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Abstract

This paper develops an R&D decision-making model in the real options framework. The model is generic enough to capture three types of uncertainty in an R&D project, namely, uncertainty of research duration and costs, market value of technology, and a competitor's technology development. I derive analytical solutions, which help practitioners and researchers to evaluate various cases of R&D investment. Further, by analyzing the model with a wide range of parameter values, I reveal the following effects of the three types of uncertainty on R&D investment: Higher uncertainty of research duration and costs, unlike market value uncertainty, speeds up investment, especially combined with a higher risk of competition. The investment timing can be U-shaped in the strength of competition because of the trade-off between the preemptive investment effect and the decreased project value effect. These results can account for empirical findings about the uncertainty-investment relation in industries with high R&D intensity and severe competition.

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1 Introduction

Research and development (R&D) investment is a key determinant of a firm's long-term growth. It is critical for a firm's management to accurately evaluate and execute an R&D project. This paper contributes to both theory and practice by developing a tractable model for evaluating and optimizing R&D investment as well as unveiling the interactions of several key features associated with R&D investment.

The difficulties of R&D decision-making lie in three types of uncertainty, namely, uncertainty of technology, market value, and competitors (e.g., see Raz, Shenhar, and Dvir (2002)). In an R&D project, technological specifications, time schedule, and budget are planned before project initiation, but, in many cases, the outcome is not as successful as planned. This is called technological uncertainty. Market uncertainty means that the market value of a newly developed technology is not deterministic but affected by both technology-specific and macroeconomic shocks on the market. An R&D project also faces a risk of competition. For example, if a competitor develops an alternative technology first, the market value of technology in progress will greatly lower.

In reality, the R&D decision-making process involving the three risks is challenging for managers. Although the net present value (NPV) method has been widely employed for R&D decision-making, a more sophisticated method, which is called the real options method, has gradually spread in the real world (e.g., Hartmann and Hassan (2006) and Baker, Dutta, and Saadi (2011)). In fact, the literature argues advantages of the real options method over the NPV method in R&D decision-making with high uncertainty. Many case studies apply the real options method to R&D project valuation (e.g., Loch and Bode-Greuel (2001), Lee and Paxson (2001), Cassimon, Backer, Engelen, Wouwe, and Yordanov (2011), and Pennings and Sereno (2011)). To my knowledge, however, the existent real options models miss any of the three types of uncertainty explained above.

Instead of a specific case study, I develop a generic and tractable real options model for R&D decision-making with the three types of uncertainty so that researchers and practitioners can analyze various cases of R&D investment using the model. This paper considers the following model: A firm can initiate an R&D project by paying a sunk cost at an arbitrary time. There is a lag between project initiation and completion (henceforth, called research duration). The firm also continues to pay costs during research duration. When the firm completes the project, the firm receives the market value of the technology.

The model takes into consideration the three types of uncertainty as follows. Technological uncertainty can be modeled by a random variable standing for research duration. Note that total cost, which increases with longer research duration, is also stochastic. With regard to market uncertainty, following the standard literature on real options, I assume that the dynamics of the market value of the technology follows a geometric Brownian motion. Lastly, as in the works of Armada, Kryzanowski, and Pereira (2011) and Lavrutich, Huisman, and Kort (2016), the model assumes hidden competition, i.e., the firm faces the risk of a competitor's technology development, which follows the Poisson arrival process. If a competitor completes an alternative technology first, a fraction of the technology value is lost.

By analytically deriving the model solutions as well as conducting numerical analysis with a wide range of parameter values, I reveal several new effects of the three types of uncertainty on R&D investment decisions. First, I show that higher uncertainty of research duration improves the project value and accelerates R&D investment. This implies that the uncertainty-investment sensitivity depends on the type of uncertainty. It is well known that the market uncertainty-investment sensitivity is negative because higher market uncertainty increases the incentive for a firm to wait for additional information. On the other hand, technological uncertainty, which will not be dissolved by waiting, has positive effects on R&D investment because the R&D project value is convex with respect to research duration. A higher risk of a competitor's technology development intensifies the convexity. Because of the convexity, higher uncertainty of research duration increases the project value and speeds up R&D investment, especially combined with severe competition.[1]

These results can potentially explain several empirical findings. For instance, Grullon, Lyandres, and Zhdanov (2012) and Kraft, Schwartz, and Weiss (2015) showed that the sensitivity of uncertainty to firm value increases for a firm with higher R&D intensity. Driver, Temple, and Urga (2006) showed that industries with high R&D intensity and severe preemptive competition tend to have positive uncertainty-investment sensitivities.

Second, I show that the effects of hidden competition on investment are not monotonic in an R&D project with research duration. It is well known that, in the absence of research duration, a greater threat of competitors accelerates investment because it decreases the value of waiting. With research duration, however, severe competition decreases the expected project value at project initiation because a competitor can potentially develop technology before project completion. When the latter effect dominates the former effect, severe competition delays R&D investment. Indeed, I show that the investment timing has U-shaped relation with the arrival rate of a competitor's technology development. I also show that the investment timing can be non-monotonic with respect to the remaining value after a competitor's technology development.

Lastly, I explain key differences from the related literature to date. A seminal work by Weeds (2002) is the most similar to this paper. She examined a real options model with uncertain research duration and rival preemption. However, in her model, a firm has complete information about its competitor while research duration is exponentially distributed. Unlike Weeds (2002), I assume that not research duration, which can be estimated internally in the firm, but a competitor's technology development, which can

[1] Although Huchzermeier and Loch (2001) also showed that higher uncertainty of time schedule can accelerate R&D investment, their model does not consider the interactions between technological uncertainty and competition.

be an unexpected and exogenous event, follows an exponential distribution. Recently, Cassimon, Backer, Engelen, Wouwe, and Yordanov (2011) and Pennings and Sereno (2011) conducted case studies of R&D projects in the pharmaceutical industry taking account of both technological and market uncertainty, but their models, which are based on European options, do not entail any implications of the optimal R&D investment timing. Their papers do not consider a risk of competition either. Thus, this paper, more so than the previous works, helps R&D decision-making with the three types of uncertainty.

The remainder of this paper is organized as follows. After Section 2 introduces the model setup, Section 3 derives the model solutions analytically. In Section 4, with numerical examples, I analyze the model in full detail and provide empirical implications. Section 5 briefly summarizes the paper.

2 Model setup

Consider a firm that has an option to initiate an R&D project by paying sunk cost I_0, such as investment costs in new facilities and equipment, which is a positive constant.[2] The project will take T years until completion, where T is a constant in Sections 3.1 and 3.2. I will also consider random variable T in Section 3.3. Throughout the paper, T is called research duration.[3] For T years, the firm continuously pays cost I_1, such as personnel expenses and experimental costs, which is a positive constant. I define total cost by

$$I = I_0 + \int_0^T \mathrm{e}^{-rt} I_1 \mathrm{d}t = I_0 + \frac{(1 - e^{-rT})I_1}{r}, \tag{1}$$

where a positive constant r is the discount rate. Total cost (1) increases in T. When T follows a random variable, total cost (1) is also stochastic. By introducing random T in Section 3.3, the model can capture technological uncertainty of research duration and costs.

At project completion, the firm receives one-shot profit $X(t)$ as the market value of the technology,[4] where as in the standard real options literature (e.g., Dixit and Pindyck

[2]This paper focuses on a fixed-size investment project for developing a new technology. It could be interesting for future research to incorporate the project type and/or size in the model. For instance, Nishihara and Ohyama (2008), based on Weeds (2002), investigated the choice between two alternative technologies, while Nishihara (2012) studied dynamic management of multiple investment projects with synergies. Several papers including Huisman and Kort (2015), Shibata and Nishihara (2015a), and Lukas, Spengler, Kupfer, and Kieckhafer (2017) examined both investment timing and sizing decisions.

[3]Some papers distinguish the lag between project inception and completion (the gestation lag) and the lag between project completion and commercial application (the application lag) (e.g., Pakes and Schankerman (1984)). For simplicity, I assume that the total lag is equal to T.

[4]$X(t)$ may be interpreted as the expected discounted value of cash flows generated by the technology. I can easily extend the model solutions in the setup, allowing for a stream of cash flows after the completion time T rather than the one-shot profit. When cash flows follow a geometric Brownian motion, the solutions in the extended model are essentially the same as the solutions in this paper.

(1994)), $X(t)$ follows a geometric Brownian motion:

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}B(t)\ (t > 0), \quad X(0) = x, \tag{2}$$

where $B(t)$ denotes the standard Brownian motion defined in the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$ and $\mu, \sigma(> 0)$ and $x(> 0)$ are constants. For convergence, I assume that $r > \mu$.[5] By introducing $X(t)$, the model can capture market uncertainty, which dynamically changes due to specific and macroeconomic shocks on the demand market.

As in the works by Armada, Kryzanowski, and Pereira (2011) and Lavrutich, Huisman, and Kort (2016), the model considers hidden competition, which means that the firm cannot observe a potential competitor until a technology developed by a competitor appears in the market. The firm knows the ex-ante risk of competition that a hidden competitor completes an alternative technology at probability $\lambda \mathrm{d}t$ for infinitesimal time interval $\mathrm{d}t$, where arrival rate λ is a positive constant. In other words, a competitor's technology development follows an exponential distribution with arrival rate λ.[6] This paper assumes that the exponential distribution is independent of the stochastic process $X(t)$. If a competitor completes a technology first, the technology value $X(t)$ will decrease to $\alpha X(t)$, where constant $\alpha \in [0, 1)$ denotes the remaining fraction after a competitor's success. Note that for $\alpha > 0$, the firm has an option of investing in the R&D project even after a competitor's success. By introducing λ and α, the model can capture the risk of a competitor's technology development.[7]

In the last part of this section, I explain key differences from the related models in the previous literature. Weeds (2002) also focused on uncertainty of research duration and a competitor's development in the real options R&D model. The model adopts a game theoretic framework with full information and assumes that research duration follows an exponential distribution. In reality, however, a firm does not exactly know the R&D progress of its competitors (cf. incomplete information in Lambrecht and Perraudin (2003) and Nishihara and Fukushima (2008)) and may not recognize which firm is a potential competitor for the project (cf. hidden competition in Armada, Kryzanowski, and Pereira (2011) and Lavrutich, Huisman, and Kort (2016)). In addition, managers do not plan exponentially distributed research duration. Because an exponential distribution has the property of being memoryless, it is used for modelling an unexpected and exogenous event

[5] For the economic rationale behind these assumptions, refer to standard textbooks such as Dixit and Pindyck (1994) and Guthrie (2009).

[6] An alternative approach for modeling competition is a game theoretic real option model (e.g., Huisman (2001), Weeds (2002), Pawlina and Kort (2006), Nishihara and Shibata (2010), Shibata (2016)). However, this approach typically imposes stronger assumptions about information of competitors. As was discussed by Armada, Kryzanowski, and Pereira (2011), a hidden competition model better fits R&D decision-making in the absence of information about competitors.

[7] A competitor's technology development may reduce research duration T and costs I_1 and I_2. This is because the firm can potentially develop a technology by utilizing the competitor's technology. I can easily derive the model solutions in the extended model, although I omit demonstrating the details.

such as a natural disaster. Accordingly, unlike Weeds (2002), I assume that not research duration, which can be estimated internally in the firm, but a competitor's technology development, which can be an unexpected and exogenous event, follows an exponential distribution.

Cassimon, Backer, Engelen, Wouwe, and Yordanov (2011) and Pennings and Sereno (2011) have conducted case studies of pharmaceutical R&D projects. Their models include both market and technological uncertainty. However, their models, which are based on European compound options, cannot provide any implications about the optimal R&D investment timing. In addition, their models do not consider uncertainty of research duration and competition.

3 Model solutions

3.1 Project value and investment timing after a competitor's success

In Sections 3.1 and 3.2, I assume that T is a constant. I consider the problem backward. Suppose that a competitor has already developed a technology in this subsection. The expected project value at the project initiation time τ is calculated as follows:

$$\mathbb{E}[\mathrm{e}^{-rT}\alpha X(\tau+T) \mid X(\tau)] = A_c X(\tau), \tag{3}$$

where

$$A_c = \alpha \mathrm{e}^{-(r-\mu)T}. \tag{4}$$

The subscript c denotes the value after a competitor's success. By (3), the project value function is expressed as

$$\begin{aligned} V_c(x) &= \sup_{\tau} \mathbb{E}[\mathrm{e}^{-r\tau}(\mathrm{e}^{-rT}\alpha X(\tau+T) - I)] \\ &= \sup_{\tau} \mathbb{E}[\mathrm{e}^{-r\tau}(A_c X(\tau) - I)] \end{aligned} \tag{5}$$

where the investment time τ is optimized over all stopping times.

As is well known (e.g., see Dixit and Pindyck (1994)), in the continuation region, $V_c(x)$ satisfies the ordinary differential equation:

$$\mu x \frac{\mathrm{d}V_c(x)}{\mathrm{d}x} + \frac{\sigma^2 x^2}{2}\frac{\mathrm{d}^2 V_c(x)}{\mathrm{d}x^2} = rV_c(x), \tag{6}$$

where $\mu x \mathrm{d}/\mathrm{d}x + \sigma^2 x^2/2\mathrm{d}^2/\mathrm{d}x^2$ corresponds to the generator of the geometric Brownian motion (2). A general solution to (6) is expressed as $V_c(x) = B_1 x^{\beta} + B_2 x^{\gamma}$, where B_1 and

B_2 are constants, and

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} (> 1), \tag{7}$$

$$\gamma = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} (< 0). \tag{8}$$

By the trivial boundary condition, i.e., $V_c(0) = 0$, I have $B_2 = 0$. For the investment threshold x_c^*, I have the boundary conditions:

$$V_c(x_c^*) = B_1 x_c^{*\beta} = A_c x_c^* - I \tag{9}$$

$$\frac{\mathrm{d}V_c(x_c^*)}{\mathrm{d}x} = B_1 \beta x_c^{*\beta - 1} = A_c, \tag{10}$$

which are called the value matching and smooth pasting conditions, respectively (e.g., see Dixit and Pindyck (1994)). By solving (9) and (10), I can derive B_1 and x_c^* as follows.

Proposition 1 *Suppose that a competitor has already developed a technology. The project value function $V_c(x)$ is given by*

$$V_c(x) = \begin{cases} \left(\dfrac{x}{x_c^*}\right)^\beta (A_c x_c^* - I) & (x < x_c^*) \\ A_c x - I & (x \geq x_c^*), \end{cases} \tag{11}$$

where the investment trigger x_c^* is defined by

$$x_c^* = \frac{\beta I}{(\beta - 1) A_c} \tag{12}$$

The optimal investment time τ_c^* is given by

$$\tau_c^* = \{t \geq 0 \mid X(t) \geq x_c^*\}. \tag{13}$$

In Proposition 1, the upper equation in (11) stands for the value of the option to invest in the R&D project after a competitor develops a technology. Because the option value is higher than the investment value, the firm waits until the technology value $X(t)$ hits the investment trigger x_c^*. Once $X(t)$ hits x_c^*, the firm initiates the R&D project, and the expected project value becomes $A_c x_c^*$. By $\partial A_c/\partial\alpha > 0, \partial A_c/\partial T < 0, \partial\beta/\partial\sigma < 0$, (11), and (12), I have $\partial V_c(x)/\partial\alpha > 0, \partial V_c(x)/\partial T < 0, \partial V_c(x)/\partial\sigma \geq 0, \partial x_c^*/\partial\alpha < 0, \partial x_c^*/\partial T > 0$, and $\partial x_c^*/\partial\sigma > 0$. In other words, a higher remaining value after a competitor's success and shorter research duration increase the project value and accelerate R&D investment, whereas higher volatility increases the option value and delays investment.

3.2 Project value and investment timing before a competitor's success

In this subsection, I consider the problem before a competitor's success. Suppose that a competitor has not yet developed a technology at the project initiation time τ. Because the technology value falls to $\alpha X(\tau+T)$ for $\tau+T>T_c$, where T_c denotes the time of a competitor's technology development, I can calculate the expected project value at τ as follows:

$$\begin{aligned}
&\mathbb{E}[\mathrm{e}^{-rT}(1_{\{\tau+T<T_c\}}+1_{\{\tau+T\geq T_c\}}\alpha)X(\tau+T)\mid X(\tau),\tau<T_c]\\
=&\mathbb{E}[\mathrm{e}^{-rT}\alpha X(\tau+T)\mid X(\tau)]+\mathbb{E}[\mathrm{e}^{-rT}(1_{\{\tau+T<T_c\}}(1-\alpha)X(\tau+T)\mid X(\tau),\tau<T_c]\\
=&\alpha\mathrm{e}^{-(r-\mu)T}X(\tau)+(1-\alpha)\mathrm{e}^{-(r-\mu)T}X(\tau)\mathbb{E}[1_{\{\tau+T<T_c\}}\mid\tau<T_c] \qquad (14)\\
=&AX(\tau), \qquad (15)
\end{aligned}$$

where in (14) I used the independence between T_c and $X(t)$, and in (15) I have

$$A=\alpha\mathrm{e}^{-(r-\mu)T}+(1-\alpha)\mathrm{e}^{-(r+\lambda-\mu)T} \qquad (16)$$

using the distribution function of the exponential distribution.

By (15), the project value function is expressed as

$$\begin{aligned}
V(x)&=\sup_{\tau}\mathbb{E}[\mathrm{e}^{-r\tau}1_{\{\tau<T_c\}}\{\mathrm{e}^{-rT}(1_{\{\tau+T<T_c\}}+1_{\{\tau+T\geq T_c\}}\alpha)X(\tau+T)-I\}+\mathrm{e}^{-T_c}1_{\{\tau\geq T_c\}}V_c(X(T_c))]\\
&=\sup_{\tau}\mathbb{E}[\mathrm{e}^{-r\tau}1_{\{\tau<T_c\}}\{AX(\tau)-I\}+\mathrm{e}^{-T_c}1_{\{\tau\geq T_c\}}V_c(X(T_c))] \qquad (17)
\end{aligned}$$

where the investment time τ is optimized over all stopping times, and T_c stands for the time of a competitor's technology development. Note that in (17), the project value changes to $V_c(X(T_c))$, which is defined by (11), at time T_c, if $T_c\leq\tau$ holds.

As is well known, in the continuation region, $V(x)$ satisfies the ordinary differential equation:

$$\mu x\frac{\mathrm{d}V(x)}{\mathrm{d}x}+\frac{\sigma^2x^2}{2}\frac{\mathrm{d}^2V(x)}{\mathrm{d}^2x}+\lambda(V_c(x)-V(x))=rV(x), \qquad (18)$$

where $\lambda(V_c(x)-V(x))$ in (18) corresponds to the fact that $V(x)$ changes to $V_c(x)$ at probability $\lambda\mathrm{d}t$ in the infinitesimal time interval $\mathrm{d}t$ in the continuation region.

Now, suppose that the investment threshold x^* before a competitor's success is lower than the investment threshold x_c^* after a competitor's success. This will be verified in Appendix A. Because of $x^*<x_c^*$, a general solution to (18) is expressed as $V(x)=V_c(x)+\tilde{B}_1x^{\tilde{\beta}}+\tilde{B}_2x^{\tilde{\gamma}}$, where $\tilde{B}_1$ and $\tilde{B}_2$ are constants, and $\tilde{\beta}$ and $\tilde{\gamma}$ are defined by

$$\tilde{\beta}=\frac{1}{2}-\frac{\mu}{\sigma^2}+\sqrt{\left(\frac{\mu}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2(r+\lambda)}{\sigma^2}}(>1), \qquad (19)$$

$$\tilde{\gamma}=\frac{1}{2}-\frac{\mu}{\sigma^2}-\sqrt{\left(\frac{\mu}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2(r+\lambda)}{\sigma^2}}(<0). \qquad (20)$$

Note that $V_c(x)$ satisfies (6) for $x < x^*(< x_c^*)$. As in Section 3.1, by the trivial boundary condition, i.e., $V(0) = 0$, I have $\tilde{B}_2 = 0$. For the investment trigger x^*, the value matching and smooth pasting conditions become

$$V(x^*) = \left(\frac{x^*}{x_c^*}\right)^{\beta} (A_c x_c^* - I) + \tilde{B}_1 x^{*\beta} = Ax^* - I, \tag{21}$$

$$\frac{\mathrm{d}V(x^*)}{\mathrm{d}x} = \left(\frac{x^*}{x_c^*}\right)^{\beta-1} \frac{\beta}{x_c^*}(A_c x_c^* - I) + \tilde{B}_1 \beta x^{*\beta-1} = A, \tag{22}$$

respectively. By solving (21) and (22), I can derive $\tilde{B}_1$ and x^* as follows.

Proposition 2 *Suppose that a competitor has not developed a technology. The project value function $V(x)$ is given by*

$$V(x) = \begin{cases} V_c(x) + \left(\dfrac{x}{x^*}\right)^{\tilde{\beta}} (Ax^* - I - V_c(x^*)) & (x < x^*) \\ Ax - I & (x \geq x^*), \end{cases} \tag{23}$$

where $V_c(x)$ is the upper equation in (11), and the investment trigger $x^* \in (0, x_c^*)$ is defined by the solution to

$$x^* = \frac{\tilde{\beta} I}{(\tilde{\beta} - 1)A} + \left(\frac{x^*}{x_c^*}\right)^{\beta} \frac{(\tilde{\beta} - \beta)(A_c x_c^* - I)}{(\tilde{\beta} - 1)A}. \tag{24}$$

The optimal investment time τ^* is given by

$$\tau^* = \{t \geq 0 \mid X(t) \geq x^*\}. \tag{25}$$

In the upper equation in (23), the project value $V(x)$ is decomposed into the project value after a competitor's success, i.e., $V_c(x)$, and the value of the option of investing in the project before a competitor's success, i.e., $(x/x^*)^{\tilde{\beta}}(Ax^* - I - V_c(x^*))$. Proposition 2 implies the R&D investment policy as follows. The firm waits for R&D investment until the technology value $X(t)$ hits the investment trigger x^*. If a competitor develops a technology first, the firm increases the investment trigger from x^* to x_c^*. When $X(t)$ hits x^* before a competitor's technology development, the firm initiates the R&D project, and the expected project value becomes Ax^*, where A includes a discount due to a risk of a competitor's success during research duration T.

By $\partial V_c(x)/\partial \alpha > 0, \partial V_c(x)/\partial T < 0, \partial V_c(x)/\partial \sigma \geq 0, \partial A/\partial \alpha > 0, \partial A/\partial \lambda < 0, \partial A/\partial T < 0, \partial \tilde{\beta}/\partial \lambda > 0, \partial \tilde{\beta}/\partial \sigma < 0$, and (23), I have $\partial V(x)/\partial \alpha > 0, \partial V(x)/\partial \lambda < 0, \partial V(x)/\partial T < 0$, and $\partial V(x)/\partial \sigma \geq 0$. In other words, a lower risk of competition, shorter research duration, and higher volatility increase the R&D project value. On the other hand, the impacts of α, λ, T, and σ on the investment trigger x^* are not clear from (24). I will reveal the impacts numerically in Section 4.

3.3 Extended model

Although so far I have treated research duration T as a constant, it is easy to extend the results in Sections 3.1 and 3.2 to a case with random variable T. In this subsection, I assume that T follows a nonnegative random variable. For tractability, I assume that T is independent of technology value $X(t)$ and a competitor's technology development time T_c. This assumption means that risks of technology, market value, and competition are not directly related to each other.

By taking expectations of (1), (4), and (16), I define

$$\bar{I} = I_0 + \frac{(1 - \mathbb{E}[e^{-rT}])I_1}{r}, \tag{26}$$

$$\bar{A}_c = \alpha \mathbb{E}[\mathrm{e}^{-(r-\mu)T}], \tag{27}$$

$$\bar{A} = \alpha \mathbb{E}[\mathrm{e}^{-(r-\mu)T}] + (1-\alpha)\mathbb{E}[\mathrm{e}^{-(r+\lambda-\mu)T}]. \tag{28}$$

By replacing I, A_c, and A with $\bar{I}, \bar{A}_c$, and $\bar{A}$, respectively, and tracing the discussions in Sections 3.1 and 3.2, I can easily derive the model solutions. Indeed, the project value function and investment trigger after a competitor's success are equal to (11) and (12), replacing I and A_c with $\bar{I}$ and $\bar{A}_c$, respectively. The project value function and investment trigger before a competitor's success are equal to (23) and (24), replacing I, A_c, and A with $\bar{I}, \bar{A}_c$, and $\bar{A}$, respectively. Clearly, the comparative statics results are unchanged from those explained after Propositions 1 and 2.

The model can capture uncertainty of technology, market value, and competitors in the R&D decision-making process, while previous models miss any of the three features. Nevertheless, I can analytically derive the R&D project value and investment timing, which help in a real-world decision-making process of R&D investment. It is one of the contributions of this paper to develop such a general and tractable model and derive the analytical solutions. In the next section, I will make another contribution to the literature by analyzing the model and revealing the interactions of the three risks in numerical examples.

4 Numerical analysis and implications

4.1 Baseline analysis

This paper does not focus on a case study of a specific R&D project. Instead, I show numerical results for a wide range of parameter values, demonstrating several properties of the project value and the investment policy in the model. I set the base parameter values in Table 1. There are several methods for estimating the market parameter values, i.e., r, μ, and σ, in a real options model (e.g., using the capital asset pricing model). For instance, Chapter 3 of Guthrie (2009) explains the details of standard calibration methods. I set r, μ, and σ in Table 1 following the standard literature.

On the other hand, the technological parameter values, i.e., research duration T and costs I_1 and I_2, as well as the competition risk parameter values, i.e., arrival rate λ and remaining fraction α, can be internally estimated by a firm. These values greatly differ depending on the project types and the industries. For example, in a project of developing a new drug, research duration is long (e.g., Kellogg and Charnes (2000), Loch and Bode-Greuel (2001), and Hartmann and Hassan (2006)). However, taking account of the fact that the average research duration is around 2 to 4 years in most of the literature (e.g., Pakes and Schankerman (1984)), I assume that T takes a value in $[2, 4]$ following a uniform distribution. The parameter values $\lambda = 0.2$ and $\alpha = 0.5$ mean that a competitor is expected to develop a technology $1/\lambda = 5$ years later and that the technology value decreases by half if a competitor develops prior to the firm.

Table 2 presents the investment triggers and project values, while Figure 1 shows the value functions $V(X(t))$ and $V_c(X(t))$. In the baseline case, the firm initiates the R&D project when the technology value $X(t)$ hits the investment trigger $x^* = 11.593$ before a competitor's technology development. The R&D project takes random duration $T \in [2, 4]$ years until completion. This delay decreases the project value by the multiplier $\mathbb{E}[\mathrm{e}^{-(r-\mu)T}] = 0.942$. A competitor may potentially develop a technology during research duration T. This risk reduces the project value further by the multiplier $\bar{A}/\mathbb{E}[\mathrm{e}^{-(r-\mu)T}] = 0.777$. As a result, the NPV at the investment time becomes $\bar{A}x^* - \bar{I} = 4.823 = 1.319\bar{I}$. In other words, considering the option value of waiting, the firm waits until the expected profit exceeds 1.319 times higher than the expected investment cost. At completion time $\tau^* + T$, the firm receives the technology value $X(\tau^* + T)$ if a competitor has not yet developed a technology. The firm receives the discounted technology value $0.5X(\tau^* + T)$ if a competitor develops a technology during research duration.

If a competitor develops a technology before $X(t)$ hits $x^* = 11.593$, the firm changes the R&D investment policy to the following. The firm invests in the R&D project once $X(t)$ hits $x_c^* = 40.658 (> x^* = 11.593)$. There is no longer risk of competition. The NPV at the investment time becomes $\bar{A}_c x_c^* - \bar{I} = 15.49 = 4.236\bar{I}$, which means that, considering the option value of waiting, the firm waits until the expected profit exceeds 4.236 times higher than the expected investment cost. Note that 4.2360 is much higher than the corresponding value 1.3188 before a competitor's success. This is because due to $\lambda = 0.2$, $\tilde{\beta} = 2.873$ is much higher than $\beta = 1.236$ (cf. (7) and (19)). It can be interpreted that a risk of competition greatly decreases the value of waiting. At completion time $\tau_c^* + T$, the firm receives the discounted technology value $0.5X(\tau_c^* + T)$.

In the following subsections, I will examine the comparative statics with respect to uncertainty of market value, research duration, and a competitor's technology development. I reveal how the interactions of the three risks affect the R&D project value and investment policy.

4.2 The impacts of market value uncertainty

This subsection presents the comparative statics with respect to market value volatility σ. Figure 2 depicts investment triggers x^* and x_c^* as well as project values $V(x)$ and $V_c(x)$ for varying levels of σ.[8] The other parameter values are set in Table 1.

I can see from Figure 2 that higher σ increases $x^*, x_c^*, V(x)$, and $V_c(x)$. The reason is that β and $\tilde{\beta}$ decrease from 1.3007 and 3.7870 to 1.1327 and 2, respectively, for $\sigma = 0.1$ to 0.4, which means that the value of waiting increases for a higher σ. These results are straightforward and consistent with the standard theory that higher market uncertainty increases the option value of waiting and delays the exercise of the option (e.g., Dixit and Pindyck (1994)). For a wide range of parameter values, I also investigated the impacts of h, λ, and α on the volatility effects. However, I could not find any significant interactions between uncertainty of market value and uncertainty of research duration or competition.

4.3 The impacts of uncertainty of research duration

This subsection presents the comparative statics with respect to uncertainty of T. I assume that T follows a uniform distribution between $[3-h, 3+h]$ with $\mathbb{E}[T] = 3$ years. Figure 3 depicts investment triggers x^* and x_c^* as well as project values $V(x)$ and $V_c(x)$ for varying levels of h. The other parameter values are set in Table 1.

I can see from Figure 3 that higher h decreases x^* and x_c^* and increases $V(x)$ and $V_c(x)$. The reason is explained as follows. Higher h increases T with respect to the convex order (e.g., see Chapter 3.A in Shaked and Shanthikumar (2007)). Then, higher h decreases $\bar{I} = I_0 + (1 - \mathbb{E}[e^{-rT}])I_1/r$ and increases $\bar{A}_c = \alpha\mathbb{E}[\mathrm{e}^{-(r-\mu)T}]$ and $\bar{A} = \alpha\mathbb{E}[\mathrm{e}^{-(r-\mu)T}] + (1-\alpha)\mathbb{E}[\mathrm{e}^{-(r+\lambda-\mu)T}]$ because $e^{-rT}, \mathrm{e}^{-(r-\mu)T}$, and $\mathrm{e}^{-(r+\lambda-\mu)T}$ are convex functions of T. Thus, interestingly, I can argue that higher uncertainty of research duration plays a positive role in decreasing the expected investment cost and increasing the expected project values, and hence, it accelerates R&D investment.

Next, I explain how a risk of competition influences the effects of uncertainty of T. Figure 4 compares the results for $\lambda = 0.2, 0.4, 0.6$, and 0.8. I omit depicting x_c^* and $V_c(x)$ because they are independent of λ. I can see from the figure that higher λ intensifies the impacts of h on x^*. The reason is that higher λ intensifies the convexity of $\bar{A} = \alpha\mathbb{E}[\mathrm{e}^{-(r-\mu)T}] + (1-\alpha)\mathbb{E}[\mathrm{e}^{-(r+\lambda-\mu)T}]$ with respect to T. Similarly, lower α intensifies the impacts of h, although I omit depicting a figure. These results suggest that a higher risk of competition magnifies the positive impacts of higher uncertainty of research duration on R&D investment.

[8]For simplicity, I change σ, taking all other parameters, r and μ as constants. This means that changes in σ have only an idiosyncratic risk component. Most of the literature, including Dixit and Pindyck (1994), presents the comparative statics under this assumption, although a few papers, including Wong (2007), examine the comparative statics assuming a relation between μ and σ.

Although uncertainty of research duration and costs is recognized as a key characteristic of an R&D project (e.g., Raz, Shenhar, and Dvir (2002)), few studies have revealed its impacts on R&D investment timing and project value.[9] In this paper, I demonstrate that higher uncertainty of research duration increases project value and accelerates investment and that severe competition intensifies the positive effects. These results contrast with the impacts of market uncertainty (cf. Section 4.2). Technological uncertainty, unlike market uncertainty, will never be dissolved by waiting, and hence, the firm has no incentive to delay investment and obtain extra information. This difference leads to the opposite effects of uncertainty.

Although my results about the positive effects of technological uncertainty are novel, many empirical observations support the results. For example, Driver, Temple, and Urga (2006) showed positive sensitivities of uncertainty to investment in industries with high R&D intensity. They also documented that positive sensitivities are observed in industries with strong first-mover advantages, i.e., high risks of competition. My results are also consistent with empirical findings by Grullon, Lyandres, and Zhdanov (2012) and Kraft, Schwartz, and Weiss (2015). Indeed, they showed that the sensitivities of uncertainty to firm values are higher for firms with higher R&D intensity.

4.4 The impacts of competitor risk

This subsection presents the comparative statics with respect to the arrival rate λ of a competitor's technology development and the remaining value α after a competitor's success. Figure 5 depicts investment triggers x^* and x_c^* as well as project values $V(x)$ and $V_c(x)$ for varying levels of λ. The other parameter values are set in Table 1. Note that x_c^* and $V_c(x)$ do not depend on λ. As explained after Proposition 2, I can see from the figure that $V(x)$ decreases in λ.

Notably, in the figure, I find that x^* is U-shaped in λ. This result can be explained by the tradeoff between the two conflicting effects. Because of $\partial \bar{A}/\partial\lambda < 0$, higher λ lowers the expected project value at project initiation. This effect delays R&D investment. On the other hand, because of $\partial\tilde{\beta}/\partial\lambda > 0$, higher λ decreases the value of waiting. This effect accelerates R&D investment. For $\lambda < 0.135$, the latter effect dominates the former, and hence, x^* decreases in λ. For $\lambda > 0.135$, the former effect dominates the latter, and hence, x^* increases in λ. In my computations for a wide range of parameter values, I always found the U-shaped relation for positive research duration T, although the level of λ for the two effects offsets changes from 0.135 with changing parameter values. For instance, shorter T decreases the former effect and hence increases the offsetting threshold λ.

[9] A notable exception is Huchzermeier and Loch (2001). They studied the effects of uncertainty of project delay in a discrete time model, but they did not reveal any interactions between uncertainty of research duration and competition.

Next, I explore the comparative statics with respect to α. Figure 6 depicts investment triggers x^* and x_c^* as well as project values $V(x)$ and $V_c(x)$ for varying levels of λ. As explained after Propositions 1 and 2, I can see from the figure that $V(x)$ and $V_c(x)$ increases in α, while x_c^* decreases in α.

The impacts of α on x^* are interesting in Figure 6. As in the effects of λ, two conflicting effects exist. Because of $\partial \bar{A}/\partial\alpha > 0$, higher α increases the expected project value at project initiation. This effect accelerates R&D investment. On the other hand, because of $\partial V_c(x)/\partial\alpha > 0$, higher α increases the project value after a competitor's success and then increases the value of waiting. This effect delays R&D investment. For the baseline parameter values, the latter effect dominates the former, and hence, x^* increases in α. However, the tradeoff changes with parameter values. For instance, Figure 7 shows the results for $\lambda = 0.2, 0.4, 0.6$, and 0.8. For $\lambda = 0.4$ and 0.6, x^* is U-shaped in α, whereas for $\lambda = 0.8$, x^* decreases in α. This is because the former effect, which is stronger for higher λ, can dominate the latter effect.

In summary, I find that R&D investment timing can be U-shaped in the strength of competition. This result is strongly contrasted with previous findings in the literature. Indeed, the standard literature has argued that a higher risk of competition decreases the value of waiting and accelerates investment. Because they do not consider any interactions between competition and research duration, the monotonic results hold. Although $\partial\bar{A}/\partial\lambda = \partial\bar{A}/\partial\alpha = 0$ holds in the absence of research duration, $\partial\bar{A}/\partial\lambda < 0$ and $\bar{A}/\partial\alpha > 0$ hold in the presence of research duration. This paper complements the previous literature by showing that a risk of competition can adversely affect R&D investment through the interactions with research duration.

5 Conclusion

In this paper, I developed a generic R&D decision-making model involving three types of uncertainty in an R&D project, namely, uncertainty of research duration, market value of technology, and technology development by a competitor. By deriving the analytical solutions in the model, this paper helps practitioners and researchers to evaluate and optimize various cases of R&D investment. In addition, by analyzing the model solutions for a wide range of parameter values, I obtained novel implications about the effects of the three risks on R&D decision making.

I showed that higher uncertainty of research duration increases the project value and speeds up R&D investment through the convexity of the project value with respect to research duration. I also showed that severe competition increases the convexity and then intensifies the positive effects of uncertainty of research duration on R&D investment. These results are novel and contrasted with the effects of market uncertainty, and they can account for several empirical findings about positive effects of uncertainty in industries with high R&D intensities and severe competition. Furthermore, I showed that R&D investment timing has a U-shaped relation with the strength of competition. Compared to the monotonic results in the previous literature, I showed that the non-monotonic results can occur through the interactions between competition and research duration.

A limitation of the model is the assumption that the three risks are independent. Without this assumption, the model does not allow analytical solutions. In practice, however, competitor risk may increase as the market value of technology increases. In such a case, one may find that the effects of market uncertainty, combined with a risk of competition, change from the normal effects. Further, the model does not consider how to finance the R&D project. In recent years, many papers, including Nishihara and Shibata (2013), Shibata and Nishihara (2015b), and Sundaresan, Wang, and Yang (2015), have investigated the effects of financial frictions on investment. R&D investment timing and project value could depend on how effectively a firm finances its project. These issues could be interesting topics for future research.

References

Armada, M., L. Kryzanowski, and P. Pereira, 2011, Optimal investment decisions for two positioned firms competing in a duopoly market with hidden competitors, *European Financial Management* 17, 305–330.

Baker, H., S. Dutta, and S. Saadi, 2011, Management views on real options in capital budgeting, *Journal of Applied Finance* 21, 18–29.

Cassimon, D., M. Backer, P. Engelen, M. Wouwe, and V. Yordanov, 2011, Incorporating technical risk in compound real option models to value a pharmaceutical R&D licensing opportunity, *Research Policy* 40, 1200–1216.

Dixit, A., and R. Pindyck, 1994, *Investment Under Uncertainty* (Princeton University Press: Princeton).

Driver, C., P. Temple, and G. Urga, 2006, Real options – delay vs. pre-emption: Do industrial characteristics matter?, *International Journal of Industrial Organization* 26, 532–545.

Grullon, G., E. Lyandres, and A. Zhdanov, 2012, Real options, volatility, and stock returns, *Journal of Finance* 67, 1499–1537.

Guthrie, G., 2009, *Real Options in Theory and Practice* (Oxford University Press: Oxford).

Hartmann, M., and A. Hassan, 2006, Application of real options analysis for pharmaceutical R&D project valuation – empirical results from a survey, *Research Policy* 35, 343–354.

Huchzermeier, A., and C. Loch, 2001, Project management under risk: Using the real options approach to evaluate flexibility in R&D, *Management Science* 47, 85–101.

Huisman, K., 2001, *Technology Investment: A Game Theoretic Real Options Approach* (Kluwer Academic Publishers: Boston).

Huisman, K., 2001, and P. Kort, 2015, Strategic capacity investment under uncertainty, *RAND Journal of Economics* 46, 376–408.

Kellogg, D., and J. Charnes, 2000, Real-options valuation for a biotechnology company, *Financial Analyst Journal* 56, 76–84.

Kraft, H., E. Schwartz, and F. Weiss, 2015, Growth options and firm valuation, NBER Working Paper 18836.

Lambrecht, B., and W. Perraudin, 2003, Real options and preemption under incomplete information, *Journal of Economic Dynamics and Control* 27, 619–643.

Lavrutich, M., K. Huisman, and P. Kort, 2016, Entry deterrence and hidden competition, *Journal of Economic Dynamics and Control* 69, 409–435.

Lee, J., and D. Paxson, 2001, Valuation of R&D real American sequential exchange options, *R&D Management* 31, 191–201.

Loch, C., and K. Bode-Greuel, 2001, Evaluating growth options as sources of value for pharmaceutical research projects, *R&D Management* 31, 231–248.

Lukas, E., T. Spengler, S. Kupfer, and K. Kieckhafer, 2017, When and how much to invest? Investment and capacity choice under product life cycle uncertainty, *European Journal of Operational Research* 260, 1105–1114.

Nishihara, M., 2012, Real options with synergies: Static versus dynamic policies, *Journal of the Operational Research Society* 63, 107–121.

Nishihara, M., and M. Fukushima, 2008, Evaluation of firm's loss due to incomplete information in real investment decision, *European Journal of Operational Research* 188, 569–585.

Nishihara, M., and A. Ohyama, 2008, R&D competition in alternative technologies: A real options approach, *Journal of the Operations Research Society of Japan* 51, 55–80.

Nishihara, M., and T. Shibata, 2010, Interactions between preemptive competition and a financing constraint, *Journal of Economics and Management Strategy* 19, 1013–1042.

Nishihara, M., and T. Shibata, 2013, The effects of external financing costs on investment timing and sizing decisions, *Journal of Banking and Finance* 37, 1160–1175.

Pakes, A., and M. Schankerman, 1984, The rate of obsolescence of patents, research gestation lags, and the private rate of return to research resources, in Z. Griliches, ed.: *R&D, Patents, and Productivity* . pp. 73–88 (University of Chicago Press: Chicago).

Pawlina, G., and P. Kort, 2006, Real options in an asymmetric duopoly: Who benefits from your competitive disadvantage?, *Journal of Economics and Management Strategy* 15, 1–35.

Pennings, E., and L. Sereno, 2011, Evaluating pharmaceutical R&D under technical and economic uncertainty, *European Journal of Operational Research* 212, 374–385.

Raz, T., A. Shenhar, and D. Dvir, 2002, Risk management, project success, and technological uncertainty, *R&D Management* 32, 101–109.

Shaked, M., and J. Shanthikumar, 2007, *Stochastic Orders* (Springer: New York).

Shibata, T., 2016, Strategic entry in a triopoly market of firms with asymmetric cost structures, *European Journal of Operational Research* 249, 728–739.

Shibata, T., and M. Nishihara, 2015a, Investment-based financing constraints and debt renegotiation, *Journal of Banking and Finance* 51, 79–92.

Shibata, T., and M. Nishihara, 2015b, Investment timing, debt structure, and financing constraints, *European Journal of Operational Research* 241, 513–526.

Sundaresan, S., N. Wang, and J. Yang, 2015, Dynamic investment, capital structure, and debt overhang, *Review of Corporate Finance Studies* 4, 1–42.

Weeds, H., 2002, Strategic delay in a real options model of R&D competition, *Review of Economic Studies* 69, 729–747.

Wong, K., 2007, The effect of uncertainty on investment timing in a real options model, *Journal of Economic Dynamics and Control 31* 31, 2152–2167.

A Proof of the existence of $x^* \in (0, x_c^*)$

Define the function $F(x)$ by

$$F(x) = \frac{\tilde{\beta} I}{(\tilde{\beta}-1)A} + \left(\frac{x}{x_c^*}\right)^{\beta} \frac{(\tilde{\beta}-\beta)(A_c x_c^* - I)}{(\tilde{\beta}-1)A} - x.$$

For $x = 0$, I have

$$F(0) = \frac{\tilde{\beta} I}{(\tilde{\beta}-1)A} > 0. \tag{29}$$

For $x = x_c^*$, I have

$$\begin{aligned}
F(x_c^*) &= \frac{\tilde{\beta} I}{(\tilde{\beta}-1)A} + \frac{(\tilde{\beta}-\beta)(A_c x_c^* - I)}{(\tilde{\beta}-1)A} - x_c^* \\
&= \frac{\beta I + (\tilde{\beta}-\beta)A_c x_c^* - (\tilde{\beta}-1)A x_c^*}{(\tilde{\beta}-1)A} \\
&< \frac{\beta I + (\tilde{\beta}-\beta)A_c x_c^* - (\tilde{\beta}-1)A_c x_c^*}{(\tilde{\beta}-1)A} \qquad (30) \\
&= \frac{\beta I - (\beta-1)A_c x_c^*}{(\tilde{\beta}-1)A} \\
&= 0, \qquad (31)
\end{aligned}$$

where in (30) I used $\tilde{\beta} > 1$ and $A_c < A$, and in (31) I used (12). By $\beta > 1$, $F(x)$ is a convex function. Then, by (29), (31), and the convexity of $F(x)$, the equation $F(x) = 0$ has a unique solution x^* in the interval $(0, x_c^*)$.

Table 1: Baseline parameter values.

r	μ	σ	T	λ	α	I_1	I_2	x
0.08	0.06	0.2	$[2, 4]$	0.2	0.5	1	1	8

Table 2: Investment triggers and project values.

x^*	x_c^*	$\bar{I}$	$\bar{A}$	$\bar{A}_c$	$V(x)$	$V_c(x)$
11.5927	40.6582	3.6566	0.7314	0.4709	2.6063	2.0764

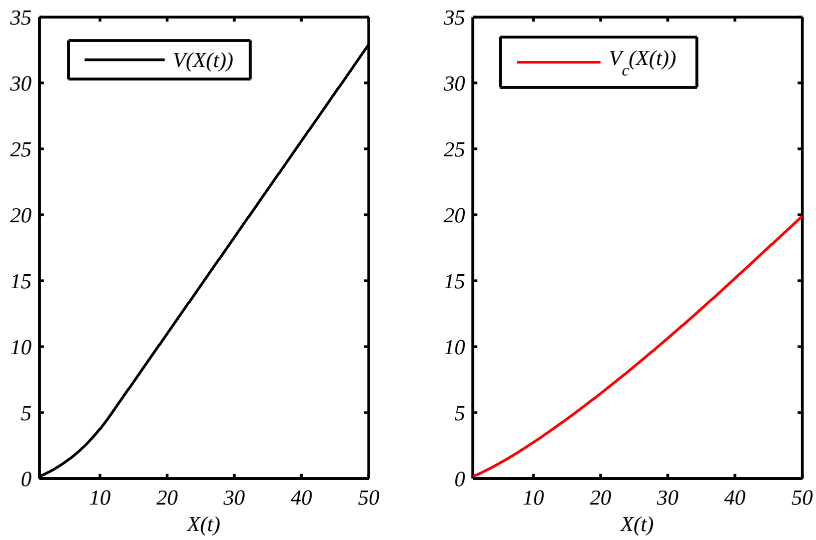

Figure 1: Value functions. The figure plots $V(X(t))$ and $V_c(X(t))$ as functions of $X(t)$. The parameter values are set in Table 1.

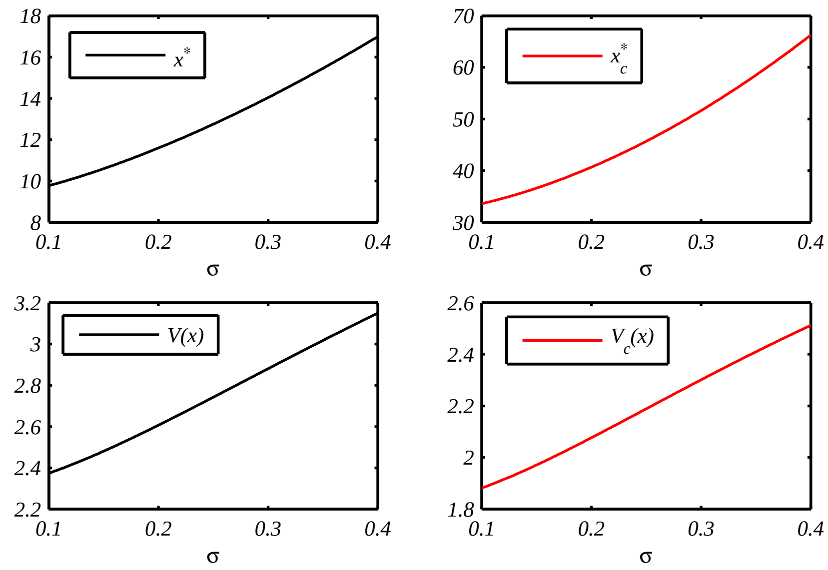

Figure 2: The impacts of market value uncertainty. This figure plots $x^*, x_c^*, V(x)$, and $V_c(x)$ for varying levels of σ. The other parameter values are set in Table 1.

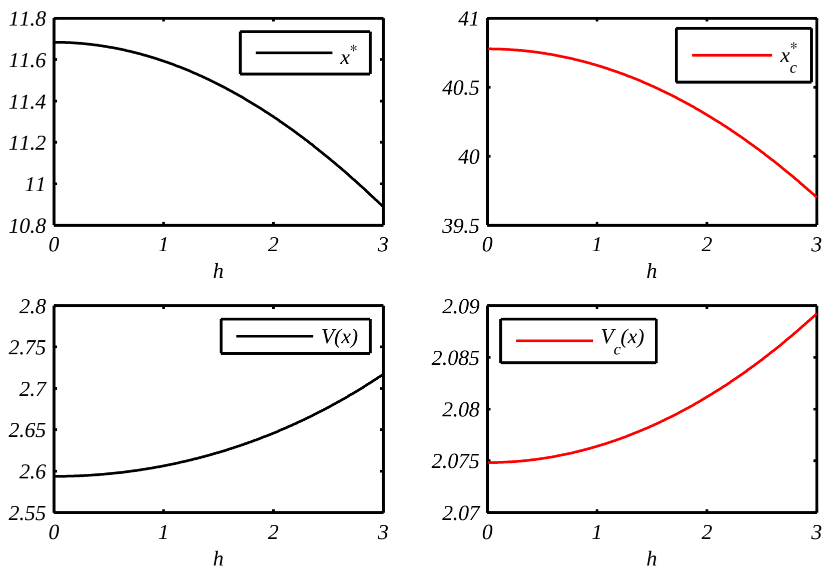

Figure 3: The impacts of uncertainty of research duration. This figure plots $x^*, x_c^*, V(x)$, and $V_c(x)$ for varying levels of h, where research duration T follows a uniform distribution with $[3-h, 3+h]$. The other parameter values are set in Table 1.

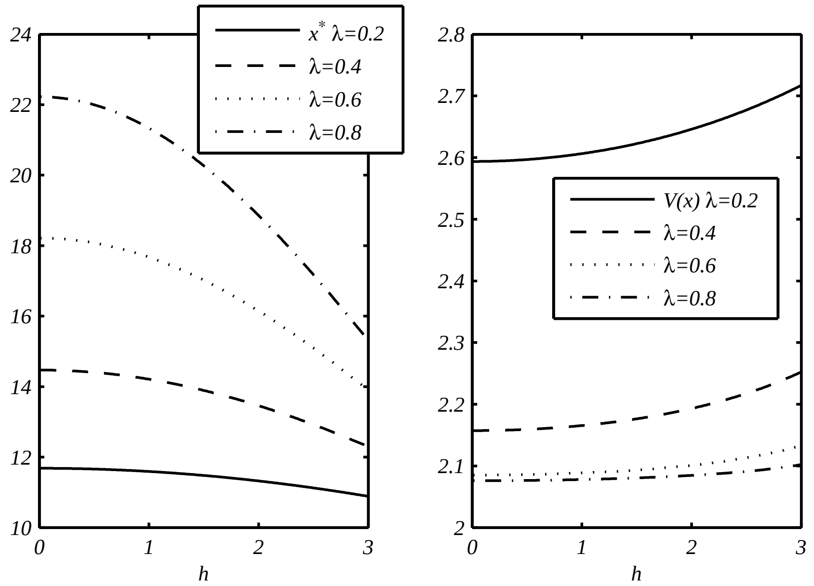

Figure 4: The impacts of uncertainty of research duration combined with the arrival rate of a competitor's technology development. The left and right panels plot x^* and $V(x)$, respectively, for varying levels of h and λ, where research duration T follows a uniform distribution with $[3-h, 3+h]$. The other parameter values are set in Table 1.

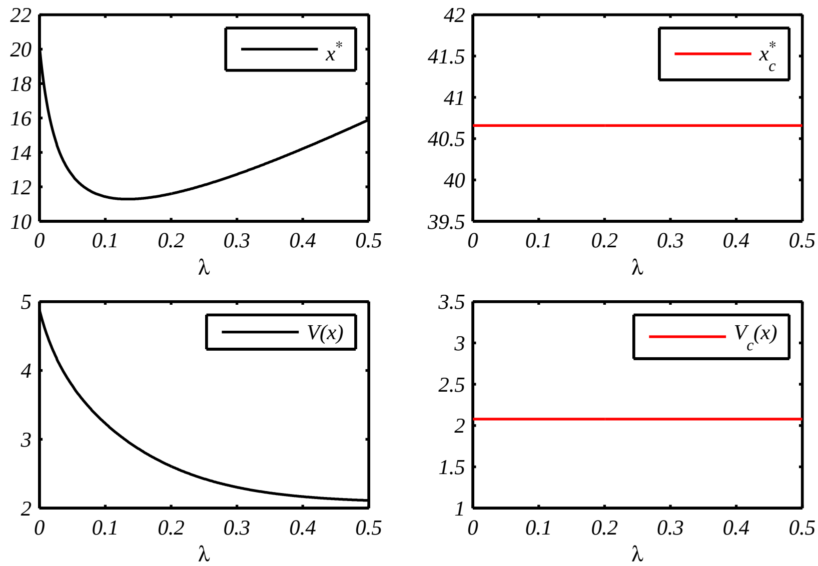

Figure 5: The impacts of the arrival rate of a competitor's technology development. This figure plots $x^*, x_c^*, V(x)$, and $V_c(x)$ for varying levels of λ. The other parameter values are set in Table 1.

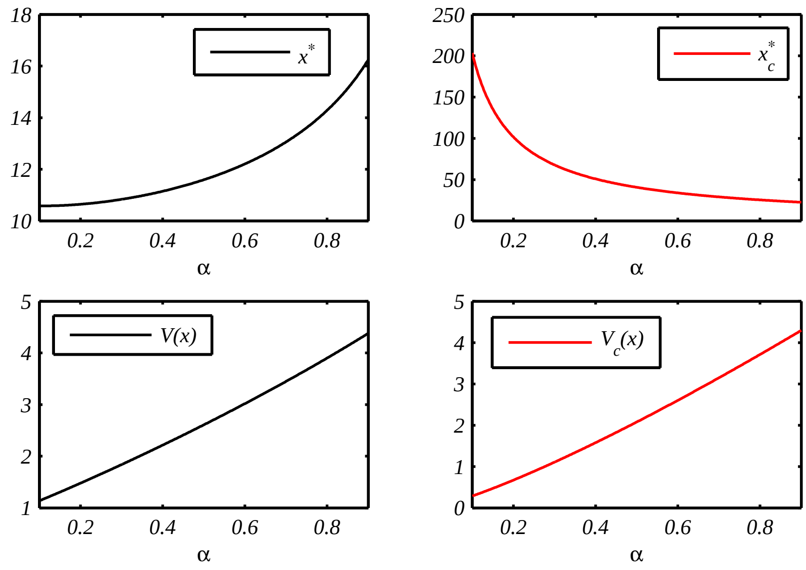

Figure 6: The impacts of the remaining value after a competitor's technology development. This figure plots $x^*, x_c^*, V(x)$, and $V_c(x)$ for varying levels of α. The other parameter values are set in Table 1.

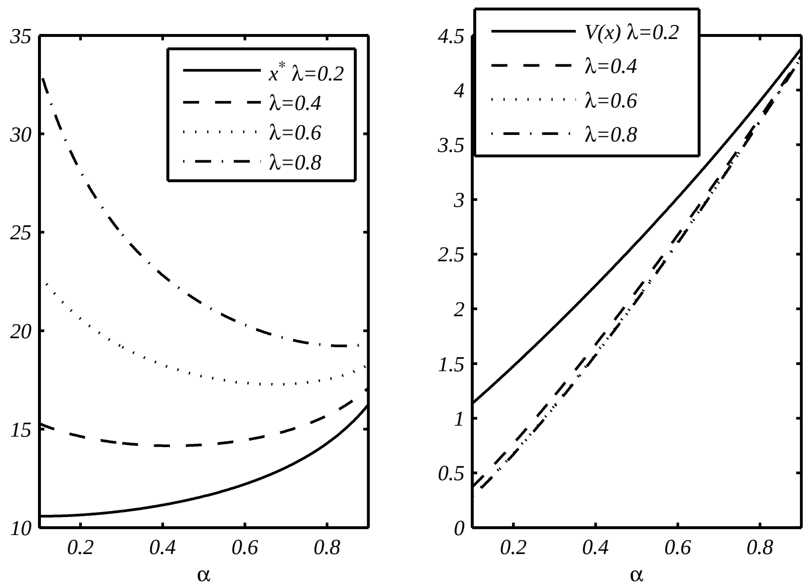

Figure 7: The impacts of the remaining value after a competitor's technology development combined with the arrival rate of a competitor's technology development. The left and right panels plot x^* and $V(x)$, respectively, for varying levels of α and λ. The other parameter values are set in Table 1.