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Abstract

This paper presents a dynamic general equilibrium model to investigate the co-evolution of employment and financial systems in the process of economic development when firms' commitment to financial and labor contracts is limited. We show that equilibrium modes of financial and labor contracts endogenously change from the informal contracting phase in which both of them are implicitly self-enforced to the formal contracting phase in which they are formally enforced and become more market-based as economies develop well. Furthermore, the formal contracting phase is irreversible in the sense that, once the economy enters that regime, it never returns back to the informal contracting phase.

Keywords: Dynamic General Equilibrium, Insider Lending, Implicit and Explicit Labor Contracts, Market Lending

JEL Classification Numbers: D86, J41, J64

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1 Introduction

In this paper we present a dynamic general equilibrium framework to investigate the co-evolution of employment and financial systems over time in the endogenous process of economic development when firms' commitment to financial and labor contracts is limited. Among others both financing and labor are the ones which play crucial factors in different phases of economic development. In particular, labor and financial arrangements are not governed by formally written contracts but they rely on non-contractual relations in emerging and growing stages of economic development. [1]

Regarding financial arrangements in the U.S. economic history, Lamoreaux (1994) reported that in the late 19th century the firms in New England relied on *insider lending* that they borrowed from the banks who had personal connections with them. As the U.S. economy expanded in the early 20th century, they however shifted toward borrowing from those whom they did not know personally, i.e., they changed toward more market-based financing. Allen (2001) also pointed out that major German banks such as Commerzbank, Dresdner and Deutsche grew rapidly in the 19th century by developing long-term relationships with industry enterprises. [2] Also, it has been often argued that the Japanese financial system after the Second World War was characterized by the bank-oriented system in which so called the main banks engaged in not only lending to the client firms but also helping their managements in several ways such as rescuing them in the case of financial distress and being represented as directors on the boards of these client firms (Aoki (1994), Aoki and Patrick (1994)). These lending practices were not necessarily based on formally written contracts but rather on informal and implicit agreements between the firms and main banks. Although such bank-oriented system supported the rapid economic growth in Japan during 1960s, it has been replacing by more market-based financing such as issuing bonds, convertibles and warrants since the late 1970s (see Hoshi and Kashyap (2004), Rajan and Zingales (2004)).

Employment systems have also evolved in the process of economic development (see for example Gordon (1985) and Jacoby (1997)). Moriguchi (2003, 2005) argued that the Japanese and U.S. firms had maintained the implicit labor contracts with employed workers during 1920s. For example, they had provided employment security, several fringe benefits (company housing, health care, etc), and so on, to employed workers but the great recession in 1930s more seriously hit the U.S. firms than the Japanese firms so that the former was forced to breach the implicit labor contracts and adopted more formal and explicit

[1] Greif (2006) investigates how informal contracting arrangements functioned in the Western economic history when formal institutions such as courts were absent. Johnson, McMillan and Woodruff (2002) and McMillan and Woodruff (1999a, 1999b) provide the evidence about the roles of non-contractual relations in developing and emerging economies. See also Macaulay (1963) for a classical observation about how non-contractual relations are widely used in practice.

[2] These German banks offered the firms low interest rates and were represented as directors on the boards of them. Such close and lasting relationships between large banks and firms contributed to the rapid expansion of the German economy between the late 19th century and the First World War. Maurer and Haber (2007) also provide related empirical evidence about Mexican banks during similar periods.

labor contracts. [3] On the other hand, the Japanese firms still maintained the implicit labor contracts even after the Second World War. [4] Although the Japanese firms relied on the implicit labor practices such as implicit employment security and seniority-based pay during the rapid growth periods in 1960s, they have introduced more performance-based pay system since 1990s (see for example Abe (2007) and Abe and Hoshi (2007)). [5]

As the above historical facts show, both employment and financial systems of an economy might change from the implicit-based contracts to the explicit-and market-based contracts over time as the economy grows and matures over time. In this paper we present a dynamic general equilibrium model to account for how employment and financial systems do not separately evolve but they interact with each other in the process of economic development. The previous literature has also emphasized the complementary roles of financial and employment systems. For example, Aoki (1994) discussed that the Japanese main bank system was institutionally complement with its employment system characterized by the imperfect labor market and long term employment relationships. Our paper extends this view further to understand the complementary roles of employment and financial systems in a dynamic perspective. To our best knowledge, our paper is the first attempt to investigate the dynamic macroeconomic implications about the co-evolution of employment and financial systems.

There are two key features of financial and labor contracts in our model: first, firms can make only limited commitment to both financial and labor contracts. Firms have the incentive to renege on not only the implicit labor contracts with employed workers but also the implicit financial contracts with lenders (financiers). Then the self-enforcing condition must be satisfied such that firms voluntarily honor the implicit labor and financial promises at the same time, which we call *dynamic enforcement* constraint. Second, firms can rely on both implicit and explicit contracts for financing and labor. Workers are motivated to work hard by the explicit wage contingent on a verifiable signal, for example their objective job performances, as well as the implicit wage contingent on an unverifiable signal, for example, the acquired firm-specific skill. Here, the verifiable signal is assumed to be less informative than the unverifiable signal regarding worker's effort choice. For example, the firm-specific skill acquired by a worker may reflect more directly his or her effort exerted for an assigned job rather than his or her job performance which is determined by the worker's firm-specific skill and further stochastic noises. Thus, the implicit wage is less costly to elicit workers' efforts than the explicit one unless enforcement problem arises. Then, firms try to use the implicit wage rather than the explicit one but the former is limited by the presence of the self-enforcing constraint. In addition, firms finance capital investment by not only *market lending* in an anonymous competitive credit market but also *insider lending* with

[3] See also Jacoby (1997) about the formation of employment relationships in the early stages of economic development in U.S.

[4] Gordon (1985) describes the history of labor relations in Japan. See also Dore (1973) for a comparison between the labor practices in the Japanese and U.K. firms.

[5] Abe and Hoshi (2007) reported that the use of seniority wages is recently declining and is being replaced by more performance-based pay in the Japanese firms. Also the Japanese firms have focused more on the importance of personnel management because they need to evaluate workers' performances in the more accurate way which are based on their wages.

particular financiers who lend to them and help their managements in the informal and unverifiable manners. Insider lending improves the managerial productivity of the client firm more than market lending. Thus firms try to use insider lending rather than market lending but, since firms cannot commit to make repayment to financiers under insider lending, its financing capacity is limited by the self-enforcing constraint again.

The novel feature of our model is that the implicit wage and insider lending interact with each other through the dynamic enforcement constraint. Each period young entrepreneurs purchase the firm ownerships from old entrepreneurs at its market price and becomes new owners of the firm. When the old entrepreneurs owning a firm breaches the implicit contracts with existing workers and financiers, all young workers newly employed by the firm may quit that firm in which case the firm cannot produce at all in the next period. This in turn triggers the punishment that the firm ownership will be not sold to the next generation at a positive price. Then, the old entrepreneurs owning a firm have stronger incentive to self-enforce the implicit wage and insider lending when they expect to sell the firm at a larger market value by honoring these contracts relative to the gain of deviating from them. [6] When firms raise more capital by borrowing from financiers via insider lending, the dynamic enforcement constraint becomes tighter so that they find it difficult to commit to pay larger implicit wages to workers. Thus, as firms invest more in capital as the economy grows, they shift to rely on market lending for financing capital investment and more on the explicit wage rather than the implicit wage.

We then show that there exist multiple equilibrium paths which change over time from the informal contracting phase in which firms rely on implicit wage and insider lending to the formal (market) contracting phase in which they resort to explicit wage and market lending. As firms depend more on explicit wage and market lending in such equilibrium paths, they invest in more capital so that the economy develops well over time. We also show that the change toward more formal and market-based contracts in both financing and labor is irreversible in the sense that the economy never returns back to the informal contracting phase once it enters the formal contracting regime together with a large capital accumulation. Thus our theoretical results confirm the aforementioned historical evidence that both financial and labor contracts tend be more formal and market-based from the informal and implicit ones as economies grow and mature over time.

These results are obtained due to the sequences of self-fulfilled expectations about the firm market values as follows. [7]

In the informal contracting phase, firms pay implicit wages to employed workers and finance capital investment by insider lending. However, capital raised by insider lending is constrained by the dynamic enforcement constraint to ensure that firms honor the agreed upon implicit financial contracts. When a firm expects a lower firm value in the next period $t + 1$, it cannot commit to larger repayments to financiers under insider lending, resulting in smaller capital investment in the current period t. Then, a lower demand for

[6]Our model is related to the literature emphasizing the role of the firm reputation as an asset. See Kreps (1990) and Tadelis (1999, 2002)) for such issues.

[7]More formally, we will define the firm value as the market price of a firm ownership divided by the aggregate outputs of the economy in the previous period.

capital investment in the current period allows more credit supply to use for purchasing the firm ownerships (firm stocks), leading to a larger firm value in the current period t. In this way, the expectation about a lower future firm value is associated with a larger current firm value. Such expectation about a low firm value in period $t + 1$ is then self-fulfilled by the expectation about a high firm value in further two periods ahead $t + 2$, and so on. These sequences of self-fulfilled expectations that high and low firm values alternate over time constitutes a part of equilibrium paths in the informal contracting phase.

On the other hand, as firms increase capital investment beyond the level constrained by insider lending, they start to finance some of capital investment by market lending in addition to insider lending. Then they enter the formal contracting phase. In that phase, when firms expect a lower firm value in the next period $t + 1$, the dynamic enforcement contract is tighten in the current period t so that they must reduce capital financed by insider lending but rely more on market lending. The increase in the demand for market lending reduces the credit supply in the whole economy which becomes available to purchase the firm ownerships. Then the firm value in the current period t must decrease. Moreover, as firms increase capital investment further, they find it difficult to commit to both insider lending and implicit wage at the same time. Then, they shift to rely on market lending and explicit wage over time. However, explicit wage is more costly than implicit wage because the former depends on more noisy signals regarding workers' efforts than the latter. This reduces the firm's flow profit, thus resulting in a further decline in the current firm value when a lower future firm value is expected to tighten the dynamic enforcement constraint. In this way, the expectation about a low firm value in the future period $t + 1$ is associated with a lower firm value in the current period t in the formal contracting phase. The expectation about a low firm value in period $t + 1$ is self-fulfilled by the expectation about a low firm value in $t + 2$, and so on. Again, these sequences of self-fulfilled expectations constitutes a part of equilibrium paths in the formal contracting phase.

The above sequences of self-fulfilled expectations about the firm values cause multiple equilibrium paths: some of them start with higher firm values in the informal contracting phase and enter the formal contracting phase along with declining firm values over time. Then the credit demand needed to purchase the firm ownerships falls over time as the firm values decline. This implies that more credit supply becomes available to finance new capital investment in the whole economy, leading to capital accumulation and economic development. Thus there exist equilibrium paths which exhibit the feature that organizational modes of labor and finance change from the informal contracting phase to the formal contracting phase over time as capital proceeds to increase over time. This theoretical insight implies that, as economies grow and mature over time, employment and financial systems co-evolve toward more explicit-and market-based ones, confirming the aforementioned historical facts.

Related Literature. The literature of relational contracts has attracted different applications of economic models in which contracts are not formally enforced but self-enforced via repeated agreements (Levin (2003)). Some of these papers have considered the implications about labor contracts (McLeod and Malcomson (1989, 1998), Malcomson (2013)). Others have investigated the features of dynamic lending contracts (for example Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2006)). Also there are some studies which deal with the interactions between explicit and relational (implicit) contracts (Baker, Gibbons and Murphy (1994), Pearce and Stacchetti (1998), Schmidt and Schnitzer (1995)). However, most of these applications have focused on relational labor contracts and relational financial contracts separately, which is in contrast to our paper addressing both of these contracts in an integrated framework. The recent paper by Barron and Li (2015) considers both relational labor and financial contracts but they do not take into account the interactions between relational and explicit contracts. Also our paper differs from them in that our main purpose is to investigate macroeconomic implications about the interactions between financial and labor contracts with limited commitment. In addition, our model is related to the papers which address the firm reputation as an asset (Kreps (1990), Tadelis (1999, 2002)). Our model builds the argument that the firm maintaining a good reputation can be sold at higher market price into the dynamic general framework and consider its macroeconomic implications.

Our paper is also related to the studies which deal with macroeconomic implications about self-enforcing debt (Hellwing and Lorenzoni (2009), Jeske (2006)) and long term labor contracts (Francois and Roberts (2003)). Again, these existing papers have focused on either self-enforcing financial contract or self-enforcing labor contract, in contrast to our paper

Furthermore, as we have already mentioned, our paper is inspired by the studies on institutional complementarity (for example Aoki (1994)). These studies have identified an economic institution as a set of complementary sub-systems. In particular Aoki (1994) discussed that the Japanese employment system characterized by the imperfect labor market and long term employment is institutionally complement with the bank-oriented financial system after the Second World War. Our paper extends this view to the dynamic framework and consider the joint evolution process of employment and financial systems over time.

The rest of the paper is organized as follows: in section 2 we set up the basic model. In section 3 we characterize the optimal financial and labor contracts given the market-determined variables. Then in section 4 we embed the optimal contracts into the dynamic general framework and in section 5 we derive full equilibrium dynamics. Section 6 concludes the paper. All formal proofs and some extensions are relegated to the appendix.

2 Model

2.1 Economic Environment

We consider an overlapping generations economy with a single good which is used both for consumption and investment. Time is discrete and extends over infinity, denoted by $t = 0, 1, 2, ...$ Each period young capitalists and young workers with one unit mass each are newly born. They live for two periods and are concerned with consumption only when old. Among one unit mass of young capitalists, $1 - N_f$ are *entrepreneurs* who run the firms and the remaining N_f are *financiers* who help financing the firms (we will explain this in more details below.) We assume that there are N firms in the economy, which are identified as N distinct projects or N physical assets, and that N is fixed over time.

Each firm has one job to hire at most one old worker for completing its production. The old worker employed by a firm exerts an unverifiable binary effort $a_t \in \{0, 1\}$. Then a firm produces the output according to the following production function:

$$y_t = h(a_t)k_t^{\gamma} \tag{1}$$

where $y_t \geq 0$ denotes the output, $h(a_t)$ the productivity of an employed old worker and $k_t \geq 0$ capital invested respectively. Here we assume that $\gamma \in (0, 1)$ and $h(1) \equiv 1 > \underline{s} \equiv h(0)$. Also we assume that capital must be invested in one period advance before production and it fully depreciates within one period.

In the economy there are three markets as follows:

Labor Market: We assume that hiring workers must be made in advance before they become old, i.e., the firms need to hire young workers although they work when old. For example, workers need to be trained when young before they become old and actually work at assigned jobs. Then we model the hiring process in the labor market in the simple way as follows: a certain number of young workers arrives at each firm in every period. Then each firm hires only one young worker among those who visited it. Since there are N firms each of which employs one worker, N young workers are employed while $1 - N$ young workers are unemployed where we assume $N \leq 1$.

Firm Ownership (Stock) Market: There is a competitive market for ownerships of the firms to be traded. Such market might be interpreted as a stock market where individuals buy and sell the stocks of a firm at a given market price. Without loss of generality, we normalize the number of ownership rights of each firm to unity. We denote by V_t the market price of a firm at which each old individual having an ownership of the firm sells it to young individuals in the end of period $t - 1$. [8]

In what follows we will also confine our attention to the case that only entrepreneurs trade the firm ownerships but financiers and workers do not. This assumption does not lose any generality because, by the no-arbitrage condition (2) defined below, every individual becomes indifferent for purchasing the ownership of a firm and not purchasing it in equilibrium (see more details below).

Credit Market: There is an anonymous competitive credit market where everyone can

[8] As an alternative scenario, we can consider the bilateral bargaining in which old individuals are randomly matched with young individuals and make the take-it-or-leave-it offer of the price at which the latter buys the firm ownerships from the former. This situation can be interpreted as so called negotiated block trade (Au, Fong and Li (2015)). This alternative setting does not substantially change the following results.

freely borrow and lend at a given market (gross) interest rate r_t: one unit of good is traded today in exchange for r_t units of good tomorrow. When the firms finance capital investment by borrowing from the competitive credit market, we call this *market lending*. Market lending is based on formally written financial contract and hence is legally enforceable. The firms can commit to make the repayment of market interest rate r_t.

There are two means for the firms to finance capital investments. One is market lending in the anonymous competitive credit market we have explained above and the other is what we call *insider lending* as follows. [9]

Insider Lending: The firms can finance capital investments from financiers who have specific knowledge to help the managements of client firms, which cannot be provided by market lending. For example, financiers not only lend to the client firms but also they provide managerial advices to the client firms and are represented as directors on the boards of them, which help improving the management efficiency of the client firms. We call these inputs provided by financiers to the client firms *relation-specific capital*. However, such relation-specific capital is hard to be verified outside particular firm-financier relationships. Thus the firms cannot commit to formal financial contracts contingent on relation-specific capital provided by financiers. In order for the firms to finance from financiers, the firms simultaneously approach financiers and offer financial contracts to them where such contracts are enforced in the self-interest way of the firms (as we will see below.)

As one possible interpretation of insider lending, one might think the bank-firm relationships typically observed in Germany and Japan: The banks, called the main banks, send directors to the client firms, own the shares of them, rescue them when they are in financially distress, and so on (see Aoki and Patrick (1994), Hoshi and Kashyap (2004) for the case in Japan). These activities performed by the main banks are not formally written agreements with the firms but implicitly enforced between them. The firms and main banks usually engage in lasting relationships which allow them to enforce the implicit financial contracts in the self-interest manner. [10]

The advantage of insider lending relative to market lending is that the former can lend to the firms in more productive manner than the latter by providing relation-specific capital as we have mentioned above. We capture such gain of insider lending by assuming that, in order to provide one unit of relation-specific capital, financiers need only $\lambda \in (0, 1)$ units of good. [11] Under market lending one unit of good is used as one unit of capital

[9] More precisely, the firms can finance capital investments by issuing equities and selling them at the firm ownership (stock) market. However, such equity financing is equivalent to borrowing from the competitive credit market as shown in Appendix B. This is because the no-arbitrage condition (2), which we introduce below, holds so that the market interest rate equals to the rate of stock return, implying that the firms are indifferent for borrowing in the credit market and issuing new equities. Thus we will not consider equity financing in what follows.

[10] In that sense the firm-bank relationships are often called relational financing (Aoki and Patrick (1994)).

[11] This assumption is justified as follows: suppose that a financier spends $\alpha > 1$ units of good for creating one unit of relation-specific capital and that one unit of relation-specific capital has its quality-adjusted

whereas only $\lambda \in (0,1)$ units of good is needed for one unit capital under insider lending. On the other hand, the disadvantage of insider lending is that, since relation-specific capital is not verifiable, the firms cannot formally commit to make repayments contingent on it. We denote by x_t relation-specific capital and by R_t repayment which the firm promise to pay to a financier under insider lending. We assume that, if the firm raises k'_t capital from market lending and x_t relation-specific capital from insider lending, its total capital $k_t \equiv k'_t + x_t$ matters for the production, i.e., $y_t = h(a_t)k_t^{\gamma}$ for $k_t = k'_t + x_t$. In other words relation-specific capital raised by insider lending is perfect substitute to capital raised by market lending.

Note also that financiers have no capacity limits for lending relation-specific capital to the firms because they can access to the competitive credit market and borrow the necessary fund at a given market interest rate r_t in order to lend relation-specific capital to the firms. Thus, even when multiple firms offer insider lending contracts to a single financier, the latter can make financial contracts and engage in insider lending with these firms at the same time.

We make the following assumptions on verifiability of capital and output: first, as we have mentioned, relation-specific capital x_t provided by financiers under insider lending is not verifiable as well as it is not verified that financier spent λx_t for creating relation-specific capital x_t. On the other hand, it is verifiable how much the firms borrowed from the credit market. Second, the total capital k_t and output y_t are assumed to be not verifiable as well. These restrictions on verifiability of capital and output imply that the firm cannot make repayments to financiers under insider lending contingent on the relation-specific capital x_t provided by the financiers and the output y_t produced by the firm. [12]

2.2 Endowment

To introduce the savings and close the model, we assume that young capitalists born in each period can access to the private technology for producing w_t units of good by

value $\rho > 1$ in the production per job. For example, if z relation-specific capital is used per job, the firm can produce the output $y = (\rho z)^{\gamma}$ per. Thus, by spending αz units of good, the financier can provide the relation-specific capital of z units which contributes to the production by its quality-adjusted value ρz. Let $x \equiv \rho z$ and assume that $\lambda \equiv \alpha/\rho < 1$. Then, λx units of good are needed to create x units of the quality-adjusted relation-specific capital. To avoid complication, we will not use the term "quality-adjusted" but we simply call $x_t \equiv \rho z_t$ the relation-specific capital in what follows.

[12] In addition, it does not help by making a repayment R_t to the financier contingent on his or her savings denoted by S_t where $w_{t-1} \geq S_t$. Let denote by $R(S_t)$ such repayment. Although the repayment can vary with the financier's savings, this does not ensure that the financier provides a relation-specific capital x_t: suppose that the firm wants to raise x_t relation-specific capital from a financier and induces him or her to save $\hat{S}_t$. Thus $w_{t-1} - \hat{S}_t \geq \lambda x_t$. If the financier follows this, he or she earns $r_t\hat{S}_t + R(\hat{S}_t)$. However, the financier can obtain at least this payoff even if he or she does not invest λx_t in relation-specific capital. Furthermore, the financier has the incentive to collude with the other party as follows: the financier passes amount of λx_t to the third party who then lends it to the credit market in the behalf of the financier. Such savings are not recorded on the account of the financier but on the account of the colluding third party so that the repayment to the financier $R(\hat{S}_t)$ is not affected. Then the financier and the third party can share the earned interest income $r_t\lambda x_t$ between them, which yields more than $r_t\hat{S}_t + R(\hat{S}_t)$ to the financier.

themselves. Here w_t depends on the aggregate productivity of the economy denoted by A_t and we assume that $w_t = LA_t$ where $L > 0$ represents the degree about how the capitalist's endowment reflects the aggregate productivity A_t. For simplicity, we assume that the aggregate productivity depends on the social knowledge embodied in the aggregate outputs produced in the current period, i,.e., $A_t \equiv \int_0^N y_t(i)di$ where $y_t(i)$ denotes the output of firm i at period t, through the learning-by-doing effect (Arrow (1962)). Then young capitalists are endowed with w_t units of good each in the beginning of period t. This simple setting is made only for avoiding unnecessary complication of the model. We can provide an alternative (and more reasonable) scenario which endogenously determines the endowment of young capitalists (see Appendix B).

2.3 Production and Moral Hazard

Each old worker is endowed with one unit of labor skill and can access to the private technology which produces ψA_t units of good by using one unit of endowed skill where $\psi > 0$. Here, as in the case of capitalists, the worker's private technology also reflects the aggregate productivity embodied in the aggregate outputs $A_t = \int_0^N y_t(i)di$. After old workers were employed by the firms, they decide whether to exert high or low effort for the productions, denoted by $a_t \in \{0, 1\}$, where $a_t = 1$ ($a_t = 0$) stands for high (low) effort. When an old worker exerts high effort $a_t = 1$, he must spend fully one unit of labor skill endowed with him. When exerting low effort $a_t = 0$, he does not needs to spend his endowed skill for the production but he can secretly spend it for his own private technology to earn ψA_t units of good.

The effort choice by old worker is non-verifiable but the firm can observe two types of the signal regarding it. One is the perfect but unverifiable signal $s_t = a_t \in \{0, 1\}$ about the worker's effort a_t. The firm can thus perfectly observe the worker's effort choice a_t but it is not verified outside the employment relationship. The other is the imperfect but verifiable signal denoted by $\sigma_t \in \{\sigma_h, \sigma_l\}$. Here high signal σ_h is realized with probability $q_a \equiv \text{Prob}(\sigma_h|a) \in (0, 1)$ conditional on an effort choice $a \in \{0, 1\}$. We assume that $1 > q_1 > q_0 > 0$, meaning that the verifiable signal σ is less informative than the unverifiable one s_t regarding the worker's effort $a_t \in \{0, 1\}$. We use the notation $\Delta q \equiv q_1 - q_0 > 0$.

We denote by $\{b_t(s_h), b_t(s_l)\}$ an implicit labor contract which specifies a wage $b_t(s)$ contingent on the worker's unverifiable signal s_t. Here $b_t(s) \geq 0$ must hold for each s by the worker's limited liability. Since the signal (effort) $s = a$ is not verified, such contract must be self-enforcing. We also denote by $\{v_t(\sigma_l), v_t(\sigma_h)\}$ an explicit labor contract which specifies a wage v_t contingent on the verifiable signal σ_t. Here $v_t(\sigma) \geq 0$ for each σ. The explicit wage $v_t(\sigma)$ is legally enforced.

More generally, we can think of worker's wage as $W_t(s, \sigma)$ which is contingent on both of these signals s and σ in the interdependent way. However, we can show that generality is not lost by restricting our attention to the additive separable form $W_t(s, \sigma) = b_t(s) + v_t(\sigma)$ (see Appendix B in more details). We can also show that $b_t(0) = v_t(\sigma_l) = 0$ and hence in what follows we will write $b_t \equiv b_t(1) \geq 0$ and $v_t \equiv v(\sigma_h) \geq 0$ and call them the implicit bonus and the explicit bonus respectively.

2.4 Observability and Quitting Option

All the wages paid to old workers and repayments paid to financiers are publicly observable. In addition, we assume that a young worker employed by a firm in period t can observe the unverifiable signal $s_t = a_t \in \{0,1\}$ of the old worker working at the same firm and the relation-specific capital x_t provided by the financier who engages in insider lending with the firm.

Furthermore, we assume that it is publicly observable whether or not the young worker employed by a firm in the previous period has been retained as the old worker by the same firm in the current period. Put differently, all other parties outside the firm can observe whether or not the young worker employed by the firm in the previous period has left the firm and has been hence not retained as old worker in the current period.

Finally, we assume that in the end of any period the firm and employed young worker simultaneously decide whether or not to exercise the quitting option: by the quitting option, it is meant that the young worker leaves the firm while the firm liquidates the firm's asset (project) in which case the firm cannot produce the outputs forever.

2.5 Market Price of Firm Ownership

The ownership rights of the firms are traded in the competitive stock market. Let V_t denote the market price of one unit ownership of a firm determined in the end of period $t-1$. Then, if a young individual purchases one unit ownership of a firm at its market price V_t in period $t-1$, he or she expects that he or she will obtain a flow profit (dividend) π_t from the production of the firm and can sell the ownership to the young generation at a market price V_{t+1} in the next period t when old. On the other hand, if he or she lends one unit of good to the anonymous credit market, then he or she earns the market interest rate r_t. Thus, the following no-arbitrage condition must be satisfied for the firm ownership (stock) market to clear: [13]

$$r_t = \frac{\pi_t + V_{t+1}}{V_t} \tag{2}$$

which means that the rate of gross return by holding one unit of the firm ownership must be equal to the gross interest rate r_t in the credit market. This can be re-written by

$$V_t = \frac{1}{r_t}\{\pi_t + V_{t+1}\}. \tag{3}$$

[13] If the left hand side of (2) is greater than its right hand side, then nobody buys the firm ownership, which implies that its market price V_t goes down to zero. But then the right hand side increases beyond $r_t > 0$. Also, if the reverse inequality holds, then young individuals demand to buy the firm ownerships as much as possible, resulting in an infinite demand. This however implies that its demand exceeds the supply and hence V_t must go up until the equality is preserved.

2.6 Timing

The events within each period t proceeds as follows:

1. The firm offers a labor contract $\{b(s_t), v(\sigma_t)\}$ to an employed old worker. Then the old worker chooses an effort $a_t \in \{0, 1\}$.

2. The firms hire young workers newly born in this period.

3. The signals of each employed old worker $s_t = a_t \in \{0, 1\}$ and $\sigma_t \in \{\sigma_h, \sigma_l\}$ are realized and observed to the firm, the old worker himself and the young worker newly employed by the firm. Then, the firm owned by old entrepreneurs produce the output $y_t = h(a_t)k_t^\gamma$ by using old worker's effort a_t and capital k_t invested in the previous period $t-1$. Then the firm decides whether or not to pay the implicit wage $b(s_t)$ to the old worker and repayment R_t to the financier who lent to the firm via insider lending.

4. After the payment decision by the firm, the newly employed young worker and the firm simultaneously decide whether or not to exercise the quitting option. If the firm exercises the quitting option, it liquidates the firm's asset and leaves the economy. If the young worker quits, he is unemployed when old.

5. It is publicly observed whether or not the young worker employed by the firm have left the firm. The old entrepreneurs owning the firm earn a flow profit π_t and sell the firm ownership to the young entrepreneurs at a market price V_{t+1}. Then the owners of the firms are changed from the old entrepreneurs to the young ones.

6. The firms owned by the young entrepreneurs simultaneously approach the young financiers and offer insider lending contracts $\{x_{t+1}, R_{t+1}\}$ to each of them, which specifies a relation-specific capital x_{t+1} to be provided by a financier and the corresponding repayment R_{t+1} to her. At the same time, the firm raises capital k'_{t+1} from market lending if necessary.

Then the economy moves to the next period $t+1$: capital k'_{t+1} and x_{t+1} invested in the previous period t will be used for the production in period $t+1$.

Remark. We are here assuming that the repayment R_t to financier is not contingent on the performance signals s_t and σ_t of employed old workers. This is a reasonable restriction because it is the realistic case that the repayment to lenders do not vary with employed workers' task performances. However, we can extend the model to allow such case and show that our main results still remain valid (see Appendix B).

Remark. Since financiers are identical, we can suppose without loss of generality that each firm approaches only one financier and offers an insider lending contract $\{x_t, R_t\}$ to her. Note that, since financiers have no capacity limit to lend to the firms, it does not matter how many firms they lend to via insider lending. In other words, each financier can lend to multiple firms at the same time. Financiers are willing to lend to a firm as long as they can earn at least what they obtain by lending to the credit market at the interest rate r_t.

In the following sections we will first derive the optimal financial and labor contracts chosen by the firm in a given period t, by taking the market interest rate r_t and the future market price of the firm V_{t+1} as exogenously given. Then we will embed the optimal contract into the dynamic general equilibrium model, which endogenously determines the equilibrium paths of the interest rates and market prices of the firms.

3 Optimal Financial and Labor Contracts

3.1 Constraints

Consider a firm in period t which is hiring an old worker retained from the previous period $t-1$. Suppose that the old worker exerted high effort $a_t = 1$ for the production. Thus the unverifiable signal (effort) $s_t = a_t$ is observed to the firm, in which case the firm must pay the implicit bonus $b_t \equiv b(1) \geq 0$ to the old worker according to the implicit wage contract. Also the firm must make the repayment R_t to the financier from whom it borrowed by insider lending.

When the firm reneges on the implicit bonus b_t to the old worker and repayment R_t to the financier following the liquidation, it can totally save these payments $b_t + R_t$. We denote by V_{t+1} the market price of the firm ownership (stock) when making the agreed upon payments b_t and R_t to the old worker and the financier. Then, for the firm to honor these payments, the following *dynamic enforcement* (DE) constraint must be satisfied

$$V_{t+1} \geq b_t + R_t. \tag{DE}$$

Otherwise, the firm does not pay b_t and R_t and then liquidates the firm's asset, yielding at least the gain of $b_t + R_t$. [14]

On the other hand, if DE is satisfied, then there exists an equilibrium in which the firm honors making the agreed upon payments b_t and R_t. To see this, suppose that the above DE holds and consider the following strategies of the firm and the young worker employed by the firm: The young worker stays in the firm in period t if the firm has paid the implicit bonus b_t to the old worker who worked at the same firm and chose high effort $a_t = 1$ as well as the firm has made the repayment R_t to the financier. However, otherwise the young worker quits the firm. Also, all the young workers who will be matched with the firm in any future period will quit the firm if they observe that some young workers

[14] One might consider an alternative timing of repayment to the financier as follows: the firm pays R_t first before the financier invests in relation-specific capital x_t. In this scenario the firm has no rooms to breach the financial contract. However, the financier will then breach the financial contract by not delivering x_t after he or she has received the repayment R_t. Such breach by the financier cannot be detected because there are no ways to punish the deviating financier: if the financier collects R_t and does not provide x_t when young, he or she will consume all the earned income without any punishment when old.

who were employed by the firm in the past periods have quit and have been hence not retained as old workers in the firm. Otherwise, they stay in the firm. The firm continues its production operation if it has paid the implicit bonus b_t to the employed old worker when the latter chose high effort $a_t = 1$, it has paid R_t to the financier in period t and it has retained the young worker employed in the previous period. Otherwise the firm liquidates its asset. Then, we can show that the strategies specified above constitute an equilibrium supported by the continuation equilibrium in which the firm and the young worker exercise the quitting option simultaneously after the firm reneged on b_t or R_t in period t. This triggers that all the young workers employed by the same firm in the future will exercise the quitting option in any future period, making the market prices of the firm down to zero in any future period (see Appendix A in more details). Thus, if DE is satisfied, it is not profitable for the firm to renege on b_t or R_t.

Next we turn to the incentive compatibility constraint for employed old workers. In what follows we will pay our attention to the case that the firm wants to implement high efforts from an employed old worker. [15] Note that $b_t(1) \equiv b_t \geq 0$ and $v_t(\sigma_h) \equiv v_t \geq 0$ but $b_t(0) = 0$ and $v_t(\sigma_l) = 0$. Then, given the firm honoring the implicit bonus b_t, the employed old worker chooses high effort $a_t = 1$ if

$$b_t + q_1 v_t \geq q_0 v_t + \psi A_t$$

where the left hand side denotes the expected payoff of the old worker when exerting high effort $a_t = 1$ while the right hand side denotes the expected payoff when exerting low effort $a_t = 0$ and spending his endowed labor skill secretly to the private technology which yields ψA_t outputs. When the old worker chooses high effort, he earns the implicit bonus b_t plus the expected explicit bonus $q_1 v_t$ while, when he exerts low effort, he obtains no implicit bonus but the expected explicit bonus $q_0 v_t$.

Thus the following old worker's incentive compatibility (WIC) must be satisfied for high effort $a_1 = 1$ to be implemented:

$$b_t + \Delta q v_t \geq \psi_t \equiv \psi A_t \tag{WIC}$$

where ψ_t denotes the opportunity cost of exerting high effort. Also, since old workers have no wealth, the following limited liability (LL) constraint must be satisfied: [16]

$$b_t \geq 0, \quad v_t \geq 0 \tag{LL}$$

In addition, each young financier born in period $t-1$ can always obtain at least what he or she earns by saving all his or her endowment w_{t-1} to the credit market, i.e., $r_t w_{t-1}$.

[15] Intuitively, when the old worker's productivity $\underline{s} = h(0)$ when he exerts low effort $a_t = 0$ is low enough, the firm has no incentive to implement low effort. See Appendix B for more details.

[16] In addition, old worker accepts the offered contract $\{b_t, v_t\}$ when the individual rationality constraint $b_t + q_h v_t \geq \psi A_t$ is satisfied where the old worker can obtain ψA_t by rejecting the contract and spending the endowed labor skill to his own private technology. However, by WIC and LL, this individual rationality constraint is satisfied so that we can ignore it.

Thus the following individual rationality (IR) constraint must be satisfied for each young financier to accept the insider lending contract $\{R_t, x_t\}$:

$$R_t + r_t(w_{t-1} - \lambda x_t) \geq r_t w_{t-1} \tag{IR}$$

Here we are assuming that young financiers do not purchase the firm ownerships. However, this is made without loss of generality because the no-arbitrage condition (2) means that it becomes indifferent for saving one unit of good at the interest rate r_t and buying one unit of the firm ownership at a market price V_t.[17] IR states that the young financier born in period $t-1$ spends λx_t units of good for providing x_t relation-specific capital under insider lending and then receives the repayment R_t. In addition, the financier saves (or borrows) the remaining amount $w_{t-1} - \lambda x_t$ to (from) the credit market which yields the interest earning $r_t(w_{t-1} - \lambda x_t)$.

The flow profit of a firm π_t is then given by

$$\pi_t \equiv k_t^\gamma - (b_t + q_1 v_t) - R_t - r_t(k_t - x_t). \tag{4}$$

Here k_t^γ is the output produced given high effort $a_t = 1$ exerted by the old worker. The second term is the expected total wage paid to an employed old worker who chooses $a_t = 1$. Also R_t is the repayment to a financier under insider lending, and $r_t(k_t - x_t)$ is the cost of market lending ($k_t \geq x_t$) respectively.

Finally, capital raised by market lending $k_t - x_t$ must be non-negative:

$$k_t \geq x_t \geq 0. \tag{NE}$$

Before proceeding further, we explain the key feature of DE constraint which plays the important role in what follows. By combining DE with WIC and IR, we obtain

$$\begin{aligned} V_{t+1} &\geq R_t + b_t \\ &\geq \lambda r_t x_t + \psi_t - \Delta q v_t. \end{aligned} \tag{5}$$

When the firm increases capital investment x_t financed by insider lending, the explicit bonus v_t must be increased as long as the above inequality (5) is binding. However, since the explicit bonus depends on more noisy signal than the implicit one, the increase in the explicit bonus v_t costs the firm more when it relies more on insider lending to finance capital. Then, as the firm invests more in capital, it shifts from insider lending to market lending. In this way, the explicit bonus v_t and relation-specific capital x_t are related with each other through DE, which causes the change from the regime of the implicit wage and insider lending to the regime of the explicit wage and market lending as we will see later.

[17] If a young financier purchases l_t^f firm ownerships, then he or she obtains $R_t + r_t(w_{t-1} - \lambda x_t - l_t^f V_t) + l_t^f(\pi_t + V_{t+1})$ when he or she accepts the insider lending contract. When he or she rejects it, he or she obtains $r_t(w_{t-1} - \tilde{l}_t^f V_t) + \tilde{l}_t^f(\pi_t + V_{t+1})$ where $\tilde{l}_t^f \neq l_t^f$ may be the case. However, since $r_t V_t = \pi_t + V_{t+1}$ by the no-arbitrage condition (2), we have the same condition as IR above.

3.2 Optimal Contracts

We next move to derive the contracts the firms offer to old workers and financiers. The firm offers a labor contract $\{b_t, v_t\}$ to an employed old worker as well as it approaches a financier and offers an insider lending contract $\{R_t, x_t\}$ in period t. We focus on the *optimal contract* that maximizes the current firm market price V_t or equivalently the flow profit π_t subject to WIC, DE, IR, LL and NE, given the market interest rate r_t and the firm's belief about its future market price V_{t+1}.

The optimal contracts depend on what beliefs the firm forms about the future market price V_{t+1}. To be consistent with the continuation equilibrium strategy specified in the previous subsection, we suppose that, when considering a contract offer $\{b_t, v_t, x_t, R_t\}$ for the production in period t, the firm has the belief about the market price V_{t+1} in the next period $t+1$ as follows: (i) V_{t+1} does not vary with what contracts the firm offers to old worker and financier in the current period t as long as the firm honors paying according to the offered contract, [18] and (ii) V_{t+1} will be zero when the firm reneges on the implicit bonus b_t or the repayment R_t because then the young worker employed in period t will quit the firm. These beliefs are consistent with the strategy of young workers specified in the previous subsection: the newly hired young worker will quit the firm if the latter reneged on the payments to the old worker and financier under the offered contract or the young worker employed in the previous period was not retained in the current period. When the participants in the stock market observe that the young worker hired by a firm has left the firm, they believe that such firm has lost the trust from employed workers, which in turn triggers the separation of all young workers in the future periods. Then, nobody purchases the ownership (stock) of such trust-losing firm in any future, resulting in zero market prices of the firm in the future.

There are two remarks on the above restrictions on the beliefs about the firm market prices as follows. First, young workers trust the firm as long as the latter does not breach the contracts offered to the employed old workers. Thus, even when the firm offers a different contract from the equilibrium one, it is not the object to be punished as long as the firm has honored the agreed upon implicit contracts. Second, the firm believes that its market prices will go down to zero in any future period once it reneged on the implicit bonus or repayment to financiers. According to the workers' strategy, all young workers will quit the trust-losing firm in any future period, resulting in zero production in the future. When the outputs of a firm will be zero forever, it is reasonable to suppose that its market price will be zero as well. In fact, a sequence $\{V_s\}_{s=t}^{\infty}$ such that $V_s = 0$ for all $s \geq t$ satisfies the no-arbitrage condition $V_t = (1/r_t)V_{t+1}$ when $\pi_s = 0$ for all $s \geq t$.

Given the above belief about the future market price of the firm V_{t+1}, the optimal contract solves the following problem for $t \geq 1$: [19]

[18] In fact, the contract used in period t does not affect the production in the next period $t+1$ so that it is reasonable to suppose that the market price V_{t+1} of the firm in the next period $t+1$ does not change with the contract in the current period t.

[19] In the initial period $t = 0$ old entrepreneurs are endowed an initial capital stock k_0 each which is used

Problem (P):

$$\max \;\; V_t = \frac{1}{r_t}\{\pi_t + V_{t+1}\}$$

subject to DE, WIC, IR, LL and NE, given r_t and V_{t+1}.

The immediate features of the optimal solution to Problem (P) are as follows: First, since a decrease in b_t can raise the firm's flow profit and weaken DE as long as WIC is satisfied, WIC must be binding, i.e., $b_t = \psi_t - \Delta q v_t$. By substituting this into the expected wage $b_t + q_1 v_t$, we obtain

$$b_t + q_1 v_t = \psi_t + q_0 v_t.$$

Second, since the reduction of R_t can increase the firm's profit and weaken DE as long as IR is satisfied, IR must be binding as well, i.e., $R_t = \lambda r_t x_t$. Then DE can be written by

$$V_{t+1} \geq R_t + b_t = \lambda r_t x_t + \psi_t - \Delta q v_t, \tag{6}$$

from which we have

$$v_t \geq (1/\Delta q)(\lambda r_t x_t + \psi_t - V_{t+1}).$$

Then, the explicit bonus v_t should be set as

$$v_t = \max\left\{\frac{1}{\Delta q}(\lambda r_t x_t + \psi_t - V_{t+1}), 0\right\}. \tag{7}$$

When the future market price of the firm V_{t+1} is larger, DE becomes weaker so that the optimal wage involves only the implicit bonus, i.e., $v_t = 0$ and $b_t = \psi_t$.

We substitute the above expression of v_t into the firm's flow profit (4) in order to re-write V_t as

$$V_t = \frac{1}{r_t}\left\{k_t^\gamma - \lambda r_t x_t - (k_t - x_t) r_t - \psi_t - \max\left\{\frac{q_0}{\Delta q}(\lambda r_t x_t + \psi_t - V_{t+1}), 0\right\} + V_{t+1}\right\}. \tag{8}$$

The optimal contract is characterized depending on whether or not DE becomes binding and whether or not the explicit bonus v_t is used. If DE can be ignored, the firm uses only insider lending to finance capital investment because it is more efficient than market lending.

For a further analysis, we define three key capital levels: first, we define the capital investment, denoted by k_t^*, which maximizes the firm's profit without DE ($k_t = x_t$), yielding the first order condition:

$$\gamma(k_t^*)^{\gamma-1} = \lambda r_t. \tag{9}$$

to produce the outputs $y_0 = h(a_0)k_0^\gamma$. When implementing high effort $a_0 = 1$, they offer the contract $\{b_0, v_0\}$ to the initial old worker to maximize the profit $\pi_0 = k_0^\gamma - (b_0 + q_1 v_0)$ subject to WIC, LL and DE: $V_1 \geq b_0$. When implementing low effort $a_0 = 0$, the profit becomes $\underline{s}k_0^\gamma$. The difference from subsequent periods is that DE in the initial period does not include capital raised by insider lending.

Second, when the firm finances capital investment only by market lending ($x_t = 0$), the optimal capital, denoted k_t^{**}, is given by the following first order condition:

$$\gamma(k_t^{**})^{\gamma-1} = r_t. \tag{10}$$

Here $k_t^* > k_t^{**}$ holds due to $\lambda < 1$. Third, we define the capital $\hat{x}_t$ which makes DE binding as follows

$$\hat{x}_t = \frac{1}{\lambda r_t}(V_{t+1} - \psi_t) \tag{11}$$

whenever $V_{t+1} \geq \psi_t$.

Throughout the paper, we will maintain the following assumption:

Assumption 1. $\lambda(1 + q_l/\Delta q) > 1$.

Assumption 1 says that the verifiable signal σ of each old worker's effort is not so informative, i.e., Δq is small, relative to the efficiency of insider lending captured by the parameter value $1 - \lambda$. One unit of capital can be raised by one unit of good via market lending but by only $\lambda \in (0, 1)$ units of good via insider lending. Thus, for financing one unit capital, $(1 - \lambda)r_t$ can be saved under insider lending relative to market lending, which is the benefit of using the former. On the other hand, when the explicit bonus v_t is paid together with insider lending, one unit increase in relation-specific capital x_t must be accompanied with the rise of the explicit bonus by the amount of $(q_l/\Delta q)\lambda r_t$ in expected term due to (7). Assumption 1 then ensures that such capital cost accompanied with the explicit bonus is greater than the benefit of insider lending $(1 - \lambda)r_t$. When Assumption 1 does not hold, the benefit of insider lending always dominates the capital cost accompanied with the explicit bonus, implying that market lending is never used for financing capital investment (see Appendix B for more details.) In order to make market lending viable, we will maintain Assumption 1 in the following analysis.

By solving Problem (P), we show the following result.

Lemma 1. *Suppose that Assumption 1 holds. The optimal contract solving Problem (P) is characterized as follows.*

(i) *Suppose that $V_{t+1} \geq \psi_t$ and $r_t^\gamma \geq (\gamma/\lambda^\gamma)(V_{t+1} - \psi_t)^{\gamma-1}$. Then capital investment is financed by only insider lending and attains the unconstrained optimum $k_t = k_t^*$ without DE. The optimal wage involves only the implicit bonus, $b_t = \psi_t$ and $v_t = 0$.*

(ii) *Suppose that $V_{t+1} \geq \psi_t$ and $(\gamma/\lambda^\gamma)(V_{t+1} - \psi_t)^{\gamma-1} > r_t^\gamma \geq \gamma\lambda^{1-\gamma}(V_{t+1} - \psi_t)^{\gamma-1}$. Then capital investment is financed by only insider lending and given by $k_t = \hat{x}_t$ where DE is binding. The optimal wage involves only the implicit bonus, $b_t = \psi_t$ and $v_t = 0$.*

(iii) *Suppose that $V_{t+1} \geq \psi_t$ and $\gamma\lambda^{1-\gamma}(V_{t+1} - \psi_t)^{\gamma-1} > r_t^\gamma$. Then total capital k_t is given by $k_t = k_t^{**}$ where capital $\hat{x}_t$ is financed by insider lending and the remaining capital*

$k_t^{**} - \hat{x}_t$ is financed by market lending. The optimal wage involves only the implicit bonus, $b_t = \psi_t$ and $v_t = 0$.

(iv) *Suppose that $\psi_t > V_{t+1} \geq 0$. Then capital is financed by only market lending and given by $k_t = k_t^{**}$ (thus $x_t = 0$). The optimal wage involves both the implicit and explicit bonus, $v_t = (1/\Delta q)(\psi_t - V_{t+1}) > 0$ and $b_t = V_{t+1} \geq 0$.*

Since the implicit bonus b_t is less costly than the explicit bonus v_t to motivate old workers, the firms want to use the former first and then the latter if necessary. Also the firms want to finance capital by insider lending which is more efficient than market lending.

The optimal contract has the different features depending on the future market price of the firm V_{t+1} and the current interest rate r_t. When the future market price V_{t+1} is expected to be high enough relative to the current interest rate r_t (case (i) of Lemma 1), DE is not binding so that the optimal capital coincides with the unconstrained optimum k_t^* and only the implicit bonus is used to motivate employed old workers. When V_{t+1} falls to the range in case (ii) of Lemma 1, DE becomes binding so that capital investment is constrained by DE and smaller than the unconstrained one $k_t = \hat{x}_t < k_t^*$ although the optimal wage still uses only the implicit bonus. When V_{t+1} falls further to the range in case (iii) of Lemma 1, DE becomes tight so that the firms must finance capital by both insider and market lending ($k_t = k_t^{**} > x_t = \hat{x}_t$) while still keeping the use of the implicit bonus. When V_{t+1} is so small that $\psi_t > V_{t+1}$ (case (iv) of Lemma 1), the firms never finance capital by insider lending but they resort to market lending fully. In addition, the firms combine the explicit bonus $v_t = (1/\Delta q)(\psi_t - V_{t+1})$ with the implicit bonus $b_t = V_{t+1}$ because DE is so severe that the implicit bonus is not sufficient to motivate workers and hence the explicit bonus must be introduced.

Remark. The optimal contract never involves both insider lending ($x_t > 0$) and the explicit bonus ($v_t > 0$). The reason for this is as follows. The explicit bonus v_t depends on the verifiable but noisy signal σ while insider lending can save the cost of raising capital and its gain is captured by the parameter value $1-\lambda$. By (7) the explicit bonus is given by $v_t = (1/\Delta q)(\lambda r_t x_t + \psi_t - V_{t+1})$ whenever the firm finances capital investment via insider lending ($x_t > 0$). However, when the verifiable signal σ is not so informative (Δq is small) as made in Assumption 1, the cost of raising relation-specific capital x_t becomes larger through increasing the explicit bonus by the term $(1/\Delta q)\lambda r_t$. Assumption 1 then implies that the effectiveness of insider lending captured by $1 - \lambda$ is so small relative to its cost $(q_l/\Delta q)\lambda$ arising from the use of the explicit bonus at the same time. Thus the firms never use the explicit bonus when they finance via insider lending. [20]

[20] In contrary to this, if we drop Assumption 1, the firms always choose insider lending but market lending never emerges (see Appendix B).

4 Full Equilibrium

We embed the optimal contract derived in the previous section into the dynamic general equilibrium framework which endogenously determines the market interest rate r_t, capital k_t, and the market price of the firm ownership V_t.

4.1 Firm Ownership (Stock) Market

As we have mentioned, all young individuals become indifferent for purchasing the firm ownerships and not purchasing them because the no-arbitrage condition (3) implies that the net return of purchasing one unit of the firm ownership becomes zero, i.e., $-r_tV_t+\pi_t+V_{t+1}=0$. Then, without loss of generality we can suppose that only young entrepreneurs purchase the firm ownerships.

4.2 Credit Market Equilibrium

In the credit market in period t there is the excess credit demand by N_f young financiers as

$$N_f(w_{t-1}-m\lambda x_t) \tag{12}$$

where each of them lends λx_t to each of m firms via insider lending. Since financiers are indifferent regarding how many firms they lend to, m is indeterminate but it is irrelevant in the following analysis. Here, note that $N_f m = N$ holds. On the other hand, the excess credit demand by $1-N_f$ young entrepreneurs is given as follows

$$(1-N_f)[w_{t-1}-l_t((k_t-x_t)+V_t)] \tag{13}$$

where each of them purchases l_t ownerships (stocks) of the firms at the price V_t and borrows capital k_t-x_t from market lending per firm they own.

Then the total excess credit demand must be zero in equilibrium:

$$N_f(w_{t-1}-m\lambda x_t)+(1-N_f)[w_{t-1}-l_t((k_t-x_t)+V_t)]=0. \tag{14}$$

Here, by noting that the total demand for the firm ownerships by young entrepreneurs $l_t(1-N_f)$ must be equal to the total ownerships of the firms, N, we have $l_t(1-N_f)=N$. [21] Also, N_f young financiers lend to m firms each so that $N_f m = N$ holds. Then, by re-arranging the above condition, we obtain the credit market equilibrium (CME):

$$w_{t-1}=N(k_t-(1-\lambda)x_t+V_t) \tag{CME}$$

This states that total resources available in period $t-1$ for the young generation to invest in new capital and purchase the firm ownerships from the old generation is given by the total

[21] Recall that there are one unit mass of ownerships per firm. Thus the total number of ownership rights of the firms should be equal to N. Note that firms have no reasons to issue new equities for financing capital investment as we have mentioned. They finance capital investment k_t-x_t which is not covered by insider lending via borrowing from the credit market but not issuing new equities.

endowment of young capitalists, w_{t-1}. Insider lending needs λx_t units of good per firm for financing x_t capital whereas market lending requires x_t units of good per firm for financing the same amount of x_t capital, which implies that the former can save $(1-\lambda)x_t$ units of good relative to the latter. Thus the net capital demand $k_t - (1-\lambda)x_t$ after subtracting such saving by insider lending must be financed via the credit market. Finally, young entrepreneurs purchase N firms from the old generation at the market price V_t, resulting in NV_t credit demand in the economy. The sum of these credit demand must be equal to the total endowment of the economy w_{t-1}, which is the credit supply of the economy.

The endowment of young capitalists w_{t-1} depends on the aggregate productivity A_{t-1} embodied in the aggregate output Ny_{t-1}, i.e., it is given by $w_{t-1} = LA_{t-1} = LNy_{t-1}$.

An equilibrium path of the economy is described as a sequence $\{V_t, k_{t-1}, x_t, b_{t-1}, v_{t-1}, r_t\}_{t=1}^{\infty}$ which solves the optimal contract and satisfies both CME and the firm market price (8), given the initial capital stock k_0.

In what follows we will assume $N = 1$ without loss of generality. Thus the aggregate output Ny_t is simply given by $y_t = k_t^{\gamma}$ and the worker's opportunity cost of choosing high effort $a_t = 1$ is given by $\psi_t \equiv \psi A_t = \psi y_t = \psi k_t^{\gamma}$ respectively.

5 Equilibrium Dynamics

5.1 Static Equilibrium

We define a new state variable $Q_t \equiv V_t / y_{t-1}$ by dividing the market price of a firm V_t in the current period t by the aggregate output $y_{t-1} = k_{t-1}^{\gamma}$ in the previous period $t-1$. We call this *the firm value* simply.

Before moving to the analysis of full equilibrium dynamics, we define a *static equilibrium* in period t as $\{Q_t, k_t, x_t, r_t, b_t, v_t\}$ given Q_{t+1} and k_{t-1} such that all these variables satisfy Lemma 1 and CME given the capital in the previous period k_{t-1} and the expectation about the firm value Q_{t+1} in the next period $t+1$. Then, we can show that there are three different regimes of static equilibrium as depicted in Figure 1.

When the future firm value Q_{t+1} is expected to be so high that $Q_{t+1} \geq \lambda\gamma + \psi$, the firms have enough incentive to honor the promised implicit bonus $b_t = \psi_t$ and repayment R_t to financiers. Thus, insider lending and implicit wage become a static equilibrium in this regime. When Q_{t+1} is in the middle range ($\lambda\gamma + \psi > Q_{t+1} \geq \psi$), DE becomes tight so that the firms find it difficult to finance capital via only insider lending. In that case the firms complement market lending with insider lending while they still maintain only the implicit bonus. When Q_{t+1} is so low that $\psi > Q_{t+1}$, the firms must rely on the explicit bonus to motivate workers but then it becomes too costly to use insider lending. As a result, the firms shift to using only market lending together with the explicit bonus.

In the following we examine these three regimes in more details.

Implicit Contract Regime. First we consider the implicit contract regime in which labor contract is based only on the implicit bonus $b_t = \psi_t$ (i.e., $v_t = 0$) and financing is based only on insider lending ($k_t = x_t$). There are further two cases: one is that DE is not binding and the other is that it becomes binding. When DE is slack, the firm chooses capital k_t to maximize its profit $k_t^\gamma - \lambda r_t k_t$ which yields the unconstrained optimum $k_t = k_t^*$ satisfying $\gamma k_t^{\gamma-1} = \lambda r_t$. By using this and the fact that $v_t = 0$ and $b_t = \psi_t$, the firm market price V_t (see (8)) is given by

$$V_t = (k_t^{1-\gamma}/\lambda\gamma)\{(1-\gamma)k_t^\gamma - \psi_t + V_{t+1}\}.$$

By using $\psi_t = \psi y_t = \psi k_t^\gamma$ and the definition of $Q_t = V_t/k_{t-1}$, this can be written by

$$Q_t = (k_t/k_{t-1}^\gamma)\{(1-\gamma) - \psi + Q_{t+1}\}. \tag{15}$$

In this regime CME becomes $w_{t-1} = Lk_{t-1}^\gamma = \lambda k_t + V_t$ because $k_t = x_t$ holds. Then we can write CME as $\lambda k_t/k_{t-1}^\gamma = L - Q_t$ which we substitute into (15) to obtain

$$\frac{Q_t}{L - Q_t} = (1-\gamma) - \psi + Q_{t+1}. \tag{16}$$

This can be a static equilibrium when DE is not binding in the optimal contract (case (i) of Lemma 1). By substituting $r_t = (1/\lambda)k_t^{\gamma-1}$ into the non-binding DE constraint, $V_{t+1} \geq \lambda r_t k_t + \psi_t$, and using the definition of Q_{t+1} and $\psi_t \equiv \psi k_t^\gamma$, we obtain

$$Q_{t+1} \geq \gamma + \psi.$$

Then we show the following.

Lemma 2. *Suppose that Assumption 1 holds. Suppose that $Q_{t+1} \geq \gamma + \psi$. Then the static equilibrium in period t has the following features: (i) DE is not binding, (ii) the firm value Q_t satisfies (16), (iii) the market interest rate is given by $r_t = (1/\lambda\gamma)k_t^{\gamma-1}$, and (iv) capital k_t is determined by $(L - Q_t)k_{t-1}^\gamma = \lambda k_t$.*

Suppose next that DE is binding in the implicit contract regime. Then, case (ii) of Lemma 1 becomes a static equilibrium. In that case the firms still finance capital investment by insider lending but DE becomes binding ($k_t = x_t = \hat{x}_t$). By using $k_t = \hat{x}_t \equiv (1/\lambda r_t)(V_{t+1} - \psi_t)$ and the definitions of Q_{t+1} and ψ_t, the condition in case (ii) of Lemma 1 is shown to be equivalent to $\gamma + \psi > Q_{t+1} \geq \lambda\gamma + \psi$. Also, CME becomes $Lk_{t-1}^\gamma = \lambda k_t + V_t$ again because of $k_t = x_t$, implying that

$$k_t = (L - Q_t)k_{t-1}^\gamma/\lambda. \tag{17}$$

Given such capital k_t, the interest rate r_t which makes DE$_t$ binding becomes

$$r_t = \frac{Q_{t+1} - \psi}{\lambda}k_t^{\gamma-1}.$$

When we substitute this into (8), we can show the following result.

Lemma 3. *Suppose that Assumption 1 holds. Suppose that $\gamma+\psi > Q_{t+1} \geq \lambda\gamma+\psi$. Then the static equilibrium in period t has the following features: (i) the firms choose capital $k_t = (L-Q_t)k_{t-1}^{\gamma}/\lambda$, (ii) the market interest rate satisfies $r_t = (Q_{t+1}-\psi)((L-Q_t)k_{t-1}^{\gamma})^{\gamma-1}/\lambda^{\gamma}$, (iii) DE_t becomes binding, (iv) the firms pay only the implicit bonus $b_t = \psi_t$ but not the explicit bonus (i.e., $v_t = 0$), and the firm value Q_t is given by*

$$\frac{Q_t}{L-Q_t} = \frac{1}{Q_{t+1}-\psi}. \tag{18}$$

Mixed Contract Regime. Next we consider the regime in which the firms use both insider and market lending ($k_t > x_t > 0$) but still maintain only the implicit bonus ($b_t = \psi_t$ and $v_t = 0$), which we call *mixed contract regime*. This occurs in case (iii) of Lemma 1, which is equivalent to $\lambda\gamma + \psi > Q_{t+1} \geq \psi$ because $r_t = \gamma k_t^{\gamma-1}$ holds in this regime. Also CME is written by

$$Lk_{t-1}^{\gamma} = k_t - (1-\lambda)x_t + V_t$$

and DE_t becomes binding, $V_{t+1} = \lambda r_t x_t + \psi_t$ which yields $r_t = (V_{t+1} - \psi_t)/\lambda x_t$. The firm chooses total capital k_t satisfying $\gamma k_t^{\gamma-1} = r_t$. By combining this with the binding DE, we obtain

$$r_t = \gamma k_t^{\gamma-1} = (V_{t+1} - \psi_t)/\lambda x_t. \tag{19}$$

Here the second line implies that $x_t = (Q_{t+1} - \psi)k_t/\lambda\gamma$. Thus CME shows that

$$(L-Q_t)k_{t-1}^{\gamma} = \left[1 - \left(\frac{1-\lambda}{\lambda\gamma}\right)(Q_{t+1}-\psi)\right] k_t. \tag{20}$$

Then we can show the following:

Lemma 4. *Suppose that Assumption 1 holds. Suppose that $\lambda\gamma+\psi > Q_{t+1} \geq \psi$. Then, the static equilibrium in period t has the following features: (i) the firms choose total capital k_t satisfying (20) and the relation-specific capital x_t satisfying $x_t = (Q_{t+1} - \psi)k_t/\lambda\gamma$, (ii) the market interest rate r_t is given by $r_t = \gamma k_t^{\gamma-1}$, (iii) DE becomes binding, (iv) the firms pay only the implicit bonus, i.e., $b_t = \psi_t$ and $v_t = 0$, and (v) the firm value Q_t satisfies*

$$\frac{Q_t}{L-Q_t} = \frac{1-\gamma+(1/\lambda)(Q_{t+1}-\psi)}{\gamma\left[1-\left(\frac{1-\lambda}{\lambda\gamma}\right)(Q_{t+1}-\psi)\right]}. \tag{21}$$

Explicit Contract Regime. Finally we consider the case (iv) of Lemma 1, $\psi_t > V_{t+1} > 0$. This is equivalent to $\psi > Q_{t+1} > 0$. In this regime the firms choose total capital k_t satisfying $\gamma k_t^{\gamma-1} = r_t$ and finance all capital k_t by market lending. The credit market equilibrium (CME) in period $t-1$ is then given by

$$Lk_{t-1}^{\gamma} = k_t + V_t \tag{22}$$

because $x_t = 0$ in this regime. This can be written by $k_t/k_{t-1}^{\gamma} = L - Q_t$, which we substitute into $r_t = \gamma k_t^{\gamma-1}$ in the expression of the firm price V_t given by (8) in order to show the following:

Lemma 5. *Suppose that Assumption 1 holds. Suppose that $\psi > Q_{t+1} > 0$. Then the static equilibrium in period t has the following features: (i) the firms finance capital investment $k_t = (L - Q_t)k_{t-1}^{\gamma}$ via only market lending but not insider lending ($x_t = 0$), (ii) the market interest rate r_t is given by $r_t = \gamma k_t^{\gamma-1}$, (iii) the firms pay the explicit bonus $v_t = (1/\Delta q)(\psi - Q_{t+1})k_t^{\gamma} > 0$ and the implicit bonus $b_t = Q_{t+1}k_t^{\gamma} > 0$, and (iv) the firm value Q_t satisfies*

$$\frac{Q_t}{L - Q_t} = \frac{1}{\gamma}\left[1 - \gamma - \left(1 + \frac{q_0}{\Delta q}\right)\psi + \left(1 + \frac{q_0}{\Delta q}\right)Q_{t+1}\right]. \tag{23}$$

5.2 Endogenous Changes in Employment and Financial Systems

Now we combine all Lemma 2-5 to obtain the full equilibrium paths of the economy. Once a path of the firm values $\{Q_t\}_{t=1}^{\infty}$ is determined, we can recover a path of capital $\{k_t\}_{t=0}^{\infty}$ from CME starting with the initial capital stock k_0. The path of relation-specific capital $\{x_t\}_{t=1}^{\infty}$ is then obtained as well by using the paths of $\{k_t\}_{t=0}^{\infty}$ and $\{Q_t\}_{t=1}^{\infty}$. We can also recover the path of the interest rates $\{r_t\}_{t=1}^{\infty}$ from using these paths. Thus it suffices to determine the path of the firm values $\{Q_t\}_{t=1}^{\infty}$ which evolves from Lemma 2-5 as follows

$$\frac{Q_t}{L - Q_t} = \begin{cases} (1/\gamma)[(1-\gamma) - \psi + Q_{t+1}] & \text{if } Q_{t+1} > \gamma + \psi \\ \dfrac{1}{Q_{t+1} - \psi} & \text{if } \gamma + \psi \geq Q_{t+1} > \lambda\gamma + \psi \\ \dfrac{1 - \gamma + (1/\lambda)(Q_{t+1} - \psi)}{\gamma\left[1 - \left(\frac{1-\lambda}{\lambda\gamma}\right)(Q_{t+1} - \psi)\right]} & \text{if } \lambda\gamma + \psi > Q_{t+1} \geq \psi \\ (1/\gamma)\{1 - \gamma - (1 + (q_0/\Delta q))\psi + (1 + (q_0/\Delta q))Q_{t+1}\} & \text{if } \psi > Q_{t+1} > 0 \end{cases}$$

given $Q_t \in [0, L)$ for all $t \geq 1$. We define the above relation between Q_t and Q_{t+1} as a function $Q_t = \Phi(Q_{t+1})$ where Φ is increasing except the interval $[\lambda\gamma + \psi, \gamma + \psi]$ which is the implicit contract regime with binding DE (see Figure 2).

Note that DE constraint becomes binding in all the regimes except the implicit contract regime with high enough firm value $Q_{t+1} > \gamma + \psi$. In all these regimes the future firm value Q_{t+1} influences capital investment k_t and worker's wages b_t and v_t in the current period t, which is in contrast to the case that DE is non-binding.

For a comparison, we will consider the benchmark case in which DE can be ignored by supposing that relation-specific capital x_t and worker's effort a_t (thus the unverifiable signal $s_t = a_t$) are verifiable. To ensure that the flow profit of the firm in the benchmark case becomes positive, we make the following assumption.

Assumption 2. $1 > \gamma + \psi$.

The equilibrium path of the economy in the benchmark is governed by $\{Q_t\}_{t=1}^{\infty}$ satisfying (16), which is same as in the implicit contract regime without binding DE. We then define a unique steady state satisfying (16) at $Q_t = Q_{t+1}$, denoted by Q^*, as follows

$$\frac{\gamma Q^*}{L - Q^*} = (1 - \gamma) - \psi + Q^*.$$

Under Assumption 2 the equilibrium path $\{Q_t\}$ in the benchmark case is unique and given by $Q_t = Q^*$ for all $t \geq 1$. [22] Thus the firm value Q_t does not change over time in the benchmark equilibrium. We also denote by k^* the corresponding steady state capital which satisfies CME:

$$(L - Q^*)(k^*)^{\gamma} = \lambda k^*.$$

Since $Q_t = Q^*$ holds for all t, CME implies that $k_{t-1}^{\gamma}(L - Q^*) = k_t$ and hence the path of capitals $\{k_t\}_{t=0}^{\infty}$ is uniquely determined as well. As a result, k_t goes to the steady state k^* as $t \to \infty$.

We now return back to the case that the relation-specific capital x_t and worker's effort a_t are not verifiable. We define the three cut off firm values as follows (see Figure 2):

$$\hat{Q} \equiv L(1 - \gamma),$$

$$Q'' \equiv \frac{L}{\gamma + 1},$$

and

$$Q' \equiv \frac{L}{\lambda\gamma + 1}$$

where $\psi = \Phi(\hat{Q})$, $\lambda\gamma + \psi = \Phi(Q')$ and $\gamma + \psi = \Phi(Q'')$ are satisfied respectively. Here we can verify that $\hat{Q} < Q'' < Q'$ hold. Then, the implicit contract regime with binding DE prevails when $Q_t \in (Q'', Q')$ and $Q_{t+1} \in (\lambda\gamma + \psi, \gamma + \psi)$ while the implicit contract regime without DE arises when $Q_t > Q''$ and $Q_{t+1} > \gamma + \psi$ respectively. Also, equilibrium mode becomes the mixed contract regime when $Q_t \in (\hat{Q}, Q')$ and $Q_{t+1} \in (\psi, \lambda\gamma + \psi)$ while it becomes the explicit contract regime when $Q_t \in (0, \hat{Q})$ and $Q_{t+1} \in (0, \psi)$ respectively.

Although equilibrium dynamics becomes complicated, we can show that the economy never moves from the mixed or explicit contract regime to the implicit contract regime in any equilibrium path.

Proposition 1. *Suppose that Assumption 1 and 2 are satisfied. Then, in any equilibrium path $Q_t < \lambda\gamma + \psi$ holds for all $t \geq T$ once $Q_T < \lambda\gamma + \psi$ in some period T.*

[22] If $Q_1 \neq Q^*$, we can show from (16) that Q_t goes to either a negative value or positive infinity under Assumption 2, either of which cannot be an equilibrium path.

Proposition 1 states that, if the economy enters the mixed or explicit contract regime in some period $T-1$ which implies that $Q_T < \lambda\gamma+\psi$, then it never enters the implicit contract regime from period T onwards, i.e., $Q_t < \lambda\gamma+\psi$ for all $t \geq T$. In this sense organizational changes in financial and labor contracts are irreversible such that, once market lending or explicit wage is adopted in some period, such market-and-explicit contract based feature of employment and financial systems persists over time.

From Proposition 1, there are only two possible candidates for equilibrium paths. First, the economy moves from the implicit contract regime to the mixed or explicit contract regime over time. Second, the economy stays in the implicit contract regime forever. In the latter case the equilibrium path either fluctuates over time in the implicit contract regime or stays in the steady state in the implicit contract regime from the initial period forever. The steady state firm value, denoted by Q^i, in the implicit contract regime is defined as follows

$$\frac{Q^i}{L-Q^i} = \begin{cases} 1-\gamma-\psi+Q^i & \text{if } Q^i > \gamma+\psi \\ 1/(Q^i-\psi) & \text{if } \lambda\gamma+\psi < Q^i < \gamma+\psi \end{cases}$$

Note that Q^i coincides with the firm value in the benchmark case, $Q^i = Q^*$, when $Q^* > \gamma+\psi$.

We then show that the firm value must be constant at the steady state value Q^i if the economy stays in the implicit contract regime forever. Thus there are no equilibrium paths which fluctuate forever in the implicit contract regime.

Proposition 2. *Suppose that Assumption 1 and 2 are satisfied. Then, in any equilibrium path in which $Q_s \neq Q^i$ for some period s, it must be that $Q_t < \lambda\gamma+\psi$ for all $t \geq T$ from some period T onward.*

Proposition 2 together with Proposition 1 implies that in any equilibrium path the economy eventually enters the mixed or explicit contract regime unless it stays in the implicit contract steady state Q^i from the initial period forever. Thus any equilibrium path must be either (i) that the economy stays in the implicit contract regime with the firm value being constant at Q^i in all periods or (ii) that the economy enters the mixed or explicit contract regime from some period onward.

The first type (i) of equilibrium path exists if the implicit contract steady state Q^i exists, which is shown to be equivalent to the condition that $Q' > \lambda\gamma+\psi$. [23]

Our next task is to provide the conditions under which the second type (ii) of equilibrium path actually exists. In particular we show that there exists an equilibrium path which starts from the implicit contract regime and then moves to the mixed contract regime and explicit contract regime over time. To this end, we make the following additional assumption.

[23] When this condition is satisfied, $\Phi(\lambda\gamma+\psi) = Q' > \lambda\gamma+\psi$ holds. Then, since Φ is increasing and convex with $\Phi(\infty) = L$ when $Q > \gamma+\psi$ and Φ is decreasing when $\lambda\gamma+\psi < Q < \gamma+\psi$, there exists a unique Q^i such that $\Phi(Q^i) = Q^i$ and $\lambda\gamma+\psi < Q^i$.

Assumption 3. $\Gamma \equiv (1 + q_0/\Delta q)\psi - (1-\gamma) > 0$.

Assumption 3 says that the verifiable signal σ is not so informative (Δq is small) again as assumed in Assumption 2. Assumption 3 ensures that $\Phi(0) > 0$ (Figure 2).

We define the two cut off values of the parameter L measuring the size of savings (the credit supply) of the economy as follows

$$\hat{L} \equiv \frac{\psi}{1-\gamma}$$

where $\hat{Q} > \psi$ holds when $L > \hat{L}$, and

$$L' \equiv (1 + \lambda\gamma)(\lambda\gamma + \psi)$$

where $Q' > \lambda\gamma + \psi$ holds when $L > L'$, respectively.

We also define (Q^e, k^e) as the steady state in the explicit contract regime, i.e., it satisfies (23) at $Q_t = Q_{t+1}$:

$$\frac{Q^e}{L - Q^e} = \frac{1}{\gamma}\left[1 - \gamma - \left(1 + \frac{q_0}{\Delta q}\right)\psi + \left(1 + \frac{q_0}{\Delta q}\right)Q^e\right]. \tag{24}$$

Then, from CME at $k_t = k_{t-1}$ and $Q_t = Q^e$, we obtain the steady state capital k^e in the explicit contract regime:

$$k^e = (L - Q^e)(k^e)^\gamma. \tag{25}$$

Then we can verify the following claim.

Lemma 6. *Suppose that $L > \hat{L}$ and Assumption 3 hold. Then the steady state in the explicit contract regime (Q^e, k^e) exists. Furthermore, Q^e is decreasing in L and $Q^e = \psi$ at $L = \hat{L}$.*

Given Lemma 6, we can show the following result.

Proposition 3. *Suppose that Assumption 1-3 and $\hat{L} > L'$ hold.*[24] *Then there exists some $\overline{L} > \hat{L}$ such that for all $L \in (\hat{L}, \overline{L})$ the following multiple equilibrium paths exist with different switching periods T_i and T_e such that the economy stays*

(i) *in the implicit contract regime for $t \leq T_i$,*

(ii) *in the mixed contract regime for $T_i < t < T_e$,*

(iii) *in the explicit contract regime for $t > T_e$*

[24] When λ is small, $\hat{L} > L'$ is satisfied.

where the switching periods T_e and T_i are not unique.

The condition $L > \hat{L} > L'$ stated in Proposition 3 ensures that $\hat{Q} > \psi$ and $Q' > \lambda\gamma + \psi$ (Figure 3). [25] By combining this with Assumption 3, $\hat{Q} > \psi$ implies that there exists a steady state in the explicit contract regime (depicted by Q^e in Figure 3). Also $L > L'$ can guarantee that the implicit contract steady state (Q^i, k^i) exists (Figure 3). In addition, $L < \overline{L}$ is used to show that the firm value in the explicit contract steady state Q^e is so large that the flow profit of the firm becomes positive in the explicit contract steady state. [26]

As we have already mentioned, equilibrium path is unique in the benchmark case in which DE can be ignored. In contrast, equilibrium paths become multiple when DE constraint becomes binding. It can be seen in Figure 3 that multiple equilibrium paths are possible with different initial firm values Q_1. The main intuition behind multiple equilibrium paths is due to the existence of the sequences of self-fulfilled expectations over time, although such mechanism works differently in the implicit contract regime and other regimes.

We first consider the implicit contract regime with binding DE and see that the expectation about a high (low) future firm value Q_{t+1} is associated with a low (high) current value Q_t. Thus high and low firm values alternate overt time and constitute an equilibrium path in this regime. To see this, suppose that the firm value Q_{t+1} is expected to be lower in the next period $t + 1$. Then, since DE becomes binding and all capital is financed by insider lending in this regime, capital investment $k_t = x_t$ in the current period t decreases (note that (DE) $V_{t+1} = \lambda r_t k_t + \psi_t$ implies that $Q_{t+1} = \lambda(r_t/y_t)k_t + \psi$.) This reduces the excess credit demand in the credit market so that more credit supply is left for financing the purchase of the firm ownership (note that (CME) $Ly_{t-1} = \lambda k_t + V_t$ implies that $L = \lambda k_t/y_{t-1} + Q_t$.) This increases the firm value Q_t in the current period t. In this way the expectation about a low future firm value Q_{t+1} results in a high current firm value Q_t. The expectation about a low firm value Q_{t+1} is self-fulfilled by the expectation about a high firm value Q_{t+2} in period $t+2$, which will be self-fulfilled by the expectation about a low future firm value Q_{t+3} in period $t+3$, and so on. Thus, the path of alternating high and low firm values becomes an equilibrium in the implicit contract regime with binding DE.

On the other hand, in the mixed contract regime, the expectation about a low firm value in the future period Q_{t+1} results in a low firm value Q_t in the current period t. To see this, suppose that the future firm value Q_{t+1} is expected to be lower. Then, since DE is binding, capital investment financed by insider lending x_t in the current period t decreases. However, in contrast to the implicit contract regime, since both insider and market lending are used in this regime, the decrease in capital financed by insider lending implies that more credit is needed to finance the remaining capital by market lending.

[25] In Figure 3 we depict the case that $\gamma + \psi > L$ so that DE is always binding in the implicit contract regime.

[26] The flow profits in the implicit contract and mixed contract regimes can be non-negative due to Assumption 2.

This increases the excess credit demand $k_t - (1-\lambda)x_t$ in the credit market in the current period t, implying that less credit supply is left for financing the purchase of the firm ownerships in the current period t. This then decreases the firm value Q_t in the current period t because of CME: $L = (k_t - (1-\lambda)x_t)/y_{t-1} + Q_t$. In this way, a low future firm value Q_{t+1} is associated with a low current firm value Q_t.

In the explicit contract regime, the expectation about a low future firm value Q_{t+1} makes the explicit bonus v_t larger in the current period t because DE is binding so that $v_t/y_t = (1/\Delta q)(\psi - Q_{t+1})$. Then, since the explicit bonus is more costly than the implicit bonus and the firm must rely on the former in this regime, the flow profit π_t in the current period t becomes lower, resulting in a decrease in the firm value Q_t in the current period t. Again, the expectation about a low future firm value is associated with a low current firm value Q_t.

Thus, in either mixed or explicit contract regime the expectation about a low firm value Q_{t+1} in period $t+1$ will be self-fulfilled by the expectation about a low firm value Q_{t+2} in period $t+2$, which will be self-fulfilled by the expectation about a low firm value Q_{t+3} in period $t+3$, and so on.

As a result, these sequences of self-fulfilled expectations about the firm values Q_t cause multiple equilibrium paths. Some of them have the feature that financial and labor contracts change between the different regimes toward more explicit-and market-based systems as shown in Proposition 3. Such equilibrium paths can be constructed by having an initial value of the market value Q_1 in the implicit contract regime and having the subsequent values of Q_t in the mixed contract regime. As we have discussed above, once Q_t enters the mixed contract regime, a decreasing sequence of Q_t can be a self-fulfilled expectation equilibrium which eventually enters the explicit contract regime. Such path can be actually an equilibrium as long as Q_t does not diverge to negative values, i.e., $Q_t \in (0, L)$ holds in any period t.

To see this last point, consider the explicit contract regime in which the firm must rely on the explicit bonus $v_t/y_t = (1/\Delta q)(\psi - Q_{t+1})$ measured in terms of the current output y_t. When the future firm value Q_{t+1} decreases by one unit amount, this raises the current explicit bonus by $(1/\Delta q)$ units. Such loss tends to be larger as the verifiable signal σ becomes more noisy (i.e., Δq is smaller), which implies that one unit increase in the future value Q_{t+1} results in more than one unit decline in the current value Q_t. This ensures that $\Delta Q_t/\Delta Q_{t+1} > 1$, although $\Delta Q_t/\Delta Q_{t+1} < 1$ holds at the steady state in the benchmark case without DE constraint. Thus, by reversing the above inequality, $\Delta Q_{t+1}/\Delta Q_t < 1$ holds in the explicit contract regime when the verifiable signal is not so informative as assumed in Assumption 1 and Assumption 3. Then, as Q_t becomes lower, a further decrease in the current value of Q_t does not reduce the next period value Q_{t+1} too much, guaranteeing that Q_t does not diverge to negative values even when it decreases over time.

5.3 Organizational Changes in Economic Development

Next we investigate how the changes in financial and labor contracts affect and are affected by capital accumulation and economic development.

As we have shown in Proposition 1 and 2, equilibrium paths are characterized as either one of the following: (i) $Q_t = Q^i$ for all $t \geq 1$ or (ii) $Q_s \neq Q^i$ for some s and $Q_t < \lambda\gamma + \psi$ for all $t \geq T$ from some period T. In the former case (i) equilibrium capital follows the dynamic equation

$$(L - Q^i)k_{t-1}^{\gamma} = \lambda k_t$$

due to CME in the implicit contract regime. Then it is verified that k_t monotonically increases over time when $(L - Q^i) > k_0^{1-\gamma}$ and k_t eventually reaches the steady state in the implicit contract regime, denoted by k^i, satisfying

$$(L - Q^i)(k^i)^{\gamma} = k^i.$$

In the latter case (ii) capital dynamics becomes more complicated because the firm value Q_t may fluctuate for some periods (but not permanently) in the implicit contract regime and then moves to the mixed contract or explicit contract regime as depicted in Figure 4. However, we know from Proposition 2 that the economy eventually enters the mixed or explicit contract regime in either which the dynamic equation Φ governing the paths of the firm values Q_t is monotone decreasing. Thus, capital dynamics becomes easier to be characterized in these regimes than in the implicit contract regime.

We denote by $g_t \equiv k_t/k_{t-1} - 1$ the net growth rate of capital. Then we show the following result.

Proposition 4. *Suppose that Assumption 1-3 hold. Suppose also that* $L > \max\{\hat{L}, L'\}$. *Then equilibrium path must be either one of the following features*:

(i) $Q_t = Q^i$ *for all* $t \geq 1$ *and capital* k_t *converges to the steady state* k^i *in the implicit contract regime. Furthermore, the net growth rate of capital* g_t *is positive if* $L - Q^i > k_0^{1-\gamma}$.

(ii) $Q_s \neq Q^i$ *for some* s *and* $Q_t \to Q^e$ *as* $t \to \infty$ *where capital* k_t *converges to the steady state* k^e *in the explicit contract regime. Furthermore, the net growth rate of capital* $g_{t+1} > g_t$ *holds if* $g_t < 0$ *and* $g_{t+1} > 0$ *holds if* $g_t > 0$ *in the mixed and explicit contract regimes.*

Thus, when the size of economy savings L is large so that $L > \max\{L', \hat{L}\}$, the economy eventually reaches either the implicit contract steady state (Q^i, k^i) or the explicit contract steady state (Q^e, k^e) in the long run.

In the second case in Proposition 4, as capital k_t increases over time, the equilibrium modes of financing and labor contracts change from the implicit contract-based to the explicit and market-based together. The key feature of this result is that dynamical changes in financial system are accompanied with the changes in employment system: as financial system moves from insider lending to market lending, employment system also moves from the implicit-based contract to the explicit-based one. Such co-evolution of financial and employment systems is caused by the interactions between the dynamic enforcement (DE) constraint and market equilibrium conditions. As we have already seen in Proposition 3, the fall of the firm values Q_t is supported by self-fulfilled expectation. Then, the credit demand for purchasing the firm ownerships falls over time as well, resulting in more credit supply used to finance larger capital investment over time. Thus capital k_t increases if its growth rate g_t is positive or the growth rate g_t increases if g_t is negative in the mixed contract and explicit contract regimes. [27]

Our theoretical insights help understand how financial and employment systems evolve over time in the process of economic development. As we have discussed, the rapid economic growth of the Japanese economy during 1960s was associated with the bank oriented financial system and implicit employment relationships. However, as the Japanese economy matured and slowed down after 1970s, they have shifted toward financing in anonymous financial markets from the bank-oriented financing (Hoshi and Kashyap (2004), Rajan and Zingales (2004)). At the same time, the implicit labor practices of the Japanese firms based on implicit employment security and seniority-based pay system have been changing to be replaced by more performance-based explicit contracts in 1990s (Abe and Hoshi (2007)). These changes in employment and financial systems might suggest that the implicit contracts in both financing and labor contexts change together toward more market-and explicit-based systems. In that sense employment and financial systems are dynamically complement with each other, which might support the aforementioned historical evidence (Abe and Hoshi (2007), Aoki (1994), and Aoki and Patrick (1994)).

5.4 Long Run Welfare

As we have shown, the economy admit multiple equilibrium paths converging to different steady states under certain conditions. In this section we investigate the comparison among the long run welfare in these different steady states. To allow multiple steady states, we maintain Assumption 1-3 and $L > \hat{L}$ which ensure that the explicit contract steady state (Q^e, k^e) always exists.

When λ is so small, $\hat{L} > L'$ holds so that $L > \hat{L} > L'$ and hence there exists the implicit contract steady state (Q^i, k^i) in addition to the explicit contract one (Q^e, k^e) (see Proposition 4). When λ is close to 1, we may have $L' > L > \hat{L}$ so that the there exists a steady state in the mixed contract regime but the implicit contract steady state (Q^i, k^i) disappears. We denote by (Q^m, k^m) such mixed contract steady state if it exists. Note that $Q^e < Q^m$ and $Q^e < Q^i$ hold.

Thus, under Assumption 1-3 and $L > \hat{L}$ the long run equilibrium is characterized by two steady states: (i) (Q^e, k^e) and (ii) (Q^i, k^i) or (Q^m, k^m). In the latter case (ii) the long run steady state becomes (Q^i, k^i) when λ is small whereas it becomes (Q^m, k^m) when λ is close to 1.

[27] However, capital may temporarily decrease when the economy moves from one regime to the other.

Now we investigate which long run equilibrium attains a higher welfare. We define the social welfare of the economy as the sum of the utilities of all individuals in a steady state. Since individuals consume only when old and their utilities are linear with respect to their consumption equal to incomes they earn, we can derive the social welfare by summing all the incomes of old capitalists and old workers in a steady state. In a steady state, $1 - N_f$ old entrepreneurs earn

$$(1 - N_f)\{r(w - l^E(k - x + V)) + l^E(\pi + r(k - x) + V)\} \tag{26}$$

where l^E denotes the number of firms owned by each old entrepreneur and satisfies $l^E(1 - N_f) = N = 1$. Also, N_f old financiers earn

$$N_f\{r(w - m\lambda x) + mR\} \tag{27}$$

where each financier lends to m firms. All old workers earn

$$NEW + (1 - N)\psi Ny \tag{28}$$

where $Ny = Nk^\gamma$ is the aggregate output, $EW \equiv b + q_1 v$ denotes the expected (average) wage per employed old worker and $(1 - N)\psi Ny$ is the income unemployed old workers earn by accessing to their private production technology.

By summing all these payoffs and using $l^E(1 - N_f) = N = 1$ and $N_f m = N = 1$, we obtain the social welfare as

$$\begin{aligned}
SW(Q, k) &\equiv r\{w - N(k - (1 - \lambda)x + V)\} + N\{\pi + r(k - x) + R + EW + V\} + (1 - N)\psi Ny \\
&= N\{k^\gamma - R - EW + R + EW + V\} + (1 - N)\psi Ny \\
&= N\{k^\gamma + V\} + (1 - N)\psi Ny \\
&= Nk^\gamma(1 + Q + (1 - N)\psi) & (29) \\
&= k^\gamma(1 + Q) & (30)
\end{aligned}$$

where we used $w = N(k - (1 - \lambda)x) + V)$ due to CME and $N = 1$.

In the implicit contract steady state (Q^i, k^i), CME implies that $(L - Q^i)k^\gamma = \lambda k$ and hence $k = k^i \equiv ((L - Q^i)/\lambda)^{1/(1-\gamma)}$. Thus the social welfare in the implicit contract steady state is given as follows

$$SW^i \equiv SW(Q^i, k^i) = \left(\frac{L - Q^i}{\lambda}\right)^{\frac{\gamma}{1-\gamma}} (1 + Q^i). \tag{31}$$

On the other hand, in the explicit contract steady state (Q^e, k^e), the social welfare is given as follows

$$SW^e \equiv SW(Q^e, k^e) = (L - Q^e)^{\frac{\gamma}{1-\gamma}}(1 + Q^e) \tag{32}$$

due to CME: $(L - Q^e)k = k^\gamma$. Note that both Q^i and Q^e are independent of λ. Finally, in the mixed steady state (Q^m, k^m), the social welfare is given by

$$SW^m \equiv SW(Q^m, k^m) = \left(\frac{L - Q^m}{1 - \frac{1-\lambda}{\lambda\gamma}(Q^m - \psi)}\right)^{\frac{\gamma}{1-\gamma}} (1 + Q^m) \tag{33}$$

by using CME in the mixed contract regime: $(L - Q^m)(k^m)^\gamma = (1 - ((1-\lambda)/\lambda\gamma))(Q^m - \psi))k^m$. Q^m depends on λ.

Then we can show the following result:

Proposition 5. *Suppose that Assumption 1-3 and $L > \hat{L}$ hold. Then, the long run welfare in the implicit contract steady state (Q^i, k^i), SW^i, is the highest among those in all the steady states when λ is small enough. However, the social welfare SW^e in the explicit contract steady state is the highest among those in all the steady states when λ is close to 1.*

When insider lending is highly efficient so that λ is small, the implicit contract steady state which involves fully insider lending achieves the highest social welfare among all the steady states. However, when the efficiency of insider lending is not so large, the implicit contract steady state or the mixed contract steady state does not attain larger welfare than the explicit contract steady state. The reason for this is as follows: in the implicit contract steady state the firm value Q^i is larger than those in other steady states, which implies that more resources must be used for financing the purchase of such high firm value in the implicit steady state. This reduces capital investment and aggregate production of the economy. On the other hand, in the explicit contract steady state the firm value Q^e is the lowest among all the steady states so that more credit supply can be used to finance larger capital investment. This raises the aggregate output of the economy. When insider lending is not so efficient, its advantage of saving the lending costs is dominated by the effect of lower aggregate outputs, showing that the explicit contract steady state achieves the largest social welfare.

6 Conclusion

In this paper we have investigated the dynamic macroeconomic implications of finance and labor contracts with limited commitment of the firms. Employed workers are motivated to work hard by not only the implicit wage but also the explicit wage. The firms also finance capital investment by not only insider lending but also market lending. For the implicit labor contract and insider lending to be self-enforcing, the dynamic enforcement (DE) constraint must be satisfied such that the firms have no incentive to renege on the implicit wage and repayment to financiers under insider lending. Then we have shown that the dynamic interactions between DE constraint and market equilibrium conditions endogenously lead to the dynamic changes in organizational modes of financial and labor contracts: in equilibrium paths finance and labor contracts move toward more explicit and market-based system over time. Furthermore, we have also shown that the market-based and explicit-based feature of employment and financial systems persists in the long run even though equilibrium paths become non-monotonic and complex.

We conclude this paper by discussing some extensions of the model left for the future research. First, in our model workers and financiers contract with firms only once so that they do not form long term relationships. Even so, the firms have the incentive not to renege on the implicit financial and labor contracts because they expect that their market prices will go down when they lose reputation by not honoring contracts with the current workers and financiers. When the firms can engage in long term relationships with workers and financiers, it becomes easier for them to enforce the implicit labor and financial contracts. Then, the implicit contract regime may be more likely to be sustained, which may affect the regime switching patterns of equilibrium paths. Second, there are other ways to model the roles of financiers than the one presented in this paper. For example, one important role by banks is to monitor the client firms at lower costs than market lending. The bank financing is then interpreted as monitored loans while market lending is the non-monitored loans by which investors do not directly monitor the firms' activities (see Baliga and Polak (2004), Tirole (2005)). Then it will be interesting to see how the bank monitoring is combined with different forms of employment contracts and how such determination of the corporate governance is linked to the process of economic development.

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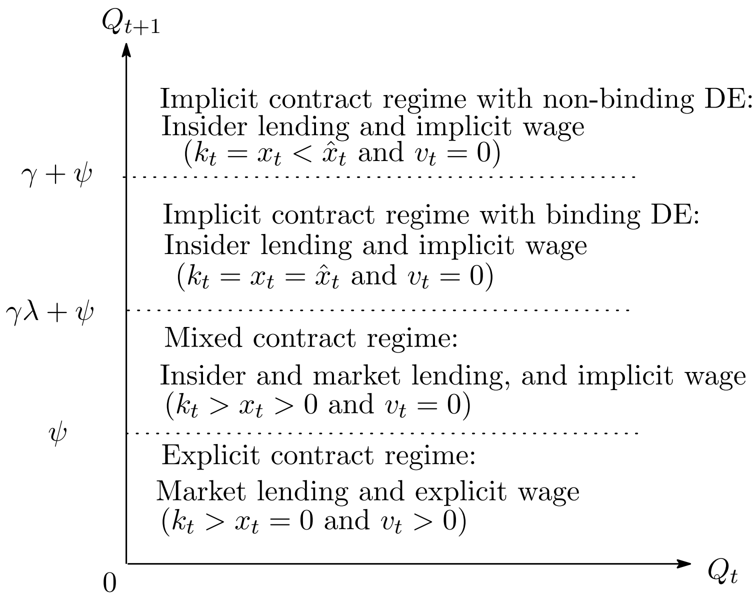

Figure 1: Different regimes of static equilibrium

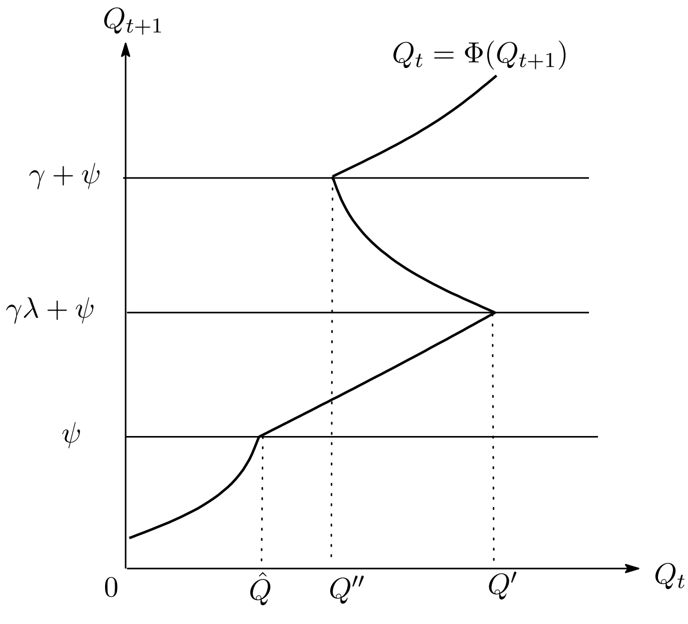

Figure 2: Equilibrium Relations between Q_t and Q_{t+1}

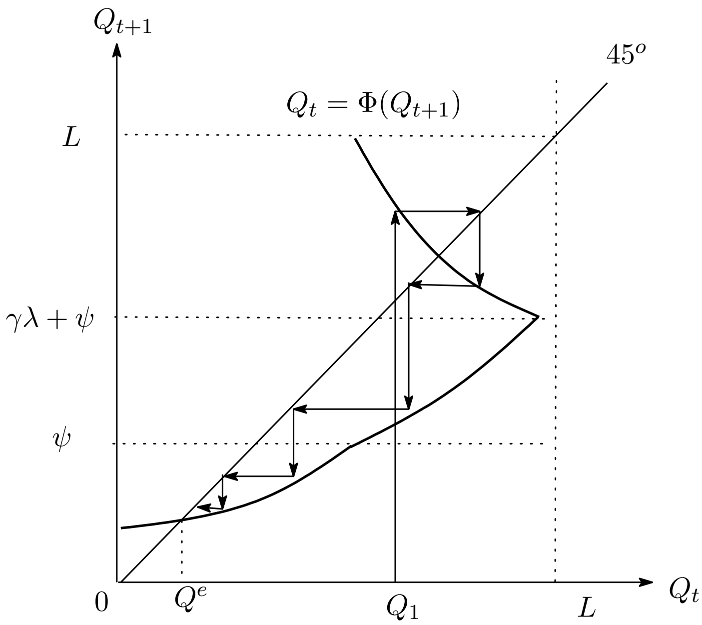

Figure 3: Endogenous Organizational Changes

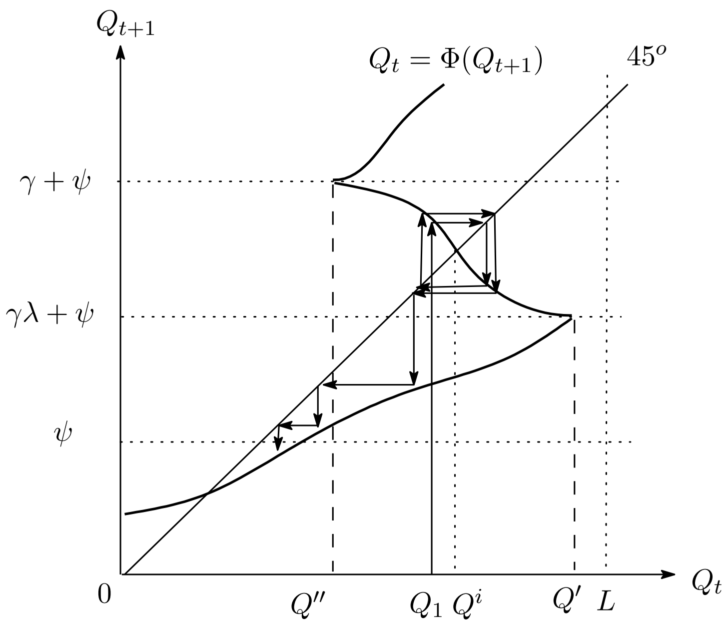

Figure 4: Irreversible Change to Mixed or Explicit Contract Regime

Appendix for Online Publication

7 Appendix A: Proofs

In this section we will provide the formal proofs for the results presented in the main text.

7.1 Sufficiency of DE

In the main text we have claimed that, if DE is satisfied, there exists an equilibrium in which the firm honors for paying the implicit bonus b_t to the old worker and repayment R_t to the financier. We will show that the strategies of the firm and the workers defined in the main text leads to such equilibrium when DE is satisfied.

Suppose that DE is satisfied. Suppose also that the firm breached to pay b_t or R_t in some period t. Then we can show that it becomes a continuation equilibrium for the firm and the young workers employed by the firm to exercise the quitting options simultaneously as follows: given the firm's liquidation, the young workers are indifferent for quitting and not quitting. Also, one optimal response of the firm to the young worker's quitting choice is to liquidate the firm's asset because it expects that the young workers who will be employed by the firm in the future periods will quit the firm according to their strategies, implying that the firm cannot produce in any future period. Then, one possible path of the firm's market prices which is consistent with zero productions in the future is that $V_s = 0$ for all $s \geq t$ which actually satisfies the no-arbitrage condition, $V_s = (1/r_t)V_{s+1}$. Thus its ownership rights will be never sold at positive prices. Here note that the young workers born in period $t+1$ can observe that the young workers employed in period t have quit the firm and have been hence not retained in the firm in period $t+1$. According to the specified strategy, the newly employed young worker will quit the firm in period $t+1$ whenever the young workers employed by the firm in period t have quit. Similarly the young workers born in period $t+2$ will observe such quitting behavior of the young workers in period $t+1$ and then will quit the firm as well, and so on. Thus, all the young workers in any period $s > t$ will quit once the firm reneges on the payments and then the young worker quit the firm.

The above continuation equilibrium ensures that the market price of the deviating firm goes down to zero in any future period $s > t$, i.e., $V_s = 0$ for all $s > t$. Thus, although the deviating firm can save the payment b_t or R_t in the current period t, it will be sold at zero market price after the deviation. Then, if DE is satisfied, such deviation is not profitable for the firm.

7.2 Proofs for Lemmas and Propositions

Proof of Lemma 1. We define the firm's flow profit in period t as

$$\Pi(k_t, x_t) \equiv k_t^{\gamma} - \lambda r_t x_t - (k_t - x_t)r_t - \psi_t - \max\left\{\frac{q_0}{\Delta q}(\lambda r_t x_t + \psi_t - V_{t+1}), 0\right\}.$$

Note that $\Pi(k_t, x_t)$ is increasing in x_t when $x_t < \hat{x}_t$ and decreasing in x_t when $x_t \geq \hat{x}_t$ under Assumption 1 where recall that $\hat{x}_t \equiv (1/\lambda r_t)(V_{t+1} - \psi_t)$ when $V_{t+1} \geq \psi_t$. Thus the optimal choice of x_t which maximizes $\Pi(k_t, x_t)$ subject to (NE) $0 \leq x_t \leq k_t$ is given by

$$x_t = x(k_t) \equiv \begin{cases} k_t & \text{if } k_t \leq \hat{x}_t \\ \hat{x}_t & \text{if } k_t > \hat{x}_t \end{cases}$$

When $\psi_t > V_{t+1}$, the optimal choice of x_t must be zero, $x(k_t) = 0$. By substituting these results into $\Pi(k_t, x_t)$, we re-define the firm's flow profit as

$$\pi(k_t) \equiv \Pi(k_t, x(k_t)) = \begin{cases} k_t^\gamma - \lambda r_t k_t - \psi_t & \text{if } k_t \leq \hat{x}_t \\ k_t^\gamma - r_t k_t - \psi_t + \frac{1-\lambda}{\lambda}(V_{t+1} - \psi_t) & \text{if } k_t > \hat{x}_t \geq 0 \\ k_t^\gamma - r_t k_t - (1 + q_0/\Delta q)\psi_t + (q_0/\Delta q)V_{t+1} & \text{if } \hat{x}_t < 0 \end{cases}$$

First we suppose that $V_{t+1} > \psi_t$ (that is, $\hat{x}_t > 0$).
(i) Suppose that $\gamma \hat{x}_t^{\gamma-1} \leq \lambda r_t$ which is equivalent to the condition stated in (i) of Lemma 1. Then $\pi(k_t)$ has a unique optimum k_t^* less than $\hat{x}_t$ and hence $v_t = 0$ holds.
(ii) Suppose that $r_t \geq \gamma \hat{x}_t^{\gamma-1} > \lambda r_t$ which is equivalent to the condition stated in (ii) of Lemma 1. Then, $\pi(k_t)$ is increasing in k_t for $k_t < \hat{x}_t$ but decreasing in k_t for $k_t > \hat{x}_t$. Thus $\pi(k_t)$ has the maximum at $k_t = \hat{x}_t$. Then DE is binding and $v_t = 0$ still holds.
(iii) Suppose that $\gamma \hat{x}^{\gamma-1} > r_t$ which is equivalent to the condition stated in (iii) of Lemma 1. Then, $\pi(k_t)$ is increasing in $k_t < \hat{x}_t$ and has a unique maximum at $k_t = k_t^{**} > \hat{x}_t$. Thus the firm invests in capital $k_t = k_t^{**}$ totally and $x_t = \hat{x}_t$ is financed by insider lending while $k_t^{**} - \hat{x}_t$ is financed by market lending. Also $v_t = (1/\Delta q)(\lambda r_t \hat{x}_t + \psi_t - V_{t+1}) = 0$ holds.

Next, suppose that $\psi_t > V_{t+1}$ (that is, $\hat{x}_t < 0$). Then the optimal choice of x_t must be $x_t = 0$ under Assumption 1. Thus we must have $v_t = (1/\Delta q)(\psi_t - V_{t+1}) > 0$, showing that the explicit bonus must be used. Then the firm's flow profit is given by

$$\pi(k_t) = k_t^\gamma - r_t k_t - (1 + q_0/\Delta q)\psi_t + (q_0/\Delta q)V_{t+1}$$

which is maximized at $k_t = k_t^{**}$. Q.E.D.

Proof of Lemma 2. The implicit contract regime with non-binding DE occurs when $\gamma \hat{x}_t^{\gamma-1} \leq \lambda r_t$ due to Lemma 1 (i). In this case $k_t = k_t^*$, i.e., $\gamma k_t^{\gamma-1} = \lambda r_t$. Thus, the above condition $\gamma \hat{x}_t^{\gamma-1} \leq \lambda r_t$ can be written by

$$\gamma \left(\frac{k_t^{1-\gamma}}{\gamma}(V_{t+1} - \psi_t) \right)^{\gamma-1} \leq \lambda r_t = \gamma k_t^{\gamma-1}$$

by using the definition of $\hat{x}_t$. This is further written by $V_{t+1} \geq \gamma k_t^\gamma + \psi_t$ so that $Q_{t+1} \geq$

$\gamma + \psi$. Then, by using the definition of V_t and CME_{t-1}, we obtain

$$\begin{aligned} V_t &= \frac{1}{r_t}\{k_t^\gamma - \lambda r_t k_t - \psi_t + V_{t+1}\} \\ &= \frac{\lambda k_t^{1-\gamma}}{\gamma}\{(1-\gamma)k_t^\gamma - \psi k_t^\gamma + V_{t+1}\} \\ &= \frac{\lambda k_t}{\gamma}\{1-\gamma-\psi+Q_{t+1}\} \\ &= \frac{k_{t-1}^\gamma(\lambda k_t/k_{t-1}^\gamma)}{\gamma}\{1-\gamma-\psi+Q_{t+1}\} \\ &= \frac{k_{t-1}^\gamma(L-Q_t)}{\gamma}\{1-\gamma-\psi+Q_{t+1}\} \end{aligned}$$

which yields

$$\frac{\gamma Q_t}{L-Q_t} = 1-\gamma-\psi+Q_{t+1}.$$

The implicit contract regime with binding DE occurs when $(\gamma/\lambda)\hat{x}_t^{\gamma-1} > r_t \geq \gamma\hat{x}_t^{\gamma-1}$ (case (ii) of Lemma 1). In this case we know $k_t = x_t = \hat{x}_t$ from Lemma 1 (ii), implying that $r_t = (1/\lambda k_t)(V_{t+1} - \psi_t)$. Then the above inequalities $(\gamma/\lambda)\hat{x}_t^{\gamma-1} > r_t \geq \gamma\hat{x}_t^{\gamma-1}$ can be written by $\gamma k_t^\gamma + \psi_t > V_{t+1} \geq \lambda\gamma k_t^\gamma + \psi_t$, which shows that $\gamma+\psi > Q_{t+1} \geq \lambda\gamma+\psi$. Then, by using the binding DE, $\psi_t = \psi k_t^\gamma$ and CME ($\lambda k_t/k_{t-1}^\gamma = L - Q_t$) together with $\pi(k_t)$, we obtain

$$\begin{aligned} V_t &= (1/r_t)\{k_t^\gamma - \psi_t - \lambda r_t k_t + V_{t+1}\} \\ &= \frac{\lambda k_t}{V_{t+1}-\psi_t}k_t^\gamma \\ &= \frac{\lambda k_t}{Q_{t+1}-\psi} \end{aligned}$$

which yields

$$Q_t = V_t/k_{t-1}^\gamma = \frac{\lambda k_t/k_{t-1}^\gamma}{Q_{t+1}-\psi}$$

implying that

$$\frac{Q_t}{L-Q_t} = \frac{1+(1-p_1)\psi}{Q_{t+1}-\psi}.$$

Q.E.D.

Proof of Lemma 3. The mixed contract regime occurs when $\gamma\hat{x}_t^{\gamma-1} > r_t$ and $V_{t+1} \geq \psi_t$ (case (iii) of Lemma 1). In this case we know $k_t = k_t^{**}$, i.e., $\gamma k_t^{\gamma-1} = r_t$ holds, and DE becomes binding at $x_t = \hat{x}_t$. Thus $\gamma\hat{x}_t^{\gamma-1} > r_t$ can be written by

$$\gamma\left(\frac{k_t^{1-\gamma}}{\lambda\gamma}(V_{t+1}-\psi_t)\right)^{\gamma-1} > r_t = \gamma k_t^{\gamma-1}$$

which is further arranged as $\lambda\gamma k_t^\gamma + \psi_t > V_{t+1}$, implying that $\lambda\gamma + \psi > Q_{t+1}$. Since $V_{t+1} \geq \psi_t$ is equivalent to $Q_{t+1} \geq \psi$, we must have $\lambda\gamma + \psi > Q_{t+1} \geq \psi$.

By using $\pi(k_t)$ in the proof of Lemma 1 and the binding DE, we have

$$V_t = (k_t^{1-\gamma}/\gamma)\left\{(1-\gamma)k_t^\gamma + (1/\lambda)(V_{t+1} - \psi_t)\right\}.$$

By dividing both sides of this expression by k_{t-1}^γ and using CME ((20) in the main text) and the definition of Q_t and Q_{t+1}, we can verify that

$$\frac{Q_t}{L - Q_t} = \frac{1 - \gamma + (1/\lambda)(Q_{t+1} - \psi)}{\gamma\left[1 - \left(\frac{1-\lambda}{\lambda\gamma}\right)(Q_{t+1} - \psi)\right]}.$$

Q.E.D.

Proof of Lemma 4. The explicit contract regime occurs when $\psi_t > V_{t+1}$, i.e., $\psi > Q_{t+1}$ (Lemma 1 (vi)). Then, by using $\pi(k_t)$, the firm's market price is given by

$$V_t = (1/r_t)\{k_t^\gamma - r_t k_t - (1 + q_0/\Delta q)\psi_t + (1 + q_0/\Delta q)V_{t+1}\}$$

where $r_t = \gamma k_t^{\gamma-1}$. By using CME ($k_t = (L - Q_t)k_{t-1}^\gamma$) and $r_t = \gamma k_t^{\gamma-1}$, we obtain

$$\begin{aligned} Q_t &= V_t/k_{t-1}^\gamma \\ &= \frac{1}{r_t k_{t-1}^\gamma}\left[(1-\gamma)k_t^\gamma - (1 + (q_0/\Delta q)\psi_t + (1 + (q_0/\Delta q))V_{t+1}\right] \\ &= \frac{k_t/k_{t-1}^\gamma}{\gamma}\left[1 - \gamma - (1 + (q_0/\Delta q))\psi + (1 + (q_0/\Delta q))Q_{t+1}\right] \\ &= \frac{L - Q_t}{\gamma}\left[1 - \gamma - (1 + (q_0/\Delta q))\psi + (1 + (q_0/\Delta q))Q_{t+1}\right] \end{aligned}$$

which yields the equilibrium value of Q_t. Q.E.D.

We provide some preliminary results to characterize the equilibrium paths.

Lemma A1. *In any equilibrium path the flow profit of the firms must be non-negative* $\pi_t \geq 0$ *in each period* $t \geq 0$.

Proof. Suppose that $\pi_t < 0$ holds in some period t. Then, the firms cannot pay out some parts of b_t, v_t and R_t from the total revenue k_t^γ in that period t. Thus the equilibrium contract $\{b_t, v_t, R_t, k_t, x_t\}$ in period t cannot be enforced. Instead, the firms must adjust ex post to make different payments $\tilde{b}_t$, $\tilde{v}_t$ and $\tilde{R}_t$ the sum of which can be covered by the total revenue k_t^γ, ensuring that the revenue is not less than the total payment ex post. Then, the firms should have offered such contract which attains a non-negative profit without loss of generality. Q.E.D.

Lemma A2. *In any equilibrium path the firms never shut down the productions in each period $t \geq 0$.*

Proof. Suppose that the firms shut down in some period t. Thus the firms make no capital investment $k_t = 0$ and earn zero profit $\pi_t = 0$ in period t. Then, the firm market price in period t satisfies $V_t = (1/r_t)V_{t+1}$ by the no-arbitrage condition. Also the credit market equilibrium in period t implies that $0 = Lk_t^{\gamma} = k_{t+1} - (1-\lambda)x_{t+1} + V_{t+1}$ and hence $k_{t+1} = V_{t+1} = 0$ (note that $k_{t+1} = k'_{t+1} + x_{t+1} \geq 0$, $k'_{t+1} \geq 0$ and $x_{t+1} \geq 0$). This shows that $V_t = 0$ by the above no-arbitrage condition. Then the credit market equilibrium in period $t-1$ in turn implies that $Lk_{t-1}^{\gamma} = k_t - (1-\lambda)x_t + V_t = 0$ and hence $k_{t-1} = 0$ because $k_t = 0$ implies $x_t = 0$ as well. Repeating this process, we have $k_s = 0$ and $V_s = 0$ for all $1 \leq s \leq t$. However, in the initial period 0 the credit market equilibrium is then given by $Lk_0^{\gamma} = k_1 - (1-\lambda)x_1 + V_1 = 0$ so that $k_0 = 0$, contradicting to the initial condition $k_0 > 0$. Thus, $k_t > 0$ holds in any period $t \geq 1$. Also, in the initial period $t = 0$ the firms can alway earn a positive profit $\underline{s}k_0^{\gamma} > 0$ by using the endowed initial capital k_0 and implementing low effort from old workers. Q.E.D.

By Lemma A1 and A2 equilibrium paths must be that the firms produce and earn non-negative profits in each period. In addition, we can show that the firms must induce high effort from employed old workers in each period in any equilibrium path when the productivity $\underline{s} = h(0)$ of shirking old worker is sufficiently small (see Proposition B1 in the appendix B below), which we assume in what follows.

Proof of Proposition 1. Suppose contrary to this claim that there exists some period T such that the equilibrium path is in the mixed or explicit contract regime at $T-1$:

$$Q_T \leq \lambda\gamma + \psi$$

but it is in the implicit contract regime at T:

$$Q_{T+1} \geq \lambda\gamma + \psi.$$

This implies that $L > \lambda\gamma + \psi$: otherwise, $Q_{T+1} \geq \lambda\gamma + \psi \geq L$ and then CME in period T must imply that $0 \geq (L - Q_{T+1})k_T^{\gamma} = \lambda k_{T+1}$ and hence $k_{T+1} \leq 0$. Then $k_{T+1} = 0$ must hold so that $k_s = 0$ and $Q_s = 0$ for all $s > T+1$ due to CME from period t onward. However, then $Q_{T+1} = 0$ must be also satisfied because we have $V_{T+1} = (1/r_{T+1})(\pi_{T+1} + V_{T+2}) = 0$ by $\pi_{T+1} = 0$ (due to $k_{T+1} = 0$) and $V_{T+2} = 0$, a contradiction to $Q_{T+1} > 0$. Thus $L > \lambda\gamma + \psi$.

Case 1: DE is binding in period T. In this case we have

$$\frac{Q_T}{L - Q_T} = \frac{1}{Q_{T+1} - \psi}$$

Since $Q_T \leq \lambda\gamma + \psi$, this implies that

$$\frac{\lambda\gamma + \psi}{L - (\lambda\gamma + \psi)} \geq \frac{1}{Q_{T+1} - \psi}.$$

We define $\tilde{Q}$ such that

$$\frac{\lambda\gamma + \psi}{L - (\lambda\gamma + \psi)} = \frac{1}{\tilde{Q} - \psi}.$$

The above two (in)equalities imply that $\tilde{Q} \leq Q_{T+1}$.

Next we show that $\tilde{Q} > Q' \equiv \frac{L}{\lambda\gamma+1}$ if $Q' > \lambda\gamma + \psi$. In fact $\tilde{Q} > Q'$ is equivalent to

$$\frac{\lambda\gamma + \psi}{L - (\lambda\gamma + \psi)} < \frac{1}{Q' - \psi}$$

which can be in turn written by

$$L > L' \equiv (\lambda\gamma + 1)(\lambda\gamma + \psi)$$

which holds if $Q' > \lambda\gamma + \psi$.

Now we consider three sub-cases as follows:

Case 1-1: $Q'' > \gamma + \psi$. Then $Q'' > \gamma + \psi \geq Q_T$ so that $Q'' > Q_T$. However, then there are no Q_{T+1} such that $Q_T = \Phi(Q_{T+1})$ and $Q_{T+1} \geq \lambda\gamma + \psi$ by definition of Q'' (Figure A1). Thus the equilibrium path never moves to the implicit contract regime in period T, a contradiction.

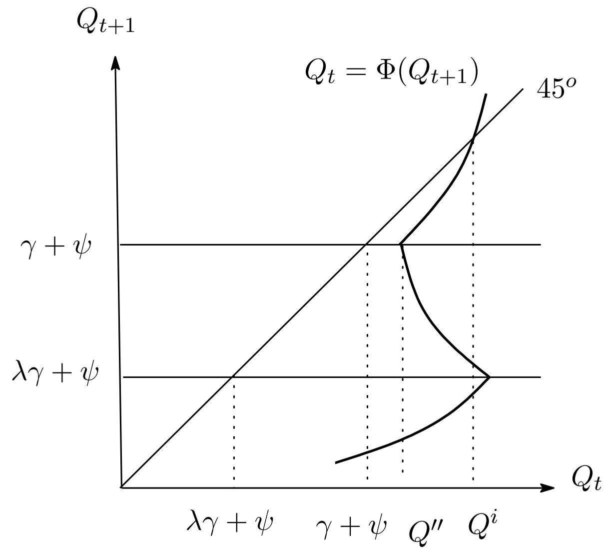

Figure A1: $Q'' > \gamma + \psi$

Case 1-2: $Q' < \lambda\gamma + \psi$. Then $Q' < \lambda\gamma + \psi \leq Q_{T+1}$ so that $Q' < Q_{T+1}$. Since in this case we have $\Phi(Q) < Q$ for all $Q > \lambda\gamma + \psi$, Q_t increases over time and $Q_t \to \infty$ as $t \to \infty$ (Figure A2).

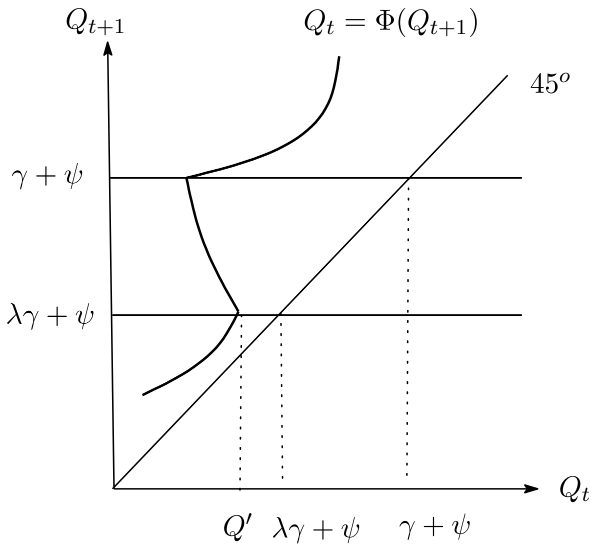

Figure A2: $Q' < \lambda\gamma + \psi$

Case1-3: $Q'' < \gamma + \psi$ and $Q' > \lambda\gamma + \psi$ (Figure A3). In this case we know that $\tilde{Q} > Q'$. Thus $Q' < \tilde{Q} \leq Q_{T+1}$. However, when $Q' < Q_{T+1}$, the next period value Q_{T+2} which satisfies $Q_{T+1} = \Phi(Q_{T+2})$ must be above $\gamma + \psi$. Then, since $\Phi(Q) < Q$ for all $Q > \gamma + \psi$ if $Q'' < \gamma + \psi$, Q_t increases over time from period $T + 1$ and $Q_t \to \infty$ as $t \to \infty$, a contradiction.

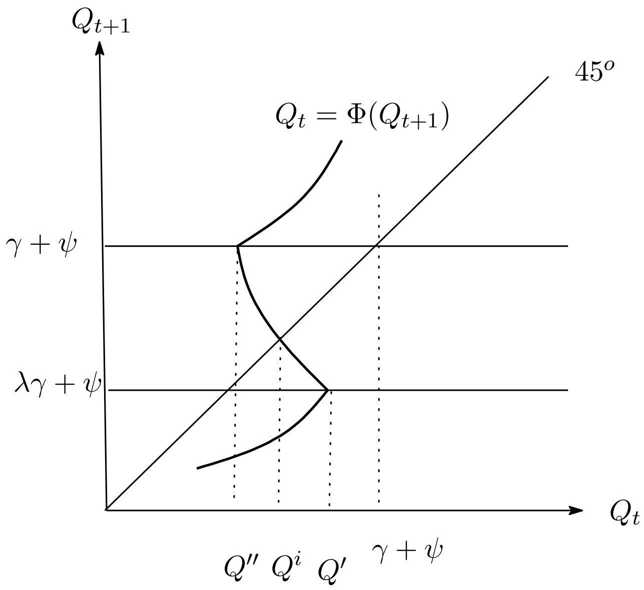

Figure A3: $Q'' < \gamma + \psi$ and $Q' > \lambda\gamma + \psi$

Case 2: DE is not binding in period T. In this case we have $Q_{T+1} > \gamma + \psi$.

Case 2-1: $Q'' > \lambda\gamma + \psi$. Then $Q'' > \lambda\gamma + \psi \geq Q_T$ so that $Q'' > Q_T$. Thus there exist no Q_{T+1} such that $Q_T = \Phi(Q_{T+1})$ and Q_{T+1} lies in the implicit contract regime in period $T + 1$.
Case 2-2: $Q'' < \lambda\gamma + \psi < Q'$. This must imply that $Q' < \gamma + \psi$. If this is not the case, we have $Q'' < \lambda\gamma + \psi$ and $Q' \geq \gamma + \psi$. However, we then obtain

$$(\lambda\gamma + 1)(\gamma + \psi) < L < (\gamma + 1)(\lambda\gamma + \psi)$$

due to the definitions of Q' and Q'', which in turn implies that $1 < \psi$. This contradicts to Assumption 2. Thus $Q' < \gamma + \psi$. Then, $Q_{T+1} > \gamma + \psi > Q'$ must imply the following: Q_{T+2} defined as $Q_{T+1} = \Phi(Q_{T+2})$ has the property that $Q_{T+2} > \gamma + \psi$. Also $Q_{T+2} > Q_{T+1}$. Thus $Q_t \to \infty$ as $t \to \infty$, a contradiction.
Case 2-3: $Q' < \lambda\gamma + \psi$. Since Q_t increases over time when $Q' < \lambda\gamma + \psi$ and $Q_{T+1} > \gamma + \psi$, we have $Q_t \to \infty$ as $t \to \infty$, a contradiction. Q.E.D.

Proof of Proposition 2. Suppose that $Q_t \neq Q^i$ holds in some period t in some equilibrium path $\{Q_t\}_{t=1}^{\infty}$ which stays in the implicit contract regime forever from the initial period.

Case 1: $Q'' > \gamma + \psi$ (Figure A1). In this case Q^i must satisfy $Q^i > \gamma + \psi$ so that DE is not binding at Q^i. Also, we must have $\Phi(Q) > Q$ for all $Q \in (\lambda\gamma + \psi, Q^i)$ and $\Phi(Q) < Q$ for all $Q > Q^i$. Thus, if $Q_t \neq Q^i$, the equilibrium path $\{Q_s\}_{s=t}^{\infty}$ from period t either diverges to a positive infinity or eventually becomes below $\lambda\gamma + \psi$. In the former the equilibrium path cannot be an equilibrium and in the latter the equilibrium path cannot stay in the implicit contract regime anymore.

Case 2: $Q' < \lambda\gamma + \psi$ (Figure A2). In this case we must have $\Phi(Q) < Q$ for all $Q > \lambda\gamma + \psi$: since $Q'' < Q' < \lambda\gamma + \psi < \gamma + \psi$, it must be that $Q'' < \gamma + \psi$ and hence $\Phi(Q) < Q$ for all $Q > \gamma + \psi$ under Assumption 2. Also $\Phi(Q) < Q$ must hold for all $Q \in (\lambda\gamma + \psi, \gamma + \psi)$ because Φ is decreasing in $Q \in (\lambda\gamma + \psi, \gamma + \psi)$ and $Q' < \lambda\gamma + \psi$. Then, the equilibrium path $\{Q_s\}_{s=t}^{\infty}$ diverges to a positive infinity as long as it stays in the implicit contract regime. This cannot be an equilibrium.

Case 3: $Q'' < \gamma + \psi$ and $Q' > \lambda\gamma + \psi$ (Figure A3). In this case Q^i satisfies $\lambda\gamma + \psi < Q^i < \gamma + \psi$. Also, $Q_\tau < \gamma + \psi$ must hold for some period τ because otherwise Q_s diverges to infinity, which cannot be an equilibrium. To see this, note that $\Phi(Q) < Q$ for all $Q \geq \gamma + \psi$ in Case 3, implying that $Q_s \to \infty$ as $s \to \infty$ if $Q_t \geq \gamma + \psi$ for all $t \geq 1$. Thus $Q_\tau < \gamma + \psi$ must hold. In what follows we suppose $\tau = t + 1$ without loss of generality so that $Q_{t+1} < \gamma + \psi$. Then, since Q_t must be in the implicit contract regime and DE must be binding in period t, we obtain

$$\frac{Q_t}{L - Q_t} = \frac{1}{Q_{t+1} - \psi}. \tag{A1}$$

First, suppose that $Q_t < Q^i$ (Figure A4). Then there must exist some integer $n \geq 1$ such that $Q_{t+n+1} < \gamma + \psi$ again: otherwise, $Q_s > \gamma + \psi$ must hold for all $s > t$ so that Q_s diverges to infinity as $s \to \infty$, which cannot be an equilibrium. Furthermore, since $Q_t < Q^i$, it must be that $Q_t < Q^i < Q_{t+n}$. Also, since Φ is decreasing in Q when $Q \in (\lambda\gamma + \psi, \gamma + \psi)$, by using $Q_t < Q^i$ and $\lambda\gamma + \psi < Q_{t+1} < \gamma + \psi$, we must have $Q_{t+1} > Q_t$. Given $Q_{t+n+1} < \gamma + \psi$, for the equilibrium path $\{Q_s\}_{s=t+n}^{\infty}$ to stay in the implicit contract regime, $Q_{t+n+1} \geq \lambda\gamma + \psi$ must hold. Thus $Q_{t+n+1} \in (\lambda\gamma + \psi, \gamma + \psi)$. Then, since the equilibrium path lies in the implicit contract regime, it must be that DE is binding in period $t + n$:

$$\frac{Q_{t+n}}{L - Q_{t+n}} = \frac{1}{Q_{t+n+1} - \psi}. \tag{A2}$$

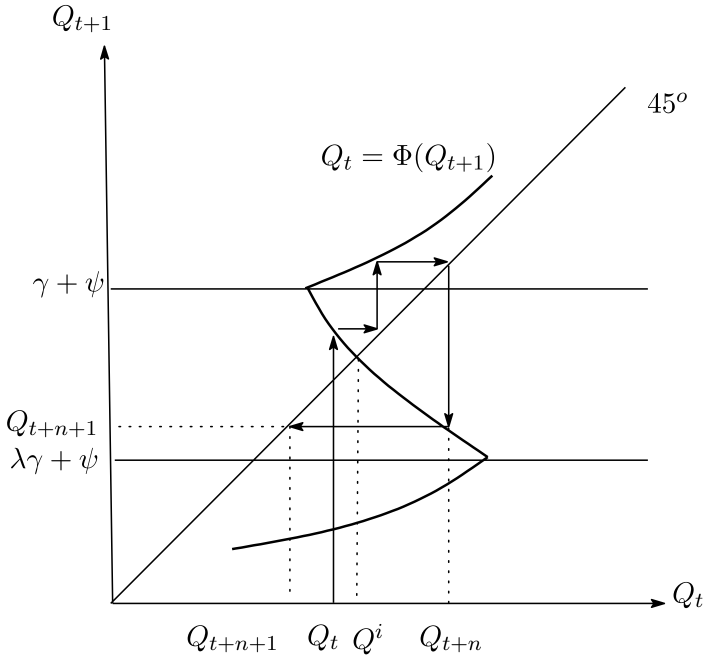

Figure A4: $Q'' < \gamma + \psi$ and $Q' > \lambda\gamma + \psi$

Furthermore, since $Q_{t+n} > Q^i$ and Φ is decreasing in $Q \in (\lambda\gamma + \psi, \gamma + \psi)$, $Q_{t+n+1} < Q^i$ must be satisfied. We then show that $Q_{t+n+1} < Q_t$. Suppose contrary to this claim that $Q_{t+n+1} \geq Q_t$. Then, by using (A2) together with $Q_{t+n} > Q_{t+1}$, we can show that

$$\frac{1}{Q_t - \psi} \geq \frac{1}{Q_{t+n+1} - \psi} = \frac{Q_{t+n}}{L - Q_{t+n}} \geq \frac{Q_{t+1}}{L - Q_{t+1}}$$

which implies that

$$\frac{1}{Q_t - \psi} \geq \frac{Q_{t+1}}{L - Q_{t+1}}. \tag{A3}$$

By combining (A1) with (A3), we then obtain

$$\frac{Q_{t+1}}{L-Q_{t+1}} \times \frac{L-Q_t}{Q_t} \leq \frac{Q_{t+1}-\psi}{Q_t-\psi}$$

which is in turn rewritten by

$$\psi L(Q_{t+1}-Q_t) \geq Q_t Q_{t+1}(Q_{t+1}-Q_t).$$

Since $Q_{t+1} > Q_t$, the above inequality shows that $\psi L \geq Q_t Q_{t+1}$. Thus $Q_t \leq \psi L/Q_{t+1}$. By using this, (A1) implies that

$$\frac{\psi L/Q_{t+1}}{L-\psi L/Q_{t+1}} \geq \frac{Q_t}{L-Q_t} = \frac{1}{Q_{t+1}-\psi}$$

which in turn shows that $\psi \geq 1$. This however contradicts to Assumption 2. Thus $Q_{t+n+1} < Q_t$ must be satisfied. The above result has established that, if $Q_t < Q^i$, $Q_{t+n+1} < Q_t$ must hold for some positive integer n. Then, $Q_{t+n+1} < Q_t < Q^i$ so that $Q_{t+n+1} < Q^i$. By a similar argument, we can show that there exists some integer $m > n$ such that $Q_{t+m+1} < Q_{t+n+1} < Q^i$. By repeating this process, we obtain a decreasing sequence $Q_{t+k(j+1)} < Q_{t+k(j)} < \cdots < Q_t < Q^i$ where $k(j)$ is a positive integer and $k(j+1) > k(j)$ for $j = 0, 1, 2, ...$ and $k(0) \equiv 0$. This sequence eventually becomes lower than $\lambda\gamma + \psi$. Then the equilibrium path $\{Q_t\}_{t=1}^{\infty}$ must leave the implicit contract regime. This is a contradiction.

Second, suppose that $Q_t > Q^i$. Since Φ is decreasing in Q when $\lambda\gamma + \psi < Q < \gamma + \psi$, $Q_t > Q^i$ and $Q_{t+1} \in (\lambda\gamma + \psi, \gamma + \psi)$ imply that $Q_{t+1} < Q^i$. Then we can use the same argument as in the case of $Q_t < Q^i$ by replacing Q_t by Q_{t+1} and starting the same steps from $Q_{t+1} < Q^i$.

Thus we have established the result that, if $Q_t \neq Q^i$ at some t, the equilibrium path must enter the mixed or explicit contract regime in some period T. Then, Proposition 1 implies that any equilibrium path must stay in the mixed contract or the explicit contract regime from period T onward and that it never returns back to the implicit contract regime. Q.E.D.

Proof of Lemma 6. By the definition of $\hat{L}$ ($\psi = \Phi(\psi)$), we have $Q^e = \psi$ at $L = \hat{L}$. Also, it can be verified that Q^e is decreasing in L under Assumption 3 because the left hand side of (24) in the main text decreases with L. Q.E.D.

Proof of Proposition 3. Suppose that Assumption 1-3 hold and that $\hat{L} > L'$. Then suppose that $L > \hat{L}$.

Recall the definition of $\hat{Q}$ such that $\hat{Q} = \Phi(\psi)$ and then note that $\hat{Q} > \psi$ where

$$\hat{Q} \equiv L(1-\gamma)$$

and $L > \hat{L} \equiv \psi/(1-\gamma)$. When $Q_{t+1} < \psi$, we have the explicit contract regime in which

$$\frac{\gamma Q_t}{L-Q_t} = 1 - \gamma - (1 + (q_0/\Delta q))\psi + (1 + (q_0/\Delta q))Q_{t+1}.$$

Then we can verify that $dQ_{t+1}/dQ_t > 0$ and $d^2Q_{t+1}/dQ_t^2 > 0$.

Also in the mixed contract regime we have

$$\frac{\gamma Q_t}{L - Q_t} = \frac{1 - \gamma + (-(1/\lambda))\psi + (1/\lambda)Q_{t+1}}{1 - \frac{1-\lambda}{\lambda\gamma}(Q_{t+1} - \psi)}$$

from which we obtain

$$Q_{t+1} = \frac{(1 + (1-\lambda)\psi/\lambda\gamma)\gamma Z_t - (1 - \gamma + (-(1/\lambda))\psi)}{((1-\lambda)/\lambda)Z_t + (1/\lambda)}$$

where $Z_t \equiv Q_t/(L - Q_t)$. Then in the mixed contract regime we can verify that

$$dQ_{t+1}/dQ_t \propto \left(\frac{1}{L - \lambda Q_t}\right)^2 > 0$$

and $d^2Q_{t+1}/dQ_t^2 > 0$. Furthermore, in the mixed contract regime $\Phi(Q) > Q$ and $\Phi' > 0$ hold due to $Q' > \lambda\gamma + \psi$ (by $L > L'$) and $\hat{Q} > \psi$ (by $L > \hat{L}$).

In the explicit contract regime we can also show that $Q_{t+1} > 0$ holds at $Q_t = 0$ (i.e., $\Phi^{-1}(0) > 0$) under Assumption 3. Recall that $\hat{Q} = \Phi(\psi)$. Then, since $\hat{Q} > \psi$ (i.e., $\Phi^{-1}(\psi) < \psi$ under $L > \hat{L}$), there exists some $Q \in (0, \psi)$, denoted by Q^e, such that $Q^e = \Phi(Q^e)$ in the explicit contract regime:

$$\frac{Q^e}{L - Q^e} = (1/\gamma)\left[1 - \gamma - (1 + q_0/\Delta q)\psi + \left(1 + \frac{q_0}{\Delta q}\right)Q^e\right].$$

Such Q^e is unique because $dQ_{t+1}/dQ_t > 0$ and $d^2Q_{t+1}/dQ_t^2 > 0$ together with $\Phi^{-1}(0) > 0$ and $\Phi(\psi) > \psi$.

Since Φ is increasing over $[0, \lambda\gamma + \psi]$ and Q^e is the unique value of Q satisfying $Q = \Phi(Q)$ over $[0, \lambda\gamma + \psi]$, we can see that there exist equilibrium paths $\{Q_t\}_{t=1}^{\infty}$ (i) starting with $Q_1 \in (\lambda\gamma + \psi, \gamma + \psi)$ in the implicit contract regime in which DE becomes binding in period $t = 0$ and (ii) converging to Q^e as $t \to \infty$ (Figure 3). Along such process, the equilibrium path $\{Q_t\}_{t=1}^{\infty}$ moves from the implicit contract regime to the mixed contract regime and then to the explicit contract regime over time. Since Q_1 is a jump variable, there exist multiple paths with different initial values of $Q_1 \in (\lambda\gamma + \psi, \gamma + \psi)$.

In the initial period $t = 0$ the firm obtains the equilibrium profit as $\pi_0 = k_0^{\gamma}[1 - \psi - \max\{(q_0/\Delta q)(\psi - Q_1), 0\}]$ when implementing high effort $a = 1$ and $\pi_0 = \underline{s}k_0^{\gamma}$ when implementing low effort $a = 0$ respectively. The firm attains the larger one of these profits. Here the initial capital k_0 is exogenously given. Thus the initial period profit is determined only by the initial firm value Q_1.

Finally, we can verify that the flow profit of the firm becomes positive in any period in the equilibrium path constructed above.

First, in the implicit contract regime when DE is not binding, the firm's flow profit is given by

$$\begin{aligned}\pi_t &= k_t^{\gamma} - \lambda r_t k_t - p_1\psi_t \\ &= k_t^{\gamma}(1 - \gamma - \psi) \\ &> 0\end{aligned}$$

due to Assumption 2. When DE is binding, it becomes

$$\begin{aligned}\pi_t &= k_t^\gamma - \lambda r_t k_t - \psi_t \\ &= k_t^\gamma - V_{t+1} \\ &= k_t^\gamma - Q_{t+1}) \\ &> k_t^\gamma(\gamma + \psi - Q_{t+1}) \\ &\geq 0\end{aligned}$$

because of Assumption 2 and $\gamma + \psi \geq Q_{t+1}$ when DE is binding.

In the mixed contract regime the flow profit of the firm is given by

$$\begin{aligned}\pi_t &= k_t^\gamma - r_t k_t + (1-\lambda) r_t x_t - \psi_t \\ &= (1-\gamma) k_t^\gamma - \psi_t + (1-\lambda) r_t x_t \\ &= k_t^\gamma (1 - \gamma - \psi) + (1-\lambda) r_t x_t \\ &> 0\end{aligned}$$

due to Assumption 2.

In the explicit contract regime the firm's expected flow profit is given by

$$\begin{aligned}\pi_t &= k_t^\gamma - r_t k_t^\gamma - (1 + q_0/\Delta q)\psi_t + (q_0/\Delta q) V_{t+1} \\ &= k_t^\gamma \left\{ q_1 \left[1 - \gamma - (Q_{t+1} + (1/\Delta q)(\psi - Q_{t+1}))\right] + (1 - q_1)\left[1 - \gamma - Q_{t+1}\right]\right\}\end{aligned}$$

where $\gamma k_t^{\gamma-1} = r_t$. Note here that the firm must pay $b_t + v_t = V_{t+1} + (1/\Delta q)(\psi_t - V_{t+1})$ when the old worker's verifiable signal is realized as $\sigma_t = \sigma_h$ while it must pay $b_t = V_{t+1}$ when $\sigma_t = \sigma_l$ respectively. In either case of $\sigma_t = \sigma_h$ or $\sigma_t = \sigma_l$ the firm's realized profit must be non-negative. Since $\psi > Q_{t+1}$, it then suffices to show that

$$1 - \gamma - (Q_{t+1} + (1/\Delta q)(\psi - Q_{t+1})) \geq 0$$

which is rewritten by

$$Q_{t+1} \geq \underline{Q} \equiv \frac{\psi - \Delta q(1-\gamma)}{1 - \Delta q} > 0.$$

Here $\underline{Q} > 0$ holds due to Assumption 3. Also $\underline{Q} < \psi$ is satisfied due to Assumption 1. Since the equilibrium sequence Q_t is decreasing over time ($Q_{t+1} \geq Q_t$) and converges to Q^e, it is enough to obtain $Q^e \geq \underline{Q}$. By Lemma 6, Q^e is decreasing in L and $Q^e = \psi$ at $L = \hat{L}$. Also $Q^e \to \underline{Q}^*$ when $L \to \infty$ where $\underline{Q}^*$ is defined as the firm value making the expected flow profit $q_1[1-\gamma-(Q+(1/\Delta q)(\psi-Q))]+(1-q_1)[1-\gamma-Q]$ zero. Then $\underline{Q} > \underline{Q}^*$ holds. Thus, we can find some $\overline{L} > \hat{L}$ such that for all $L \in (\hat{L}, \overline{L})$ we have $\psi > Q^e > \underline{Q}$ because $\underline{Q}^* < \underline{Q} < \psi$. Q.E.D.

Proof of Proposition 4. Suppose that Assumption 1-4 and $L > \max\{\hat{L}, L'\}$ are satisfied.

By Proposition 2, we know that any equilibrium path must be either $Q_t = Q^i$ for all $t \geq 1$ or $Q_t \neq Q^i$ for some period t.

(i) In the former case ($Q_t = Q^i$ for all $t \geq 1$) the equilibrium path must stay in the implicit contract regime forever from the initial period $t = 0$. Then it follows from CME that $(L - Q^i)k_t^\gamma = \lambda k_{t+1}$ for $t = 0, 1, 2, ...$ and hence $k_t \to k^i$ as $t \to \infty$. If the initial capital k_0 satisfies $(L - Q^i)k_0^\gamma > \lambda k_0$, we have $k_t \leq k^i$ for all $t \geq 0$ because $(L - Q^i)k^\gamma > \lambda k$ for all $k < k^i$. Then, $\lambda k_t = (L - Q^i)k_{t-1}^\gamma > \lambda k_{t-1}$ so that $k_t > k_{t-1}$ and hence $g_t > 0$ for all $t \geq 1$.

(ii) Second, when $Q_t \neq Q^i$ holds for some t in an equilibrium path, such path must enter the mixed or explicit contract regime from some period T onward and never returns back to the implicit contract regime anymore. Since $L > \max\{\hat{L}, L'\}$, $\Phi(Q) > Q$ holds for all $Q \in [\psi, \lambda\gamma + \psi]$. Thus, if the equilibrium path lies in the mixed contract regime in some period, the firm value Q_t declines from that period over time and eventually enters the explicit contract regime in which $Q_t < \psi$. Then, $Q_t \to Q^e$ holds as $t \to \infty$, which also implies that $k_t \to k^e$ in such limit.

Furthermore, when the economy enters the mixed contract regime, $Q_t = \Phi(Q_{t+1})$ satisfies

$$Q_{t+1} = \frac{\left(1 + \frac{1-\lambda}{\lambda\gamma}\psi\right)\frac{\gamma Q_t}{L-Q_t} - \eta}{\frac{1-\lambda}{\lambda}\frac{Q_t}{L-Q_t} + \frac{1}{\lambda}}$$

where we define $\eta \equiv 1 - \gamma + (1 - p_1 - (1/\lambda))$. Then we have

$$\begin{aligned} 1 - \frac{1-\lambda}{\lambda\gamma}(Q_{t+1} - \psi) &= \frac{L - Q_t}{\gamma Q_t}\left[\eta + (1/\lambda)Q_{t+1}\right] \\ &= \frac{L - Q_t}{\gamma Q_t}\left[\eta + \frac{\left(1 + \frac{1-\lambda}{\lambda\gamma}\psi\right)\frac{\gamma Q_t}{L-Q_t} - \eta}{(1-\lambda)\frac{Q_t}{L-Q_t} + 1}\right] \end{aligned}$$

where the last bracket term can be re-written by

$$\left[\eta + \frac{\left(1 + \frac{1-\lambda}{\lambda\gamma}\psi\right)\frac{\gamma Q_t}{L-Q_t} - \eta}{(1-\lambda)\frac{Q_t}{L-Q_t} + 1}\right] = \left[\frac{\eta(1-\lambda) + \gamma + \frac{1-\lambda}{\lambda}\psi}{L - \lambda Q_t}\right] Q_t$$

Thus CME in the mixed contract regime implies that

$$\begin{aligned} k_{t-1}^\gamma &= \frac{1}{(L - Q_t)}\left[1 - \frac{1-\lambda}{\lambda\gamma}(Q_{t+1} - \psi)\right] k_t \\ &= \left[\frac{\eta(1-\lambda) + \gamma + \frac{1-\lambda}{\lambda}\psi}{L - \lambda Q_t}\right] k_t \end{aligned}$$

so that

$$\begin{aligned} k_t &= \gamma\left[\frac{L - \lambda Q_t}{\eta(1-\lambda) + \gamma + \frac{1-\lambda}{\lambda}\psi}\right] k_{t-1}^\gamma \\ &\equiv G(Q_t)k_{t-1}^\gamma \end{aligned}$$

The right hand side of the above expression is decreasing in Q_t. Since Q_t decreases over time in the mixed contract regime, we can show that $k_{t+1}/k_t^\gamma = G(Q_{t+1}) > G(Q_t) = k_t/k_{t-1}^\gamma$, which implies that $(g_{t+1}+1) > (g_t+1)^\gamma$ for $g_t \equiv k_t/k_{t-1} - 1$. Then g_t increases whenever $g_t < 0$ and $g_{t+1} > 0$ holds as long as $g_t > 0$. This shows the desired result.

In the explicit contract regime CME implies that

$$k_t = (L - Q_t)k_{t-1}^\gamma.$$

Since Q_t decreases over time when $Q_t < \psi$, we have $(g_{t+1}+1) > (g_t+1)^\gamma$ again, yielding the desired result as well. Q.E.D.

Proof of Proposition 5. When $\lambda \to 0$, we have $\hat{L} > L'$. Then, under Assumption 1-3 and $L > \hat{L}$, there are the steady states only in the implicit contract regime (Q^i, k^i) and the explicit contract regime (Q^e, k^e). Then $SW^i > SW^e$ holds as $\lambda \to 0$ because Q^i and Q^e are independent of λ.

Next we take λ close to 1. When $L > L'$ still holds, the long run steady states are only the implicit contract steady state and the explicit contract steady states again. In this case the long run welfare in the implicit contract regime becomes

$$\lim_{\lambda\to 1} SW^i = (L - Q^i)^{\frac{\gamma}{1-\gamma}}(1 + Q^i)$$

When $L' > L$, the implicit contract steady state no longer exists but the mixed contract steady state (Q^m, k^m) exists in addition to the explicit contract steady state. The steady state welfare in the mixed contract regime SW^m becomes

$$\lim_{\lambda\to 1} SW^m = \lim_{\lambda\to 1}\left(\frac{L - Q^m}{1 - \frac{1-\lambda}{\lambda\gamma}(Q^m - \psi)}\right)^{\frac{\gamma}{1-\gamma}}(1 + Q^m) = (L - Q^m)^{\gamma/(1-\gamma)}(1 + Q^m)$$

where $Q^m \to Q^*$ as $\lambda \to 1$ and recall that Q^* satisfies

$$\frac{\gamma Q^*}{L - Q^*} = 1 - \gamma - \psi + Q^*.$$

On the other hand, the steady state welfare in the explicit contract regime still exists and is given by SW^e which is independent of λ. Also $Q^e < Q^m$ holds.

We now compute from $SW(Q) \equiv (L - Q)^{\gamma/(1-\gamma)}(1 + Q)$ that

$$\begin{aligned} SW'(Q) &= -\frac{\gamma}{1-\gamma}(L - Q)^{\frac{\gamma}{1-\gamma}-1}(1 + Q) + (L - Q)^{\frac{\gamma}{1-\gamma}} \\ &= (L - Q)^{\frac{\gamma}{1-\gamma}-1}\left\{-\frac{\gamma}{1-\gamma}(1 + Q) + L - Q\right\} \end{aligned}$$

The bracket term in the above expression of $SW'(Q)$ is decreasing in Q. Thus $SW''(Q) < 0$ at any Q satisfying $SW'(Q) = 0$ (note that $L > Q$ must hold in any steady state. Thus $L \neq Q$.) This shows that SW is a quasi-concave function. Also, we have $SW'(Q^e) \leq 0$ if and only if $(1-\gamma)L - \gamma \leq Q^e$. Since the realized profit of the firm must be non-negative in the steady state in the explicit contract regime, it must be that $Q^e \geq \underline{Q}$ for the cut off value $\underline{Q}$ defined in the proof of Proposition 3. This implies that

$$\begin{aligned} \frac{\gamma \underline{Q}}{L-\underline{Q}} &\geq \left[1-\gamma-(1+q_0/\Delta q)\psi+(q_0/\Delta q)\underline{Q}\right]+\underline{Q} \\ &\geq \underline{Q} \end{aligned}$$

because the expected flow profit, expressed by the above bracket term, becomes non-negative at $\underline{Q}$. Thus, $\underline{Q} \geq L-\gamma$ which shows that $Q^e \geq \underline{Q} \geq L-\gamma$. Then $Q^e \geq (1-\gamma)L-\gamma$ is satisfied. This implies that $SW'(Q^e) \leq 0$. Since $Q^e \leq Q^m \to Q^*$ and $Q^e \leq Q^i$, we obtain that $SW(Q^e) = SW^e > SW^i = SW(Q^i)$ and $SW^e > SW(Q^m) = SW^m$. Q.E.D.

8 Appendix B: Extensions

In this appendix we investigate several extensions of the model.

8.1 Separable Wage

A wage of an old worker can be written by $W(s,\sigma)$ in more general form, which depends on the perfect but unverifiable signal $s = a \in S \equiv \{0,1\}$ and the imperfect but verifiable signal $\sigma \in \Sigma \equiv \{\sigma_h, \sigma_l\}$ in the interdependent way. We however show that we can restrict our attention to the additive separable form $W(s,\sigma) = b(s)+v(\sigma)$ without loss of generality as we have considered in the main text.

To show this, suppose that $W(s,\sigma)$ satisfies DE, WIC and LL. Then define a new wage scheme as follows:

$$v_t(\sigma) \equiv \min_{s\in S} W_t(s,\sigma)$$

for an explicit wage scheme and

$$b_t(s) \equiv \sum_{\sigma\in\Sigma} q(\sigma|s)[W_t(s,\sigma) - \min_{s\in S} W_t(s,\sigma)]$$

for an implicit wage scheme respectively. Here, $q(\sigma|s) \in (0,1)$ denotes the probability of the verifiable signal σ being realized conditional on an unverifiable signal (effort) $s = a$ respectively. By construction, $v_t(\sigma) \geq 0$ and $b_t(s) \geq 0$ hold for all $s \in S$ and $\sigma \in \Sigma$.

Then we have

$$\sum_{\sigma\in\Sigma} q(\sigma|s)[b_t(s) + v_t(\sigma)] = \sum_{\sigma\in\Sigma} q(\sigma|s) W_t(s,\sigma)$$

for any perfect unverifiable signal (effort) $s = a$ so that the new scheme still implements the same effort as the original one does. Also, the expected wage of the new wage scheme is same as the original one as well.

Finally, we check that the new wage scheme satisfies DE. Since the original wage scheme W satisfies DE, we have

$$V_{t+1} \geq R_t + W_t(s,\sigma) - \min_{s\in S} W_t(s,\sigma)$$

for each $\sigma \in \Sigma$. This states that the firm has no incentive to renege on the worker's wage $W_t(s,\sigma)$ and the repayment R_t to the financier by claiming that the worker's unverifiable signal $s \in S$ was the one such that the wage is the lowest, i.e., $\min_s W_t(s,\sigma)$, rather than paying the agreed upon one $W_t(s,\sigma)$, for each realization of the two signals $s \in S$ and $\sigma \in \Sigma$.

Then, by using the definition of b_t and taking the expectation about the above inequality DE over σ, we obtain

$$\begin{aligned} V_{t+1} &\geq R_t + \sum_{\sigma\in\Sigma} q(\sigma|s)[W_t(s,\sigma) - \min_{s\in S} W_t(s,\sigma)] \\ &= R_t + b_t(s) \\ &\geq R_t + b_t(s) - \min_{s\in S} b_t(s). \end{aligned}$$

which implies that the new wage scheme b_t satisfies DE: under the bonus scheme b_t, the firm can renege on b_t and R_t by claiming that the worker's signal s was the one such that the bonus is the lowest, i.e., $\min_s b_t(s)$, which is however less profitable than paying the agreed upon implicit wage $b_t(s)$ for each realization of $s \in S$.

8.2 Equity Financing

We have assumed in the main text that the firms borrow the remaining capital investment $k_t - x_t$ from the credit market which is not covered by insider lending. Alternatively, the firms can issue new equities to finance some part of $k_t - x_t$. However, such equity financing is irrelevant as long as the no-arbitrage condition holds.

We denote by n_t the number of equities issued by a firm (owned by young entrepreneurs) in the end of period $t-1$ for financing some k''_t of $k_t - x_t$ which will be used for production in period t while the remaining $k'_t \equiv k_t - x_t - k''_t$ is borrowed from the credit market. n_0 denotes the number of existing equities held by initial old individuals in period 0, which is exogenously given. An equity contract issued in period s specifies a stream of dividends $\{d_t^s\}_{t=s}^{\infty}$ over time such that each shareholder has the financial claim to receive d_t^s in period $t \geq s$ from the firm. Note that $\Pi_t = \sum_{s=0}^{t} n_s d_t^s$ holds where $\Pi_t = k_t^{\gamma} - r_t k'_t - R_t - E[b_t + v_t]$ denotes the profit of a firm which should be shared between shareholders owning $\sum_{s=0}^{t} n_s$ total equities of the firm in period t. Here $E[b_t + v_t]$ denotes the expected wage paid to an employed old worker.

The period t value of an equity issued in period $s \leq t$, denoted by V_t^s, satisfies the no-arbitrage condition:

$$r_t V_t^s = d_t^s + V_{t+1}^s. \tag{B1}$$

Consider the total value of equities of a firm owned by the existing (young) individuals in period $t-1$ *before* they issue new equities for financing a capital investment k''_t for the production in period t. We define such total equity value of the existing shareholders as $\tilde{V}_t \equiv \sum_{s=0}^{t-1} n_s V_t^s$. The young shareholders who own the firm in period $t-1$ behave to maximize their total payoffs represented by the total value $\tilde{V}_t$.

By multiplying (B1) by n_t^s and summing over s from $s=0$ to $s=t$, we obtain

$$r_t \sum_{s=0}^{t} n_s V_t^s = \sum_{s=0}^{t} n_s d_t^s + \sum_{s=0}^{t} n_s V_{t+1}^s.$$

which can be written by

$$\begin{aligned} r_t(\tilde{V}_t + n_t V_t^t) &= \Pi_t + \sum_{s=0}^{t} n_s V_{t+1}^s \\ &= \Pi_t + \tilde{V}_{t+1} \end{aligned}$$

Since newly issued equities are used to finance k_t'', we obtain $n_t V_t^t = k_t''$. By using this, the above expression can be further written by

$$\begin{aligned} r_t \tilde{V}_t &= \Pi_t - r_t k_t'' + \tilde{V}_{t+1} \\ &= k_t^\gamma - r_t(k_t'' + k_t') - R_t - E[b_t + v_t] + \tilde{V}_{t+1} \\ &= k_t^\gamma - r_t(k_t - x_t) - R_t - E[b_t + v_t] + \tilde{V}_{t+1} \end{aligned}$$

which is exactly same as the value V_t defined in the main text. The young shareholders owing the firm in period $t-1$ thus has the same objective V_t to maximize as we have considered in the main text.

Finally, we check that the credit market equilibrium (CME) is not essentially changed from the one in the main text. Suppose that each of $1-N_f$ young entrepreneurs born in period $t-1$ purchases l_t^s units of equities issued in period s. Note that $(1-N_f)l_t^s = Nn_s$ must hold because the total demand in period $t-1$ for the equity issued in period s should be equal to its total supply. Since the period $t-1$-price of an equity issued in period s is given by V_t^s, each of those young entrepreneurs spends $\sum_{s=0}^{t} l_t^s V_t^s$ for purchasing the total equities in period $t-1$. Also the young entrepreneurs born in period $t-1$ must spend $k_t' \equiv k_t - x_t - k_t''$ new capital investment by market lending per firm. Thus, since there are totally $\sum_{s=0}^{t} n_s$ units of equities per firm which have been issued up to period $t-1$, each of those young entrepreneurs must spend $k_t' / \sum_{s=0}^{t} n_s$ per equity he or she owns. Since he or she owns l_t^s units for period t-equity issued in period s, he or she must spend

$$\sum_{s=0}^{t} l_t^s \frac{(k_t - x_t - k_t'')}{\sum_{s=0}^{t} n_s}$$

for financing new capital investment by market lending. Thus, the young entrepreneurs born in period $t-1$ have the excess credit demand as follows

$$(1-N_f)\left\{ w_{t-1} - \sum_{s=0}^{t} l_t^s V_t^s - \sum_{s=0}^{t} l_t^s \frac{(k_t - x_t - k_t'')}{\sum_{s=0}^{t} n_s} \right\}$$

which is equivalent to

$$\begin{aligned}
&(1-N_f)w_{t-1} - \sum_{s=0}^{t}(1-N_f)l_t^s V_t^s - \sum_{s=0}^{t}(1-N_f)l_t^s \frac{(k_t - x_t - k_t'')}{\sum_{s=0}^{t} n_s} \\
&= (1-N_f)w_{t-1} - N\sum_{s=0}^{t} n_s V_t^s - N(k_t - x_t - k_t'') \\
&= (1-N_f)w_{t-1} - N\left\{\sum_{s=0}^{t-1} n_s V_t^s + n_t V_t^t + (k_t - x_t - k_t'')\right\} \\
&= (1-N_f)w_{t-1} - N\{\tilde{V}_t + k_t'' + (k_t - x_t - k_t'')\} \\
&= (1-N_f)w_{t-1} - N(\tilde{V}_t + (k_t - x_t)).
\end{aligned}$$

This is essentially same as the excess credit demand of the young entrepreneurs which we have obtained in the main text.

Since the young financiers born in period $t-1$ have the excess credit demand as $N_f(w_{t-1} - m\lambda x_t)$ where m stands for the number of the firms each financier lends to, it is not changed from the one given in the main text. Since $N_f m = N$ holds, the total excess credit demand in the economy in period $t-1$ is given by

$$w_{t-1} - N\{k_t - (1-\lambda)x_t + \tilde{V}_t\}$$

which must be zero in CME. This condition is same as the CME given in the main text.

Hence, equity financing does not affect the results obtained in the main text.

8.3 Endogenous Endowment

For simplicity, in the main text we have assumed that young capitalists and old workers can access to the private technology which produces the outputs by themselves: young capitalists can produce $w_t = LA_t$ units of good which becomes their endowment where $A_t = Ny_t$ denotes the social knowledge embodied in the aggregate outputs y_t. Old workers can secretly access to the private technology which produces ψA_t units of good by spending endowed labor skill.

In this section we will provide a microeconomic foundation for this setting.

In stead of assuming the private technology, we introduce a perfectively competitive labor market in which young capitalists and old workers supply their endowed labor units to earn the competitive market wage, which becomes the sources for the credit supply of the economy. In addition to the prefect competitive labor market there is another labor market in which old workers are hired by the firms as considered in the basic model. They arrive at each firm every period and one worker among those who visited a firm is hired by the firm. The employed old workers work by choosing effort $a \in \{0, 1\}$ as in the main model.

We assume that each young capitalist (entrepreneur or financier) is endowed with one unit of labor while each old worker is endowed with $B > 0$ units of labor. They can inelastically supply their endowed labor units to the perfectly competitive labor market to earn the market wage, denoted by w_t. The old workers who are employed by the firms can decide whether to choose high effort ($a_t = 1$) by spending B units of labor for the production of the firm or to choose low effort ($a_t = 0$) by secretly spending the endowed labor to the perfectly competitive labor market to earn Bw_t outside incomes.

We then slightly change the basic model as follows: There is the final good sector in which many final good producers produce the final good, of which price is normalized to unity. The firms modeled in the main text are reinterpreted as the ones producing the ideas to be used for the production of the final good. In what follows, when we refer to "firm", it means the firm producing the ideas while, when we refer to "producer", it means the producer producing the final good respectively. We suppose that the final good sector is the decentralized matching market in which the final good producers and the firms producing the ideas are randomly matched with each other. When they are matched in the end of period $t-1$, the firm invests in capital k_t to produce the ideas $y_t = h(a_t)k_t^\gamma$ in period t. In the beginning of period t, the final good producer who is matched with a firm expects the ideas developed by the matched firm, denoted by $\hat{y}_t$, and then employs the competitive labor L_t from the perfectly competitive labor market. By combining the ideas y_t with the employed labor L_t, a matched pair of a final good producer and a firm can produce the final good outputs $Y_t = y_t L_t$. At the production stage, the final good producer is assumed to observe the firm's ideas y_t and then enter the bargaining stage in which they split the final good outputs Y_t according to a constant share; the firm obtains $\beta \in (0, 1)$ share of Y_t while the final good producer obtains the remaining share $1 - \beta$ respectively.

The final good producer matched with a firm thus obtains the profit $(1-\beta)Y_t - w_t L_t = (1-\beta)\hat{y}_t L_t - w_t L_t$. Then, the profit maximization yields $(1-\beta)\hat{y}_t = w_t$. In the rational expectation equilibrium we have $y_t = \hat{y}_t$ so that

$$w_t = (1-\beta)y_t = (1-\beta)k_t^\gamma$$

provided $a_t = 1$ as considered in the main text.

The firm matched with a final good producer obtains the share of the final good outputs as $\beta Y_t = \beta k_t^\gamma$ because $L_t = 1 + (1-N)B = 1$ holds in the competitive labor market. Note that all young capitalists supply their endowed labor to the competitive labor market but $1 - N$ old workers who are not matched with the firms work in the competitive labor market. However, since we assume $N = 1$, $L_t = 1$ holds. Then, the firm matched with a final good producer earns the profit as

$$\pi_t = \beta k_t^\gamma - r_t(k_t - x_t) - \lambda r_t x_t - (b_t + q_0 v_t)$$

which is essentially same as the one derived in the main text.

Each old worker who is matched with a firm earns the expected wage as

$$b_t + q_0 v_t = \psi_t + (q_0/\Delta q)\max\{\psi_t - V_{t+1}, 0\}$$

where $\psi_t \equiv Bw_t = B(1-\beta)k_t^\gamma$.

The credit market equilibrium is modified as follows: one unit mass of young capitalists earn the competitive market wage w_{t-1} in period $t-1$ which become the savings of the economy. Thus, the credit market clears when

$$w_{t-1} = k_t - (1-\lambda)x_t + V_t$$

which is same as in the main text. Here, $w_{t-1} = (1-\beta)k_t^{\gamma}$.

By defining the firm value as $V_t/\beta k_t^{\gamma}$, we can obtain the equilibrium equations governing the firm values $\{Q_t\}_{t=1}^{\infty}$ in the different regimes which are similar to what we have derived in the main text, by replacing L by $(1-\beta)/\beta$ and defining $\psi \equiv B(1-\beta)/\beta$.

We can then ensure Assumption 1 made in the main text by taking $\beta \in (0,1)$ close to 1 and/or small B so that $1-\gamma > \psi \equiv (1-\beta)B/\beta$. Also, Assumption 2 and 3 are all satisfied for small Δq, given ψ. We have also made the conditions $\hat{L} > L'$ in Proposition 3. This is modified as $\psi/(1-\gamma) > (1+\lambda\gamma)(\lambda\gamma+\psi)$ which is satisfied as well for small B (and/or β close to 1) and small $\lambda \in (0,1)$. The condition $L > \hat{L}$ made in Proposition 3 then holds when $1-\gamma > B$, by replacing L by $(1-\beta)/\beta$ because $(1-\beta)/\beta > \hat{L} \equiv \psi/(1-\gamma) = B(1-\beta)/\beta(1-\gamma)$ which is rewritten by $1-\gamma > B$.

8.4 Contingent Repayment

In the main text we have assumed that the repayment to a financier is not contingent on the worker's performance signals. This is a realistic assumption because it is rarely observed in the real world that the repayment to a bank varies with the task performances of workers who are employed by the client firm borrowing from the bank. It is hence reasonable to assume that financier's repayment is independent of the worker's signals. However, in a theoretical interest it is possible to allow the financier's repayment R_t to be contingent on worker's signals $s = a \in \{0,1\}$ and $\sigma \in \{\sigma_l, \sigma_h\}$. In this subsection we will consider such extension.

Let denote by $R_t(s, \sigma, x_t'')$ the repayment to a financier in period t, which is contingent on the worker's signals $s = a$ and σ, when the financier provides a relation-specific capital x_t''. Suppose that the financial contract specifies the repayment $R_t(s, \sigma, x_t'') \geq 0$ to the financier and the relation-specific capital x_t to be provided by the financier. [28]

Define

$$\tilde{R}_t(s,\sigma) \equiv R_t(s,\sigma,x_t) - \min_{\sigma,z} R_t(s,\sigma,z) \geq 0$$

and set a new repayment scheme:

$$R_t''(a) \equiv \begin{cases} E_\sigma[\tilde{R}_t(a,\sigma)|a] & \text{if } z = x_t \\ 0 & \text{otherwise} \end{cases}$$

[28] The repayment $R_t(s, \sigma, x_t'')$ must be non-negative. Otherwise, for some s, σ and x_t, the financier is required to make a positive payment to the firm. However, then the financier reneges on that payment because, even in doing so, there are no ways to punish the financier: the financier can always refuse to pay and secure at least the payoff $r_t(w_{t-1} - \lambda x_t)$ while, if she makes a payment $R_t < 0$, she earns $R_t + r_t(w_{t-1} - \lambda x_t)$ which is less than the former payoff.

Here $E_\sigma[\cdot|a]$ denotes the expectation over σ conditional on the perfect but unverifiable signal $s = a$.

Then, since the original scheme $R_t(\cdot, \cdot, x_t)$ implements the provision of relation-specific capital x_t, the new repayment scheme does so:

$$\begin{aligned}\lambda r_t(x_t - z) &\leq E_\sigma[R_t(a, \sigma, x_t) - R_t(a, \sigma, z)|a] \\ &\leq E_\sigma[R_t(a, \sigma, x_t) - \min_{\sigma,z} R_t(a, \sigma, z)|a] \\ &= E_\sigma[\tilde{R}_t(a, \sigma)|a]\end{aligned}$$

for all $z \neq x_t$. Thus, $\lambda r_t(x_t - z) \leq E_\sigma[\tilde{R}_t(a, \sigma)|a_t] = R''_t(a_t)$ for all $z \neq x_t$. This is the desired result. In particular, we have $R''_t(a_t) \geq \lambda r_t x_t$ by setting $z = 0$, which implies that the new repayment scheme satisfies IR of the financier. Also, the new repayment scheme achieves a lower expected repayment than the original one because $R''_t(a_t) = E_\sigma[\tilde{R}_t(a_t, \sigma)|a_t] \leq E_\sigma[R_t(a_t, \sigma, x_t)|a_t]$ (note that $\min_{\sigma,z} R_t(s, \sigma, z) \geq 0$.) Then, generality is not lost by focusing on the repayment scheme $R_t(a)$ which is contingent only on the perfect unverifiable signal $s = a \in \{0, 1\}$ of the worker but not on the verifiable signal $\sigma \in \Sigma$.

Then, when the old worker chooses $\hat{a}_t \in \{0, 1\}$ in equilibrium, the financier can fully anticipate that she will surely face the repayment $R_t(\hat{a}_t)$ on the equilibrium path. Thus, the IR of the financier is given by $R_t(\hat{a}_t) \geq \lambda r_t x_t$ which is same as one in the main text. Also, DE constraint is not changed because $V_{t+1} \geq R_t(\hat{a}_t) + b_t$ must hold following the worker's effort choice $\hat{a}_t$ on the equilibrium path. Thus, the equilibrium contract is not changed at all even when we allow the repayment of financier R_t to be contingent on the signals of the employed old worker.

8.5 Dropping Assumption 1

When Assumption 1 is dropped, the firms always use insider lending but never resort to market lending at all. Thus Assumption 1 is necessary for market lending to emerge in an equilibrium path. This is shown as follows: the flow profit defined in the proof of Lemma 1, $\Pi(k_t, x_t)$, is always increasing in x_t, given k_t, when Assumption 1 is not satisfied. Thus the firm always chooses $x_t = k_t$ so that it never uses market lending at all.

8.6 Sub-Optimality of Implementing Low Effort $a_t = 0$

In the main text we have assumed that the firms always implement high effort $a_t = 1$ from employed old workers. In this subsection we will show that it becomes never optimal for the firm to induce old workers to choose low effort when the worker's productivity in that case $\underline{s} \equiv h(0)$ is sufficiently small.

Specifically we make the following assumption:

Assumption B1. $(1 - \gamma)(\underline{s}/\lambda)^{\gamma/(\gamma-1)} > (1 + q_0/\Delta q)\psi + \underline{s}$.

Assumption B1 is satisfied when $\underline{s}$ is small enough. Then we can show the following.

Proposition B1. *Suppose that Assumption B1 holds. Then, in any equilibrium path* $a_t = 1$ *must hold for all* $t \geq 1$. [29]

Proof. Suppose that there exists an equilibrium path in which the firms implement low effort $a_t = 0$ from matched old workers in some period t. Then, the firm's flow profit in period t becomes

$$\underline{\Pi}(k_t, x_t) \equiv \underline{s}k_t^{\gamma} - r_t k_t + (1-\lambda) r_t x_t$$

in the equilibrium in which the firms implement low effort $a_t = 0$ because the firms pay zero wage for implementing $a_t = 0$. Here $V_{t+1} \geq \lambda r_t x_t$ so that $x_t \leq \hat{x}_t \equiv V_{t+1}/\lambda r_t$. Since $\underline{\Pi}$ is increasing in x_t, we have $x_t = k_t$ when $k_t \leq \hat{x}_t$ and $x_t = \hat{x}_t$ when $k_t > \hat{x}_t$ respectively. Thus the firm's flow profit is written by

$$\underline{\pi}(k_t) \equiv \begin{cases} \underline{s}k_t^{\gamma} - \lambda r_t k_t & \text{if } k_t \leq \hat{x}_t \\ \underline{s}k_t^{\gamma} - r_t k_t + (1-\lambda)(V_{t+1}/\lambda) & \text{if } k_t > \hat{x}_t \end{cases}$$

The firms choose k_t to maximize $\underline{\pi}(k_t)$ in the equilibrium. There are three cases which might happen in the equilibrium.

Case 1: The firms choose the equilibrium capital $k_t = \hat{x}_t$. That is, $r_t > \gamma \underline{s} \hat{x}_t^{\gamma-1} > \lambda r_t$. Then the equilibrium profit becomes $\underline{\pi}_t = \underline{s}\hat{x}_t^{\gamma} - V_{t+1}$.

Suppose that some firm deviates from the equilibrium by implementing high effort $a_t = 1$ from matched old workers and borrowing all capital z_t from market lending. Such deviation yields at least the flow profit as follows:

$$\begin{aligned} \pi_t'' &\equiv \max_{z_t} z_t^{\gamma} - r_t z_t - \psi_t - \max\{(q_0/\Delta q)(\psi_t - V_{t+1}), 0\} \\ &= (1-\gamma) z_t^{\gamma} - \psi_t - \max\{(q_0/\Delta q)(\psi_t - V_{t+1}), 0\} \end{aligned}$$

where $\psi_t = \psi y_t = \psi \hat{x}_t^{\gamma}$ in the supposed equilibrium and z_t satisfies $\gamma z_t^{\gamma-1} = r_t$. Since $\underline{s}\gamma \hat{x}_t^{\gamma-1} > \lambda r_t = \lambda \gamma z_t^{\gamma-1}$, we obtain $z_t > (\underline{s}/\lambda)^{1/(\gamma-1)} \hat{x}_t$. Thus the deviation profit becomes

$$\begin{aligned} \pi_t'' &\geq (1-\gamma)(\underline{s}/\lambda)^{\gamma/(\gamma-1)} \hat{x}_t^{\gamma} - \psi_t - \max\{(q_0/\Delta q)(\psi_t - V_{t+1}), 0\} \\ &\geq (1-\gamma)(\underline{s}/\lambda)^{\gamma/(\gamma-1)} \hat{x}_t^{\gamma} - (1 + (q_0/\Delta q))\psi_t \\ &= (1-\gamma)(\underline{s}/\lambda)^{\gamma/(\gamma-1)} \hat{x}_t^{\gamma} - (1 + (q_0/\Delta q))\psi \hat{x}_t^{\gamma} \end{aligned}$$

which can be larger than $\underline{\pi}_t$ under Assumption B1.

[29] In the initial period $t = 0$ the firms may implement low effort from employed old workers ($a_0 = 0$). However, this does not change the equilibrium path of the firms values $\{Q_t\}_{t=1}^{\infty}$: the credit market equilibrium in $t = 0$ is written by $Ly_0 = k_1 - (1-\lambda)x_1 + V_1$ so that $L = (k_1 - (1-\lambda)x_1)/y_0 + Q_1$. Also the initial firm value is given by $Q_1 = (y_1/r_1 y_0)(\pi_1/y_1 + Q_2)$ where the term $(y_1/r_1 y_0)$ depends only on Q_1 and ψ_1 by using the above credit market equilibrium. Thus the initial output y_0 and hence initial old workers' effort $a_0 = 0$ do not affect the equilibrium paths of the firm values.

Case 2: The firms choose the equilibrium capital $k_t < \hat{x}_t$. Then, $\underline{s}\gamma k_t^{\gamma-1} = \lambda r_t$. The equilibrium profit becomes $\underline{\pi}_t = (1-\gamma)\underline{s}k_t^\gamma$. Also $\psi_t = \psi y_t = \psi k_t^\gamma$ holds.

Consider the same deviation by some firm as in Case 1. The deviating firm chooses capital z_t satisfying $\gamma z_t^{\gamma-1} = r_t = (\underline{s}/\lambda)\gamma k_t^{\gamma-1}$. Thus $z_t = (\underline{s}/\lambda)^{1/(\gamma-1)}k_t$. Then the deviation profit is at least

$$\pi_t'' \geq (1-\gamma)(\underline{s}/\lambda)^{\gamma/(\gamma-1)}k_t^\gamma - (1+(q_0/\Delta q))\psi k_t^\gamma$$

which is larger than the equilibrium profit $\underline{\pi}_t = (1-\gamma)\underline{s}k_t^\gamma$ under Assumption B1.

Case 3: The firms choose the equilibrium capital $k_t > \hat{x}_t$. Then the firms choose $\underline{s}\gamma k_t^{\gamma-1} = r_t$ and earns the flow profit $\underline{\pi}_t = (1-\gamma)\underline{s}k_t^\gamma + (1-\lambda)(V_{t+1}/\lambda)$. Since $k_t > \hat{x}_t$, we then have

$$\begin{aligned}\underline{\pi}_t &= (1-\gamma)\underline{s}k_t^\gamma + (1-\lambda)r_t\hat{x}_t \\ &\leq (1-\gamma)\underline{s}k_t^\gamma + (1-\lambda)r_t k_t \\ &= (1-\gamma)\underline{s}k_t^\gamma + (1-\lambda)\underline{s}\gamma k_t^\gamma \\ &= (1-\gamma\lambda)\underline{s}k_t^\gamma\end{aligned}$$

The same deviation as in Case 1 gives the capital choice z_t satisfying $\gamma z_t^{\gamma-1} = r_t = \gamma\underline{s}k_t^{\gamma-1}$. Thus the deviation yields at least the following flow profit

$$\pi_t'' \geq (1-\gamma)\underline{s}^{\gamma/(\gamma-1)}k_t^\gamma - (1+(q_0/\Delta q))\psi_t$$

where $\psi_t = \psi k_t^\gamma$. Thus the deviation profit can be larger than the equilibrium one if

$$(1-\gamma)\underline{s}^{\gamma/(\gamma-1)} - (1+(q_0/\Delta q))\psi > (1-\gamma\lambda)\underline{s}$$

which is satisfied under Assumption B1. Q.E.D.

Next we show the following result.

Proposition B2. *Suppose that Assumption 1-3 and $\hat{L} > L'$ hold. Then, for sufficiently small $\underline{s}$, there exists some $\overline{L} > \hat{L}$ such that for all $L \in (\hat{L}, \overline{L})$ there exists the equilibrium path which has the same features as in Proposition 3 and $a_t = 1$ for all $t \geq 1$.*

Proof. Take the equilibrium path considered in Proposition 3. Then we show that the firms have no incentive to implement low effort in any period in such equilibrium path, when $\underline{s}$ is sufficiently small. If some firm deviates to implement low effort $a_t = 0$ from matched old workers and invests in capital k_t'', it would obtain the profit as $\underline{\pi}(k_t'')$ defined in the proof of Proposition B1. Then, we obtain

$$\underline{\pi}(k_t'') \leq F(r_t) \equiv \max_k \underline{s}k^\gamma - \lambda r_t k.$$

Case 1: In the implicit contract regime with non-binding DE the equilibrium firm's flow profit becomes $\pi_t = (1-\gamma-\psi)k_t^\gamma$ where the equilibrium capital k_t satisfies $\gamma k_t^{\gamma-1} = \lambda r_t$.

Given this equilibrium interest rate, the upper bound for the deviation profit $F(r_t)$ can be written by

$$F(r_t) = \tilde{F}(k_t) \equiv (1-\gamma)\underline{s}^{1/(1-\gamma)}\lambda^{\gamma/(\gamma-1)}k_t^\gamma$$

where k attaining $F(r_t)$, denoted by $\underline{k}_t$, satisfies $\underline{s}\gamma\underline{k}_t^{\gamma-1} = \lambda r_t$ so that $\underline{k}_t = (\underline{s}/\lambda)^{1/(1-\gamma)}k_t$. However, then $\tilde{F}(k_t) \leq \pi_t = (1-\gamma-\psi)k_t^\gamma$ holds if

$$1-\gamma-\psi \geq (1-\gamma)\underline{s}^{1/(1-\gamma)}. \tag{B6}$$

When DE is binding in the implicit contract regime, $k_t = \hat{x}_t \equiv V_{t+1}/\lambda r_t$ where $r_t > \gamma k_t^{\gamma-1} > \lambda r_t$ is satisfied. In this case the capital $\underline{k}_t$ attaining $F(r_t)$ satisfies $\underline{s}\gamma\underline{k}_t^{\gamma-1} = \lambda r_t > \lambda\gamma k_t^{\gamma-1}$ so that $\underline{k}_t < (\underline{s}/\lambda)^{1/(1-\gamma)}k_t$. Then we obtain

$$\begin{aligned}
F(r_t) &= \tilde{F}(k_t) \\
&= \underline{s}(1-\gamma)\underline{k}_t^\gamma \\
&\leq (1-\gamma)\underline{s}(\underline{s}/\lambda)^{\gamma/(1-\gamma)}k_t^\gamma \\
&\leq \pi_t \equiv k_t^\gamma - V_{t+1} \\
&= k_t^\gamma(1-Q_{t+1}) \\
&\leq k_t^\gamma(1-\gamma-\psi)
\end{aligned}$$

if

$$1-\gamma-\psi \geq (1-\gamma)\lambda^{\gamma/(\gamma-1)}\underline{s}^{1/(1-\gamma)}. \tag{B7}$$

Case 2: In the mixed contract regime the equilibrium flow profit is given by $\pi_t \equiv (1-\gamma)k_t^\gamma - \psi_t + (1-\lambda)V_{t+1}/\lambda$ where $\gamma k_t^{\gamma-1} = r_t$. The upper bound for the deviation profit $F(r_t)$ is then written by

$$\begin{aligned}
F(r_t) &= \tilde{F}(k_t) \\
&\equiv \underline{s}(1-\gamma)\underline{k}_t^\gamma \\
&= (1-\gamma)\lambda^{\gamma/(\gamma-1)}\underline{s}^{1/(1-\gamma)}k_t^\gamma
\end{aligned}$$

because $\underline{k}_t$ satisfies $\underline{s}\gamma\underline{k}_t^{\gamma-1} = \lambda r_t = \lambda\gamma k_t^{\gamma-1}$. The right hand side of the above expression is smaller than the equilibrium profit π_t if (B7) holds.

Case 3: In the explicit contract regime the equilibrium profit is given by

$$\pi_t = [1-\gamma-(1+(q_0/\Delta q)))\psi + (q_0/\Delta q)Q_{t+1}]k_t^\gamma$$

and the equilibrium capital (interest rate) satisfies $\gamma k_t^{\gamma-1} = r_t$. Then the upper bound for the deviation profit $\tilde{F}(k_t)$ is lower than the equilibrium profit if

$$1-\gamma-(1+(q_0/\Delta q))\psi + (q_0/\Delta q)Q_{t+1} \geq (1-\gamma)\lambda^{\gamma/\gamma-1}\underline{s}^{1/(1-\gamma)} \tag{B8}$$

which is rearranged as

$$Q_{t+1} \geq \underline{Q}^* \equiv (\Delta q/q_0)\left\{(1+(q_0/\Delta q))\psi - (1-\gamma) + (1-\gamma)\lambda^{\gamma/(\gamma-1)}\underline{s}^{1/(1-\gamma)}\right\} \tag{B9}$$

Since the firm value Q_t decreases over time and converges to the steady state Q^e in the equilibrium path considered in Proposition 3, (B9) is satisfied if $Q^e \geq \underline{Q}^*$ holds. This condition is equivalent to

$$\frac{\gamma \underline{Q}^*}{L - \underline{Q}^*} > (1-\gamma)\lambda^{\gamma/(\gamma-1)}\underline{s}^{1/(1-\gamma)} + \underline{Q}^*.$$

When $\underline{s} \to 0$, $\underline{Q}^* \to L - \gamma$. Thus, it suffices to obtain $Q^e > L - \gamma$ which is satisfied as we have seen in the proof of Proposition 3 (recall that this was ensured by $L < \overline{L}$ so that $Q^e \geq \underline{Q}$ in the proof of Proposition 3).

Summarizing all the above results, in the equilibrium path derived in Proposition 3 the firms have no incentive to deviate for implementing low effort $a_t = 0$ from matched old workers in any period. Q.E.D.