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Takanori Ago† Tadashi Morita‡

Takatoshi Tabuchi§ Kazuhiro Yamamoto¶

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Abstract

This study analyzes the interplay between the agglomeration of economic activities and interregional differences in working hours, which are typically longer in large cities, as they are normally more developed than small cities. For this purpose, we develop a two-region model with endogenous labor supply. Although we assume a symmetric distribution of immobile workers, the symmetric equilibrium breaks in the sense that firms may agglomerate when trade costs are intermediate and labor supply is elastic. We also show that the price index is always lower, while labor supply, per capita income, real wages, and welfare are always higher in the more agglomerated region.

JEL Classification: R23, F16

Key words: elastic labor supply; agglomeration; symmetry break

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†School of Commerce, Senshu University, E-mail address: ago@isc.senshu-u.ac.jp

‡Faculty of Economics, Kindai University, E-mail address: t-morita@kindai.ac.jp

§Faculty of Economics, University of Tokyo, E-mail address: ttabuchi@e.u-tokyo.ac.jp

¶Graduate School of Economics, Osaka University. E-mail address: yamamoto@econ.osaka-u.ac.jp

1 Introduction

Large cities often have more varieties of consumption goods and services, where people have fewer children and work longer hours (Morita and Yamamoto, 2014) earning more salaries in order to enjoy such rich urban life.[1] During the era of Industrial Revolution, we observe two phenomena: the economic activities are agglomerated in cities and working hours in cities are longer than that in rural area. Hohenberg and Lees (1985) showed that in the process of Industrial Revolution, geographic agglomeration of economic activities has progressed. Urban agglomeration materialized in an increase of urbanization rate and the formation of industrial clusters in the core of Europe like London that have been by and large sustained (Martin and Ottaviano, 2001). On the other hand, Voth (2000) investigated the working hours in England during the Industrial Revolution and reported that the total working hours per year was 3,350 hours in London and 3,211 in the North parts of England in 1760. de Vries (1994) called the increase in working hours in the 18th century in the United Kingdom an "Industrious Revolution." He argued that since the variety of consumption goods had increased during this period, workers worked harder in order to earn more income to pay for the growing number of consumption goods available.

Thus, in the process of Industrial Revolution, the working hours increased because of the rise in the real wage, which enabled them to purchase consumption varieties. At the same time, the industrial agglomeration was progressed, and the working hours was raised, especially in London. The purpose of this paper is to construct a model in which industrial agglomeration brings about the increase in working hours, which becomes one reason for agglomeration of economic activities in the process of Industrial Revolution.

Based on the foregoing, this study analyzes the interplay between the agglomeration of

[1] Rosenthal and Strange (2008) show that among professionals, working hours are longer in larger cities, which are comparable with more developed countries. Gicheva (2013) also shows that young highly educated workers work longer hours to pursue career advancement and to earn higher wages based on the 1979 cohort of the National Longitudinal Survey of Youth. According to 2006 Survey on Time Use and Leisure Activities in Japan, the working time (average time spent on work) is longer in denser prefectures which consist of (more developed) large cities.

economic activities and interregional differences in working hours by using the framework of new economic geography pioneered by Krugman (1991). For this purpose, we develop a model of new economic geography by introducing endogenous labor supply without the interregional migration of labor. More specifically, we construct a two-region model with one differentiated good sector. Each agent is spatially immobile and chooses the optimal amount of labor supply as well as the consumption of the good. An increase in labor supply brings about disutility due to the labor burden, whereas it raises wage income. Therefore, each agent determines labor supply at which marginal disutility by labor equals the real wage, which is defined by the nominal income over the price index in the region.

Our main finding is that even if two regions are identical, the symmetric configuration of firms breaks if the elasticity of labor supply with respect to real wage is sufficiently high. That is, the emergence of an endogenous agglomeration is possible without assuming the spatial mobility of labor. This finding is in sharp contrast to studies of new trade theory such as Krugman (1980), where this symmetry never breaks.

The mechanism that brings about endogenous agglomeration occurs is as follows. The real wage is higher in a region that has more manufacturing firms. If firms agglomerate more, the price index decreases further, and thus, the real wage rises further. That is, the relative value of nominal income to labor disutility goes up. Since our model assumes an elastic labor supply unlike the familiar models that incorporate a fixed labor supply, the amount of labor supply rises in the agglomerated region. This leads to higher per capita income and a larger market size, which attracts manufacturing firms to the region. In summary, labor supply has a positive correlation with agglomeration of manufacturing firms, whereas migration of workers leads to agglomeration of firms in the new economic geography framework.

When the symmetry breaks, we have an asymmetric distribution of firms as a stable equilibrium, where the amount of labor supply is shown to be larger, while the nominal wage earning and per capita total income are higher in the agglomerated region. We also show that individual welfare is higher in the agglomerated region, implying that the higher nominal income and lower price index dominate the higher labor disutility in the agglomerated region.

From the theoretical perspective, we construct a model of endogenous agglomeration without interregional migration. According to Combes et al. (2008, p. 166), "Moving beyond the Krugman model in search of alternative explanations appears to be warranted in order to understand the emergence of large industrial regions in economies characterized by a low spatial mobility of labor." In this study, we consider that labor supply changes based on workers' choice of working hours rather than because of the relocation of firms. Workers prefer to adjust working hours than changing firms through interregional migration in the short run when shocks occur in the labor market. According to Nakajima and Tabuchi (2011), the annual gross migration between prefectures was 2.9% of the Japanese population for 1954-2005 and that between states was 1.1% of the U.S. population for 1989–2004. Braunerhjelm et al. (2000) also report the existence of low spatial labor mobility in EU countries.

Some studies have examined the endogenous agglomeration of firms without labor migration. Krugman and Venables (1995) introduce the input-output linkages that yield the agglomeration of firms in the absence of migration. Amiti and Pissarides (2005) assume training costs for skill formation, which serves as a proxy for labor migration, resulting in the emergence of firm agglomeration. Picard and Toulemonde (2006) consider labor unions that introduce wage rigidities so that unionized and high-wage firms agglomerate in a region in the absence of labor mobility. Our study shows that an increase in working hours fosters agglomeration of firms under immobile labor, which is consistent with the above-mentioned facts in the process of Industrial Revolution.

Corsetti et al. (2007) is closely related with our paper. They construct a two country, monopolistic competition model with elastic labor supply. Their focus is on the effects of technological progress on terms of trade and welfare, while our focus is on the endogenous agglomeration induced by the elastic labor supply. Ago et al. (2016) also consider elastic labor supply focusing on international differentials in productivity and working hours in the framework of international trade, whereas this paper considers interregional or intercity trade with migration of firms, so that agglomeration emerges.

The remainder of the paper is organized as follows. In section 2, we present the basic model. Sections 3 and 4 assume the same population size while section 5 assumes different population sizes. In section 3, we characterize and examine the symmetric equilibrium of firm distribution. We show that when the symmetric equilibrium is unstable, asymmetric equilibria exist. In section 4, we analyze such asymmetric equilibria. Section 5 considers regional asymmetry in the sense that population size or productivity differs between regions. Section 6 concludes the paper.

2 The model

The economy consists of two regions, denoted $r = 1, 2$ and a manufacturing sector producing a differentiated good. Let L_r be the mass of immobile workers in region r, and n be the mass of mobile capital in the economy. We assume that one unit of capital is needed as a fixed requirement to produce each variety meaning that the total number of varieties of a differentiated good is n, which is exogenously given.

The preferences of an agent located in region $r = 1, 2$ are given by:

$$U_r = \left[\int_0^n x_r(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} - \frac{\theta}{\theta+1} l_r^{\frac{\theta+1}{\theta}}, \qquad \sigma > 1, \ \theta > 0, \tag{1}$$

where $x_r(i)$ is the consumption of a variety indexed i in region r and l_r is the amount of labor supply, which reduces the utility since supplying labor reduces leisure time in region r. Each agent supplies labor and earns hourly wage w_r, which is used to purchase the good. She chooses the amount of labor supply, l_r, as well as the consumption of each variety, $x_r(i)$. Therefore, labor supply is elastic. We follow Corsetti et al. (2007) and the elasticity of labor supply with respect to real wage θ is constant.

In addition to the wage, she receives rewards from capital holding, a. Her income constraint is given by

$$a + w_r l_r = \int_0^n p_r(i) x_r(i) di, \tag{2}$$

where $p_r(i)$ is the price of variety i sold in region r.

From (1) and (2), we find the labor supply to be

$$l_r = \left(\frac{w_r}{P_r} \right)^\theta \tag{3}$$

where

$$P_r = \left[\int_0^n p_r(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}$$

is the price index, σ is the elasticity of the substitution between differentiated varieties. We assume $\sigma > 1$ and $\theta > 0$ to satisfy the second-order conditions for utility maximization. Equation (3) shows that labor supply is an increasing function of the real wage. On the one hand, when the nominal wage w_r increases, each agent raises labor supply in order to purchase the good. On the other hand, when price index P_r goes up, the value of real income goes down, which reduces labor supply.

We also find the individual demand for variety i produced in region r and consumed in region s as follows:

$$x_{rs}(i) = (a + w_s l_s) \frac{p_{rs}(i)^{-\sigma}}{P_s^{1-\sigma}} = \left(a + w_s^{1+\theta} P_s^{-\theta} \right) \frac{p_{rs}(i)^{-\sigma}}{P_s^{1-\sigma}}, \tag{4}$$

where the second equality is derived from the substitution of (3). Because of the symmetry of each variety, we drop i hereafter.

The interregional trade of the good incurs an iceberg type trade cost. If $\tau > 1$ units of the good are exported between two regions, only one unit reaches the destination. We define $\phi_{rs} = \phi \equiv \tau^{1-\sigma} < 1$ if $r \neq s$ and $\phi_{rs} = 1$ if $r = s$. The price index in region r can be expressed as

$$P_r = (n_r p_{rr}^{1-\sigma} + n_s p_{sr}^{1-\sigma})^{\frac{1}{1-\sigma}} \tag{5}$$

for $r, s = 1, 2$ $(r \neq s)$.

To produce x units of a differentiated good, mx units of labor are needed in addition to one unit of capital. The rewards from capital holding are the profits of firms. We assume that each agent has an equal share of capital, therefore, the total rewards from capital are equally shared by all agents. The profit of a manufacturing firm in region r is described as

$$\pi_r = (p_{rr} - mw_r)x_{rr}L_r + (p_{rs} - m\tau w_r)x_{rs}L_s, \tag{6}$$

where individual demand x_{rs} is given by (4) and the reward from capital holding per agent is given by

$$a = \frac{n_1\pi_1 + n_2\pi_2}{L_1 + L_2}. \tag{7}$$

Each manufacturing firm sets prices, p_{rr} and p_{rs}, to maximize the profits. The prices of the good are computed as

$$p_{rr} = \frac{\sigma m}{\sigma - 1}w_r, \qquad p_{rs} = \frac{\sigma m\tau}{\sigma - 1}w_r. \tag{8}$$

By substituting (8) into (6), we have

$$\begin{aligned}\pi_r &= \left(\frac{\sigma m}{\sigma - 1}w_r - mw_r\right)x_{rr}L_r + \left(\frac{\sigma m\tau}{\sigma - 1}w_r - m\tau w_r\right)x_{rs}L_s \\ &= \frac{mw_r}{\sigma - 1}\left(x_{rr}L_r + \tau x_{rs}L_s\right). \end{aligned} \tag{9}$$

Total labor supply and the total labor demand in region r are l_rL_r and $n_rm\left(x_{rr}L_r + x_{rs}L_s\tau\right)$, respectively. Thus, the labor market clearing condition in region r is expressed as

$$l_rL_r = n_rm\left(x_{rr}L_r + x_{rs}L_s\tau\right), \tag{10}$$

where the LHS of this equation represents the labor supply and the RHS the labor demand.

By plugging (10) into (9), we obtain

$$\pi_r = \frac{w_r l_r}{n_r} \frac{L_r}{\sigma - 1}. \tag{11}$$

Hence, we have shown that *the profit of a firm is proportional to the sales per firm*, $w_r l_r / n_r$, which comprises the wage bill $w_r l_r$ and the number n_r of firms. The profit is in proportion to the former, while inversely proportional to the latter. In the agglomerated region, the denominator n_r of (11) is larger, implying keen competition among firms there. To attain a spatial equilibrium, the numerator $w_r l_r$ of (11) should also be larger in the agglomerated region. This fact means that firms in the agglomerated region should offer a higher wage bill $w_r l_r$ to secure larger labor supply, which is due to a larger number of firms.

In the spatial equilibrium, the profit of each firm is the same between regions. That is, the spatial equilibrium conditions are given by

$$\Delta\pi \equiv \pi_1 - \pi_2 = 0, \tag{12}$$

and the labor market clearing condition (10). They lead to

$$\frac{n_1/L_1}{n_2/L_2} = \frac{w_1 l_1}{w_2 l_2}. \tag{13}$$

If the home market effect $n_1/L_1 > n_2/L_2$ is exhibited, then per capita income $a + w_1 l_1$ is higher in the larger region.

Lemma 1 *Per capita income is higher in a region with more firms relative to population.*

If an agglomerated region is interpreted as a more developed region (i.e., a large city), then this agrees with the stylized facts in the urban economy: income per capita is higher in larger cities.

However, the wage is not necessarily higher in the larger region. As shown in Appendix

A, we have

$$\frac{dw}{d\lambda} \lesseqgtr 0 \Leftrightarrow \phi \lesseqgtr \frac{1}{2\sigma - 1} \tag{14}$$

where $\lambda \equiv n_1/n$ and $w \equiv w_1/w_2$, which are the endogenous variables to be determined by the two spatial equilibrium conditions, (10) and (12), with (3), (4), (7), and (11). We choose the labor in region 2 as numeràire so that $w = w_1$.[2] Since (14) holds for an arbitrary given λ, we can say the following.

Proposition 2 *If trade costs are high $\phi < 1/(2\sigma - 1)$, the nominal wage, w_r is lower in the agglomerated region. Otherwise $\phi > 1/(2\sigma - 1)$, the nominal wage is higher in the agglomerated region.*

It is somewhat surprising that the wage is lower in the agglomerated region, which usually does not occur under new economic geography or new trade theory with immobile workers.

We explain in section 4.2 that excess labor supply because of the increase in λ reduces the wage near autarky. A similar intuition can be applied to high trade costs $\phi < 1/(2\sigma - 1)$. We also explain in section 4.2 that excess labor demand because of the increase in λ raises the wage near free trade. A similar intuition is applied for low trade costs $\phi > 1/(2\sigma - 1)$.[3]

Firms migrate to a region with a higher profit, meaning that ad hoc dynamics may be given by

$$\dot{\lambda} = \Delta\pi.$$

Finally, plugging (3), (7), and (11) into utility (1) yields the indirect utility:

$$V_r = \frac{a + w_r l_r}{P_r} - \frac{\theta}{\theta + 1}\left(\frac{w_r}{P_r}\right)^{\theta+1}, \tag{15}$$

[2] One may wonder if region 2 is empty. However, such a fully agglomerated equilibrium never arises as we show in section 3.

[3] In the real world, nominal wage is higher in the agglomerated regions. This is because domestic interregional trade costs are relatively low. Thus, $\phi > 1/(2\sigma - 1)$ my be satisfied in the real world.

where a and P_r are expressed by λ and w_r.

3 Symmetric equilibrium

To focus on the symmetric equilibrium, we set an equal population size of regions $L_1 = L_2$, which is normalized to 1 in this section and the next section. It is apparent that there always exists a symmetric equilibrium for any values of the parameters. However, this equilibrium can be stable or unstable depending on the parameter values. Substituting (3) and (11) into (12), totally differentiating it with respect to λ, and evaluating it at the symmetric equilibrium, we can obtain the stability condition as follows:

$$\begin{aligned} \left.\frac{d\Delta\pi}{d\lambda}\right|_{(\lambda,w)=(1/2,1)} &= \left.\frac{\partial\Delta\pi}{\partial\lambda} + \left(\frac{\partial\Delta\pi}{\partial P_1}\frac{dP_1}{d\lambda} + \frac{\partial\Delta\pi}{\partial P_2}\frac{dP_2}{d\lambda}\right) + \frac{\partial\Delta\pi}{\partial w}\frac{dw}{d\lambda}\right|_{(\lambda,w)=(1/2,1)} \\ &= Cg\left(\phi\right) < 0, \end{aligned} \tag{16}$$

where $dw/d\lambda$ is computed by applying the implicit function theorem to (10), which is a function of λ and w. C is a positive constant and

$$g\left(\phi\right) \equiv -\left(2\sigma-1\right)\left(2\theta+1\right)\phi^2 + 2\left[\left(2\sigma-1\right)\theta + 3\sigma - 2\sigma^2\right]\phi - 1.$$

From $g\left(\phi\right)$, we have the following.

Proposition 3 *The symmetric equilibrium is stable if $\theta < \theta_{\text{break}}$ and unstable if $\theta > \theta_{\text{break}}$, where*

$$\theta_{\text{break}} \equiv \frac{\left[\left(2\sigma-1\right)\phi+1\right]\left[\left(2\sigma-3\right)\phi+1\right]}{2\left(2\sigma-1\right)\left(1-\phi\right)\phi}$$

is the solution of $g\left(\phi\right) = 0$.

In Figure 1, the blue curve represents $\theta = \theta_{\text{break}}$. The symmetric equilibrium is stable below it and unstable above it. Furthermore, we can show that the symmetric equilibrium never breaks for a sufficiently inelastic labor supply such that

$$\theta < \theta_B \equiv \sigma - 1 + \frac{2\sqrt{(\sigma-1)\,\sigma}}{2\sigma - 1}.$$

This result corresponds to the familiar result under an inelastic labor supply $\theta = 0$ in Krugman (1980), among others. When θ is small, labor supply is inelastic with respect to the real wage. Suppose that some manufacturing firms move to a region. The price index in the region that attracts firms decreases, which raises labor supply from (3). However, such an expansion of labor supply is small because labor supply is inelastic. On the contrary, labor demand increases according to the number of firms. Further, the tight labor market forces wage to rise, and thus the profits of firms reduce, which ensures the stability of the symmetric equilibrium.

Thus, *the symmetry break requires an elastic labor supply (large θ).*[4] Suppose θ is large enough and labor supply is elastic with respect to the real wage. Firms can expect large labor supply and agents can expect a higher real wage, which expands the market size in the destination region. More precisely, if $\theta > \theta_B$, the symmetric equilibrium is unstable when ϕ is in the interval of (ϕ_{B1}, ϕ_{B2}), where ϕ_{B1} and ϕ_{B2} are the solutions of $g(\phi) = 0$ and satisfy $0 < \phi_{B1} < \phi_{B2} < 1$. Otherwise, the symmetric configuration is a stable equilibrium.

Next, we check the possibility of a fully agglomerated equilibrium, $\lambda = 1$. If this is the case, the substitution of (4) into (9) yields the profit differential

$$\Delta\,\pi|_{\lambda=1} = (\pi_1 - \pi_2)|_{\lambda=1} = \frac{w_1 l_1}{(\sigma-1)n}\left[1 - \left(\frac{w_1}{w_2}\right)^{\sigma-1}\right].$$

However, because labor supply in region 2 is $l_2 = 0$, the wage in region 2 is $w_2 = 0$ from (3). Hence, $\Delta\,\pi|_{\lambda=1} = -\infty$, which violates the equilibrium condition. Therefore, full agglomera-

[4] According to the literature such as Tabuchi and Thisse (2011), σ is between 1.9 and 7.6 depending on the definition of industry: the wider it is, the smaller σ is. If $\sigma = 3$, the threshold of the labor supply elasticity is $\theta_B = 2.98$. The labor supply elasticity in the literature is between 1 (Domeij and Floden, 2006) and 3.8 (Imai and Keane, 2004) implying that the symmetric equilibrium can be stable or unstable.

tion is never an equilibrium.[5] Stated differently, manufacturing production is always carried out in both regions by immobile workers, whose labor supply is positive. Otherwise, they earn no income and consume no good.

We have seen that the symmetric equilibrium is unstable if $\theta > \theta_{\text{break}}$ and that the fully agglomerated equilibrium never exists. Nevertheless, an equilibrium for any continuous utilities always exists, as shown by Ginsburgh *et al.* (1985), and a stable equilibrium always exists, as shown by Tabuchi and Zeng (2004).This finding suggests the existence of a partially agglomerated equilibrium that is stable if $\theta > \theta_{\text{break}}$.

4 Asymmetric equilibrium

We explore asymmetric equilibria by examining the conditions of labor market clearing and spatial equilibrium, Taking the logarithm of the ratio of (20) to (19) in Appendix A, we can reduce the equilibrium conditions to the following single equation

$$f(R) \equiv \log\left[\left(\frac{1-\phi R}{R-\phi} R^{\frac{\theta}{\sigma-1}}\right)^{\frac{\sigma-1}{\theta-\sigma+2}} \frac{1+\phi(2\sigma-1)R}{R+\phi(2\sigma-1)}\right] = 0. \tag{17}$$

Because the numerator of $f'(R)$ is quartic in R, there are at most five solutions. Because $f(R)$ is symmetric about $R = 1$, the number of solutions is one, three, or five, which is determined by the two thresholds θ_{break} and θ_j. When $\theta = \theta_j(\sigma, \phi)$, there is a repeated root of $f(R) = 0$ in the interval of $(1, 1/\phi)$ as well as in the interval of $(\phi, 1)$. Then, we can establish the following proposition.

Proposition 4 *Let $\lambda = 1/2$ be the symmetric equilibrium and $\lambda_1 \in (0, 1/2)$ be the asymmetric equilibrium.*

(i) If $\theta < \theta_j$, there is a unique stable equilibrium $\lambda = 1/2$;

[5] $w_2 = 0$ implies zero marginal cost under the CES setting. That is, the profit-maximizing price is zero, which leads to infinite demand and profits. Hence, each firm has an incentive to migrate to the empty region.

(ii) If $\theta_j < \theta < \theta_{\text{break}}$, there are five equilibria, three of which are stable and given by $\lambda = \lambda_1, 1/2, 1 - \lambda_1$;

(iii) If $\theta > \theta_{\text{break}}$, there are three equilibria, two of which are stable and given by $\lambda = \lambda_1, 1 - \lambda_1$;

The proof is contained in Appendix B.

Next, consider a thought experiment that the trade freeness ϕ steadily increases. As seen in Figure 1, as ϕ increases, there are two transitions of stable equilibria:

(a) when $\theta > \theta_B$, first symmetric equilibrium, then asymmetric equilibrium, and finally symmetric equilibrium again;

(b) when $\theta < \theta_B$, the equilibrium is always symmetric.

In case (a), there are two subcases:

(a1) when θ is sufficiently large, the first bifurcation is tomahawk while the second pitchfork;

(a2) when θ is intermediate, both bifurcations are pitchfork.

That is, as trade costs steadily fall, the spatial distribution of economic activities is initially dispersed, then partially agglomerated, and then dispersed again given $\theta > \theta_B$. The above mentioned transition is drawn as the red arrow in Figure 1. For a given $\theta > \theta_B$, falling trade costs move along the arrow, where the stable equilibrium distribution of firms runs from dispersion to partial agglomeration and then redispersion. It is worth noting that the agglomeration force is strong for intermediate trade costs compared with small and large ones.

4.1 Decomposition into the four effects

We can decompose the effects of the relocation of manufacturing firms to region 1 on the profit differential $\Delta\pi$ in the neighborhood of the symmetric equilibrium $(\lambda^*, w^*) = (1/2, 1)$ in the stability condition (16). The first term of (16) is negative. An increase in the number of firms brings about the *competition effect*: the higher number of firms, the lower are profits. The second term of (16) is positive. An increase in the number of firms also generates the *price index effect*: an increase in the number of firms lowers the price index in region 1 and raises the price index in region 2. When the price index is lowered in region 1, agents increase labor supply, which expands the market and raises profits. The third term of (16) is through the change in the wage. Since $\partial \Delta \pi / \partial w > 0$ always holds, the change $dw/d\lambda$ through the labor market clearing condition (10) matters. Figure 2 illustrates the labor market in region 1, where the upward sloping curve is the labor supply function given by the LHS of (10) and where the downward sloping curve is the labor demand function derived from the RHS of (10).[6] Further, there is a unique intersection point of the two curves, which is the equilibrium (l_1^*, w_1^*). Figure 2(A) illustrates the shift in labor supply l_1^S due to the increase in λ, while Figure 2(B) presents the shift in labor demand l_1^D due to the increase in λ. The supply curve l_1^S shifts right because $\partial l_1^S / \partial \lambda \geq 0$ and this decreases the wage rate. We name this effect the *excess labor supply effect*. When λ increases, the number of firms in region 1 increases, which lowers the price index in region 1. When the price index in region 1 is lowered, agents in region 1 increase labor supply, since at the given nominal wage, the real wage in region 1 rises. Then, excess labor supply emerges with the increase in λ.

The demand curve l_1^D can shift right or left following the increase in the number of firms in region 1. We name this effect the *excess labor demand effect*. When λ increases, the number of firms is raised, which increases the labor demand. However, the increase in λ lowers the price index in region 1, which decreases labor demand there, since competition among firms in region 1 intensifies. If the former effect dominates the latter, the demand curve l_1^D shifts right and excess labor demand emerges as λ increases. On the contrary, if the latter effect outweighs the former, the demand curve shifts left. The increase in λ may increase or decrease the equilibrium wage depending on the shifts in the two curves.

It can be shown below that $dw/d\lambda < 0$ if the *excess labor supply effect* is strong, whereas $dw/d\lambda > 0$ if the *excess labor demand effect* is positive and strong. These two effects are new

[6] From (3), the labor supply function is $l_1^S = k \left[\lambda + \phi (1 - \lambda) w^{\sigma - 1} \right]^{\frac{\theta}{\sigma - 1}}$, where $\partial l_1^S / \partial w > 0$ and $k \equiv \left(\frac{\sigma - 1}{\sigma m} \right)^{\theta} n^{\frac{\theta}{\sigma - 1}}$. Using (13), the labor demand function is given by $l_1^D = \frac{k\lambda}{1 - \lambda} \frac{\left(\phi \lambda w^{1-\sigma} + 1 - \lambda \right)^{\frac{\theta}{\sigma - 1}}}{w}$, where $\partial l_1^D / \partial w < 0$.

and do not exist in standard models with exogenous labor supply. Analysis of the indirect impact is somewhat complicated because we have to consider the labor market clearing condition.

The strength of these four effects depends on the trade freeness ϕ as shown below. We examine the two extreme cases of near autarky and near free trade in the vicinity of the symmetric equilibrium $(\lambda^*, w^*) = (1/2, 1)$.

4.2 Near autarky $\phi \approx 0$ and near free trade $\phi \approx 1$

When the trade cost is sufficiently high, (16) is given by

$$\lim_{\phi \to 0} \left. \frac{d\Delta\pi}{d\lambda} \right|_{(\lambda,w)=(1/2,1)} = \left[\frac{\partial \Delta\pi}{\partial \lambda} + \underbrace{\left(\frac{\partial \Delta\pi}{\partial P_1} \frac{dP_1}{d\lambda} + \frac{\partial \Delta\pi}{\partial P_2} \frac{dP_2}{d\lambda} \right)}_{+} \right] + \left. \underbrace{\frac{\partial \Delta\pi}{\partial w}}_{+} \underbrace{\frac{\partial w}{\partial \lambda}}_{-} \right|_{(\lambda,w)=(1/2,1)} < 0.$$

The bracketed term is positive, implying that the price index effect dominates the competition effect, which tends to break the symmetry. However, the product of the second terms is negative, which implies that the excess labor supply effect dominates the excess labor demand effect. When trade costs are sufficiently large, in both countries the real wage is low. Then, an increase in the number of firms raises the labor supply compared to labor demand and *the excess labor supply effect dominates the other effects* for a prohibitive trade cost as a whole, meaning that the symmetry does not break near autarky $\phi \in [0, \phi_{B1})$.

When the trade cost is sufficiently small, (16) is given by

$$\lim_{\phi \to 1} \left. \frac{d\Delta\pi}{d\lambda} \right|_{(\lambda,w)=(1/2,1)} = \left[\frac{\partial \Delta\pi}{\partial \lambda} + \underbrace{\left(\frac{\partial \Delta\pi}{\partial P_1} \frac{dP_1}{d\lambda} + \frac{\partial \Delta\pi}{\partial P_2} \frac{dP_2}{d\lambda} \right)}_{-} \right] + \left. \underbrace{\frac{\partial \Delta\pi}{\partial w}}_{+} \underbrace{\frac{\partial w}{\partial \lambda}}_{+} \right|_{(\lambda,w)=(1/2,1)} < 0.$$

The bracketed term is negative, meaning that the price index effect is dominated by the competition effect, which stabilizes the symmetric equilibrium. When trade costs are sufficiently small, market competition in both countries becomes severe and then the competition effect is large. The positive second term means that an increase in λ hardly affects the price index P_1 but raises the wage w_1 due to excess labor demand, which destabilizes the symmetric equilibrium. The inequality implies that *the competition effect outweighs the other effects* for costless trade, and hence, the symmetry is a stable equilibrium near free trade $\phi \in (\phi_{B2}, 1]$.

Next, by examining the differential indices, we are able to establish the following results.

Proposition 5 *In the asymmetric equilibrium, price index P_r^* is always lower, while labor supply l_r^*, wage earning $w_r^* l_r^*$, per capita nominal income $a^* + w_r^* l_r^*$, real wage w_r^*/P_r^*, and welfare V_r^* are always higher in the agglomerated region.*

The proof is presented in Appendix C. Price index P_r^* is lower in the agglomerated region because more firms supply varieties without trade costs. In this region, the relative value of the nominal wage to the price index is higher, which raises labor supply l_r^* from (3).[7]

Consequently, wage earning $w_r^* l_r^*$, per capita income $a^* + w_r^* l_r^*$ and real wage w_r^*/P_r^* are higher in the agglomerated region. These higher values outweigh the disutility from labor supply, which leads to higher welfare V_r^* in this region.

Proposition 5 states that labor supply is larger in the agglomerated region with higher nominal wage earning and per capita income. This is consistent with the facts presented in the introduction. Large labor supply brings about higher per capita income in this region, which expands its market size. This in turn attracts manufacturing firms. As a result, workers enjoy better access to a large market and are better off with a higher real wage and welfare in the agglomerated region.

Finally, we check if the home market effect is exhibited in the presence of an elastic labor supply. This effect is normally defined as "a more-than-proportional relationship between a country's share of the world production of a good and its share of world demand for the same good" (Crozet and Trionfetti, 2008). By using the equilibrium condition (13), we can

[7] This result is consistent with Rosenthal and Strange (2008) as we mentioned in the introduction. They show that working hours are longer in larger cities among professionals.

easily show that if $\lambda > 1/2$, then

$$\frac{n_1}{n_2} > \frac{(a + w_1 l_1)\, L_1}{(a + w_2 l_2)\, L_2}$$

always holds. Furthermore, the home market effect is also defined that "countries tend to export those kinds of products for which they have relatively large domestic demand" (Krugman, 1980). This is true if the following ratio exceeds 1:

$$\frac{n_1 p_{12} x_{12}}{n_2 p_{21} x_{21}} = \frac{n_1 w_1^{1-\sigma}}{n_2 w_2^{1-\sigma}} \left(\frac{P_1}{P_2}\right)^{1-\sigma} \frac{a + w_1 l_1}{a + w_2 l_2} \tag{18}$$

We know from Proposition 5 that $P_1^{1-\sigma} > P_2^{1-\sigma}$, which implies $n_1 w_1^{1-\sigma} > n_2 w_2^{1-\sigma}$ because

$$P_1^{1-\sigma} - P_2^{1-\sigma} = (1 - \phi) \left(\frac{m\sigma}{\sigma - 1}\right)^{1-\sigma} \left(n_1 w_1^{1-\sigma} - n_2 w_2^{1-\sigma}\right).$$

We also know from Proposition 5 that $w_1 l_1 > w_2 l_2$. Thus, the three terms on the RHS of (18) are greater than 1 for all $\lambda > 1/2$. Hence, the home market effect is necessarily exhibited even under an elastic labor supply.

5 Asymmetric regions

5.1 Different sized regions

So far, the mass of immobile workers was the same between regions. In this section, we consider the case of different population sizes between regions, $L_1 > L_2$, to explore the size effect on the spatial distribution of economic activities.

By using the parameter values of $(L_1, L_2, \sigma, \theta, n, m) = (2, 1, 3, 2, 1, 1)$, the interregional differential indices are plotted in Figures 3 and 4. In Figure 3, the blue curve is region 1's firm share $\lambda (= n_1/n)$, the red curve is the nominal wage differential w_1/w_2, and the yellow curve is the real wage differential $(w_1/P_1) / (w_2/P_2)$. In Figure 4, the blue curve is the utility differential V_1/V_2, the red curve is the differential in working hours l_1/l_2, and the yellow curve is the price index differential P_2/P_1. It is worth noting that all the curves are inverted U-shaped. The property remains the same for different parameter values.

Several observations can be made from these figures. First, as to the firm share, we observe $n_1/n_2 > L_1/L_2$ for all $0 < \phi < 1$, implying that the home market effect is always exhibited, as confirmed by most studies in new trade theory. Second, the nominal wage in the larger region is smaller for small ϕ, but larger for large ϕ, which is in accord with Proposition 2. Therefore, if trade costs are low, this result is consistent with the real world. Third, the price index is always lower, while labor supply, wage earning, per capita nominal income, real wage, and welfare are always higher in the larger region for all $0 < \phi < 1$. This finding is in accord with Proposition 5. Note that the second and third results are based on different population sizes, while Propositions 2 and 5 are based on the same population size. Finally, all the differential indices converge when the two regions are fully integrated $\phi = 1$.

5.2 Different productivity regions

Next, we consider the case that the marginal productivity in region 1 is higher than region 2. Setting the parameter values as $(L, \sigma, \theta, n, m_1, m_2) = (1, 3, 2, 1, 0.8, 1)$, several interregional differential indices are drawn in Figures 5 and 6. In Figure 5, the blue curve is region 1's firm share $\lambda(= n_1/n)$, the red curve is the nominal wage differential w_1/w_2, and the yellow curve is the real wage differential $(w_1/P_1) / (w_2/P_2)$. In Figure 6, the blue curve is the utility differential V_1/V_2, the red curve is the differential in working hours l_1/l_2, and the yellow curve is the price index differential P_2/P_1. As in the case of different productivity, all the curves are inverted U-shaped. The property remains the same for different parameter values.

From Figures 5 and 6, we can obtain some observations. First, the nominal wage in the high productivity region is smaller for small ϕ, but larger for large ϕ. Second, the price index is always lower, while labor supply, wage earning, per capita nominal income, real wage, and welfare are always higher in the high productivity region for all $0 < \phi < 1$. At last, from the blue curve in Figure 5, firms agglomerate in the high productivity region.

6 Conclusion

In this study, we introduced an elastic labor supply into the framework of new economic geography and examined the impacts of trade costs on the equilibrium outcomes of working hours and the spatial distribution of economic activities. Despite the symmetric distribution of immobile workers between two regions, we found that when trade costs are intermediate and labor supply is sufficiently elastic, the symmetry breaks. This finding is in sharp contrast to the body of literature on new economic geography. We also showed that the price index is always lower, whereas labor supply, wage earning, per capita income, real wage, and welfare are always higher in the agglomerated region.

When the labor supply elasticity is small, symmetric equilibrium tends to be stable. This implies that an elastic labor supply is not a strong agglomeration force. For the emergence of agglomeration, labor migration may be needed as in the new economic geography framework. However, when the labor supply elasticity is 3.8 as in Imai and Keane (2004), the symmetric equilibrium is unstable. This implies that the elastic labor supply is one of the important agglomeration forces.

We can present some future directions of research. The introduction of labor migration into our model would be an important future extension. It might also be important to incorporate commuting costs into our model because they may be regarded as a part of working hours for workers. It is of interest how government policies on income tax or commuting subsidy affect the social welfare.

References

[1] Ago, T., T. Morita, T. Tabuchi, and K. Yamamoto (2016) Endogenous labor supply and international trade, forthcoming in *International Journal of Economic Theory.*

[2] Amiti, M., and C. A. Pissarides (2005) Trade and industrial location with heterogeneous labor, *Journal of International Economics* 67, 392-412.

[3] Braunerhjelm, P., R. Faini, V. Norman, F. Ruane, and P. Seabright (2000) *Integration and the Regions of Europe: How the Right Policy Can Prevent Polarization.* London: Center for Economic Policy Research.

[4] Combes, P.-P., T. Mayer, and J.-F. Thisse (2008) *Economic Geography: The integration of Regions and Nations*, Princeton: Princeton University Press.

[5] Corsetti, G., P. Martin, and P. Pesenti (2007) Productivity, terms of trade and the 'home market effect' *Journal of International Economics* 73, 99-127

[6] Crozet, M. and F. Trionfetti (2008) Trade costs and the Home Market Effect, *Journal of International Economics* 76, 309-321.

[7] de Vries, J. (1994) The industrial revolution and the industrious revolution, *Journal of Economic History* 54, 249-70.

[8] Domeij, D. and M. Floden (2006) The labor-supply elasticity and borrowing constraints: Why estimates are biased, *Review of Economic Dynamics* 9, 242-262.

[9] Ginsburgh, V., Y.Y. Papageorgiou, and J.-F. Thisse (1985) On existence and stability of spatial equilibria and steady-states. *Regional Science and Urban Economics* 15, 149-158.

[10] Hohenberg, P. and Lees, L. (1985) *The Making of Urban Europe 1000-1950*, Cambridge, MA: Harvard University Press.

[11] Imai, S. and Keane, M. P., (2004) Intertemporal labor supply and human capital accumulation *International Economic Review*, 45, 601-641.

[12] Krugman, P. (1980). Scale economies, product differentiation and the pattern of trade. *American Economic Review* 70, 950-59.

[13] Krugman, P. (1991) Increasing returns and economic geography. *Journal of Political Economy* 99, 483-99.

[14] Krugman, P. and Venables, A. (1995) Globalization and the inequality of nations. *Quarterly Journal of Economics* 110, 857-880.

[15] Kuhn, P. and F. Lozano (2008) The expanding workweek? Understanding trends in long work hours among U.S. men, 1979–2006. *Journal of Labor Economics* 26:2, 311-343.

[16] Martin, P. and G.I.P. Ottaviano (2001) Growth and agglomeration. *International Economic Review*, 42, 947–968.

[17] Morita, T. and K. Yamamoto (2017) Economic geography, endogenous fertility and agglomeration, forthcoming in *Japanese Economic Review.*

[18] Nakajima, K. and T. Tabuchi (2011) Estimating interregional utility differentials. *Journal of Regional Science* 51, 31-46.

[19] Picard, P. M. and E. Toulemonde (2006) Firms agglomeration and unions. *European Economic Review* 50, 669-694.

[20] Rosenthal, S.S. and W.C. Strange (2008) Agglomeration and hours worked. *Review of Economics and Statistics* 90, 105-118.

[21] Tabuchi, T. and J.-F. Thisse (2011) A new economic geography model of central places. *Journal of Urban Economics* 69, 240-252.

[22] Tabuchi, T. and D.-Z. Zeng (2004) Stability of spatial equilibrium. *Journal of Regional Science* 44, 641-660.

[23] Voth , H.-J. (2000) *Time and Work in England 1750-1830*, Oxford: Clarendon Press.

Appendix A: Derivation of (14)

Solving (5) for n_1 and n_2 and substituting them into the spatial equilibrium condition (12) and the labor market clearing condition (10), we get the following two equations

$$W = \frac{R+\phi\left(2\sigma-1\right)}{1+\phi\left(2\sigma-1\right)R} \tag{19}$$

$$W = \left(\frac{1-\phi R}{R-\phi}R^{\frac{\theta}{\sigma-1}}\right)^{\frac{\sigma-1}{\theta-\sigma+2}} \tag{20}$$

with two variables $W \equiv w^{\sigma-1}$ and

$$R \equiv \left(\frac{P_1}{P_2}\right)^{\frac{1}{\sigma-1}} = \frac{\phi\lambda+\left(1-\lambda\right)W}{\lambda+\phi\left(1-\lambda\right)W} \in (\phi, 1/\phi). \tag{21}$$

First, suppose $\phi = 1/(2\sigma-1)$. Then, $W = R = 1$ always holds from (19). Next, suppose $\phi \neq 1/(2\sigma-1)$. From (19) and (21), λ can be expressed as a function of W:

$$\lambda = \frac{\left[2\sigma\phi W - \left(2\sigma\phi^2-\phi^2+1\right)\right]W}{\sigma\phi W^2 - \left(2\sigma\phi^2-\phi^2+1\right)W\lambda + \sigma\phi^2} \tag{22}$$

Differentiating it, we have

$$\frac{d\lambda}{dW} = \frac{\sigma\phi\left[4\sigma\phi W - \left(2\sigma\phi^2-\phi^2+1\right)\left(W^2+1\right)\right]}{2\left[\sigma\phi W^2 - \left(2\sigma\phi^2-\phi^2+1\right)W\lambda + \sigma\phi\right]^2}$$

When $\phi < 1/(2\sigma-1)$, $d\lambda/dW = 0$ has no real solution of W. This implies the sign of $d\lambda/dW$ does not change. Since $d\lambda/dW|_{W=1} < 0$, $d\lambda/dW$ should be negative for all W.

On the other hand, when $\phi > 1/(2\sigma-1)$, $d\lambda/dW = 0$ has two real solutions, which are denoted by W_l and W_h, where $W_l < W_h$. Let W_0 be the solution of $\lambda = 0$ in (22) and W_1 be the solution of $\lambda = 1$ in (22). We readily have $W_l < W_0 < 1 < W_1 < W_h$. Since $d\lambda/dW|_{W=1} > 0$, $d\lambda/dW$ should be positive for all W.

Appendix B: Proof of Proposition 4

We get

$$\begin{aligned} f(1) &= 0 \\ f(1/\phi) &= \begin{cases} -\infty & \sigma - 2 > \theta \\ +\infty & \sigma - 2 < \theta \end{cases} \end{aligned}$$

Computing the derivative of $f(R)$ and factorizing it, we have

$$f'(R) = \frac{df_n(R)}{df_d(R)},$$

where $df_n(R)$ is quartic in R and

$$df_d(R) \equiv (\sigma - 2 - \theta)(R - \phi)(1 - \phi R)[R + (2\sigma - 1)\phi][1 + (2\sigma - 1)\phi R]R.$$

The sign of $df_d(R)$ is determined by the sign of $\sigma - 2 - \theta$.

Because $\theta_{\text{break}} > \sigma - 2$, three cases may arise.

(i) $\theta \leq \sigma - 2$,

When $\theta = \sigma - 2$, the term in parentheses in (17), $f_1(R) \equiv \frac{1-\phi R}{R-\phi}R^{\frac{\theta}{\sigma-1}}$, should be equal to 1. Otherwise, $f(R)$ is either $-\infty$ or $+\infty$. Because $f_1'(R) < 0$ for all $R \in (\phi, 1/\phi)$ and $f_1(1) = 0$, there exists a unique equilibrium $R = 1$.

When $\theta < \sigma - 2$, we can show that $\partial df_n(R)/\partial\theta < 0$, we get $df_n(R) > df_n(R)|_{\theta=\sigma-2} > 0$. On the other hand, $df_d(R)$ is positive, and thus $f'(R)$ is positive for all $R \in (\phi, 1/\phi)$. Hence, there exists a unique equilibrium $R = 1$.

(ii) $\theta_{\text{break}} > \theta > \sigma - 2$.

We know that $f(1) = 0$, $f(1/\phi) = -\infty$ for $\theta > \sigma - 2$, and that $df_d(R) < 0$ for all $\theta > \sigma - 2$. We also have $df_n(1) < 0$ for all $\sigma - 2 < \theta < \theta_{\text{break}}$. Hence, the number of solutions in the interval of $(1, 1/\phi)$ is even. However, because $df_n(R)$ is quartic in R, there are at most five solutions in $f(R) = 0$. Since $f(1) = 0$ and since $f(R)$ is symmetric about $R = 1$, the number of solutions in $f(R) = 0$ in the interval of $(1, 1/\phi)$ is either zero or two. More precisely, the number of equilibria in the interval of $(\phi, 1/\phi)$ is one if $\theta < \min\{\theta_1, \theta_{\text{break}}\}$ and five if $\theta_1 < \theta < \theta_{\text{break}}$, where θ_1 is a function of σ and ϕ.

(iii) $\theta > \theta_{\text{break}}$.

We know that $f(1) = 0$, $f(1/\phi) = -\infty$ for $\theta > \sigma - 2$, and $f'(1) > 0$ for all $\theta > \theta_{\text{break}}$. Hence, the number of solutions in $f(R) = 0$ in the interval of $(1, 1/\phi)$ is one, and thus, the number of equilibria is three.

Appendix C: Proof of Proposition 5

Assume $\lambda^* > 1/2$.

(i) Proof of $P_1^* < P_2^*$. From (21), we immediately have

$$\text{sgn}\left(\lambda^* - \frac{1}{2}\right) = \text{sgn}(1 - R^*) = \text{sgn}(P_2^* - P_1^*).$$

(ii) Proof of $l_1^* > l_2^*$. From (3), we get

$$\frac{l_1^*}{l_2^*} = \left(\frac{w^* P_2^*}{P_1^*}\right)^{\theta} = \left(\frac{W^*}{R^*}\right)^{\frac{\theta}{\sigma-1}},$$

where W^* is a function of R^* given by (19). Because $\partial\left(W^*/R^*\right)/\partial R^* < 0$ and $W^*/R^*|_{R^*=1} = 1$ hold, we have

$$\operatorname{sgn}\left(\lambda^* - \frac{1}{2}\right) = \operatorname{sgn}\left(1 - R^*\right) = \operatorname{sgn}\left(l_1^* - l_2^*\right).$$

(iii) Proof of $w_1^* l_1^* > w_2^* l_2^*$. We showed in Lemma 1.

(iv) Proof of $w_1^*/P_1^* > w_2^*/P_2^*$. This is obvious from $l_1^* > l_2^*$ together with (3).

(v) Proof of $V_1^* > V_2^*$. From (15), we get

$$V_1^* - V_2^* = \frac{P_2^*}{2\left(\sigma-1\right)\left(\theta+1\right)\left(R^*\right)^{\frac{1}{\sigma-1}}}\left\{\left(\theta+1\right)\left[1-\left(R^*\right)^{\frac{2}{\sigma-1}}\left(m^*\right)^{\frac{\theta+1}{\theta}}\right] - \left(2\sigma+\theta-1\right)\left(R^*\right)^{\frac{1}{\sigma-1}}\left[1-\left(m^*\right)^{\frac{\theta+1}{\theta}}\right]\right\}$$

where $m^* \equiv l_1^*/l_2^*$. Since $\partial\left(V_1^* - V_2^*\right)/\partial m^* > 0$ holds for all $R^* < 1$, we have $V_1^* - V_2^* > V_1^* - V_2^*|_{m^*=1} > 0$ for all $R^* < 1$, i.e., for all $\lambda^* > 1/2$.

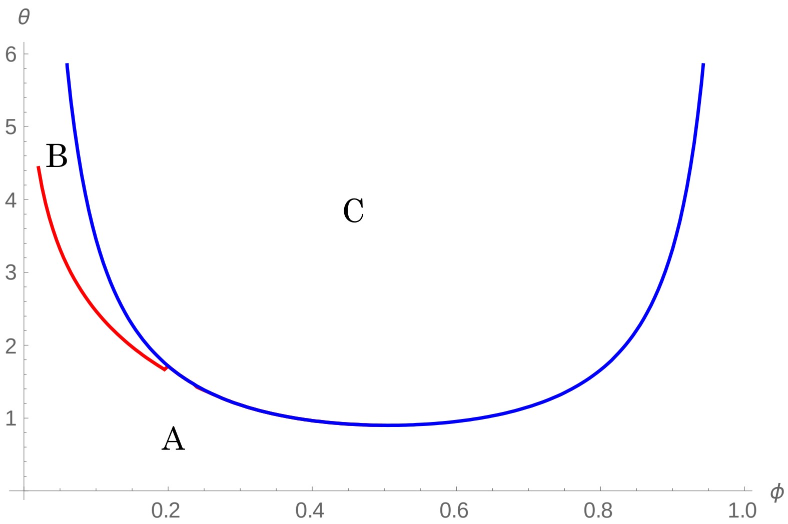

Figure 1: One symmetric stable equilibrium (A), three stable equilibria (B), and two asymmetric stable equilibria (C)

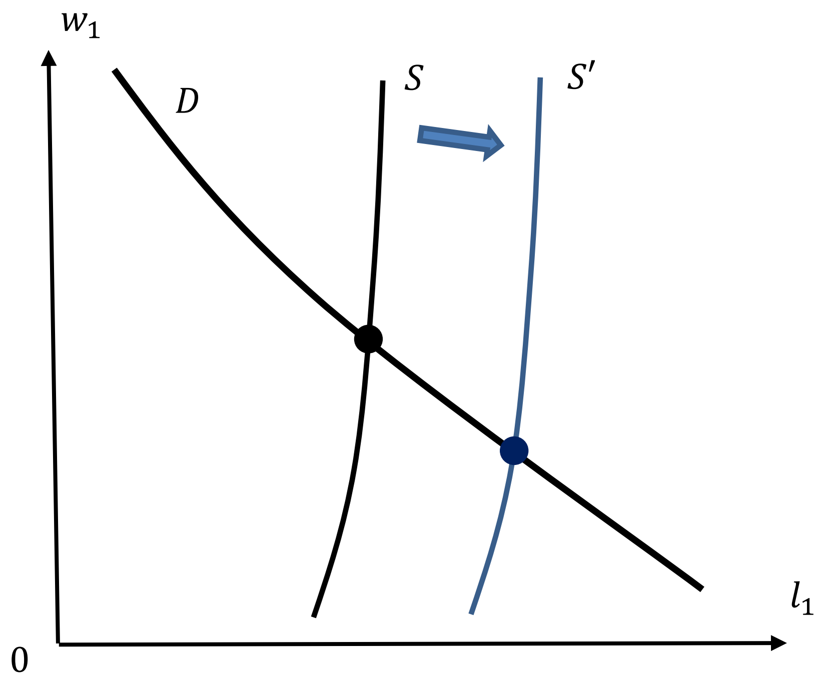

Figure 2(A): Supply shift due to excess labor supply effect

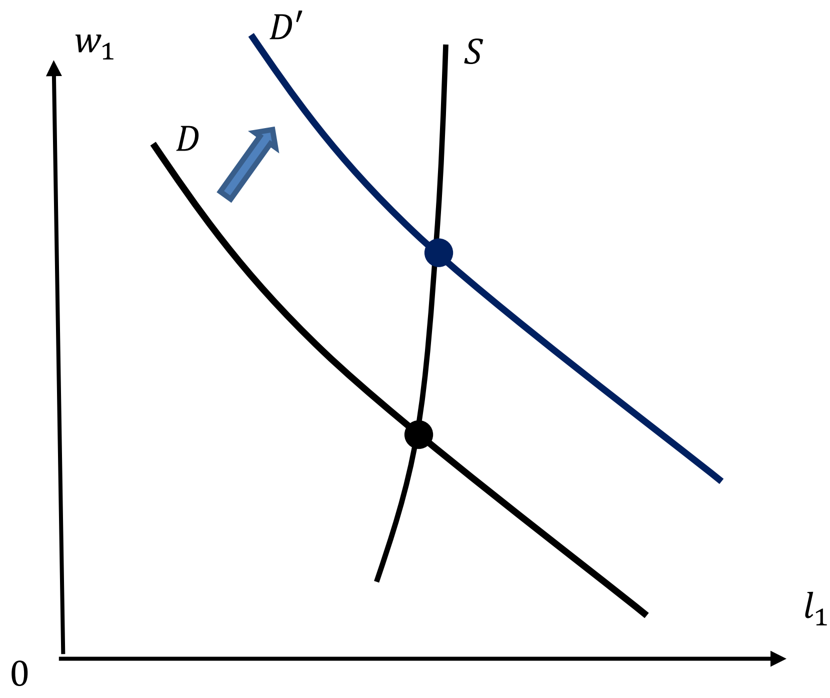

Figure 2(B): Demand shift due to excess labor demand effect

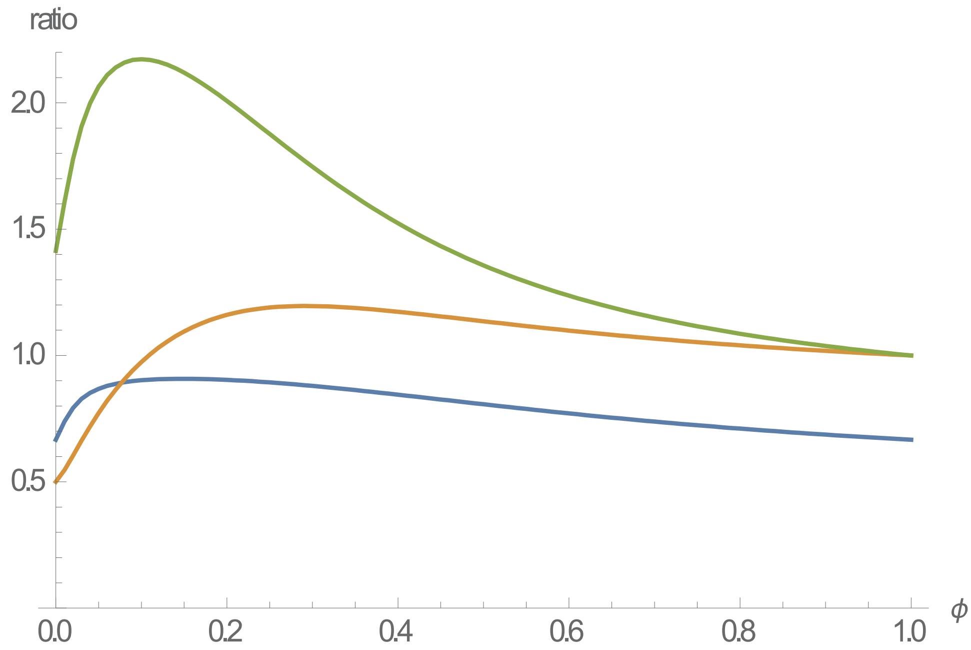

Figure 3: Interregional differentials when L_1/L_2= 2 (λ blue, w_1/w_2 red, $(w_1/P_1)/(w_2/P_2)$ yellow)

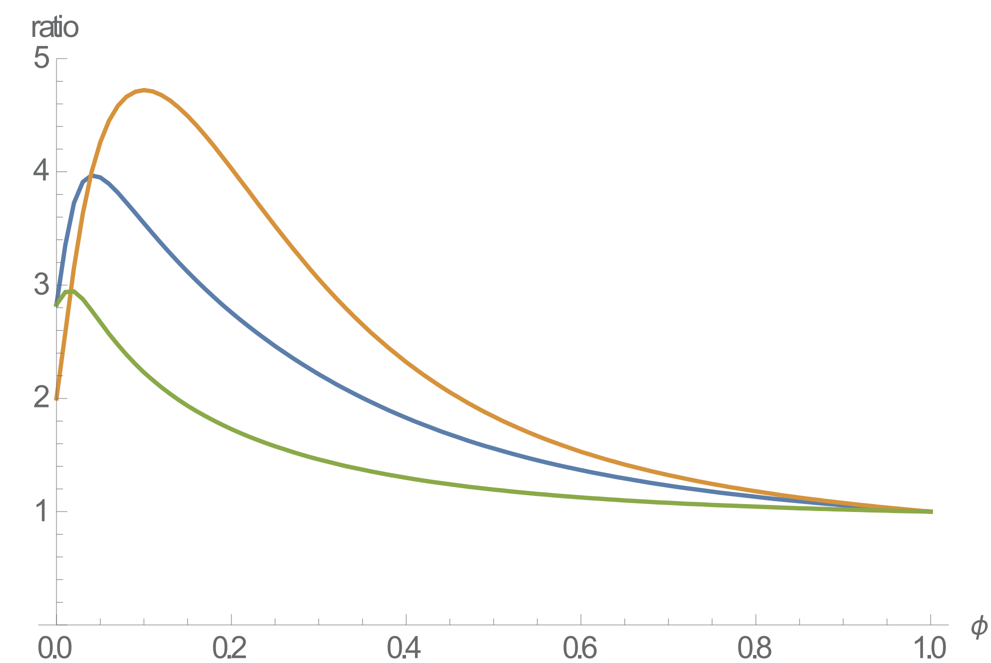

Figure 4: Interregional differentials when L_1/L_2= 2 (V_1/V_2 blue, l_1/l_2 red, P_2/P_1 yellow)

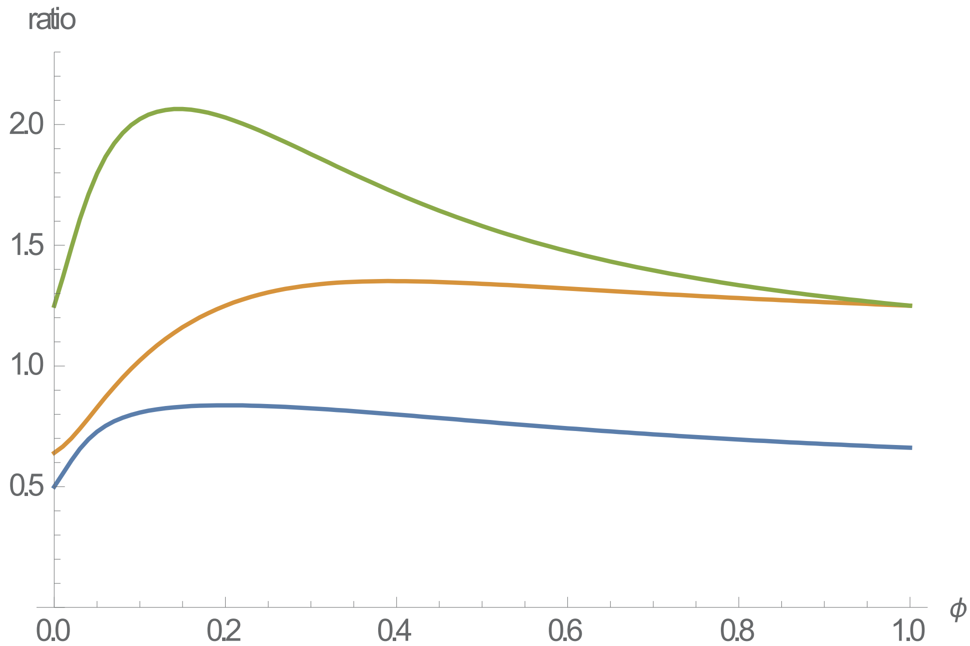

Figure 5: Interregional differentials when m_1/m_2= 0.8 (λ blue, w_1/w_2 red, $(w_1/P_1)/(w_2/P_2)$ yellow)

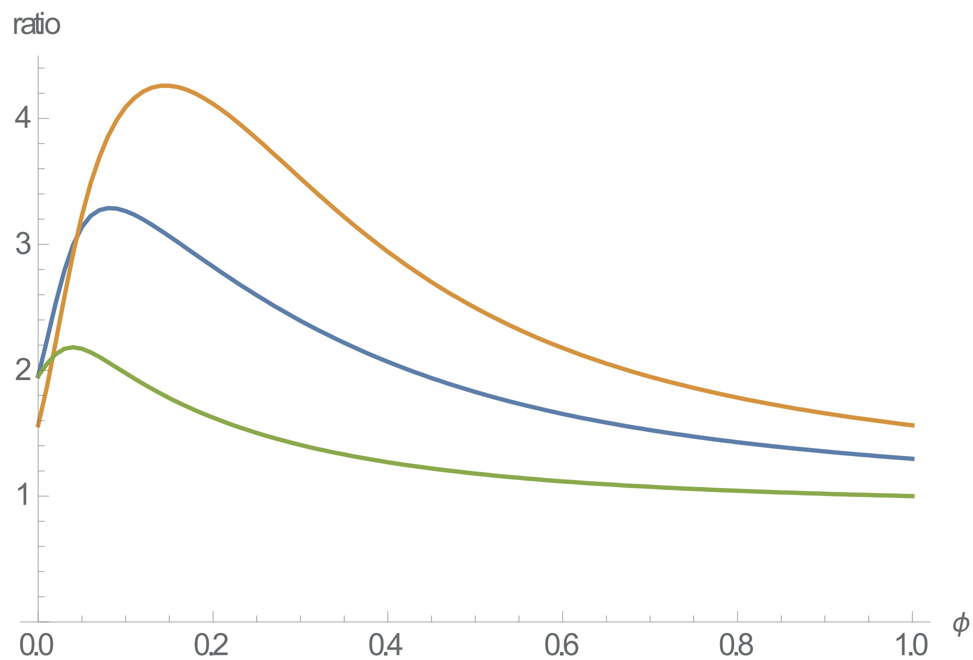

Figure 6: Interregional differentials when m_1/m_2= 0.8 (V_1/V_2 blue, l_1/l_2 red, P_2/P_1 yellow)