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Abstract

We consider a cross-country difference of age gap in voter turnout and its impact on fiscal policymaking in a multi-country, overlapping-generations model. We present conflict over fiscal policy between successive generations (i.e., the young and elderly). We show that higher turnout of the elderly in voting may have a non-monotone effect on the size of government debt, depending on voters' inter-temporal elasticity of substitution of public expenditure.

Keywords: fiscal policy; voter turnout; public debt; probabilistic voting; small open economies.

JEL Classification: D70; E62; H63

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1 Introduction

The Organisation for Economic Co-operation and Development (OECD, 2007) reports that the voter turnout of young people is, on average, 20 points lower than that of individuals aged 65 years and over. In particular, it indicates a considerable difference in the age gap in voter turnout among OECD countries (see Table 1). This trend is also reported by Smets (2012), who surveys national election data for the following 10 countries: Canada, Denmark, Finland, Germany, Great Britain, Italy, the Netherlands, Norway, Sweden, and the United States. She defines the age gap as the turnout difference between younger (aged ≤ 35 years) and older voters (aged > 35 years) and reports that in the 2000s, the gap was around or above 20% in Canada, Finland, Norway, Great Britain, and the United States, whereas it was around or below 10% in the other five countries.

[Table 1 is here.]

The age gap in voter turnout could affect fiscal policy formation (Salavov, 2006). In particular, it should pressurize politicians to shift the fiscal burden from the present generation to future generations. This pressure incentivizes the government to finance a part of government expenditure through the issue of public bonds. This in turn may increase the debt-to-gross domestic product (GDP) ratio in the long run (Song, Storesletten, and Zilibotti, 2012). However, this prediction does not seem to fit well with observations in OECD countries. Figure 1 depicts the correlation between the age gap in voter turnout and government debt-to-GDP ratio. The age gap is measured by the percentage-point difference in voting rates between older voters (aged>55 years) and younger voters (aged 16–35 years). No remarkable correlation could be observed from the data when the two outliers, Great Britain and Japan, were omitted from the sample.

[Figure 1 is here.]

Against this background, the present study provides a simple dynamic politico-economic model that enables us to explain various relationships between the age gap in voter turnout and debt-to-GDP ratio observed in OECD countries. For the analysis, we use the model of Song, Storesletten, and Zilibotti (2012), who propose a dynamic politico-economic model of public debt. Their model assumes that the world economy consists of a unit mass of small open economies and that each country is populated by overlapping generations who live for two periods, that is, the working young and retired old, both of whom benefit from public goods provision financed by tax and public bond issues. The authors employ the probabilistic voting method developed by Lindbeck and Weibull (1987) to demonstrate fiscal policymaking. Under this voting system, the government in each period chooses fiscal policy that maximizes the weighted average utility of voters (i.e., the young and old). We extend the framework of Song, Storesletten, and Zilibotti (2012) by assuming cross-country differences in the rates of abstention from voting. In particular, we assume that voter turnout rates differ between the young and old as well as among countries.

Within this framework, we consider an increased turnout rate (i.e., a lower abstention rate) of the old. A higher turnout rate of the old is associated with a larger weight on the old in the political objective. This induces the government to respond more strongly to the demand of the retired old. Given that the retired old do not have a tax burden but benefit from public goods, the government responds to the request from the old by raising the tax rate and issuing more public bonds. Therefore, a larger age gap in voter turnout works to increase public goods provision and thus raise the tax and public debt burdens on the working young.

Higher public goods provision today makes the young prefer higher public goods in their old age because they want to smooth public goods consumption over their life cycles. The government responds to this demand of the young by restraining the issue of public bonds and compensates for the loss of revenue by raising the tax rate. This in turn lowers debt repayment costs, increases the budget in the next period, and thereby enables the government to increase public goods provision in the next period. Therefore, the higher voter turnout of the old definitely increases the tax rate, while it creates two opposing effects on public bond issues.

We show that the negative effect on public bond issues depends on the magnitude of the inter-temporal elasticity of substitution of public goods. A lower elasticity implies less demand for future public goods provision by the young. In other words, the government is less inclined to control public bond issues as elasticity decreases. Our analysis shows that there is a critical value of elasticity, such that the negative effect of the voter turnout rate of the old on public bond issues is outweighed by its positive effect when elasticity is below the critical value. In addition, the analysis shows that there is another critical value of elasticity such that the negative effect outweighs the positive effect. Furthermore, the two opposing effects balance each other at some voter turnout rate of the old when elasticity lies between the two critical values. This case could be viewed as representing the non-monotone relationship between the age gap in voter turnout and debt-to-GDP ratio.

We further analyze the effect of the age gap in voter turnout on public bond issues by focusing on the parameter representing the preferences for public goods, as in Song, Storesletten, and Zilibotti (2012). The preferences may affect the two opposing effects of the voter turnout rate of the old on public bond issues in the following ways. First, greater preferences for public goods increase the weight on the old's utility of public goods, thereby strengthening the positive effect on public bond issues. Second, greater preferences for public goods increase the weights on the young's utility of public goods in their youth and old age. However, greater preferences for public goods do not affect the negative effect on public bond issues because the weights on the two periods of life increase to the same degree. Therefore, the positive effect on public bond issues becomes larger as the preferences for public goods increase.

Our results contribute to the literature in the following two ways. First, the present study extends the model of Song, Storesletten, and Zilibotti (2012), who examine the relationship between the political power of the old and debt-to-GDP-ratio. They show that an increase in the relative political power of older agents, caused mainly by population aging, leads to a higher steady-state level of public debt. Their model prediction, however, does not seem to fit well with the observation in OECD countries. The present study instead focuses on the age gap in voter turnout and shows various relationships between this age gap and the size of government debt, which may fit well with the evidence in OECD countries.[1]

Second, the present study is related to the political economy of population aging and government expenditure. As the population ages, the median voter becomes older and hence is more willing to support larger government expenditure. Several studies support this view from the political economy perspective (e.g., Gradstein and Kaganovich, 2004; Bassetto, 2008; Gonzalez-Eiras and Niepelt, 2008, 2012). However, an effect in the opposite direction could be expected because aging reduces the willingness of the working generation to accept larger government expenditure (Razin, Sadka, and Swagel, 2002; Galasso and Profeta, 2007). These studies assume a balanced government budget, and thus leave the issue of government debt unresolved. The present study focuses on government debt rather than government expenditure as a measure of government size and shows a possible negative effect of the age gap in voter turnout on the size of government.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 introduces the political mechanism of fiscal policy formation and demonstrates fiscal policy in voting. Section 4 characterizes a steady-state equilibrium and shows how the cross-country debt distribution is affected by the age gap in voter turnout, preferences for public goods, and inter-temporal elasticity of substitution of public goods. Section 5 provides concluding remarks.

2 Model

Our model is based on that developed by Song, Storesletten, and Zilibotti (2012). Time is discrete and is denoted by $t = 0,\ 1,\ 2, \cdots$. The world economy consists of a unit mass of small open countries. Each country is populated by overlapping generations of agents

[1]The present study is also related to Röhrs (2016), who focuses on the absence of commitment in voting as a source of inefficiency in a closed economy. However, it differs from hers in that we show the importance of the age gap in voter turnout for explaining the cross-country difference in fiscal policy.

who live for two periods: they work in the first period and retire in the second period. There is no population growth and each generation has a unit mass. We assume that countries are identical except for voter turnout rates in order to shed light on their role in shaping fiscal policy.

2.1 Utility Maximization

Each agent is assumed to receive utility from private consumption and publicly provided goods. The utility of a type-i young agent in country $j = [0, 1]$ born in period t is

$$U^y_{j,t} = \frac{(c^y_{j,t})^{1-\sigma} - 1}{1-\sigma} + \theta \cdot \frac{(g_{j,t})^{1-\sigma} - 1}{1-\sigma} + \beta \cdot \left\{ \frac{(c^o_{j,t+1})^{1-\sigma} - 1}{1-\sigma} + \theta\lambda \cdot \frac{(g_{j,t+1})^{1-\sigma} - 1}{1-\sigma} \right\},$$

where $c^y_{j,t}$ is consumption during youth, $c^o_{j,t+1}$ is consumption during old age, $g_{j,t}$ is public goods provision in period t, $\beta \in (0, 1]$ is a discount factor, and $\theta (> 0)$ and $\theta\lambda (> 0)$ capture the preference weights on public goods for the young and old, respectively. In particular, the parameter λ captures the relative strength of the preference for public goods of the old agents. The parameter $\sigma (> 0)$ is an inverse of the inter-temporal elasticity of substitution. A higher σ implies a lower elasticity (de la Croix and Michel, 2002, p.11).

Each young agent supplies one unit of labor inelastically and earns exogenous wage w. The individual budget constraints of type-i agents during youth and old age are given by

$$c^y_{j,t} + s_{j,t} \leq (1 - \tau_{j,t}) \cdot w,$$
$$c^o_{j,t+1} \leq R s_{j,t},$$

where $s_{j,t}$ represents savings, $\tau_{j,t}$ is an income tax rate in period t, and R is the (endogenous) world interest rate. Following Song, Storesletten, and Zilibotti (2012), we focus on stationary equilibria and thus, we characterize an allocation of each country as a function of a constant R.

We solve the utility-maximization problem and obtain the following consumption and savings functions:

$$\begin{aligned}
c^y_{j,t} &= \frac{1}{1+\beta\left(\beta R\right)^{(1-\sigma)/\sigma}} \cdot w \cdot \left(1-\tau_{j,t}\right), \\
c^o_{j,t+1} &= \frac{\beta\left(\beta R\right)^{(1-\sigma)/\sigma}}{1+\beta\left(\beta R\right)^{(1-\sigma)/\sigma}} \cdot R \cdot w \cdot \left(1-\tau_{j,t}\right), \\
s_{j,t} &= \frac{\beta\left(\beta R\right)^{(1-\sigma)/\sigma}}{1+\beta\left(\beta R\right)^{(1-\sigma)/\sigma}} \cdot w \cdot \left(1-\tau_{j,t}\right).
\end{aligned}$$

Ignoring irrelevant terms, we can express the indirect utility function of a type-i young agent as follows:

$$\begin{aligned}
V^y_{j,t} = V^y\left(\tau_{j,t}, g_{j,t}, g_{j,t+1}\right) \equiv \frac{1}{1-\sigma} \cdot \bigg[&\left\{1+\beta\left(\beta R\right)^{(1-\sigma)/\sigma}\right\}^{\sigma} \cdot \left\{w \cdot \left(1-\tau_{j,t}\right)\right\}^{1-\sigma} \\
&+\theta \cdot \left(g_{j,t}\right)^{1-\sigma} + \beta\lambda\theta \cdot \left(g_{j,t+1}\right)^{1-\sigma}\bigg].
\end{aligned} \tag{1}$$

The first term in the square brackets on the right-hand side denotes the utility of consumption during youth and old age, the second term denotes the utility of public goods during youth, and the third term denotes the utility of public goods during old age.

The indirect utility function of old agents in period t is expressed as

$$V^o_{j,t} = V^o\left(g_{j,t}\right) \equiv \frac{1}{1-\sigma} \cdot \lambda\theta \cdot \left(g_{j,t}\right)^{1-\sigma}, \tag{2}$$

where the irrelevant terms are omitted from the expression. Old agents each have the same indirect utility function regardless of their type because their savings during youth are predetermined and an individual's utility function is assumed to be additively separable.

2.2 Government Budget Constraint

Government bonds are traded in international asset markets. Given inherited debt for young agents, $b_{j,t}$, the government of country j in period t chooses an income tax rate, $\tau_{j,t}$, public goods expenditure, $g_{j,t}$, and a new issue of government bonds, $b_{j,t+1}$, subject to the following government budget constraint:

$$b_{j,t+1} = g_{j,t} + Rb_{j,t} - \tau_{j,t}w. \tag{3}$$

where the revenue shortfall, $g_{j,t}+Rb_{j,t}-\tau_{j,t}w$, could be covered by issuing new government bonds, $b_{j,t+1}$.

Governments are committed to not repudiating debt. Thus, sovereign debt cannot exceed the present value of the maximum feasible tax revenue (i.e., the natural debt limit). In an environment with a constant interest rate and exogenous wages, the tax revenue in period t is maximized at $\tau_{j,t} = 1$. Therefore, the natural debt limit, denoted by $\bar{b}$, is identical across countries, and is given by

$$b_{j,t+1} \leq \sum_{s=1}^{\infty} \frac{w}{R^s} = \frac{w}{R-1} \equiv \bar{b} \ \text{ for all } j.$$

Hereafter, we omit time index t and use recursive notation with x' denoting next period x.

3 Politico-Economic Equilibrium

This section introduces the political mechanism of fiscal policy formation in the presence of an age gap in voter turnout.

3.1 Probabilistic Voting with Abstention

The political mechanism in the present model is based on the probabilistic voting developed by Lindbeck and Weibull (1987). In this voting scheme, there is electoral competition between two office-seeking candidates, denoted by, for example, A and B. Each candidate announces a set of fiscal policies subject to the government budget constraint and the natural debt limit to maximize his/her probability of winning the election. As we demonstrate in Appendix A.1, the two candidates' platforms converge in equilibrium to the same fiscal policy that maximizes the weighted-average utility of voters. Formally, the political objective function in country j is given by

$$\Omega(\tau_j, g_j, b_j'; \omega(p_j), R) = \omega_j V^o(g_j) + (1 - \omega_j) V^y(\tau_j, g_j, b_j'),$$

where ω_j presents a relative political weight on the old in country j.

The present study differs from standard probabilistic voting in that some individuals abstain from voting. In particular, voter turnout rates differ between the young and old. For example, among the young's (old's) votes in country j, proportion q_j^y (q_j^o) participates in voting, while proportion $1 - q_j^y$ ($1 - q_j^o$) abstains from voting. Therefore, the relative political weight of the old, denoted by ω_j, is specified as

$$\omega_j \equiv \frac{q_j^o}{q_j^o + q_j^y}, \text{ i.e., } \omega(p_j) \equiv \frac{p_j}{1 + p_j},$$

where $p_j \equiv q_j^o / q_j^y$ denotes a relative turnout of the old. A higher relative turnout of the old implies a larger relative political weight on them. The weights on the young and old are equal if there is no abstention.[2]

The current fiscal policy affects the future fiscal policy through its effect on debt accumulation. Each candidate takes this effect into account when shaping his/her fiscal policy. To capture such an inter-temporal effect, we here use the concept of stationary Markov-perfect equilibrium (SMPE) described by Song, Storesletten, and Zilibotti (2012), which is defined as follows:

Definition 1. *An SMPE comprises an interest rate R, a stationary debt distribution $\{b_j\}_j$, a debt rule $b_j' = B(b_j; \omega(p_j), R)$, a government expenditure rule $g_j = G(b_j; \omega(p_j), R)$, and a tax rule $\tau_j = T(b_j; \omega(p_j), R)$, such that the following two conditions hold:*

[2] One way to endogenize the voter turnout rates is to assume a rational voting choice (see, for example, Dhillon and Peralta, 2002). However, the present study assumes exogenous voter turnout rates because the focus is on the effect of the difference in the voter turnout rates on fiscal policy rather than the mechanism behind the difference.

(i) $\langle B(b_j;\omega(p_j),R), G(b_j;\omega(p_j),R), T(b_j;\omega(p_j),R)\rangle = \arg\max_{\{\tau_j,g_j,b'_j\}}$ $\Omega(\tau_j,g_j,b'_j;\omega(p_j),R)$,*and the government's budget constraint and natural debt limit are satisfied:*

$$\begin{aligned} B(b_j;\omega(p_j),w,R) &= G(b_j;\omega(p_j),R) + Rb_j - T(b_j;\omega(p_j),R)w, \\ B(b_j;\omega(p_j),R) &\leq \frac{w}{R-1} \equiv \bar{b}. \end{aligned}$$

(ii) *the world asset market clears,*

$$\int_j s_j dj = \int_j b'_j dj,$$

where $b'_j = B(b_j;\omega(p_j),R)$ *and* $s_j = \frac{\beta(\beta R)^{\frac{1-\sigma}{\sigma}}}{1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}} \cdot w \cdot (1-\tau_j)$.

The timing of events is as follows. (i) The two candidates, A and B, simultaneously and non-cooperatively announce their electoral platforms. (ii) The election is held. (iii) The elected candidate implements his/her announced policy platform. (iv) Given the policy platform, each individual sets savings and consumption from the viewpoint of utility maximization. (v) Finally, international financial markets clear.

3.2 Voting on Fiscal Policy

Hereafter, we take the market-clearing interest rate as given for a while, and we solve the model by backward induction. Since we have solved the utility-maximization problem already, we are now ready to find the equilibrium fiscal policy. Given R, the first-order conditions with respect to g_j and τ_j are

$$(g_j)^{-\sigma} = -f(\omega(p_j)) \cdot (g'_j)^{-\sigma} \cdot \frac{\partial G(b'_j;\omega(p_j),w,R)}{\partial b'_j}, \tag{4}$$

$$\frac{1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}}{1-\tau_j} = \left(\frac{\theta\beta\lambda}{f(\omega(p_j))}\right)^{\frac{1}{\sigma}} \cdot w \cdot \frac{1}{g_j}, \tag{5}$$

where

$$f(\omega(p_j)) \equiv \frac{(1-\omega(p_j))\beta\lambda}{(1-\omega(p_j))+\omega(p_j)\lambda}, \tag{6}$$

and $g_j = G(b_j;\omega(p_j),w,R)$, $g_j' = G(b_j';\omega(p_j),w,R)$, $\tau_j = T(b_j;\omega(p_j),w,R)$, and $b_j' = g_j + Rb_j - \tau_j w \equiv B(b_j;\omega(p_j),w,R)$. The derivation of (4) and (5) is given in Appendix A.2.

Condition (4) is a generalized Euler equation for public goods provision. The government chooses public goods provision to equate the marginal benefit on the left-hand side to the marginal cost on the right-hand side. The term $\partial G(b_j';\omega(p_j),w,R)/\partial b_j'$ on the right-hand side captures the *disciplining effect* exercised by the young voters; this is qualitatively similar to that demonstrated in Song, Storesletten, and Zilibotti (2012). This effect implies that the young agents' concern for future public goods provision restrains government expenditure and thus, prevents the government from running up too much public bond issues. In particular, the term $f(\omega(p_j))$ on the right-hand side of (4) indicates that the disciplining effect decreases as the political power of the old increases.

Condition (5) states that the government chooses the tax rate to equate the marginal cost of taxation to its marginal benefit. A higher tax rate lowers the consumption of the young. The left-hand side of (5) presents this negative effect of taxation. On the other hand, a higher tax rate increases tax revenue. This incentivizes the government to reduce current public bond issues, which, in turn, increases future public goods provision. The right-hand side of (5) represents this positive effect of taxation. We should note that, as observed on the right-hand side, the relative turnout of the old, p_j, affects the shape of the fiscal policy through the decision on the tax rate. This point is investigated further in the following analysis.

Based on the guess-and-verify method, we find policy functions that satisfy conditions (4) and (5), as demonstrated in the following lemma.

Lemma 1. *Given R and b_j, country j's policy functions in SMPE are given by*

$$
\begin{aligned}
G(b_j;\omega(p_j),R) &= \gamma_j^* \cdot (\bar{b} - b_j), \\
T(b_j;\omega(p_j),R) &= 1 - \left\{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right\} \cdot \left\{\frac{f(\omega(p_j))}{\beta\lambda\theta}\right\}^{\frac{1}{\sigma}} \cdot \frac{1}{w} \cdot \gamma_j^* \cdot (\bar{b} - b_j), \\
B(b_j;\omega(p_j),R) &= \bar{b} - \left\{f(\omega(p_j))\gamma_j^*\right\}^{\frac{1}{\sigma}} \cdot (\bar{b} - b_j),
\end{aligned}
$$

where $\gamma_j^* (> 0)$ satisfies following condition:

$$
R - \gamma_j^* \cdot \left[1 + \left\{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right\} \cdot \left\{\frac{f(\omega(p_j))}{\beta\lambda\theta}\right\}^{\frac{1}{\sigma}}\right] = \left\{f(\omega(p_j))\gamma_j^*\right\}^{\frac{1}{\sigma}}. \tag{7}
$$

Proof. See Appendix A.3.

The government expenditure on public goods is linearly related to the stock of public debt. In particular, higher public debt is associated with lower public goods provision, because debt accumulation increases debt repayment costs and thus, reduces the available resources for the government. To compensate for this loss, the government increases the tax rate and public bond issues. This incentivizes the future government to issue more public bonds to repay the increased debt. This might result in debt accumulation up to the natural debt limit. We formally investigate this possibility in the next section.

4 Steady State

Here, we focus on a steady-state equilibrium in which the relevant terms are constant across periods, and we assume it is stable. Our task is to determine the world interest rate that satisfies Definition 1(ii) and the associated debt distribution that satisfies Definition 1(i). However, an analytical solution is not available, except for the case of $1/\sigma = 2$, as demonstrated in the next subsection. Therefore, we employ the following analysis strategy. First, in Subsection 4.1, we pin down the world interest rate, R, for a given world distribution of debt. Next, in Subsection 4.2, we determine the world distribution of debt for a given R. Finally, in Subsection 4.3, we solve for R and the debt distribution simultaneously.

4.1 World Interest Rate

For the purpose of analysis, we rewrite the law of motion of debt in Lemma 1 as follows:

$$\bar{b} - b'_j = \phi^*_j \cdot \left(\bar{b} - b_j\right),$$

where

$$\phi^*_j = \phi^*\left(\omega(p_j), R\right) \equiv \left\{f(\omega(p_j))\gamma^*_j\right\}^{\frac{1}{\sigma}}, \quad j \in [0,1]. \tag{8}$$

We define $\bar{\phi}$ as

$$\bar{\phi} \equiv \max\left\{\phi^*\left(\omega(p_j), R\right)\right\}_{j\in[0,1]}.$$

Same as the model in the main text, the steady-state equilibrium interest rate, R^*, satisfies $\bar{\phi} = 1$.Substituting $\bar{\phi} = 1$ into (7) enables us to express the condition that determines the equilibrium interest rate as follows:

$$R^* = 1 + \frac{1}{f(\omega(p_{j^*}))} \cdot \left[1 + \left\{1 + \beta(\beta R^*)^{\frac{1-\sigma}{\sigma}}\right\} \cdot \left\{\frac{f(\omega(p_{j^*}))}{\beta\lambda\theta}\right\}^{\frac{1}{\sigma}}\right], \tag{9}$$

where j^* denotes a country that accumulates debt below $\bar{b}$ and satisfies $\bar{\phi} = 1$. Other countries accumulate debt up to $\bar{b}$ and attain $\bar{\phi} < 1$. We should note here that we take the distribution of debt as given, and thus, country j^* is not specified at this moment.

The following lemma establishes the condition for the existence and uniqueness of the steady-state equilibrium interest rate, R^*, for a given distribution of debt in the world economy.

Lemma 2. *Given the distribution of debt, there is a unique steady-state equilibrium interest rate, $R^* \geq 1$, satisfying (9) if $1/\sigma \in (0,2)$ or if $1/\sigma = 2$ and $\theta > f\left(\omega_{j_*}\right)^{1/2}/\lambda$.*

Proof. See Appendix A.4.

The assumption about $1/\sigma$ is supported by the empirical studies of Kydland and Prescott (1982), Hall (1988), Campbell and Mankiw (1989), Browning, Hansen, and Heckman (1999), and Campbell (1999), who report $1/\sigma \in (0,1]$, and those of Hansen and Singleton (1982), Attanasio and Weber (1989), and Vissing-Jorgensen and Attanasio (2003), who report $1/\sigma \in (1,2]$. Following these estimations, hereafter, we assume $1/\sigma \in (0,2]$, which is also employed by Song, Storesletten and Zilibotti (2012). While this assumption narrows the scope of our model, it enables us to solve the model in a tractable way. When $\sigma = 1/2$, we add the assumption of $\theta > f\left(\omega_{j^*}\right)^{1/2}/\lambda$ to ensure that the gross interest rate R^* lies within the range $(1,\infty)$.

4.2 International Differences in Voter Turnout and Public Debt Distribution

The policy function $B(b_j;\omega(p_j),R)$ in Lemma 1 suggests that the relative political weight on the old has an effect on the public debt distribution through ϕ_j^*. To isolate its effect from the interest rate effect, we take R as given, and investigate the effect of $\omega(p_j)$ on ϕ_j^*.

Lemma 3. *Given R, the following result holds:*

$$\begin{cases} \frac{\partial \phi_j^*}{\partial \omega(p_j)} \leq 0 & \text{if } \Psi(R,\omega(p_j)) \geq \frac{1}{\sigma} - 1, \\ \frac{\partial \phi_j^*}{\partial \omega(p_j)} > 0 & \text{otherwise}, \end{cases}$$

where

$$\Psi(R,\omega(p_j)) \equiv \frac{1}{1+\beta\left(\beta R\right)^{(1-\sigma)/\sigma}} \cdot \left\{ \frac{\beta\lambda\theta}{f\left(\omega(p_j)\right)} \right\}^{1/\sigma} > 0.$$

Proof. See Appendix A.5.

[Figure 2 is here.]

Let ω_{max} and ω_{min} denote the maximum and minimum values of $\omega(p_j)$ ($0 < \omega_{min} < \omega_{max} < 1$), respectively. Figure 2 illustrates the relationship between $\omega(p_j)$ and ϕ_j within the range $[\omega_{min}, \omega_{max}]$. In particular, panel (a) illustrates the case of $1/\sigma - 1 \leq \Psi(R, \omega_{min})$

where $\partial \phi_j / \partial \omega(p_j) < 0$ holds for any $\omega(p_j) \in [\omega_{min},\ \omega_{max}]$. This case implies that countries with the lowest $\omega(p_j)$ ($\omega(p_j) = \omega_{min}$) have the highest ϕ_j ($\phi_j = \bar{\phi} = 1$). Panel (c) demonstrates the case of $\Psi(R, \omega_{max}) \leq 1/\sigma - 1$ where $\partial \phi_j / \partial \omega_j > 0$ holds for any $\omega(p_j) \in [\omega_{min},\ \omega_{max}]$. In this case, countries with the highest $\omega(p_j)$ ($\omega(p_j) = \omega_{max}$) have the highest ϕ_j ($\phi_j = \bar{\phi} = 1$).

Panel (b) depicts the intermediate case between the cases of panels (a) and (c), where $\Psi(R, \omega_{min}) < 1/\sigma - 1 < \Psi(R, \omega_{max})$. There is a threshold level of $\omega(p_j)$ denoted by $\widetilde{\omega}(R)$, which satisfies

$$\begin{aligned} &\Psi(R, \widetilde{\omega}(R)) = \frac{1}{\sigma} - 1 \qquad (10) \\ \Leftrightarrow\ &\widetilde{\omega}(R) = \frac{1 - \frac{1}{\theta} \cdot \left[\frac{1-\sigma}{\sigma} \cdot \left\{1 + \beta\left(\beta R\right)^{(1-\sigma)/\sigma}\right\}\right]^{\sigma}}{1 - \lambda - \frac{1}{\theta} \cdot \left[\frac{1-\sigma}{\sigma} \cdot \left\{1 + \beta\left(\beta R\right)^{(1-\sigma)/\sigma}\right\}\right]^{\sigma}} \in (\omega_{min},\ \omega_{max}) \end{aligned}$$

In this case, $\partial \phi_j / \partial \omega_j \geq 0$ holds when $\omega(p_j) \in [\omega_{min},\ \widetilde{\omega}(R)]$, whereas $\partial \phi_j / \partial \omega_j < 0$ holds when $\omega(p_j) \in (\widetilde{\omega}(R),\ \omega_{max}]$. Therefore, countries with $\omega(p_j) = \widetilde{\omega}(R)$ have the highest ϕ_j where $\phi_j = \bar{\phi} = 1$.

The result described so far is summarized in the following proposition.

Proposition 1. *Given the world interest rate, R, the steady-state distribution of public debt is as follows.*

- **(i)** *When $1/\sigma - 1 \leq \Psi(R, \omega_{min})$, countries with $\omega(p_j) = \omega_{min}$ accumulate public debt below $\bar{b}$, whereas other countries accumulate public debt up to $\bar{b}$.*
- **(ii)** *When $\Psi(R, \omega_{min}) < 1/\sigma - 1 < \Psi(R, \omega_{max})$, countries with $\omega(p_j) = \widetilde{\omega}(R) \in (\omega_{min},\ \omega_{max})$ accumulate public debt below $\bar{b}$, whereas other countries accumulate public debt up to $\bar{b}$.*
- **(iii)** *When $\Psi(R, \omega_{max}) \leq 1/\sigma - 1$, countries with $\omega(p_j) = \omega_{max}$ accumulate public debt below $\bar{b}$, whereas other countries accumulate public debt up to $\bar{b}$.*

The result in Proposition 1 suggests that the relative turnout of the old, p_j, and the inter-temporal elasticity of substitution, $1/\sigma$, affect the steady-state distribution of public debt in the world economy. To understand their impacts, we first consider the role of the relative voter turnout of the old in shaping fiscal policy. Higher turnout of the old increases their political influence in policymaking. This in turn incentivizes the government to respond more strongly to the request from the old. Because the current public goods provision is the only concern for the old, the government increases it by raising the tax rate as well as by increasing public bond issues.

However, higher public goods provision today makes the young prefer higher public goods in the future because they want to smooth the utility of public goods across periods from the viewpoint of utility maximization. In addition, the government responds to this preference of the young by restraining the issue of public bonds, and compensates for the loss of the revenue by raising the tax rate. Therefore, higher voter turnout of the old definitely increases the tax rate, while it creates two opposing effects on public bond issues.

The negative effect on public bond issues depends on the magnitude of the inter-temporal elasticity of substitution, represented by $1/\sigma$. A lower $1/\sigma$ implies a lower inter-temporal elasticity of substitution, thereby providing less incentive for the government to control public bond issues. In other words, the negative effect becomes less effective as the inter-temporal elasticity of substitution decreases. In particular, when $1/\sigma$ is low such that $1/\sigma - 1 \leq \Psi\left(R, \omega_{\min}\right)$, the negative effect is outweighed by the positive effect. As a result, we obtain an equilibrium in which all the countries except the country with the lowest voter turnout of the old accumulate debt up to the natural debt limit, as we describe in Proposition 1(i) (see panel (a) of Figure 2).

The result is reversed when $1/\sigma$ is high such that $1/\sigma - 1 \geq \Psi\left(R, \omega_{\max}\right)$ (see panel (c) of Figure 2). In this case, the negative effect outweighs the positive effect because of the high inter-temporal elasticity of substitution. As a result, we obtain an equilibrium in which all countries except the country with the highest voter turnout of the old accumulate debt up to the natural debt limit (Proposition 1(iii)). Finally, when the elasticity lies between$\Psi(R, \omega_{\min})$ and $\Psi(R, \omega_{\max})$, there is a critical value of p_j, defined by $\omega(p_j) = \tilde{\omega}(R)$ such that the two opposing effects balance each other. Below (above) the critical value, the positive effect outweighs (is outweighed by) the negative effect, thereby resulting in an equilibrium in which all the countries except the country with $\omega(p_j) = \tilde{\omega}(R)$ accumulates debt up to the limit (see panel (b) of Figure 2).

4.3 Stationary Markov-perfect Equilibrium

Based on the result established thus far, we are now ready to demonstrate the SMPE. We first solve the model analytically for the case of $1/\sigma = 2$, and then, we solve it numerically for the case of $1/\sigma \in (0, 2)$.

We first consider the case of $1/\sigma = 2$. For the characterization of the equilibrium, we focus on θ, representing the preference of public goods, and introduce the following two threshold values of θ :

$$\begin{aligned}
\underline{\theta} &\equiv \frac{1}{\lambda} \cdot \left[2f(\omega_{\max}) + \left(\left(1+\beta^2\right)/\beta^2\right) \cdot \left(f(\omega_{\max})\right)^2\right]^{1/2}, \\
\overline{\theta} &\equiv \frac{1}{\lambda} \cdot \left[2f(\omega_{\min}) + \left(\left(1+\beta^2\right)/\beta^2\right) \cdot \left(f(\omega_{\min})\right)^2\right]^{1/2},
\end{aligned}$$

where $\underline{\theta} < \overline{\theta}$ holds. Condition (9) yields the following world interest rate when $1/\sigma = 2$:

$$R^* = 1 + \frac{1 + (1+\beta^2) \cdot \left\{\frac{f(\omega(p_j))}{\beta\lambda\theta}\right\}^2}{f(\omega(p_j)) - \beta^2 \cdot \left\{\frac{f(\omega(p_j))}{\beta\lambda\theta}\right\}^2}.$$

Accompanied by the result in Proposition 1, this equilibrium interest rate leads to the following result:

Proposition 2. *Suppose that $1/\sigma = 2$ and $\theta > f(\omega_{max})^{1/2}/\lambda$ hold. The world distribu-*

tion of debt is

$$\begin{cases} b_j < \bar{b} \text{ for } \omega_j = \omega_{\min} \text{ and } b_j = \bar{b} \text{ for } \omega_j \neq \omega_{\min} & \text{if} \quad \theta \in \left[\overline{\theta}, \infty\right), \\ b_j < \bar{b} \text{ for } \omega_j = \widetilde{\omega}(R^*) \text{ and } b_j = \bar{b} \text{ for } \omega_j \neq \widetilde{\omega}(R^*) & \text{if} \quad \theta \in \left(\underline{\theta}, \overline{\theta}\right), \\ b_j < \bar{b} \text{ for } \omega_j = \omega_{\max} \text{ and } b_j = \bar{b} \text{ for } \omega_j \neq \omega_{\max} & \text{if} \quad \theta \in \left(f(\omega_{max})^{1/2}/\lambda, \underline{\theta}\right], \end{cases}$$

where $\widetilde{\omega}(R^*)$ satisfies

$$\widetilde{\omega}(R^*) = \frac{1 - \frac{\beta + \sqrt{\beta^2 + \theta^2\lambda^2(1+\beta^2)}}{\theta^2\lambda}}{1 - \lambda - \frac{\beta + \sqrt{\beta^2 + \theta^2\lambda^2(1+\beta^2)}}{\theta^2\lambda}}.$$

The corresponding world interest rate is

$$R^* = \begin{cases} 1 + \frac{1 + \left(1+\beta^2\right) \cdot \{f(\omega_{\min})/\beta\lambda\theta\}^2}{f(\omega_{\min}) - \beta^2 \cdot \{f(\omega_{\min})/\beta\lambda\theta\}^2} & \text{if} \quad \theta \in \left[\overline{\theta}, \infty\right), \\ 1 + 2 \cdot \frac{1 + \left\{1 + \theta^2\lambda^2(1+\beta^2)/\beta^2\right\}^{1/2}}{\theta^2\lambda^2} & \text{if} \quad \theta \in \left(\underline{\theta}, \overline{\theta}\right), \\ 1 + \frac{1 + \left(1+\beta^2\right) \cdot \{f(\omega_{\max})/\beta\lambda\theta\}^2}{f(\omega_{\max}) - \beta^2 \cdot \{f(\omega_{\max})/\beta\lambda\theta\}^2} & \text{if} \quad \theta \in \left(f(\omega_{max})^{1/2}/\lambda, \underline{\theta}\right]. \end{cases}$$

Proof. See Appendix A.6.

The result in Proposition 2 suggests that the parameter θ, representing the preferences for public goods, also affects the world distribution of debt. To understand the effect of θ in detail, recall the two opposing effects of the relative voter turnout of the old, ω, on public bond issues, which are described in the previous section. The preferences for public goods might or might not affect these two opposing effects of ω in the following ways. First, a larger θ increases the weight on the old's utility of public goods, thereby strengthening the positive effect of ω on public bond issues. Second, a larger θ increases the weights on the young's utility of public goods in their youth and old age. Because the weights on the two periods of life increase to the same degree, an increase in θ does not affect the young's incentive for consumption smoothing behavior. This implies that the negative effect of ω is not affected by an increase in θ.

Consequently, the positive effect of ω increases as θ increases. In particular, when θ is high such that $\theta \geq \overline{\theta}$, the positive effect of ω outweighs its negative effect. As a result, we obtain an equilibrium in which all countries except the country with the lowest voter turnout of the old, $\omega_j = \omega_{\min}$, accumulate public debt up to the natural debt limit. The result is reversed when θ is low such that $\theta \leq \underline{\theta}$. In this case, the negative effect of ω outweighs its positive effect, and thus, we obtain an equilibrium in which all countries except the country with the highest turnout of the old, $\omega_j = \omega_{\max}$, accumulate public debt up to the natural debt limit. Finally, when θ lies between $\underline{\theta}$ and $\overline{\theta}$, there is a critical value of ω_j, defined by $\omega_j = \widetilde{\omega}(R^*)$, such that the two opposing effects balance each other. Figure 3 illustrates three possible cases shown in Proposition 2.

[Figure 3 is here.]

The results in Propositions 1 and 2 suggest that the country with lower voter turnout of the old is more likely to accumulate public debt below the natural debt limit and to become a lender as $1/\sigma$ becomes lower and θ becomes higher. To confirm this, we solve the model numerically and show that the statement holds for $1/\sigma \in (0, 2)$. Figure 4 shows that the country with lowest voter turnout of the old, denoted by $\omega_{\min}$, accumulates public debt below the natural debt limit and becomes a lender when $1/\sigma$ is low and θ is high. On the other hand, the country with the highest voter turnout of the old, denoted by $\omega_{\max}$, accumulates public debt below the natural debt limit and becomes a lender when $1/\sigma$ is high and θ is low. Therefore, the present study suggests that the age gap in voter turnout is a key determinant of the distribution of public debt in the world economy, and that the impact of the age gap heavily depends on the preferences for public goods as well as the inter-temporal elasticity of substitution.

[Figure 4 is here.]

5 Conclusion

In this study, we investigate the effect of the age gap in voter turnout on fiscal policy-making and show that this age gap has two opposing effects on public bond issues. First, the higher voter turnout of the old increases their political influence. This incentivizes the government to increase public goods provision today by issuing more public bonds because the old do not bear the cost of future debt repayment. Second, higher public goods provision today makes the young prefer higher public goods provision in the future from the viewpoint of public goods consumption smoothing. Furthermore, the government responds to this request by the young by restraining the issue of public bonds today. In particular, this negative effect on public bond issues becomes less effective as the inter-temporal elasticity of substitution decreases. The net effect of the two opposing effects depends on the magnitude of the inter-temporal elasticity of substitution. This is the main cause of the non-monotone relationship between the age gap and size of government debt.

We demonstrate the above result by adopting several assumptions that make the analysis tractable. In particular, we assume inelastic labor supply and thus we ignore the labor–leisure choice by households. While this assumption enables us to derive an analytical solution, it results in an equilibrium that overly portrays the cross-country difference: all countries except one are distinguished by debt up to the natural debt limit. We might resolve this problem by introducing the labor–leisure choice of households; however, here, we retain the assumption of inelastic labor supply to simplify the presentation.

We also assume no cross-country difference in population growth rates but instead focus on the age gap in voter turnout. Introducing the difference in population growth rates is expected to produce a similar effect as the difference in age gaps because the relative political weights to the young and elderly vary as population growth rates change. However, this prediction is not fulfilled, as demonstrated in Appendix B. We show that a higher population growth rate is associated with a higher level of public debt. Therefore, the difference in abstention from voting, rather than the difference in population growth rates, is crucial for explaining the non-monotone effect of voter turnout on the size of government debt.

A Proofs

A.1 Derivation of the Political Objective Function $\Omega(\cdot)$

The description of probabilistic voting follows that in Persson and Tabellini (2000). The population consists of two distinct groups, $J \in \{y, o\}$, representing the young and old, respectively. Since there is no population growth, the population share of each group is 1/2. An electoral competition takes place between two office-seeking candidates, denoted by A and B. Each candidate announces a set of fiscal policies $\{b'_j,\ \tau_j,\ g_j\}$, subject to the government budget constraint, $b'_j = g_j + Rb_j - \tau_j w$, and to the natural debt limit, $b'_j \leq \bar{b}$.

At the time of the election, voters base their voting decisions on economic policy announcements as well as the two candidates' ideologies. Specifically, voter k in group J prefers candidate A if

$$V^J(\tau_{j,A}, g_{j,A}, b'_{j,A}) > V^J(\tau_{j,B}, g_{j,B}, b'_{j,B}) + \sigma^{kJ} + \delta,$$

where V^J is the indirect utility of group J, $\{\tau_{j,A}, g_{j,A}, b'_{j,A}\}$ and $\{\tau_{j,B}, g_{j,B}, b'_{j,B}\}$ are policy sets that are announced by candidates A and B, respectively. Within each group, voters differ according to their ideology toward the two candidates. The parameter σ^{kJ}, which measures voter k's individual ideological bias toward candidate B, is an individual-specific parameter that can take a negative or positive value. A positive value of σ^{kJ} implies that voter k is biased in favor of party B, whereas voters with $\sigma^{kJ} = 0$ are ideologically neutral, that is, they care only about economic policy. We assume that this parameter has a uniform distribution on $[-1/(2\phi),\ 1/(2\phi)]$. In addition, voters' decisions are affected by the candidates' average popularity. The parameter δ measures the average (relative) popularity of candidate B in the population as a whole, and has a uniform distribution on $[-1/(2\psi),\ 1/(2\psi)]$.

The timing of events is as follows. (i) Candidates A and B simultaneously and non-cooperatively announce their electoral platforms: $\{\tau_{j,A}, g_{j,A}, b'_{j,A}\}$ and $\{\tau_{j,B}, g_{j,B}, b'_{j,B}\}$.

(ii) The actual value of δ is realized. (iii) Elections are held. (iv) The elected candidate implements his/her announced policy platform.

In the present framework, some voters abstain from voting. The voter turnout rate of group J is denoted by $q_j^J \in (0,1), J \in \{y, o\}$, which is independent from their ideological bias, σ^{kJ}. For example, among the young in country j, proportion $1 - q_j^y$ participates in voting, whereas proportion q_j^y abstains from voting.

A *swing voter* is one whose ideological bias, given the candidates' platform, makes him/her indifferent between the two parties. The swing voter's ideology in group J $(J = y, o)$, σ^J, satisfies

$$\sigma^J = V^J(\tau_{j,A}, g_{j,A}, b'_{j,A}) - V^J(\tau_{j,B}, g_{j,B}, b'_{j,B}) - \delta. \tag{A.1}$$

Recall that q^J denotes the voting rate of group J; this is independent from their ideological bias, σ^{kJ}. Candidate A's actual vote share is

$$\begin{aligned}\Gamma_A &\equiv \frac{1}{q_j^y + q_j^o} \times \left[\underbrace{q_j^o \phi \cdot \left(\sigma^o + \frac{1}{2\phi} \right)}_{\text{old}} + \underbrace{q_j^y \phi \cdot \left(\sigma^y + \frac{1}{2\phi} \right)}_{\text{young}} \right] \\ &= \frac{\phi}{q_j^y + q_j^o} \cdot \sum_J q_j^J \cdot \left(\sigma^J + \frac{1}{2\phi} \right). \end{aligned} \tag{A.2}$$

Using Eqs. (A.1) and (A.2), we compute candidate A's probability of winning the election as follows:

$$\begin{aligned} P^A &= \text{Prob}_\delta \left[\Gamma^A \geq \frac{1}{2} \right] \\ &= \text{Prob}_\delta \left[\sum_J q_j^J \cdot \left(V^J(\tau_{j,A}, g_{j,A}, b'_{j,A}) - V^J(\tau_{j,B}, g_{j,B}, b'_{j,B}) - \delta + \frac{1}{2\phi} \right) \geq \frac{q_j^y + q_j^o}{2\phi} \right] \\ &= \text{Prob}_\delta \left[\frac{1}{q_j^y + q_j^o} \cdot \sum_J q_j^J \left\{ V^J(\tau_{j,A}, g_{j,A}, b'_{j,A}) - V^J(\tau_{j,B}, g_{j,B}, b'_{j,B}) \right\} \geq \delta \right] \\ &= \frac{1}{2} + \psi \sum_J \frac{q_j^J}{q_j^y + q_j^o} \cdot \left\{ V^J(\tau_{j,A}, g_{j,A}, b'_{j,A}) - V^J(\tau_{j,B}, g_{j,B}, b'_{j,B}) \right\}. \end{aligned}$$

Candidate B wins the election with probability $P^B = 1 - P^A$,

$$\begin{aligned} P^B &= 1 - P^A \\ &= \frac{1}{2} + \psi \sum_J \frac{q_j^J}{q_j^y + q_j^o} \cdot \left\{ V^J(\tau_{j,B}, g_{j,B}, b'_{j,B}) - V^J(\tau_{j,A}, g_{j,A}, b'_{j,A}) \right\}. \end{aligned}$$

Each candidate chooses a policy platform to maximize his/her probability of winning the election, given the other candidate's policy platform. The two candidates' choices satisfy the following equations:

$$(\tau_{j,A}^*,\ g_{j,A}^*,\ b_{j,A}'^*) = \arg \max_{\tau_{j,A}, g_{j,A}, b'_{j,A}} \sum_J \frac{q_j^J}{q_j^y + q_j^o} \cdot \left\{ V^J(\tau_{j,A}, g_{j,A}, b'_{j,A}) - V^J(\tau_{j,B}^*,\ g_{j,B}^*,\ b_{j,B}'^*) \right\}$$

$$(\tau_{j,B}^*,\ g_{j,B}^*,\ b_{j,B}'^*) = \arg \max_{\tau_{j,B}, g_{j,B}, b'_{j,B}} \sum_J \frac{q_j^J}{q_j^y + q_j^o} \cdot \left\{ V^J(\tau_{j,B}, g_{j,B}, b'_{j,B}) - V^J(\tau_{j,A}^*,\ g_{j,A}^*,\ b_{j,A}'^*) \right\}.$$

The expressions suggest that the two candidates' platforms converge in equilibrium to the same fiscal policy that maximizes the weighted-average utility of each group,

$$\begin{aligned} (\tau_{j,A}^*,\ g_{j,A}^*,\ b_{j,A}'^*) &= (\tau_{j,B}^*,\ g_{j,B}^*,\ b_{j,B}'^*) \\ &= \arg\ max_{\tau_j, g_j, b'_j}\ \omega(p_j) V^o(g_j) + (1 - \omega(p_j)) V^y(\tau_j, g_j, b'_j), \end{aligned}$$

where $p_j \equiv q_j^o / q_j^y$ and $\omega(p_j) \equiv p_j / 1 + p_j$. This is the objective function given in Subsection 3.1.

■

A.2 Derivation of Eqs. (4) and (5)

We substitute Eq. (B.4) in Eq. (B.5) and reformulate the political objective function as follows:

$$\Omega(\tau_j, g_j, g_j'; \omega(p_j), w, R) = \omega(p_j) \cdot \frac{1}{1-\sigma} \cdot \lambda\theta \cdot (g_j)^{1-\sigma}$$
$$+ (1-\omega(p_j)) \cdot \frac{1}{1-\sigma} \cdot \left[\left\{ 1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}} \right\}^{\sigma} \cdot \{w(1-\tau_j)\}^{1-\sigma} + \theta \cdot (g_j)^{1-\sigma} + \beta\lambda\theta \cdot (g_j')^{1-\sigma} \right]. \tag{A.3}$$

We differentiate (A.3) with respect to g_j and τ_j and obtain

$$\{(1-\omega(p_j)) + \omega(p_j)\lambda\}\theta(g_j)^{-\sigma} + (1-\omega(p_j))\beta\lambda\theta(g_j')^{-\sigma} \cdot \frac{\partial g_j'}{\partial b_j'} \cdot \frac{\partial b_j'}{\partial g_j} = 0, \tag{A.4}$$

$$(1-\omega(p_j)) \left\{ 1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}} \right\}^{\sigma} (-1)(1-\tau_j)^{-\sigma} w^{1-\sigma}$$
$$+ (1-\omega(p_j))\beta\lambda\theta(g_j')^{-\sigma} \cdot \frac{\partial g_j'}{\partial b_j'} \cdot \frac{\partial b_j'}{\partial \tau_j} = 0. \tag{A.5}$$

The government budget constraint in (3) implies $\partial b_j'/\partial g_j = 1$. We use this condition to rewrite (A.4) as in (4). In addition, we rewrite (A.5) as

$$\frac{\left\{ 1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}} \right\}^{\sigma}}{(1-\tau_j)^{\sigma}} \cdot w^{1-\sigma} = (-1)\beta\lambda\theta(g_j')^{-\sigma} \frac{\partial g_j'}{\partial b_j'} \cdot w$$
$$= \theta \cdot \frac{(1-\omega(p_j)) + \omega(p_j)\lambda}{1-\omega(p_j)} \cdot w \cdot \frac{1}{(g_j)^{\sigma}}.$$

where we derive the first equality by using $\partial b_j'/\partial \tau_j = -w$, and the second equality by using (4). By rearranging the terms, we obtain Eq. (5).

■

A.3 Proof of Lemma 1

Recall the first-order conditions with respect to g_j and τ_j, given by (4) and (5), respectively. We substitute (5) into the government budget constraint and obtain

$$b_j' = g_j + Rb_j - w \cdot \left[1 - \left\{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right\} \cdot \left\{\frac{f(\omega(p_j))}{\beta\lambda\theta}\right\}^{\frac{1}{\sigma}} \cdot \frac{1}{w} \cdot g_j\right],$$

or

$$b_j' = g_j + Rb_j - (R-1)\bar{b} + \left\{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right\} \cdot \left\{\frac{f(\omega(p_j))}{\beta\lambda\theta}\right\}^{\frac{1}{\sigma}} \cdot g_j, \tag{A.6}$$

where we derive the second expression by using $\bar{b} \equiv w/(R-1)$.

To find the solution satisfying (4), (5), and (A.6), we estimate

$$g_j' = \gamma_j \cdot \left(\bar{b} - b_j'\right), \tag{A.7}$$

where γ_j is an undetermined coefficient. We substitute (A.7) into (4) and obtain

$$\left(\frac{\gamma_j \cdot \left(\bar{b} - b_j'\right)}{g_j}\right)^{\sigma} = \gamma_j f(\omega(p_j)),$$

or

$$b_j' = \bar{b} - (\gamma_j)^{\frac{1}{\sigma}-1} \cdot \{f(\omega(p_j))\}^{\frac{1}{\sigma}} \cdot g_j.$$

Plugging this expression into (A.6) and rearranging the terms, we obtain

$$g_j = \frac{R \cdot \left(\bar{b} - b_j\right)}{1 + \left\{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right\} \cdot \left\{\frac{f(\omega(p_j))}{\beta\lambda\theta}\right\}^{\frac{1}{\sigma}} + (\gamma_j)^{\frac{1}{\sigma}-1} \cdot \{f(\omega(p_j))\}^{\frac{1}{\sigma}}}.$$

Our estimate is verified if, for a given R, γ_j satisfies the following condition:

$$\gamma_j = \frac{R}{1 + \left\{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right\} \cdot \left\{\frac{f(\omega(p_j))}{\beta\lambda\theta}\right\}^{\frac{1}{\sigma}} + (\gamma_j)^{\frac{1}{\sigma}-1} \cdot \{f(\omega(p_j))\}^{\frac{1}{\sigma}}},$$

or

$$R-\gamma_j\cdot\left[1+\left\{1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right\}\cdot\left\{\frac{f(\omega(p_j))}{\beta\lambda\theta}\right\}^{\frac{1}{\sigma}}\right]=\{\gamma_j f(\omega(p_j))\} \tag{A.8}$$

The left-hand side of (A.8), denoted by LHS, is decreasing in γ_j with $LHS|_{\gamma_j=0}=R$ and $\lim_{\gamma_j\to\infty} LHS=-\infty$. The right-hand side of (A.8), denoted by RHS, is an increasing function of γ_j with $RHS|_{\gamma_j=0}=0$ and $\lim_{\gamma_j\to\infty} RHS=\infty$. Therefore, there is a unique $\gamma_j(>0)$ satisfying (A.8). We obtain the corresponding tax and debt policy functions by substituting $g_j=\gamma_j\cdot\left(\bar{b}-b_j\right)$ into (5) and (A.6), respectively.

■

A.4 Proof of Lemma 2

First, suppose that $1/\sigma\le 1$ holds. We denote the right-hand side of (9) by $RHS(R^*)$ with $RHS(1)>1$. Moreover, $RHS(R^*)$ is decreasing in R^* and satisfies $\lim_{R^*\to\infty} RHS(R^*)\in(1,+\infty)$. Therefore, when $1/\sigma\le 1$, there is a unique $R^*\in(1,+\infty)$ satisfying Eq. (9).

Next, suppose that $1/\sigma\in(1,2)$ holds. The first and second-order differentials of $RHS(\cdot)$ with respect to R^* are

$$\begin{aligned}\frac{\partial RHS(R^*)}{\partial R^*}&=\frac{(1-\omega_j)+\omega_j\lambda}{(1-\omega_j)\,\beta\lambda}\cdot(\beta)^{\frac{1}{\sigma}}\cdot\left(\frac{1-\omega_j}{\theta\{(1-\omega_j)+\omega_j\lambda\}}\right)^{\frac{1}{\sigma}}\\&\quad\times\left(\frac{1}{\sigma}-1\right)\cdot(R^*)^{\frac{1}{\sigma}-2}>0,\\\frac{\partial^2 RHS(R^*)}{\partial R^{*2}}&=\frac{(1-\omega_j)+\omega_j\lambda}{(1-\omega_j)\,\beta\lambda}\cdot(\beta)^{\frac{1}{\sigma}}\cdot\left(\frac{1-\omega_j}{\theta\{(1-\omega_j)+\omega_j\lambda\}}\right)^{\frac{1}{\sigma}}\\&\quad\times\left(\frac{1}{\sigma}-1\right)\cdot\left(\frac{1}{\sigma}-2\right)\cdot(R^*)^{1/\sigma-3}<0,\end{aligned}$$

where $\lim_{R^*\to+\infty}\partial RHS(R^*)/\partial R^*=0$ holds when $1/\sigma\in(1,2)$. Therefore, there is a unique R^* satisfying Eq. (9) when $1/\sigma\in(1,2)$.

Finally, suppose that $1/\sigma=2$ holds. Then, we can derive R^* directly by solving (9) for R^* as follows:

$$R^* = 1 + \frac{1}{f(\omega_j)} \cdot \left[1 + \{1 + \beta(\beta R^*)\} \cdot \left\{ \frac{f(\omega_j)}{\beta\lambda\theta} \right\}^2 \right],$$

or

$$R^* = \frac{1 + f(\omega_j) + \left\{ \frac{f(\omega_j)}{\beta\lambda\theta} \right\}^2}{f(\omega_j) - \beta^2 \cdot \left\{ \frac{f(\omega_j)}{\beta\lambda\theta} \right\}^2} = 1 + \frac{1 + (1+\beta^2) \cdot \left\{ \frac{f(\omega_j)}{\beta\lambda\theta} \right\}^2}{f(\omega_j) - \beta^2 \cdot \left\{ \frac{f(\omega_j)}{\beta\lambda\theta} \right\}^2}, \tag{A.9}$$

where $R^* > 1$ holds if and only if $\theta > \{f(\omega_j)\}^{1/2}/\lambda$.

A.5 Proof of Lemma 3

Recall that γ_j^* satisfies (7). We differentiate this with respect to γ_j^* and $\omega(p_j)$ and obtain

$$\begin{aligned}
&\frac{\partial \gamma_j^*}{\partial \omega(p_j)} \\
&= -\frac{\frac{1}{\sigma}\left(\gamma_j^*\right)^{1/\sigma} \{f(\omega(p_j))\}^{1/\sigma - 1} f'(\omega(p_j)) + \gamma_j^* \left\{1 + \beta\left(\beta R\right)^{1/\sigma - 1}\right\} \left(\frac{1}{\theta\beta\lambda}\right)^{1/\sigma} \frac{1}{\sigma} \{f(\omega(p_j))\}^{1/\sigma - 1} f'(\omega(p_j))}{\frac{1}{\sigma}\left(\gamma_j^*\right)^{1/\sigma - 1} \{f(\omega(p_j))\}^{1/\sigma} + \left[1 + \left\{1 + \beta\left(\beta R\right)^{1/\sigma - 1}\right\} \left(\frac{f(\omega(p_j))}{\theta\beta\lambda}\right)^{1/\sigma}\right]} \\
&= -\gamma_j^* \{f(\omega(p_j))\}^{-1} f'(\omega(p_j)) \cdot \frac{\left(\gamma_j^*\right)^{1/\sigma - 1} \{f(\omega(p_j))\}^{1/\sigma} + \left\{1 + \beta\left(\beta R\right)^{1/\sigma - 1}\right\} \left(\frac{f(\omega(p_j))}{\theta\beta\lambda}\right)^{1/\sigma}}{\left(\gamma_j^*\right)^{1/\sigma - 1} \{f(\omega(p_j))\}^{1/\sigma} + \sigma\left[1 + \left\{1 + \beta\left(\beta R\right)^{1/\sigma - 1}\right\} \left(\frac{f(\omega(p_j))}{\theta\beta\lambda}\right)^{1/\sigma}\right]},
\end{aligned} \tag{A.10}$$

where $f'(\omega(p_j)) < 0$ holds from (6).

With the use of (8) and (A.10), we obtain

$$\frac{\partial \phi_j^*}{\partial \omega(p_j)} = \frac{1}{\sigma} \cdot \left(f(\omega(p_j)) \cdot \gamma_j^*\right)^{\frac{1-\sigma}{\sigma}} \cdot \left\{ f'(\omega(p_j)) \cdot \gamma_j^* + f(\omega(p_j)) \cdot \frac{\partial \gamma_j^*}{\partial \omega(p_j)} \right\}$$
$$= \frac{1}{\sigma} \cdot \left(f(\omega(p_j)) \cdot \gamma_j^*\right)^{\frac{1-\sigma}{\sigma}} \cdot f'(\omega(p_j)) \cdot \gamma_j^*$$
$$\times \left[1 - \frac{\left(\gamma_j^*\right)^{1/\sigma - 1} \left\{ f(\omega(p_j)) \right\}^{1/\sigma} + \left\{ 1 + \beta (\beta R)^{1/\sigma - 1} \right\} \left(\frac{f(\omega(p_j))}{\theta \beta \lambda} \right)^{1/\sigma}}{\left(\gamma_j^*\right)^{1/\sigma - 1} \left\{ f(\omega(p_j)) \right\}^{1/\sigma} + \sigma \left[1 + \left\{ 1 + \beta (\beta R)^{1/\sigma - 1} \right\} \left(\frac{f(\omega(p_j))}{\theta \beta \lambda} \right)^{1/\sigma} \right]} \right]. \tag{A.11}$$

The expression in (A.11) leads to the following condition:

$$\frac{\partial \phi_j^*}{\partial \omega(p_j)} \gtreqless 0 \Leftrightarrow \sigma \left[1 + \left\{ 1 + \beta (\beta R)^{1/\sigma - 1} \right\} \cdot \left(\frac{f(\omega(p_j))}{\theta \beta \lambda} \right)^{\frac{1}{\sigma}} \right] \lesseqgtr \left\{ 1 + \beta (\beta R)^{1/\sigma - 1} \right\} \cdot \left(\frac{f(\omega(p_j))}{\theta \beta \lambda} \right)^{\frac{1}{\sigma}}$$
$$\Leftrightarrow \frac{1}{1 + \beta (\beta R)^{1/\sigma - 1}} \cdot \left[\frac{\theta \cdot (1 - \omega(p_j)) + \omega(p_j) \lambda}{1 - \omega(p_j)} \right]^{1/\sigma} \lesseqgtr \frac{1 - \sigma}{\sigma}.$$

■

A.6 Proof of Proposition 2

(i) Suppose that $\phi^*(\omega_{min}, R^*) = \bar{\phi} = 1$. Eq. (A.9) leads to the following equilibrium interest rate:

$$R^* = 1 + \frac{1 + (1 + \beta^2) \cdot \left\{ \frac{f(\omega_{min})}{\beta \lambda \theta} \right\}^2}{f(\omega_{min}) - \beta^2 \cdot \left\{ \frac{f(\omega_{min})}{\beta \lambda \theta} \right\}^2} (> 1). \tag{A.12}$$

As demonstrated in Proposition 1(i), $\phi^*(\omega_{min}, R^*) = \bar{\phi} = 1$ holds when $1/\sigma - 1 \le \Psi(R^*, \omega_{min})$ is satisfied. We substitute $1/\sigma = 2$ and (A.12) into this condition and obtain

$$
\begin{aligned}
& 1 \le \Psi(R^*, \omega_{min}) = \frac{1}{1+\beta^2 R^*} \cdot \left\{ \frac{\beta\lambda\theta}{f(\omega_{min})} \right\}^2 \\
\Leftrightarrow\ & 1 + \beta^2 \left[1 + \frac{1 + (1+\beta^2) \cdot \left\{ \frac{f(\omega_{min})}{\beta\lambda\theta} \right\}^2}{f(\omega_{min}) - \beta^2 \cdot \left\{ \frac{f(\omega_{min})}{\beta\lambda\theta} \right\}^2} \right] \le \left\{ \frac{\beta\lambda\theta}{f(\omega_{min})} \right\}^2 \\
\Leftrightarrow\ & (1+\beta^2) \cdot \left\{ \frac{f(\omega_{min})}{\beta\lambda\theta} \right\}^2 + \beta^2 \cdot \frac{\left\{ \frac{f(\omega_{min})}{\beta\lambda\theta} \right\}^2 + (1+\beta^2) \cdot \left\{ \frac{f(\omega_{min})}{\beta\lambda\theta} \right\}^4}{f(\omega_{min}) - \beta^2 \cdot \left\{ \frac{f(\omega_{min})}{\beta\lambda\theta} \right\}^2} \le 1 \\
\Leftrightarrow\ & (1+\beta^2) \cdot \left\{ \frac{f(\omega_{min})}{\beta\lambda\theta} \right\}^2 \cdot f(\omega_{min}) - (1+\beta^2)\beta^2 \cdot \left\{ \frac{f(\omega_{min})}{\beta\lambda\theta} \right\}^4 \\
& \quad + \beta^2 \left\{ \frac{f(\omega_{min})}{\beta\lambda\theta} \right\}^2 + (1+\beta^2)\beta^2 \cdot \left\{ \frac{f(\omega_{min})}{\beta\lambda\theta} \right\}^4 \le f(\omega_{min}) - \beta^2 \cdot \left\{ \frac{f(\omega_{min})}{\beta\lambda\theta} \right\}^2 \\
\Leftrightarrow\ & (1+\beta^2) \cdot \left\{ \frac{f(\omega_{min})}{\beta\lambda\theta} \right\}^2 \cdot f(\omega_{min}) + \beta^2 \left\{ \frac{f(\omega_{min})}{\beta\lambda\theta} \right\}^2 \le f(\omega_{min}) - \beta^2 \cdot \left\{ \frac{f(\omega_{min})}{\beta\lambda\theta} \right\}^2 \\
\Leftrightarrow\ & (1+\beta^2) \cdot \{f(\omega_{min})\}^3 + \beta^2 \cdot \{f(\omega_{min})\}^2 \le \beta^2\lambda^2\theta^2 f(\omega_{min}) - \beta^2 \cdot \{f(\omega_{min})\}^2 \\
\Leftrightarrow\ & (1+\beta^2) \cdot \{f(\omega_{min})\}^2 + 2\beta^2 f(\omega_{min}) \le \beta^2\lambda^2\theta^2 \\
\Leftrightarrow\ & \theta \ge \overline{\theta} \equiv \frac{1}{\lambda} \cdot \left[2f(\omega_{\min}) + \left(\left(1+\beta^2\right)/\beta^2 \right) \cdot \left(f(\omega_{\min}) \right)^2 \right]^{1/2}.
\end{aligned}
\tag{A.13}
$$

Therefore, when the condition in (A.13) is satisfied, countries with $\omega(p_j) = \omega_{min}$ accumulate public debt below $\bar{b}$.

(ii) Suppose that $\phi^*(\widetilde{\omega}(R^*), R^*) = \bar{\phi} = 1$. Eq. (A.9) leads to the following equilibrium interest rate:

$$
R^* = 1 + \frac{1 + (1+\beta^2) \cdot \left\{ \frac{f(\widetilde{\omega}(R^*))}{\beta\lambda\theta} \right\}^2}{f(\widetilde{\omega}(R^*)) - \beta^2 \cdot \left\{ \frac{f(\widetilde{\omega}(R^*))}{\beta\lambda\theta} \right\}^2} (> 1). \tag{A.14}
$$

When $1/\sigma = 2$, (10) and (6) are reformulated as

$$
\widetilde{\omega}(R) = \frac{1 - \frac{1}{\theta} \cdot (1+\beta^2 R)^{1/2}}{1 - \lambda - \frac{1}{\theta} \cdot (1+\beta^2 R)^{1/2}}, \tag{A.15}
$$

and

$$f(\widetilde{\omega}(R)) = \frac{\beta\theta\lambda}{(1+\beta^2 R)^{1/2}}, \tag{A.16}$$

respectively.

Substituting (A.16) into (A.14) leads to

$$R^* = 1 + \frac{1 + (1+\beta^2)\frac{1}{1+\beta^2 R^*}}{\frac{\beta\theta\lambda}{(1+\beta^2 R^*)^{1/2}} - \beta^2 \frac{1}{1+\beta^2 R^*}}.$$

This yields two solutions for R^*. The solution satisfying $R^* > 1$ is taken as equilibrium interest rate

$$R^* = 1 + 2 \cdot \frac{1 + \sqrt{1 + \frac{\theta^2\lambda^2(1+\beta^2)}{\beta^2}}}{\theta^2\lambda^2} > 1. \tag{A.17}$$

Proposition 1(ii) shows that $\phi^*(\widetilde{\omega}(R^*), R^*) = \bar{\phi} = 1$ holds if $\Psi(R^*, \omega_{min}) < 1/\sigma - 1 < \Psi(R^*, \omega_{max})$ is satisfied. With $1/\sigma = 2$ and (A.17), the first inequality condition, $\Psi(R^*, \omega_{min}) < 1/\sigma - 1$, is reformulated as

$$1 > \Psi(R^*, \omega_{min}) = \frac{1}{1+\beta^2 R^*} \cdot \left\{ \frac{\beta\theta\lambda}{f(\omega_{min})} \right\}^2,$$

or

$$\Leftrightarrow \; \theta < \overline{\theta} \equiv \frac{1}{\lambda} \cdot \left[2f(\omega_{\min}) + \left(\left(1+\beta^2\right)/\beta^2 \right) \cdot \left(f(\omega_{\min}) \right)^2 \right]^{1/2}.$$

The second inequality condition, $1/\sigma - 1 < \Psi(R^*, \omega_{max})$, is reformulated as

$$1 < \Psi(R^*, \omega_{max}) = \frac{1}{1+\beta^2 R^*} \cdot \left\{ \frac{\beta\theta\lambda}{f(\omega_{max})} \right\}^2,$$

or

$$\theta > \underline{\theta} \equiv \frac{1}{\lambda} \cdot \left[2f(\omega_{\max}) + \left(\left(1+\beta^2\right)/\beta^2 \right) \cdot \left(f(\omega_{\max}) \right)^2 \right]^{1/2}.$$

By substituting (A.17) into $\widetilde{\omega}(R^*)$, we obtain the corresponding value of $\widetilde{\omega}(R^*)$ as

$$\widetilde{\omega}(R^*) = \frac{1 - \dfrac{\beta + \sqrt{\beta^2 + \theta^2\lambda^2(1+\beta^2)}}{\theta^2\lambda}}{1 - \lambda - \dfrac{\beta + \sqrt{\beta^2 + \theta^2\lambda^2(1+\beta^2)}}{\theta^2\lambda}},$$

(iii) Suppose that $\phi^*_{max} = \bar{\phi} = 1$. Eq. (A.9) leads to

$$R^* = 1 + \frac{1 + (1+\beta^2) \cdot \left\{ \dfrac{f(\omega_{max})}{\beta\lambda\theta} \right\}^2}{f(\omega_{max}) - \beta^2 \cdot \left\{ \dfrac{f(\omega_{max})}{\beta\lambda\theta} \right\}^2} (> 1). \tag{A.18}$$

Proposition 1(iii) shows that $\phi^*_{max} = \bar{\phi} = 1$ holds when $\Psi(R^*, \omega_{max}) \leq 1/\sigma - 1$ is satisfied. We substitute $1/\sigma = 2$ and (A.18) into this condition and obtain

$$\begin{aligned}
1 \geq \Psi(R^*, \omega_{max}) &= \frac{1}{1+\beta^2 R^*} \cdot \left\{ \frac{\beta\lambda\theta}{f(\omega_{max})} \right\}^2 \\
\Leftrightarrow\ 1 + \beta^2 & \left[1 + \frac{1 + (1+\beta^2) \cdot \left\{ \dfrac{f(\omega_{max})}{\beta\lambda\theta} \right\}^2}{f(\omega_{max}) - \beta^2 \cdot \left\{ \dfrac{f(\omega_{max})}{\beta\lambda\theta} \right\}^2} \right] \leq \left\{ \frac{\beta\lambda\theta}{f(\omega_{max})} \right\}^2 \\
\Leftrightarrow\ \theta \leq \underline{\theta} &\equiv \frac{1}{\lambda} \cdot \left[2f(\omega_{\max}) + \left(\left(1+\beta^2\right)/\beta^2 \right) \cdot \left(f(\omega_{\max})\right)^2 \right]^{1/2}. \qquad \text{(A.19)}
\end{aligned}$$

Therefore, if (A.19) is satisfied, countries with $\omega(p_j) = \omega_{min}$ accumulate public debt below $b_{min} < \bar{b}$. Finally, from (A.18), $R^* > 1$ holds if and only if $\theta > \{f(\omega_{max})\}^{1/2}/\lambda$.

■

B An Alternative Setting: Cross-Country Difference in Population Growth Rates

In this appendix, we assume away the cross-country difference in the age gap in voter turnout. Instead, we assume a cross-country difference in population growth rates. Under this alternative assumption, we show that a higher population growth rate is associated with a higher level of public debt. The model presented below is based on that developed by Arawatari (2018).

B.1 Model

We retain the notation used in the main text. The present model differs from the one in the main text in that there is a cross-country difference in population growth rates. The population growth rate of country j is denoted by n_j. Let us assume that n_j is drawn from a finite-valued set, $n_j \in \{\underline{n}, n^1, n^2, \cdots, \bar{n}\} \equiv \Gamma_N$, $\underline{n} > -1$. The size of the population born in period t is denoted by $N_{j,t}$: $N_{j,t+1} = (1+n_j)N_{j,t}$.

B.1.1 Utility Maximization

The utility-maximization problem of young agents is given by

$$\max_{c_t^y, c_{t+1}^o} U_{j,t}^y = \frac{(c_{j,t}^y)^{1-\sigma}-1}{1-\sigma} + \theta \cdot \frac{(g_{j,t})^{1-\sigma}-1}{1-\sigma} + \beta \cdot \left\{ \frac{(c_{j,t+1}^o)^{1-\sigma}-1}{1-\sigma} + \theta \cdot \frac{(g_{j,t+1})^{1-\sigma}-1}{1-\sigma} \right\},$$
$$\text{s.t. } c_{j,t}^y + s_{j,t} \leq (1-\tau_{j,t}) \cdot w,$$
$$c_{j,t+1}^o \leq R s_{j,t},$$

where $g_{j,t}$ is public goods provision per capita. To simplify the analysis, we assume $\lambda = 1$.

Solving the problem leads to the following consumption and savings functions:

$$c_{j,t}^y = \frac{1}{1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}} \cdot w \cdot (1-\tau_{j,t}), \tag{B.1}$$
$$c_{j,t+1}^o = \frac{\beta(\beta R)^{\frac{1-\sigma}{\sigma}}}{1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}} \cdot R \cdot w \cdot (1-\tau_{j,t}), \tag{B.2}$$
$$s_{j,t} = \frac{\beta(\beta R)^{\frac{1-\sigma}{\sigma}}}{1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}} \cdot w \cdot (1-\tau_{j,t}). \tag{B.3}$$

Ignoring the irrelevant terms, we can express the indirect utility functions of type-i

young agents and old agents as follows:

$$
\begin{aligned}
V^y_{j,t} &= \frac{1}{1-\sigma} \cdot \left[\left\{ 1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}} \right\}^{\sigma} \cdot \{ w \cdot (1-\tau_{j,t}) \}^{1-\sigma} + \theta \cdot (g_{j,t})^{1-\sigma} + \beta\theta \cdot (g_{j,t+1})^{1-\sigma} \right] \\
&= V^y(\tau_{j,t}, g_{j,t}, g_{j,t+1}), \qquad \text{(B.4)} \\
V^o_{j,t} &= \frac{1}{1-\sigma} \cdot \theta \cdot (g_{j,t})^{1-\sigma} \\
&= V^o(g_{j,t}). \qquad \text{(B.5)}
\end{aligned}
$$

B.1.2 Government Budget Constraint

The government budget constraint is given by

$$
b_{j,t+1} = \frac{g_{j,t}}{1+n_j} + \frac{Rb_{j,t}}{1+n_j} - \frac{\tau_{j,t} w}{2+n_j}.
$$

A lower population growth rate results in a smaller tax base in the future. Therefore, the natural debt limit, denoted by $\bar{b}_j$, differs across countries. Country i's limit, $\bar{b}_j$, is defined as follows:

$$
(N_{j,t+1} + N_{j,t}) \cdot b_{j,t+1} \le \sum_{s=1}^{\infty} \frac{N_{j,t+s} \cdot w}{R^s} \;\Leftrightarrow\; b_{j,t+1} \le \frac{1+n_j}{2+n_j} \cdot \frac{w}{R-(1+n_j)} \equiv \bar{b}_j.
$$

B.2 Politico-Economic Equilibrium

This section explains the political mechanism of fiscal policy formation in the presence of a cross-country difference in population growth rates.

B.2.1 Probabilistic Voting with Abstention

Recall that the sizes of the young and old in country j are $N_{j,t}$ and $N_{j,t-1}$, respectively. Consider two political candidates, A and B, as in Appendix A.1. Candidate A's actual vote share is

$$\begin{aligned}
\Gamma_A &= \frac{1}{N_{j,t}+N_{j,t-1}} \cdot \left\{ \underbrace{N_{j,t-1}\cdot\phi\cdot\left(\sigma^o+\frac{1}{2\phi}\right)}_{old} + \underbrace{N_{j,t}\cdot\phi\cdot\left(\sigma^y+\frac{1}{2\phi}\right)}_{young} \right\} \\
&= \frac{\phi}{2+n_j}\cdot\left\{\left(\sigma^o+\frac{1}{2\phi}\right)+(1+n_j)\cdot\left(\sigma^y+\frac{1}{2\phi}\right)\right\}. \qquad \text{(B.6)}
\end{aligned}$$

Therefore, we can compute candidate A's probability of winning the election as follows:

$$\begin{aligned}
P^A &= \mathrm{Prob}_\delta\left[\Gamma^A \geq \frac{1}{2}\right] \\
&= \frac{1}{2}+\psi\cdot\left[\frac{1}{2+n_j}\cdot\{V^o(g_{j,A})-V^o(g_{j,B})\}\right. \\
&\qquad \left. +\frac{1+n_j}{2+n_j}\cdot\left\{V^y(\tau_{j,A},g_{j,A},b'_{j,A})-V^y(\tau_{j,B},g_{j,B},b'_{j,B})\right\}\right].
\end{aligned}$$

Candidate B wins the election with probability $P^B = 1-P^A$,

$$\begin{aligned}
P^B &= 1-P^A \\
&= \frac{1}{2}+\psi\cdot\left[\frac{1}{2+n_j}\cdot\{V^o(g_{j,B})-V^o(g_{j,A})\}\right. \\
&\qquad \left. +\frac{1+n_j}{2+n_j}\cdot\left\{V^y(\tau_{j,B},g_{j,B},b'_{j,B})-V^y(\tau_{j,A},g_{j,A},b'_{j,A})\right\}\right].
\end{aligned}$$

These expressions suggest that the two candidates' platforms converge in equilibrium to the same fiscal policy that maximizes the weighted average utility of each group,

$$(\tau^*_{j,A},\ g^*_{j,A},\ b'^*_{j,A}) = (\tau^*_{j,B},\ g^*_{j,B},\ b'^*_{j,B}) = \arg\max_{\tau,g,b'}\ \omega(n_j)V^o(g)+(1-\omega(n_j))V^y(\tau,g,b').$$

where $\omega(n_j)\equiv 1/(2+n_j)$, denoting the relative political weight of the old, is a decreasing function of the population growth rate. The weights on the young and old are equal if there is no population growth.

Definition B1. *An SMPE comprises an interest rate R, a stationary debt distribution*

$\{b_j\}_j$, a debt rule $b'_j = B(b_j; n_j, R)$, a government expenditure rule $g_j = G(b_j; n_j, R)$, and a tax rule $\tau_j = T(b_j; n_j, R)$, such that the following two conditions hold:

(i) $\langle B(b_j; n_j, R), G(b_j; n_j, R), T(b_j; n_j, R)\rangle = \arg\max_{\{\tau_j, g_j, b'_j\}}$ $\Omega(\tau_j, g_j, b'_j; n_j, R)$, *and the government's budget constraint and natural debt limit are satisfied:*

$$B(b_j; n_j, R) = \frac{G(b_j; n_j, R)}{1+n_j} + \frac{Rb_j}{1+n_j} - \frac{T(b_j; n_j, R)w}{2+n_j},$$
$$B(b_j; n_j, R) \leq \frac{1+n_j}{2+n_j} \cdot \frac{w}{R-(1+n_j)} \equiv \bar{b}_j.$$

(ii) *the world asset market clears,*

$$\int_j s_j dj = \int_j b'_j dj,$$

where $b'_j = B(b_j; n_j, R)$ and $s_j = \frac{\beta(\beta R)^{\frac{1-\sigma}{\sigma}}}{1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}} \cdot w \cdot (1 - T(b_j; n_j, R))$.

B.2.2 Voting on Fiscal Policy

Hereafter, we take the market-clearing interest rate as given and solve the model by backward induction. Since we have solved the utility-maximization problem already, we are now ready to find the equilibrium fiscal policy. Given R, the first-order conditions with respect to g_j and τ_j are

$$\left(\frac{g'_j}{g_j}\right)^{\sigma} = -\frac{1}{1+n_j} \cdot \frac{1+n_j}{2+n_j} \cdot \beta \cdot \frac{\partial G(b'_j; n_j, R)}{\partial b'_j}, \tag{B.7}$$
$$\frac{1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}}{1-\tau_j} = \theta^{\frac{1}{\sigma}} \cdot w \cdot \frac{1}{g_j}, \tag{B.8}$$

where $g_j = G(b_j; n_j, R)$, $g'_j = G(b'_j; n_j, R)$, $\tau_j = T(b_j; n_j, R)$, and $b'_j = g_j + Rb_j - \tau_j w \equiv B(b_j; n_j, R)$.

The population growth rate affects the generalized Euler equation for public goods provision in Eq. (B.7) via the two terms, $1/(1+n_j)$ and $(1+n_j)/(2+n_j)$. The term

$1/(1+n_j)$ indicates that a higher population growth rate weakens the disciplining effect expressed by the term $\partial G(b_j'; n_j, R)/\partial b_j'$. Population growth implies an expansion of the tax base in the next period and thus reduces the per capita fiscal burden for the next generation. This incentivizes current politicians to issue more public debt. The term $(1+n_j)/(2+n_j)$ is the share of the young in the population, indicating their relative political power in voting. A higher population growth rate implies the greater political power of the young and thus strengthens the disciplining effect. Hence, population growth has two opposing effects on the disciplining effect (i.e., public debt issuance), but the former effect outweighs the latter one in the present framework.

Lemma B1. *Given R and b_j, country j's policy functions in the SMPE are given by*

$$
\begin{aligned}
G(b_j; n_j, R) &= (1+n_j) \cdot \gamma_j^* \cdot (\bar{b}_j - b_j), \\
T(b_j; n_j, R) &= 1 - \left\{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right\} \cdot \theta^{-\frac{1}{\sigma}} \cdot \frac{1+n_j}{w} \cdot \gamma_j^* \cdot (\bar{b}_j - b_j), \\
B(b_j; n_j, R) &= \bar{b}_j - (\gamma_j^*)^{\frac{1}{\sigma}} \cdot \left(\frac{1+n_j}{2+n_j} \cdot \beta\right)^{\frac{1}{\sigma}} \cdot (\bar{b}_j - b_j),
\end{aligned}
$$

where $\gamma_j^* (> 0)$ satisfies the following condition:

$$
\frac{R}{1+n_j} - \gamma_j^* \cdot \left[1 + \frac{1+n_j}{2+n_j} \cdot \left\{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right\} \cdot \theta^{-\frac{1}{\sigma}}\right] = (\gamma_j^*)^{\frac{1}{\sigma}} \cdot \left(\frac{1+n_j}{2+n_j} \cdot \beta\right)^{\frac{1}{\sigma}}. \quad \text{(B.9)}
$$

Proof.

We substitute (B.8) into the government budget constraint and obtain

$$
b_j' = \frac{g_j}{1+n_j} + \frac{Rb_j}{1+n_j} - \frac{w}{2+n_j} \cdot \left[1 - \left\{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right\} \cdot \theta^{-\frac{1}{\sigma}} \cdot \frac{1}{w} \cdot g_j\right],
$$

or

$$
b_j' = \frac{g_j}{1+n_j} + \frac{Rb_j}{1+n_j} - \frac{R-(1+n_j)}{1+n_j} \cdot \bar{b}_j + \frac{1}{2+n_j} \cdot \left\{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right\} \cdot \theta^{-\frac{1}{\sigma}} \cdot g_j, \quad \text{(B.10)}
$$

where we derive the second expression by using $w/(2+n_j) = \{R-(1+n_j)\}\bar{b}_j/(1+n_j)$.

To find the solution satisfying (B.7), (B.8), and (B.10), we estimate

$$g_j' = (1+n_j) \cdot \gamma_j \cdot \left(\bar{b}_j - b_j'\right), \tag{B.11}$$

where γ_j is an undetermined coefficient. We substitute (B.11) into (B.7) and obtain

$$b_j' = \bar{b}_j - \frac{1}{1+n_j} \cdot (\gamma_j)^{\frac{1}{\sigma}-1} \cdot \left(\frac{1+n_j}{2+n_j} \cdot \beta\right)^{\frac{1}{\sigma}} \cdot g_j.$$

By plugging this expression into (B.10) and rearranging the terms, we obtain

$$g_j = \frac{R \cdot (\bar{b}_j - b_j)}{1 + \frac{1+n_j}{2+n_j} \cdot \left\{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right\} \cdot \theta^{-\frac{1}{\sigma}} + (\gamma_j)^{\frac{1}{\sigma}-1} \cdot \left(\frac{1+n_j}{2+n_j} \cdot \beta\right)^{\frac{1}{\sigma}}}.$$

Our estimate is verified if, for a given R, γ_j satisfies the following condition:

$$(1+n_j) \cdot \gamma_j = \frac{R}{1 + \frac{1+n_j}{2+n_j} \cdot \left\{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right\} \cdot \theta^{-\frac{1}{\sigma}} + (\gamma_j)^{\frac{1}{\sigma}-1} \cdot \left(\frac{1+n_j}{2+n_j} \cdot \beta\right)^{\frac{1}{\sigma}}},$$

or

$$\frac{R}{1+n_j} - \gamma_j \cdot \left[1 + \frac{1+n_j}{2+n_j} \cdot \left\{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right\} \cdot \theta^{-\frac{1}{\sigma}}\right] = \gamma_j^{\frac{1}{\sigma}} \left(\frac{1+n_j}{2+n_j} \cdot \beta\right)^{\frac{1}{\sigma}}. \tag{B.12}$$

The left-hand side of (B.12), denoted by LHS, is decreasing in γ_j with $LHS|_{\gamma_j=0} = R/(1+n_j)$ and $\lim_{\gamma_j \to \infty} LHS = -\infty$. The right-hand side of (B.12), denoted by RHS, is an increasing function of γ_j with $RHS|_{\gamma_j=0} = 0$ and $\lim_{\gamma_j \to \infty} RHS = \infty$. Therefore, there is a unique $\gamma_j (> 0)$ satisfying (B.12). We obtain the corresponding tax and debt policy functions by substituting $g_j = \gamma_j \cdot \left(\bar{b} - b_j\right)$ into (B.8) and (B.10), respectively.

■

B.3 Steady State

B.3.1 World Interest Rate

For the analysis, we rewrite the law of motion of debt in Lemma 1 as follows:

$$\bar{b}_j - b_j' = \phi_j^* \cdot \left(\bar{b}_j - b_j\right),$$

where

$$\phi_j^* = \phi^*(n_j, R) \equiv (\gamma_j^*)^{\frac{1}{\sigma}} \left(\frac{1+n_j}{2+n_j} \cdot \beta\right)^{\frac{1}{\sigma}}, \quad j \in [0,1]. \tag{B.13}$$

We define $\bar{\phi}$ as

$$\bar{\phi} \equiv \max \left\{\phi^* \left(n_j, R\right)\right\}_{j \in [0,1]}.$$

Following the procedure in the main text, we determine the steady-state world interest rate as follows. First, there is no country with $\phi^* \left(n_j, R\right) > 1$. Otherwise, some countries would accumulate ever-increasing surpluses, while the other counties could accumulate debt up to $\bar{b}$ at most. This condition prevents the international asset market from clearing. Second, there is no R such that $\bar{\phi} < 1$. Otherwise, all countries would accumulate public debt up to $\bar{b}$, which prevents the international asset market from clearing. Thus, the steady-state equilibrium interest rate, R^*, satisfies $\bar{\phi} = 1$.

Substituting $\bar{\phi} = 1$ into (B.9) enables us to express the condition that determines the equilibrium interest rate as follows:

$$\frac{R^*}{1+n_{j^*}} = 1 + \frac{2+n_{j^*}}{(1+n_{j^*})\beta} \cdot \left[1 + \frac{1+n_{j^*}}{2+n_{j^*}} \cdot \left\{1 + \beta(\beta R^*)^{\frac{1-\sigma}{\sigma}}\right\} \cdot \theta^{-\frac{1}{\sigma}}\right], \tag{B.14}$$

where j^* denotes a country that accumulates debt below $\bar{b}$ and satisfies $\bar{\phi} = 1$. Other countries accumulate debt up to $\bar{b}$ and attain $\bar{\phi} < 1$. We take the distribution of debt as given, and thus country j^* is not specified at this moment.

Hereafter, we assume $1/\sigma \in (0,2)$ as in the main analysis. The following lemma establishes the condition for the existence and uniqueness of the steady-state equilibrium interest rate, R^*, for a given distribution of debt in the world economy.

Lemma B2. *Assume* $1/\sigma \in (0, 2)$. *Given the distribution of debt, there is a unique steady-state equilibrium interest rate,* $R^* \geq 1$, *satisfying* (B.14).

Proof.

First, suppose that $1/\sigma \leq 1$ holds. We denote the right-hand side of (B.14) by $RHS(R^*)$ with $RHS(1) > 1/(1 + n_{j^*})$. Moreover, $RHS(R^*)$ is decreasing in R^* and satisfies $\lim_{R^* \to \infty} RHS(R^*) \in (1, +\infty)$. Therefore, when $1/\sigma \leq 1$, there is a unique $R^* \in (1, +\infty)$ satisfying Eq. (B.14).

Next, suppose that $1/\sigma \in (1, 2)$ holds. The first- and second-order differentials of $RHS(\cdot)$ with respect to R^* are

$$
\begin{aligned}
&\frac{\partial RHS(R^*)}{\partial R^*} = \beta^{\frac{1}{\sigma}-1}\theta^{-\frac{1}{\sigma}} \cdot \frac{1-\sigma}{\sigma} \cdot (R^*)^{\frac{1-2\sigma}{\sigma}} > 0, \\
&\frac{\partial^2 RHS(R^*)}{\partial (R^*)^2} = \beta^{\frac{1}{\sigma}-1}\theta^{-\frac{1}{\sigma}} \cdot \frac{1-\sigma}{\sigma} \cdot \frac{1-2\sigma}{\sigma} \cdot (R^*)^{\frac{1-2\sigma}{\sigma}} < 0, \\
&\lim_{R^* \to \infty} \frac{\partial RHS(R^*)}{\partial R^*} = 0.
\end{aligned}
$$

Therefore, there is a unique $R^* \in (1, +\infty)$ satisfying Eq. (B.14) when $1/\sigma \in (1, 2)$.

■

B.4 International Differences in the Population Growth Rate and Public Debt Distribution

The policy function $B(b_j; n_j, R)$ in Lemma 1 suggests that the relative political weight on the old influences the public debt distribution through ϕ_j^*. To isolate its effect from the interest rate effect, we take R as given and investigate the effect of n_j on ϕ_j^*. We obtain the following proposition.

Proposition B1. *Countries with the lowest population growth rate,* $n_j = \underline{n}$, *accumulate public debt below* $\bar{b}$, *whereas the other countries accumulate public debt up to* $\bar{b}$.

Proof.

Recall that γ_j^* satisfies (B.12). We differentiate this with respect to γ_j^* and n_j and obtain

$$\frac{\partial \gamma_j^*}{\partial n_j} = -\frac{\left(\gamma_j^*\right)^{\frac{1}{\sigma}} \beta^{\frac{1}{\sigma}} \frac{1}{\sigma} \left(\frac{1+n_j}{2+n_j}\right)^{\frac{1}{\sigma}-1} \frac{1}{(2+n_j)^2} + \gamma_j^* \frac{1}{(2+n_j)^2} \left\{1 + \beta \left(\beta R\right)^{\frac{1-\sigma}{\sigma}}\right\} \theta^{-\frac{1}{\sigma}} + \frac{R}{(1+n_j)^2}}{\frac{1}{\sigma} \left(\gamma_j^*\right)^{\frac{1}{\sigma}-1} \left(\frac{1+n_j}{2+n_j}\beta\right)^{\frac{1}{\sigma}} + 1 + \frac{1+n_j}{2+n_j} \left\{1 + \beta \left(\beta R\right)^{\frac{1-\sigma}{\sigma}}\right\} \theta^{-\frac{1}{\sigma}}}. \tag{B.15}$$

With the use of (B.13) and (B.15), we obtain

$$\begin{aligned}
\frac{\partial \phi_j^*}{\partial n_j} &= \frac{1}{\sigma} (\gamma_j^*)^{\frac{1}{\sigma}-1} \frac{\partial \gamma_j^*}{\partial n_j} \left(\frac{1+n_j}{2+n_j}\beta\right)^{\frac{1}{\sigma}} + \left(\gamma_j^*\right)^{\frac{1}{\sigma}} \beta^{\frac{1}{\sigma}} \frac{1}{\sigma} \left(\frac{1+n_j}{2+n_j}\right)^{\frac{1}{\sigma}-1} \frac{1}{(2+n_j)^2} \\
&= \frac{1}{\sigma} (\gamma_j^*)^{\frac{1}{\sigma}-1} \left(\frac{1+n_j}{2+n_j}\beta\right)^{\frac{1}{\sigma}} \cdot \left\{\frac{\partial \gamma_j^*}{\partial n_j} + \frac{\gamma_j^*}{(1+n_j)(2+n_j)}\right\} \\
&= \frac{1}{\sigma} (\gamma_j^*)^{\frac{1}{\sigma}-1} \left(\frac{1+n_j}{2+n_j}\beta\right)^{\frac{1}{\sigma}} \cdot \frac{\gamma_j^*}{(1+n_j)(2+n_j)} \\
&\quad \times \left\{1 - \frac{\frac{1}{\sigma}(\gamma_j^*)^{\frac{1}{\sigma}-1} \left(\frac{1+n_j}{2+n_j}\beta\right)^{\frac{1}{\sigma}} + \frac{1+n_j}{2+n_j} \left\{1 + \beta \left(\beta R\right)^{\frac{1-\sigma}{\sigma}}\right\} \theta^{-\frac{1}{\sigma}} + \frac{2+n_j}{\gamma_j^*(1+n_j)} R}{\frac{1}{\sigma}(\gamma_j^*)^{\frac{1}{\sigma}-1} \left(\frac{1+n_j}{2+n_j}\beta\right)^{\frac{1}{\sigma}} + 1 + \frac{1+n_j}{2+n_j} \left\{1 + \beta \left(\beta R\right)^{\frac{1-\sigma}{\sigma}}\right\} \theta^{-\frac{1}{\sigma}}}\right\} \\
&= \frac{1}{\sigma} (\gamma_j^*)^{\frac{1}{\sigma}-1} \left(\frac{1+n_j}{2+n_j}\beta\right)^{\frac{1}{\sigma}} \cdot \frac{\gamma_j^*}{(1+n_j)(2+n_j)} \\
&\quad \times \frac{1 - \frac{2+n_j}{\gamma_j^*(1+n_j)} R}{\frac{1}{\sigma}(\gamma_j^*)^{\frac{1}{\sigma}-1} \left(\frac{1+n_j}{2+n_j}\beta\right)^{\frac{1}{\sigma}} + 1 + \frac{1+n_j}{2+n_j} \left\{1 + \beta \left(\beta R\right)^{\frac{1-\sigma}{\sigma}}\right\} \theta^{-\frac{1}{\sigma}}}.
\end{aligned}$$

From (B.9), we obtain

$$\begin{aligned}
&\frac{2+n_j}{\gamma_j^*(1+n_j)} \cdot R \\
&\quad = (2+n_j) + (1+n_j) \left\{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right\} \theta^{-\frac{1}{\sigma}} + (2+n_j)(\gamma_j^*)^{\frac{1}{\sigma}-1} \left(\frac{1+n_j}{2+n_j}\beta\right)^{\frac{1}{\sigma}} > 1.
\end{aligned}$$

Therefore, we obtain $\partial \phi_j^* / \partial n_j < 0$.

Recall that countries with the highest ϕ_j^* accumulate debt below $\bar{b}$ and satisfy $\bar{\phi} = 1$ (Section B.1). With $\partial \phi_j^* / \partial n_j < 0$, we can conclude that countries with $n_j = \underline{n}$ accumulate public debt below $\bar{b}$, whereas the other countries accumulate public debt up to $\bar{b}$. This finding implies that a higher population growth rate is associated with a higher level of public debt.

■

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Country	**Age gap in voter turnout**	**Country**	**Age gap in voter turnout**
ITA	−3.51	USA	14.37
BEL	−1.49	FRA	15.75
AUS	−0.35	IRL	17.82
NLD	1.18	PRT	18.61
DEU	2.11	POL	19.17
SWE	2.64	SVN	19.55
HUN	4.62	ISR	19.76
DNK	5.74	CHE	19.87
ESP	7.17	FIN	20.33
CZE	11.06	KOR	22.79
NOR	12.29	JPN	25.20
CAN	13.12	GBR	38.18
NZL	13.71	Average	12.79

Table 1: The percentage-point difference in voting rates between those aged +55 years and those aged 16–35 years among OECD countries. A higher percentage difference in voting rates implies greater political power of old citizens. Data source: OECD Society at a Glance 2011.

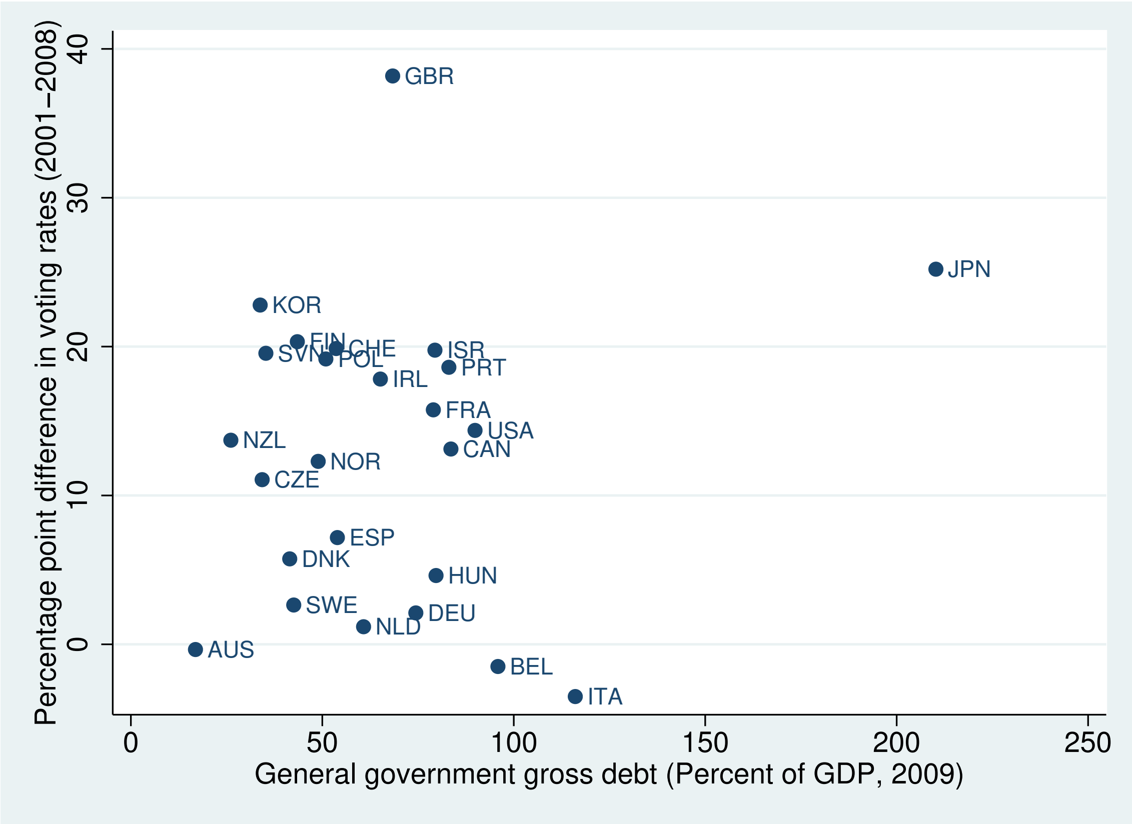

Figure 1: This scatter plot shows the relationship between the percentage-point difference in voting rates between those aged +55 years and those aged 16–35 years and the government debt-to-GDP ratio. A higher percentage difference in voting rates implies greater political power of old citizens. Data source: OECD Society at a Glance 2011, International Monetary Fund, World Economic Outlook Database, April 2012

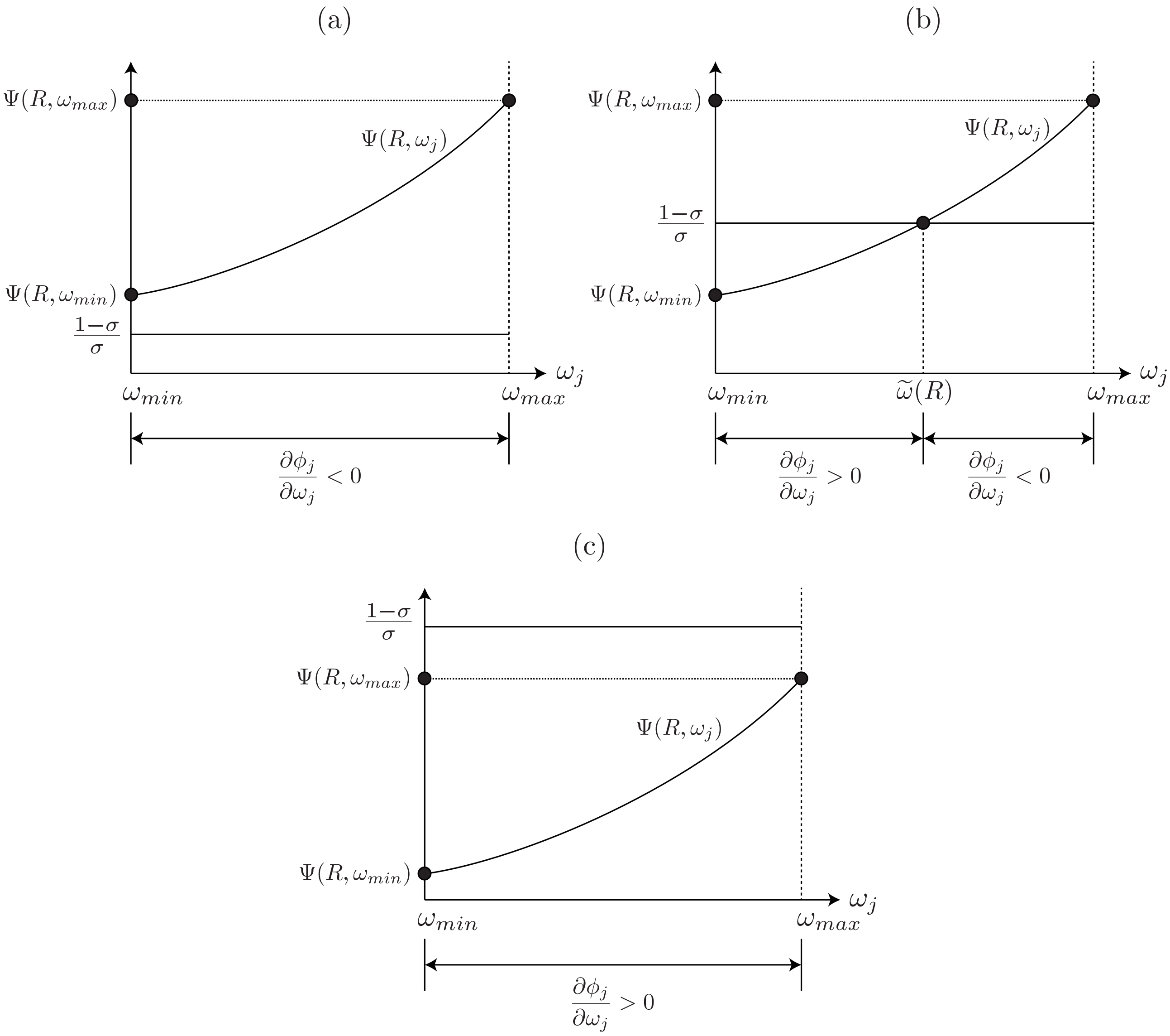

Figure 2: This figure shows the relationship between $\omega(p_j)$ and ϕ_j. Panel (a) shows the case of $(1-\sigma)/\sigma \leq \Psi(R, \omega_{min})$; panel (b) shows the case of $\Psi(R, \omega_{min}) < (1-\sigma)/\sigma < \Psi(R, \omega_{max})$; and panel (c) shows the case of $\Psi(R, \omega_{max}) \leq (1-\sigma)/\sigma$.

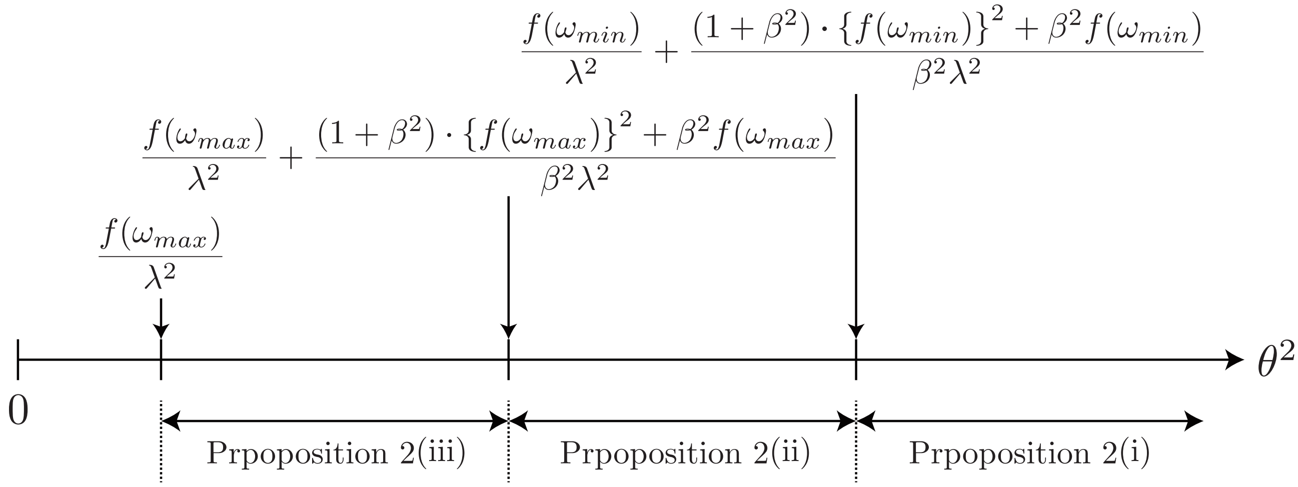

Figure 3: The equilibrium pattern under $\sigma = 1/2$.

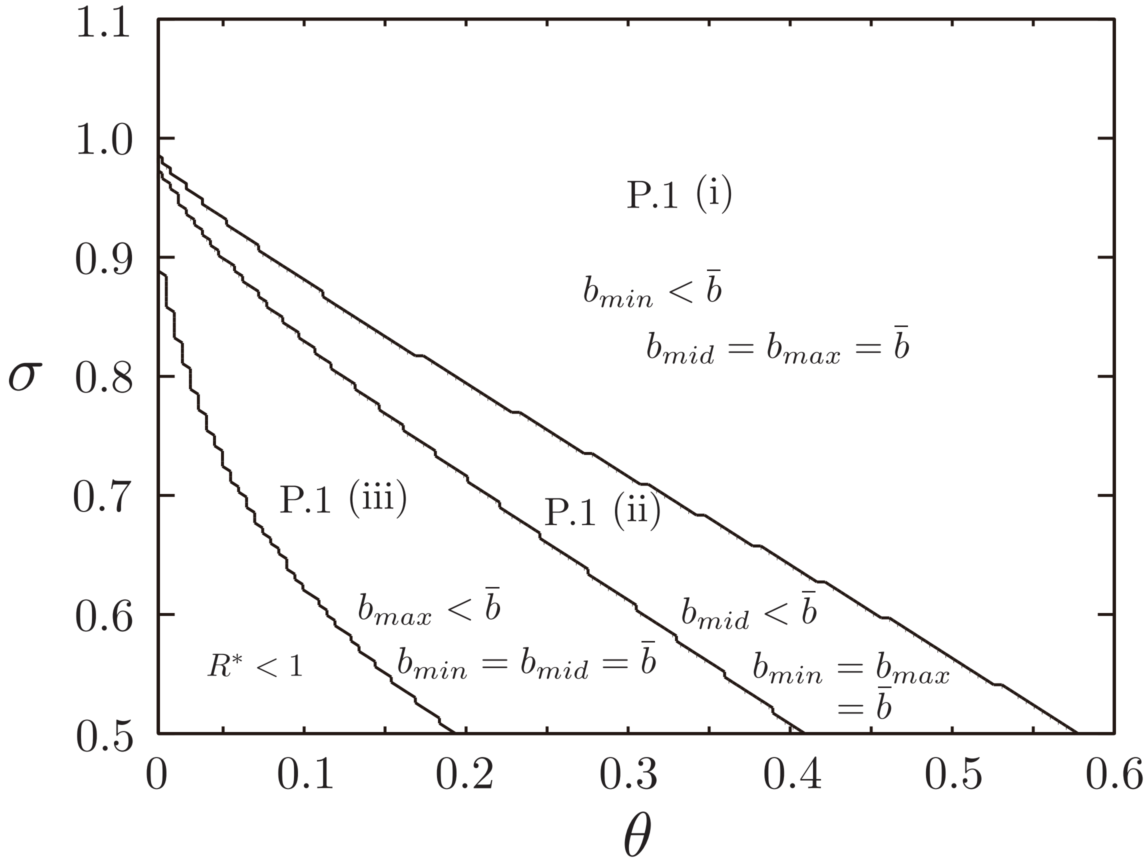

Figure 4: A numerical example of the equilibrium patterns.