worse than tax financing.

A Proofs

A.1 Proof of Lemma 1

The first-order conditions with respect to x_t and $\hat{b}_{t+1}$ are:

$$x_t : \frac{(1+\beta)(1+n)}{i_t-(1+n)x_t+(1+n)\hat{b}_{t+1}} + \frac{\{\beta(\alpha-1)+\gamma\alpha\}(1+n)}{i_t-(1+n)x_t-\frac{1+n}{\beta}\hat{b}_{t+1}} = \frac{(\beta+\gamma)(1-\alpha)\eta}{x_t}, \quad (12)$$

$$\hat{b}_{t+1} : \frac{(1+\beta)(1+n)}{i_t-(1+n)x_t+(1+n)\hat{b}_{t+1}} \leq \frac{\{\beta(\alpha-1)+\gamma\alpha\}\frac{1+n}{\beta}}{i_t-(1+n)x_t-\frac{1+n}{\beta}\hat{b}_{t+1}}, \quad (13)$$

where an equality holds in (13) if $\hat{b}_{t+1} > 0$, while an inequality holds if $\hat{b}_{t+1} = 0$.

Assume that $\hat{b}_{t+1} > 0$. We substitute (13) with an equality into (12), and obtain:

$$\frac{\{\beta(\alpha-1)+\gamma\alpha\}\frac{1+n}{\beta}}{i_t-(1+n)x_t-\frac{1+n}{\beta}\hat{b}_{t+1}} + \frac{\{\beta(\alpha-1)+\gamma\alpha\}(1+n)}{i_t-(1+n)x_t-\frac{1+n}{\beta}\hat{b}_{t+1}} = \frac{(\beta+\gamma)(1-\alpha)\eta}{x_t},$$

or

$$x_t = \frac{(\beta+\gamma)(1-\alpha)\eta}{(1+n)\left[\{\beta(\alpha-1)+\gamma\alpha\}(1+\beta)+\beta(\beta+\gamma)(1-\alpha)\eta\right]} \cdot \left[\beta i_t-(1+n)\hat{b}_{t+1}\right]. \quad (14)$$

We substitute (14) into (13). After rearranging the terms, we obtain the policy function of $\hat{b}_{t+1}$ as

$$\hat{b}_{t+1} = \frac{(1+\beta)\beta-\{\beta(\alpha-1)+\gamma\alpha\}}{(1+n)\left[1+(\beta+\gamma)(\alpha+(1-\alpha)\eta)\right]} i_t.$$

We then substitute this into (14) to obtain the policy function of x_t as $x_t = X_0 \cdot i_t$. Therefore, we obtain

$$x_t = X_0 \cdot i_t \text{ and } \hat{b}_{t+1} = B_0 \cdot i_t,$$

where X_0 and B_0 are defined in Lemma 1.

To obtain the policy function of τ_t, recall the government budget constraint, which is reformulated as $(1+n)\hat{b}_{t+1} + \tau_t(1-\alpha)A(k_t)^\alpha h_t = (1+n)x_t + \alpha A(k_t)^{\alpha-1}\hat{b}_t$. Plugging the policy function of x_t and $\hat{b}_{t+1}$ into this constraint and using $b_t = \hat{b}_t h_t$, we have:

$$\begin{aligned}\tau_t(1-\alpha)A(k_t)^\alpha h_t &= (1+n)X_0 i_t + \alpha A(k_t)^{\alpha-1} b_t h_t - (1+n)B_0 i_t \\ &= (1+n)(X_0-B_0)\left\{(1-\alpha)A(k_t)^\alpha h_t - \alpha A(k_t)^{\alpha-1} b_t h_t\right\} + \alpha A(k_t)^{\alpha-1} b_t h_t \\ &= (1+n)(X_0-B_0)(1-\alpha)A(k_t)^\alpha h_t + \{1-(1+n)(X_0-B_0)\}\alpha A(k_t)^{\alpha-1} b_t h_t.\end{aligned}$$

Dividing both sides by $(1-\alpha)A(k_t)^\alpha h_t$, we obtain:

$$\tau_t = (1+n)(X_0-B_0) + \frac{\alpha}{1-\alpha}\{1-(1+n)(X_0-B_0)\}\frac{b_t}{k_t}.$$

■

A.2 Proof of Proposition 1

First, suppose that $\gamma \geq \beta \{(1+\beta)+(1-\alpha)\}/\alpha$ (i.e., $B_0 = 0$) holds. We obtain $\hat{b}_{t+1} = 0$ from Lemma 1, and the corresponding tax rate as follows:

$$\tau_t = \begin{cases} (1+n)X_0 + \frac{\alpha}{1-\alpha}\{1-(1+n)X_0\}\frac{b_0}{k_0} & \text{for } t=0, \\ (1+n)X_0 & \text{for } t \geq 1. \end{cases}$$

The tax rate from period 1 onward satisfies $\tau_t \in (0,1)$ because $(1+n)X_0 \in (0,1)$ holds; and the tax rate in period 0 satisfies $\tau_0 \in (0,1)$ if

$$0 < (1+n)X_0 + \frac{\alpha}{1-\alpha}\{1-(1+n)X_0\}\frac{b_0}{k_0} < 1.$$

Given that $1-(1+n)X_0 > 0$, the first inequality holds if $b_0/k_0 > 0$; the second inequality holds if $b_0/k_0 < (1-\alpha)/\alpha$. Thus, $\tau_t \in (0,1)$ holds if $b_0/k_0 \in (0, (1-\alpha)/\alpha)$.

We can use the policy functions of x_t and τ_t to reformulate the capital market clearing condition in (5) as follows:

$$k_{t+1} = \begin{cases} \Psi_K \cdot \left[(1-\alpha)A(k_0)^{\alpha} - \alpha A(k_0)^{\alpha-1} b_0\right]^{1-\eta} & \text{for } t=0, \\ \Psi_K \cdot \left[(1-\alpha)A(k_t)^{\alpha}\right]^{1-\eta} & \text{for } t \geq 1. \end{cases}$$

Given $k_0(>0)$ and $b_0(>0)$, a unique sequence of $\{k_t\}$ satisfies the above condition. The condition for $t \geq 1$ suggests that the sequence stably converges to the unique steady state.

Next, suppose that $\gamma \leq \beta\{(1+\beta)+(1-\alpha)\}/\alpha$ (i.e., $B_0 > 0$) holds. We first derive the conditions for which $k_{t+1} > 0, b_{t+1} > 0$, and $\tau_t \in (0,1)$ hold along the equilibrium path. Second, we show the existence and uniqueness of the steady-state equilibrium. Finally, we show that the unique steady-state equilibrium is a sink.

Step 1.

Recall the capital and debt accumulation equations given by:

$$k_{t+1} = \Psi_K \cdot \left[(1-\alpha)A(k_t)^{\alpha} - \alpha A(k_t)^{\alpha-1} b_t\right]^{1-\eta} = \Psi_K \cdot \left(\frac{i_t}{h_t}\right)^{1-\eta}, \tag{15}$$

$$b_{t+1} = \Psi_B \cdot \left[(1-\alpha)A(k_t)^{\alpha} - \alpha A(k_t)^{\alpha-1} b_t\right]^{1-\eta} = \Psi_B \cdot \left(\frac{i_t}{h_t}\right)^{1-\eta}, \tag{16}$$

respectively. $k_{t+1} > 0$ and $b_{t+1} > 0$ hold along the equilibrium path if $i_t/h_t > 0$ for all $t, \Psi_K > 0$, and $\Psi_B > 0$.

In period 0, we reformulate the term i_0/h_0 as:

$$\frac{i_0}{h_0} = (1-\alpha)A(k_0)^{\alpha} \cdot \left[1 - \frac{\alpha}{1-\alpha} \cdot \frac{b_0}{k_0}\right].$$

Thus, $i_0/h_0 > 0$ holds if

$$\frac{b_0}{k_0} < \frac{1-\alpha}{\alpha}. \tag{17}$$

In period $t \geq 1$, the term i_t/h_t is rewritten as

$$\begin{aligned}
\frac{i_t}{h_t} &= (1-\alpha)A\,(k_t)^\alpha \cdot \left[1 - \frac{\alpha}{1-\alpha} \cdot \frac{b_t}{k_t}\right] \\
&= (1-\alpha)A\,(k_t)^\alpha \cdot \left[1 - \frac{\alpha}{1-\alpha} \cdot \frac{\Psi_B}{\Psi_K}\right] \\
&= (1-\alpha)A\,(k_t)^\alpha \cdot \frac{\left[\left\{1-(1+n)X_0 - \frac{1+n}{\beta}B_0\right\} - (1+n)\frac{\alpha(1+\beta)}{(1-\alpha)\beta}B_0\right]}{1-(1+n)X_0 - \frac{1+n}{\beta}B_0},
\end{aligned}$$

where we obtain the second line using (15) and (16), and the third line using the definition of Ψ_K and Ψ_B.

The expression above suggests that $i_t/h_t > 0$ holds for $t \geq 1$ if the following condition holds:

$$B_0 > 0 \text{ and } \left\{1-(1+n)X_0 - \frac{1+n}{\beta}B_0\right\} - (1+n)\frac{\alpha(1+\beta)}{(1-\alpha)\beta}B_0 > 0. \tag{18}$$

In addition, the definitions of Ψ_K and Ψ_B suggest that $\Psi_K > 0$ and $\Psi_B > 0$ hold if (18) holds. After some manipulation, we have:

$$B_0 > 0 \Leftrightarrow \gamma < \frac{\beta}{\alpha}\left\{(1+\beta) + (1-\alpha)\right\}, \tag{19}$$

$$\left\{1-(1+n)X_0 - \frac{1+n}{\beta}B_0\right\} - (1+n)\frac{\alpha(1+\beta)}{(1-\alpha)\beta}B_0 > 0 \Leftrightarrow \frac{\beta}{\alpha}(1+\alpha\beta) < \gamma. \tag{20}$$

Therefore, $k_{t+1} > 0$ and $b_{t+1} > 0$ for $t \geq 0$ if (17), (19), and (20) hold.

Next, consider the tax rate:

$$\tau_t = (1+n)\left(X_0 - B_0\right) + \frac{\alpha}{1-\alpha} \cdot \left\{1-(1+n)\left(X_0 - B_0\right)\right\} \cdot \frac{b_t}{k_t},$$

where

$$1-(1+n)\left(X_0 - B_0\right) > 0.$$

Given $b_0/k_0 > 0$, $\tau_0 > 0$ holds if $X_0 - B_0 > 0$, that is, if

$$\frac{\beta\left\{(1+\beta) + (1-\eta)(1-\alpha)\right\}}{\alpha + (1-\alpha)\eta} < \gamma. \tag{21}$$

For $t \geq 1$, $\tau_t > 0$ holds if $X_0 - B_0 > 0$ and $b_t/k_t > 0$, that is, if (19), (20), and (21) hold.

To find the condition for $\tau_t < 1$, let us first consider the period-0 tax rate. By direct calculation, we have:

$$\tau_0 < 1 \Leftrightarrow \frac{b_0}{k_0} < \frac{1-\alpha}{\alpha} \Leftrightarrow (17).$$

For period $t \geq 1$, $b_t/k_t = \Psi_B/\Psi_K$ holds. We thus have:

$$\tau_t < 1 \Leftrightarrow \frac{\alpha}{1-\alpha} \cdot \frac{\Psi_B}{\Psi_K} < 1 \Leftrightarrow \frac{\beta(1+\alpha\beta)}{\alpha} < \gamma \Leftrightarrow (20).$$

The results established thus far suggest that $k_{t+1} > 0$, $b_{t+1} > 0$, and $\tau_t \in (0,1)$ hold if (17), (19), (20), and (21) hold, that is, if:

$$\max\left\{\frac{\beta\left\{(1+\beta)+(1-\alpha)(1-\eta)\right\}}{\alpha+(1-\alpha)\eta}, \frac{\beta(1+\alpha\beta)}{\alpha}\right\} < \gamma < \frac{\beta\left\{(1+\beta)+(1-\alpha)\right\}}{\alpha},$$

and

$$\frac{b_0}{k_0} \in \left(0, \frac{1-\alpha}{\alpha}\right).$$

Step 2.

The steady-state pair of (k, b) satisfies:

$$\begin{aligned} k &= \Psi_K \cdot \left[(1-\alpha)A\,(k)^{\alpha} - \alpha A\,(k)^{\alpha-1}\,b\right]^{1-\eta}, \\ b &= \Psi_B \cdot \left[(1-\alpha)A\,(k)^{\alpha} - \alpha A\,(k)^{\alpha-1}\,b\right]^{1-\eta}. \end{aligned}$$

These equations lead to $k/b = \Psi_K/\Psi_B$, that is, $b = (\Psi_B/\Psi_K)\cdot k$. Plugging this into $k = \Psi_K \cdot \left[(1-\alpha)A\,(k)^{\alpha} - \alpha A\,(k)^{\alpha-1}\,b\right]^{1-\eta}$, and rearranging the terms, we obtain a unique value of k given by:

$$k = \left[\Psi_K \cdot \left\{(1-\alpha)A - \alpha A\frac{\Psi_B}{\Psi_K}\right\}^{1-\eta}\right]^{1/(1-\alpha(1-\eta))}. \tag{22}$$

The corresponding value of b is also uniquely determined using $b = (\Psi_B/\Psi_K)\,k$.

Step 3.

Recall the law of motions of capital and debt:

$$\begin{aligned} k_{t+1} &= \Psi_K \cdot \left[(1-\alpha)A\,(k_t)^{\alpha} - \alpha A\,(k_t)^{\alpha-1}\,b_t\right]^{1-\eta}, \\ b_{t+1} &= \Psi_B \cdot \left[(1-\alpha)A\,(k_t)^{\alpha} - \alpha A\,(k_t)^{\alpha-1}\,b_t\right]^{1-\eta}. \end{aligned}$$

Differentiating these with respect to k and b and evaluating them at the steady state, we obtain:

$$\begin{bmatrix} dk_{t+1} \\ db_{t+1} \end{bmatrix} = J \begin{bmatrix} dk_t \\ db_t \end{bmatrix},$$

where

$$J \equiv \begin{bmatrix} \Psi_K \cdot \hat{\imath}(k,b)\cdot \alpha(1-\alpha)A(k)^{\alpha-2}(k+b) & -\Psi_K \cdot \hat{\imath}(k,b)\cdot \alpha A(k)^{\alpha-1} \\ \Psi_B \cdot \hat{\imath}(k,b)\cdot \alpha(1-\alpha)A(k)^{\alpha-2}(k+b) & -\Psi_B \cdot \hat{\imath}(k,b)\cdot \alpha A(k)^{\alpha-1} \end{bmatrix},$$

and

$$\hat{\imath}(k,b) \equiv \left[(1-\alpha)A(k)^{\alpha} - \alpha A(k)^{\alpha-1}b\right]^{-\eta}\cdot(1-\eta).$$

For any $(k,b) \gg 0$, the trace and determinant of J are

$$\begin{cases} \mathrm{tr}J = \alpha A(k)^{\alpha-1}\cdot \hat{\imath}(k,b)\cdot\left[(1-\alpha)\Psi_K - \alpha\Psi_B\right] > 0, \\ \det J = 0. \end{cases}$$

Here, the sign of the term $(1-\alpha)\Psi_K - \alpha\Psi_B$ is

$$(1-\alpha)\Psi_K - \alpha\Psi_B > 0 \Leftrightarrow \gamma > \frac{\beta}{\alpha}(1+\alpha\beta),$$

where $\gamma > \frac{\beta}{\alpha}(1+\alpha\beta)$ is satisfied with the assumption in Proposition 5. Moreover, it is true that:

$$\Delta \equiv (\text{tr}J)^2 - 4\text{det}J = (\text{tr}J)^2 > 0.$$

Therefore, J has two distinct positive eigenvalues, denoted by λ_1 and $\lambda_2(> \lambda_1)$, at the steady state.

We obtain the eigenvalues of J by solving the following equation:

$$p(\lambda) \equiv (\lambda)^2 - (\text{tr}J)\,\lambda + \text{det}J = 0.$$

The solution of $p(\lambda) = 0$ is

$$\lambda_1 = 0 \text{ and } \lambda_2 = \text{tr}J(> 0).$$

The remaining task is to show $\lambda_2 < 1$. For this purpose, recall the law of motion of capital in the steady state, $k = \Psi_K \cdot \left[(1-\alpha)A\,(k)^{\alpha} - \alpha A\,(k)^{\alpha-1}\,b\right]^{1-\eta}$. This is reformulated as:

$$\hat{\imath}(k,b) = (1-\eta)\left(\frac{k}{\Psi_K}\right)^{\frac{-\eta}{1-\eta}}. \tag{23}$$

Using (22) and (23), we rewrite $\lambda_2 =$tr$J$ as follows:

$$\begin{aligned}
\lambda_2 &= \text{tr}J \\
&= \alpha A(k)^{\alpha-1}(1-\eta)\left(\frac{k}{\Psi_K}\right)^{\frac{-\eta}{1-\eta}}\left[(1-\alpha)\Psi_K - \alpha\Psi_B\right] \\
&= \alpha A\,(\Psi_K)^{\frac{-1}{1-\eta}}\frac{1}{(1-\alpha)A - \alpha A\frac{\Psi_B}{\Psi_K}}(1-\eta)\left(\frac{1}{\Psi_K}\right)^{\frac{-\eta}{1-\eta}}\left[(1-\alpha)\Psi_K - \alpha\Psi_B\right] \\
&= \alpha(1-\eta) \in (0,1),
\end{aligned}$$

where the first line comes from (23) and the second line is derived using (22).

■

A.3 Proof of Proposition 2.

(i) In both cases, the policy function of x is $x_t = X_0 \cdot i_t$, where $i_t \equiv \left\{(1-\alpha)A(k_t)^{\alpha} - \alpha A(k_t)^{\alpha-1}\right\}h_t$. Given that $Y_t = A(k_t)^{\alpha}h_tN_t$, we compute the ratio $N_{t+1}x_t/Y_t$ as follows:

$$\begin{aligned}
\left.\frac{N_{t+1}x_t}{Y_t}\right|_{\text{tax}} &= \frac{(1+n)X_0\left\{(1-\alpha)A\,(k_t)^{\alpha} - \alpha A\,(k_t)^{\alpha-1}\,b_t\right\}h_t}{A\,(k_t)^{\alpha}\,h_t} \\
&= \begin{cases} (1+n)X_0\left\{(1-\alpha) - \alpha b_0/k_0\right\} & \text{for } t = 0, \\ (1+n)X_0(1-\alpha) \text{ for } t \geq 1, \end{cases}
\end{aligned}$$

and

$$\left.\frac{N_{t+1}x_t}{Y_t}\right|_{\text{debt}} = \frac{(1+n)X_0\left\{(1-\alpha)A\left(k_t\right)^{\alpha} - \alpha A\left(k_t\right)^{\alpha-1} b_t\right\} h_t}{A\left(k_t\right)^{\alpha} h_t}$$
$$= (1+n)X_0\left\{(1-\alpha) - \alpha\, b_t/k_t|_{\text{debt}}\right\},$$

where $b_t > k_t$ holds for any equilibrium path. Therefore, we obtain the first part of Proposition 2.

(ii) Recall that the tax rates in the tax-finance and debt-finance cases are given by:

$$\tau_t|_{\text{tax}} = \begin{cases} (1+n)X_0 + (1-(1+n)X_0)\frac{\alpha}{1-\alpha}\frac{b_0}{k_0} & \text{for } t=0, \\ (1+n)X_0 & \text{for } t \geq 1, \end{cases}$$

and

$$\tau_t|_{\text{debt}} = \begin{cases} (1+n)\left(X_0 - B_0\right) + \frac{\alpha}{1-\alpha}\cdot\left\{1-(1+n)\left(X_0 - B_0\right)\right\}\cdot\frac{b_0}{k_0} & \text{for } t=0, \\ (1+n)\left(X_0 - B_0\right) + \frac{\alpha}{1-\alpha}\cdot\left\{1-(1+n)\left(X_0 - B_0\right)\right\}\cdot\frac{\Psi_B}{\Psi_K} & \text{for } t \geq 1. \end{cases}$$

For $t=0$, $\tau_0|_{\text{tax}}$ and $\tau_0|_{\text{debt}}$ are compared as follows:

$$\tau_0|_{\text{tax}} \gtreqless \tau_0|_{\text{debt}} \Leftrightarrow \frac{b_0}{k_0} \lesseqgtr \frac{1-\alpha}{\alpha}.$$

Assuming $\frac{b_0}{k_0} < \frac{1-\alpha}{\alpha}$ in Proposition 1, we obtain $\tau_0|_{\text{tax}} > \tau_0|_{\text{debt}}$.

For $t \geq 1$, $\tau_t|_{\text{tax}}$ and $\tau_t|_{\text{debt}}$ are compared as follows:

$$\begin{aligned} \tau_t|_{\text{tax}} \gtreqless \tau_t|_{\text{debt}} &\Leftrightarrow 1 \gtreqless \frac{\alpha}{1-\alpha}\left\{1-(1+n)\left(X_0 - B_0\right)\right\}\frac{1+\beta}{\beta\left\{1-(1+n)X_0 - \frac{1+n}{\beta}B_0\right\}} \\ &\Leftrightarrow \beta\left\{1-(1+n)X_0 - \frac{1+n}{\beta}B_0\right\} \gtreqless \frac{\alpha}{1-\alpha}(1+\beta)\left\{1-(1+n)\left(X_0 - B_0\right)\right\} \\ &\Leftrightarrow \gamma \gtreqless \frac{\beta(1-\alpha)}{\alpha} + \frac{(1+\beta)^2}{1-\alpha}, \end{aligned}$$

where the second line comes from $1-(1+n)X_0 - \frac{1+n}{\beta}B_0 > 0$, and the third line comes from $(\beta+\gamma)\,\alpha/\beta - 1 > 0$, which holds under the assumption in Proposition 1. Therefore, we obtain the second part of Proposition 2.

■

A.4 Proof of Proposition 3

The policy function of x and the capital accumulation equation in the steady state for the tax-finance case are given by

$$x = X_0\cdot(1-\alpha)A(k)^{\alpha}h,$$
$$k = \left(\Psi_K|_{B_0=0}\right)\cdot\left\{(1-\alpha)A(k)^{\alpha}\right\}^{1-\eta}.$$

Combining these equations, we obtain:

$$x = X_0(1-\alpha)A\left(\Psi_K|_{B_0=0}\right)^{\frac{\alpha}{1-\alpha(1-\eta)}}\{(1-\alpha)A\}^{\frac{\alpha(1-\eta)}{1-\alpha(1-\eta)}}h.$$

Substituting this into $h' = D(x)^{\eta}(h)^{1-\eta}$ leads to:

$$h' = D\cdot\left[X_0(1-\alpha)A\left(\Psi_K|_{B_0=0}\right)^{\frac{\alpha}{1-\alpha(1-\eta)}}\{(1-\alpha)A\}^{\frac{\alpha(1-\eta)}{1-\alpha(1-\eta)}}h\right]^{\eta}(h)^{1-\eta},$$

or

$$\left.\frac{h'}{h}\right|_{\text{tax}} = D\cdot\left[X_0\{(1-\alpha)A\}^{\frac{1}{1-\alpha(1-\eta)}}\left(\Psi_K|_{B_0=0}\right)^{\frac{\alpha}{1-\alpha(1-\eta)}}\right]^{\eta}. \tag{24}$$

Next, recall the set of the three equations for the debt-finance case and evaluate them at the steady state:

$$\begin{aligned} k &= \Psi_K\cdot\left[(1-\alpha)A(k)^{\alpha}-\alpha A(k)^{\alpha-1}b\right]^{1-\eta},\\ b &= \Psi_B\cdot\left[(1-\alpha)A(k)^{\alpha}-\alpha A(k)^{\alpha-1}b\right]^{1-\eta},\\ \frac{h'}{h} &= D(X_0)^{\eta}\cdot\left[(1-\alpha)A(k)^{\alpha}-\alpha A(k)^{\alpha-1}b\right]^{\eta}. \end{aligned}$$

The first two equations imply

$$k = (\Psi_K)^{\frac{1}{1-\alpha(1-\eta)}}\left[(1-\alpha)A-\alpha A\frac{\Psi_B}{\Psi_K}\right]^{\frac{1-\eta}{1-\alpha(1-\eta)}}.$$

Plugging this into the first equation, we obtain:

$$\left[(1-\alpha)A(k)^{\alpha}-\alpha A(k)^{\alpha-1}b\right]^{\eta} = (\Psi_K)^{\frac{\alpha\eta}{1-\alpha(1-\eta)}}\left[(1-\alpha)A-\alpha A\frac{\Psi_B}{\Psi_K}\right]^{\frac{\eta}{1-\alpha(1-\eta)}}.$$

We substitute this into the third equation and obtain:

$$\left.\frac{h'}{h}\right|_{\text{debt}} = D(X_0)^{\eta}(\Psi_K)^{\frac{\alpha\eta}{1-\alpha(1-\eta)}}\left[(1-\alpha)A-\alpha A\frac{\Psi_B}{\Psi_K}\right]^{\frac{\eta}{1-\alpha(1-\eta)}}. \tag{25}$$

A direct comparison of (24) and (25) leads to:

$$\begin{aligned} \left.\frac{h'}{h}\right|_{\text{tax}} \gtrless \left.\frac{h'}{h}\right|_{\text{debt}} &\Leftrightarrow \left(\Psi_K|_{B_0=0}\right)^{\alpha}(1-\alpha)A \gtrless (\Psi_K)^{\alpha}\left[(1-\alpha)A-\alpha A\frac{\Psi_B}{\Psi_K}\right]\\ &\Leftrightarrow \left[\frac{\beta}{1+\beta}\cdot\frac{1-(1+n)X_0}{(1+n)D(X_0)^{\eta}}\right]^{\alpha}(1-\alpha)A\\ &\gtrless \left[\frac{\beta}{1+\beta}\cdot\frac{1-(1+n)X_0-\frac{1+n}{\beta}B_0}{(1+n)D(X_0)^{\eta}}\right]^{\alpha}\left[(1-\alpha)A-\alpha A\frac{\Psi_B}{\Psi_K}\right]\\ &\Leftrightarrow [1-(1+n)X_0]^{\alpha}(1-\alpha)A \gtrless \left[1-(1+n)X_0-\frac{1+n}{\beta}B_0\right]^{\alpha}\left[(1-\alpha)A-\alpha A\frac{\Psi_B}{\Psi_K}\right], \end{aligned}$$

where we obtain the second line from the definitions of $\Psi_K|_{B_0=0}$ and Ψ_K.

Given that $B_0 > 0$ and $X_0 > 0$, we have:

$$1-(1+n)X_0 > 1-(1+n)X_0 - \frac{1+n}{\beta}B_0,$$
$$(1-\alpha)A > (1-\alpha)A - \alpha A\frac{\Psi_B}{\Psi_K}.$$

Therefore, we find that $\frac{h'}{h}\big|_{\text{tax}} > \frac{h'}{h}\big|_{\text{debt}}$.

$\blacksquare$

A.5 Proof of Proposition 4

Recall the indirect utility function of the period-t middle-aged, (7). Given k_t, b_t, and h_t, we substitute the corresponding policy functions into (7) and obtain

$$\begin{aligned}
V_{t,\text{tax}} &= \{1+(\beta+\gamma)(\alpha+(1-\alpha)\eta)\}\ln\left\{(1-\alpha)A(k_t)^{\alpha} - \alpha A(k_t)^{\alpha-1}b_t\right\} \\
&\quad + (1-\eta)(\beta+\gamma)(1-\alpha)\ln h_t + C_{tax} \\
V_{t,\text{debt}} &= \{1+(\beta+\gamma)(\alpha+(1-\alpha)\eta)\}\ln\left\{(1-\alpha)A(k_t)^{\alpha} - \alpha A(k_t)^{\alpha-1}b_t\right\} \\
&\quad + (1-\eta)(\beta+\gamma)(1-\alpha)\ln h_t + C_{debt},
\end{aligned}$$

where

$$\begin{aligned}
C_{tax} &\equiv \{1+(\beta+\gamma)\alpha\}\ln(1-(1+n)X_0) + \eta(\beta+\gamma)(1-\alpha)\ln X_0 + C_1, \\
C_{debt} &\equiv (1+\beta)\ln[1-(1+n)X_0+(1+n)B_0] + + \{\beta(\alpha-1)+\gamma\alpha\}\ln\left\{1-(1+n)X_0 - \frac{1+n}{\beta}B_0\right\} \\
&\quad + (\beta+\gamma)(1-\alpha)\eta\ln X_0 + C_1,
\end{aligned}$$

and

$$V_{t,\text{tax}} \lesseqgtr V_{t,\text{debt}} \Leftrightarrow C_{tax} \lesseqgtr C_{debt}.$$

Notice that $C_{tax} < C_{debt}$ holds because we assume the parametric condition that realizes $b > 0$, that is, $V_{t,\text{tax}} < V_{t,\text{debt}}$.

In the steady state, the expressions of $V_{t,\text{tax}}$ and $V_{t,\text{debt}}$ are reformulated as

$$\begin{aligned}
V_{tax} &= \{1+(\beta+\gamma)(\alpha+(1-\alpha)\eta)\}\ln\underbrace{(1-\alpha)A(k_{tax})^{\alpha}}_{(\#1)} \\
&\quad + (1-\eta)(\beta+\gamma)(1-\alpha)\ln\underbrace{\left(\lim_{t\to\infty}h_{t,tax}\right)}_{(\#2)} + C_{tax}, \\
V_{debt} &= \{1+(\beta+\gamma)(\alpha+(1-\alpha)\eta)\}\ln\underbrace{\left\{(1-\alpha)A(k_{debt})^{\alpha} - \alpha A(k_{debt})^{\alpha-1}b\right\}}_{(\#3)} \\
&\quad + (1-\eta)(\beta+\gamma)(1-\alpha)\ln\underbrace{\left(\lim_{t\to\infty}h_{t,debt}\right)}_{(\#4)} + C_{debt},
\end{aligned}$$

where k_{tax} and k_{debt} denote the steady-state capital in the tax-finance and the debt-finance cases, respectively. While $C_{tax} < C_{debt}$ holds under the present parametric condition, the term (#2) outweighs the term (#4) as demonstrated in Proposition 3. In addition, in the steady state, we obtain

$$\begin{aligned}&(1-\eta)(\beta+\gamma)(1-\alpha)\ln\left(\lim_{t\to\infty} h_{t,tax}\right) + C_{tax}\\ &> (1-\eta)(\beta+\gamma)(1-\alpha)\ln\left(\lim_{t\to\infty} h_{t,debt}\right) + C_{debt},\end{aligned}$$

because the difference between $h_{t,tax}$ and $h_{t,debt}$ becomes larger as time passes. Therefore, $V_{tax} > V_{debt}$ holds in the steady state if (#1) is larger than (#3), that is, if $k_{tax} > k_{debt}$.

To show $k_{tax} > k_{debt}$, recall that k_{tax} satisfies the capital-market-clearing condition, $k_{tax} = \Psi_K|_{B_0=0} \cdot \{(1-\alpha)A\,(k_{tax})^{\alpha}\}^{1-\eta}$, that is,

$$k_{tax} = \left(\tilde{\Psi}_K\right)^{\frac{1}{1-\alpha(1-\eta)}} \cdot [(1-\alpha)A]^{\frac{1-\eta}{1-\alpha(1-\eta)}},$$

and k_{debt} is given by

$$k_{debt} = (\Psi_K)^{\frac{1}{1-\alpha(1-\eta)}} \cdot \left[(1-\alpha)A - \alpha A\frac{\Psi_B}{\Psi_K}\right]^{\frac{1-\eta}{1-\alpha(1-\eta)}},$$

from Eq. (22). Because $\Psi_B > 0$, $\Psi_K > 0$, and $\Psi_K|_{B_0=0} > \Psi_K$ hold under the present parametric condition, we obtain $k_{tax} > k_{debt}$.

■

A.6 Calibration of Parameters

We fix the share of capital at $\alpha = 1/3$, following Song, Storesletten, and Zilibotti (2012) and Lancia and Russo (2016). Each period lasts 30 years; this assumption is standard in quantitative analyses of the two-period overlapping-generations model (see, for example, Gonzaelz-Eiras and Niepelt, 2009; Lanica and Russo, 2016). Our selection of β is 0.98, which is also standard in the literature. Since the agents in the present model plan over generations that span 30 years, we discount the future by $(0.98)^{30}$.

We assume an annual gross population growth rate of 1.007, which is the OECD average rate during $2000-2010$.[1] This assumption implies that the net population growth rate for 30 years is $(1.007)^{30} - 1 \simeq 0.233$. For η, the estimate in Card and Krueger (1992) implies an elasticity of school quality of 0.12. In addition, recent simulation studies suggest that η is in the rage of $0.1 - 0.3$ (Cardak, 2004) and $0.05 - 0.15$ (Glomm and Ravikumar, 1998). Following these earlier studies, we here set at $\eta = 0.12$.

[1]Source: World Development Indicators (http://data.worldbank.org/products/wdi) (Accessed on May 24, 2016).

For γ, we focus on the income tax revenue-to-GDP ratio in the steady state, $\tau whN/Y = (1-\alpha)\tau$, or

$$\frac{\tau whN}{Y} = (1-\alpha)\cdot\left[(1+n)(X_0-B_0) + \frac{\alpha}{1-\alpha}\left\{1-(1+n)(X_0-B_0)\right\}\frac{\Psi_B}{\Psi_K}\right].$$

Given $\alpha = 1/3$, $\beta = (0.98)^{30}$, $1+n = (1.007)^{30}$, and $\eta = 0.12$, we can solve this expression for γ by using the ratio $\tau whN/Y$ observed in OECD countries. In particular, we use the data on "Tax on personal income," defined as the taxes levied on the net income (gross income minus allowable tax reliefs) and capital gains of individuals.[2] The OECD average during 2000–2010 is 0.08665. We can determine γ by solving the above expression and obtain $\gamma = 3.47$.

The productivity of final goods, A, is normalized at $A = 1$. For the productivity of human capital D, we use the data on the per capita GDP gross growth ratio of 1.072, which is the OECD average during $2000-2010$.[3] We substitute this data and the values of α, β, n, η, γ, and A into the following equation that expresses the per capita growth rate in the debt-finance case:

$$\frac{h'}{h} = D\cdot(X_0)^{\eta}\cdot(\Psi_K)^{\frac{\alpha\eta}{1-\alpha(1-\eta)}}\cdot\left\{(1-\alpha)-\alpha A\frac{\Psi_B}{\Psi_K}\right\}^{\frac{\eta}{1-\alpha(1-\eta)}}.$$

We solve this expression for D and obtain $D = 14.21$.

■

A.7 Supplementary Explanation for Section 4

We substitute (10) into (11) to reformulate the problem as

$$\max_{\{\tilde{d},k',\tilde{x}\}} \ln\left[A(k)^{\alpha} - \frac{1}{1+n}\tilde{d} - (1+n)k'D(\tilde{x})^{\eta} - (1+n)\tilde{x}\right]$$
$$+\frac{\beta}{\theta}\ln\tilde{d} + \gamma\alpha\ln k' + \left\{(\beta+\gamma)+\frac{\theta}{1-\theta}(1+\beta+\gamma)\right\}\eta\ln\tilde{x} + \theta\cdot SW(k').$$

The first-order conditions with respect to $\tilde{d}, k'$, and $\tilde{x}$ are

$$\tilde{d}: \frac{1/(1+n)}{\tilde{c}} = \frac{\beta/\theta}{\tilde{d}}, \tag{26}$$

$$k': \frac{(1+n)D(\tilde{x})^{\eta}}{\tilde{c}} = \frac{\gamma\alpha}{k'} + \theta\cdot SW'(k'), \tag{27}$$

$$\tilde{x}: \frac{\eta(1+n)k'D(\tilde{x})^{\eta-1}+(1+n)}{\tilde{c}} = \frac{\left\{(\beta+\gamma)+\frac{\theta}{1-\theta}(1+\beta+\gamma)\right\}\eta}{\tilde{x}}. \tag{28}$$

[2]Source: OECD, Tax on personal income (indicator). doi:10.1787/94af18d7-en (Accessed on June 6, 2016).

[3]Source: World Development Indicators (http://data.worldbank.org/products/wdi) (Accessed on May 24, 2016).

We make the guess $SW(k') = \phi_0 + \phi_1 \ln k'$, where ϕ_0 and ϕ_1 are undetermined coefficients. For this guess, (27) becomes

$$(1+n)D(\tilde{x})^{\eta}k' = (\gamma\alpha + \theta\phi_1)\,\tilde{c}. \tag{29}$$

With (28) and (29), we obtain

$$(1+n)\tilde{x} = \left[(\beta+\gamma) + \frac{\theta}{1-\theta}(1+\beta+\gamma) - (\gamma\alpha+\theta\phi_1)\right]\eta\tilde{c}. \tag{30}$$

Substitution of (26), (29), and (30) into the resource constraint in (10) leads to

$$\tilde{c} = \frac{1}{\phi}A\,(k)^{\alpha},$$

where

$$\phi \equiv \left(1+\frac{\beta}{\theta}\right) + (\gamma\alpha+\theta\phi_1)(1-\eta) + \left\{(\beta+\gamma) + \frac{\theta}{1-\theta}(1+\beta+\gamma)\right\}\eta. \tag{31}$$

The policy functions of $\tilde{d}, \tilde{x}$, and k' are

$$\begin{aligned}
\tilde{d} &= (1+n)\cdot\frac{\beta}{\theta}\cdot\frac{1}{\phi}A\,(k)^{\alpha}, \\
\tilde{x} &= \frac{1}{1+n}\cdot\left[\phi - \left\{\left(1+\frac{\beta}{\theta}\right) + (\gamma\alpha+\theta\phi_1)\right\}\right]\cdot\frac{1}{\phi}A\,(k)^{\alpha}, \qquad (32)\\
k' &= \frac{\gamma\alpha+\theta\phi_1}{(1+n)D\left[\frac{1}{1+n}\cdot\left\{\phi - \left(\left(1+\frac{\beta}{\theta}\right)+(\gamma\alpha+\theta\phi_1)\right)\right\}\right]^{\eta}}\cdot\left(\frac{1}{\phi}A\,(k)^{\alpha}\right)^{1-\eta}. \qquad (33)
\end{aligned}$$

Substituting these policy functions into the Bellman equation gives

$$SW(k) = Cons\,(\phi_0,\phi_1) + \alpha\phi\ln k,$$

where $Cons\,(\phi_0,\phi_1)$ includes constant terms. The guess is verified if $\phi_0 = Cons\,(\phi_0,\phi_1)$ and $\alpha\phi = \phi_1$. Therefore, ϕ_1 is given by

$$\phi_1 = \frac{\alpha}{1-\alpha\theta(1-\eta)}\cdot\left[\left(1+\frac{\beta}{\theta}\right) + \gamma\alpha(1-\eta) + \left\{(\beta+\gamma)+\frac{\theta}{1-\theta}(1+\beta+\gamma)\right\}\eta\right], \tag{34}$$

and the corresponding policy function of $\tilde{x}$ is obtained by substituting (31) and (34) into (32). We can also compute the steady-state capital level, growth rate given by $h'/h = D(\tilde{x})^{\eta}$, and education-GDP ratio given by $(1+n)x/y$, by using (31) – (34).

■

B Supplementary Materials

B.1 Supplementary Explanation for Subsection 6.1

The utility maximization problem of the middle in the presence of private education is

$$\begin{aligned}
&\max \ln c_t + \beta \ln d_{t+1} + \gamma \ln w_{t+1} h_{t+1} \\
&\text{s.t. } c_t + s_t + (1+n)e_t \le (1-\tau_t) w_t h_t, \\
&d_{t+1} \le R_{t+1} s_t, \\
&h_{t+1} = D \left(x_t\right)^{\eta} \left(e_t\right)^{\delta} \left(h_t\right)^{1-\eta-\delta}.
\end{aligned}$$

Solving this problem leads to

$$\begin{aligned}
s_t &= \frac{\beta}{1+\beta+\gamma\delta} \cdot (1-\tau_t) w_t h_t, c_t = \frac{1}{1+\beta+\gamma\delta} \cdot (1-\tau_t) w_t h_t, \text{ and} \\
e_t &= \frac{\gamma\delta}{(1+n)\left(1+\beta+\gamma\delta\right)} \cdot (1-\tau_t) w_t h_t. \qquad (35)
\end{aligned}$$

The capital-market-clearing condition is

$$(1+n) \cdot \left(k_{t+1} h_{t+1} + \hat{b}_{t+1}\right) = \frac{\beta}{1+\beta+\gamma\delta} \cdot (1-\tau_t) w_t h_t.$$

This is reformulated as

$$\begin{aligned}
k_{t+1} = &\frac{1}{(1+n) D \left(x_t\right)^{\eta} \left(\frac{\gamma\delta}{(1+n)(1+\beta+\gamma\delta)} \cdot \left(i_t - (1+n)x_t + (1+n)\hat{b}_{t+1}\right)\right)^{\delta} \left(h_t\right)^{1-\eta-\delta}} \\
&\times \left[\frac{\beta}{1+\beta+\gamma\delta} \left(i_t - (1+n)x_t + (1+n)\hat{b}_{t+1}\right) - (1+n)\hat{b}_{t+1}\right]. \qquad (36)
\end{aligned}$$

We substitute (35) and (36) into the utility function and obtain the indirect utility function of the middle as follows:

$$\begin{aligned}
V_t = &\left\{1+\beta+\delta\left(\gamma+\beta\left(1-\alpha\right)-\gamma\alpha\right)\right\} \ln\left(i_t - (1+n)x_t + (1+n)\hat{b}_{t+1}\right) \\
&+ \left\{\beta(\alpha-1)+\gamma\alpha\right\} \ln\left(i_t - (1+n)x_t - \frac{1+\gamma\delta}{\beta}(1+n)\hat{b}_{t+1}\right) + (\beta+\gamma)(1-\alpha)\eta \ln x_t,
\end{aligned}$$

where irrelevant terms are omitted from the expression.

We solve the maximization problem of V_t and obtain the following policy functions:

$$\begin{aligned}
\hat{b}_{t+1} &= \tilde{B}_0 \cdot i_t, \\
x_t &= \tilde{X}_0 \cdot i_t,
\end{aligned}$$

where

$$\begin{aligned}
\tilde{B}_0 &\equiv \max\left\{0, \tfrac{\{(1+\beta)\beta-(\beta(\alpha-1)+\gamma\alpha)\}+\delta\cdot\{\beta(\beta+\gamma)(1-\alpha)-\gamma(\beta(\alpha-1)+\gamma\alpha)\}}{(1+n)(1+\gamma\delta)\cdot[1+(\beta+\gamma)\{\alpha+(\eta+\delta)(1-\alpha)\}]}\right\}, \\
\tilde{X}_0 &\equiv \tfrac{\eta(\beta+\gamma)(1-\alpha)}{(1+n)\cdot[1+(\beta+\gamma)\{\alpha+(\eta+\delta)(1-\alpha)\}]}.
\end{aligned}$$

We compare $\tilde{X}_0$ and X_0 and obtain

$$\tilde{X}_0 < X_0 \Leftrightarrow 0 < \delta(\beta+\gamma)(1-\alpha),$$

which holds for any positive parameter values. We also compare $\tilde{B}_0$ and B_0 and obtain

$$\begin{aligned}\tilde{B}_0 \gtrless B_0 \Leftrightarrow \beta&\underbrace{\{(\beta+\gamma)(1-\alpha)-\gamma(1+\beta)\}}_{(*1)}\cdot[1+(\beta+\gamma)(\alpha+(1-\alpha)\eta)]\\ &\gtrless (1+\gamma\delta)(\beta+\gamma)(1-\alpha)\underbrace{[(1+\beta)\beta-(\beta(\alpha-1)+\gamma\alpha)]}_{(*2)}.\end{aligned}$$

The term marked by (*1) is negative while the term marked by (*2) is positive under the assumption in Proposition 5. Therefore, we obtain $\tilde{B}_0 < B_0$.

■

B.2 Supplementary Explanation for Subsection 6.2

B.2.1 Utility Maximization

We consider the utility maximization of the middle-aged. The problem is

$$\begin{aligned}&\max \ln c_t + \beta \ln d_{t+1} + \gamma \ln w_{t+1}h_{t+1}\\ &\text{s.t. } c_t + s_t \le (1-\tau_t)\,w_t h_t\\ &d_{t+1} \le R_{t+1}s_t + p_{t+1}.\end{aligned}$$

Solving this problem leads to the following saving function

$$s_t = \frac{\beta}{1+\beta}\left[(1-\tau_t)\,w_t h_t - \frac{p_{t+1}}{\beta R_{t+1}}\right],$$

and the indirect utility function of the middle-aged:

$$V_t^m = (1+\beta)\ln\left[(1-\tau_t)\,w_t h_t + \frac{p_{t+1}}{R_{t+1}}\right] + \beta \ln R_{t+1} + \gamma \ln w_{t+1}h_{t+1} + \left(\ln\frac{1}{1+\beta} + \beta\ln\frac{\beta}{1+\beta}\right). \tag{37}$$

The capital market clearing condition becomes

$$(1+n)\cdot\left(k_{t+1}h_{t+1} + \hat{b}_{t+1}\right) = \frac{\beta}{1+\beta}\cdot\left[(1-\tau_t)w_t h_t - \frac{p_{t+1}}{\beta R_{t+1}}\right]. \tag{38}$$

We use (38), the factor market clearing conditions and the government budget constraint to reformulate V_t^m in (37) as

$$\begin{aligned}V_t^m \simeq\ &(1+\beta)\ln\left[i_t - (1+n)x_t - \frac{p_t}{1+n} + (1+n)\hat{b}_{t+1} + \frac{p_{t+1}}{\alpha A\,(k_{t+1})^{\alpha-1}}\right]\\ &+\{\beta(\alpha-1)+\gamma\alpha\}\ln\left[i_t - (1+n)x_t - \frac{p_t}{1+n} - \frac{(1+n)\hat{b}_{t+1}}{\beta} - \frac{p_{t+1}}{\beta\alpha A\,(k_{t+1})^{\alpha-1}}\right]\\ &+(\beta+\gamma)\,(1-\alpha)\,\eta\ln x_t + (\beta+\gamma)\,(1-\alpha)\,(1-\eta)\ln h_t,\end{aligned} \tag{39}$$

where irrelevant terms are omitted from the expression.

The indirect utility of the old is

$$V_t^o = \beta \ln\left(R_t s_{t-1} + p_t\right) + \gamma \ln w_t h_t.$$

We substitute the capital market clearing condition, $(1+n)\left(k_t h_t + \hat{b}_t\right) = s_{t-1}$, and the factor market clearing condition into V_t^o and obtain

$$V_t^o = \beta \ln\left[j_t + \frac{p_t}{1+n}\right] + \beta \ln(1+n) + \gamma \ln\left(1-\alpha\right) A\left(k_t\right)^{\alpha} h_t, \tag{40}$$

where j_t is defined as $j_t \equiv \alpha A\left(k_t\right)^{\alpha-1} \cdot \left(k_t h_t + \hat{b}_t\right)$. By substituting (39) and (40) into $\Omega_t = \omega V_t^o + (1+n)(1-\omega)V_t^m$ and rearranging the terms, we obtain

$$\begin{aligned}
\Omega_t \simeq{}& \omega\beta \ln\left(j_t + \frac{p_t}{1+n}\right) \\
&+ (1+n)(1-\omega)\left(1+\beta\right) \ln\left[i_t - (1+n)x_t - \frac{p_t}{1+n} + (1+n)\hat{b}_{t+1} + \frac{p_{t+1}}{\alpha A\left(k_{t+1}\right)^{\alpha-1}}\right] \\
&+ (1+n)(1-\omega)\left\{\beta(\alpha-1) + \gamma\alpha\right\} \ln\left[i_t - (1+n)x_t - \frac{p_t}{1+n} - \frac{(1+n)\hat{b}_{t+1}}{\beta} - \frac{p_{t+1}}{\beta\alpha A\left(k_{t+1}\right)^{\alpha-1}}\right] \\
&+ (1+n)(1-\omega)\left(\beta+\gamma\right)\left(1-\alpha\right)\eta \ln x_t,
\end{aligned}$$

where irrelevant terms are omitted from the expression.

B.2.2 Solution to the Problem of Maximizing Ω_t

The period-t government's problem is to choose $\left\{p_t, x_t, \hat{b}_{t+1}\right\}$ that maximizes Ω_t. However, we should note that the next-period old-age benefit, p_{t+1}, which is chosen by the next-period government, is included in the period-t political objective function. We need to speculate on the next-period government's choice, which could be affected by the current policy choice. Based on the expression of Ω_t, it is natural to conjecture that $p_{t+1} = P(i_{t+1}, j_{t+1})$. In particular, we conjecture a linear policy function, $p_{t+1} = P_0 i_{t+1} + P_1 j_{t+1}$, where P_0 and P_1 are constant.

Given this conjecture, the present value of the period-$t+1$ old-age benefit in (39) becomes

$$\begin{aligned}
\frac{p_{t+1}}{\alpha A\left(k_{t+1}\right)^{\alpha-1}} &= P_0 \cdot \left(\frac{1-\alpha}{\alpha} k_{t+1}h_{t+1} - \hat{b}_{t+1}\right) + P_1 \cdot \left(k_{t+1}h_{t+1} + \hat{b}_{t+1}\right) \\
&= \left(P_0 \frac{1-\alpha}{\alpha} + P_1\right) \frac{1}{1+n} \frac{\beta}{1+\beta} \left[i_t - (1+n)x_t - \frac{p_t}{1+n} - \frac{(1+n)\hat{b}_{t+1}}{\beta} - \frac{p_{t+1}}{\beta\alpha A\left(k_{t+1}\right)^{\alpha-1}}\right] \\
&\quad + \left(-P_0 + P_1\right)\hat{b}_{t+1},
\end{aligned}$$

where the expression in the second line is obtained by substituting (38) into the expression in the first line. By rearranging the terms, we obtain

$$\frac{p_{t+1}}{\alpha A\left(k_{t+1}\right)^{\alpha-1}} = \frac{\left(P_0\frac{1-\alpha}{\alpha}+P_1\right)\frac{1}{1+n}\frac{\beta}{1+\beta}\left[i_t-(1+n)x_t-\frac{p_t}{1+n}-\frac{(1+n)\hat{b}_{t+1}}{\beta}\right]+\left(-P_0+P_1\right)\hat{b}_{t+1}}{1+\left(P_0\frac{1-\alpha}{\alpha}+P_1\right)\frac{1}{1+n}\frac{1}{1+\beta}}. \tag{41}$$

We substitute (41) into V_t^m in (39) to obtain

$$\begin{aligned}\Omega_t \simeq\ & \omega\beta\ln\left(j_t+\frac{p_t}{1+n}\right)+(1+n)(1-\omega)\left(1+\beta\right)\ln Z_{0t}\\ & (1+n)(1-\omega)\left\{\beta(\alpha-1)+\gamma\alpha\right\}\ln Z_{1t}\\ & (1+n)(1-\omega)\left(\beta+\gamma\right)\left(1-\alpha\right)\eta\ln x_t,\end{aligned}$$

where

$$\begin{aligned}Z_{0t} &\equiv \left\{1+\left(P_0\frac{1-\alpha}{\alpha}+P_1\right)\frac{1}{1+n}\right\}\left(i_t-(1+n)x_t-\frac{p_t}{1+n}\right)+\left\{1+\frac{-P_0+P_1}{1+n}\right\}(1+n)\hat{b}_{t+1},\\ Z_{1t} &\equiv \beta\left(i_t-(1+n)x_t-\frac{p_t}{1+n}\right)-\left\{1+\frac{-P_0+P_1}{1+n}\right\}(1+n)\hat{b}_{t+1}.\end{aligned}$$

We further guess at $1+\frac{-P_0+P_1}{1+n}=0$ and solve the problem of maximizing Ω_t. After some calculation, we obtain the policy function of p_t as follows:

$$p_t = \frac{(1+n)\left[i_t-\frac{(1+n)(1-\omega)}{\omega\beta}\left\{1+\left(\beta+\gamma\right)\left(\alpha+(1-\alpha)\eta\right)\right\}j_t\right]}{1+\frac{(1+n)(1-\omega)}{\omega\beta}\left\{1+\left(\beta+\gamma\right)\left(\alpha+(1-\alpha)\eta\right)\right\}}.$$

Thus, the guess is verified if P_0 and P_1 are given by

$$\begin{aligned}P_0 &= \frac{(1+n)}{1+\frac{(1+n)(1-\omega)}{\omega\beta}\left\{1+\left(\beta+\gamma\right)\left(\alpha+(1-\alpha)\eta\right)\right\}},\\ P_1 &= \frac{(-1)\left(1+n\right)\frac{(1+n)(1-\omega)}{\omega\beta}\left\{1+\left(\beta+\gamma\right)\left(\alpha+(1-\alpha)\eta\right)\right\}}{1+\frac{(1+n)(1-\omega)}{\omega\beta}\left\{1+\left(\beta+\gamma\right)\left(\alpha+(1-\alpha)\eta\right)\right\}}.\end{aligned}$$

These expressions imply that the guess $1+\frac{-P_0+P_1}{1+n}=0$ is also verified.

■

B.3 Supplementary Explanation for Subsection 6.4

In the tax-finance case, we have $\hat{b}_{t+1}=0$ for all t. Given $k_{t+1}=k$ for all t, the solution of maximizing V_t is

$$x_t = \frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)}\left(wh_t-R\hat{b}_t\right).$$

Therefore, the growth rate is

$$\left.\frac{h_{t+1}}{h_t}\right|_{tax} = D \cdot \left[\frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)} \left(w \left. h_t \right|_{tax} - R\hat{b}_t \right) \frac{1}{\left. h_t \right|_{tax}} \right]^{\eta},$$

and the indirect utility is

$$\begin{aligned} \left. V_t \right|_{tax} &= (1+\beta+\gamma\eta) \ln \left. i_t \right|_{tax} + \{\beta(\alpha-1)+\gamma\alpha\} \ln k + \gamma(1-\eta) \ln \left. h_t \right|_{tax} \\ &+ (1+\beta) \ln \frac{1+\beta}{1+\beta+\gamma\eta} + \gamma\eta \ln \frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)} + C_0. \end{aligned}$$

In the debt-finance case, the solution of maximizing V_t is

$$\begin{aligned} \hat{b}_{t+1} &= \bar{b} \cdot h_t, \\ x_t &= \frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)} \left(i_t + (1+n)\bar{b}_t \right), \end{aligned}$$

where $\bar{b} \equiv w/(R-(1+n))$. Therefore, the growth rate is

$$\left.\frac{h_{t+1}}{h_t}\right|_{debt} = D \cdot \left[\frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)} \left(\left. i_t \right|_{tax} + (1+n)\bar{b}_t \right) \frac{1}{\left. h_t \right|_{debt}} \right]^{\eta},$$

and the indirect utility function is

$$\begin{aligned} \left. V_t \right|_{debt} &= (1+\beta+\gamma\eta) \ln \left(\left. i_t \right|_{debt} + (1+n)\bar{b}_t \right) + \{\beta(\alpha-1)+\gamma\alpha\} \ln k + \gamma(1-\eta) \ln \left. h_t \right|_{debt} \\ &+ (1+\beta) \ln \frac{1+\beta}{1+\beta+\gamma\eta} + \gamma\eta \ln \frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)} + C_0. \end{aligned}$$

These expressions suggest that in period 0, we have

$$\left.\frac{h_1}{h_0}\right|_{tax} < \left.\frac{h_1}{h_0}\right|_{debt} \text{ and } \left. V_0 \right|_{tax} < \left. V_0 \right|_{debt}.$$

From period 1 onward, we have

$$\begin{aligned} \left. x_t \right|_{debt} &= \frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)} \left(wh_t - R\bar{b}h_t + (1+n)\bar{b}h_t \right) \\ &= \frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)} \left(wh_t - \frac{R-(1+n)}{R-(1+n)} wh_t \right) \\ &= 0, \end{aligned}$$

implying that $\left. h_{t+1}/h_t \right|_{debt} = 0$ and $\left. V_t \right|_{debt} = -\infty$ for $t \geq 1$.

■

References

[1] de la Croix, D., and Michel, P., 2002. *A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations.* Cambridge University Press, Cambridge, United Kingdom.

[2] de la Croix, D., and Doepke, M., 2004. Public versus private education when differential fertility matters. *Journal of Development Economics* 73, 607–629.

[3] Ghosh, S., and Mourmouras, I.A., 2004a. Endogenous growth, welfare and budgetary regimes. *Journal of Macroeconomics* 26, 623–635.

[4] Ghosh, S., and Mourmouras, I.A., 2004b. Debt, growth and budgetary regimes. *Bulletin of Economic Research* 56, 241–250.

[5] Glomm, G., 2004. Inequality, majority voting and the redistributive effects of public education funding *Pacific Economic Review* 9, 93–101.

[6] Glomm, G., and Kaganovich, M., 2003. Distributional effects of public education in an economy with public pensions. *International Economic Review*, 44, 917-937.

[7] Glomm, G., and Kaganovich, M., 2008. Social security, public education and the growth-inequality relationship. *European Economic Review* 52, 1009–1034.

[8] Glomm, G., and Ravikumar, B., 1992. Public versus private investment in human capital endogenous growth and income inequality. *Journal of Political Economy*, 100, 818-834.

[9] Glomm, G., and Ravikumar, B., 1995. Endogenous public policy and multiple equilibria. *European Journal of Political Economy* 11, 653–662.

[10] Glomm, G., and Ravikumar, B., 2001. Human capital accumulation and endogenous public expenditure. *Canadian Journal of Economics* 34, 807–826.

[11] Glomm, G., and Ravikumar, B., 2003. Public education and income inequality. *European Journal of Political Economy* 19, 289–300.

[12] Gonzalez-Eiras, M., and Niepelt, D., 2012. Aging, government budgets, retirement, and growth. *European Economic Review* 56, 97–115.

[13] Gradstein, M., and Justman, M., 1996. The political economy of mixed public and private schooling: A dynamic analysis. *International Tax and Public Finance* 3, 297-310.

[14] Gradstein, M., and Justman, M., 2002. Education, social cohesion, and economic growth. *American Economic Review* 92, 1192-1204.

[15] Gradstein, M., and Kaganovich, M., 2004. Aging population and education finance. *Journal of Public Economics* 88, 2469–2485.

[16] Greiner, A., and Semmler, W., 2000. Endogenous growth, government debt and budgetary regimes. *Journal of Macroeconomics* 22, 363–384.

[17] Greiner, A., 2008. Human capital formation, public debt and economic growth. *Journal of Macroeconomics* 30, 415-427.

[18] Greiner, A., 2012. Public capital, sustainable debt and endogenous growth. *Research in Economics* 66, 230–238.

[19] Iturbe-Ormaetxe, I., and Valera, G., 2012. Social security reform and the support for public education. *Journal of Population Economics* 25, 609–634.

[20] Kaganovich, M., and Meier, V., 2012. Social security systems, human capital, and growth in a small open economy. *Journal of Public Economic Theory* 14, 573–600.

[21] Kemnitz, A., 2000. Social security, public education, and growth in a representative democracy. *Journal of Population Economics* 13, 443–462.

[22] Kunze, L., 2014. Life expectancy and economic growth. *Journal of Macroeconomics* 39, 54–65.

[23] Lambrecht, S., Michel, P., and Vidal, J.P., 2005. Public pensions and growth. *European Economic Review* 49, 1261–1281.

[24] Lancia F., and Russo, A., 2016. Public education and pensions in democracy: A political economy theory. *Journal of the European Economic Association* 14, 1038-1073.

[25] Lindbeck, A.J., and Weibull, W., 1987. Balanced-budget redistribution as the outcome of political competition. *Public Choice* 52, 273–297.

[26] Minea, A., and Villieu, P., 2009. Borrowing to finance public investment? The 'Golden Rule of Public Finance' reconsidered in an endogenous growth setting. *Fiscal Studies* 30, 103–133.

[27] Naito, K., 2012. Two-sided intergenerational transfer policy and economic development: A politico-economic approach. *Journal of Economic Dynamics and Control* 36, 1340–1348.

[28] Ono, T., 2015. Public education and social security: A political economy approach. *Economics of Governance* 16, 1–25.

[29] Ono, T., and Uchida, Y., 2016. Pensions, education, and growth: A positive analysis. *Journal of Macroeconomics* 48, 127–143.

[30] Poutvaara, P., 2006. On the political economy of social security and public education. *Journal of Population Economics* 19, 345–365.

[31] Saint-Paul, G., and Verdier, T., 1993. Education, democracy and growth. *Journal of Development Economics* 42, 399-407.

[32] Soares, J., 2006. A dynamic general equilibrium analysis of the political economy of public education. *Journal of Population Economics* 19, 367–389.

[33] Song, Z., Storesletten, K., and Zilibotti, F., 2012. Rotten parents and disciplined children: A politico-economic theory of public expenditure adn debt. *Econometrica* 80, 2785–2803. http://www.econ.uzh.ch/dam/jcr:00000000-492b-dd34-ffff-fffff8cbd06d/ssz_altruism_141015.pdf (July 28, 2016).

[34] Yakita, A., 2008. Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting. *Journal of Public Economics* 92, 897–914.

[35] Zhang, J., 2003. Optimal debt, endogenous fertility, and human capital externalities in a model with altruistic bequests. *Journal of Public Economics* 87, 1825-1835.

[36] Zhang, J., Zhang, J., and Lee, R., 2003. Rising longevity, education, savings, and growth. *Journal of Development Economics* 70, 83-101.

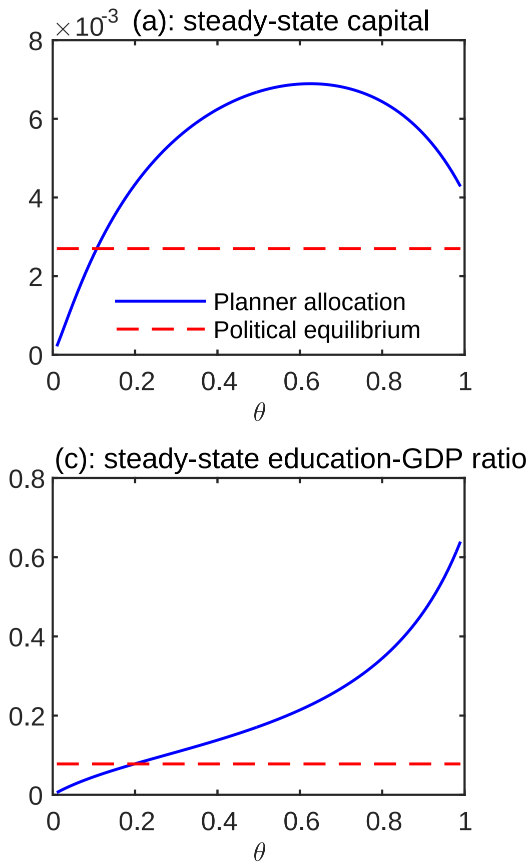

Figure 1: Panel (a): steady-state capital level and θ. Panel (b): steady-state growth rate and θ. Panel (c): education-GDP ratio and θ.

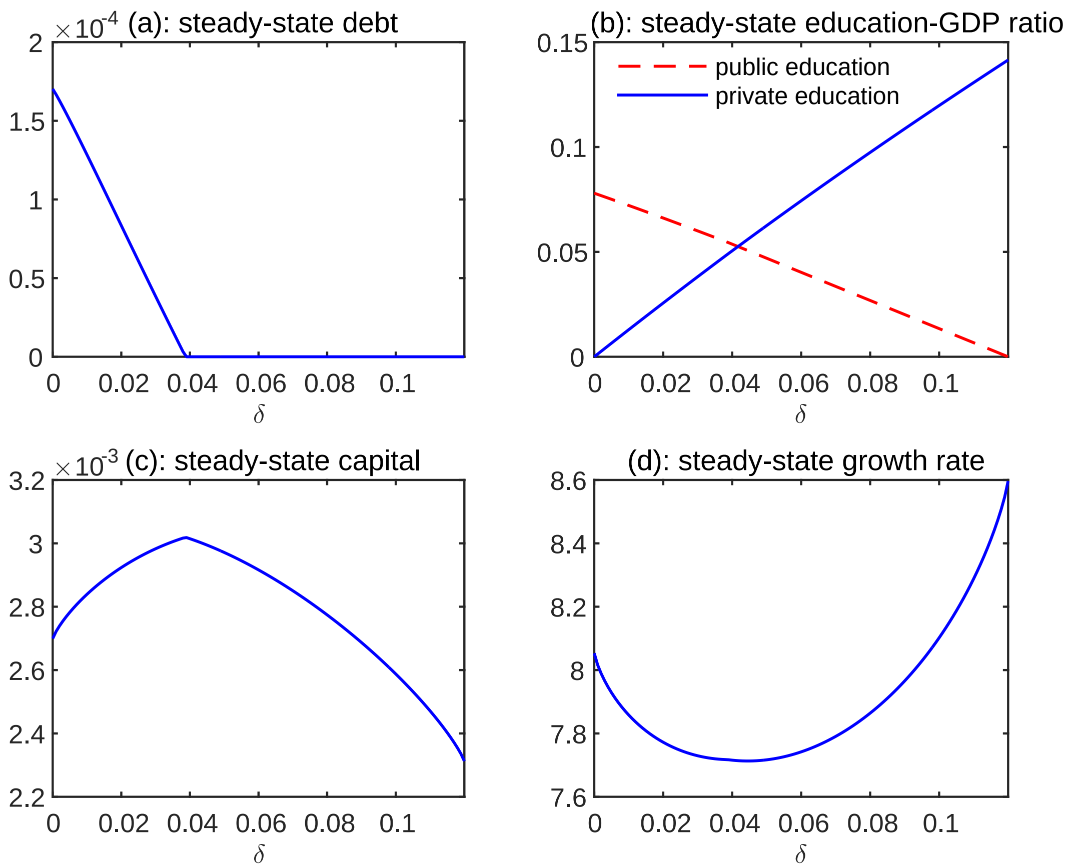

Figure 2: Panel (a): steady-state debt and δ. Panel (b) steady-state education-GDP ratio and δ. Panel (c): steady-state capital and δ. Panel (d): steady-state growth rate and δ.