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Abstract

I introduce financial market friction into a neoclassical growth model. I consider a moral hazard problem between bankers and workers in the macroeconomic model. Using the model, this study analyzes how capital adequacy requirements for banks affect the economy. I show that strengthening capital adequacy requirements is desirable for an economy whose financial market has not developed sufficiently. Regulatory authorities should pull up the minimum capital adequacy ratio in a country whose financial market has not developed sufficiently. Moreover, there is no need to change the minimum capital adequacy ratio in a country whose financial market has developed sufficiently even if the economy experiences a recession.

Keywords: Capital adequacy requirements; Economic Growth; Financial Intermediaries; Macro-prudential policies

JEL Classification Codes: E44, G21, G28

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1 Introduction

Since the financial crisis, macro-prudential policies that aim to make the financial sector more resilient have been discussed all over the world. Above all, many researchers and policy institutions have discussed capital adequacy requirements. The Bank for International Settlements (BIS) revised the Basel II market risk framework in 2009. Moreover, the BIS revised the minimum capital requirements for market risk framework in 2016. Basel III is intended to strengthen bank capital requirements. Then, these reforms have raised minimum capital adequacy ratio. Moreover, many researchers take a view that minimum capital adequacy ratios should be pro-cyclical.[1] Against this background, there are many studies on the analysis of capital adequacy requirements in macroeconomic models after the 2007–2008 financial crisis.[2] Benigno et al. (2010), Bianchi and Mendoza (2010) and Kannan et al.(2012) study how policy injection such as macro-prudential policies including capital adequacy requirements affect over borrowing in macroeconomic models. Angeloni and Faia (2013), Unsal (2013), Angelini et al. (2014), Medina and Roldos (2014), Baker and Wurgler (2015), and Collard et al (2017) study the interaction of macro-prudential and monetary policies. They study capital adequacy requirements as one of macro-prudential policies using recent macroeconomic models, but they do not focus on the relationship between financial frictions and capital adequacy requirements.

The current study examines how capital adequacy requirements affect the economy in a macroeconomic model. I introduce financial friction into a macroeconomic model with bankers and workers. Undertaking empirical research, Furfine (2000) and Francis and Osborne (2012) study capital adequacy requirements for banks in the US and European countries. They focus on the targeted capital ratios such as the minimum capital adequacy ratio. Thus, I introduce capital adequacy requirements in the macroeconomic model by setting the minimum capital adequacy ratio. Using this model, I find that regulatory authorities should not change minimum capital adequacy ratios

[1]Some studies examine whether the leverage ratio is counter-cyclical. For instance, He and Krishnamurthy (2008) and Gertler and Kiyotaki (2015) show that the capital adequacy ratio is pro-cyclical, that is, the leverage ratio is counter-cyclical. On the other hand, Adrian and Shin (2009) show that the leverage ratio is pro-cyclical. The current study shows the former case.

[2]Capital adequacy requirements existed before the financial crisis. Rochet (1992), Blum and Hellwig (1995), and Barth et al. (2004) study it in macroeconomic models.

even if the economy experiences a recession in the country whose financial market has developed sufficiently. On the contrary, in a country whose financial market has not developed sufficiently, regulatory authorities should pull up the minimum capital adequacy ratio. This policy makes the capital adequacy ratio determined in market equilibrium equal to the minimum capital adequacy ratio and thereby realizes higher levels of consumption and output.

The model I develop is a simple macroeconomic model with financial market friction. Following Gertler and Kiyotaki (2010), I introduce financial market friction into a macroeconomic model. Gertler et al. (2012), Gertler and Kiyotaki (2015), and Aoki et al. (2016) extend the model of Gertler and Kiyotaki (2010) in order to analyze bank runs, the monetary policies in emerging countries, and macro-prudential policies, respectively. Gertler et al. (2012) is relevant to the current study since they analyze macro-prudential policies; however, they do not analyze capital adequacy requirements. To incorporate capital adequacy requirements in such a model and examine how these requirements affect the level of workers' investment, bankers' investment, aggregate capital, and thereby consumption, this study simplifies the model of Gertler and Kiyotaki (2010).[3] In the current model, there is only one production sector and there is no capital goods sector. Owing to this simplification, using the current model, although I cannot analyze how the economic crisis affects the economy through changes of asset prices. I can analyze how a productivity shock affects consumption and output. For the productivity shock changes the spread of lending returns and deposit costs, and this shock affects both the production sector and the banking sector independently, as well as the above studies.[4] This effect does not emerge in a neoclassical growth model with no financial market frictions. In the current model, I analyze how capital adequacy requirements affect macroeconomic variables, such as investment, consumption, and output, using the neoclassical growth model with financial market friction.

The remainder of this paper is organized as follows. Section 2 presents a basic structure of the model. It describes the behavior of households and banks, final goods production, and market equilibrium. Section 3 describes the model's dynamic system and Section 4 analyzes the properties of a steady state. Concluding remarks are offered in Section 5.

[3]Technically, incorporating capital adequacy requirements in such a model leads to more cases of dynamic systems.

[4]From this view, the current study adopts the spirit of the macroeconomic model with financial market friction, as in Kiyotaki and Moore (1997).

2 Model

I consider a closed economy in which time is discrete. Following Gertler and Kiyotaki (2010), I introduce financial market friction into a macroeconomic model.[5] There is a representative household with a continuum of members of measure unity. The members consist of bankers and workers. I explain the behavior of the households in the following subsection.

2.1 Households

Workers supply labor to a final good sector. They use their wage earnings for savings and consumption. They save by depositing their assets with bankers and managing the capital market. I assume there is a disadvantage of workers relative to bankers in the financing business.[6] Moreover, workers are lenders in the capital market.[7] Specifically, in order to manage capital in the capital market, workers require the following extra management costs while bankers do not:

$$f\left(K^h\right) = \frac{1}{2} \cdot \omega \cdot \left(K^h(t)\right)^2, \tag{1}$$

where K^h represents capital holdings by workers at the end of period t and $\omega > 0$ is a parameter reflecting the disadvantage of workers relative to bankers in the financing business. Workers are under the following no-arbitrage condition:

$$r\left(t+1\right) - f'\left(K^h\right) = r^d\left(t+1\right), \tag{2}$$

where $r\left(t+1\right)$ is a rental price in the capital market and $r^d\left(t+1\right)$ represents the rate of returns on deposits. The left-hand side of equation (2) represents the returns on managing capital in the capital market. The right-hand side of equation (2) represents the rate of returns on deposits with banks.

Rewriting equation (2), I obtain

$$r\left(t+1\right) - r^d\left(t+1\right) = \omega \cdot K^h(t). \tag{3}$$

[5] Gertler et al. (2012), Gertler and Kiyotaki (2015), and Aoki et al. (2016) introduce financial market friction into a macroeconomic model following Gertler and Kiyotaki (2010).

[6] Gertler and Kiyotaki (2015) and Aoki et al. (2016) adopt the same assumption and the same function of management cost.

[7] I assume the cost is infinity when workers are borrowers in the capital market.

Next, I describe the representative households' problem. At each period, with probability $1-\sigma$, bankers retire their banking business and with the same probability $1-\sigma$, workers become the new bankers. Therefore, the ratio of workers to bankers is constant and thus, the total population is constant in this model. When a banker becomes a worker, the banker brings the net worth of banking to the household. When a worker becomes a banker, the representative household gives a part of its savings to the new banker for start-up funds. I consider a moral hazard problem in which bankers misbehave instead of investing their capital. This capital consists of deposits from workers and the bankers' own net worth.[8] Thus, workers do not deposit all of their assets but lend them in the capital market even if they need extra management costs of capital.

The representative households maximize its expected utility subject to a budget constraint, as follows:[9]

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln C(t), \tag{4}$$

subject to

$$\begin{aligned} & C(t) + f\left(K^h(t)\right) + S(t) \\ =& w(t) \cdot L(t) + r(t) \cdot K^h(t-1) + r^d(t) \cdot D(t-1) \\ +& (1-\sigma) \cdot \left[r(t) \cdot K^b(t-1) - r^d(t) \cdot D(t-1)\right] + \lambda \cdot S(t), \end{aligned} \tag{5}$$

with

$$S(t) = K^h(t) - (1-\delta) \cdot K^h(t-1) + D(t) - D(t-1), \tag{6}$$

where $C(t)$ is consumption in period t; $\beta \in (0,1)$ denotes the discount factor; $S(t)$ represents savings in period t; $w(t)$ is the wage in period t; $L(t)$ is the labor supply in period t; $D(t)$ represents deposits in period t; $K^b(t)$ is the capital investment by bankers; $\lambda \in (0,1)$ is the proportion of savings used for the net worth of new bankers; and $\delta \in (0,1)$ is the rate of depreciation of capital. Here, $1-\sigma \in (0,1)$ is the probability that bankers become workers and the probability that workers become bankers. I assume that $L(t)$ is 1 hereafter.

The first-order conditions of the maximization for consumption, the capital investment of workers

[8] I describe the moral hazard problem in detail in the next subsection.

[9] I assume a logarithmic utility function for simplification.

and the deposits, imply

$$\frac{q(t)}{q(t+1)} = \frac{C(t+1)}{\beta \cdot C(t)}, \tag{7}$$

$$\frac{q(t)}{q(t+1)} = \frac{(1-\delta)\cdot(1-\lambda)+r(t+1)}{1-\lambda+\omega\cdot K^h(t)}, \tag{8}$$

$$\frac{q(t)}{q(t+1)} = \frac{(1-\lambda)+\sigma\cdot r^d(t+1)}{1-\lambda}, \tag{9}$$

where $q(t)$ is Lagrange multiplier. From equations (3) and (9), I obtain

$$\frac{q(t)}{q(t+1)} = \frac{(1-\lambda)+\sigma\cdot r(t+1)-\sigma\cdot\omega\cdot K^h(t)}{1-\lambda}. \tag{10}$$

From equations (8) and (10), I obtain

$$\begin{aligned}\omega^2\cdot\sigma\left(K^h(t)\right)^2 &+ \left[-\sigma\cdot\omega\cdot r(t+1)-(1-\sigma)\cdot\omega\cdot(1-\lambda)\right]\cdot K^h(t)\\ &+(1-\lambda)\left[r(t+1)\cdot(1-\sigma)-\delta\cdot(1-\lambda)\right]=0.\end{aligned} \tag{11}$$

for $K^h(t)>0$ and $D(t)>0$.

2.2 Banks

Bankers maximize the discounted sum of expected value of their own net worth. The problem of a banker who exits the bank at the end of period t and brings net worth back to the household is

$$\max V(t) = E_t\left[\sum_{j=1}^{\infty}\beta^t\cdot\sigma^{j-1}\cdot(1-\sigma)\cdot n(t+j)\right], \tag{12}$$

subject to

$$n(t)+d(t)=k^b(t), \tag{13}$$

and

$$n(t)=r(t)\cdot k^b(t-1)-r^d(t)\cdot d(t-1), \tag{14}$$

where $n(t)$ is the net worth of each banker at the end of period t, $d(t)$ represents funds from households' deposits of each banker at the end of period t, and $k^b(t)$ represents the investment of each banker at the end of period t. Equations (13) and (14) are constraints on the flow of funds. Equation (13) represents the balance sheet condition while equation (14) is the evolution of a banker's net worth.

I consider the following moral hazard problem, following Gertler and Kiyotaki (2010). After bankers collect deposits from workers, bankers can leave the bank with the funds and divert a part of the funds for their private benefit. A proportion of the funds that can be diverted is $\theta \in (0, 1)$. Thus, the incentive comparative constraint can be written as

$$V(t) \geq \theta \cdot k^b(t). \tag{15}$$

The light-hand side of equation (15) is the value of investment of bankers' funds. The right-hand side of equation (15) is the value of diverting these funds.

I introduce capital adequacy requirements into this model. The rule is that bankers must keep the ratio of their net worth to risky assets, that is, investment must be larger than $\overline{\phi}$. Let $\phi(t)$ denote the capital adequacy ratio, $\phi(t) \equiv \frac{n(t)}{k^b(t)}$. Formally, the capital adequacy requirements in this model are described as follows:

$$\phi(t) \geq \overline{\phi}. \tag{16}$$

Generally, the value of the banker at the end of period t satisfies the following Bellman equation:

$$V(t) = \beta \cdot (1 - \sigma) \cdot n(t + 1) + \beta \cdot \sigma \cdot V(t + 1). \tag{17}$$

As in Gertler and Kiyotaki (2010), to solve the decision problem, I guess that the value function is linear; that is

$$V(t) = \iota(t) \cdot k^b(t) - \nu(t) \cdot d(t), \tag{18}$$

where $\iota(t) > 0$ is the marginal return of the banker's investment and $\nu(t) > 0$ is the marginal cost of deposits.

Let $\mu(t)$ be defined such that $\mu(t) \equiv \iota(t) - \nu(t)$. Substituting this definition and equation (13) into equation (18), I obtain

$$V(t) = \mu(t) \cdot k^b(t) + \nu(t) \cdot n(t) \geq \theta k^b(t) \tag{18'}$$

Substituting equation (18') into (15), I obtain

$$\phi(t) \geq \frac{\theta - \mu(t)}{\nu(t)}, \tag{19}$$

where $\phi(t) = \frac{n(t)}{k^b(t)}$.

Equation (19) is binding if $0 < \mu(t) < \theta$, and thus, equations (16) and (19) yield[10]

$$\frac{\theta - \mu(t)}{\nu(t)} = \hat{\phi}(t) \equiv \begin{Bmatrix} \phi(t), & \text{when} & \phi(t) \geq \overline{\phi}, \\ \overline{\phi}, & \text{when} & \phi(t) < \overline{\phi}, \end{Bmatrix}. \tag{20}$$

From equation (20) and the definition of $\mu(t)$, I obtain

$$\iota(t) = \theta + \nu(t) \cdot \left[1 - \hat{\phi}(t)\right]. \tag{21}$$

I arrange the flow constraint on funds. First, equation (14) can be rewritten as

$$d(t) = \frac{r(t+1)}{r^d(t+1)} \cdot k^b(t) - \frac{n(t+1)}{r^d(t+1)}. \tag{22}$$

Then, substituting equation (22) into equation (13), I obtain

$$n(t+1) = \left[r(t+1) - r^d(t+1)\right] \cdot k^b(t) + r^d(t+1) \cdot n(t). \tag{23}$$

After I combine the conjectured value function (18'), the Bellman equation (17), and equation (23), I verify that the value function is linear in $k^b(t)$ and $n(t)$ if $\mu(t)$ and $\nu(t)$ satisfy[11]

$$\mu(t) = \beta \cdot \Omega(t+1) \cdot \left[r(t+1) - r^d(t+1)\right], \tag{24}$$

$$\nu(t) = \beta \cdot \Omega(t+1) \cdot r^d(t+1), \tag{25}$$

where

$$\Omega(t+1) \equiv (1-\sigma) + \frac{\sigma \cdot \mu(t+1)}{\hat{\phi}(t+1)} + \sigma \cdot \nu(t+1). \tag{26}$$

Let $\Omega(t+1)$ be the marginal value of net worth at period $t+1$. From equations (24), (25), and the definition of $\mu(t)$, I obtain

$$\iota(t) = \beta \cdot \Omega(t+1) \cdot r(t+1). \tag{27}$$

Substituting equations (25) and (27) into equation (21), I obtain

$$\beta \cdot \Omega(t+1) \cdot \left[r(t+1) - \left(1 - \hat{\phi}(t)\right) \cdot r^d(t+1)\right] = \theta, \tag{28}$$

where $\hat{\phi}(t)$ is given by equation (20).

[10] The incentive comparative constraint (15) can be written as $\nu(t)n(t) \geq (\theta - \mu(t))k^b(t)$. Since $\nu(t)$ and $n(t)$ are positive values, if $\theta - \mu(t) < 0$, this constraint cannot be binding, that is, $v(t)n(t) > (\theta - \mu(t))k^b(t)$.

[11] See Appendix A.

From equations (21) and (26), and the definition of $\mu(t)$, I obtain

$$\Omega(t+1) = \frac{(1-\sigma)\cdot\hat{\phi}(t+1) + \sigma\cdot\theta}{\hat{\phi}(t+1)}, \ \ \forall t. \tag{29}$$

Substituting equations (25), (27), and (29) into equation (21), I obtain the relationship between the return of investment $r(t+1)$ and the payments for deposits $r^d(t+1)$, as follows:

$$r(t+1) = \left[\frac{\theta\cdot\hat{\phi}(t+1)}{\beta\cdot\Big((1-\sigma)\cdot\hat{\phi}(t+1)+\sigma\cdot\theta\Big)}\right] + \Big(1-\hat{\phi}(t)\Big)\cdot r^d(t+1). \tag{30}$$

In equilibrium, the solution of the maximization problem for the banker, (30) and the no-arbitrage condition (3) must be satisfied. Thus, from equations (3) and (30), I obtain the return of capital investment $r(t+1)$ as the function of the capital adequacy ratio, $\hat{\phi}(t)$ and $\hat{\phi}(t+1)$, where $\hat{\phi}(\cdot)$ is given by equation (20):

$$r(t+1) = -\left(\frac{1-\hat{\phi}(t)}{\hat{\phi}(t)}\right)\cdot\omega\cdot K^h(t) + \frac{\theta}{\beta\cdot\Big((1-\sigma)\cdot\hat{\phi}(t+1)+\sigma\cdot\theta\Big)}\cdot\left(\frac{\hat{\phi}(t+1)}{\hat{\phi}(t)}\right), \tag{31}$$

where $\omega > 0$ and $0 < \sigma < 1$.

Since in equilibrium the optimal conditions for workers and bankers, equations (11) and (31), are satisfied, I obtain the following equation:[12]

$$\hat{\phi}(t+1) = \begin{Bmatrix} \Psi\left(K^h(t),\hat{\phi}(t)\right), & \text{when} \quad \hat{\phi}(t) \geq \overline{\phi}, \\ \overline{\phi}, & \text{when} \quad \hat{\phi}(t) < \overline{\phi}, \end{Bmatrix}, \tag{32}$$

where

$$\Psi\left(K^h(t),\hat{\phi}(t)\right) \equiv \frac{\beta\cdot\sigma\cdot\theta\cdot\Gamma\left(K^h(t),\hat{\phi}(t)\right)}{\theta\cdot\omega\cdot\hat{\phi}(t)\cdot K^h(t) - (1-\sigma)\cdot(1-\lambda)\theta - \beta\cdot(1-\sigma)\cdot\Gamma\left(K^h(t),\hat{\phi}(t)\right)}, \tag{33}$$

and

$$\Gamma\left(K^h(t),\hat{\phi}(t)\right) \equiv \omega^2\cdot\sigma\cdot\left(K^h(t)\right)^2 - \omega\cdot K^h(t)\cdot(1-\lambda)\cdot(1-\sigma) - \delta\cdot\hat{\phi}(t)\cdot(1-\lambda)^2.$$

Equations (32) and (33) imply that the capital adequacy ratio at period $t+1$, $\hat{\phi}(t+1) = \frac{n(t+1)}{k^b(t+1)}$ depends on the capital adequacy ratio at period t, $\hat{\phi}(t) = \frac{n(t)}{k^b(t)}$ and the capital investment of workers, $K^h(t)$ under the capital adequacy requirements, $\hat{\phi}(t) > \overline{\phi}$.

[12]The derivation of (32) is given in appendix B.

2.3 Final Goods Producer

The final goods are produced by capital and labor. The production function is as follows:

$$Y(t) = A \cdot K(t)^{\alpha} \cdot L(t)^{1-\alpha}, \tag{34}$$

where $Y(t)$ is aggregate output at period t, A is a parameter of aggregate productivity, $K(t)$ is aggregate capital used for production at period t, and $L(t)$ is aggregate labor supply at period t with $\alpha \in (0,1)$. For simplicity, I normalize the number of workers in each period as one, $L(t) = 1$.

Perfect competition prevails in the final goods sector. I take the final good as a numeraire. The optimal conditions of the profit maximization are

$$r(t) = \alpha \cdot \frac{Y(t)}{K(t)}, \tag{35}$$

$$w(t) = \alpha \cdot Y(t). \tag{36}$$

2.4 Market Equilibrium

Output is consumed, invested, or used to pay the cost of managing the household's capital. Thus, the following holds:

$$Y(t) = C(t) + I(t) + f\left(K^h(t)\right), \tag{37}$$

with

$$I(t) = K(t+1) - K(t) + \delta \cdot K(t). \tag{38}$$

The market equilibrium for capital ownership implies

$$K(t) = K^h(t-1) + K^b(t-1), \tag{39}$$

where $K(t)$ is the aggregate capital in period t, $K^h(t-1)$ is the aggregate capital holdings of workers at the end of $t-1$, and $K^b(t-1)$ is the aggregate capital holdings of bankers at the end of $t-1$ with $K^b(t-1) \equiv \int k^b(t-1)$.

The competitive equilibrium is described by the four state variables, $\left(K^h(t), K^b(t), D(t), N(t)\right)$, the three price variables $\left(w(t), r(t), r^d(t)\right)$, and the two jump variables $(Y(t), C(t))$.

3 Dynamic System

To describe the dynamic system of the model, I define the ratio of bankers' lending to aggregate capital as $\eta(t) \equiv \frac{K^b(t-1)}{K(t)}$, the ratio of workers' capital holdings to aggregate capital becomes $1 - \eta(t) \equiv \frac{K^h(t-1)}{K(t)}$, the ratio of consumption to aggregate capital as $x(t) \equiv \frac{C(t-1)}{K(t)}$, and the ratio of bankers' net worth to aggregate capital as $B(t) \equiv \frac{N(t-1)}{K(t)}$ with $N(t) \equiv \int n(t)$.

The no-arbitrage condition (1), the relationship among the return of investment, $r(t)$, the capital adequacy ratio, $\hat{\phi}(t)$ (31), the goods market-clearing condition (37), the capital accumulation (38), and the definitions of $x(t)$, $\eta(t)$ and $B(t)$ yield [13]

$$\begin{aligned}
&\left(\frac{B(t+1)}{\eta(t+1)} \cdot \frac{\eta(t)}{B(t)}\right) \cdot \left[\frac{\theta \cdot ((1-\lambda+\omega(1-\eta(t))K(t)) - \alpha\beta \cdot x(t))}{\alpha\beta\left[(1-\sigma) \cdot \frac{B(t+1)}{\eta(t+1)} + \sigma\theta\right]}\right] + \frac{K(t+1)}{K(t)} \\
&+ \omega \cdot \frac{(1-\eta(t+1))^2 \cdot (K(t+1))^2}{K(t)} \\
&= \frac{x(t) \cdot \left[\beta \cdot (1-\delta) \cdot (1-\lambda) - \beta \cdot \omega \cdot (1-\eta(t)) \cdot K(t) \cdot \left(\frac{1-\frac{B(t)}{\eta(t)}}{\frac{B(t)}{\eta(t)}}\right)\right]}{(1-\lambda) + \omega \cdot (1-\eta(t)) \cdot K(t)}. \\
&- \frac{\omega}{\alpha} \cdot (1-\eta(t)) \cdot K(t) \cdot \frac{\left(1-\frac{B(t)}{\eta(t)}\right)}{\frac{B(t)}{\eta(t)}} + (1-\delta).
\end{aligned} \tag{40}$$

Combining the optimization conditions for the representative households' problem, (7) and (8) with the relationship among the return of investment, $r(t)$ and the capital adequacy ratio, $\hat{\phi}(t)$ (31), I obtain[14]

$$\begin{aligned}
&\frac{x(t+1)}{x(t)} \cdot \frac{K(t+1)}{K(t)} \cdot (1-\lambda) - \frac{\sigma\theta \frac{B(t+1)}{\eta(t)} \frac{\eta(t+1)}{B(t)}}{(1-\sigma)\frac{B(t+1)}{\eta(t+1)} + \sigma\theta} \\
&= \beta\,(1-\lambda) - \beta \cdot \left(\frac{\sigma}{\frac{B(t)}{\eta(t)}} \cdot \omega\,(1-\eta(t))\,K(t)\right).
\end{aligned} \tag{41}$$

From the no-arbitrage condition (3), the flow constraint of funds for bankers, (23), the relationship between $r(t+1)$ and $K^h(t)$, (31), and the definitions of $x(t)$, $\eta(t)$ and $B(t)$, I obtain[15]

$$\frac{K(t+1)}{K(t)} \cdot \frac{\eta(t+1)}{\eta(t)} \cdot \beta \cdot \left[(1-\sigma)\frac{B(t+1)}{\eta(t+1)} + \sigma\theta\right] = \theta. \tag{42}$$

Note that $\frac{B(t+1)}{\eta(t+1)} = \hat{\phi}(t)$, the capital adequacy ratio.

[13] See Appendix C.

[14] See Appendix D.

[15] See Appendix E.

Taking one-period lag of equation (32) and using the definitions of $x(t)$, $\eta(t)$ and $B(t)$, I obtain

$$\frac{B(t+1)}{\eta(t+1)} = \begin{Bmatrix} \Psi\left(\eta(t), B(t), K(t)\right), & \text{when} \quad \frac{B(t)}{\eta(t)} \geq \overline{\phi}, \\ \overline{\phi}, & \text{when} \quad \frac{B(t)}{\eta(t)} < \overline{\phi}, \end{Bmatrix}, \tag{43}$$

where

$$\begin{aligned} &\Psi\left(\eta(t), B(t), K(t)\right) \\ &\equiv \frac{\beta \cdot \sigma \cdot \theta \cdot \Gamma\left(\eta(t), B(t), K(t)\right)}{\theta \cdot \omega \cdot \frac{B(t)}{\eta(t)} \cdot (1-\eta(t)) \cdot K(t) - (1-\sigma) \cdot (1-\lambda)\theta - \beta \cdot (1-\sigma) \cdot \Gamma\left(\eta(t), B(t), K(t)\right)}, \end{aligned} \tag{44-1}$$

with

$$\begin{aligned} &\Gamma\left(\eta(t), B(t), K(t)\right) \\ &\equiv \omega^2 \cdot \sigma \cdot \left((1-\eta(t)) \cdot K(t)\right)^2 - \omega \cdot (1-\eta(t)) \cdot K(t) \cdot (1-\lambda) \cdot (1-\sigma) \\ &- \delta \cdot \frac{B(t)}{\eta(t)} \cdot (1-\lambda)^2. \end{aligned} \tag{44-2}$$

The above equations (40), (41), (42), and (43) constitute the dynamic system of x, η, B and K, that describes the economy.

4 Steady-State Analysis

I consider the steady-state economy. Let y_{ss} denote the level of steady state of variable y. I focus on the following four variables in steady state for interpretation of the economy and comparative statics analysis: the capital adequacy ratio ϕ_{ss}, the workers' management capital K_{ss}^h, the bankers' lending capital K_{ss} and the consumption C_{ss}. The steady state level of these four variables are determined by the above difference equations which determines the dynamic system in the economy (40), (41), (42) and (43) as follows.

Using (40), (41) and (42) and (43), the steady state level of the above four variables determined by the following equations:

$$\theta \cdot \left(\frac{(1-\lambda)\omega K_{ss}^h - \alpha\beta x_{ss}}{\alpha\beta[(1-\sigma)\phi_{ss} + \sigma\theta]}\right) + \frac{\omega\left(K_{ss}^h\right)^2}{K_{ss}} = \frac{x_{ss}(\beta(1-\delta)(1-\lambda) - \beta\omega K_{ss}^h \frac{1-\phi_{ss}}{\phi_{ss}})}{1-\lambda+\omega K_{ss}^h} - \frac{\omega\left(K_{ss}^h\right)}{\alpha}\frac{1-\phi_{ss}}{\phi_{ss}} - \delta; \tag{40ss}$$

$$(1-\lambda)-\frac{\sigma\theta}{(1-\sigma)\phi_{ss}+\sigma\theta}=\beta(1-\lambda)-\frac{\beta\sigma\omega K_{ss}^{h}}{\phi_{ss}}; \tag{41ss}$$

$$\beta((1-\sigma)\phi_{ss}+\sigma\theta)=\theta; \tag{42ss}$$

$$\frac{B_{ss}}{\eta_{ss}}=\phi_{ss}\equiv\left\{\begin{array}{ll}\Psi(K_{ss}^{h}), & \text{when } \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}\geq\overline{\phi} \\ \overline{\phi}, & \text{when } \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}<\overline{\phi}\end{array}\right\}. \tag{43ss}$$

where

$$\Psi_{ss}\left(K_{ss}^{h},\phi_{ss}\right) \equiv\frac{\beta\cdot\sigma\cdot\theta\cdot\Gamma_{ss}\left(K_{ss}^{h},\phi_{ss}\right)}{\theta\cdot\omega\cdot\phi_{ss}\cdot K_{ss}^{h}-(1-\sigma)\cdot(1-\lambda)\theta-\beta\cdot(1-\sigma)\cdot\Gamma_{ss}\left(K_{ss}^{h},\phi_{ss}\right)}, \tag{44-1ss}$$

with

$$\Gamma_{ss}\left(K_{ss}^{h},\phi_{ss}\right)\equiv\omega^{2}\cdot\sigma\cdot\left(K_{ss}^{h}\right)^{2}-\omega\cdot K_{ss}^{h}\cdot(1-\lambda)\cdot(1-\sigma)-\delta\cdot\phi_{ss}\cdot(1-\lambda)^{2}. \tag{44-2ss}$$

First, equation (42) yields the steady-state level of capital adequacy ratio, ϕ_{ss}:

$$\frac{B_{ss}}{\eta_{ss}}=\phi_{ss}\equiv\left\{\begin{array}{ll}\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}, & \text{when } \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}\geq\overline{\phi} \\ \overline{\phi}, & \text{when } \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}<\overline{\phi},\end{array}\right\}. \tag{45}$$

Since θ is a parameter reflecting the degree of financial friction of the economy, equation (45) yields the following lemma 1.[16] I define $\hat{\phi}_{ss}$ as $\hat{\phi}_{ss}\equiv\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}$.

Lemma 1 *The capital adequacy ratio in the steady state ϕ_{ss} is higher in the economy with larger financial friction θ.*

In addition, this lemma implies that the leverage ratio in the steady state $\frac{1}{\phi_{ss}}$ is lower in the economy with larger financial friction θ.

The level of households' capital in the steady state, K_{ss}^{h}, must satisfy (41) and (45) with $x(t)=x(t+1)=x_{ss}$, $(1-\eta(t))\cdot K(t)=(1-\eta(t+1))\cdot K(t+1)=K_{ss}^{h}$ and $\frac{B(t)}{\eta(t)}=\frac{B(t+1)}{\eta(t+1)}=\phi_{ss}$.

From (41) and (45), I obtain[17]

$$K_{ss}^{h}=\left\{\begin{array}{ll}\frac{\theta\cdot(1-\beta\sigma)\cdot[\beta\sigma-(1-\lambda)(1-\beta)]}{\omega\beta^{2}\cdot\sigma}\equiv\hat{K}_{ss}^{h}, & \text{when } \hat{\phi}_{ss}\geq\overline{\phi} \\ \frac{\overline{\phi}\left(\sigma\beta-(1-\beta)(1-\lambda)[(1-\sigma)\overline{\phi}+\sigma\theta]\right)}{\beta\sigma\omega[(1-\sigma)\overline{\phi}+\sigma\theta]}\equiv\overline{K}_{ss}^{h}, & \text{when } \hat{\phi}_{ss}<\overline{\phi},\end{array}\right\}, \tag{46}$$

where $\hat{\phi}_{ss}\equiv\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}$.

[16] I assume that $\theta<\beta$, especially β is close to 1, then $\phi_{ss}\in(0,1)$ is satisfied in equation (45).

[17] Appendix F.

Lemma 2 *In the steady state, the capital adequacy ratio* ϕ_{ss} *and the households' management capital* K^h_{ss} *are given by equations (45) and (46), respectively. Let* $\hat{\lambda}_1$, $\hat{\beta}_1$, *and* $\overline{\phi}_1$ *be such that* $\hat{\beta}_1 \equiv \frac{1}{1+\sigma}$, $\hat{\lambda}_1 \equiv \frac{1-\beta-\beta\sigma}{1-\beta}$, *and* $\overline{\phi}_1 \equiv \frac{\sigma\theta\cdot[1-(1-\lambda)\cdot(1-\beta)]}{(1-\lambda)\cdot(1-\beta\cdot(1-\delta))\cdot(1-\sigma)}$. *The conditions for the existence of* K^h_{ss} *are as follows:*

(i) when $\hat{\phi}_{ss} > \overline{\phi}$, $\lambda > \hat{\lambda}_1$ *and* $\beta L \hat{\beta}_1$.

(ii) when $\hat{\phi}_{ss} \leq \overline{\phi}$, $\overline{\phi} < \overline{\phi}_1$.

Proof. See Appendix G. ■

Lemma 2 implies that when the minimum capital adequacy ratio $\overline{\phi}$ is excessively high, the households' management capital K^h_{ss} in the steady state does not exist. Moreover, Lemma 2 implies that the households' management capital K^h_{ss} in the steady state exists if the net worth of a new banker, λ is sufficiently large and the discount factor β is sufficiently low. These conditions for the existence of K^h_{ss} do not depend on a parameter of financial friction, θ.[18]

The following lemmas 3 and 4 give the properties of K^h_{ss}.

Lemma 3 *The households' capital* K^h_{ss} *increases as the degree of financial friction* θ.

Proof. Differentiating both $\hat{K}^h_{ss}$ and $\overline{K}^h_{ss}$ with respect to θ, respectively. ■

Lemma 3 implies K^h_{ss} decreases as the financial market develops in the economy.

Lemma 4 *Consider the case in which* $\hat{\phi}_{ss} \leq \overline{\phi}$ *where* $\hat{\phi}_{ss} = \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}$. *Let* $\overline{\phi}_2$ *be a positive value such that satisfies the following equation:* $Z_1 \cdot (\overline{\phi}^2) + Z_2 \cdot \overline{\phi} = Z_3$ *where*

$Z_1 \equiv -(1-\sigma)^2 \cdot [2-(1-\beta)(1-\lambda)]$; $Z_2 \equiv -2(1-\sigma)\sigma\theta$; $Z_3 \equiv -\sigma^2\theta^2[(1-(1-\beta)(1-\lambda)]$.

(i) The households' capital $\overline{K}^h_{ss}$ *increases as the minimum capital adequacy ratio* $\overline{\phi}$ *increases, if* $0 < \overline{\phi} \leq \overline{\phi}_2$.

(ii) $\overline{\phi}_2$ *increases as* θ *increases.*

Proof. Appendix H ■

Lemma 4 implies that if the economy faces a large financial friction, θ, the households' capital $\overline{K}^h_{ss}$ increases as the minimum capital adequacy ratio $\overline{\phi}$ increases. If the the economy faces small financial friction, θ, the households' capital $\overline{K}^h_{ss}$ decreases as the minimum capital adequacy ratio $\overline{\phi}$ increases.

[18] Whether the capital adequacy requirements are satisfied depends on a parameter of financial friction θ.

Since $\frac{B(t+1)}{\eta(t+1)} = \frac{B(t)}{\eta(t)} = \phi_{ss}$, $(1-\eta(t)) \cdot K(t) = K_{ss}$, $\eta(t) \cdot K_{ss} = K^b_{ss}$ in the steady state, equation (43) yield

$$K^b_{ss} = \frac{\sigma \cdot K^h_{ss} \cdot \left[\beta\omega \cdot [\sigma\theta + (1-\sigma)\phi_{ss}] \cdot K^h_{ss} - \theta\phi^2_{ss}\right] + (1-\lambda) \cdot \phi_{ss} \cdot [(1-\sigma)(1-\phi_{ss}) - \delta\beta\sigma\theta]}{\beta \cdot [(1-\sigma) + \phi_{ss}] \cdot [\sigma\theta + (1-\sigma) \cdot \phi_{ss}]}, \tag{47}$$

where ϕ_{ss} is given by equation (45) and K^h_{ss} is given by equation (46).

Since K^b_{ss} depends on K^h_{ss} fron equation (47), I examine the relationship between the level of households' capital and the level of bankers' capital in the steady state.

Lemma 5 *First, consider the case in which* $\overline{\phi} < \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \equiv \hat{\phi}_{ss}$.
Let $\hat{\lambda}_2$ *be such that* $\hat{\lambda}_2 \equiv \frac{2\beta(1-\sigma)^2 \cdot [\beta-(1-\beta(1-\delta))] - \theta^2(1-\beta\sigma) \cdot [(2-\beta)(1-\beta\sigma)\theta + \beta^2(1-\sigma)]}{2\beta(1-\sigma)^2 \cdot (1-\beta(1-\delta))}$. *The amount of bankers' lending* $\hat{K}^b_{ss}$ *increases as the amount of households' investment* $\hat{K}^h_{ss}$ *increases, if* $\hat{\lambda}_2 \geq \lambda$.
Second, consider the case in which $\hat{\phi}_{ss} \leq \overline{\phi}$.
Let $\overline{\phi}_3$ *be such that* $\overline{\phi}_3 \equiv \frac{\theta \cdot [(2-\beta) - \beta\sigma \cdot (1-\lambda) \cdot (1-\beta(1-\delta))]}{[\theta \cdot (1-\beta) + (1-\sigma) \cdot \beta \cdot (1-\lambda)(1-\beta(1-\delta))]}$. *The amount of bankers' lending* $\overline{K}^b_{ss}$ *increases as the amount of workers' investment* $\overline{K}^h_{ss}$ *increases, if* $\overline{\phi}_3 \geq \overline{\phi}$.

Proof. Appendix I. ■

As the capital of households K^h_{ss}increases, the interest rate spread $(r_{ss} - r^d_{ss})$ widens from equation (3); however, the interest rate spread widens and the capital of bankers does not increase, if λ is sufficiently large or the minimum capital adequacy ratio $\overline{\phi}$ is sufficiently high. First part of lemma 5 implies that the amount of lending of bankers K^b_{ss} increases as the capital of households' management K^h_{ss} increases if a parameter of the initial net worth of new bankers, λ is sufficiently small in the case in which $\overline{\phi} < \hat{\phi}_{ss}$. Second part of lemma 5 implies that the amount of lending of bankers K^b_{ss} increases as the investment of households K^h_{ss} increases if the minimum capital adequacy ratio, $\overline{\phi}$ is sufficiently low in the case in which $\phi_{ss} = \overline{\phi}$.

Since $K_{ss} = K^b_{ss} + K^h_{ss}$, equations (46) and (47) determine the level of aggregate capital in the steady state, K_{ss}. As the amount of households' management capital K^h_{ss} increases, the management cost increases from equation (1). As the amount of households' management capital K^h_{ss} increases, the amount of bankers' lending K^b_{ss} increases from lemma 3 and then the increase of K^h_{ss}decreases the amount of aggregate capital K_{ss}.

Lemma 5 and $K_{ss} = K^b_{ss} + K^h_{ss}$ give the following lemma 6.

Lemma 6 *First, consider the case in which $\overline{\phi} < \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \equiv \hat{\phi}_{ss}$.*

Let $\hat{\theta}_1$ be such that satisfies the following equation for a positive value:

$$\sigma\theta^2(1-\beta\sigma)^2 - \beta(1-\sigma)(1-\beta\sigma)\theta = \beta^2(1-\sigma)^3.$$

The aggregate capital $\hat{K}_{ss}$ increases as the households' investment $\hat{K}^h_{ss}$ increases, if $0 < \theta < \hat{\theta}_1$.

Second, consider the case in which $\hat{\phi}_{ss} \leq \overline{\phi}$. Let $\overline{\theta}_1$ be such that $\overline{\theta}_1 \equiv \frac{\beta(1-\sigma)}{\sigma}$

Let $\overline{\phi}_4$ be such that satisfies the following equation for a positive value:

$$(\theta\sigma - \beta(1-\sigma)) \cdot (\overline{\phi})^2 - \beta \cdot [\sigma\theta + (1-\sigma)^2] \cdot \overline{\phi} = \beta\sigma\theta(1-\sigma).$$

The aggregate capital $\overline{K}_{ss}$ increases as the households' investment $\overline{K}^h_{ss}$ increases, if $\overline{\theta}_1 > \theta$ or $\overline{\theta}_1 \leq \theta$ and $0 < \overline{\phi} < \overline{\phi}_4$.

Proof. Appendix J. ■

First part of lemma 6 implies that the aggregate capital K_{ss} increases as the capital of households K^h_{ss} increases if a parameter of financial friction, θ is sufficiently small, regardless of the minimum capital adequacy ratio, $\overline{\phi}$. Second part of lemma 6 implies that the aggregate capital K_{ss} increases as the capital of households K^h_{ss} increases if the financial system is not sufficiently developed and the minimum capital adequacy ratio, $\overline{\phi}$ is sufficiently low in the case in which $\phi_{ss} = \overline{\phi}$.

Since in the steady state $x(t+1) = x(t) = x_{ss}$ and $K(t+1) = K(t) = K_{ss}$, equation (40) yields

$$x_{ss} = \left\{ \begin{array}{ll} \frac{\omega\cdot\hat{K}^h_{ss}\cdot\left[\left(\frac{1-\hat{\phi}_{ss}}{\hat{\phi}_{ss}}\right)+\frac{\hat{K}^h_{ss}}{K_{ss}}\right]-(1-\delta)-\left(\frac{\theta}{\alpha\beta\cdot[(1-\sigma)\cdot\hat{\phi}_{ss}+\sigma\theta]}\right)}{\beta\cdot(1-\delta)\cdot(1-\lambda)-\beta\cdot\omega\cdot\hat{K}^h_{ss}\cdot\left(\frac{1-\hat{\phi}_{ss}}{\hat{\phi}_{ss}}\right)+\left(\frac{\theta}{(1-\sigma)\cdot\hat{\phi}_{ss}+\sigma\theta}\right)} & \text{when} \quad \hat{\phi}_{ss} \geq \overline{\phi} \\ \frac{\omega\cdot\overline{K}^h_{ss}\cdot\left[\left(\frac{1-\overline{\phi}}{\overline{\phi}}\right)+\frac{\overline{K}^h_{ss}}{\overline{K}_{ss}}\right]-(1-\delta)-\left(\frac{\theta}{\alpha\beta\cdot[(1-\sigma)\cdot\overline{\phi}+\sigma\theta]}\right)}{\beta\cdot(1-\delta)\cdot(1-\lambda)-\beta\cdot\omega\cdot\overline{K}^h_{ss}\cdot\left(\frac{1-\overline{\phi}}{\overline{\phi}}\right)+\left(\frac{\theta}{(1-\sigma)\cdot\overline{\phi}+\sigma\theta}\right)} & \text{when} \quad \hat{\phi}_{ss} < \overline{\phi} \end{array} \right\}, \qquad (48)$$

where ϕ_{ss} is given by equation (45) and K^h_{ss} is given by equation (46).

Since x is the ratio of consumption and aggregate capital, $x_{ss} \equiv \frac{C_{ss}}{K_{ss}}$, multiplying both sides of equation (48) by K_{ss} yields

$$C_{ss} = \left\{ \begin{array}{ll} \frac{\left[\omega\cdot\hat{K}^h_{ss}\cdot\left(\frac{1-\hat{\phi}_{ss}}{\hat{\phi}_{ss}}\right)-(1-\delta)-\left(\frac{\theta}{\alpha\beta\cdot[(1-\sigma)\cdot\hat{\phi}_{ss}+\sigma\theta]}\right)\right]\cdot\hat{K}_{ss}+\omega\cdot\left(\hat{K}^h_{ss}\right)^2}{\beta\cdot(1-\delta)\cdot(1-\lambda)-\beta\omega\cdot\hat{K}^h_{ss}\cdot\left(\frac{1-\hat{\phi}_{ss}}{\hat{\phi}_{ss}}\right)+\left(\frac{\theta}{(1-\sigma)\cdot\hat{\phi}_{ss}+\sigma\theta}\right)} & \text{when} \quad \hat{\phi}_{ss} \geq \overline{\phi} \\ \frac{\left[\omega\cdot\overline{K}^h_{ss}\cdot\left(\frac{1-\overline{\phi}}{\overline{\phi}}\right)-(1-\delta)-\left(\frac{\theta}{\alpha\beta\cdot[(1-\sigma)\cdot\overline{\phi}+\sigma\theta]}\right)\right]\cdot\overline{K}_{ss}+\omega\cdot\left(\overline{K}^h_{ss}\right)^2}{\beta\cdot(1-\delta)\cdot(1-\lambda)-\beta\omega\cdot\overline{K}^h_{ss}\cdot\left(\frac{1-\overline{\phi}}{\overline{\phi}}\right)+\left(\frac{\theta}{(1-\sigma)\cdot\overline{\phi}+\sigma\theta}\right)} & \text{when} \quad \hat{\phi}_{ss} < \overline{\phi} \end{array} \right\}, \qquad (49)$$

where ϕ_{ss} is given by equation (45) and K^h_{ss} is given by equation (46) and ϕ_{ss} and K^h_{ss} can be described as the functions of parameters. The following proposition 1 and Figure 1 show the condition of existence for the steady state level of consumption C_{ss}.

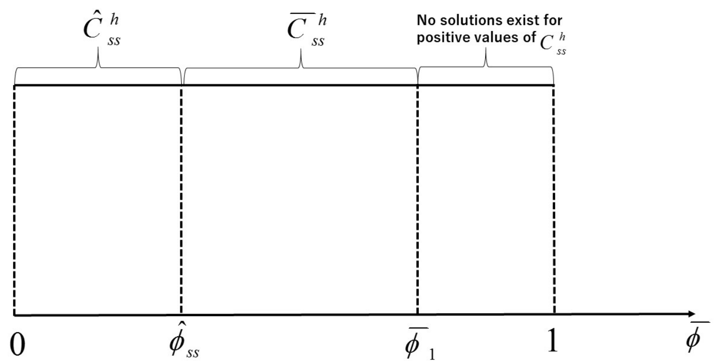

Figure 1: the steady state level of consumption

Proposition 1 *Let $\hat{\lambda}_1$, $\hat{\beta}_1$ and $\overline{\phi}_1$ be such that $\hat{\beta}_1 \equiv \frac{1}{1+\sigma}$, $\hat{\lambda}_1 \equiv \frac{1-\beta-\beta\sigma}{1-\beta}$ and $\overline{\phi}_1 \equiv \frac{\sigma\theta\cdot[1-(1-\lambda)\cdot(1-\beta)]}{(1-\lambda)\cdot(1-\beta)\cdot(1-\sigma)}$. The conditions for the existence of C_{ss} are as follows:*

(i) when $\hat{\phi}_{ss} > \overline{\phi}$, $\lambda > \hat{\lambda}_1$ and $\beta < \hat{\beta}_1$.

(ii) when $\hat{\phi}_{ss} \leq \overline{\phi}$, $\overline{\phi} < \overline{\phi}_1$.

Proof. Since C_{ss} exist if K^h_{ss} and ϕ_{ss} exist, lemma 2 gives the above conditions. ■

The following lemma 7 and proposition 2 show the properties of the steady state level of consumption, C_{ss}.

Lemma 7 *The steady-state level of consumption increases as the steady-state level of households' investment, $\frac{dCss}{dK^h ss} > 0$ regardless of the level of minimum capital adequacy ratio, $\overline{\phi}$.*

Proof. Appendix K. ■

Proposition 2 *The steady-state level of consumption increases as the steady-state level of households' investment, $\frac{dCss}{d\theta} > 0$ regardless of the level of minimum capital adequacy ratio, $\overline{\phi}$.*

Proof. Lemma 3 and lemma 7 give the above property. ■

Proposition 2 implies C_{ss} decreases as its financial market develops with lower θ. Intuitively, if its financial market does not develop sufficiently, the households decide to manage their funds in the capital market but they reduce deposits with bankers. Moreover, there are few room to invest by both bankers and workers since the tightness of the financial market cause the demand for capital to diminish. Thus, even if θ is high, that is, the financial market does not develop, households consume more.

Since I obtain the steady state level of the following variables: ϕ_{ss}, $K^b_{ss} = \eta_{ss} K_{ss}$, $K_{ss} = K^b_{ss} + K^h_{ss}$, and $x_{ss} \equiv \frac{C_{ss}}{K_{ss}}$, I examine how the capital adequacy requirements affect the economy in the model. First, I examine how the minimum capital adequacy ratio, $\overline{\phi}$ affects the steady state level of consumption $\overline{C}^h_{ss}$ when $\phi_{ss} = \overline{\phi}_{ss}$. From equation (49), when $\phi_{ss} = \hat{\phi}_{ss}$, the minimum capital adequacy ratio $\overline{\phi}$ does not affect the consumption $\hat{C}_{ss}$.

Since lemma 7 shows that $\frac{dC_{ss}}{dK^h_{ss}} > 0$ regardless of the range of parameters. Using these results and (49), I investigate the conditions that $\overline{C}_{ss}$ is larger than $\hat{C}_{ss}$. Since $\hat{C}_{ss}$ does not depend on $\overline{\phi}$ and $\overline{C}_{ss}$ is achieved when $\hat{\phi}_{ss} < \overline{\phi}$, if $\overline{C}_{ss}$ increases as $\overline{\phi}$ is higher in a rage $\hat{\phi}_{ss} < \overline{\phi}$, $\overline{C}_{ss}$ is higher than $\hat{C}_{ss}$. The following proposition 3 and figures 2 and 3 summarize the above arguments.

Proposition 3 *Consider the case in which $\hat{\phi}_{ss} \leq \overline{\phi}$ where $\hat{\phi}_{ss} = \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}$.*

Let $\overline{\phi}_2$ be a positive value such that satisfies the following equation: $Z_1 \cdot (\overline{\phi}^2) + Z_2 \cdot \overline{\phi} = Z_3$ where $Z_1 \equiv -(1-\sigma)^2 \cdot [2 - (1-\beta)(1-\lambda)]$; $Z_2 \equiv -2(1-\sigma)\sigma\theta$; $Z_3 \equiv -\sigma^2\theta^2[1-(1-\beta)(1-\lambda)]$.

The steady state level of consumption $\overline{C}_{ss}$ increases as the minimum capital adequacy ratio $\overline{\phi}$ increases, if $\overline{\phi} \leq \overline{\phi}_2$ where $\overline{\phi}_2$ is an increase function of financial friction θ.

Proof. Lemma 7 shows $\frac{dC_{ss}}{dK^h_{ss}} > 0$ and Lemma 4 gives the condition of the sign of $\frac{dK^h_{ss}}{d\overline{\phi}}$. Thus, these lemma give the above property of $\frac{dC_{ss}}{d\overline{\phi}}$.[19] ■

The result of proposition 3 as shown in Figures 2 and 3. Figure 2 depicts the case $\hat{\phi}_{ss} \leq \overline{\phi}_2$ and Figure 3 depicts the case $\overline{\phi} > \overline{\phi}_2$. From proposition 3, since $\overline{\phi}_2$ increases as θ, $\overline{\phi}_2$ shift to the left. It implies that an economy whose financial market has not developed sufficiently with high θ is shown in as in Figure 2 and an economy whose financial market has developed sufficiently with low θ is shown in as in Figure 3. Suppose an economy at $\overline{\phi} = \hat{\phi_{ss}}$ in Figure 2. If regulatory authorities raise the minimum capital adequacy ratio from $\hat{\phi}_{ss}$ to $\overline{\phi}$ close to $\overline{\phi}_2$, the steady state level of consumption changes from $\hat{C}_{ss}$ to $\overline{C}_{ss}$ and this $\overline{C}_{ss}$ is higher than $\hat{C}_{ss}$ as shown in Figure 2. Thus, proposition 3 and Figure 2 imply that if the economy faces the large financial friction θ, regulatory authorities should raise the minimum capital adequacy ratio ϕ in order to achieve the higher level of consumption in the steady state and thus the higher growth rate of the economy. Moreover, if $\overline{\phi}_2 < 1$, the regulatory authorities should raise $\overline{\phi}$ and set $\overline{\phi} = \overline{\phi}_2$ to achieve the maximum of the steady state level of consumption C_{ss}.

[19] $\overline{\phi}$ is a function of θ and it increases as θ increases.

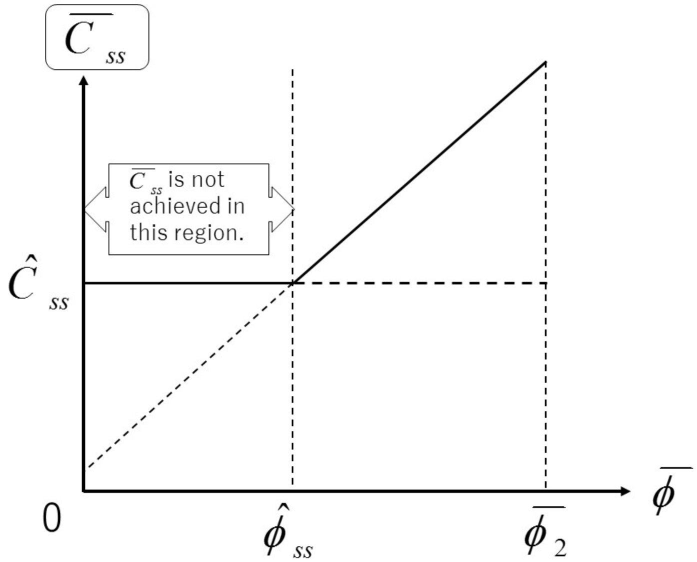

Figure 2: An economy whose financial market has not developed sufficiently (high θ)

On the other hand, proposition 3 and figure 3 imply that regulatory authorities should not raise the minimum capital adequacy ratio $\overline{\phi}$ in order to achieve in an economy whose financial market has developed sufficiently with less θ. Suppose an economy at $\overline{\phi} = \overline{\phi_2}$ in Figure 3. If regulatory authorities raise the minimum capital adequacy ratio, the steady state level of consumption changes from $\hat{C}_{ss}$ to $\overline{C}_{ss}$ and this $\overline{C}_{ss}$ is lower than $\hat{C}_{ss}$ as shown in Figure 3. In this case, there is no need to change the minimum capital adequacy ratio ϕ in order to achieve the higher steady state level of consumption. Thus, proposition 3 and Figure 3 imply that if the economy faces less financial friction θ, regulatory authorities should not change or lower the minimum capital ratio if the regulatory authorities aims to achieve the higher steady state consumption level and thus the higher growth rate of the economy.

Finally, let us consider the case where a negative productivity shock occurs at that economy. Substituting the relationship between the interest rate r_{ss} and the capital holdings by workers K_{ss}^h, (31), ϕ_{ss} given by equation (45) and K_{ss}^h given by equation (48) into the relationship between r_{ss} and the aggregate capital K_{ss} (35), the aggregate capital in the steady state K_{ss} can be determined:

$$K_{ss}^b + K_{ss}^h = (\alpha A)^{\frac{1}{1-\alpha}} \cdot \left[-\left(\frac{1-\phi_{ss}}{\phi_{ss}} \right) \omega K_{ss}^h + \frac{\theta}{\beta[(1-\sigma)\phi_{ss} + \sigma\theta]} \right]^{\frac{-1}{1-\alpha}}, \tag{50}$$

where K_{ss}^h is given by (46) and K_{ss}^b is given by (47).

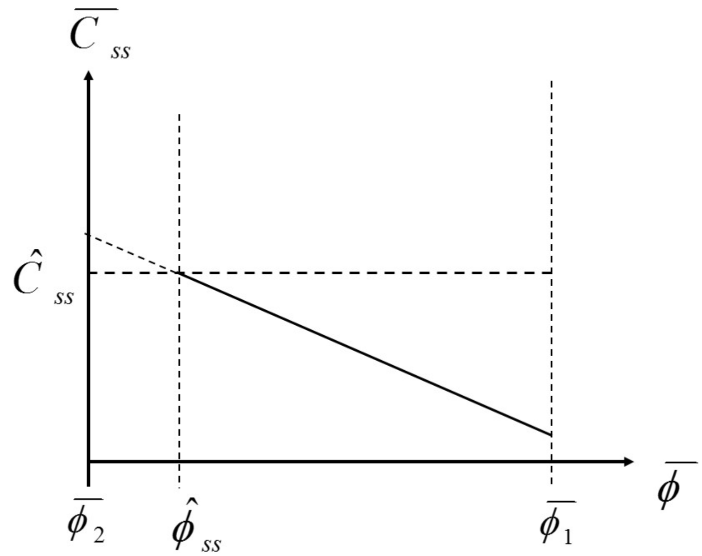

Figure 3: An economy whose financial market has developed sufficiently (low θ)

From equation and (50), I obtain

$$\frac{dK_{ss}^h}{dA} = \left(\frac{dK_{ss}^h}{dK_{ss}}\right)^{-1} \cdot \frac{1}{1-\alpha} \cdot K_{ss} + \frac{1}{A}. \tag{51}$$

Equation (51) tells us that the negative productivity shock decreases the steady state level of households' capital investment if it decreases the steady state level of aggregate capital. Using equation (51), lemma 6 and lemma 7, I obtain $\frac{dC_{ss}}{dA} > 0$. Thus, even if A declines, the above arguments about proposition 3, Figure 2 and 3 can be applied to the economy. In detail , when the negative productivity shock occurs in the economy, proposition 3 implies that deregulation have a good effect on the economy in only the country where the financial market sufficiently develops with less θ. On the other hand, even if the negative productivity shock occurs in the economy, proposition 3 implies that regulatory authorities should raise the minimum capital adequacy ratio $\overline{\phi}$ in order to achieve the higher steady state level of consumption in only the country where the financial market has not sufficiently developed with higher θ.

One of the key mechanism behind this result is the decreases of bankers' net worth in the non-production sector at the negative productivity shock. Because of financial frictions, the decreases of the aggregate capital has an ambiguous effect on the consumption in this model. If there is no financial friction, when the economy experiences a recession with decreases of aggregate capital, the investment and consumption decline. In conclusion, when regulatory authorities decide to change the minimum capital adequacy ratio, they should take care about the degree of financial development in order to achieve higher level of consumption in the steady state. Thus, the one of the contribution of this study is to show that there is a relationship between the degree of financial development and capital adequacy requirements.

5 Concluding Remarks

I introduce financial market frictions into a simple macroeconomic model. Using the current model, this study analyzes how capital adequacy requirements for banks affect the economy. I show that in the economy with larger financial frictions, regulatory authorities should raise the minimum capital adequacy requirements in order to improve the steady state level of consumption. Even if the economy faces a negative productivity shock, they should raise the minimum capital adequacy requirements in order to improve the steady state level of consumption in an economy with larger financial frictions. This result implies that relaxing the rule when recession is not always optimal for consumers. The condition for the above case depends on the degree of financial friction. Moreover, I show that when a negative shock on productivity occurs, deregulation have a good effect on the economy only in the country where the financial market sufficiently develops. Because of financial frictions, the decreases of the aggregate capital has an ambiguous effect on the economy in this model.

I examine the steady state in the economy; however, I have not examined the transition of the economy. This work is left for future work. Moreover, for future works, I calibrate this model and compare the real economy. Recently, many researchers and institutions have interests on the discussion about Basel III. To discuss this, I must extend this model to open economy. This is also left for future works.

Appendix

A Derivation of (24), (25) and (26)

Substituting equation (18') into $V(t)$ in left-hand side of (17) and $V(t+1)$ in the right-side hand of (17), I obtain

$$\mu(t)k^b(t) + \nu(t)n(t) = \beta(1-\sigma)n(t+1) + \beta\sigma\mu(t+1)k^b(t+1) + \beta\sigma\nu(t+1)n(t+1).$$

Since $\frac{n(t+1)}{k^b(t+1)} \equiv \hat{\phi}(t+1)$ from (20), substituting this definition into the above equation, I obtain

$$\mu(t)k^b(t) + \nu(t)n(t) = \beta(1-\sigma)n(t+1) + \beta\sigma\mu(t+1)\frac{n(t+1)}{\hat{\phi}(t+1)} + \beta\sigma\nu(t+1)n(t+1).$$

Substituting $n(t+1)$ given by equation (23) into the above equation, I obtain

$$\mu(t)k^b(t) + \nu(t)n(t) = \beta\Omega(t+1)\left(\left[r(t+1) - r^d(t+1)\right]k^b(t) + r^d(t+1)n(t)\right), \tag{52}$$

where

$$\Omega(t+1) \equiv (1-\sigma) + \frac{\sigma\mu(t+1)}{\hat{\phi}(t+1)} + \sigma\nu(t+1). \tag{26}$$

Since the coefficient of the left-hand side in (52) is equivalent to the coefficient of the right-hand side in (52), if the guess is correct, I obtain the following equations:

$$\mu(t) = \beta\Omega(t+1)\left[r(t+1) - r^d(t+1)\right], \tag{24}$$

$$\nu(t) = \beta\Omega(t+1)r^d(t+1), \tag{25}$$

where

$$\Omega(t+1) \equiv (1-\sigma) + \frac{\sigma\mu(t+1)}{\hat{\phi}(t+1)} + \sigma\cdot\nu(t+1). \tag{26}$$

B Derivation of (32)

Substituting equation (31) into equation (11), I obtain

$$
\begin{aligned}
&\omega^2 \cdot \sigma \cdot \left(K^h(t)\right)^2 - (1-\sigma)\cdot\omega\cdot(1-\lambda)\cdot K^h(t) - \delta\cdot(1-\lambda)^2 \\
+&\sigma\cdot\omega\cdot K^h(t)\cdot\left(\frac{1-\hat{\phi}(t)}{\hat{\phi}(t)}\right)\cdot\omega\cdot K^h(t) - \frac{\sigma\cdot\omega\cdot K^h(t)\cdot\theta\cdot\hat{\phi}(t+1)}{\beta\cdot[(1-\sigma)\cdot\hat{\phi}(t+1)+\sigma\cdot\theta]\cdot\hat{\phi}(t)} \\
+&\frac{(1-\lambda)\cdot(1-\sigma)\cdot\theta\cdot\hat{\phi}(t+1)}{\beta\cdot[(1-\sigma)\cdot\hat{\phi}(t+1)+\sigma\theta]\cdot\hat{\phi}(t)} - (1-\lambda)\cdot(1-\sigma)\cdot\left(\frac{1-\hat{\phi}(t)}{\hat{\phi}(t)}\right)\cdot\omega\cdot K^h(t) \\
&= 0
\end{aligned}
$$

The above equation can be rewritten as

$$
\hat{\phi}(t+1) = \Psi\left(K^h(t),\hat{\phi}(t)\right) \equiv \frac{\beta\cdot\sigma\cdot\theta\cdot\Gamma\left(K^h(t),\hat{\phi}(t)\right)}{\theta\cdot\omega\cdot\hat{\phi}(t)\cdot K^h(t) - (1-\sigma)\cdot(1-\lambda)\theta - \beta\cdot(1-\sigma)\cdot\Gamma\left(K^h(t),\hat{\phi}(t)\right)}, \tag{33}
$$

with

$$
\Gamma\left(K^h(t),\hat{\phi}(t)\right) \equiv \omega^2\cdot\sigma\cdot\left(K^h(t)\right)^2 - \omega\cdot K^h(t)\cdot(1-\lambda)\cdot(1-\sigma) - \delta\cdot\hat{\phi}(t)\cdot(1-\lambda)^2.
$$

Since $\hat{\phi}(t+1)$ is determined by equation (33) as long as the capital adequacy requirement (16) is satisfied, that is, as far as $\hat{\phi}(t+1) > \overline{\phi}$, I obtain equation (32).

C Derivation of (40)

Taking one period lag of equation (31), substituting this into equation (35), and dividing it by $K(t)$, I obtain

$$
\frac{Y(t)}{K(t)} = \frac{1}{\alpha}\cdot\left[\frac{\hat{\phi}(t-1)-1}{\hat{\phi}(t-1)}\right]\cdot\omega\cdot K^h(t-1) + \frac{\theta}{\alpha\beta\cdot\left[(1-\sigma)\cdot\hat{\phi}(t)+\sigma\theta\right]}\cdot\left(\frac{\hat{\phi}(t)}{\hat{\phi}(t-1)}\right). \tag{35'}
$$

The definition of $x(t)$ yields

$$
\frac{C(t)}{K(t)} = \frac{C(t-1)}{K(t)}\cdot\frac{C(t)}{C(t-1)} = x(t)\cdot\frac{C(t)}{C(t-1)}. \tag{53}
$$

Moreover, equations (7) and (8) yield

$$
\frac{C(t)}{C(t-1)} = \beta\cdot\frac{q(t-1)}{q(t)} = \frac{\beta\cdot(1-\delta)\cdot(1-\lambda)+\beta\cdot r(t)}{(1-\lambda)+\omega\cdot K^h(t-1)}. \tag{54}
$$

Substituting (54) into (53), I obtain

$$\frac{C(t)}{K(t)} = \beta \cdot x(t) \cdot \left[\frac{(1-\delta)\cdot(1-\lambda)+r(t)}{(1-\lambda)+\omega\cdot K^h(t-1)}\right], \tag{55}$$

where $r(t)$ is given by the function of $K^h(t-1)$, $\hat{\phi}(t)$, and $\hat{\phi}(t-1)$ in equation (31).

Substituting (35), (38), (55) and (1) into the market clearing condition of the goods market and dividing it by $K(t)$, I obtain

$$\frac{1}{\alpha}\cdot r(t) = \beta \cdot x(t) \cdot \left[\frac{(1-\delta)\cdot(1-\lambda)+r(t)}{(1-\lambda)+\omega\cdot K^h(t-1)}\right] + \frac{K(t+1)}{K(t)} - 1 + \delta + \frac{\omega}{2}\cdot\frac{\left(K^h(t)\right)^2}{K(t)}, \tag{56}$$

where $r(t)$ is given by the function of $K^h(t-1)$and $\hat{\phi}(t), \hat{\phi}(t-1)$ in equation (31).

Substituting $r(t)$ given by (31) into (56) and using the definition of $1-\eta(t)$, I can rewrite (56) as

$$\begin{aligned}
&-\beta\cdot x(t)\cdot\left[\frac{-\left(\frac{1-\hat{\phi}(t-1)}{\hat{\phi}(t-1)}\right)\cdot\omega\cdot K^h(t-1)+\frac{\theta}{\beta\cdot[(1-\sigma)\cdot\hat{\phi}(t)+\sigma\theta]}\cdot\frac{\hat{\phi}(t)}{\hat{\phi}(t-1)}+(1-\delta)\cdot(1-\lambda)}{(1-\lambda)+\omega\cdot K^h(t-1)}\right]\\
&+(1-\delta)+\frac{\theta}{\alpha\beta\cdot[(1-\sigma)\cdot\hat{\phi}(t)+\sigma\theta]}\cdot\frac{\hat{\phi}(t)}{\hat{\phi}(t-1)}-\frac{\omega}{\alpha}\cdot K^h(t-1)\cdot\frac{\left(1-\hat{\phi}(t-1)\right)}{\hat{\phi}(t-1)}\\
&=\frac{K(t+1)}{K(t)}+\omega\cdot\frac{(1-\eta(t+1))^2\cdot(K(t+1))^2}{K(t)}.
\end{aligned} \tag{56'}$$

The left-hand side of equation (56') depends on $x(t)$, $\hat{\phi}(t-1), \hat{\phi}(t), \eta(t)$ and $K(t)$ and the right-hand side of equation (56') depends on $K(t+1), K(t)$ and $\eta(t+1)$.

Using the definition of $\hat{\phi}(t)$, (56') can be rewritten as

$$\begin{aligned}
&\left(\frac{B(t+1)}{\eta(t+1)}\cdot\frac{\eta(t)}{B(t)}\right)\cdot\left[\frac{\theta\cdot((1-\lambda+\omega(1-\eta(t))K(t))-\alpha\beta\cdot x(t))}{\alpha\beta\left[(1-\sigma)\cdot\frac{B(t+1)}{\eta(t+1)}+\sigma\theta\right]}\right]+\frac{K(t+1)}{K(t)}\\
&+\omega\cdot\frac{(1-\eta(t+1))^2\cdot(K(t+1))^2}{K(t)}\\
&=\frac{\beta x(t)\cdot\left[(1-\delta)\cdot(1-\lambda)-\omega\cdot(1-\eta(t))\cdot K(t)\cdot\left(\frac{1-\frac{B(t)}{\eta(t)}}{\frac{B(t)}{\eta(t)}}\right)\right]}{(1-\lambda)+\omega\cdot(1-\eta(t))\cdot K(t)}.\\
&-\frac{\omega}{\alpha}\cdot(1-\eta(t))\cdot K(t)\cdot\frac{\left(1-\frac{B(t)}{\eta(t)}\right)}{\frac{B(t)}{\eta(t)}}+(1-\delta).
\end{aligned} \tag{40}$$

The left-hand side of equation (40) depends on $x(t)$, $B(t+1), \eta(t+1)$ and $K(t)$ and the right-hand side of equation (56') depends on $x(t), K(t), B(t)$ and $\eta(t)$.

D Derivation of (41)

From the definition of $x(t)$, I obtain

$$\frac{x(t+1)}{x(t)} \equiv \frac{\frac{C(t)}{K(t+1)}}{\frac{C(t-1)}{K(t)}} = \frac{C(t)}{\beta \cdot C(t-1)} \cdot \beta \cdot \left(\frac{K(t+1)}{K(t)}\right)^{-1}. \tag{57}$$

Substituting (7) and (8) into (57), I obtain

$$\frac{x(t+1)}{x(t)} = \frac{q(t-1)}{q(t)} \cdot \beta \cdot \left(\frac{K(t+1)}{K(t)}\right)^{-1}. \tag{57'}$$

Substituting (10) and (31) into (57'), I obtain

$$\frac{x(t+1)}{x(t)} = \beta \cdot \left(\frac{K(t+1)}{K(t)}\right)^{-1} \cdot \frac{(1-\lambda) - \left(\frac{\sigma}{\hat{\phi}(t-1)}\right) \cdot \omega \cdot K^h(t-1) + \frac{\sigma\theta \cdot \frac{\hat{\phi}(t)}{\hat{\phi}(t-1)}}{\beta \cdot [(1-\sigma)\cdot\hat{\phi}(t)+\sigma\theta]}}{1-\lambda}. \tag{57''}$$

Multiplying $\frac{K(t+1)}{K(t)}$ by the both sides of equation (57") and using $\hat{\phi}(t-1) \equiv \frac{B(t)}{\eta(t)}$ and $K^h(t-1) = (1-\eta(t)) \cdot K(t)$, I obtain

$$\begin{aligned} &\frac{x(t+1)}{x(t)} \cdot \frac{K(t+1)}{K(t)} \cdot (1-\lambda) - \frac{\sigma\theta \frac{B(t+1)}{\eta(t)} \frac{\eta(t+1)}{B(t)}}{(1-\sigma)\frac{B(t+1)}{\eta(t+1)} + \sigma\theta} \\ &= \beta\,(1-\lambda) - \beta \cdot \left(\frac{\sigma}{\frac{B(t)}{\eta(t)}} \cdot \omega\,(1-\eta(t))\,K(t)\right). \end{aligned} \tag{41}$$

E Derivation of (42)

The constraint of flow of funds of bankers (23) can be rewritten as the following equation for the aggregate variables:

$$N(t+1) = \left[r(t+1) - r^d(t+1)\right] \cdot K^b(t) + r^d(t+1) \cdot N(t). \tag{23'}$$

Substituting equation (3) into equation (23'), I obtain

$$N(t+1) = \omega \cdot K^h(t) \cdot K^b(t) + r(t+1) \cdot N(t) - \omega \cdot K^h(t) \cdot N(t). \tag{23''}$$

Taking one lag of (23") and using $N(t-1) = B(t) \cdot K(t)$ and $K^b(t-1) = \eta(t) \cdot K(t)$, I obtain

$$B(t+1) \cdot K(t+1) = \omega \cdot K^h(t-1) \cdot \eta(t) \cdot K(t) + r(t) \cdot B(t) \cdot K(t) - \omega \cdot K^h(t-1) \cdot B(t) \cdot K(t). \tag{58}$$

Dividing the both sides of (58) by $K(t)$, I obtain

$$B(t+1)\cdot\frac{K(t+1)}{K(t)}=\omega\cdot K^h(t-1)\cdot\eta(t)+r(t)\cdot B(t)-\omega\cdot K^h(t-1)\cdot B(t). \tag{58'}$$

From the definition of $\hat{\phi}(t)$, $B(t)=\eta(t)\cdot\hat{\phi}(t-1)$. Substituting it into (58'), I obtain

$$\eta(t+1)\cdot\hat{\phi}(t)\cdot\frac{K(t+1)}{K(t)}=\omega\cdot K^h(t-1)\cdot\eta(t)+r(t)\cdot\eta(t)\cdot\hat{\phi}(t-1)-\omega\cdot K^h(t-1)\cdot\eta(t)\cdot\hat{\phi}(t-1). \tag{58''}$$

Multiplying the both sides of (58") by $\frac{1}{\eta(t)\cdot\hat{\phi}(t-1)}$, I obtain

$$\frac{\eta(t+1)}{\eta(t)}\cdot\frac{\hat{\phi}(t)}{\hat{\phi}(t-1)}\cdot\frac{K(t+1)}{K(t)}=\omega\cdot K^h(t-1)\cdot\left(\frac{1-\hat{\phi}(t-1)}{\hat{\phi}(t-1)}\right)+r(t). \tag{59}$$

Substituting (31) into (59), I obtain

$$\frac{\eta(t+1)}{\eta(t)}\cdot\frac{\hat{\phi}(t)}{\hat{\phi}(t-1)}\cdot\frac{K(t+1)}{K(t)}=\omega\cdot K^h(t-1)\cdot\left(\frac{1-\hat{\phi}(t-1)}{\hat{\phi}(t-1)}\right)+\left(\frac{\hat{\phi}(t-1)-1}{\hat{\phi}(t-1)}\right)\cdot\omega\cdot K^h(t-1)+\frac{\theta\cdot\frac{\hat{\phi}(t)}{\hat{\phi}(t-1)}}{\beta\cdot[(1-\sigma)\cdot\hat{\phi}(t)+\sigma\theta]}. \tag{59'}$$

Substituting $\hat{\phi}(t)=\frac{B(t+1)}{\eta(t+1)}$ into (59'), I obtain

$$\frac{\eta(t+1)}{\eta(t)}\cdot\frac{K(t+1)}{K(t)}=\frac{\theta}{\beta\cdot[(1-\sigma)\cdot\frac{B(t+1)}{\eta(t+1)}+\sigma\theta]}.$$

$$\leftrightarrow\frac{K(t+1)}{K(t)}\cdot\frac{\eta(t+1)}{\eta(t)}\cdot\beta\cdot\left[(1-\sigma)\frac{B(t+1)}{\eta(t+1)}+\sigma\theta\right]=\theta. \tag{42}$$

F Derivation of (46)

In the steady state, $\frac{x(t+1)}{x(t)}=1$, $\frac{K(t+1)}{K(t)}=1$, $(1-\eta(t))\cdot K(t)=K^h_{ss}$, $\frac{B(t+1)}{B(t)}\cdot\frac{\eta(t+1)}{\eta(t)}=1$, and $\frac{B(t+1)}{\eta(t+1)}=\phi_{ss}$. Then, substituting these into (41), I obtain

$$(1-\lambda)-\frac{\sigma\theta}{(1-\sigma)\phi_{ss}+\sigma\theta}=\beta(1-\lambda)-\frac{\beta\sigma\omega K^h_{ss}}{\phi_{ss}}; \tag{41ss}$$

From (45), $(1-\sigma)\hat{\phi}_{ss}+\sigma\theta=\frac{\theta}{\beta}$ when $\phi_{ss}=\hat{\phi}_{ss}=\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}$ and $\phi_{ss}=\overline{\phi}$, when $\hat{\phi}_{ss}<\overline{\phi}$. Substituting into the above equations into the above equation in each case, I obtain

$$K^h_{ss}=\left\{\begin{array}{ll}\frac{\theta\cdot(1-\beta\sigma)\cdot[\sigma\beta-(1-\lambda)(1-\beta)]}{\omega\cdot\beta^2\sigma}\equiv\hat{K}^h_{ss}, & \text{when}\quad\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}\geq\overline{\phi}\\ \frac{\overline{\phi}[\sigma\theta-(1-\beta)(1-\lambda)[(1-\sigma)\overline{\phi}+\sigma\theta]}{\beta\sigma\omega[(1-\sigma)\overline{\phi}+\sigma\theta]}\equiv\overline{K}^h_{ss}, & \text{when}\quad\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}<\overline{\phi},\end{array}\right\}. \tag{46}$$

G Proof of Lemma 2

Proof. First, I consider the case in which $\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} > \overline{\phi}$. From equation (46), for the existence of K_{ss}^h for a positive value, the following condition must be satisfied;

$$\frac{\theta \cdot (1-\beta\sigma) \cdot [\sigma\beta - (1-\lambda)(1-\beta)]}{\omega \cdot \beta^2 \cdot \sigma} > 0.$$

The above inequality is satisfied if

$$\begin{aligned}
&[\beta\sigma - (1-\lambda)(1-\beta)] > 0 \\
&\leftrightarrow \lambda > \frac{1-\beta-\beta\sigma}{1-\beta} \\
&\leftrightarrow \lambda < \hat{\lambda}_1,
\end{aligned}$$

where $\hat{\lambda}_1 \equiv \frac{1-\beta-\beta\sigma}{1-\beta}$.

Since $\lambda > 0$, $\hat{\lambda}_1 > 0$ must be satisfied;

$$\begin{aligned}
&\hat{\lambda}_1 > 0 \\
&\leftrightarrow 1-\beta-\beta\sigma > 0 \\
&\leftrightarrow \beta < \frac{1}{1+\sigma} \equiv \hat{\beta}_1.
\end{aligned}$$

Since $\delta > 0$ and $\sigma > 0$, $\hat{\beta}_1$ satisfies $0 < \hat{\beta}_1 < 1$.

Thus, if $\lambda > \hat{\lambda}_1$ and $\beta < \beta_1$, then K_{ss}^h exists.

Second, I consider the case in which $\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \leq \overline{\phi}$. From equation (46), for the existence of K_{ss}^h for a positive value, the following condition must be satisfied;

$$\overline{K}_{ss}^h = \frac{\overline{\phi}[\sigma\theta - (1-\beta)(1-\lambda)[(1-\sigma)\overline{\phi} + \sigma\theta]}{\beta\sigma\omega[(1-\sigma)\overline{\phi} + \sigma\theta]} > 0.$$

Since $1-\sigma > 0$, the above inequality can be rewritten as follows:

$$\begin{aligned}
&\sigma\theta - (1-\beta)(1-\lambda)[(1-\sigma)\overline{\phi} + \sigma\theta] > 0 \\
&\leftrightarrow \frac{\sigma\theta[1-(1-\beta)(1-\lambda)]}{(1-\lambda)\cdot(1-\beta)\cdot(1-\sigma)} > \overline{\phi} \\
&\leftrightarrow \overline{\phi}_1 > \overline{\phi},
\end{aligned}$$

where $\overline{\phi}_1 \equiv \frac{\sigma\theta[1-(1-\beta)(1-\lambda)]}{(1-\lambda)\cdot(1-\beta)\cdot(1-\sigma)}$.

Since $1-\beta>0$, $1-\lambda>0$ and $1-\sigma>0$, then $\overline{\phi}_1>0$.

Hence, $\phi_{ss}=\overline{\phi}$, $\overline{K}^h_{ss}$ exists if $\frac{\theta\cdot(1-\beta\sigma)}{\beta\cdot(1-\sigma)}\geq\overline{\phi}<\overline{\phi}_1$.

■

H Proof of Lemma 3

Proof. Consider the case in which $\overline{\phi}<\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}\equiv\hat{\phi}_{ss}$. Differentiating equation (46) with respect to $\overline{\phi}$, I obtain

$$\frac{\sigma\theta[1-(1-\beta)(1-\lambda)]-2(1-\sigma)\overline{\phi}\beta\sigma\omega[(1-\sigma)\overline{\phi}+\sigma\theta]-(1-\sigma)\overline{\phi}[\sigma\theta-(1-\beta)(1-\lambda)[(1-\sigma)\overline{\phi}+\sigma\theta]\beta\omega\theta}{\omega^2\cdot\sigma\dot{\beta}^2\cdot\left[(1-\sigma)\overline{\phi}+\sigma\theta\right]^2}. \tag{60}$$

(60) implies that $\frac{d\overline{K}^h_{ss}}{d\overline{\phi}}>0$ if

$$\begin{aligned}&-\left(-2(1-\sigma)^2+(1-\beta)(1-\lambda)(1-\sigma)^2\right)\cdot(\overline{\phi}^2)-(-2(1-\sigma)\sigma\theta)\cdot\overline{\phi}\\&>-\sigma^2\theta^2[1-(1-\beta)(1-\lambda)].\end{aligned} \tag{61}$$

Let Z_1, Z_2,andZ_3, be such that $Z_1\equiv-2(1-\sigma)^2+(1-\beta)(1-\lambda)(1-\sigma)^2$; $Z_2\equiv-2(1-\sigma)\sigma\theta$; $Z_3-\sigma^2\theta^2[1-(1-\beta)(1-\lambda)].\equiv$. Due to the condition for existence of $\overline{K}^h_{ss}$ from (46) and $\beta\in(0,1)$ and thus Z_3 takes a negative value. Since $\beta\in(0,1)$ and $\sigma\in(0,1)$, $Z_1<0$ and $Z_2<0$.

Let $\overline{\phi}_2$ be a positive value such that satisfies the following equation: $Z_1\cdot(\overline{\phi}^2)+Z_2\cdot\overline{\phi}=Z_3$. As shown in Figure 4, (61) is satisfied if $0<\overline{\phi}<\overline{\phi}_2$. The sign of $\frac{dK^h_{ss}}{d\overline{\phi}}>0$ can be satisfied if $\overline{\phi}_2$ is sufficiently high.

(ii) Since $\overline{\phi}_2$ satisfies $Z_1\cdot(\overline{\phi}^2)+Z_2\cdot\overline{\phi}=Z_3$, $\overline{\phi}_2$ increases as Z_3 decreases. In addition, Z_3 decreases as θ become higher. Thus, $\overline{\phi}_2$ is an increase function that increases of θ.

Hence, $\frac{d\overline{K}^h_{ss}}{d\overline{\phi}}>0$ if θ is high sufficiently such that $\overline{\phi}<\overline{\phi}_2$

■

I Proof of Lemma 5

Proof. Differentiating equation (47) with respect to K^h_{ss}, I obtain

$$\frac{dK^b_{ss}}{dK^h_{ss}}=\frac{\sigma\cdot\left[\beta\omega K^h_{ss}\cdot(\sigma\theta+(1-\sigma)\phi_{ss})-\theta(\phi_{ss})^2\right]}{\beta\cdot[(1-\sigma)+\phi_{ss}]\cdot[\sigma\theta+(1-\sigma)\cdot\phi_{ss}]}. \tag{62}$$

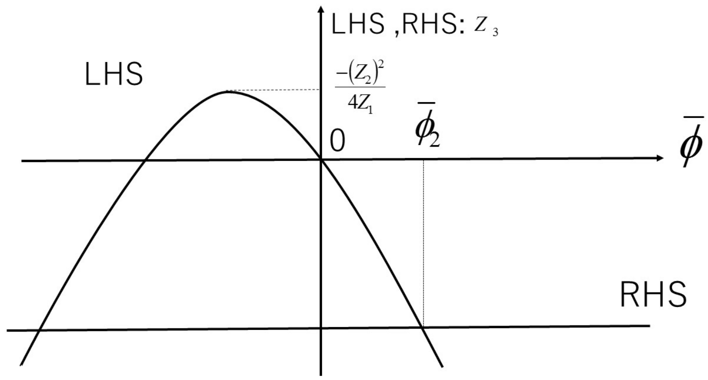

Figure 4: Equation $Z_1 \cdot (\overline{\phi}^2) + Z_2 \cdot \overline{\phi} = Z_3$

From (62), $\frac{dK^b_{ss}}{dK^h_{ss}} > 0$ if

$$\beta\omega K^h_{ss} > \frac{\theta \cdot (\phi_{ss})^2}{\sigma\theta + (1-\sigma)\cdot\phi_{ss}}. \tag{63}$$

First, I consider the case in which $\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} > \overline{\phi}$. Substituting $K^h_{ss} = \hat{K}^h_{ss}$ and $\phi_{ss} = \hat{\phi}_{ss}$ from (45) and (46) into the condition (63), I obtain

$$\begin{aligned} & 2\beta\omega\hat{K}^h_{ss} > \frac{\theta \cdot (\hat{\phi}_{ss})^2}{\sigma\theta + (1-\sigma)\cdot\hat{\phi}_{ss}} \\ \leftrightarrow & \frac{2\beta(1-\sigma)^2 \cdot [\beta - (1-\beta(1-\delta))] - \theta^2(1-\beta\sigma)\cdot[(2-\beta)(1-\beta\sigma)\theta + \beta^2(1-\sigma)]}{2\beta(1-\sigma)^2\cdot(1-\beta(1-\delta))} > \lambda. \end{aligned} \tag{64}$$

Let $\hat{\lambda}_2$ be such that $\hat{\lambda}_2 \equiv \frac{4\beta(1-\sigma)^2\cdot[\beta-(1-\beta(1-\delta))]-\theta^2(1-\beta\sigma)\cdot[(2-\beta)(1-\beta\sigma)\theta+\beta^2(1-\sigma)]}{2\beta(1-\sigma)^2\cdot(1-\beta(1-\delta))}$. From (64), $\frac{d\hat{K}^b_{ss}}{d\hat{K}^h_{ss}} > 0$ if $\hat{\lambda}_2 > \lambda$.

Second, I consider the case in which $\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \leq \overline{\phi}$. Substituting $K^h_{ss} = \overline{K}^h_{ss}$ and $\phi_{ss} = \overline{\phi}_{ss}$ from (45) and (46) into the condition (63), I obtain

$$\begin{aligned} & 2\beta\omega\overline{K}^h_{ss} > \frac{\theta \cdot (\overline{\phi}_{ss})^2}{\sigma\theta + (1-\sigma)\cdot\overline{\phi}_{ss}} \\ \leftrightarrow & \frac{\theta\cdot[(2-\beta) - 2\beta\sigma\cdot(1+(1-\sigma)\lambda)\cdot(1-\beta(1-\delta))]}{[\theta\cdot(1-\ \beta) + (1-\sigma)\cdot 2\beta\cdot(1+(1-\sigma)\lambda)(1-\beta(1-\delta))]} > \overline{\phi}. \end{aligned} \tag{65}$$

Let $\overline{\phi}_2$ be such that $\overline{\phi}_2 \equiv \frac{\theta\cdot[(2-\beta)-2\beta\sigma\cdot(1+(1-\sigma)\lambda)\cdot(1-\beta(1-\delta))]}{\left[\theta\cdot\left(1-\frac{\beta}{2}\right)+(1-\sigma)\cdot\beta\cdot(1+(1-\sigma)\lambda)(1-\beta(1-\delta))\right]}$.From (65), $\frac{d\overline{K}^b_{ss}}{d\overline{K}^h_{ss}} > 0$ if $\overline{\phi}_2 > \overline{\phi}$. ■

J Proof of Lemma 6

Proof. Since $K_{ss} = K_{ss}^h + K_{ss}^b$, equation (47) yields

$$K_{ss} = \frac{K_{ss}^h \cdot \beta \cdot [(1-\sigma)+\phi_{ss}] \cdot [\sigma\theta + (1-\sigma)\cdot\phi_{ss}]}{\beta \cdot [(1-\sigma)+\phi_{ss}] \cdot [\sigma\theta + (1-\sigma)\cdot\phi_{ss}]} + \frac{\sigma \cdot K_{ss}^h \cdot \left[\beta\omega \cdot [\sigma\theta + (1-\sigma)\phi_{ss}] \cdot K_{ss}^h - \theta\phi_{ss}^2\right] + (1+(1-\sigma)\lambda)\cdot\phi_{ss}\cdot[(1-\sigma)(1-\phi_{ss}) - \delta\beta\sigma\theta]}{\beta \cdot [(1-\sigma)+\phi_{ss}] \cdot [\sigma\theta + (1-\sigma)\cdot\phi_{ss}]}. \tag{66}$$

Differentiating equation (66) with respect to K_{ss}^h, I obtain

$$\frac{dK_{ss}}{dK_{ss}^h} = \frac{\beta \cdot [\sigma\theta + (1-\sigma)\phi_{ss}] \cdot [(1-\sigma) + \phi_{ss} + 2\sigma\omega K_{ss}^h] - \theta\sigma \cdot (\phi_{ss})^2}{\beta \cdot [(1-\sigma)+\phi_{ss}] \cdot [\sigma\theta + (1-\sigma)\cdot\phi_{ss}]}. \tag{67}$$

(67) implies that $\frac{dK_{ss}}{dK_{ss}^h} > 0$ if

$$K_{ss}^h > \frac{\theta\sigma(\phi_{ss})^2 - \beta\cdot[\sigma\theta + (1-\sigma)\phi_{ss}]\cdot[(1-\sigma)+\phi_{ss}]}{\beta\sigma\omega\cdot[\sigma\theta + (1-\sigma)\cdot\phi_{ss}]}. \tag{68}$$

Let Λ be such that $\Lambda \equiv \frac{\theta\sigma(\phi_{ss})^2 - \beta\cdot[\sigma\theta+(1-\sigma)\phi_{ss}]\cdot[(1-\sigma)+\phi_{ss}]}{\beta\sigma\omega\cdot[\sigma\theta+(1-\sigma)\cdot\phi_{ss}]}$. Then, (62') is satisfied if

$$\begin{aligned}
&\Lambda < 0 \\
\leftrightarrow &\theta\sigma(\phi_{ss})^2 - \beta\cdot[\sigma\theta + (1-\sigma)\phi_{ss}]\cdot[(1-\sigma)+\phi_{ss}] < 0 \\
\leftrightarrow &(\theta\sigma - \beta(1-\sigma))\cdot(\phi)_{ss}^2 - \beta\cdot[\sigma\theta + (1-\sigma)^2]\cdot\phi_{ss} < \beta\sigma\theta(1-\sigma).
\end{aligned} \tag{69}$$

First, consider the case in which $\overline{\phi} < \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \equiv \hat{\phi}_{ss}$.

Substituting $\hat{\phi}_{ss}$ into (69), I obtain

$$\sigma\theta^2(1-\beta\sigma)^2 - \beta(1-\sigma)(1-\beta\sigma)\theta < \beta^2(1-\sigma)^3. \tag{69-1}$$

Let $\hat{\theta}_2$ be such that satisfies the following equation for a positive value:

$$\sigma\theta^2(1-\beta\sigma)^2 - \beta(1-\sigma)(1-\beta\sigma)\theta = \beta^2(1-\sigma)^3. \tag{69-1'}$$

Similarly, let $\hat{\theta}_1$ be such that satisfies equation (69-1') for a negative value.

As shown in Figure 5, (69-1) is satisfied if $\theta < \hat{\theta}_2$. Since $\theta > 0$, $\frac{dK_{ss}}{dK_{ss}^h} > 0$ if $0 < \theta < \hat{\theta}_2$.

Second, consider the case in which $\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \leq \overline{\phi}$. Substituting $\overline{\phi}_{ss}$ into (69), I obtain

$$(\theta\sigma - \beta(1-\sigma))\cdot(\overline{\phi})^2 - \beta\cdot[\sigma\theta + (1-\sigma)^2]\cdot\overline{\phi} < \beta\sigma\theta(1-\sigma). \tag{69-2}$$

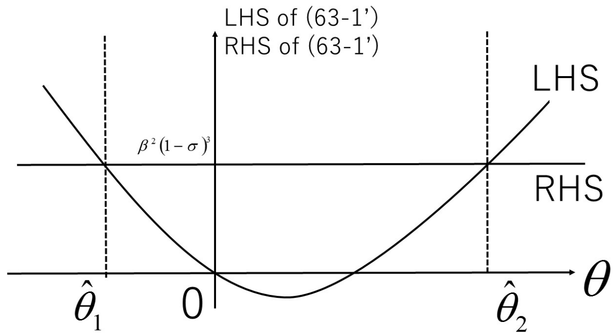

Figure 5: Equation (69-1')

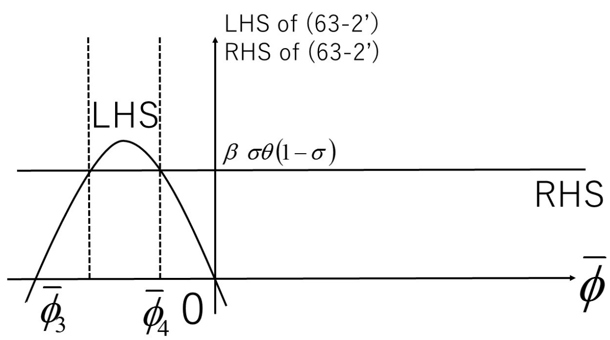

Figure 6: Equation (69-2') with $\overline{\theta}_1 > \theta$

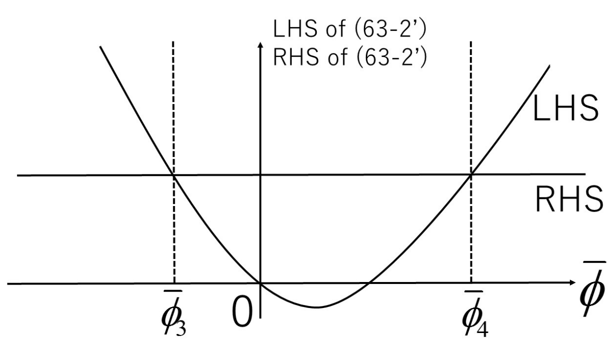

Figure 7: Equation (69-2') with $\overline{\theta}_1 \leq \theta$

Let $\overline{\phi}_3$ and $\overline{\phi}_4$ be such that satisfies the following equation with $\overline{\phi}_3 < \overline{\phi}_4$:

$$(\theta\sigma - \beta(1-\sigma)) \cdot (\overline{\phi})^2 - \beta \cdot [\sigma\theta + (1-\sigma)^2] \cdot \overline{\phi} = \beta\sigma\theta(1-\sigma). \tag{69-2'}$$

The left-hand side of (69-2') is convex upwards if $(\theta\sigma - \beta(1-\sigma)) < 0$ as shown in Figure 2. The left-hand side of (69-2') is convex downwards if $(\theta\sigma - \beta(1-\sigma)) > 0$ as shown in Figure 6. Let $\overline{\theta}_1$ be such that $\overline{\theta}_1 \equiv \frac{\beta(1-\sigma)}{\sigma}$. Then, $(\theta\sigma - \beta(1-\sigma)) < 0$ can be rewritten as $\overline{\theta}_1 > \theta$.

As shown in Figures 5 and 6, $\frac{d\overline{K}_{ss}}{d\overline{K}^h_{ss}} > 0$, if $\overline{\theta}_1 > \theta$ or $\overline{\theta}_1 \leq \theta$ and $0 < \overline{\phi} < \overline{\phi}_4$.

■

K Proof of Lemma 7

Proof. Differentiating equation (49) with respect to K^h_{ss}, I obtain

$$\begin{aligned}\frac{dCss}{dK^hss} = & \frac{\left[\omega \cdot \left(\frac{1-\phi_{ss}}{\phi_{ss}}\right) \cdot K^h_{ss} - (1-\delta) - \left(\frac{\theta}{\alpha\beta[(1-\sigma)\phi_{ss}+\sigma\theta]}\right)\right] \cdot \frac{dK}{dK^h_{ss}}}{\beta(1-\delta)(1+(1-\sigma)\lambda) - \beta\omega K^h_{ss} \cdot \left(\frac{1-\phi_{ss}}{\phi_{ss}}\right) + \left(\frac{\theta}{(1-\sigma)\phi_{ss}+\sigma\theta}\right)} \\ & + \frac{\beta\omega \cdot \left(\frac{1-\phi_{ss}}{\phi_{ss}}\right) \cdot \left(\left[\omega \cdot K^h_{ss} \cdot \left(\frac{1-\phi_{ss}}{\phi_s s}\right) - (1-\delta) - \left(\frac{\theta}{\alpha\beta \cdot [(1-\sigma) \cdot \phi_{ss}+\sigma\theta]}\right)\right] \cdot K_{ss} + \omega \cdot \left(K^h_{ss}\right)^2\right)}{\left[\beta(1-\delta)(1+(1-\sigma)\lambda) - \beta\omega K^h_{ss} \cdot \left(\frac{1-\phi_{ss}}{\phi_{ss}}\right) + \left(\frac{\theta}{(1-\sigma)\phi_{ss}+\sigma\theta}\right)\right]^2}.\end{aligned} \tag{70}$$

Since $\left(\left[\omega \cdot K^h_{ss} \cdot \left(\frac{1-\phi_{ss}}{\phi_s s}\right) - (1-\delta) - \left(\frac{\theta}{\alpha\beta \cdot [(1-\sigma) \cdot \phi_{ss}+\sigma\theta]}\right)\right] \cdot K_{ss} + \omega \cdot \left(K^h_{ss}\right)^2\right)$ is positive due to C_{ss} is non-negative in equation (49), the second term of the right-hand side in equation (70) is positive.

The first-term of the right-hand side in equation (70) is positive if

$$\underbrace{\left[\omega \cdot \left(\frac{1-\phi_{ss}}{\phi_{ss}}\right) \cdot K^h_{ss} - (1-\delta) - \left(\frac{\theta}{\alpha\beta[(1-\sigma)\phi_{ss}+\sigma\theta]}\right)\right]}_{(*)} \cdot \frac{dK}{dK^h_{ss}} > 0. \tag{71}$$

Substituting K^h_{ss} and ϕ_{ss} from (45) and (46) into the term $(*)$ of (70), I confirm this term is negative. Due to the condition of existence of K^h_{ss} from lemma 2, β is sufficiently large such that $\beta_1 < \beta$ and then the term $(*)$ of (71) is close to 0. Thus, I obtain the inequality (71). Because the first term of (69) is sufficiently small and close to 0, and the second term of (70) is positive regardless of the minimum capital adequacy ratio $\overline{\phi}$, I obtain $\frac{dC_{ss}}{dK^h_{ss}} > 0$. ■

References

[1] Adrian, T., & Shin, H. (2009). Money, Liquidity and Monetary Policy. *mimeo, Federal Reserve Bank of New York and Princeton University.*

[2] Angelini, P., Neri, S., & Panetta, F. (2014). Interaction between Capital Requirements and Monetary Policy. *Journal of Money, Credit and Banking*, 46 (6), 1073–112.

[3] Angeloni, I., & Faia, E. (2013). Capital Regulation and Monetary Policy with Fragile Banks. *Journal of Monetary Economics*, 60 (3), 311–324.

[4] Aoki, K., Benigno, G., & Kiyotaki, N. (2016). Monetary and Financial Policies in Emerging Markets. *mimeo, Princeton University.* Baker, M., & Wurgler, J. (2015). Do Strict Capital Requirements Raise the Cost of Capital? Banking Regulation and the Low Risk Anomaly, *American Economic Review*, 105 (5), 315–320.

[5] Barth, J.R, Caprio Jr., G., & Levine, R. (2004). Banking Regulation and Supervision. *Journal of Financial Intermediation*, 13, 205–248.

[6] Benigno, G., Chen, H., Otrok, C., Rebucci, A., & Young, E. (2010). Financial Crises and Macro-Prudential Policies. *Journal of International Economics*, 89 (2), 453–470.

[7] Bianchi, J., & Mendoza, E.G.(2010). Overborrowing, Financial Crises and 'Macro-Prudential' Taxes. *NBER Working Paper*, No. 1609.

[8] Blum, J., & Hellwig, M. (1995). The Macroeconomic Implications of Capital Adequacy Requirements for Banks. *European Economic Review*, 39, 739–749.

[9] Collard, F., Dellas, H., Diba, B., & Loisel, O. (2017). Optimal Monetary and Prudential Policies. *American Economic Journal: Macroeconomics*, 9 (1), 40–87.

[10] Francis, W.B., & Osborne, M,. (2012). Capital Requirements and Bank Behavior in the UK: Are There Lessons for International Capital Standards?. *Journal of Banking and Finance*, 36, 803–816.

[11] Furfine, C. (2000). Evidence on the Response of US Banks to Changes in Capital Requirements. *BIS Working Papers*, No. 88.

[12] Gertler, M., & Kiyotaki, N. (2010). Financial Intermediation and Credit Policy in Business Cycle Analysis. *Handbook of Monetary Economics*, 3 (3), 547–599.

[13] Gertler, M., & Kiyotaki, N. (2015). Banking Liquidity and Bank Runs in an Infinite Horizon Economy. *American Economic Review*, 105 (7), 2011–2043.

[14] Gertler, M., Kiyotaki, N., & Queralto, A. (2012). Financial Crises, Bank Risk Exposure and Government Financial Policy. *Journal of Monetary Economics*, 59, S17–S34.

[15] He, Z., & Krishnamurthy, A. (2008). Intermediary Asset Pricing. *American Economic Review*, 103 (2), 732–770.

[16] Kannan, P., Rabanal, A., & Scott, A. (2012). Monetary and Macroprudential Policy Rules in a Model with House Price Booms. *B.E. Journal of Macroeconomics*, 12 (1), Article 16.

[17] Kiyotaki, N., & Moore, J. (1997). Credit Cycles. *Journal of Political Economy*, 105 (2), 211–248.

[18] Medina, J.P., & Roldos, J. (2014). Monetary and Macroprudential Policies to Manage Capital Flows. *IMF Working Paper*, WP/14/30.

[19] Rochet, J.C. (1992). Capital Requirements and the Behavior of Commercial Banks. *European Economic Review*, 36 (5), 1137–1170.

[20] Unsal, F. (2013). Capital Flows and Financial Stability: Monetary Policy and Macroprudential Responses. *International Journal of Central Banking*, 9 (1), 232–285.