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Abstract

This paper examines the effects of the negative interest rate policy (NIRP) introduced by the Bank of Japan in January 2016. It has effectively stimulated private residential investment, and in lowering long-term interest rates, it has likely supported private nonresidential investment. There is also reason to believe that it likely stopped the appreciation of the yen and arrested the downward trend in Japanese stock prices around August 2016. Overall, we find that the NIRP has had expansionary effects, and therefore serves as a legitimate policy tool in alleviating Japan's zero-interest rate lower bound, notwithstanding some potential negative side effects.

Keywords: Negative Interest Rates, Residential/Nonresidential Investment, Foreign Exchange Rates

JEL Classification Number: E52

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1. Introduction

Over the period starting in June 2015 until June 2016, the nominal effective exchange rate of the Japanese yen appreciated by 19.4%, largely because of exogenous negative shocks from abroad, and many other Japanese macroeconomic indicators deteriorated, including the rate of inflation. In response to this weakening of the economy, the Bank of Japan (BOJ) decided to adopt a negative interest rate policy (NIRP) in January 2016, following the example set by the European Central Bank (ECB) and three other European central banks.[1] Since then, the BOJ has started charging private financial institutions a fee of 0.1% on the portion of their reserves kept with it.

As it has only been about a year since the BOJ introduced the policy, it is still too early to draw any firm conclusions on the effectiveness of NIRP given the small sample size of the necessary data. However, this does not lessen in any way the urgent need on the policy side to evaluate the effectiveness of this important policy tool. Problematically, the NIRP is an unprecedented macroeconomic policy with little supporting economic theory or empirical evidence. The purpose of this paper is to preliminarily report on the effects of the NIRP, as recently introduced by the BOJ, on the Japanese economy, and to discuss any policy implications.

Our tentative conclusion is that the NIRP overall has empirically observable expansionary effects. It therefore serves as a legitimate policy tool in alleviating zero-interest rate lower bounds, notwithstanding potential negative side effects. The remainder of the paper is organized as follows. Section 2 presents a simple analytic model to consider the effects of the NIRP, comprising a four-asset model based on Tobin (1969), Yabushita (2009), and Honda (2014). Section 3 discusses some limitations of this model, and Section 4 provides some empirical evidence on the effects of the NIRP on the Japanese economy. In Section 5, we provide an interpretation of the statistical facts and discuss some policy implications. The Appendix includes details of some of the mathematical results presented in Section 2.

2. A Simple Analytic Model

In this section, we first explain a simple analytic model in which banks act exogenously and serve as "mechanical tunnels" to transmit monetary policy shocks from the monetary authorities to financial market variables. The assumption of a mechanical tunnel is purely for analytic convenience, and is not the case in reality. We relax this assumption and discuss its implications in Section 3.

2.1 Four-Asset Model

We assume the economy comprises four sectors: a private sector, a foreign sector, a government, and a central bank. The private sector includes both financial and nonfinancial institutions. For analytical purposes, we regard the income account variables as exogenous in determining portfolio choice behavior, and find the market equilibrium for the stock of assets conditional upon the assumed values of output, income, and the remaining flow variables. We also assume prices remain constant and equal the numeraire of one throughout the period. This strategy is identical to that employed when constructing the well-known liquidity preference/money supply equilibrium (LM) curve in macroeconomics. The key behavioral assumption here is that spending and portfolio decisions are independent.

Extending the models in Tobin (1969), Yabushita (2009) and Honda (2014), we consider a model with four assets, namely, money, bonds, stocks and foreign assets.

2.2 Demand for Assets

The respective demands for money (M), bonds (B), stocks (V), and foreign assets (F) depend on their relative asset yields, gross domestic product (GDP) (Y), and the given wealth (W^S):

$$
\begin{aligned}
&\text{Money:} && M = M(c, i, r, z, Y, W^S) \\
&\text{Bonds:} && B = B(c, i, r, z, Y, W^S) \\
&\text{Stocks:} && V = V(c, i, r, z, Y, W^S) \\
&\text{Foreign assets:} && F = F(c, i, r, z, Y, W^S). \qquad (1)
\end{aligned}
$$

Money in this model is central bank money, or currency plus private bank demand deposits held at the central bank. In this model, we assume that the central bank pays interest (c) on reserves (or money).[2] Variable c is exogenous, and its domain extends from minus to plus infinity. When negative, c is the nominal carrying cost of reserves. Economic agents then hold money up to some nonpositive point for transaction purposes, even if its return is negative.

We also assume that bonds, stocks, and foreign assets yield returns of an interest rate (i), stock returns (r), and foreign asset returns (z), respectively. The domains of these variables also extend from minus to plus infinity. The expected rate of return from holding foreign assets z is then the sum of two components, the interest rate on foreign bonds and the expected rate of change in the exchange rate $E[\Delta e/e]$, where $E[*]$ denotes the expectation operator. We assume z is exogenous throughout.[3]

Bank lending is assumed to be a variant of corporate bond lending, and is therefore included in B. For simplicity, we ignore or mechanically treat bank-lending behavior in this model.[4] We also assume that the assets are gross substitutes in standard microeconomics terminology. That is, the demand for each asset varies directly with its own rate of return and inversely with all other rates of return. The own-derivatives of the respective demand functions

$$(\partial M/\partial c, \partial B/\partial i, \partial V/\partial r, \partial F/\partial z),$$

are then positive, and the cross-derivatives nonpositive.

The total demand for the four assets sums to the total demand for wealth in economy W:

$$W = M(c, i, r, z, Y, W^S) + B(c, i, r, z, Y, W^S) + V(c, i, r, z, Y, W^S) + F(c, i, r, z, Y, W^S). \quad (2)$$

Such that the total demand for assets W is a function of all variables on the right-hand side of equation (2).

When any one of the returns, c, i, r, or z, changes, the demand for each asset reacts, but the sum of the changes in demand for each asset are assumed zero. That is:

$$M_j + B_j + V_j + F_j = 0 \text{ for } j = 1, 2, 3, 4, \quad (3)$$

where subscript j denotes the partial derivatives of the demand functions, M, B, V, F, with respect to j-th argument on the right-hand side of equation (1).

Just as there is a budget constraint in standard microeconomics, we have a balance sheet constraint. That is, we assume that the total demand for assets W in (2) is equal to the exogenous total supply of assets W^S:

$$W = W^S.$$

As a result, when exogenous total wealth W^S increases, we have the following balance sheet restriction:

$$M_6 + B_6 + V_6 + F_6 = 1, \tag{4}$$

where the subscript denotes the partial derivative with respect to total wealth W^S, and the 6-th argument in the respective functions for $M, B, V, and\ F$ in equation (1). We also assume that all four assets are normal goods, such that:

$$M_6 > 0, B_6 > 0, V_6 > 0, and\ F_6 > 0\ .$$

2.3 Supply of Assets

We assume that the central bank exogenously supplies money stock, M^S. Firms provide the supply of stocks, qK^S, where q and K^S denote the market value price of one unit of physical capital and the stock of physical capital, respectively, or alternatively, the stock price and the total number of stocks outstanding, respectively.

The government and firms supply bonds, $P^B B^S$, where P^B and B^S denote the market value of one unit of bonds and the total quantity of bonds outstanding, respectively. We assume that the respective total quantities of stocks and bonds outstanding in the economy, K^S and B^S, are exogenously given. However, their market prices, q and P^B, are endogenously determined through arbitrage, as explained below.

The total supply of foreign assets is given by eF^S, where e and F^S are the exchange rate (measured in yen per unit of foreign currency) and the total balance of foreign assets (measured in foreign currency), respectively. We assume that the quantity of foreign assets outstanding, F^S, is exogenously given, but that the exchange rate e is endogenously determined.

2.4 Inverse Relations between Market Prices and Returns

We assume an inverse relation between the bond price, P^B, and the interest rate, i:

$$dP^B/di<0. \tag{5}$$

Similarly, we also assume that an inverse relation also holds for capital stocks:

$$dq/dr<0, \tag{6}$$

However, unlike conventional models, we assume that the domains for variables i and r are from minus to plus infinity.

The rationale behind inequalities (5) and (6) is as follows. Suppose an economic agent holds one unit of bonds. The agent has two possible choices. One is that he/she sells the bond immediately in the market. In this case, he/she obtains the current market price of the bond, P^B. The other is that he/she holds the bond and expects to earn the stream of fixed income produced by this bond in the future. The current value of the future stream of fixed income is discounted by the bond market interest rate i. We assume arbitrage works between these two choices, such there must be an inverse relation between the bond price P^B and the market interest rate i, as in inequality (5). One simple example is a consol bond with a return of one yen per year, such that the market value P^B =1/i. There is indeed an inverse relation between the price of bonds P^B and the interest rate i in this case.

In a similar manner, suppose an economic agent holds one unit of physical capital that produces a real return R (assumed exogenous) each year. Once again, the agent has two possible choices. One is that he/she sells the physical capital in the market. In this case, he/she receives q, the current market price of equity. The other choice is that he/she holds the capital permanently and expects to earn a stream of fixed real returns R in the future. We then discount the current value of the future stream of fixed real returns by the rate of return on capital stocks r, where r is the rate of return on stocks required for market investors to hold capital stocks in their portfolios. Assuming arbitrage between the two choices, we have an equation $q=R/r$. Hence, we have an inequality (6).

We assume the reproduction cost of one unit of physical capital is one, and remains constant throughout this analysis. Hence, stock prices q in our model also represent Tobin's q, which is the ratio of the market value of capital to its reproduction cost.

2.5 Market Equilibrium

The following four equations give the market equilibrium conditions:

$$M^S = M(c, i, r, z, Y, W^S), \tag{7}$$

$$P^B B^S = B(c, i, r, z, Y, W^S), \tag{8}$$

$$qK^S = V(c, i, r, z, Y, W^S), \tag{9}$$

$$eF^S = F(c, i, r, z, Y, W^S), \tag{10}$$

with all but one of these conditions automatically satisfied when the other three are met because of the balance sheet constraint:

$$M^S + P^B B^S + qK^S + eF^S = W^S = W, \tag{11}$$

where W^S denotes the total supply of wealth. Therefore, we only have to consider any three of the above four equations; we choose equations (7), (8), and (10). The three endogenous variables are the interest rate i, returns on capital stocks r, and the foreign exchange rate e. Variables P^B, q, and W^S are also endogenous because of conditions (5), (6), and (11), respectively. The remaining variables in the system, (7) through (10), are exogenous.

Substituting equation (11) into equations (7), (8), and (10), we have:

$$M^S = M(c, i^*, r^*, z, Y, M^S + P^B(i^*)B^S + q(r^*)K^S + e^*F^S), \tag{12}$$

$$P^B(i^*)B^S = B(c, i^*, r^*, z, Y, M^S + P^B(i^*)B^S + q(r^*)K^S + e^*F^S), \tag{13}$$

$$e^*F^S = F(c, i^*, r^*, z, Y, M^S + P^B(i^*)B^S + q(r^*)K^S + e^*F^S), \tag{14}$$

where superscript * indicates the equilibrium value.

2.6 The Effects of an Increase in Interest on Reserves c

We interpret a reserve carrying cost increase as a decrease in the interest on reserves c across its negative domain. Hence, we are interested in the effects of an exogenous decrease in interest on reserves c on the equilibrium endogenous variables (i^*, r^*, e^*).

Assuming a smooth differentiable function of our system of equations, we investigated the comparative statics of an exogenous increase in interest on reserves c. The appendix shows that an increase in the interest on reserves raises the interest rate, $di^*/dc>0$, increases the required rate of returns from stocks, $dr^*/dc>0$, and appreciates the value of the domestic currency, $de^*/dc<0$.

Lowering the negative interest rate on reserves further into a more negative range by the central bank then leads to lower interest rates on bonds. It also reduces the required rate of returns from stocks and depreciates the value of the domestic currency. However, these conclusions are subject to important qualifications, which are discussed in the following section.

3. Limitations of the Overly Simplified Model

3.1 Exogenous Financial Sector

In the above overly simplified model, private banks play no role in the transmission mechanism of monetary policy shocks. Instead, they are exogenous and mechanical actors. In reality, this is not the case as a decrease in the interest on reserves removes net profits from banks, which will react to this exogenous negative shock.

First, with reduced profits, banks may become more cautious in taking risks and thus less willing to make loans to their customers. The magnitude of the loss of their profits, due to the central bank's policy change, could be large. Indeed, there is the possibility that the incentive of private banks to avoid taking risks in making loans could overwhelm the expansionary monetary policy intent of the central bank. In that case, lowering the already negative interest rate on reserves further would not increase, but rather decrease private bank lending, contrary to the central bank's intention.

Second, if private banks attempt to avoid any reduction in their net profits, they will pass their losses on to depositors, and charge larger fees on deposits. Depositors would then cease depositing their cash into banks and instead hoard any surplus cash. This sort of disintermediation is likely to cause significant troubles in real terms in the economy, and would avoided at all costs by the monetary authorities.[5]

3.2 Expectations

Variables in financial markets are forward-looking, and expectations play an important role in the real world. However, for simplicity, our model ignores the role of expectations. When expectations change, they could shift the demand for assets in the equations in (1). Strictly speaking, when we then wish to analyze the effects of a negative interest policy, we should specify the demands for money M, bonds B, stocks V, and foreign assets F as:

$$M = M(c, i, r, z, Y, W^S, \varphi^M(c)),$$

$$B = B(c, i, r, z, Y, W^S, \varphi^B(c)),$$

$$V = V(c, i, r, z, Y, W^S, \varphi^V(c)),$$

$$F = F(c, i, r, z, Y, W^S, \varphi^F(c)), \tag{15}$$

instead of the corresponding equations in (1), where $\varphi^M(c)$, $\varphi^B(c)$, $\varphi^V(c)$, and $\varphi^F(c)$ denote the impacts of an increase of interest on reserves on the demands for money, bonds, stocks, and foreign assets, respectively, through changes in expectations among market participants. In such a complicated model with expectations, our standard results, $di^*/dc>0$, $dr^*/dc>0$, and $de^*/dc<0$, may no longer hold.

Our simple model with no expectations certainly has some limitations for real-world analysis. It is certainly desirable to extend our model and to incorporate expectations formally. However, this is beyond the scope of the current paper, and remains an open question.

4. Some Tentative Empirical Evidence

Given the small passage of time since the introduction of the NIRP by the BOJ in January 2016, little data have been accumulated. However, in this section, we attempt to provide some tentative empirical evidence on the effects of the NIRP using the available data.

4.1. Immediate Impact on Asset Markets

Figure 1 depicts the immediate impact of the NIRP on the stock market. There are three possible types of reaction, being favorable to the market, unfavorable, or mixed.

<FIGURE 1 ABOUT HERE>

Panels (a) and (b) plot the stock market price indexes for the banking and insurance industries, respectively, relative to the Nikkei Average Stock Price. We standardize all indexes to one as of January 28, 2016 to correspond with the time the BOJ announced the NIRP. In anticipation of a more severe market environment, the stock indexes for both the banking and insurance industries dropped sharply relative to the Nikkei Average.

By contrast, the real estate industry received news of the NIRP more favorably. Panels (c) and (d) plot the stock market price index of the real estate industry and Japan-Real Estate Investment Trust (J-REIT) index, respectively. As shown, the plots of the stock market prices of both real estate-related industries lie above the Nikkei Average.

Panels (e) and (f) depict the reactions of the indexes of securities and commodity futures, and other financial services (such as leasing), respectively. In these industries, it appears the news was favorable at first, but then a more negative change was seen in response.

4.2. Effects on Financial Variables

4.2.1. Market Interest Rates

The BOJ announced the NIRP in January 2016, and it came into effect in February that same year. Figure 2 illustrates that both short- (1-month LIBOR; solid line) and long-term (10-year government bonds; dashed line) interest rates fell sharply (some 50 basis points for 10-year government bonds) and became negative as soon as the NIRP was announced. Subsequently, the long-term interest rate quickly increased to about zero percent and remained there until September 2016 when the BOJ began to use the long-term interest rate as an operating target and set it to zero percent.

<FIGURE 2 ABOUT HERE>

<FIGURE 3 ABOUT HERE>

4.2.2. Foreign Exchange Rates

Figure 3 depicts the movements of the nominal effective yen exchange rate using a 26-country index (2010 = 100). Largely because of exogenous shocks outside Japan, the yen's effective exchange rate appreciated by 19.4% from June 2015 to June 2016. This appreciation weakened the competiveness of Japanese firms, and the macro indicators for core machinery, retail sales, and production deteriorated over this period. Concerned about the possibility that the slowdown in spending might lower prices, the BOJ decided to increase the purchase of exchange-traded funds (ETFs) at its July 2016 meeting. Although we will need to wait until more data is available to confirm whether this was because of the expansionary monetary policy conducted in July 2016 and/or the implementation of the NIRP since January 2016, the appreciating trend in the yen, evident since June 2015, finally halted around August 2016, as shown in Figure 3.

After August 2016, the yen's effective exchange rate began to depreciate. There are two possible reasons for this. First, with Donald Trump being elected as the new U.S. president in November 2016, his expected expansionary fiscal policy stance seems to have contributed to a surge in the value of the U.S. dollar. Second, in December

2016, the U.S. Federal Reserve raised the operating target of the federal funds rate to a range of 0.5% to 0.75%, which also contributed to the U.S. dollar appreciation evident as at February 2017. Looking at the movements of the nominal effective exchange rate of the euro, we can discern a similar depreciation in the euro relative to the U.S. dollar over the same period, and this may reconfirm our speculation concerning the reasons for the depreciating yen.

4.2.3. Stock Prices

Figure 4 provides a graph of the Tokyo Stock Price Index (TOPIX), showing that it moved closely with the nominal effective foreign exchange rate after 2015. The TOPIX stopped falling in mid-2016, and rose sharply after November 2016. Again, we await rigorous analysis to determine whether the NIRP truly contributed to the turnaround of stock prices in the middle of 2016, as for the effective foreign exchange rate.

<FIGURE 4 ABOUT HERE>

4.3. Effects on Real Variables

4.3.1. Production

In this subsection, we observe the movements of real variables and examine their correlation with the NIRP. Figure 5 plots the index of industrial production (IIP). As shown, the IIP displays a close correlation with movement in the financial variables, including the effective exchange rate and stock prices. For the most part, the IIP has declined throughout 2015, but turned around and began to climb toward the middle of

2016. Around November 2016, there was a further surge in production.

<FIGURE 5 ABOUT HERE>

4.3.2. Residential Investment

Table 1 is the official statement by the Japanese government on preliminary estimates (released February 13, 2017) on seasonally adjusted real GDP for the second (April–June) and third (July–September) quarters of 2016, respectively.

<TABLE 1 ABOUT HERE>

As shown in the first and third columns of Table 1, private residential investment grew by 3.3% and 2.4% over the quarter (percent change from previous quarter), respectively.[6] The values in parentheses for private residential investment shown in the second and fourth columns in Table 1 are both about 0.1, suggesting that this increase in private residential investment increased GDP growth by 0.1% per quarter (about 0.4% annually). These jumps in private residential investment clearly coincided with the NIRP.

4.3.3. Nonresidential Investment

Despite the sharp appreciation of the yen, nonresidential investment grew by +1.3% in the second quarter (the first column in Table 1). The growth rate is negative in the third quarter (-0.3% in the third column of Table1), but relatively small in magnitude, likely owing to the substantial reduction in the long-term interest rate under the NIRP. Indeed, the Ministry of Finance's "Financial Statements Statistics of Corporations by Industry (April–June, 2016)" reports that the growth rate of fixed investment over the previous quarter was a respectable +3.1%, seemingly despite a 3.5% fall in sales and a 10.0% decline in earnings in the second quarter of 2016.[7] We conjecture that the lower long-term interest rate associated with the NIRP supported private fixed investment.

4.3.4. Exports

The growth rate in exports of goods and services in the second quarter of 2016 (the first column in Table 1) was as weak as –1.2% because of the sharp appreciation in the yen. Around August 2016, the yen appreciation halted, and the growth of exports of goods and services improved to +2.1% in the third quarter (the third column in Table 1).

5. Some Interpretation and Policy Implications

This final section summarizes our main findings, provides an interpretation, and discusses their policy implications. First, although as small as –0.1% in magnitude, the introduction of the NIRP in January 2016 substantiality increased residential investment and thereby supported the growth of the overall Japanese economy. Second, the introduction of the NIRP lowered the long-term interest rate by roughly 50 basis points, which likely supported private nonresidential investment.

Third, there are at least two reasons to believe that NIRP likely stopped the appreciating trend in the yen around August 2016. The first is some statistical evidence by Honda and Inoue (2015). Using Granger causality tests and impulse response analysis, they showed that differences in U.S. and Japanese bond yields have significantly affected, with some lag, the yen–dollar exchange rate over the last 30 years. Given that the NIRP substantially lowered both long- and short-term interest rates in Japan, the NIRP must have widened the spread in yields, pushing the exchange rate toward a yen depreciation.

The second reason more directly relates to the introduction of the NIRP in January 2016. Figure 6 plots the flow of funds accounts for foreign securities in the insurance and pension funds sector over the period 2011 to 2016. Clearly, there is a large increase in the first quarter of 2016, immediately after the introduction on NIRP. The average of the 20 data points available for the period from the first quarter of 2011 to the fourth quarter of 2015 is 56.8 billion yen, while that for the last three data points is 4,838.3 billion yen. The difference of –4,781.5 billion yen is significantly different from zero at the 1%, level, thereby supporting the argument that the NIRP involved significant portfolio rebalancing effects.[8] Facing lower yields on domestic securities, the insurance and pension funds sector increased its purchase of foreign securities, and in purchasing foreign assets, they must have bought U.S. dollars and raised the price of the dollar against the yen.[9]

<FIGURE 6 ABOUT HERE>

Given this, we surmise that it was indeed the NIRP that increased the yield margin between U.S. and Japanese securities and arrested the rapidly appreciating yen in August 2016. As Hamada et al. (2010, pp. 30–40) correctly argue, changes in exchange rates have a great impact on the Japanese real economy. This holds in the present case as well. If there were no NIRP in January 2016, the growth of the Japanese economy would probably have been much weaker in the second quarter of 2016.

Fourth, NIRP was also likely to have contributed to slowing the downward trend in stock prices around August 2016. As demonstrated in our analytic model in Section 2, a NIRP has expansionary effects on stock prices, and these combined with real estate stimulate the real sector through various channels.

Overall, we found the NIRP has had significant expansionary effects on the Japanese economy.[10] It is therefore a legitimate policy tool for alleviating Japan's zero-interest rate lower bound, notwithstanding the potential negative side effects discussed earlier.

Finally, using the four-asset model, Honda (2014) showed that an increase in central bank money has expansionary policy effects. The present paper demonstrates that a decrease in the interest rate on reserves has the same qualitative effects as an increase in central bank money. As these comparative statics results are independent, it implies that each policy tool has independent policy effects.

Appendix: Effects of an exogenous increase in the interest on reserves *c*

This appendix provides the basis for the three inequalities, $\frac{di^*}{dc} > 0, \frac{dr^*}{dc} > 0, and\ \frac{de^*}{dc} < 0,$ asserted in Subsection 2.6. There are three endogenous variables, (i^*, r^*, e^*), and three equations in (12), (13), and (14). Differentiating equations (12), (13), and (14) with respect to *c*, we have:

$$-\begin{bmatrix} \mathrm{M}_1 \\ B_1 \\ \mathrm{F}_1 \end{bmatrix}$$

$$= \begin{bmatrix} \mathrm{M}_2 + \mathrm{M}_6\mathrm{B}^\mathrm{S}(\mathrm{dP}^\mathrm{B}/\mathrm{di}) & \mathrm{M}_3 + \mathrm{M}_6K^S(\mathrm{dq}/\mathrm{dr}) & \mathrm{M}_6\mathrm{F}^\mathrm{S} \\ \mathrm{B}_2 - (1-\mathrm{B}_6)\mathrm{B}^\mathrm{S}(\mathrm{dP}^\mathrm{B}/\mathrm{di}) & \mathrm{B}_3 + \mathrm{B}_6K^S(\mathrm{dq}/\mathrm{dr}) & \mathrm{B}_6\mathrm{F}^\mathrm{S} \\ \mathrm{F}_2 + \mathrm{F}_6\mathrm{B}^\mathrm{S}(\mathrm{dP}^\mathrm{B}/\mathrm{di}) & \mathrm{F}_3 + \mathrm{F}_6K^S(\mathrm{dq}/\mathrm{dr}) & (\mathrm{F}_6 - 1)\mathrm{F}^\mathrm{S} \end{bmatrix} \begin{bmatrix} (\mathrm{di}^*/\mathrm{dc}) \\ (\mathrm{dr}^*/\mathrm{dc}) \\ (\mathrm{de}^*/\mathrm{dc}) \end{bmatrix}, \text{(A1)}$$

where M_j, B_j, V_j, and F_j (*j* = 1, 2, 3, 4, 5, 6) denote the partial derivatives of demand for money *M*, bonds *B*, stocks *V*, and foreign assets *F*, with respect to the *j*-th argument in (1), respectively. Solving this system of equations, we obtain:

$$\frac{\mathrm{di}^*}{\mathrm{dc}} = -\mathrm{F}^\mathrm{S}[F_3(M_6B_1 - M_1B_6) + M_3\{B_1(1-F_6) + B_6F_1\} + B_3\{M_1(F_6-1) - M_6F_1\} + (M_6B_1 - M_1B_6)K^S(\mathrm{dq}/\mathrm{dr})]/\Delta, \qquad \text{(A2)}$$

$$\frac{\mathrm{dr}^*}{\mathrm{dc}} = -F^S[M_1B_2(1-F_6) + M_1B_6F_2 - M_2B_1(1-F_6) - M_6B_1F_2 - M_2B_6F_1 + M_6B_2F_1 - \{M_1(1-B_6-F_6) + M_6B_1 + M_6F_1\}B^S(\mathrm{dp}^\mathrm{B}/\mathrm{di})]/\Delta,$$

(A3)

$$\frac{\mathrm{de}^*}{\mathrm{dc}} = -[M_1(B_2F_3 - B_3F_2) + B_1(\mathrm{M}_3F_2 - \mathrm{M}_2F_3) + F_1(M_2B_3 - M_3B_2) + \{M_1(B_6F_3 - F_3 - B_3F_6) + B_1(M_3F_6 - M_6F_3) + F_1(M_6B_3 + M_3 - M_3B_6)\}\mathrm{B}^{\mathrm{S}}(\mathrm{dp}^{\mathrm{B}}/\mathrm{di}) + \{M_1(B_2F_6 - B_6F_2) + B_1(M_6F_2 - M_2F_6) + F_1(M_2B_6 - M_6B_2)\}K^S(\mathrm{dq}/\mathrm{dr}) + (M_6F_1 - F_6M_1)\mathrm{B}^{\mathrm{S}}K^S(\mathrm{dp}^{\mathrm{B}}/\mathrm{di})(\mathrm{dq}/\mathrm{dr})]/\Delta, \quad \text{(A4)}$$

where Δ denotes the determinant of the (3×3) matrix on the right-hand side of equation (A1), and is given by:

$$\Delta = \mathrm{F}^{\mathrm{S}}\Bigg[\mathrm{M}_2\{\mathrm{B}_3(\mathrm{F}_6 - 1) - \mathrm{B}_6\mathrm{F}_3\} + \mathrm{M}_3\{\mathrm{B}_6\mathrm{F}_2 + \mathrm{B}_2(1 - \mathrm{F}_6)\} + \mathrm{M}_6(\mathrm{B}_2\mathrm{F}_3 - \mathrm{B}_3\mathrm{F}_2) - M_6(B_3 + F_3)\mathrm{B}^{\mathrm{S}}\left(\frac{\mathrm{dp}^{\mathrm{B}}}{\mathrm{di}}\right) + M_3(F_6 + B_6 - 1)B^S\left(\frac{dp^B}{di}\right) + (\mathrm{M}_6B_2 - M_2B_6)K^S(\mathrm{dq}/\mathrm{dr}) - \mathrm{M}_6\mathrm{B}^{\mathrm{S}}K^S(\mathrm{dp}^{\mathrm{B}}/\mathrm{di})(\mathrm{dq}/\mathrm{dr})\Bigg]. \quad \text{(A5)}$$

First, making use of equations (3) and (4), and the assumption of gross substitutes between the demands for assets, we can show each of the seven terms on the right-hand side of equation (A5) is negative. Hence, Δ is negative. Second, we can show that each of the four terms in the numerator on the right-hand side of equation (A2) is also negative. Therefore, we obtain inequality $\frac{di^*}{dc} > 0$.

Third, we can also show that the sum of the first six terms in the square brackets on the right-hand side of equation (A3) is positive, the sum of the three terms in the curly brackets on the right-hand side of equation (A3) is also positive, and thus the numerator of the right-hand side of equation (A3) is negative. Therefore, we have inequality $\frac{dr^*}{dc} > 0$.

Finally, the sum of the first three terms in the square brackets on the right-hand side of equation (A4) is negative, the sum of the next three terms in the curly brackets on the right-hand side of equation (A4) is positive, the sum of the three terms in the curly brackets in front of $K^S(\mathrm{dq/dr})$ on the right-hand side of equation (A4) is also positive, and $(M_6F_1 - F_6M_1)$ is negative. Therefore, we have inequality $\frac{\mathrm{de}^*}{\mathrm{dc}} < 0$.

Footnotes

1. The BOJ also took additional expansionary monetary policy measures to increase the purchase of ETFs at its July 2016 meeting.
2. We take c to be a weighted average of the interest payments on reserves at the central bank and the return on the currency, which is zero.
3. This assumption is composed of two parts. The first is that the interest rate on foreign bonds is exogenous to investors. The second is that the expected rate of change in the exchange rate remains the same, or E[Δe/e] is constant, throughout the period. Alternatively, the time horizon in our model is the period over which the expected rate of change in the exchange rate is unchanged. Also, note that the condition, E[Δe/e] is constant, is equivalent to the assumption that the elasticity of the expected exchange rate at the end of the period with respect to the current exchange rate is one, or $(\frac{d\hat{e}}{de})(e/\hat{e}) = 1$, where $\hat{e}$ denotes the expected exchange rate at the end of the period.
4. This is one of the limitations of the present model. It remains an open question to incorporate private bank behavior into our model as an endogenous variable.
5. There are at least three ways of eliminating or alleviating these side effects. First, facing the introduction of fees on deposits, depositors may reduce the amount of deposits, but may instead increase their risky investments in their portfolio choices, and stimulate the real economy. Second, under lower interest rate environments, some banks with relatively strong balance sheet positions might aggressively offer lower lending rates to gain new customers, even if they incur certain losses in the short run. Third, new financial agents might emerge and enter these financial markets. These include new nonbank and/or foreign financial firms that do not currently exist in Japanese financial markets.
6. Multiplied by four, these provide rough estimates of the annual growth rates.
7. Financial Statements Statistics of Corporations by Industry (April–June, 2016) from the Ministry of Finance covers only large corporations with capital exceeding 10 million yen.
8. This statistical result is robust to changes in the sample size. Varying the size of the first sample from three to 20 does not alter the result of the rejection of the null hypothesis of equal means at the 1% significance level.
9. There is a conundrum concerning the interpretation of the statistical data in the flow of funds accounts for the insurance and pension funds sector. While we observe a distinct difference in the mean between before and after the introduction of the NIPR in January 2016 in the BOJ flow accounts data, as shown in Figure 6, there is a much less marked difference in the corresponding stock accounts data. To solve this little mystery, recall that there was a sharp depreciation in the USD against the JPY over the period from June 2015 until August 2016. With the USD depreciation, the value of dollar assets in the stock accounts fell, and this obscured the increase in the flow of foreign securities in the data.
10. See also Honda (2017) for a similar discussion of the effects of the NIPR in the Euro area.

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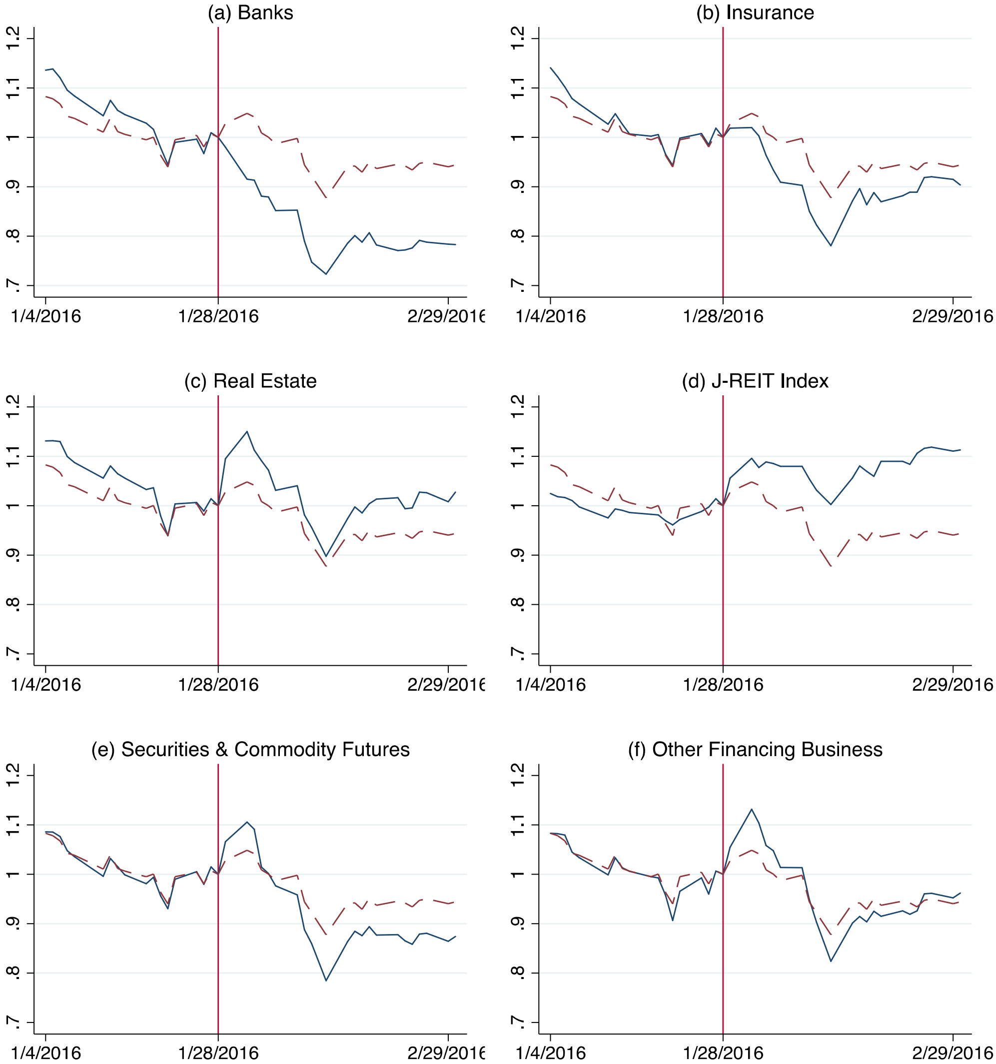
Table 1: Real GDP Growth Rate and Its Components

	2016Q2		2016Q3	
GDP	0.4		0.3	
Private Consumption	0.2	(0.1)	0.3	(0.2)
Private Residential Investment	3.3	(0.1)	2.4	(0.1)
Private Non-Resi. Investment	1.3	(0.2)	-0.3	(-0.1)
Government Consumption	-1.1	(-0.2)	0.3	(0.0)
Public Investment	1.1	(0.1)	-0.7	(-0.0)
Exports of Goods and Services	-1.2	(-0.2)	2.1	(0.4)
Imports of Goods and Services	-1.0	(0.2)	-0.2	(0.0)

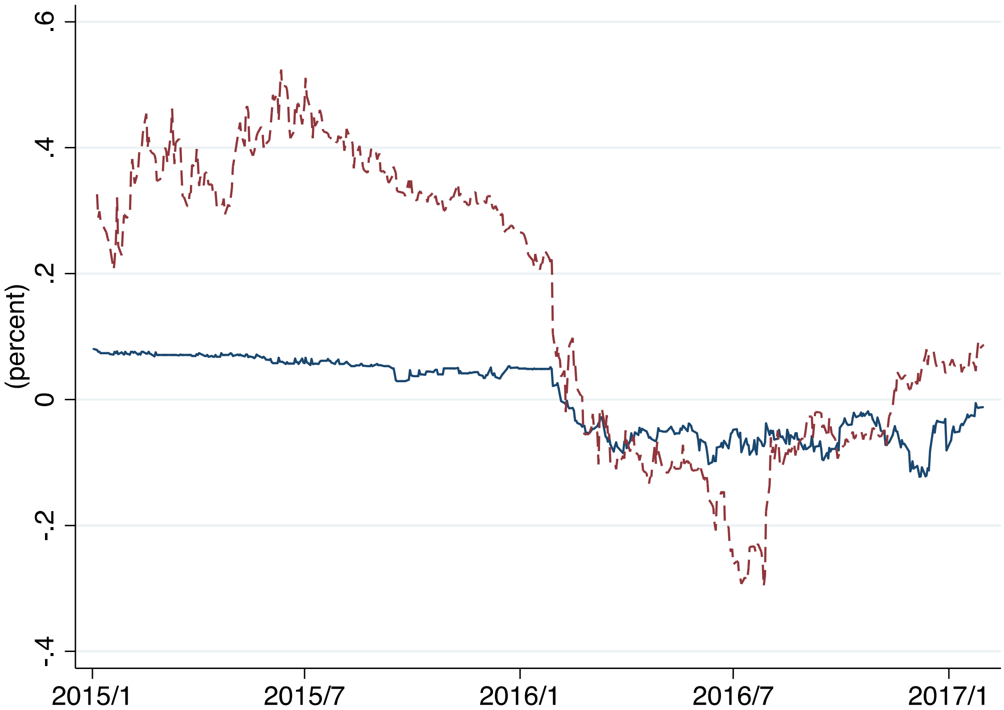
Notes: Data source is seasonally-adjusted quarterly series (percent changes from the previous quarter) released at February 13, 2017 by the Cabinet Office, Government of Japan. The figures in parentheses indicate contributions to changes in GDP.

Figure 1: Impact of NIRP on the Stock Market



Notes: All indexes are standardized to one as of January 28, 2016 to correspond with the time the BOJ announced the NIRP. Dashed lines in all panels denote the Nikkei Stock Average.

Figure 2: Short-term and Long-term Interest Rates



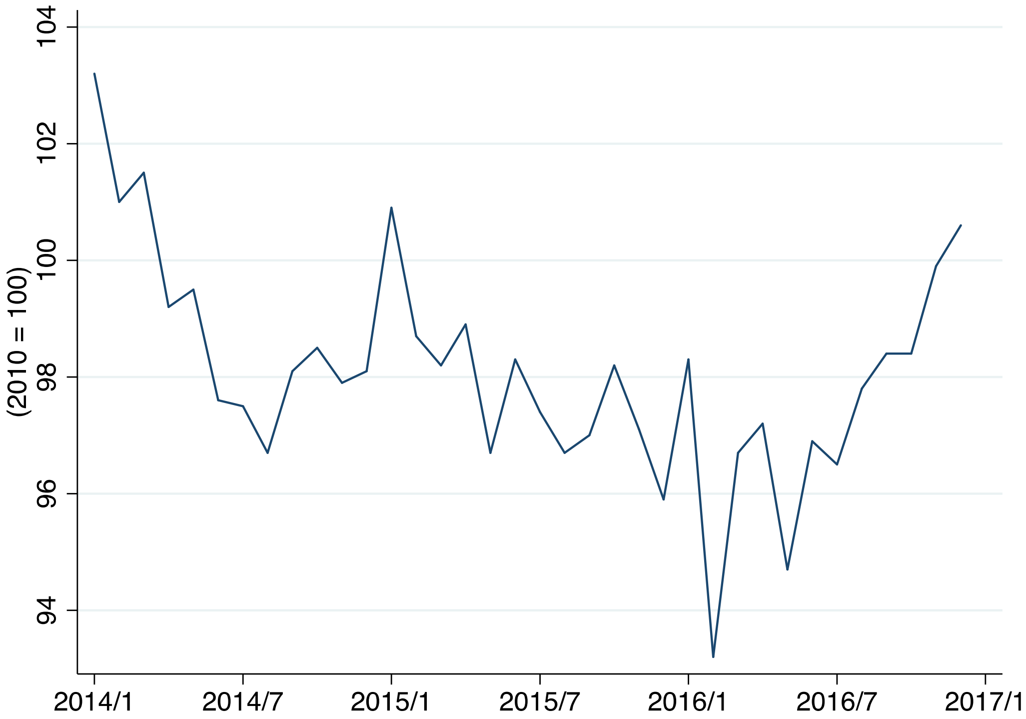
Notes: Solid line denotes the short-term interest rate (one-month LIBOR based on Japanese Yen) and dashed line denotes long-term interest rate (10-year Japanese government bond rate).

Figure 3: Nominal Effective Yen Exchange Rate

Notes: Data source is monthly series, which is standardized to 100 at 2010, released on the web site of the Bank of Japan. The higher value means the more appreciated the currency.

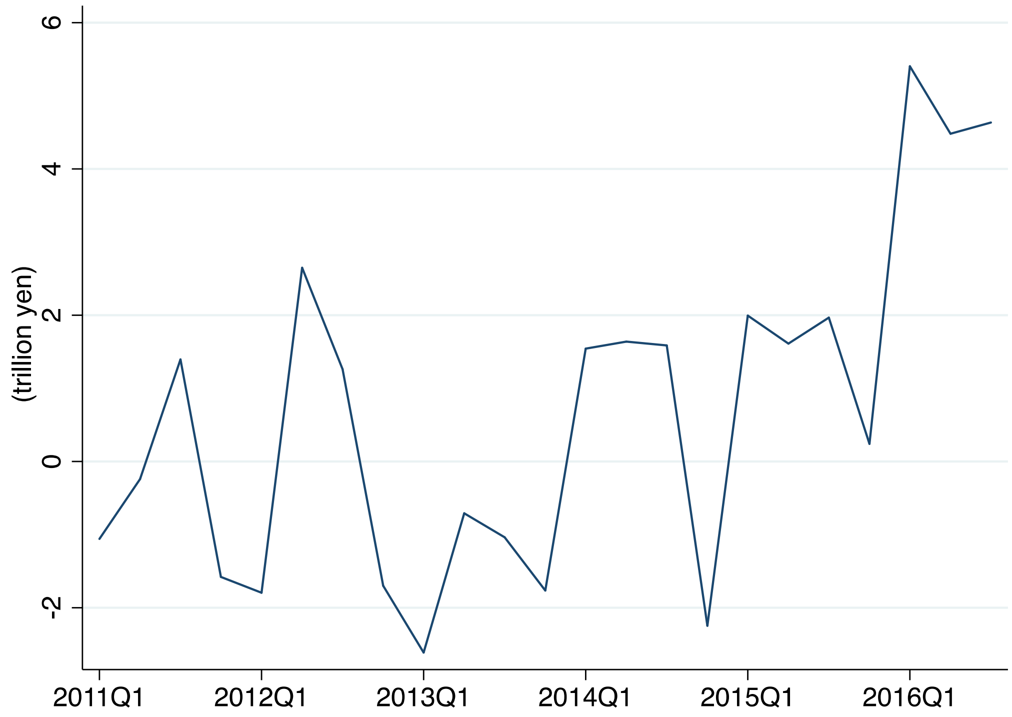
Figure 4: Tokyo Stock Price Index (TOPIX)

Figure 5: Index of Industrial Production



Notes: Data source is seasonally-adjusted monthly series released on the web site of the Ministry of Economy, Trade and Industry.

Figure 6: Flow of Funds of Foreign Securities in the Insurance and Pension Funds Sector



Notes: Data source is “Outward investment in securities” by the “Insurance and pension funds” sector in the Flow of Funds Accounts compiled by the Bank of Japan.