Discussion Papers In Economics And Business

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January 2017

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Abstract

This study presents a two-class, successive generations model with human capital accumulation and the choice to opt out of public education. The model demonstrates the mutual interaction between inequality and education choice and shows that the interaction leads to two locally stable steady-state equilibria. The existence of multiple stable equilibria implies a negative correlation between inequality and enrollment in public education, which is consistent with evidence from OECD countries. This study also presents a welfare analysis using data from OECD countries and shows that introducing a compulsory public education system leaves the first generation worse off, though improves welfare for future generations of individuals in a lower class. The results also suggest that the two equilibria are not Pareto-ranked.

- JEL Classification Numbers: D70, H52, I24
  Key words: Public education, opting out, inequality.

*Tetsuo Ono acknowledges financial support from Grant-in-Aid for Scientific Research (C) from the Ministry of Education, Culture, Sports, Science and Technology of Japan No. 15K03509. Yuki Uchida acknowledges financial support from Grant-in-Aid for JSPS Fellows of the Japan Society for the Promotion of Science No. 15J01375.

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1 Introduction

Compulsory school attendance laws, enforced in nearly all developed countries, require parents to have their children attend public or private school for a designated period. Public school is entirely funded by local and state taxes, whereas private schools obtain funding by charging their students tuition. Parents can choose either option depending on their income and preferences. Because public schooling is a kind of government intervention, higher income parents who benefit less from public schooling are more likely to choose private schools. Therefore, we expect an association between higher inequality and higher enrollments in private education institutions, as observed in data from OECD countries illustrated in Figure 1.

[Figure 1 here.]

de la Croix and Doepke (2009) develop a political economy theory that attempts to explain the relationship between inequality and private education and show that their theory is consistent with data from states in the U.S. and cross-country evidence. They assume an exogenous income distribution and analyze the effect of expanding inequality on education choices; however, their analysis implies a reverse effect, that is, that education affects inequality. Recent studies suggest that the reverse effect is also important (Saint-Paul and Verdier, 1993; Zhang, 1996) and exists across and within countries over time (De Gregorio and Lee, 2002; Teulings and van Rens, 2008).

The presence of the reverse effect implies that there is a mutual interaction of inequality and education choice over time—inequality affects adults' choice of education and policy, and this, in turn, determines inequality within the next generation. Cardak (2004a, 2004b) attempts to demonstrate this mutual interaction of education choice and inequality in a two-period overlapping-generations model. In particular, Cardak (2004a) calibrates the model to the U.S. economy, and shows by simulation that the coexistence of public and private education polarizes the income distribution. He therefore focuses on education choice and inequality within a country over time.

The present study instead focuses on the cross-country differences and aims to clarify the causes of the differences observed in Figure 1 from the political economy point of view. For this purpose, we follow a simple two-class, successive generations model with human capital accumulation as in Gradstein and Justman (1996) and de la Croix and Doepke (2004). We extend their frameworks by introducing the choice to opt out of public education as Cardak (2004a, 2004b) does. In particular, the model in this study has two types of family dynasties classified according to their level of human capital (i.e., low-type and high-type). Agents from either type of family enter adulthood with a stock of human capital invested by their parents, earn after-tax income, and obtain utility from consumption and their children's human capital. Agents compare the maximized utility under each type of education, and choose the one with the highest value.

Every adult agent votes on public education expenditures in each period. This study assumes that most families are of the low-type. We compute the low-type's preferred public education expenditure and analyze the corresponding education choice by adult agents. We show that the low-type adults always choose public education because they pay less than they receive from public education. However, the high-type's decision depends on income inequality. As inequality increases, the income discrepancy between the two types increases, and so does the high-type's tax burden. Therefore, high-type adults opt out of public education when inequality is high, while they choose public education when inequality is low.

The current education choice and expenditures influence human capital formation, which, in turn, determines inequality in the next generation. We demonstrate this mutual interaction between inequality and education across generations and show that the interaction leads to two, locally stable steady-state equilibria. One steady state shows a polarized income distribution with high-type agents opting out of public education, in line with Cardak (2004a). The other steady state has perfect equality and full enrollment in public education. This study is novel in that it shows the existence of multiple stable equilibria that imply that higher inequality is associated with lower enrollment in public schools. This model prediction is consistent with the evidence presented in Figure 1.

To investigate the welfare implications of the model, we compare the two steady states in terms of utility by considering an economic environment in which the equilibrium converges to the higher-inequality steady state. We then introduce an alternative education system into this environment, that is, a compulsory public school system that prohibits students from opting out of public school. This system forces the economy into the lower-inequality steady state. Therefore, we can evaluate the multiple, stable steady-state equilibria by comparing the higher-inequality steady state in the mixed education system and the steady state in the compulsory public school system.

We show by simulation that every generation of the high-type is made worse off by the introduction of compulsory public schooling, since expenditures on education depart from their optimal levels. However, the new system has a mixed effect on the low-type agents. The first generation is made worse off since the per-capita public education expenditure decreases. From generation 2 onward, there is a positive effect from the compulsory public schooling on human capital formation. This effect may outweigh the negative effect of the decrease in the per capita public education expenditure. The result suggests an intergenerational tradeoff and that the two equilibria are not Pareto-ranked. The result also suggests that the shift from a mixed education system to a compulsory public school system that aims to improve equality, is not Pareto-improving.

This study is related to three strands of literature. The first is the static analyses of public and private education choices (e.g., Stiglitz, 1974; Epple and Romano, 1996; Glomm and Ravikumar, 1998; Hoyt and Lee, 1998; Bearse, Glomm and Patterson, 2005; de la Croix and Doepke, 2009; Arcalean and Schiopu, 2015). This study advances this earlier work by demonstrating the dynamic interaction of inequality and education choice, and in particular, complements Cardak's (2004a, 2004b) work by showing the existence of multiple stable equilibria that fit the cross-country evidence from OECD countries.

The second is the dynamic inequality analyses in given (public or private) education regimes (e.g., Glomm and Ravikumar, 1992; Saint-Paul and Verdier, 1993; Gradstein and Justman, 1997; Benabou, 2000; de la Croix and Doepke, 2004; Galor, Moav, and Vollrath, 2009). However, this study departs from prior work by allowing for endogenous education choice accompanied by voting on education policy. Gradstein and Justman (1996) and Ono (2016) conduct a similar analysis, though focus on private education as a supplement to public education. The present study instead focuses on the ability to opt out of public education, which leads to novel implications for the multiplicity and efficiency of equilibria.

The third strand relates to political economy analyses of redistribution and private education (Hassler, Rodriguez Mora, Storesletten, and Zilibotti, 2003; Hassler, Storesletten, and Zilibotti, 2007; Arawatari and Ono, 2009, 2013). In earlier frameworks, multiple, self-fulfilling expectations of agents on future in-cash redistribution policies create two types of equilibria: one characterized by low inequality and high redistribution, the other characterized by high inequality and low redistribution. This multiple-equilibria story implies a negative correlation between inequality and redistribution. While this is relevant to our present study, these earlier works consider private education and in-cash transfers, while our study instead focuses on in-kind public education provision and allows for private education as an alternative choice.

The remainder of this paper is organized as follows. Section 2 describes the model. Section 3 considers agents' voting behavior and describes the political equilibrium in each period. Section 4 shows the existence and stability of a steady-state equilibrium and clarifies the role of structural parameter values in the determination of inequality, individual education choice, and policy. Section 5 presents a welfare analysis of the political equilibria in addition to considering the welfare implications of a compulsory public schooling system as an alternative. Section 6 provides concluding remarks. All proofs are provided in Appendix.

2 Model

We consider a discrete-time, successive generations economy beginning at time 1. The economy is populated by individuals who live in two periods (youth and adulthood) and belong to one of two types of family dynasties indexed by $i \in \{L, H\}$. This assumption simplifies the real economy, but it enables us to demonstrate the dynamic motion of inequality in a tractable way.

A type-i adult in period 1 is endowed with h_1^i units of human capital, where $0 < h_1^L < h_1^H$. Thus, type-L and type-H individuals in period-1 have low and high human capital, respectively. As demonstrated below, members of the type-H endogenously choose more education for their children than members of type-L, so there is always inequality within the two types. However, the extent of this inequality is determined endogenously through individuals' choices.

Each adult produces one child; thus, the population remains constant from generation to generation. The fraction of type-L individuals within each generation is ϕ, leaving $1-\phi$ as the proportion of type-H individuals, where ϕ is constant across generations and satisfies $0.5 < \phi < 1$. Therefore, type-L individuals are the majority in the economy in every period, which reflects the real-world right-skewed income distribution.

2.1 Preferences and Budget Constraints

Upon entering adulthood at time t, a type-i individual has a stock of human capital h_t^i that defines his or her effective labor capacity. He or she then inelastically supplies his or her human capital to firms to receive wages. We assume that wages are normalized to one in each period, implying that labor income is equal to the human capital level.

A type-i adult of generation t derives utility from his or her current consumption, c_t^i, and from his or her child's anticipated future income, h_{t+1}^i. Consequently, we can express the agent's preferences with the following utility function

$$u_t^i = \ln c_t^i + \gamma \ln h_{t+1}^i,$$

where $\gamma (> 0)$ is a common parameter that reflects the bequest motive. We employ this logarithmic utility function to make our analysis more manageable.

Adults have a choice between public and private education for their children, which they choose based on maximum utility. However, regardless of their choice, they must pay income taxes to finance public education. Therefore, the budget constraint of a type-i adult in period t is

$$c_t^i + e_t^i \leq (1-\tau_t) h_t^i,$$

where $e_t^i (\geq 0)$ denotes type-i's private education expenditure in period t, and τ_t is the period-t income tax rate.

Let $q_t^i \in \{0,1\}$ denote a binary variable representing type-i's education choice: $q_t^i = 0$ when choosing private education, and $q_t^i = 1$ when choosing public education. The child's level of education, h_{t+1}^i, is determined by his or her parents' human capital, h_t^i, and the parents' choice of schooling, either x_t or e_t^i, where x_t is per capita public education. In particular, we assume $h_{t+1}^i = D\left(h_t^i\right)^{1-\eta}\left(q_t^i x_t + \left(1-q_t^i\right)e_t^i\right)^{\eta}$ where $D(>0)$ is the total factor productivity of human capital and $\eta \in (0,1)$ is the elasticity of schooling. The expression is reformulated as

$$h_{t+1}^i = \begin{cases} D\left(h_t^i\right)^{1-\eta}\left(e_t^i\right)^{\eta} & \text{if} \quad q_t^i = 0, \\ D\left(h_t^i\right)^{1-\eta}\left(x_t\right)^{\eta} & \text{if} \quad q_t^i = 1. \end{cases}$$

We assume the following with respect to γ and η.

Assumption 1. $\gamma\eta \in (0,1)$.

Assumption 1 is satisfied as long as $\gamma \in (0,1)$. In Section 5, we estimate γ based on data from OECD countries and find that $\gamma = 0.151$. This estimate fits well with Cardak's (2004a) estimate of 0.13 and de la Croix and Deopke's (2004) estimate of 0.169.

2.2 Education Choice

Given the tax rate, public education, and his or her human capital, each adult chooses consumption and education to maximize his or her utility subject to the budget constraint. In particular, he or she compares the maximum utility of each education choice and chooses the option with the highest value.

Suppose that a type-i adult chooses private education, $q_t^i = 0$. He or she solves the utility maximization problem by allocating disposable income between private education and consumption as in the following:

$$e_t^i = \frac{\gamma\eta}{1+\gamma\eta}(1-\tau_t)h_t^i,$$
$$c_t^i = \frac{1}{1+\gamma\eta}(1-\tau_t)h_t^i.$$

The type-i adult's utility from providing private education for his or her child, denoted by $V_{e,t}^i$, is

$$V_{e,t}^i = (1+\gamma\eta)\ln(1-\tau_t)h_t^i + \gamma \ln D\left(h_t^i\right)^{1-\eta} + \gamma\eta \ln \mu, \tag{1}$$

where

$$\mu \equiv \frac{\gamma\eta}{(1+\gamma\eta)^{(1+\gamma\eta)/\gamma\eta}}.$$

Alternatively, suppose that the type-i adult chooses public education, $q_t^i = 1$. He or she chooses $e_t^i = 0$, and thus consumes all disposable income. In this case, the type-i adult's utility from choosing public education for his or her child, denoted by $V_{x,t}^i$, is

$$V_{x,t}^i = \ln(1-\tau_t)h_t^i + \gamma \ln D\left(h_t^i\right)^{1-\eta} + \gamma\eta \ln x_t. \tag{2}$$

Given a set of policies, (x_t, τ_t), each adult chooses between education alternatives for his or her child to maximize utility. Therefore, type-i adults choose public education if and only if the following condition holds:

$$V_{x,t}^i > V_{e,t}^i \Leftrightarrow \mu(1-\tau_t)h_t^i < x_t. \tag{3}$$

We assume that each adult chooses private education when the two alternatives are indifferent. Therefore, the type-i's education choice is

$$q_t^i = \begin{cases} 1 & \text{if} \quad V_{x,t}^i > V_{e,t}^i \Leftrightarrow \mu(1-\tau_t)h_t^i < x_t, \\ 0 & \text{if} \quad V_{x,t}^i \le V_{e,t}^i \Leftrightarrow \mu(1-\tau_t)h_t^i \ge x_t. \end{cases} \tag{4}$$

The timing of events in period t is as follows: First, adult agents vote on public education, x_t. Given the voting outcome, the tax rate τ_t is set to satisfy the government's budget constraint. Second, given x_t and τ_t, each agent chooses either public or private education to maximize his or her utility. In choosing private education, agents decides how to divide their disposable income between consumption and private education subject to their budget constraints. We follow the backward induction approach to solve this multi-stage game. In particular, we first solve the second-stage problem in Section 3, and then solve the first-stage problem in Section 4.

3 Economic Equilibrium

We define an economic equilibrium in the present model as follows.

Definition 1. Given a sequence of public education expenditure, $\{x_t\}_{t=1}^{\infty}$, an *economic equilibrium* is a sequence of choices and allocations, $\{q_t^i\}_{i=L,H}^{t=1,\dots,\infty}$ and $\left\{c_t^i, e_t^i, h_{t+1}^i\right\}_{i=L,H}^{t=1,\dots,\infty}$; and a sequence of taxes, $\{\tau_t\}_{t=1}^{\infty}$, with the initial condition, $h_1^i, i = L, H$, such that in each period,

(i) given h_t^i, x_t, and τ_t, a type-i agent chooses q_t^i and the corresponding c_t^i and e_t^i to maximize his or her utility;

(ii) given h_t^i, x_t, and q_t^i, τ_t is set to satisfy the government budget constraint, $\left\{q_t^L\phi + q_t^H(1-\phi)\right\}x_t = \tau_t h_t$;

(iii) given h_t^i, x_t, and (q_t^i, e_t^i), which satisfy conditions (i) and (ii), h_{t+1}^i is determined by $h_{t+1}^i = D\left(h_t^i\right)^{1-\eta}\left(q_t^i x_t + \left(1-q_t^i\right)e_t^i\right)^{\eta}$.

To find the period-t economic equilibrium solution, we introduce an inequality index ρ_t: Let h_t denote the average human capital in period t, $h_t \equiv \phi h_t^L + (1-\phi)h_t^H$, and let ρ_t denote the ratio of h_t^L to h_t,

$$\rho_t \equiv \frac{h_t^L}{h_t} \in (0,1].$$

The index ρ_t suggests that a larger (smaller) ρ_t implies a lower (higher) income inequality between the high-type and low-type groups, and thus a more equal (unequal) society. Using this inequality index and the definition for average human capital, we can rewrite the ratio h_t^H/h_t as

$$\frac{h_t^H}{h_t} = \frac{1-\phi\rho_t}{1-\phi}.$$

Therefore, we replace the two state variables, h_t^L and h_t^H with h_t and ρ_t in the following analysis.

Using the definition of h_t and ρ_t, we can reformulate the condition in (4) and obtain the corresponding pair of education choices in the economic equilibrium. First, suppose that both types of adults choose private education, $\left(q_t^L, q_t^H\right) = (0,0)$. Condition (ii) in Definition 1 implies that the government budget constraint is reduced to $\tau_t = 0$ because no agent will choose public education. Substituting $\tau_t = 0$ in (4) and rearranging the terms, we obtain

$$q_t^L = 0 \text{ if } x_t \leq \mu h_t^L \text{ and } q_t^H = 0 \text{ if } x_t \leq \mu h_t^H.$$

Therefore, we have $\left(q_t^L, q_t^H\right) = (0,0)$ if $x_t \leq \mu h_t^L$, or

$$\left(q_t^L, q_t^H\right) = (0,0) \text{ if } x_t \leq x_t^{00} \equiv \mu\rho_t h_t, \tag{5}$$

where the superscript "00" of x_t^{00} implies $\left(q_t^L, q_t^H\right) = (0,0)$.

Second, suppose that only type-H adults choose public education, $\left(q_t^L, q_t^H\right) = (0,1)$. The government budget constraint is $\tau_t = (1-\phi)x_t/h_t$. Substituting this into (4), we obtain

$$\begin{aligned} q_t^L &= 0 \text{ if } x_t \leq \mu\cdot\left(1-\frac{(1-\phi)x_t}{h_t}\right)\cdot h_t^L = \frac{\mu\rho_t}{1+\mu(1-\phi)}h_t, \\ q_t^H &= 1 \text{ if } \mu\cdot\left(1-\frac{(1-\phi)x_t}{h_t}\right)\cdot h_t^H = \frac{\mu(1-\phi\rho_t)/(1-\phi)}{1+\mu(1-\phi\rho_t)} < x_t. \end{aligned}$$

There is no x_t that satisfies both conditions. Therefore, there is no economic equilibrium with $\left(q_t^L, q_t^H\right) = (0,1)$.

Third, suppose that only type-L adults choose public education, $\left(q_t^L, q_t^H\right) = (1,0)$. The government budget constraint is $\phi x_t = \tau_t h_t$. Substituting this into (4) and rearranging the terms, we obtain

$$\begin{aligned} q_t^L &= 1 \text{ if } x_t^{10} \equiv \frac{\mu\rho_t}{1+\mu\phi\rho_t}h_t < x_t, \\ q_t^H &= 0 \text{ if } x_t \leq \bar{x}_t \equiv \frac{\mu(1-\phi\rho_t)/(1-\phi)}{1+\mu\phi(1-\phi\rho_t)/(1-\phi)}h_t, \end{aligned}$$

where the superscript "10" of x_t^{10} means $\left(q_t^L, q_t^H\right) = (1,0)$, and the bar of $\bar{x}_t$ indicates the upper limit of x_t, which induces the type-H adults to choose private education. Therefore, we obtain

$$\left(q_t^L, q_t^H\right) = (1,0) \text{ if } x_t^{10} < x_t \leq \bar{x}_t. \tag{6}$$

Finally, suppose that both types of adults choose public education, $\left(q_t^L, q_t^H\right) = (1,1)$. The government budget constraint is then $x_t = \tau_t h_t$. Following the same procedure above, we obtain

$$q_t^L = 1 \text{ if } \mu \cdot \left(1 - \frac{x_t}{h_t}\right) \cdot h_t^L < x_t,$$
$$q_t^H = 1 \text{ if } \mu \cdot \left(1 - \frac{x_t}{h_t}\right) \cdot h_t^H < x_t.$$

Therefore, we have $\left(q_t^L, q_t^H\right) = (1,1)$ if $\mu \cdot (1 - x_t/h_t) \cdot h_t^H < x_t$, or

$$\left(q_t^L, q_t^H\right) = (1,1) \text{ if } x_t^{11} \equiv \frac{\mu\left(1-\phi\rho_t\right)/(1-\phi)}{1+\mu\left(1-\phi\rho_t\right)/(1-\phi)} h_t < x_t, \tag{7}$$

where the superscript "11" of x_t^{11} indicates $\left(q_t^L, q_t^H\right) = (1,1)$.

The analysis thus far suggests that there are three possible cases of education choice: $\left(q_t^L, q_t^H\right) = (0,0)$, $(1,0)$, and $(1,1)$. The choice is affected by the four threshold values of x_t, denoted by x_t^{00}, x_t^{10}, $\bar{x}_t$, and x_t^{11}. The order of these values depends on the inequality ρ_t. In particular, there are three critical values of ρ_t, denoted by ρ^l, ρ^m, and ρ^h, where $0 < \rho^l < \rho^m < \rho^h < 1$, such that

$$\begin{cases} x_t^{00} \lesseqgtr x_t^{11} \Leftrightarrow \rho_t \lesseqgtr \rho^l, \\ x_t^{00} \lesseqgtr \bar{x}_t \Leftrightarrow \rho_t \lesseqgtr \rho^m, \\ x_t^{10} \lesseqgtr x_t^{11} \Leftrightarrow \rho_t \lesseqgtr \rho^h. \end{cases} \tag{8}$$

We give the proof of (8) in Appendix A.1. In addition, direct calculation leads to

$$\begin{cases} x_t^{10} < x_t^{00}, \\ x_t^{10}, x_t^{11} < \bar{x}_t, \end{cases} \tag{9}$$

Figure 2 illustrates the four cases of ρ_t that classifies the ordering of the four threshold values of x_t and the corresponding choice of education by each type of adult. We summarize the four cases in Figure 2 in Figure 3, which is precisely stated in the following proposition:

[Figure 2 here.]

[Figure 3 here.]

Proposition 1. *There is a unique period-t economic equilibrium if any of the following three conditions hold: (i)* $x_t \leq \min\left\{x_t^{10}, x_t^{11}\right\}$, *(ii)* $\max\left\{\bar{x}_t, x_t^{00}\right\} < x_t$, *or (iii)* $\rho_t \in \left(0, \rho^l\right)$ *and* $x_t \in \left[x_t^{00}, x_t^{11}\right]$. *Otherwise, there are multiple period-t economic equilibria.*

The result in Proposition 1 suggests that the economic equilibrium is unique if the level of public education is low or high. If not, there are multiple equilibria or a unique equilibrium. In order to understand this result, let us first consider the case of a low x such that $x_t \leq \min\{x_t^{10}, x_t^{11}\}$. Due to the low level of public education expenditure, the tax rate could be reduced to satisfy the government budget constraint, regardless of education choice. This implies a low tax burden, making private education more affordable even for the type-L adults. Therefore, there is a unique economic equilibrium where both types of adults choose private education if $x_t \leq \min\{x_t^{10}, x_t^{11}\}$.

Second, consider the case of a high x such that $\max\{\bar{x}_t, x_t^{00}\} < x_t$. The government is required to set a high tax rate to satisfy the government budget constraint. This creates a negative income effect, which in turn makes private education less affordable, even for the type-H adults. Thus, there is a unique economic equilibrium where both types of adults choose public education if $\max\{\bar{x}_t, x_t^{00}\} < x_t$.

Finally, for the intermediate case such that $\min\{x_t^{10}, x_t^{11}\} < x_t \leq \max\{\bar{x}_t, x_t^{00}\}$, the uniqueness or multiplicity of equilibria depend on the tax rate that satisfies the government budget constraint. For example, consider a low inequality case such that $\rho_t \in \left(0, \rho^l\right)$ holds, as illustrated in Panel (a) of Figure 2, and focus on the public education expenditure level with $x_t \in (x_t^{11}, \bar{x}_t]$. If the tax rate is low enough that $\tau_t = \phi x_t/h_t$, then the type-H adults can afford to invest privately in education, and this choice is consistent with the condition of $\tau_t = \phi x_t/h_t$. However, if the tax rate is high enough that $\tau_t = x_t/h_t$, then they find it optimal to choose public rather than private education from the view of utility maximization. This choice is actually consistent with $\tau_t = x_t/h_t$.

4 Political Equilibrium

Based on the characterization of the economic equilibrium in Section 3, in this section, we demonstrate voting on education policy. We assume that adults vote sincerely since every agent has zero mass and thus no individual vote can change the outcome. In addition, in each period t, adult agents determine public education through a political process of majority voting. Assuming $\phi > 0.5$, type-L adults constitute the majority. Therefore, the political objective function in period t, denoted by Ω_t, is the indirect utility function of adult type-L agents.

Definition 2. Given ρ_t, a *period-t political equilibrium* is a level of public education expenditure, x_t such that x_t maximizes the type-L adults' utility subject to each type's education choice, as well as the corresponding consumption functions and government budget constraints.

We write the period-t political objective function according to the pair of education choices, $\left(q_t^L, q_t^H\right)$ demonstrated in (5), (6), and (7). Recall that the government budget constraint is

$$\tau_t h_t = \left\{ \begin{array}{ccc} \phi x_t & \text{if} & \left(q_t^L, q_t^H\right) = (1,0), \\ x_t & \text{if} & \left(q_t^L, q_t^H\right) = (1,1), \end{array} \right\}$$

and substitute this into the indirect utility function for the type-L adults. Then, the political objective function becomes:

$$\Omega_t = \left\{ \begin{array}{lll} \Omega_{00,t} \equiv V_{e,t}^L\big|_{\tau_t=0} = (1+\gamma\eta)\ln h_t\rho_t + \gamma \ln D\left(h_t\rho_t\right)^{1-\eta} + \gamma\eta\ln\mu & \text{if} & \left(q_t^L, q_t^H\right) = (0,0), \\ \Omega_{10,t} \equiv V_{x,t}^L\big|_{\tau_t=\phi x_t/h_t} = \ln\left(h_t - \phi x_t\right)\rho_t + \gamma \ln D\left(h_t\rho_t\right)^{1-\eta} + \gamma\eta\ln x_t & \text{if} & \left(q_t^L, q_t^H\right) = (1,0), \\ \Omega_{11,t} \equiv V_{x,t}^L\big|_{\tau_t=x_t/h_t} = \ln\left(h_t - x_t\right)\rho_t + \gamma \ln D\left(h_t\rho_t\right)^{1-\eta} + \gamma\eta\ln x_t & \text{if} & \left(q_t^L, q_t^H\right) = (1,1). \end{array} \right. \tag{10}$$

The functions $\Omega_{00,t}$, $\Omega_{10,t}$, $\Omega_{11,t}$ have the following properties: $\Omega_{00,t}$ is independent of x_t because both types opt out of public education, whereas $\Omega_{10,t}$ and $\Omega_{11,t}$ depend on x_t because either or both types choose public education. In particular, the solutions that maximize $\Omega_{10,t}$ and $\Omega_{11,t}$ are, respectively:

$$\left\{ \begin{array}{c} \arg\max \Omega_{10,t} = x_t^* \equiv \frac{\gamma\eta}{\phi(1+\gamma\eta)} h_t, \\ \arg\max \Omega_{11,t} = x_t^{**} \equiv \frac{\gamma\eta}{1+\gamma\eta} h_t \left(< x_t^*\right). \end{array} \right. \tag{11}$$

In addition, the following conditions hold:

$$\left\{ \begin{array}{c} \Omega_{11,t} < \Omega_{10,t} \ \forall x_t > 0, \\ x_t^{00} < x_t^{**} = \arg\max \Omega_{11,t}, \\ \Omega_{10,t}\big|_{x_t = x_t^{00}} < \Omega_{00,t}, \\ \Omega_{00,t} < \Omega_{11,t}\big|_{x_t = x_t^{**}} < \Omega_{10,t}\big|_{x_t = x_t^*}, \end{array} \right. \tag{12}$$

where the proof is given in Appendix A.3.

Given h_t and ρ_t, the period-t political equilibrium solution is $x_t = \arg\max \Omega_t$ and the corresponding education choices in (5), (6), and (7). The tax rate is set to satisfy the government budget constraint. In the following, we consider two cases: a low-inequality state, $\rho_t \in [\rho^m, 1)$, where $\bar{x}_t \leq x_t^{00}$ holds, and a high-inequality state, $\rho_t \in (0, \rho^m)$, where $x_t^{00} < \bar{x}_t$ holds.

4.1 A Low-inequality State: $\rho_t \in [\rho^m, 1)$

The education choice when $[\rho^m, 1)$ is depicted in Panels (c) and (d) of Figure 2. Figure 4 shows the corresponding political objective function. For illustration, we use the properties of Ω_t in (11) and (12).

[Figure 4 here.]

When x_t is below (above) the critical value x_t^{00}, the government expects that both types of adults will choose private (public) education, and the adults actually make that choice. We should note that when x_t is below x_t^{00}, there are at most three economic equilibria. However, the government finds it optimal to expect that both types of adults choose private education because the choice attains the highest utility for $x_t \leq x_t^{00}$. Thus, the political objective function is

$$\Omega_t = \begin{cases} \Omega_{00,t} & \text{if} \quad 0 < x_t \leq x_t^{00}, \\ \Omega_{11,t} & \text{if} \quad x_t^{00} < x_t < h_t. \end{cases}$$

The solution that maximizes Ω_t is $x_t^{**} = \arg\max \Omega_{11,t}$ because $\Omega_{11,t}|_{x_t = x_t^{**}} > \Omega_{00,t}$ holds, as shown in (12).

Lemma 1. *For $\rho_t \in [\rho^m, 1)$, the period-t* the *voting solution is $x_t^{**} = \arg\max \Omega_{11,t}$.*

Type-L adults pay less than they receive from public education and thus prefer public education over private education. As decisive voters, they choose per capita public education expenditures given their expectations of type-H voters' choices. Type-H adults may prefer private education to public education because they pay more than they receive from public education. However, their costs to provide public education in terms of utility decrease as ρ_t increases; that is, as their income level relative to the average, $h_t^H/h_t = (1-\phi\rho_t)/(1-\phi)$, decreases. In particular, if ρ_t is above ρ^m, the benefits in terms of utility outweigh the costs of public education to type-H, and type-H adults find it optimal to choose public education. Therefore, when inequality is low enough that $\rho^m \leq \rho_t < 1$, it is optimal for type-L adults to choose a per capita public education expenditure of $x_t^{**} = \arg\max \Omega_{11,t}$, given the expectation that type-H adults also choose public education.

4.2 A High-inequality State: $\rho_t \in (0, \rho^m)$

Panels (a) and (b) of Figure 2 show the education choice when $\rho_t \in (0, \rho^m)$. Figure 5 illustrates the corresponding political objective function. Because the government finds it optimal to expect the education choice that attains the highest utility, the political objective function when $\rho_t \in (0, \rho^m)$ is:

$$\Omega_t = \begin{cases} \Omega_{00,t} & \text{if} \quad 0 < x_t \leq x_t^{00}, \\ \Omega_{10,t} & \text{if} \quad x_t^{00} < x_t \leq \bar{x}_t, \\ \Omega_{11,t} & \text{if} \quad \bar{x}_t < x_t < h_t. \end{cases}$$

A main difference from the previous case is that the political objective might be maximized at $x_t \in (x_t^{00}, \bar{x}_t]$, where type-$H$ adults opt out of public education while type-L adults do not. Either type-L adults could opt out, or neither could opt out depending on the inequality.

[Figure 5 here.]

To find a political equilibrium solution, consider the following two cases: $x_t^* \leq \bar{x}_t$ as illustrated in Panel (a) of Figure 5, and $x_t^* > \bar{x}_t$ as illustrated in Panel (b) of Figure 5. Consider first the case when $x_t^* \leq \bar{x}_t \Leftrightarrow \rho_t \leq \rho^* \equiv (1 - \gamma\eta(1-\phi)/\mu\phi)/\phi$. As the figure shows, Ω_t is maximized at $x_t^* = \arg\max \Omega_{10,t}$. At this public education level, the type-H adults opt out of public education while the type-L do not. This case arises as a political equilibrium outcome when inequality is high enough that $\rho_t \in (0, \rho^*)$. This set is non-empty if and only if $\phi > \gamma\eta/(\mu + \gamma\eta)$. Therefore, there is a period-t political equilibrium with $(q_t^L, q_t^H) = (1, 0)$ and $x_t = x_t^*$ if $\rho_t \in (0, \rho^m)$ and $\phi \in (\gamma\eta/(\mu + \gamma\eta), 1)$.

Next, consider the case when $x_t^* > \bar{x}_t \Leftrightarrow \rho_t > \rho^*$. As the figure shows, there are two candidates for the period-t voting solution: one is $x_t = \bar{x}_t$, where the type-H opt out of public education, and the other is $x_t = \arg\max \Omega_{11,t} = x_t^{**}$, where both types choose public education. The type-L adults, as decisive voters, choose either to attain the highest utility. Appendix A.4 shows that there is a critical value of ρ_t, denoted by $\rho^{**} \in (\rho^*, \rho^m)$, such that $\Omega_{10,t}|_{x_t=\bar{x}_t} \lesseqgtr \Omega_{11,t}|_{x_t=x_t^{**}} \Leftrightarrow \rho_t \gtreqless \rho^{**}$. The following lemma summarizes the results thus far.

Lemma 2. *Assume $\rho_t \in (0, \rho^m)$. Given ρ_t and h_t, the period-t voting solution is*

$$\arg\max \Omega_t = \begin{cases} x_t^{**} = \arg\max \Omega_{11,t} & \text{if} \quad \rho_t \in (\rho^{**}, \rho^m), \\ x_t^* = \arg\max \Omega_{10,t} & \text{if} \quad \rho_t \in (0, \rho^*] \text{ and } \phi \in \left(\frac{\gamma\eta}{\mu+\gamma\eta}, 1\right), \\ \bar{x}_t & \text{if} \quad \rho_t \in (\max(0, \rho^*), \rho^{**}]. \end{cases}$$

Consider first a situation where $\rho_t \in (0, \rho^*]$ and $\phi \in \left(\frac{\gamma\eta}{\mu+\gamma\eta}, 1\right)$ hold: type-H adults are endowed with a sufficiently high income level but they have a low share of the population in their generation. They prefer private to public education due to its positive income effect. Given this type-H choice, $\Omega_{10,t}$ is the type-L's indirect utility function. As decisive voters, they choose per capita public education expenditure x_t that maximizes $\Omega_{10,t}$, $x_t^* = \arg\max \Omega_{10,t}$. As Panel (a) of Figure 5 shows, this choice is feasible if $x_t^* \leq \bar{x}_t$. In the current situation, the condition $x_t^* \leq \bar{x}_t$ actually holds because a low share of type-H individuals in the generation is equivalent to a high share of those of type-L and thus implies a low per-capita level of public education.

Next, consider a situation where $\rho_t \in (\max(0, \rho^*), \rho^{**}]$ holds: inequality is high but less severe than that observed in the first case. The type-H adults still prefer private to public education, but the type-L adults cannot choose an "interior" solution, $x_t^* = \arg\max \Omega_{10,t}$. Their choice is constrained by the upper limit, $\bar{x}_t$. We hereafter refer to $\bar{x}_t$ as a "corner" solution. Finally, if inequality is low enough that $\rho_t \in (\rho^{**}, \rho^m)$, the type-$H$ adults choose public education and the political objective is maximized at $x_t^{**} = \arg\max \Omega_{11,t}$.

We should note that the corner solution arises when the proportion of type-L adults is low enough that $\phi \in (1/2, \gamma\eta/(\mu+\gamma\eta)]$. A low ϕ implies a small tax burden for each agent for a given level of public education expenditure x. This lowers the marginal cost of public education, thereby inducing the type-L adults to prefer a higher public education expenditure. However, type-H adults will opt out when the public education expenditure is below $\bar{x}_t$. If the expenditure is above $\bar{x}_t$, type-H adults prefer public education to private education. Therefore, the upper limit, $\bar{x}_t$, constraints the type-L's choice of public education as long as type-H adults opt out of public education.

4.3 Voting Outcome and Education Choice

Summarizing the results in Lemmas 1 and 2, we obtain the voting solution in period t and the corresponding education choice.

Proposition 2. *Given the inequality index ρ_t, the period-t voting solution is*

$$\arg\max \Omega_t = \begin{cases} x_t^{**} & \text{if} \quad \rho_t \in (\rho^{**}, 1), \\ x_t^{*} & \text{if} \quad \rho_t \in (0, \rho^{*}] \text{ and } \phi \in \left(\frac{\gamma\eta}{\mu+\gamma\eta}, 1\right), \\ \bar{x}_t & \text{otherwise.} \end{cases}$$

The corresponding education choice is

$$\left(q^L, q^H\right) = \begin{cases} (1,1) & \text{if} \quad \rho_t \in (\rho^{**}, 1), \\ (1,0) & \text{if} \quad \rho_t \in (0, \rho^{**}]. \end{cases}$$

Proposition 2 states that type-H adults choose public education if inequality is low enough that $\rho_t \in (\rho^{**}, 1)$; otherwise, they choose private education. A small ρ_t implies a high income disparity between the two types of adults, so type-H adults owe a large tax burden. In particular, if $\rho_t \leq \rho^{**}$, the negative income effect dominates the positive effect of public education provision. This incentivizes type-H adults to opt out of public education. However, if inequality is low enough that $\rho_t > \rho^{**}$, the positive effect of public education provision dominates the negative income effect, and type-H adults choose public education.

5 Steady-state Equilibrium

The analysis in the previous section demonstrates public education expenditure as a political outcome for a given inequality index (ρ_t). The current public education influences human capital formation, which in turn determines inequality in the next generation (ρ_{t+1}). To consider the dynamic interaction between inequality and public education, we demonstrate the inequality index movement across periods and the existence and stability of a *steady-state equilibrium* in which $\rho_{t+1} = \rho_t$ holds along the equilibrium path. Based on the description of the equilibrium, we examine how the structural parameters ϕ, γ, and η affect steady-state inequality and the choice of education.

Given an initial condition, $\rho_1 (> 0)$, the political equilibrium sequence $\{\rho_t\}$ is characterized by the first-order difference equation, $\rho_{t+1} = P(\rho_t)$, where

$$P(\rho_t) = \begin{cases} P_{11}(\rho_t) \equiv \left[\phi + (1-\phi)^{\eta}\left(\frac{1}{\rho_t} - \phi\right)^{1-\eta}\right]^{-1} & \text{if } \rho_t \in (\rho^{**}, 1), \\ P_{10}(\rho_t) \equiv \left[\phi + \left(\frac{\phi}{1+\gamma\eta}\right)^{\eta}(1-\phi\rho_t)\left(\frac{1}{\rho_t}\right)^{1-\eta}\right]^{-1} & \text{if } \rho_t \in (0, \rho^{*}] \text{ and } \phi \in \left(\frac{\gamma\eta}{\mu+\gamma\eta}, 1\right), \\ \bar{P}_{10}(\rho_t) \equiv \left[\phi + (1-\phi)(1+\gamma\eta)^{1/\gamma}\left(\frac{1-\phi\rho_t}{(1-\phi)\rho_t}\right)^{1-\eta}\right]^{-1} & \text{otherwise.} \end{cases}$$

where the subscripts "11" and "10" in $P(\cdot)$ imply $\left(q^L, q^H\right) = (1,1)$ and $(1,0)$, respectively. The three cases correspond to those in Proposition 1. Appendix A.5 provides the derivation of $P_{11}(\rho_t)$, $P_{10}(\rho_t)$ and $\bar{P}_{10}(\rho_t)$.

A closer analysis of $P(\cdot)$ reveals that the function has the following properties (see Appendix A.6 for the formal proof of the following statement). First, $P_{11}(\cdot)$, $P_{10}(\cdot)$ and $\bar{P}_{10}(\cdot)$ are strictly increasing in ρ_t. Second, $P_{10}(\cdot) \gtreqless \bar{P}_{10}(\cdot)$ if and only if $\rho_t \gtreqless \rho^*$. Third, $\bar{P}_{10}(\cdot) < P_{11}(\cdot)\ \forall \rho_t \in (0,1)$. Fourth, $P_{11}(\cdot)$ satisfies $P_{11}(1) = 1$ and $P'_{11}(1) = 1-\eta \in (0,1)$. Fifth, $\bar{P}_{10}(\cdot)$ satisfies $\bar{P}_{10}(0) = 0$ and $\lim_{\rho\to 0}\left(\partial\bar{P}_{10}(\cdot)/\partial\rho_t\right) = \infty$; $P_{10}(\cdot)$ satisfies $P_{10}(0) = 0$, and $\lim_{\rho\to 0}(\partial P_{10}(\cdot)/\partial\rho_t) = \infty$. These properties imply that (i) there is a locally stable steady-state equilibrium with $\rho = 1$, and (ii) $P(\cdot)$ is strictly increasing in ρ_t but discontinuous at $\rho_t = \rho^{**}$. Figure 6 illustrates the possible patterns of $P(\cdot)$ when $\phi \in (\gamma\eta/(\mu+\gamma\eta), 1)$. The $\phi \in (1/2, \gamma\eta/(\mu+\gamma\eta)]$ case is qualitatively similar, but the threshold value ρ^* is negative and thus irrelevant. From the figure, we obtain the following proposition:

[Figure 6 here.]

Proposition 3. *If $\bar{P}_{10}(\rho^{**}) > \rho^{**}$, there is a unique stable steady-state equilibrium with $\rho = 1$; if $\bar{P}_{10}(\rho^{**}) \leq \rho^{**}$, there are two locally stable steady-state equilibria, one with $\rho \in (0, \rho^{**}]$ and the other with $\rho = 1$.*

The unique stable steady-state equilibrium is distinguished by perfect equality between the two types of agents and 100% enrollment in public school. However, another type of equilibrium exists when the multiple stable steady states are realized, distinguished by the presence of income inequality and type-H agents' opting out of public education. Thus, the multiple steady states imply that higher inequality is associated with lower enrollment in public school. This model prediction is consistent with empirical evidence from OECD countries.

For an intuitive interpretation of the condition $\bar{P}_{10}(\rho^{**}) \gtrless \rho^{**}$, we reformulate it as

$$\bar{P}_{10}(\rho^{**}) \gtrless \rho^{**} \Leftrightarrow \frac{1-\phi\rho^{**}}{1-\phi} \cdot \frac{1}{\rho^{**}} \gtrless (1+\gamma\eta)^{1/\gamma\eta}, \tag{13}$$

where ρ^{**}, defined in Subsection 4.2, satisfies $\Omega_{10,t}|_{x_t=\bar{x}_t} = \Omega_{11,t}|_{x_t=x_t^{**}}$, or

$$\left(\frac{1-\phi\rho^{**}}{1-\phi}\right)^{\gamma\eta} = 1 + \mu\phi \cdot \frac{1-\phi\rho^{**}}{1-\phi}, \ \mu \equiv \frac{\gamma\eta}{(1+\gamma\eta)^{(1+\gamma\eta)/\gamma\eta}}. \tag{14}$$

Eq. (14) indicates that ρ^{**} is a function of ϕ and $\gamma\eta$, $\rho^{**} = \rho^{**}(\phi, \gamma\eta)$. Thus, we can illustrate the condition in (13) in a $\phi - \gamma\eta$ space, as Figure 7 shows. The figure suggests that the model is more likely to produce multiple steady-state equilibria if ϕ and $\gamma\eta$ are lower.

[Figure 7 here.]

To explain this argument, we first consider the effect of $\gamma\eta$. A low γ means a low weight attached to the utility of children's human capital, and a low η means low elasticity in human capital with respect to public education expenditure. These factors imply that type-H agents determine a low benefit from public education in terms of utility. This in turn induces type-H agents to opt out of public education. Therefore, a low $\gamma\eta$ encourages an equilibrium with $\rho < 1$.

Next, we consider the role of ϕ in the steady-state equilibria outcome. Recall the definition of $\rho_{t+1} \equiv h_{t+1}^L/h_{t+1}$, or,

$$\rho_{t+1} \equiv \frac{h_{t+1}^L}{h_{t+1}} = \frac{h_{t+1}^L}{\phi h_{t+1}^L + (1-\phi)h_{t+1}^H}.$$

This expression indicates that the parameter ϕ has two effects in the determination of ρ_{t+1}. First, given h_{t+1}^H, a lower ϕ implies a larger proportion of type-H agents. This leads to a higher average human capital, h_{t+1}, and thus a lower $\rho_{t+1} \equiv h_{t+1}^L/h_{t+1}$ for a given h_{t+1}^L. Second, a lower ϕ implies a lower aggregate public education expenditure and thus a lower tax burden on type-H agents. This produces a positive income effect on private education expenditure by type-H agents. This, in turn, increases type-H's human capital, h_{t+1}^H, and the average human capital, h_{t+1}, and thus decreases ρ_{t+1}. Because of these two negative effects, the model produces an equilibrium with $\rho_{t+1} < 1$ if ϕ is low.

However, an economy with low $\gamma\eta$ and ϕ also has an equilibrium with $\rho = 1$ provided that the initial condition of ρ is high. A higher ρ implies a lower income gap and thus lower income for type-H agents. Because of this negative income effect, type-H agents find it optimal to choose public education over private education. Therefore, there is also an equilibrium with $\rho = 1$ for low values of $\gamma\eta$ and ϕ.

Thus far, we assume that human capital productivity, represented by D, is common between the two types of agents. However, D may represent a durable productive asset such as generic ability, technology transfer, or business succession that children inherit from parents. Based on this view, we can alternatively assume that the distribution of D is positively correlated with human capital: $D^H > D^L$, where D^i $(i = H, L)$ is type-i's human capital productivity (Gradstein and Justman, 1996). This assumption implies that, on average, children born to higher-income families are endowed with greater human capital productivity (Behrman and Taubman, 1989).

Under this alternative assumption, the law of motion of human capital when $(q^L, q^H) = (x, x)$ is reformulated as:

$$\rho_{t+1} = \left[\phi + \frac{D^H}{D^L} (1-\phi)^\eta \left(\frac{1}{\rho_t} - \phi \right)^{1-\eta} \right]^{-1}.$$

This equation implies a stable steady-state equilibrium with $\rho < 1$, which seems more realistic than the equilibrium with $\rho = 1$, which assumes $D^H = D^L = D$. However, the qualitative results remain unchanged. Therefore, for the tractability of analysis, we keep the assumption of $D^H = D^L = D$ in the following analysis.

6 Welfare Analysis

We use simulations to investigate the model's welfare implications. In the analysis, we set parameters ϕ, η, γ, and D as in the following. First, recall that $1-\phi$ is the fraction of type-H agents and only type-H agents opt out of public schooling. The fraction $1-\phi$ therefore represents enrollment in private institutions as a percentage. We set $1-\phi$ at 0.147 because the average rate in OECD countries was 14.7% in 2011. Second, the estimate in Card and Krueger (1992) implies an elasticity of school quality of 0.12. In addition, recent simulation studies suggest that η is in the range of $0.1 - 0.3$ (Cardak, 2004) and $0.05 - 0.15$ (Glomm and Ravikumar, 1998). Following these earlier results, we here set $\eta = 0.15$.

For γ, we focus on the public education expenditure-GDP ratio in the steady-state equilibrium distinguished by $\rho = 1$. The ratio in this equilibrium is $x/h = \gamma\eta/(1+\gamma\eta)$. Given $\eta = 0.15$, we can estimate γ by using the average ratio x/h observed in selected OECD countries. Table 1 lists 25 OECD countries and their Gini coefficients (Panel (a)) and the percentage of enrollment in private secondary education (Panel (b)) in 2011. We chose the countries in the top 50% in Panel (a) and those in the top 50% in Panel (b) and define this set as those that attain the steady state with $\rho = 1$. These countries are Slovenia, Norway, the Czech Republic, Finland, and the Netherlands. The average ratio in these countries was 2.22% in 2011[1], so we can now determine γ by solving $0.022 = 0.15\gamma/(1 + 0.15\gamma)$ for γ: $\gamma \simeq 0.151$. This estimate fits well with that of 0.13 by Cardak (2004) and that of 0.169 by de la Croix and Doepke (2004). Finally, we normalize D as $D = 1$.

[Table 1 here.]

6.1 Utility Gap

We set the initial condition h_1^L and h_1^H to attain an equilibrium path that converges to the unequal steady state with $\rho < 1$. Figure 8 plots the utility gap between type-L and type-H agents along this equilibrium path. The ratio of type-L's utility to type-H's utility represents the gap. Given the logarithmic form of the utility function, both types' utility functions take negative values in the following numerical analysis. Hence, a ratio greater than one implies that type-L's utility is lower than that of type-H. For example, if $V^L = -1.2$ and $V^H = -1.0$, then the ratio is 1.2. A higher ratio implies a wider utility gap.

[Figure 8 here.]

We look at the utility gap along the transition path and investigate how the utility gap changes in response to the spread of the initial inequality by considering a mean-preserving reduction of type-L's initial human capital, where a reduction in h_1^L is associated with an increase in h_1^H, keeping the average h_1 unchanged. Figure 8 plots the ratio from generation 1 to generation 80 for three cases of initial conditions. The solid curve illustrates the baseline case and the dashed (dot-and-dash) curve illustrates a higher (lower) initial inequality case. The figure shows that the utility gap widens as the initial inequality increases, but the difference between the three cases disappears in the long run.

6.2 Compulsory Public School System

To further investigate the welfare implications, we introduce a compulsory public school system as an alternative education regime and compare it to the mixed education system analyzed thus far in terms of utility. In the present framework, the *compulsory public school system* prohibits students from opting out of public school. Due to the limited choice of education, the system forces the economy into a steady state with $\rho = 1$. This is identical to the steady state with $\rho = 1$ in the mixed education system. Therefore, we can evaluate the multiple stable steady-state equilibria in Proposition 2 in terms of

[1]Source: UNESCO statistics (http://data.uis.unesco.org/, February 14, 2016).

utility by comparing the equilibrium with $\rho < 1$ in the mixed education system and the equilibrium with $\rho = 1$ in the compulsory public school system.

For the analysis, we take the ratio of utility in the mixed education system to that in the compulsory public school system and plot it from generation 1 to generation 80 in Figure 9. Panels (a), (b), and (c) illustrate the ratios for type-L agents, type-H agents, and social welfare (the population-weighted average utility of the two types of agents), respectively. As in Figure 8, the ratio of more (less) than 1 implies that the utility in the compulsory public school system is higher (lower) than that in the mixed education system.

[Figure 9 here.]

To interpret the result in Figure 9, we first note the indirect utility of type-L agents:

$$V^L_{mix,t} = \ln\left(h_t - \phi x^H_t\right)\frac{h^L_t}{h_t} + \gamma \ln D\left(h^L_t\right)^{1-\eta} + \gamma\eta \ln x^H_t,$$
$$V^L_{comp,t} = \ln\left(h_t - x^{**}_t\right)\frac{h^L_t}{h_t} + \gamma \ln D\left(h^L_t\right)^{1-\eta} + \gamma\eta \ln x^{**}_t,$$

where $V^L_{mix,t}$ and $V^L_{comp,t}$ are the indirect utility in the mixed education system and that in the compulsory public education system, respectively.

The expressions show that introducing compulsory public education has two opposing effects on type-L's utility. First, the tax burden decreases from ϕx^H_t to x^{**}_t. Second, per capita public education expenditure decreases from x^H_t to x^{**}_t. The numerical result in Panel (a) shows that the latter negative effect outweighs the former positive one, so introducing the compulsory public education system makes type-L agents in generation 1 worse off.

From generation 2 onward, there is an additional positive effect via the human capital formation generated by the compulsory public school system. The terms h^L_t/h_t and $D\left(h^L_t\right)^{1-\eta}$ in the above expressions illustrate this effect. This positive effect increases as the initial inequality decreases. In addition, this effect amplifies the tax reduction effect. Therefore, for the baseline case and the low initial inequality case, introducing compulsory public education makes generations from 2 onward better off. However, for the case of high initial inequality, it takes a long time to realize this welfare improvement because the negative effect remains stronger as the initial inequality increases. Type-L agents' welfare improves only from generation 70 onward.

Panel (b) plots the ratio of type-H from generation 1 to 80. In the current setting, they choose private education in the mixed education regime. Thus, their indirect utility is:

$$V^H_{mix,t} = \ln\left(h_t - \phi x^H_t\right)\frac{h^H_t}{h_t} + \gamma \ln D\left(h^H_t\right)^{1-\eta} + \gamma\eta \ln\left(h_t - \phi x^H_t\right)\frac{h^H_t}{h_t} + \gamma\eta \ln \mu,$$

$$V^H_{comp,t} = \ln\left(h_t - x^{**}_t\right)\frac{h^H_t}{h_t} + \gamma \ln D\left(h^H_t\right)^{1-\eta} + \gamma\eta \ln x^{**}_t,$$

where $V^H_{mix,t}$ and $V^H_{comp,t}$ are the indirect utility in the mixed education system and that in the compulsory public education system, respectively.

As in the case with type-L, introducing compulsory public education has two opposing effects on type-H's utility in the initial period: the tax burden decreases from ϕx^H_t to x^{**}_t and the expenditure for human capital formation decreases from $\left(h_t - \phi x^H_t\right) h^H_t/h_t$ to x^{**}_t. The numerical result in Panel (b) shows that the latter negative effect outweighs the former positive one, so the change makes type-H agents in generation 1 worse off. From generation 2 onward, these agents are also worse off because there is an additional negative effect through the delay of human capital formation generated by the compulsory public school system. The terms h^H_t/h_t and $D\left(h^H_t\right)^{1-\eta}$ in the above expressions illustrate this effect. The numerical result suggests that the shift from the mixed education to the compulsory public school system is not Pareto-improving.

Finally, we investigate the effect of the compulsory public school system on social welfare. The introduction decreases social welfare in period 1 because both types of agents are worse off. However, the effect on welfare from period 2 onward depends on the initial inequality: welfare improves earlier as the initial inequality decreases, as illustrated in Panel (c). The result suggests that the social welfare ranking of the multiple equilibria depends on the initial inequality condition.

7 Conclusion

This study presents a political economy theory to explain why countries with higher inequality are associated with lower enrollment in public education. We base the theory on a two-class (high and low), successive-generations model with human capital accumulation and the choice to opt out of public education accompanied by voting on education policy. This condition creates multiple, locally stable steady-state equilibria: one with low inequality and high enrolment in public education and the other with high inequality and low enrolment in public education. This study is novel in that it shows the negative correlation observed in OECD countries in the mutual interaction of inequality and education.

From the equity viewpoint, it is desirable to attain the low-inequality steady state. One path to this steady state involves introducing compulsory public schooling. We used a simulation to investigate the welfare implications of introducing this reform and find that it makes high-income families worse off, while improving the lot of future generations of low-income families at the expense of the current generation. The results suggest that the multiple equilibria are not Pareto-ranked, and that the shift from the existing mixed education system to a compulsory public school system is not Pareto-improving.

We demonstrated these results by making several assumptions that make the analysis tractable. In particular, we assume two classes and that the low-type agents constitute the majority in every period. This assumption enables an analytical solution to the model and an illustration of the multiple locally stable steady-state equilibria observed in empirical studies. A future extension could include probabilistic voting, a la Lindbeck and Weibull (1987), to reflect the preferences of both types of agents. While this type of analysis is ideal, the analysis is rather complicated and best left for future research.

A Appendix

A.1 Proof of (8)

Recall the definition of x_t^{00}, x_t^{10}, $\bar{x}_t$, and x_t^{11} in the text. We compare these as follows:

$$\begin{aligned}
x_t^{00} \gtrless x_t^{11} &\Leftrightarrow 0 \gtrless f(\rho_t) \equiv \mu\phi(\rho_t)^2 - (1+\mu)\rho_t + 1, \\
x_t^{00} \gtrless \bar{x}_t &\Leftrightarrow 0 \gtrless g(\rho_t) \equiv \mu\phi^2(\rho_t)^2 - (1+\mu\phi)\rho_t + 1, \\
x_t^{10} \gtrless x_t^{11} &\Leftrightarrow 0 \gtrless h(\rho_t) \equiv \mu\phi(1-\phi)(\rho_t)^2 - (1+\mu(1-\phi))\rho_t + 1,
\end{aligned}$$

where (i) $f(0) = g(0) = h(0) > 0$, (ii) $f(\cdot) < g(\cdot) < h(\cdot)$ for any $\rho_t \in (0,1]$, and (iii) $f'(\cdot) < 0, g'(\cdot) < 0$, and $h'(\cdot) < 0$ for any $\rho_t \in (0,1)$. As illustrated in Figure A.1, there are three critical values of ρ_t, denoted by ρ^l, ρ^m, and ρ^h, where $0 < \rho^l < \rho^m < \rho^h < 1$, such that $f(\rho^l) = 0$, $g(\rho^m) = 0$, and $h(\rho^h) = 0$. From Figure A.1, we obtain (8).

[Figure A.1 here.]

■

A.2 Proof of Proposition 1

Suppose that $\rho_t \in (0, \rho^l)$. Figure A.1 shows that in this case, $x_t^{00} < x_t^{11}$, $x_t^{00} < \bar{x}_t$, and $x_t^{10} < x_t^{11}$ holds. With the condition in (9), we obtain $x_t^{10} < x_t^{00} < x_t^{11} < \bar{x}_t$ as illustrated in Panel (a) of Figure 2. The figure shows that there is a unique economic equilibrium if $x_t \in (0, x_t^{10})$, $(x_t^{00}, x_t^{11}]$, or $(\bar{x}_t, h_t)$; otherwise, there are multiple economic equilibria.

Following the same procedure, we can show the uniqueness or multiplicity of the economic equilibria for the remaining three cases, $\rho_t \in [\rho^l, \rho^m)$, $[\rho^m, \rho^h)$, and $[\rho^h, 1)$. There is a unique economic equilibrium if any of the following three conditions hold: (i) $\rho_t \in [\rho^l, \rho^m)$ and $x_t \in (0, x_t^{10}]$ or $(\bar{x}_t, h_t)$ as illustrated in Panel (b) of Figure 2; (ii) $\rho_t \in [\rho^m, \rho^h)$ and $x_t \in (0, x_t^{10}]$ or (x_t^{00}, h_t) as illustrated in Panel (c) of Figure 2; and (iii) $\rho_t \in [\rho^h, 1)$ and $x_t \in (0, x_t^{11}]$ or (x_t^{00}, h_t) as illustrated in Panel (d) of Figure 2. Proposition 1 summarizes the results established thus far.

■

A.3 Proof of (12)

The first condition, $\Omega_{11,t} < \Omega_{10,t}\ \forall x_t > 0$, is immediate from the definition of $\Omega_{11,t}$ and $\Omega_{10,t}$. We show the second condition, $x_t^{00} < x_t^{**}$, with a direct comparison:

$$x_t^{00} < x_t^{**} \Leftrightarrow \mu\rho_t h_t < \frac{\gamma\eta}{1+\gamma\eta} h_t \Leftrightarrow \rho_t < (1+\gamma\eta)^{1/\gamma\eta},$$

which holds for any $\rho_t < 1$ and $\gamma\eta \in (0, 1)$.

To show the third condition, $\Omega_{10,t}|_{x_t=x_t^{00}} < \Omega_{00,t}$, we compare $\Omega_{10,t}|_{x_t=x_t^{00}}$ and $\Omega_{00,t}$, and obtain

$$\begin{aligned}\Omega_{10,t}|_{x_t=x_t^{00}} < \Omega_{00,t} &\Leftrightarrow \ln(1-\phi\mu\rho_t)\,\rho_t h_t + \gamma\eta\ln\mu\rho_t h_t < (1+\gamma\eta)\ln\rho_t h_t + \gamma\eta\ln\mu \\ &\Leftrightarrow \ln(1-\phi\mu\rho_t) < 0.\end{aligned}$$

The last inequality holds since $\ln(1-\phi\mu\rho_t) < \ln 1 = 0$.

To show the fourth condition, we first compare $\Omega_{00,t}$ and $\Omega_{11,t}|_{x_t=x_t^{**}}$ and obtain

$$\begin{aligned}\Omega_{00,t} < \Omega_{11,t}|_{x_t=x_t^{**}} &\Leftrightarrow (1+\gamma\eta)\ln\rho_t h_t + \gamma\eta\ln\mu < \ln\left(1-\frac{\gamma\eta}{1+\gamma\eta}\right)\rho_t h_t + \gamma\eta\ln\frac{\gamma\eta}{1+\gamma\eta}h_t \\ &\Leftrightarrow \gamma\eta\ln\rho_t < 0,\end{aligned}$$

where the last inequality holds since $\ln\rho_t < \ln 1 = 0$. The inequality $\Omega_{11,t}|_{x_t=x_t^{**}} < \Omega_{10,t}|_{x_t=x_t^{*}}$ is immediate since $\Omega_{11,t}|_{x_t=x_t^{**}} < \Omega_{10,t}|_{x_t=x_t^{**}} < \Omega_{10,t}|_{x_t=x_t^{*}}$.

■

A.4 Proof of Lemma 2

The text provides the following statement:

$$\arg\max\Omega_t = x_t^* \text{ if } \rho_t \in (0, \rho^*] \text{ and } \phi \in \left(\frac{\gamma\eta}{\mu+\gamma\eta}, 1\right).$$

The remaining task is to show that there is $\rho^{**} \in (\rho^*, \rho^m)$ such that

$$\Omega_{10,t}|_{x_t=\bar{x}_t} \lesseqgtr \Omega_{11,t}|_{x_t=x_t^{**}} \Leftrightarrow \rho_t \gtreqless \rho^{**}, \tag{15}$$

where

$$\begin{aligned}\Omega_{10,t}|_{x_t=\bar{x}_t} &= \ln(h_t - \phi\bar{x}_t)\,\rho_t + \gamma\ln D\,(h_t\rho_t)^{1-\eta} + \gamma\eta\ln\bar{x}_t, \\ \Omega_{11,t}|_{x_t=x_t^{**}} &= \ln(h_t - x_t^{**})\,\rho_t + \gamma\ln D\,(h_t\rho_t)^{1-\eta} + \gamma\eta\ln x_t^{**}.\end{aligned}$$

A direct comparison of $\Omega_{10,t}|_{x_t=\bar{x}_t}$ and $\Omega_{11,t}|_{x_t=x_t^{**}}$ leads to

$$
\begin{aligned}
&\Omega_{10,t}|_{x_t=\bar{x}_t} \lesseqgtr \Omega_{11,t}|_{x_t=x_t^{**}} \\
&\qquad \Leftrightarrow \ln\left(h_t - \phi\frac{\mu^{\frac{1-\phi\rho_t}{1-\phi}}}{1+\mu\phi\frac{1-\phi\rho_t}{1-\phi}}h_t\right) + \gamma\eta\ln\frac{\mu^{\frac{1-\phi\rho_t}{1-\phi}}}{1+\mu\phi\frac{1-\phi\rho_t}{1-\phi}}h_t \\
&\qquad \gtreqless \ln\left(h_t - \frac{\gamma\eta}{1+\gamma\eta}h_t\right) + \gamma\eta\ln\frac{\gamma\eta}{1+\gamma\eta}h_t \\
&\qquad \Leftrightarrow \ln\frac{\left(\mu\frac{1-\phi\rho_t}{1-\phi}\right)^{\gamma\eta}}{\left(1+\mu\phi\frac{1-\phi\rho_t}{1-\phi}\right)^{1+\gamma\eta}} \gtreqless \ln\frac{(\gamma\eta)^{\gamma\eta}}{(1+\gamma\eta)^{\gamma\eta}} \\
&\qquad \Leftrightarrow \underbrace{\left[\frac{1-\phi\rho_t}{1-\phi}\right]^{\gamma\eta/(1+\gamma\eta)}}_{LHS} \gtreqless \underbrace{1+\mu\phi\frac{1-\phi\rho_t}{1-\phi}}_{RHS},
\end{aligned}
\tag{16}
$$

where LHS and RHS in (16) are increasing in ρ_t.

At $\rho_t = \rho^* \equiv (1-\gamma\eta(1-\phi)/\mu\phi)/\phi$,

$$
LHS|_{\rho_t=\rho^*} > RHS|_{\rho_t=\rho^*} \Leftrightarrow \left(\frac{1}{\phi}\right)^{\gamma\eta/(1+\gamma\eta)} > 1,
$$

which holds for any $\phi \in (0,1)$ and $\gamma\eta \in (0,1)$. It also holds that

$$
\lim_{\rho\to\rho^m} LHS < \lim_{\rho\to\rho^m} RHS \Leftrightarrow \Omega_{10,t}|_{x_t=\bar{x}_t,\rho=\rho^m} < \Omega_{11,t}|_{x_t=x_t^{**},\rho=\rho^m},
$$

where the second inequality condition holds as shown in Lemma 1. Therefore, there is a unique ρ_t, denoted by $\rho^{**} \in (\rho^*, \rho^m)$, that satisfies (16) with an equality.

To summarize, the results thus are:

$$
\arg\max\Omega_t = \begin{cases} x_t^{**} = \arg\max\Omega_{11,t} & \textit{if} \quad \rho^{**} < \rho_t < \rho^m, \\ x_t^{*} = \arg\max\Omega_{10,t} & \textit{if} \quad \rho_t \in (0,\rho^*] \text{ and } \phi \in \left(\frac{\gamma\eta}{\mu+\gamma\eta}, 1\right), \\ \bar{x}_t & \textit{if} \quad \max(0,\rho^*) < \rho_t \le \rho^{**}, \end{cases}
$$

where

$$
\max(0,\rho^*) = \begin{cases} 0 & \text{if} \quad \phi \in \left(\frac{1}{2}, \frac{\gamma\eta}{\mu+\gamma\eta}\right], \\ \rho^* & \text{if} \quad \phi \in \left(\frac{\gamma\eta}{\mu+\gamma\eta}, 1\right), \end{cases}
$$

because $\rho^* \gtreqless 0 \Leftrightarrow \phi \gtreqless \frac{\gamma\eta}{\mu+\gamma\eta}$.

■

A.5 Derivation of $P_{11}(\cdot)$, $\bar{P}_{10}(\cdot)$, and $P_{10}(\cdot)$

First, assume $\rho_t \in (\rho^{**}, 1)$: both types of agents choose public education, $(q^L, q^H) = (1,1)$. The average human capital in period $t+1$ is

$$
\begin{aligned}
h_{t+1} &= \phi h_{t+1}^L + (1-\phi)h_{t+1}^H \\
&= \phi D\left(h_t^L\right)^{1-\eta}(x_t)^\eta + (1-\phi)D\left(h_t^H\right)^{1-\eta}(x_t)^\eta.
\end{aligned}
$$

Using this expression, we can reformulate $\rho_{t+1} = h_{t+1}^L / h_{t+1}$ as

$$\begin{aligned}
\rho_{t+1} &= \frac{D\left(h_t^L\right)^{1-\eta}(x_t)^\eta}{\phi D\left(h_t^L\right)^{1-\eta}(x_t)^\eta + (1-\phi) D\left(h_t^H\right)^{1-\eta}(x_t)^\eta} \\
&= \left[\phi + (1-\phi)\left(\frac{h_t^H}{h_t^L}\right)^{1-\eta}\right]^{-1} \\
&= \left[\phi + (1-\phi)^\eta\left(\frac{1}{\rho_t} - \phi\right)^{1-\eta}\right]^{-1},
\end{aligned}$$

where the equality in the third line comes from $h_t^H / h_t^L = (1/\rho_t - \phi)/(1-\phi)$.

Next, assume $\rho_t \in (0, \rho^{**}]$: type-$L$ agents choose public education and type-H agents choose private education. Type-H's human capital equation is

$$\begin{aligned}
h_{t+1}^H &= D\left(h_t^H\right)^{1-\eta}\left(\frac{\gamma\eta}{1+\gamma\eta}(1-\tau_t)\, h_t^H\right)^\eta \\
&= D h_t^H \left(\frac{\gamma\eta}{1+\gamma\eta}\right)^\eta \left(1 - \frac{\phi x_t}{h_t}\right)^\eta,
\end{aligned}$$

where the first equality comes from the private education function, $e_t^H = \gamma\eta(1-\tau_t)\, h_t^H / (1+\gamma\eta)$, and the second equality comes from the government budget constraint, $\phi x_t = \tau_t h_t$. With $h_{t+1}^L = D\left(h_t^L\right)^{1-\eta}(x_t)^\eta$, the period $t+1$ inequality index, ρ_{t+1}, becomes

$$\begin{aligned}
\rho_{t+1} &= \frac{D\left(h_t^L\right)^{1-\eta}(x_t)^\eta}{\phi D\left(h_t^L\right)^{1-\eta}(x_t)^\eta + (1-\phi) D h_t^H \left(\frac{\gamma\eta}{1+\gamma\eta}\right)^\eta \left(1 - \frac{\phi x_t}{h_t}\right)^\eta} \\
&= \left[\phi + (1-\phi)\frac{h_t^H \left(\frac{\gamma\eta}{1+\gamma\eta}\right)^\eta \left(1 - \frac{\phi x_t}{h_t}\right)^\eta}{\left(h_t^L\right)^{1-\eta}(x_t)^\eta}\right]^{-1}. \qquad (17)
\end{aligned}$$

Assume the corner solution,

$$x_t = \bar{x}_t \equiv \frac{\mu \frac{1-\phi\rho_t}{1-\phi}}{1 + \mu\phi\frac{1-\phi\rho_t}{1-\phi}} h_t.$$

Substituting this into (17) and rearranging the terms, we obtain $\bar{P}_{10}(\cdot)$, as in the text. Alternatively, assume the interior solution, $x_t = x_t^* = \gamma\eta h_t / \phi(1+\gamma\eta)$. Substituting this into (17) and rearranging the terms, we obtain $P_{10}(\cdot)$, as in the text.

■

A.6 Properties of $P_{11}(\cdot)$, $\bar{P}_{10}(\cdot)$, and $P_{10}(\cdot)$

(i) Claim 1: $P_{11}(\cdot)$, $\bar{P}_{10}(\cdot)$, and $P_{10}(\cdot)$ are strictly increasing inρ_t.

This claim is immediate from the expressions of $P_{11}(\cdot)$, $\bar{P}_{10}(\cdot)$, and $P_{10}(\cdot)$ in the text.

(ii) Claim 2: $P_{10}(\cdot) \gtreqless \bar{P}_{10}(\cdot)$ if and only if $\rho_t \gtreqless \rho^*$.

We directly compare $P_{10}(\cdot)$ and $\bar{P}_{10}(\cdot)$ and obtain

$$P_{10}(\cdot) \gtreqless \bar{P}_{10}(\cdot) \Leftrightarrow (1-\phi)(1+\gamma\eta)^{1/\gamma}\left(\frac{1-\phi\rho_t}{(1-\phi)\rho_t}\right)^{1-\eta} \gtreqless \left(\frac{\phi\rho_t}{1+\gamma\eta}\right)^{\eta}\left(\frac{1-\phi\rho_t}{\rho_t}\right)$$

$$\Leftrightarrow \rho_t \gtreqless \rho^* \equiv \frac{1}{\phi}\cdot\left[1-\frac{1-\phi}{\phi}\cdot\frac{\gamma\eta}{\mu}\right].$$

(iii) Claim 3: $\bar{P}_{10}(\cdot) < P_{11}(\cdot)\ \forall \rho_t \in (0,1)$.

We directly compare $\bar{P}_{10}(\cdot)$ and $P_{11}(\cdot)$ and obtain

$$\begin{aligned}
&\bar{P}_{10}(\cdot) < P_{11}(\cdot)\\
&\quad\Leftrightarrow (1-\phi)^{\eta}\left(\frac{1}{\rho_t}-\phi\right)^{1-\eta} < (1-\phi)(1+\gamma\eta)^{1/\gamma}\left(\frac{1-\phi\rho_t}{(1-\phi)\rho_t}\right)^{1-\eta}\\
&\quad\Leftrightarrow 1 < (1+\gamma\eta)^{1/\gamma},
\end{aligned}$$

which holds for any $\gamma\eta \in (0,1)$.

(iv) Claim 4: $P_{11}(\cdot)$ satisfies $P_{11}(1) = 1$ and $P_{11}'(1) = 1-\eta \in (0,1)$.

$P_{11}(1) = 1$ is immediate from the definition of $P_{11}(\cdot)$ in the text. The first differentiation of $P_{11}(\cdot)$ with respect to ρ is

$$P_{11}'(\rho_t) = \left[\phi + (1-\phi)^{\eta}\left(\frac{1}{\rho_t}-\phi\right)^{1-\eta}\right]^{-2}(1-\phi)^{\eta}(1-\eta)\left(\frac{1}{\rho_t}-\phi\right)^{-\eta}\frac{1}{(\rho_t)^2}.$$

We evaluate this at $\rho_t = 1$ to obtain $P_{11}'(1) = 1-\eta \in (0,1)$.

(v) Claim 5: $\bar{P}_{10}(\cdot)$ satisfies $\bar{P}_{10}(0) = 0$ and $\lim_{\rho\to 0}\left(\partial \bar{P}_{10}(\cdot)/\partial\rho_t\right) = \infty$; $P_{10}(\cdot)$ satisfies $P_{10}(0) = 0$ and $\lim_{\rho\to 0}(\partial P_{10}(\cdot)/\partial\rho_t) = \infty$.

We obtain $\bar{P}_{10}(0) = 0$ and $P_{10}(0) = 0$ by directly substituting $\rho_t = 0$ into $\bar{P}_{10}(\cdot)$ and $P_{10}(\cdot)$. To show $\lim_{\rho\to 0}\left(\partial\bar{P}_{10}(\cdot)/\partial\rho_t\right) = \infty$, we differentiate $\bar{P}_{10}(\cdot)$ with respect to ρ_t. After rearranging the terms, we obtain

$$\frac{\partial \bar{P}_{10}(\cdot)}{\partial \rho_t} = \frac{(1-\phi)^{\eta}(1+\gamma\eta)^{1/\gamma}(1-\eta)}{\left[\phi + (1-\phi)(1+\gamma\eta)^{1/\gamma}\left(\frac{1-\phi\rho_t}{(1-\phi)\rho_t}\right)^{1-\eta}\right]^2\cdot(\rho_t)^2\cdot\left(\frac{1-\phi\rho_t}{\rho_t}\right)^{\eta}},$$

or,

$$\begin{aligned}
\frac{\partial \bar{P}_{10}(\cdot)}{\partial \rho_t} &= (1-\phi)^{\eta}(1+\gamma\eta)^{1/\gamma}(1-\eta)\times\Big[(\phi)^2(\rho_t)^{2-\eta}(1-\phi\rho_t)^{\eta}\\
&\quad +2\phi(1-\phi)(1+\gamma\eta)^{1/\gamma}\frac{\rho_t(1-\phi\rho_t)}{1-\phi} + \left\{(1-\phi)(1+\gamma\eta)^{1/\gamma}\right\}^2\left(\frac{1}{1-\phi}\right)^{2(1-\eta)}(\rho_t)^{\eta}\Big]^{-1}.
\end{aligned}$$

Evaluating this at $\rho_t = 0$, we obtain $\lim_{\rho_t\to 0}\bar{P}_{10}'(\cdot) = +\infty$.

To show $\lim_{\rho \to 0} \left(\partial P_{10}\left(\cdot\right) / \partial \rho_t \right) = \infty$, we follow the same procedure described above. Differentiating $P_{10}\left(\cdot\right)$ with respect to ρ_t yields

$$\begin{aligned}\frac{\partial P_{10}\left(\cdot\right)}{\partial \rho_t} &= \left(\frac{\phi}{1+\gamma\eta}\right)^{\eta}\left[\phi\rho_t + \left(1-\eta\right)\left(1-\phi\rho_t\right)\right] \\ &\times \left[\left(\phi\right)^2\left(\rho_t\right)^{2-\eta} + 2\phi\left(\frac{\phi}{1+\gamma\eta}\right)^{\eta}\left(1-\phi\rho_t\right)\rho_t + \left(\frac{\phi}{1+\gamma\eta}\right)^{2\eta}\left(1-\phi\rho_t\right)^2\left(\rho_t\right)^{\eta}\right]^{-1}.\end{aligned}$$

We evaluate this at $\rho_t = 0$ and obtain

$$\lim_{\rho_t \to 0}\frac{\partial P_{10}\left(\cdot\right)}{\partial \rho_t} = \left(\frac{\phi}{1+\gamma\eta}\right)^{\eta}\left[0 + \left(1-\eta\right)\left(1-0\right)\right] \times \left(0\right)^{-1} = +\infty.$$

■

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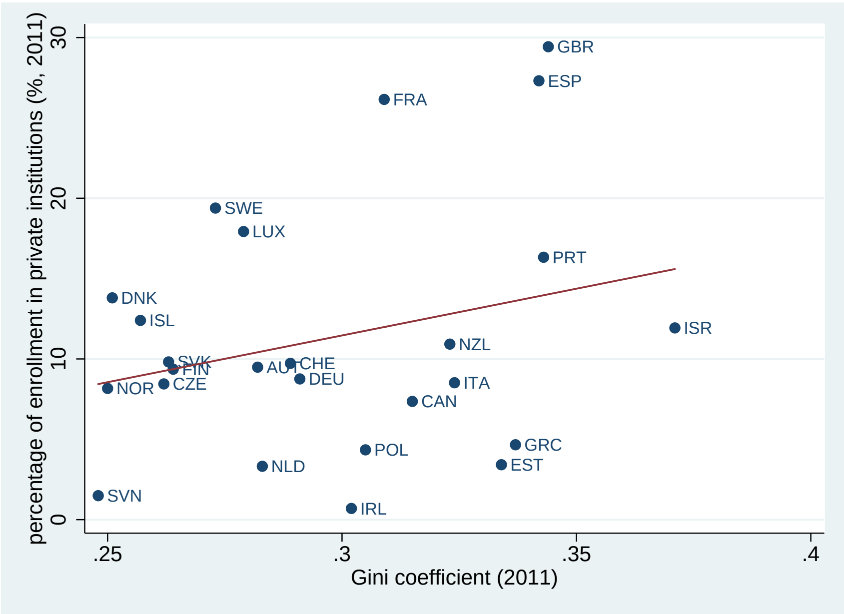

Figure 1: Gini co-efficient and enrollment (percentage) in private secondary education institutions in 2011. Source: World Bank Indicator (http://data.worldbank.org/indicator); OECD Statics (http://stats.oecd.org/).

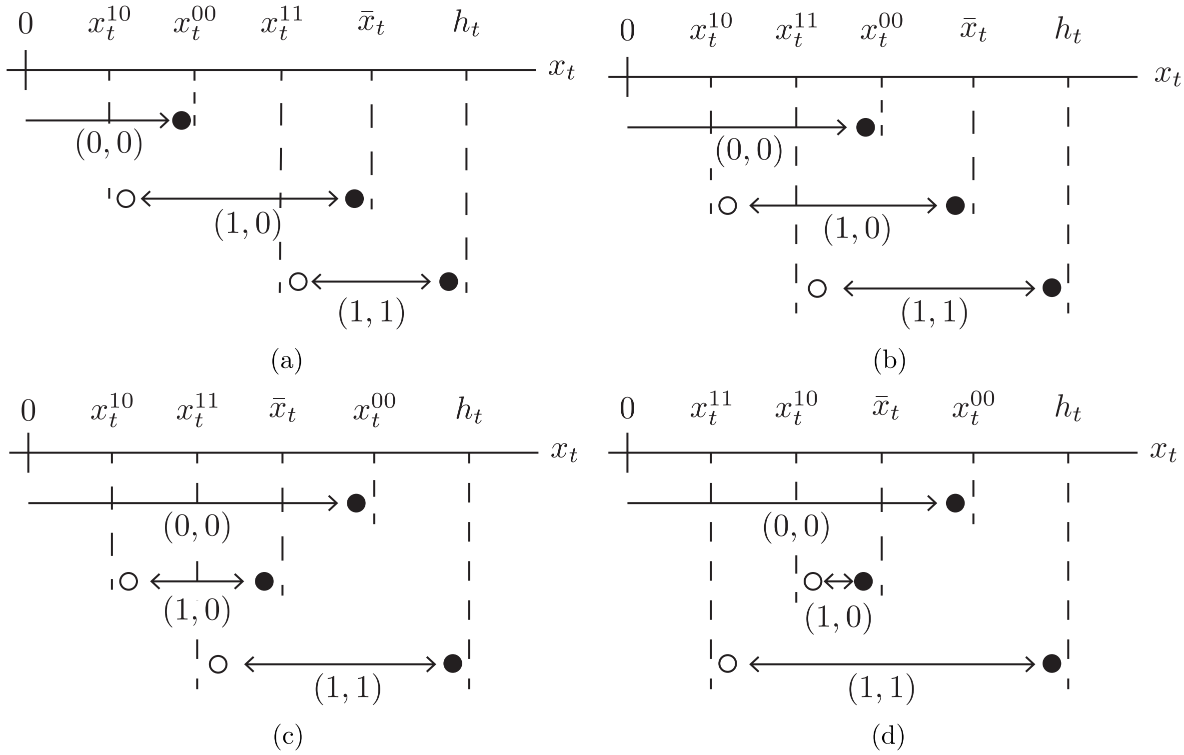

Figure 2: $\rho_t \in \left(0, \rho^l\right)$ case (Panel (a)); $\rho_t \in \left[\rho^l, \rho^m\right)$ case (Panel (b)); $\rho_t \in \left[\rho^m, \rho^h\right)$ case (Panel (c)); $\rho_t \in \left[\rho^h, 1\right]$ case (Panel (d)).

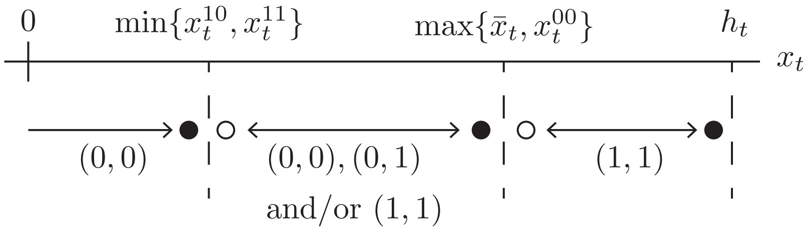

Figure 3: Education choice $\left(q_t^L, q_t^H\right)$ classified according to x_t.

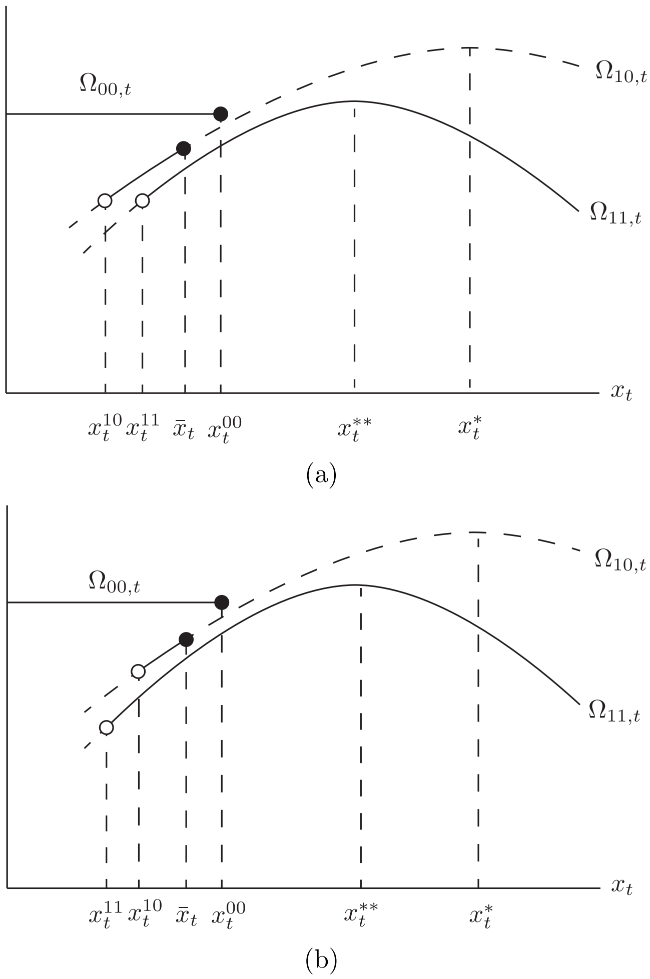

Figure 4: Illustration of the political objective function for the $\rho_t \in \left[\rho^m, \rho^h\right)$ case (Panel (a)) and the $\rho_t \in \left[\rho^h, 1\right]$ case (Panel (b)).

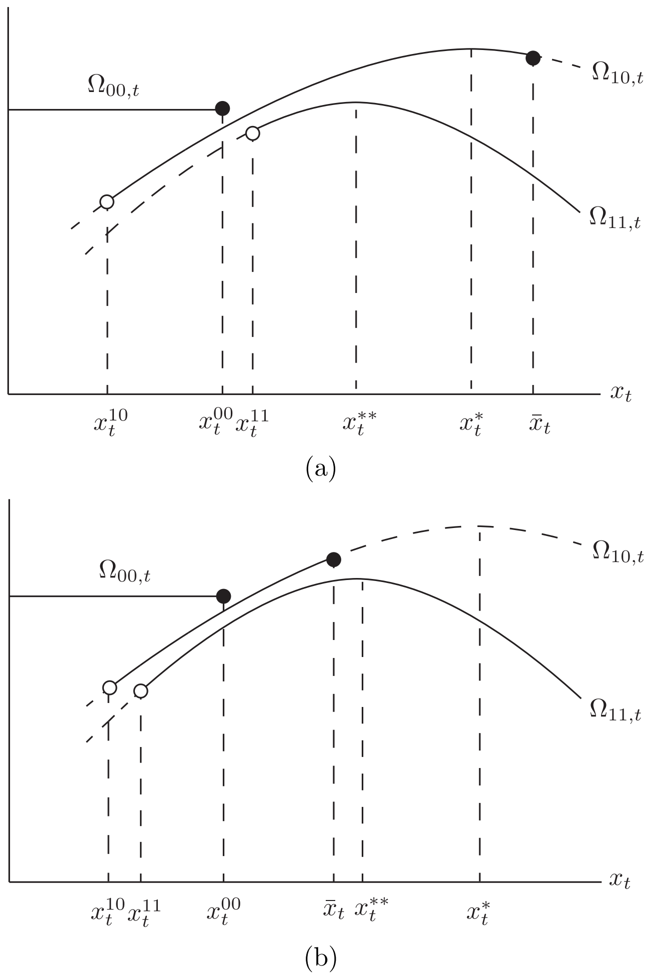

Figure 5: Illustration of the political objective function for the $\rho_t \in \left(0, \rho^l\right)$ case (Panel (a)) and the $\rho_t \in \left[\rho^l, \rho^m\right)$ case (Panel (b)).

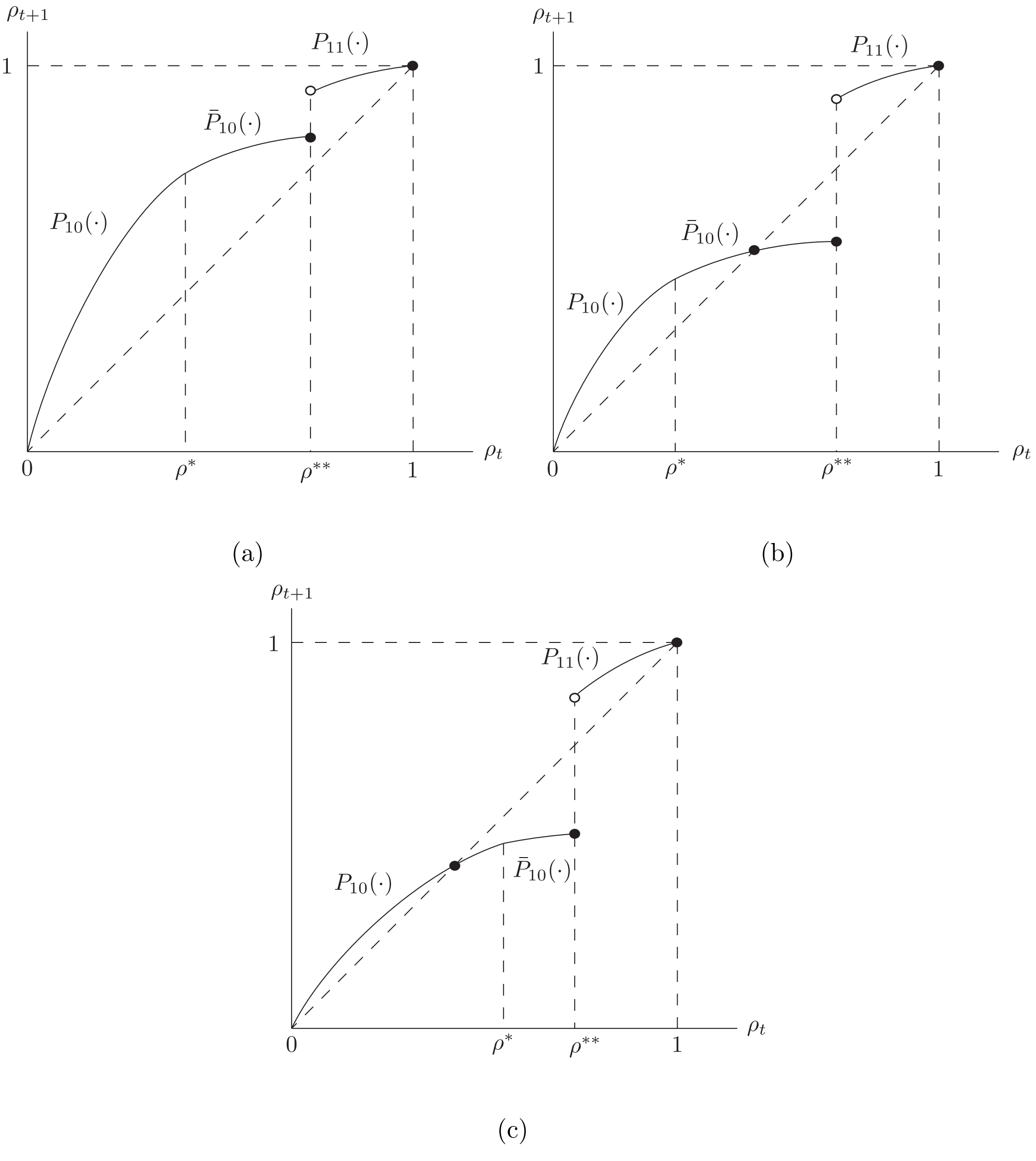

Figure 6: $\bar{P}_{10}(\rho^{**}) > \rho^{**}$ case (Panel (a)) and $\bar{P}_{10}(\rho^{**}) \leq \rho^{**}$ case (Panels (b) and (c)).

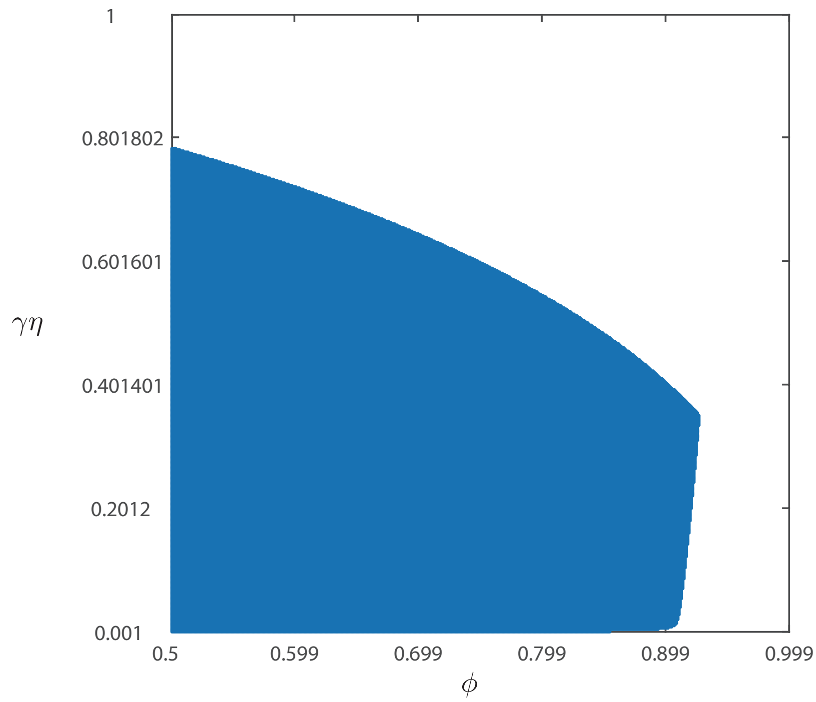

Figure 7: Multiple steady-state equilibria for the shaded area; a unique steady-state equilibrium for the non-shaded area.

(a)

Slovenia	0.248
Norway	0.25
Denmark	0.251
Iceland	0.257
Czech Republic	0.262
Slovak Republic	0.263
Finland	0.264
Belgium	0.273
Sweden	0.273
Luxembourg	0.279
Austria	0.282
Netherland	0.283
Switzerland	0.289
Germany	0.291
Ireland	0.302
Poland	0.305
France	0.309
Canada	0.315
New Zealand	0.323
Italy	0.324
Estonia	0.334
Greece	0.337
Spain	0.342
Portugal	0.343
United Kingdom	0.344
Israel	0.371

(b)

Ireland	0.69839
Slovenia	1.49191
Netherland	3.32049
Estonia	3.41722
Poland	4.34424
Greece	4.66068
Canada	7.35733
Norway	8.16549
Czech Republic	8.44604
Italy	8.52017
Germany	8.7558
Finland	9.36146
Austria	9.48843
Switzerland	9.72711
Slovak Republic	9.81426
New Zealand	10.92371
Israel	11.93098
Iceland	12.39742
Denmark	13.80195
Portugal	16.32851
Luxembourg	17.92853
Sweden	19.38773
France	26.15038
Spain	27.30868
United Kingdom	29.42805
Belgium	68.60273

Table 1: Gini coefficients (Panel (a)) and enrollment (percentage) in private secondary education institutions (Panel (b)) in selected OECD countries in 2011. Source: World Bank Indicator (http://data.worldbank.org/indicator); OECD Statics (http://stats.oecd.org/).

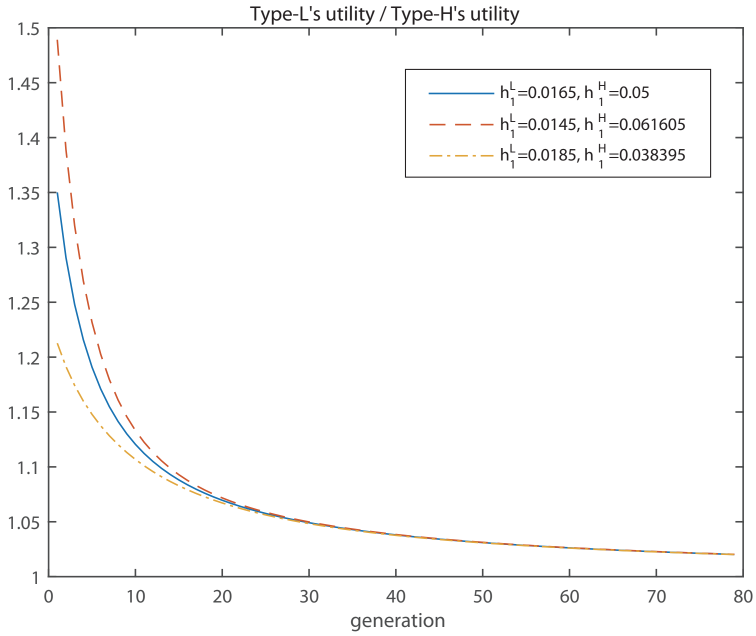

Figure 8: Ratio of type-L's utility to type-H's utility from generation 1 to 80.

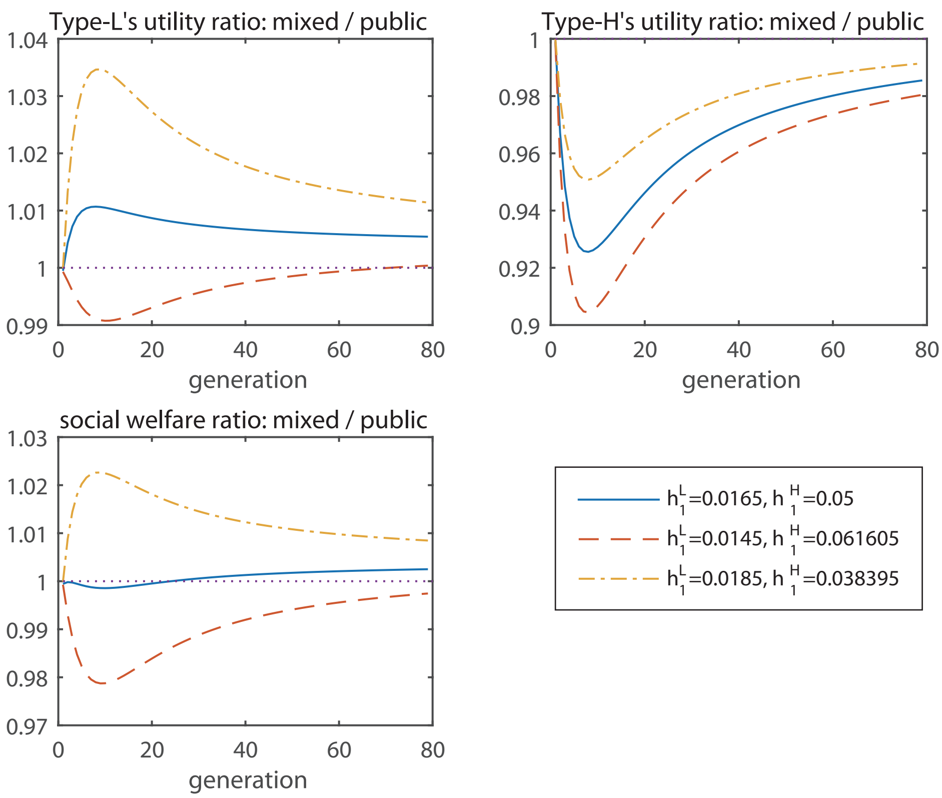

Figure 9: Ratio of mixed education systems to compulsory public school systems in terms of type-L's utility (Panel (a)), type-H's utility (Panel (b)), and social welfare (Panel (c)).

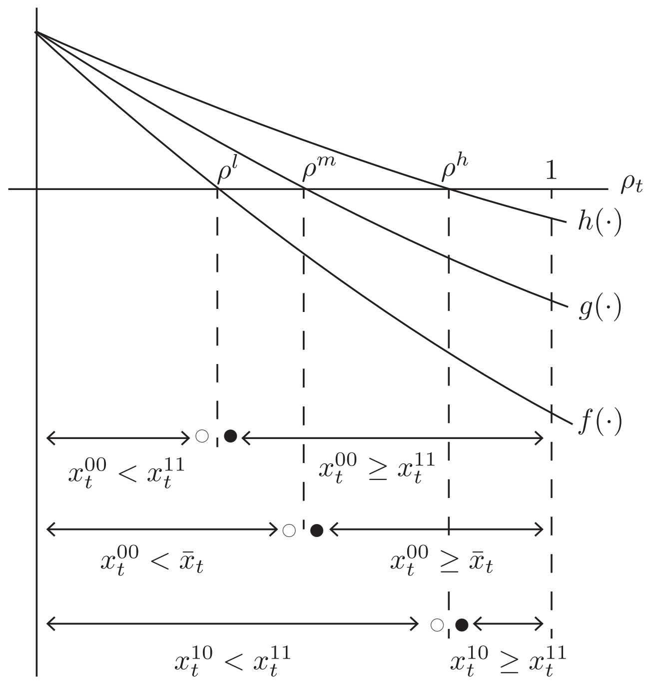

Figure A.1: Illustration for Condition (8).