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Abstract

In this paper we investigate a wide class of principal–agent problems with moral hazard and target budgets. The latter requires that the principal fixes a total budget for the wages paid to agents regardless of their outputs realized ex post. Target budgets are relevant not just because they are exogenous institutional constraints in some cases, but they can also endogenously arise in other cases, especially when agents' performances are not verifiable and thus the principal needs subjective evaluations. Although target budgets impose an additional constraint, we show the irrelevance theorem that the principal is never worse off using target budgets when there are at least two risk-neutral agents. Even when all agents are risk averse, we also show that the similar irrelevance result asymptotically holds if the number of agents is sufficiently large. Furthermore, we characterize optimal contracts when the target budget becomes a tight constraint so that the irrelevance result cannot be applied.

Keywords: Moral Hazard, Multiple Agents, Subjective Evaluation, Target Budgets

JEL Classification Numbers: D82, D86

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1 Introduction

1.1 Motivation and Results

Performance-based compensation is necessary to provide the right incentives to agents whose actions are not publicly observable (Holmström (1979), Grossman and Hart (1983)). However, in many real-life situations, compensation is limited by predetermined budgets, as we show in the several examples below. The presence of such budget constraints restricts the set of available contracts and this conflicts with the incentive provision considered in standard models of moral hazard.

In this paper we investigate a wide class of principal–agent problems with moral hazard and target budgets. A principal hires multiple agents who work on behalf of herself. Although the principal cannot directly observe the actions chosen by agents, she can observe their performance signals, called outputs herein. In addition to the incentive compatibility (IC) and individual rationality (IR) constraints, we introduce the target budget constraint that requires that once the principal determines a total budget for the sum of the wages paid to all agents ex ante, she must follow the budget regardless of the agents' outputs realized ex post.

Target budgets are relevant for many organizational problems in real world. First and perhaps most importantly, they endogenously arise when the principal needs to evaluate agents subjectively based on the privately observed outputs (we discuss more details about the related literature on this issue later). Empirical evidence shows that workers are commonly evaluated by subjective rather than objective performance measures (see Murphy (1993), Bushman, Indjejikian, and Smith (1996), and Gibbs, Merchant, Van Der Stede, and Vargus (2009) for related evidence). Under such subjective evaluations, the principal has an incentive to manipulate the outputs of agents so as to reduce the wage payments to them. Such opportunistic behavior by the principal in turn undermines the ex ante incentives of agents to work hard. Getting rid of the incentive for manipulation, the principal needs to commit the total amount of wage payments whatever outputs are realized ex post.[1] This is because if the total wages of agents vary with their outputs, the principal has an incentive to misreport the outputs unless observed outputs require her to pay the minimum amount.

Second, target budgets are also relevant as exogenous institutional constraints even when agents' outputs are objective and verifiable. For example, according to their accounting and budgeting policies, firms or divisions within a firm may meet the annual budget constraint for

[1]Although the principal may burn money to commit herself to paying a fixed amount, money burning is never optimal, as we show later.

the wages paid to employees. Governments also set fiscal budgets for the payrolls of officials working in the public sector. Similarly, some universities allocate budgets among different academic units according to their performances. For example, over last three decades several universities in U.S. have introduced a budget allocating rule, called Responsibility Center Management (RCM), in order to induce different academic units to compete for students (Whalen (1991)). Under RCM, academic units of an university are distributed tuition revenues from the university's budget based on verifiable outcomes such as the number of attracted students (Wilson (2002)). Such a budget allocating rule creates the incentives for academic units to improve educational qualities. As a related example, in several countries governments or independent institutes allocate fixed budgets for research and teaching funding among different public schools and universities based on their research and teaching outcomes. In U.K. an independent institute called Higher Education Funding Council for England (HEFCE) receives a public funding from the U.K government and allocate it among different universities for research and teaching. In particular, the allocation of research funding is based on the research qualities of universities which are evaluated periodically by Research Assessment Exercise (RAE).[2]

What are the optimal contracts when a target budget is imposed in addition to the standard IC and IR constraints? We call a contract *the third-best contract* when it maximizes the principal's payoff subject to all IC, IR and target budget constraints. If there are no target budget constraints, standard moral hazard models suggest that when agents are technologically independent of each other, the optimal contract should be separately and independently designed for each agent. However, when the total amount of wages is fixed at a predetermined constant, the optimal contracts for agents can no longer be independent, since the wage of some agent must depend on outputs of others, even if they are technologically independent of each other. When the wage of some agent, say agent i, increases as his output rises, the wage of at least one other agent, say agent $j \neq i$, must be reduced to make the total wages of all agents constant.

Such wage interdependency complicates the characterization of the third-best contract. Our main objective in this paper is to address this problem and analyze how the presence of target budgets affects the characterization of third-best contracts.

Our findings are summarized as follows. First, we show the irrelevance theorem that the principal is never worse off by target budgets as long as there are at least two risk-neutral agents. Even in the presence of target budgets, the principal can achieve the second-best

[2]For more details, see the web site of HEFCE: http://www.hefce.ac.uk/funding/annallocns/.

payoff,[3] which she would obtain if the target budget constraint were absent.

From this efficiency result, we can obtain a significant implication about subjective evaluations. As already mentioned, our model is applied to the case of subjective evaluations that agents' outputs are not publicly revealed. In that case, the principal may manipulate her reports about privately observed outputs. One traditional approach to address this issue is to consider the long-term relationships between the principal and agents, which ensures that the principal will be punished in the future if she breaches the current promise (Levin (2002), Rayo (2007), Mukherjee and Vasconcelo (2011), and Ishihara (2017)). However, these previous studies have not fully investigated what the principal can attain in the static benchmark. Indeed, they have simply stated that the principal can implement only the least costly actions from agents in the static case. For example, Levin (2002) states as follows: *"In this environment the static equilibrium is straightforward. Because the firm cannot commit to reward performance, workers will do no more than the minimum, and the firm will do best not to produce at all" (p. 1079)).*

Although this statement is true when there is a single agent, the same conclusion may not hold when there are multiple agents. Indeed, the papers cited above all employ models with multiple agents. Do static contracts always work worse than relational contracts do as these papers have mentioned? Our efficiency result shows that this is not the case. In the static environment in which agents' outputs are not verifiable (more severely, they are privately observed by the principal), the total wages of agents must be fixed at a constant. However, the principal is never worse off from the second-best case with fully verifiable outputs if there are at least two risk-neutral agents.[4] Thus, there is no further room to improve efficiency by relational contracting. We highlight this result in comparison with the existing literature on relational contracts with multiple agents, which has emphasized the advantage of relational contracting over static contracts (see the studies cited above).

The intuition behind our efficiency result is as follows. There must be at least two agents whose wages are interdependent as already discussed. For the ease of exposition, suppose that there are two risk neutral agents while all others are risk averse.[5] The principal can only offer risk neutral agents interdependent wage schemes, while offering the second-best independent contracts to risk-averse agents. Then, risk-averse agents face the same expected

[3] We use the terminology *the second best* because the principal is constrained by the IC constraints of agents ensuring that any contract must induce agents to choose the right actions owing to their self-interest. This is distinguished from the *first best*, which is the optimal outcome achieved when agents' actions are directly contractible.

[4] Most studies of relational contracts with multiple agents have focused on the case of risk-neutral agents.

[5] We can easily adapt our argument to the case of more than two risk neutral agents.

payoffs as those they would obtain in the second-best situation if no target budget constraint were present. They thus choose the second-best actions and obtain the second-best payoffs. In addition, the wage schemes of two risk-neutral agents are constructed such that they are paid not only according to piece rates depending on their own individual outputs; they also receive an equal share of the *residual wage*, which is defined as the fixed total payroll minus the sum of the piece rate wages paid to all agents. Thus, risk-neutral agents serve to balance total wage payments, thereby ensuring that the principal ends up paying an ex ante fixed amount regardless of the realized outputs. Since risk-neutral agents can absorb all the risk of the residual wage, no additional costs are associated with the risks caused under such wage schemes. Hence, the principal can achieve the second-best payoff even when she faces a target budget constraint.

We can also connect our efficiency result to the literature on rank-order contracts (e.g., Lazear and Rosen (1981), Malcomson (1984)). Rank-order contracts motivate agents while keeping the total prizes paid to them constant: the agent who performs the best obtains the highest prize, the agent who performs the second highest obtains the second highest prize, and so on. Since total prizes are fixed, the principal has no incentive to misreport the outputs of agents even when they are not verifiable. However, our efficiency result shows that rank-order contracts never become optimal; rather, all agents except risk-neutral ones should be offered piece rate wages that depend only on their own outputs. Thus, it is not optimal to use relative performance evaluations such as rank-order contracts for all agents when the total budget for their wages is fixed.

We next turn to the case of at most one risk-neutral agent so that the above efficiency result no longer applies. Although it is complicated to fully characterize the third-best contract in general settings, we provide the useful characterization result that the third-best contract is given by a tractable linear formula, which we call the *simple sharing rule*: the wage of each agent is a linear combination of two parts. One is the piece rate wage which depends only on his own output, and the other is the share of the residual wage defined as the difference between a fixed total wage and the sum of all agents' piece rate wages. We show that this simple sharing rule becomes optimal if and only if agents' preferences over income lotteries are represented by utility functions with constant absolute risk aversion (CARA). The shares of the residual wage are determined by the relative magnitudes of agents' risk aversion. More risk-averse agents are rewarded by more piece rate wages but a lower share of the residual wage.

Beyond the specific forms of utility functions, it is difficult to characterize the third-best contract in general. However, we show that such a contract has simple and important features in the asymptotic case as the number of agents becomes sufficiently large. In particular, we show that under the third-best contract, most agents are compensated according to almost their own individual outputs but not those of others when the number of agents is sufficiently large. While the wage of each agent may depend on the outputs of others, such effects become negligible when the number of agents is sufficiently large. In this way, most agents are rewarded according to almost their piece rates or individual performance evaluation (hereafter, IPE). Hence, relative performance evaluations such as rank-order contracts tend to have no value asymptotically in large organizations.

We show this asymptotic result as follows. Contrary to the above claim, if a large proportion of agents is offered wage schemes that are not IPE under the third-best contract, the principal can then increase her payoff by enlarging the set of agents offered IPE wage schemes. To this end, the principal divides the set of all agents into two subsets. The agents belonging to one set are offered IPE wage schemes, which induce them to choose the same actions as those under the original third-best contract. All the other agents belonging to the remaining subset either randomly face an IPE wage scheme or equally share the residual wage defined as the difference between the fixed total payroll and the wages paid to the agents who work under IPE schemes. The agents paid residual wages serve to balance the total wage payments of all agents regardless of the outputs realized. This random wage scheme imposes a risk on some agents because, with a positive probability, they must share the residual wage depending on the outputs of others. However, each of them needs to bear only a small share of such risk as the number of agents is large because of the Law of Large Numbers. In this way, by enlarging the set of agents offered IPE wage schemes, the principal can improve efficiency by reducing the risk that agents would otherwise incur under the original third-best contract.

As a corollary of the above result, we also show that the principal can approximate the second-best payoff on average as the number of agents becomes sufficiently large. When there are many agents, those who equally share the residual wage incur only low risk. Thus, agents face virtually the same wage schemes as the second-best ones when the number of agents is sufficiently large. Then, the principal succeeds in achieving the second-best payoff in the limit as the number of agents tends to infinity. We show this asymptotic efficiency result by using the simple sharing rule mentioned above. Therefore, this rule is shown to be asymptotically optimal in large organizations, even when no particular restrictions are made on the utility functions of agents and their production technologies.

1.2 Related Literature

Three strands of the literature are related to our study. As already discussed, our model is connected to the literature on subjective evaluations. First, several recent studies have considered models of relational contracts with multiple agents in which their outputs are not verifiable and hence not contractible (see Malcomson (2013) for a recent survey on the developments in relational contracts). Mukherjee and Vasconcelo (2011) and Ishihara (2017) investigate the job design problem of how agents are responsible for different tasks and how they are formed as teams. Levin (2002) finds the value of making relational contracts multilateral, which benefits the principal more than making relational contracts with separate agents independently. Rayo (2007) focuses on the relational incentives in a team where agents contribute to the team's outputs repeatedly. Kvaløy and Olsen (2006) also consider team-based incentives in the dynamic relationships among the principal and multiple agents. Our main insight is that even in the static environment, the principal can recover the second-best payoff that would be attained if formal contracts contingent on outputs were enforced. This is in sharp contrast to the studies above that implicitly assume that static contracts cannot induce agents to work efficiently when their outputs are not verifiable.

Second, some studies have focused on target budgets as a solution to the problem of subjective evaluations in static settings. One approach to this issue is to use money burning (MacLeod (2003), Kambe (2006)).[6] The principal commits herself to pay a fixed amount whatever outputs are realized. However, such fixed expenditure is not always equal to what the agent receives. When an agent's output is low, the principal pays a low bonus from a fixed budget and discard the remaining amount of the budget. Since the principal always spends the same expenditure regardless of the realized output of the agent, she has no incentive to lie about agent's outputs. In addition, the agent is given the right incentive to work hard because his wage can vary with his output. However, money burning may be an unrealistic solution to the problem of subjective evaluations because it is inefficient for contracting parties to discard useful resources or pay third parties ex post. In this paper we show that, even when we allow money burning so that total wages of agents can be less than what the principal pays, it is never used at the optimal contract which endogenously satisfies the budget-balancing condition that the principal pays the same amount as what agents totally receive. This can thus avoid wasteful resource destruction.

Third, rank-order contracts are also connected to the role of target budgets in static settings (Lazear and Rosen (1981), Malcomson (1984)). Under rank-order contracts, agents are paid according to the ranking determined by their relative performance and the total

[6] See also Fuchs (2007) for a dynamic extension of these static models.

prizes paid to all agents are fixed. It is known that rank-order contracts work well to motivate agents even when total budgets for prizes are fixed if two important restrictions are imposed: (i) agents are risk neutral and (ii) environments are symmetric in the sense that agents are homogeneous and play a symmetric equilibrium strategy. Instead of imposing these strong restrictions, we consider more general principal–agent problems with target budget constraints in which agents are allowed to be heterogeneous. Then, we show in our Propositions 1, 2, and 3 that the optimal contract has a different feature from rank-order contracts. Under rank-order contracts, the wage schedule of an agent has a (discontinuously) larger slope with respect to his or her outputs when his or her peers perform worse. However, we show that the third-best contract that solves the moral hazard problem with the target budget constraint has no such property; rather, how the wage of each agent is sensitive to his output is independent of those of other agents.

Furthermore, except for the case that all agents are risk-neutral, rank-order contracts cannot attain the second best when the outputs of agents are statistically independent of each other. This is because the wage of each agent depends on the outputs of others under rank-order contracts and hence risk-averse agents incur higher risk under rank-order contracts than under piece rate contracts, which depend only on their own individual performances. We avoid this problem by ensuring that the risk-neutral agents share the risk of variations in the residual wage equally, while keeping the total wage of all agents constant.

Our asymptotic efficiency result is also related to Green and Stokey (1983) and Malcomson (1986), who show that rank-order contracts asymptotically perform at least as well as piece rate contracts do as the number of agents becomes sufficiently large.[7] However, these studies focus only on the symmetric case in which agents are identical and follow a symmetric equilibrium strategy. Rank-order contracts may not perform better than piece rates do when agents are heterogeneous because in such a case, the former must manage the different effort incentives of heterogeneous agents by using the unique prize structure for all agents.[8] On the contrary, our efficiency result is valid in more general environments that allow heterogeneous agents and impose no symmetric restrictions on the equilibrium behaviors of agents.

The remaining sections are organized as follows. In Section 2, we set up the basic model. In Section 3, we show that the principal can fully implement the second-best payoff as long

[7]Green and Stokey (1983) also show that rank-order contracts sometimes perform better than piece rate contracts do when agents can privately observe some common shock affecting their individual performances.

[8]For example, under the standard rank-order tournament, the winner's prize does not depend on whoever wins (i.e., the identity of the winner).

as there are at least two risk-neutral agents even when she faces a target budget constraint. In Section 4, we characterize the third-best contract when there is at most one risk-neutral agent. Then, we show that the third-best contract becomes a tractable linear formula if and only if agents' preferences over income lotteries are represented by CARA utility functions. In Section 5, we show the asymptotic result. First, we show that the third-best contract must entail the property that most agents must be compensated according to almost their individual outputs when the number of agents is sufficiently large. Second, we show that the principal can approximate the second-best payoff by adopting a simple wage scheme as the number of agents is sufficiently large.

2 Model

2.1 Principal–Agent Environment with Moral Hazard

We consider a static moral hazard problem with one risk-neutral principal and N (risk-averse or risk-neutral) agents. With slight notational abuse, we use the same symbol N to denote the set of agents. In what follows, we also use a feminine pronoun for the principal and a masculine pronoun for agents. Agent i chooses action $a_i \in A \subset \mathbb{R}$, which stochastically determines his performance signal, called output $y_i \in Y \subset \mathbb{R}$. Agent i's action a_i is observable only to himself (i.e., it is not contractible). On the contrary, agent i's output y_i is observed by the principal. As discussed in the Introduction, the realization of agents' outputs $(y_1, ..., y_N)$ may be subjective evaluations by the principal. In this respect, they may be privately observed only by the principal and hence not verifiable.

To save notation, we assume that the sets of actions A and outputs Y are the same for all agents, although this is not essential for the following analysis. Further, although some of the results that follow hold both when the output of agent y_i is discrete and when it is continuous, we fix Y to be a finite set. In particular, we assume that Y has K distinct elements ($K \geq 2$) and denote by y^k a generic element of Y, i.e., $Y \equiv \{y^1, ..., y^K\}$. Later, we discuss how we can drop this finiteness assumption for some results. Furthermore, our results can be extended to the case that an agent's action set A and output set Y are multidimensional, although we do not pursue such a case to avoid complicated notation.

The outputs of agents $y_1, ..., y_N$ are independently distributed. We denote by $p_i(y_i|a_i) \in (0, 1)$ the probability of agent i's output being y_i conditional on his action $a_i \in A$. Here, $\sum_{y \in Y} p_i(y|a) = 1$ for all $a \in A$. We denote by $P(\mathbf{y}|\mathbf{a}) \equiv \prod_{i=1}^{N} p_i(y_i|a_i)$ the joint probability of an output profile $\mathbf{y} = (y_1, ..., y_N)$ of N agents conditional on their action profile $\mathbf{a} =$

$(a_1, ..., a_N)$.[9] In what follows, we use the notation $E_{\mathbf{y}}[\,\cdot\,|\mathbf{a}]$ to denote the expectation over output profile $\mathbf{y} \in Y^N$ of N agents conditional on their action profile $\mathbf{a} \in A^N$. Similarly, by letting $y_{-i} \equiv (y_1, ..., y_{i-1}, y_{i+1}, ..., y_N)$ and $a_{-i} \equiv (a_1, ..., a_{i-1}, a_{i+1}, ..., a_N)$, we denote by $E_{y_{-i}}[\,\cdot\,|a_{-i}]$ the expectation over the outputs of others than agent i, $y_{-i} \in Y^{N-1}$, conditional on an action profile a_{-i} of those agents. We also denote $E_{y_i}[\cdot|a_i]$ the expectation over agent i's output y_i conditional on his action a_i.

As mentioned in the Introduction, we rule out money burning; hence, the sum of the transfers made by the principal and agents must balance ex post regardless of the outputs $\mathbf{y} \in Y^N$ realized. Let $t_i(\mathbf{y}) \in \mathbb{R}$ denote the net transfer received by agent i and $t_p(\mathbf{y}) \in \mathbb{R}$ that by the principal when output profile $\mathbf{y} \in Y^N$ is realized. Then, we require that $\sum_{i=1}^N t_i(\mathbf{y}) + t_p(\mathbf{y}) = 0$ for any $\mathbf{y} \in Y^N$. Thus, without loss of generality, we set $t_i(\mathbf{y}) \equiv w_i(\mathbf{y})$, which is interpreted as the wage agent i receives, and $t_p(\mathbf{y}) = -\sum_{i=1}^N w_i(\mathbf{y})$ for any $\mathbf{y} \in Y^N$.

Agent i has the utility function defined on his wage income w_i and action a_i, denoted by $U_i : \mathbb{R} \times A \to \mathbb{R}$, and his utility is given by

$$U_i(w_i, a_i) \tag{1}$$

which is assumed to be increasing and concave with his wage w_i. Agent i obtains the reservation utility $\overline{U}_i$ when he rejects the contract offered by the principal.

The principal obtains her private benefit or revenue from the outputs of agents $\mathbf{y} \in Y^N$, denoted by $R(\mathbf{y})$. We assume that the principal's private benefit R is observable only to herself and hence non-verifiable. Given the wage profile $\mathbf{w} \equiv \{w_i\}_{i=1}^N$ paid to agents, the principal obtains the following expected payoff:

$$E_{\mathbf{y}}[R(\mathbf{y})|\mathbf{a}] - \sum_{i=1}^N w_i \tag{2}$$

2.2 Second-Best Contract

We begin with the standard moral hazard problem as the benchmark case: the principal maximizes her expected payoff (see (2)) subject to the IC and IR constraints (Grossman and Hart (1983)). The principal makes contracts contingent on the realization of outputs $\mathbf{y} \in Y^N$. The wage scheme for agent i is defined as a mapping $w_i : Y^N \to \mathbb{R}$, which specifies his wage w_i contingent on the realization of the output profiles $\mathbf{y} \in Y^N$ of agents. We denote by $\{w_i, a_i\}$ a contract for agent i, where w_i is the wage scheme and a_i is an action instructed for agent i to choose.

[9] Throughout the paper, we use a bold letter to denote a vector of the variables.

In this benchmark, the principal chooses contracts $\{w_i, a_i\}_{i=1}^N$ to solve the following *second-best problem*:

Problem SB

$$\max_{\mathbf{w},\mathbf{a}} E_{\mathbf{y}}[R(\mathbf{y})|\mathbf{a}] - \sum_{i=1}^{N} E_{\mathbf{y}}[w_i(\mathbf{y})|\mathbf{a}]$$

subject to

$$E_{\mathbf{y}}[U_i(w_i(\mathbf{y}), a_i)|a_i, a_{-i}] \geq E_{\mathbf{y}}[U_i(w_i(\mathbf{y}), a_i'')|a_i'', a_{-i}], \quad \text{for any } a_i'' \neq a_i \tag{IC}$$

$$E_{\mathbf{y}}[U_i(w_i(\mathbf{y}), a_i)|a_i, a_{-i}] \geq \overline{U}_i \tag{IR}$$

Here, IC denotes the incentive compatibility constraint that agents choose the desired actions $\mathbf{a}$ as a Nash equilibrium and IR is the individual rationality constraint ensuring that each agent accepts the offered contract. We call a contract that solves Problem SB *the second-best contract*. We also call the principal's payoff attained under the second-best contract *the second-best payoff*.

Since the outputs of agents are statistically independent of each other and there are no technological externalities between their actions, one might think that the second-best contract for agent i should be contingent only on his own output y_i. We call wage scheme w_i *independent performance evaluation* (IPE) or piece rate when it depends solely on agent i's output, y_i, that is, $w_i = w_i(y_i)$.

We denote by $\{\hat{w}_i, \hat{a}_i\}_{i=1}^N$ the second-best contract solving Problem SB and make the following assumption.

Assumption 1. *There exists an IPE contract $\{\hat{a}_i, \hat{w}_i\}_{i=1}^N$ that solves Problem SB where each agent i's wage scheme $\hat{w}_i$ depends only on his own output y_i.*

Assumption 1 holds when the agent's utility function satisfies the so-called risk independence condition commonly assumed in standard principal–agent models. This condition means that the agent's preference over income lotteries is independent of his action. Such condition holds if and only if the utility function of agent i takes the form $U_i(w_i, a_i) \equiv u_i(w_i)H_i(a_i) - G_i(a_i)$ for some functions u_i, H_i, and G_i (Keeney (1973)).

Assumption 1 is weaker than the additively separable utility function used in the literature. Indeed, we can show that the second-best contract becomes IPE without loss of generality when the risk independence condition is satisfied.

Lemma 1. *Suppose that the utility function of each agent i takes the form given by $U_i(w_i, a_i) = u_i(w_i)H_i(a_i) - G_i(a_i)$. Then, the second-best contract $\hat{w}_i$ for agent i, which solves Problem SB, depends only on his own output y_i, that is, $\hat{w}_i(y_i)$.*

Proof. See the Appendix.

Under Assumption 1, we confine our attention to the second-best contract, which is IPE: $w_i = \hat{w}_i(y_i)$ for each agent i. With this in mind, we consider the cost minimization problem for implementing action $a_i \in A$ from agent i in the second-best problem as follows:

Problem M-SB

$$\min_{w_i} \ E_{y_i}[w_i(y_i)|a_i]$$

subject to

$$E_{y_i}[U_i(w_i(y_i), a_i)|a_i] \geq E_{y_i}[U_i(w_i(y_i), a_i'')|a_i''], \ \ \forall \, a_i'' \neq a_i \tag{IC}$$

$$E_{y_i}[U_i(w_i(y_i), a_i)|a_i] \geq \overline{U}_i \tag{IR}$$

We denote by $\hat{w}_i(\cdot; a_i)$ the optimal solution to the above problem for implementing action $a_i \in A$ from agent i, provided the constraint set is non-empty. In particular, with slight notational abuse, by dropping argument $\hat{a}_i$, we denote by $\hat{w}_i(\cdot) \equiv \hat{w}_i(\cdot; \hat{a}_i)$ the second-best wage scheme for implementing the second-best action $\hat{a}_i$.

The principal pays the following total expected wages of N agents under the second-best contract $\{\hat{w}_i, \hat{a}_i\}_{i=1}^N$:

$$\hat{W} \equiv \sum_{i=1}^{N} E_{y_i}[\hat{w}_i(y_i)|\hat{a}_i]$$

and obtains the following second-best payoff:

$$\hat{\Pi} \equiv E_{\mathbf{y}}[R(\mathbf{y})|\hat{\mathbf{a}}] - \sum_{i=1}^{N} E_{y_i}[\hat{w}_i(y_i)|\hat{a}_i].$$

3 Target Budget and Efficiency Result

We now turn to the case that the principal faces a target budget constraint as well as IC and IR constraints. The principal first determines a target budget $\overline{W}$ for the sum of the wages paid to all agents. She cannot change this target budget regardless of the outputs of agents realized ex post once it is predetermined ex ante. As discussed in the Introduction, one relevant case for this is subjective evaluations under which the realization of agents' outputs $\mathbf{y}$ is observable only to the principal, meaning that the total wages of agents must be constant regardless of their outputs realized.

Target budgets require that the total wages of agents must be fixed for any realization of their outputs as follows:

$$\sum_{i=1}^{N} w_i(\mathbf{y}) = \overline{W} \quad \text{for any } \mathbf{y} \in Y^N. \tag{FTW}$$

Then, the principal solves the following problem, which we call the *third-best problem*:

Problem TB

$$\max_{\mathbf{a},\mathbf{w},\overline{W}} \quad E_{\mathbf{y}}[R(\mathbf{y})|\mathbf{a}] - \overline{W}$$

subject to FTW defined above together with

$$E_{\mathbf{y}}[U_i(w_i(\mathbf{y}), a_i)|a_i, a_{-i}] \geq E_{\mathbf{y}}[U_i(w_i(\mathbf{y}), a_i'')|a_i'', a_{-i}], \quad \text{for any } a_i \neq a_i''. \tag{IC}$$

and

$$E_{\mathbf{y}}[U_i(w_i(\mathbf{y}), a_i)|\mathbf{a}] \geq \overline{U}_i \tag{IR}$$

for all $i \in N$.

Since FTW is an additional constraint, the principal is never better off from the second-best case. However, FTW may not constrain the principal at all. Indeed, we show that the second-best contract $\{\hat{w}_i, \hat{a}_i\}_{i=1}^N$ solves the above Problem TB when there are at least two risk-neutral agents. In other words, FTW does not cause any efficiency loss compared with the second-best case.

We say that agent i is risk-neutral if his utility function U_i is represented by

$$U_i(w_i, a_i) = H_i(a_i)w_i - G_i(a_i)$$

for some functions H_i and G_i. Thus, $E_{y_i}[U_i(w_i(y_i), a_i)|a_i] = U_i(E_{y_i}[w_i(y_i)|a_i], a_i)$ for any action $a_i \in A$ and any wage scheme w_i when agent i is risk-neutral. We denote by $I_r \subseteq N$ the set of risk-neutral agents and by $L \equiv \# I_r$ the number of them.

Then, we show that the principal can fully attain the second-best payoff even when she faces the target budget constraint (FTW) as long as there are at least two risk neutral agents. Before proceeding, we give an intuition behind this result by using a simple example. Suppose that there are only two agents and they are all risk neutral ($N = \{1, 2\}$ and $L = 2$).

Let $U_i(w_i, a_i) = w_i - G_i(a_i)$ denote agent i's payoff. Recalling that $\hat{w}_i$ denotes the second-best wage scheme for agent $i = 1, 2$, we define the wage scheme for agent i as follows

$$w_i(y_i, y_j) = \tilde{w}_i(y_i) + (1/2)\{\overline{W} - \tilde{w}_i(y_i) - \tilde{w}_j(y_j)\}$$

for $i, j = 1, 2$ and $i \neq j$, where we set

$$\tilde{w}_i(y_i) \equiv 2\hat{w}_i(y_i) - E_{y_i}[\hat{w}_i(y_i)|\hat{a}_i]$$

for $i = 1, 2$. We also set $\overline{W} = \sum_{i=1,2} E[\hat{w}_i(y_i)|\hat{a}_i]$ which is equal to the total expected wage at the second-best optimum. Then, by its definition, $w_1(y_1, y_2) + w_2(y_2, y_1) = \overline{W}$ holds for any $(y_1, y_2) \in Y^2$ so that FTW is satisfied. Furthermore, we can readily see that $E[w_i(y_i, y_j)|a_i, \hat{a}_j] = E_{y_i}[\hat{w}_i(y_i)|a_i]$ for any action $a_i \in A$. Thus, agent i faces the same expected wage as what he obtains under the second-best scheme whatever actions he chooses, given agent j choosing the second-best action $\hat{a}_j$. Then, both agents choose the second-best actions so as to maximize their expected payoffs, $E_{y_i}[\hat{w}_i(y_i)|a_i] - G_i(a_i)$, and obtain the second-best payoffs.

The key of the above argument is that agent i is offered a piece rate contract $\tilde{w}_i(y_i)$, which is contingent only on his own output y_i, and equally shares the residual wage $\overline{W} - \sum_i \tilde{w}_i(y_i)$ defined as the fixed budget $\overline{W}$ minus total wages. Such a sharing scheme ensures that total wages of agents become a constant at $\overline{W}$ irrespective of their outputs $\mathbf{y} \in Y^2$ while providing them the right incentives to choose the second-best actions.

When there are more than two risk neutral agents together with risk averse agents, we can generalize the above argument by offering the similar sharing schemes only to risk neutral agents while offering the second-best (and hence piece rate) wage schemes to risk averse agents.

Thus, we show the following result.

Proposition 1. *Suppose that $L \geq 2$. Then, the principal can exactly attain the second-best payoff $\hat{\Pi}$ even when the total wages of agents must be fixed for any realization of their outputs.*

Proof. We set the wage scheme of risk-averse agent n by the second-best one $\hat{w}_n$:

$$w_n(y_n) \equiv \hat{w}_n(y_n), \quad n \notin I_r$$

Then, agent $n \notin I_r$ chooses the second-best action $\hat{a}_n$ and obtains the same expected payoff $E_{y_n}[U_n(\hat{w}_n(y_n), \hat{a}_n)|\hat{a}_n]$ as that under the second-best contract.

We now consider risk-neutral agent $i \in I_r$. We define the wage scheme for risk-neutral agent $i \in I_r$ as follows:

$$w_i(y_i, y_{-i}) \equiv \tilde{w}_i(y_i) + (1/L)\left\{\hat{W} - \sum_{n \in N \setminus I_r} \hat{w}_n(y_n) - \sum_{j \in I_r, j \neq i} \tilde{w}_j(y_j) - \tilde{w}_i(y_i)\right\} \quad (3)$$

where $\tilde{w}_i$ is defined as

$$\tilde{w}_i(y_i) \equiv \frac{L}{L-1}\hat{w}_i(y_i) - \frac{1}{L-1}E_{y_i}[\hat{w}_i(y_i)|\hat{a}_i]. \quad (4)$$

Here,

$$\hat{W} \equiv \sum_{n=1}^{N} E_{y_n}[\hat{w}_n(y_n)|\hat{a}_n]$$

denotes the total expected wages of all agents under the second-best contract. Note that $\tilde{w}_i$ is well defined because of $L \geq 2$. Moreover, $E_{y_i}[\tilde{w}_i(y_i)|\hat{a}_i] = E_{y_i}[\hat{w}_i(y_i)|\hat{a}_i]$ for each $i \in I_r$. In addition, the wage profile $(w_1, ..., w_N)$ defined above satisfies FTW:

$$\sum_{n=1}^{N} w_n(\mathbf{y}) = \hat{W} \quad \text{for any } \mathbf{y} \in Y^N$$

Given this, risk-neutral agent $i \in I_r$ obtains the following expected wage conditional on others choosing the second-best actions $\hat{a}_{-i}$:

$$\begin{aligned}
&E_{\mathbf{y}}[w_i(y_i, y_{-i})|a_i, \hat{a}_{-i}] \\
&= E_{\mathbf{y}}\left[\frac{L-1}{L}\tilde{w}_i(y_i) + \frac{1}{L}\left(\hat{W} - \sum_{n \notin I_r} \hat{w}_n(y_n) - \sum_{n \in I_r, n \neq i} \tilde{w}_n(y_n)\right) \Big| a_i, \hat{a}_{-i}\right] \\
&= E_{y_i}[\hat{w}_i(y_i)|a_i] - (1/L)E_{y_i}[\hat{w}_i(y_i)|\hat{a}_i] + (1/L)\left(\hat{W} - \sum_{n \neq i} E_{y_n}[\hat{w}_n(y_n)|\hat{a}_n]\right) \\
&= E_{y_i}[\hat{w}_i(y_i)|a_i] \quad (5)
\end{aligned}$$

for any $a_i \in A$ because $\sum_{n=1}^{N} E_{y_n}[\hat{w}_n(y_n)|\hat{a}_n] = \hat{W}$ holds. Thus, risk-neutral agent i obtains the expected payoff as follows:

$$\begin{aligned}
E_{\mathbf{y}}[U_i(w_i(\mathbf{y}), a_i)|a_i, \hat{a}_{-i}] &= U_i(E_{\mathbf{y}}[w_i(\mathbf{y})|a_i, \hat{a}_{-i}], a_i) \\
&= U_i(E_{y_i}[\hat{w}_i(y_i)|a_i], a_i) \\
&= E_{y_i}[U_i(\hat{w}_i, a_i)|a_i] \quad (6)
\end{aligned}$$

for any action $a_i \in A$. This payoff is the same as that attained under the second-best contract $\hat{w}_i$. Thus, risk-neutral agent i chooses the second-best action $\hat{a}_i$ provided all others choose the second-best actions $\hat{a}_{-i}$. The risk-neutral agents also obtain the same expected payoffs under the second-best contract. Thus, all agents obtain at least the reservation utilities, meaning that IR is satisfied. In this way, the principal can implement the second-best actions $\hat{\mathbf{a}}$ at the same total wage cost as that under the second-best contract. Q.E.D.

We obtain Proposition 1 by generalizing the intuition aforementioned in the example of two risk neutral agents. To avoid a trivial case, suppose that the second-best action $\hat{a}_i$ is not the least costly one for each $i \in N$. Since the total wages of all agents must be fixed (FTW), whenever all agents are induced to choose different actions from the least costly ones, at least two agents must be offered the wage schemes, which depend not only on their own outputs but also on those of others. To see this, suppose that all agents but agent i are offered wage schemes that depend solely on their individual outputs $w_j(y_j)$ for any $j \neq i$. Then, FTW implies $w_i = \overline{W} - \sum_{j \neq i} w_j(y_j)$; hence, agent i's wage depends only on the outputs of others. However, agent i never works hard. Thus, there must be a different agent $j \neq i$ whose wage depends on the outputs of others y_{-j}. In this way, to make total wages constant, at least two agents must be offered some interdependent wage schemes that vary with the outputs of others.

When there are at least two risk-neutral agents, the principal can have these agents bear all the risk under interdependent wage schemes. On the contrary, risk-averse agents are offered the second-best wage schemes $\{\hat{w}_i\}_{i \notin I_r}$, which are independent of the outputs of others. Thus, they incur no additional risk and choose the second-best actions. Risk-neutral agent i is offered the wage scheme (given by (3)) consisting of an IPE wage $\tilde{w}_i(y_i)$, which is slightly modified from the second-best one $\hat{w}_i$, and an equal share of the residual wage $(1/L)\{\hat{W} - \sum_{n \notin L} \tilde{w}_n(y_n) - \sum_{n \in L} \hat{w}_n(y_n)\}$. Then, risk-neutral agents absorb all the risk of the residual wage and are induced to choose the second-best actions. By construction, the principal ends up paying the second-best cost $\hat{W}$ regardless of the outputs realized.

We derive several implications from Proposition 1 as follows. First and most importantly, Proposition 1 implies that the principal incurs no efficiency loss from the target budget constraint. Hence, she can achieve the second-best payoff even when she must fix the total wages of agents because of the problem of subjective evaluations. This has a notable implication for the role of relational contracting, which can alleviate the problem of subjective evaluations. Studies of relational contracts with multiple agents have implicitly assumed that the principal cannot motivate agents to work efficiently at the static benchmark when their performance signals are not verifiable (e.g., Levin (2002), Rayo (2007) and Ishihara (2017)). Indeed, most papers have focused on the benefits of repeated transactions between the principal and agents, which can prevent the former from reneging on promised payments to the latter. However, in contrast to these studies, we show that the principal can attain the second-best payoff that she would obtain if the outputs of agents were verifiable even in the static environment in which they are not verifiable (and even when they are privately observed by the principal). Since subjective evaluations never cause efficiency loss even in the static setting, there are no gains from relational contracting even when the principal and agents contract repeatedly over time. Proposition 1 therefore suggests that we reconsider the roles of static contracts in dynamic settings as well as the value of relational contracting when the principal hires multiple agents whose outputs are not verifiable.

Second, Proposition 1 implies that when all agents are risk-neutral, the principal can implement the first-best even if she faces a target budget constraint. The first-best is defined as the outcome attained when agents' actions are verifiable. Suppose that all agents are risk neutral and their utility functions are given by $U_i(w_i, a_i) = w_i - G_i(a_i)$ without loss of generality.[10] Then, the first-best action profile, denoted by $\mathbf{a}^{fb} = (a_1^{fb}, ..., a_N^{fb})$, is defined to maximize the total expected surplus of the principal and N agents:

$$\Pi^{fb} \equiv \max_{\mathbf{a} \in A^N} \ E_{\mathbf{y}}[R(\mathbf{y})|\mathbf{a}] - \sum_{i=1}^{N} G_i(a_i) \tag{7}$$

Even when agents' actions are not verifiable, the first best is still implemented by some IPE wage schemes as long as their outputs are verifiable. For example, we can consider the following wage scheme for agent i:

$$w_i^{fb}(y_i) \equiv E_{y_{-i}}[R(y_i, y_{-i})|a_{-i}^{fb}] + T_i \tag{8}$$

where T_i is set such that the IR of agent i holds as an equality:

$$E_{\mathbf{y}}[R(\mathbf{y})|\mathbf{a}^{fb}] + T_i - G_i(a_i^{fb}) = \overline{U}_i \tag{9}$$

Then, agents choose the first-best actions $\mathbf{a}^{fb}$ given the above wage schemes. Thus, the second-best contract that solves Problem SB coincides with the first best. In other words, one of the second-best contracts solving Problem SB becomes $\hat{w}_i(y_i) = w_i^{fb}(y_i)$ for each agent $i \in N$, as defined in (8). Note that we can dispense with Assumption 1 when all agents are risk-neutral because in that case the IPE wage scheme w_i^{fb} defined in (8) solves Problem SB.

Then, Proposition 1 implies that even if the principal faces a target budget constraint (FTW), the first-best is implemented when all agents are risk-neutral.

[10]More generally, we can consider the utility function $U_i = H_i(a_i)w_i - G_i(a_i)$ for some function H_i, although we set $H_i(a_i) = 1$ for simplicity.

Corollary. *Suppose that all agents are risk-neutral. Then, the principal can attain the first-best payoff* Π^{fb} *even when the total wages of agents must be fixed for any realization of their outputs.*

Third, Proposition 1 is robust to the specifications of utility functions, action and output spaces, and the probability distributions of outputs. Proposition 1 holds as long as there exists an IPE contract solving Problem SB. This encompasses the standard additively separable utility function $U_i(w, a) = u_i(w) - G_i(a)$, finite and continuous actions, as well as finite and continuous outputs. We also do not impose any technical restrictions on the probability distributions of agents' outputs such as the monotone likelihood ratio condition. Moreover, we allow agents' actions and outputs to be multidimensional.

Fourth, our efficiency result does not rely on money burning (MacLeod (2003), Kambe (2006)). When multiple agents exist, the principal can commit herself to paying a fixed total wage to all agents without burning money. This is a desirable feature of optimal contracts because it is inefficient for contracting parties to discard useful resources or pay third parties ex post. Rank-order contracts also have a similar feature in that agents are paid different prizes depending on their relative performance ranking and these prizes sum to a constant (Lazear and Rosen (1981), Malcomson (1984, 1986)). However, such scheme cannot yield the second-best payoff to the principal in an environment in which some agents are risk-averse because such agents incur higher risk under rank-order contracts than under piece rate contracts. The contract we constructed in the proof of Proposition 1 can avoid such additional risk while implementing the second-best payoff.[11]

4 Third-Best Contract

In this section, we turn to the case not covered by Proposition 1 by assuming that there is at most one risk-neutral agent. We maintain this assumption throughout the remaining sections. Then, we investigate the properties of the third-best contract for solving Problem TB.

Here, we explicitly assume the risk independence condition that agents' preferences over income lotteries are independent of their actions. That is, we make the following assumption.[12]

[11] Only when all agents are risk-neutral do rank-order contracts work as well as piece rate contracts do.

[12] Note that Assumption 2 implies Assumption 1 because of Lemma 1.

Assumption 2. *(i)* $U_i(w_i, a_i) \equiv u_i(w_i) - G_i(a_i)$ *for each agent* $i \in N$*, where* $\arg\min_{a \in A} G_i(a) = 0$ *for each* $i \in N$*. (ii) There exists some* w *such that* $u_i(w) - G_i(0) = \overline{U}_i$ *for all* i*.*

We normalize the least costly action that minimizes the action cost $G_i(a)$ to zero for any agent. The assumption that the least costly action is the same for all agents is merely made for simplicity. The following result also holds even when we consider a more general utility function as $u_i(w)H_i(a) - G_i(a)$ for some function H_i, although we set $H_i(a) \equiv 1$ for all $a \in A$ to simplify the notation.

Given Assumption 2, we consider the problem of implementing action profile $\mathbf{a}$ at the minimum total wage $\overline{W}$ subject to IC, IR, and FTW defined in Section 3. We denote by $\overline{W}(\mathbf{a})$ the optimal value of $\overline{W}$ in this minimization problem for implementing $\mathbf{a}$.[13] The principal chooses action profile $\mathbf{a}$ to maximize her expected payoff $E[R(\mathbf{y})|\mathbf{a}] - \overline{W}(\mathbf{a})$. We call this action profile *the third-best action profile*, denoted by $\mathbf{a}^* \in A^N$. To make the problem non-trivial, we assume that the constraint set is non-empty for some $\mathbf{a} > \mathbf{0}$ and that the third-best actions $\mathbf{a}^*$ satisfy $\mathbf{a}^* > \mathbf{0}$.

Under Assumption 2, we can write IC and IR in Problem TB as follows:

$$\sum_{\mathbf{y}} P(\mathbf{y}|a_i^*, a_{-i}^*) u_i(w_i(\mathbf{y})) - G_i(a_i^*) \geq \sum_{\mathbf{y}} P(\mathbf{y}|a, a_{-i}^*) u_i(w_i(\mathbf{y})) - G_i(a) \text{ for } a \neq a_i^*,\ i \in N \tag{IC}$$

$$\sum_{\mathbf{y}} P(\mathbf{y}|a_i^*, a_{-i}^*) u_i(w_i(\mathbf{y})) - G_i(a_i^*) \geq \overline{U}_i \text{ for } i \in N \tag{IR}$$

where $P(\mathbf{y}|a_i, a_{-i}^*) \equiv p_i(y_i|a_i) \prod_{j \neq i} p_j(y_j|a_j^*)$ denotes the joint probability of output profile $\mathbf{y}$ conditional on action profile (a_i, a_{-i}^*).

Then, the third-best contract $\{w_i(\mathbf{y})\}_{i=1}^N$ should solve the following.

Problem TB

$$\min \overline{W}$$

subject to FTW, IC, and IR for implementing $\mathbf{a} = \mathbf{a}^*$.

In what follows, we maintain the assumption that both the action and the output sets, A and Y, are finite (we discuss some extensions to the continuous case in the Appendix).

[13] If the constraint set satisfying IC, IR, and FTW is empty for $\mathbf{a} \in A^N$, we define $\overline{W}(\mathbf{a}) = \infty$. Note also that $\overline{W}(\mathbf{0}) = \sum_{i=1}^N (G_i(0) + \overline{U}_i)$ holds when the principal implements the least costly action, zero, from all agents. The wage schemes that achieve $\overline{W}(\mathbf{0})$ exist from Assumption 2 (ii). Thus, $\overline{W}(\mathbf{0})$ is well defined.

We denote by $M+1$ the number of all possible actions each agent can choose, $\#A = M+1$, and then make the following assumption about the probability distributions of outputs and action costs.

Assumption 3. *There exist no non-negative M dimensional vectors $(\rho(a))_{a\in A, a\neq a_i^*}$ of which the elements are not all zero simultaneously and $\sum_{a\neq a_i^*}\rho(a) = 1$ such that*

$$\sum_{a\neq a_i^*}\rho(a)p_i(y_i|a) = p_i(y_i|a_i^*), \quad \textit{for each } y_i \in Y$$

and

$$\sum_{a\neq a_i^*}\rho(a)G_i(a) \leq G_i(a_i^*).$$

Assumption 3 states that by deviating from the targeted action a_i^* and choosing any mixed strategy ρ over his actions, agent i cannot induce the same probability distributions over his output y_i as when he follows the third-best action a_i^* at a lower action cost. Similar conditions have been often imposed on principal–agent problems in the literature (e.g., Hermalin and Katz (1993)). Assumption 3 is fairly weak in the sense that it is generically satisfied when the number of possible outputs $\#Y = K$ is larger than that of possible actions $\#A = M+1$.

We now turn to characterize the third-best contract. The key factor that affects the third-best contract is the piece rate wage of each agent i, defined as

$$\xi_i(y_i) \equiv \ln\left(\lambda_i + \sum_{a\neq a_i^*}\mu_i(a)\left(1 - \frac{p_i(y_i|a)}{p_i(y_i|a_i^*)}\right)\right) \tag{10}$$

for some non-negative constants $\lambda_i \geq 0$ and $\mu_i(a) \geq 0$ for each $a \neq a_i^*$, respectively.

To see what ξ_i means, it is useful to consider how the second-best contract $\hat{w}_i$ is characterized. The second-best contract is the solution to Problem M-SB defined in the previous section. Under Assumption 2, the second-best wage scheme $\hat{w}_i(\cdot; a_i^*)$ for implementing the third-best action a_i^* from agent i is given by the familiar formula (Holmström (1979), Grossman and Hart (1983)):

$$1/u_i'(\hat{w}_i(y_i; a_i^*)) = \lambda_i^* + \sum_{a\neq \hat{a}_i^*}\mu_i^*(a)\left(1 - \frac{p_i(y_i|a)}{p_i(y_i|a_i^*)}\right)$$

for non-negative Lagrange multipliers λ_i^* and $\mu_i^*(a)$ for each $a \neq a_i^*$ associated with IR and IC in Problem M-SB, respectively. By taking the logarithm of the right-hand side of this formula, we obtain essentially the same expression as $\xi_i(y_i)$ defined in (10), although the constant terms λ_i and $\mu_i(a)$ may differ from λ_i^* and $\mu_i^*(a)$.

On the other hand, in the third-best contracting problem, Problem TB, the principal must take into account an additional constraint, FTW. To solve this problem, we replace FTW by its weak inequality, that is, $\overline{W} \geq \sum_{i=1}^N w_i(\mathbf{y})$, which we call FTW$''$, and then consider the relaxed problem with IC, IR, and FTW$''$. When a certain constraint qualification is satisfied, as we see in the formal proof,[14] the optimal solution to this relaxed problem must satisfy the Kurash–Kuhn–Tucker (KKT) conditions as follows:

$$P(\mathbf{y}|\mathbf{a}^*)u_i'(w_i(\mathbf{y}))\left\{\lambda_i + \sum_{a\neq a_i^*}\mu_i(a)\left(1-\frac{p_i(y_i|a)}{p_i(y_i|a_i^*)}\right)\right\} = \eta(\mathbf{y}) \tag{11}$$

where $\lambda_i \geq 0$ denotes the Lagrange multiplier associated with the IR constraint for agent i, $\mu_i(a) \geq 0$ the Lagrange multiplier associated with the IC constraint for agent i's action $a \neq a_i^*$, and $\eta(\mathbf{y}) \geq 0$ the Lagrange multiplier associated with FTW$''$ for output profile $\mathbf{y} \in Y^N$. Here, $P(\mathbf{y}|\mathbf{a}^*) = \prod_{i=1}^N p_i(y_i|a_i^*)$ denotes the joint probability of N agents' outputs conditional on the third-best action profile $\mathbf{a}^*$.

Then, we can show that $\eta(\mathbf{y}) > 0$ for all $\mathbf{y} \in Y^N$ and thus FTW$''$ becomes binding for any $\mathbf{y} \in Y^N$.[15] This implies that the relaxed problem coincides with the original problem TB. Thus, we can use the above KKT conditions for any pair of agents $i \neq j$ to obtain

$$\frac{u_i'(w_i(\mathbf{y}))}{u_j'(w_j(\mathbf{y}))} = \frac{\lambda_j + \sum_{a\neq a_j}\mu_j(a)\left(1-\frac{p_j(y_j|a)}{p_j(y_j|a_j^*)}\right)}{\lambda_i + \sum_{a\neq a_i^*}\mu_i(a)\left(1-\frac{p_i(y_i|a)}{p_i(y_i|a_i^*)}\right)} = e^{\xi_j(y_j)-\xi_i(y_i)}. \tag{12}$$

Hence, in the third-best contract the ratio between the marginal income utilities of the two agents $u_i'(w_i)/u_j'(w_j)$ must be equal to the relative magnitudes of the likelihood ratios of their outputs measured by $\xi_j(y_j) - \xi_i(y_i)$.

In this way, agent i is motivated via only $\xi_i(y_i)$ in the second-best case because $\xi_i(y_i)$ varies with his output y_i. On the contrary, in the third-best case, the differences between the piece rate wages of agents $\xi_i(y_i) - \xi_j(y_j)$ play an important role. Indeed, we show the following lemma.

[14] We check that the so-called Slater condition is satisfied in the Appendix.

[15] The intuition behind this result is as follows. The principal can change the wage of any agent $w_i(y_i, y_{-i})$ contingent on the outputs of others y_{-i} to keep his IC and IR unchanged. That is, it is possible to have IC and IR unaltered by changing $w_i(y_i, y''_{-i})$ and $w_i(y_i, y'_{-i})$ for $y''_{-i} \neq y'_{-i}$ for a given y_i. Such wage variations lower total wages whenever FTW$''$ is binding for some output profile $\mathbf{y}'' = (y_i, y_{-i})$ but not for another profile $\mathbf{y}' = (y_i, y'_{-i})$. This can reduce total wages $\overline{W}$ further.

Lemma 2. *Suppose that Assumptions 2 and 3 hold. Then, the third-best contract for agent i, denoted by $w_i(\mathbf{y})$, is given by a function of the differences in piece rate wages $(\xi_i(y_i) - \xi_j(y_j))_{j \neq i}$, defined as $Q_i : Y^N \to \mathbb{R}$ and given by*

$$w_i(\mathbf{y}) = Q_i(\xi_i(y_i) - \xi_1(y_1), ..., \xi_i(y_i) - \xi_N(y_N)) \tag{13}$$

where $\xi_i(y_i)$ is defined in equation (10) and Q_i is strictly increasing in $\xi_i(y_i)$ and strictly decreasing in $\xi_j(y_j)$, $j \neq i$.

Proof. See the Appendix.

Lemma 2 shows that under the third-best contract, the wage of each agent depends only on the differences in piece rate wages between his own $\xi_i(y_i)$ and those of others $\xi_j(y_j)$, that is, $\xi_i(y_i) - \xi_j(y_j)$ for $j \neq i$, and strictly increasing in his own piece rate $\xi_i(y_i)$ but decreasing in the piece rates of others $\xi_j(y_j)$. Thus, each agent is compensated more when his piece rate wage $\xi_i(y_i)$ increases but less as those of his peers $\xi_j(y_j)$ increase. This is intuitive because the total wage of all agents must be fixed while they are motivated to work hard.

4.1 Third-Best Contract with CARA Utility Functions

Although deriving more detailed properties of the third-best contract Q_i in general is challenging, we provide the conditions under which Q_i is simplified. In particular, we provide a tractable formula for the third-best contract when agents' preferences over income lotteries are represented by utility functions with a constant degree of absolute risk aversion (or CARA). In that case, the third-best contract for agent i is given by the following *simple sharing rule*:

$$w_i(\mathbf{y}) = \beta_i \xi_i(y_i) + \alpha_i \left\{ \overline{W} - \sum_{j=1}^{N} \beta_i \xi(y_j) \right\} \tag{14}$$

for some constants $\beta_i > 0$ and α_i such that $\sum_i \alpha_i = 1$, where $\xi_i(y_i)$ represents the piece rate wage of agent i defined as in equation (10).

We suppose that risk-averse agents have CARA utility functions as follows:

$$u_i(w) = \frac{1}{r_i}(1 - e^{-r_i w}) \tag{15}$$

where $r_i > 0$ denotes the CARA of agent i. In the main text, we suppose that all agents are risk-averse to avoid unnecessary complication. In the Appendix, we consider the case that one agent is risk-neutral, while all the others are risk-averse. (Note that we are assuming that there is at most one risk-neutral agent.)

As we have shown in Lemma 2, the third-best contract for agent i depends on differences between his own piece rate wage and those of others, that is, $\xi_i(y_i) - \xi_j(y_j)$. For general income-utility functions u_i, it is too complicated to see how the third-best contract reflects these differences. However, under CARA utility functions, the ratio between the marginal income-utilities of agent i and j is simplified to $u_i'(w_i)/u_j'(w_j) = e^{r_j w_j(\mathbf{y}) - r_i w_i(\mathbf{y})}$. By substituting this into the optimality condition (12), we obtain

$$r_i w_i(\mathbf{y}) - r_j w_j(\mathbf{y}) = \xi_i(y_i) - \xi_j(y_j). \tag{16}$$

We then sum this over all $j \neq i$ to show that that the third-contracts for agents are determined by a linear combination of piece rate wages $\xi_k(y_k)$, $k = 1, 2, ..., N$, thus yielding the result that the simple sharing rule (14) becomes optimal.

We thus show the following result.

Proposition 2. *Suppose that Assumptions 2 and 3 hold and that agents' preferences over income lotteries are represented by CARA utility functions (see (15)). Then, the third-best contract* $\{w_i(\mathbf{y})\}_{i=1}^N$ *that solves Problem TB is given by the simple sharing rule as follows:*

$$w_i(\mathbf{y}) = (1/r_i)\xi_i(y_i) + \frac{1/r_i}{\sum_{l=1}^N (1/r_l)} \left\{ \overline{W} - \sum_{l=1}^N (1/r_l)\xi_l(y_l) \right\}$$

where $\xi_i(y_i)$ *is defined as in (10).*

Proof. See the Appendix.

Here, the simple sharing rule shown in Proposition 2 corresponds to the one defined in (14) by setting $\alpha_i = (1/r_i)/\sum_{j=1}^N (1/r_j)$ and $\beta_i = (1/r_i)$.

Proposition 2 provides a tractable characterization of the third-best contract. Each agent obtains his piece rate wage $(1/r_i)\xi_i(y_i)$, which depends only on his output y_i in addition to a constant share, $(1/r_i)/\sum_{l=1}^N (1/r_l)$, of the residual wage defined as the fixed total wage minus the sum of the piece rate wages of all agents, that is, $\overline{W} - \sum_{l=1}^N (1/r_l)\xi_l(y_l)$. These shares of agents are determined by the relative magnitudes of their risk aversion. More risk-averse agents are rewarded with more piece rate wages based on their own outputs but a lower share of the residual wage.

Three important remarks are in order. First, one might think that rank-order contracts are an effective way to motivate agents while keeping the total budget fixed (Lazear and

Rosen (1981), Malcomson (1984)). The principal commits herself to fixed prizes, which are paid to agents based on the relative ranking of their outputs. The rewards of agents are given by step functions under rank-order contracts: if the output of an agent rises slightly higher than those of his peers, his reward jumps to a higher prize, whereas it never changes as it slightly increases more. Put differently, the difference in or "slope" of agent i's wages under rank-order contracts, $w_i(y_i'', y_{-i}) - w_i(y_i', y_{-i})$, with respect to his output $y_i'' \neq y_i'$ discontinuously changes with the outputs of others y_{-i}. However, the third-best contract of agent i that solves Problem TB has no such property; rather, the wage slope $w_i(y_i'', y_{-i}) - w_i(y_i', y_{-i})$ is given by $(\sum_{j \neq i}(1/r_j)/\sum_{l=1}^{N}(1/r_l))(1/r_i)\xi_i(y_i)$, which is independent of the outputs of the other agents y_{-i}.

Second, when we impose standard conditions such as the monotone likelihood ratio property (MLRP) and convexity of distribution function condition (CDFC), we can show that $w_i(\mathbf{y})$ is non-decreasing in i's own output y_i but non-increasing in the outputs of others y_{-i}. The MLRP is stated formally as follows.

(MLRP). *$p_i(y_i|a')/p_i(y_i|a'')$ is non-increasing in y_i given $a'' > a'$ for each $i \in N$.*

In addition, the CDFC is given as follows:

(CDFC). *Let $F_i(z|a) \equiv \sum_{y_i \leq z} p_i(y_i|a)$ be the cumulative distribution function of $p_i(y|a)$. Then, $F_i(z|a)$ is a convex function of action $a \in A$ for each $i \in N$.*

Under the MLRP, the piece rate wage defined as $\xi_i(y_i)$ is verified to be non-decreasing in y_i as long as $\mu_i(a) > 0$ holds only for less costly actions $a \neq a_i^*$ such that $G_i(a) < G_i(a^*)$. This last condition is ensured because no agent has an incentive to choose more costly actions than a_i^* under the CDFC as is well known in standard moral hazard models (e.g., Grossman and Hart (1983)). We provide a more detailed argument in the Appendix.

Third, when N is larger, the optimal contract $w_i(\mathbf{y})$ for agent i becomes asymptotically the same as the piece rate wage $(1/r_i)\xi_i(y_i)$. To see this, note that as $N \to \infty$, the second term of the optimal wage scheme given in Proposition 2, $(1/r_i)/\sum_l(1/r_l)\{\overline{W} - \sum_l(1/r_l)\xi_l(y_l)\}$, converges to some constant in probability because of the Law of Large Numbers. We generalize this last point beyond the specific utility functions in Section 5.

Next, we show the converse of Proposition 2: the third-best contract is given by the simple sharing rule (14) only if agents' preferences over income lotteries are represented by

CARA.

Recall that the third-best contract Q_i is said to be the simple sharing rule when it is given by a linear combination of the differences in piece rate wages $(\xi_i(y_i) - \xi_j(y_j))_{j\neq i}$ in a certain specific form defined as in (14). In equation (14), we impose $\sum_i \alpha_i = 1$ and $\alpha_i \sum_{l\neq i} \beta_l = \beta_i \sum_{l\neq i} \alpha_l$ (or equivalently $\alpha_i \sum_{l\neq i} \beta_i = \beta_i(1-\alpha_i)$). This last condition is required for the optimal contract to be consistent with Lemma 2; in other words, it should depend only on the differences in piece rate wages $(\xi_i(y_i) - \xi_j(y_j))_{j\neq i}$. Indeed, under this condition, we can rewrite equation (14) as follows:

$$w_i(\mathbf{y}) = \alpha_i \overline{W} + \sum_{j\neq i} \alpha_i \beta_j \{\xi_(y_i) - \xi_j(y_j)\} \tag{17}$$

so that the third-best contract depends only on differences in piece rate wages $(\xi_i(y_i) - \xi_j(y_j))_{j\neq i}$ in the linear fashion. Note also that since Q_i is strictly increasing in $\xi_i(y_i)$ as shown in Lemma 2, we must have $\alpha_i\beta_j > 0$ for each i and j, which implies that $\beta_j \neq 0$ for any j.

The simple sharing rule which is a linear combination of the piece rate wages $\{\xi_i(y_i)\}_{i=1}^N$ is sufficient to manage agents' incentives when agents' preferences over income lotteries are represented by CARA utility functions so that their risk attitudes do not change with their incomes. One might however think that, when agents' income-utility functions are not CARA so that they exhibit income effects, the linearity of the simple sharing rule becomes no longer optimal. In fact, we we show the converse of Proposition 2.

Proposition 3. *Suppose that Assumptions 2 and 3 hold. Suppose also that $N \geq 3$. Then, the simple sharing rule given in (17) becomes the third-best contract only if agents' preferences over income lotteries exhibit CARA.*

Proof. See the Appendix.

We need $N \geq 3$ to show Proposition 3. In the case of two agents ($N = 2$), the principal may be able to optimally manage agents' incentives by using the simple sharing rule even when they do not have CARA utility functions. This is the case when two agents are symmetric. To see this, let $(1/2)\overline{W} + (1/2)\beta(\xi(y_i) - \xi(y_j))$ be the simple sharing rule for agent i. Then, the other agent $j \neq i$ faces $(1/2)\overline{W}+(1/2)\beta(\xi(y_j)-\xi(y_i))$, which is symmetric to the wage scheme of agent i. Thus, only the single term $\beta(\xi(y_i)-\xi(y_j))$ can perfectly work to manage the effort incentives of both agents at the same time by symmetry. A more detailed example is given by the following.

Example: There are only two symmetric risk-averse agents, meaning that the third-best contract is also symmetric and is given by $w^*(y_i, y_j)$. Here, by symmetry, we have $w^*(y, y) = \overline{W}/2$ for any $y_i = y_j = y$. Suppose that $Y = \{\underline{y}, \overline{y}\}$. Then, given the original contract $w^*(y_i, y_j)$, we find $\beta \neq 0$ such that

$$\begin{aligned} w^*(\overline{y}, \underline{y}) &= (1/2)\beta\xi(\overline{y}) + (1/2)\{\overline{W} - \beta\xi(\underline{y})\} \\ &= (1/2)\overline{W} + (1/2)\beta\{\xi(\overline{y}) - \xi(\underline{y})\} \end{aligned}$$

where $\xi(y) = \ln(\lambda + \sum_{a \neq a^*} \mu(a)(1 - p(y|a)/p(y|a^*)))$ and we suppose that $\xi(\overline{y}) \neq \xi(\underline{y})$. Then, we define the simple sharing rule as

$$w(y_i, y_j) \equiv (1/2)\overline{W} + (1/2)\beta\{\xi(y_i) - \xi(y_j)\}.$$

Note that $w(y, y) = \overline{W}/2$ for any $y_i = y_j = y$. We also have

$$\begin{aligned} w(\underline{y}, \overline{y}) &= (1/2)\overline{W} + (1/2)\beta\{\xi(\underline{y}) - \xi(\overline{y})\} \\ &= (1/2)\overline{W} - (1/2)\{\xi(\overline{y}) - \xi(\underline{y})\} \\ &= \overline{W} - w(\overline{y}, \underline{y}) \\ &= w(\underline{y}, \overline{y}) \\ &= w^*(\underline{y}, \overline{y}) \end{aligned}$$

by using $\overline{W} = w(\overline{y}, \underline{y}) + w(\underline{y}, \overline{y})$. Thus, agents face the same wage scheme as the original one. Hence, their incentives are unchanged and they obtain the same expected payoffs. The principal can then achieve the same total wage $\overline{W}$. This implies that the simple sharing rule becomes the third-best contract even when agents' preferences over income lotteries are not necessarily represented by CARA utility functions.

5 Asymptotic Results

5.1 Optimality of Almost Piece Rate Contracts

Next, we turn to the case of more general utility functions beyond CARA. Although it is difficult to characterize third-best contracts in general environments, we show that the third-best contract becomes almost piece rates or individual performance evaluation (IPE) when the number of agents is sufficiently large.

In this subsection, we maintain Assumption 2 but drop Assumption 3. We also maintain the basic assumption that output set Y is finite with K distinct elements, $\#Y = K$, while we allow an agent's action to be discrete or continuous.

Moreover, we impose the following condition on the income utility function u_i.

Assumption 4. *(i) u_i is continuously differentiable, increasing and concave with $u_i(\infty) = \overline{u}$ ($\overline{u}$ may be ∞) and $\lim_{w \to w_{\min}} u_i(w) = -\infty$ for some $w_{\min}$ ($w_{\min}$ may be $-\infty$). (ii) $\lim_{w \to w_{\min}} u_i'(w) = +\infty$ for risk averse agent i.*

Recall that the third-best contract $\mathbf{w}^* = (w_1^*, w_2^*, .., w_N^*)$ satisfies IC and IR as follows:

$$E_y[u_i(w_i^*(y_i, y_{-i}))|a_i^*, a_{-i}^*] - G_i(a_i^*) \geq E_y[u_i(w_i^*(y_i, y_{-i}))|a_i, a_{-i}^*] - G_i(a_i), \quad \forall\, a_i \neq a_i^* \quad \text{(IC)}$$

$$E_y[u_i(w_i^*(y_i, y_{-i}))|a_i^*, a_{-i}^*] - G_i(a_i^*) \geq \overline{U}_i \quad \text{(IR)}$$

The third-best contract for agent i, $w_i^*(y_i, y_{-i})$, may depend on the outputs of others than i, y_{-i}. We define a less risky wage scheme $\tilde{w}_i$ by eliminating the risk caused by the dependency of w_i^* on y_{-i} as follows:

$$u_i(\tilde{w}_i(y_i)) = E_{y_{-i}}[u_i(w_i^*(y_i, y_{-i}))|a_{-i}^*] \quad (18)$$

for each $y_i \in Y$ conditional on the third-best action profile of others than agent i, a_{-i}^*. Since u_i is a concave function, we have $\tilde{w}_i(y_i) \leq E_{y_{-i}}[u_i(w_i^*(y_i, y_{-i}))|a_{-i}^*]$ for each $y_i \in Y$ and each $i \in N$.

We say that for a given $\varepsilon > 0$, the wage scheme w_i^* satisfies ε-IPE when

$$\varepsilon \geq E_{y_{-i}}[w_i^*(y_i, y_{-i})|a_{-i}^*] - \tilde{w}_i(y_i) \ \ (\geq 0) \quad (19)$$

for all $y_i \in Y$. When $\varepsilon \to 0$, we say that a wage scheme w_i^* is "almost" IPE because the impact of y_{-i} on w_i^* becomes negligible. We also call a wage scheme w_i^* *asymptotic non-IPE* or simply *non-IPE* when some $\varepsilon'' > 0$ and $y_i \in Y$ exist such that for all large N,

$$E_{y_{-i}}[w_i^*(y_i, y_{-i})|a_{-i}^*] - \tilde{w}_i(y_i) \geq \varepsilon''.$$

By letting $\lfloor x \rfloor$ denote the largest integer that does not exceed a given number x, we show the following result.

Proposition 4. *Suppose that Assumptions 2 and 4 hold. Then, for any $\alpha \in (0,1)$ and any $\varepsilon > 0$, there exists some N_0 such that for all $N \geq N_0$, at least $\lfloor \alpha N \rfloor$ agents must be offered ε-IPE wage schemes at the third-best contract solving Problem TB.*

Proof. See the Appendix.

The main insight of Proposition 4 is that most agents are compensated according to "almost" IPE wage schemes ($\varepsilon \to 0$) when the number of them N is sufficiently large. Although the optimal wage scheme w_i^* of agent i may depend on the outputs of others y_{-i}, such an effect asymptotically vanishes as $N \to \infty$ (so that $\varepsilon \to 0$). In other words, most agents are virtually rewarded by piece rate contracts that depend on their own individual performances. This result implies that the value of using rank-order contracts vanishes in large organizations. We discuss in more detail below how our result is related to existing studies that show the asymptotic optimality of rank-order contracts (Green and Stokey (1983), Malcomson (1986)).

To understand the intuition behind Proposition 4, suppose, contrary to the claim, that the wages of only a limited proportion of agents are asymptotically IPE when N is sufficiently large. That is, most agents are paid according to non-IPE wage schemes. Then, we modify the third-best contract $\mathbf{w}^*$ in the following two respects, which can improve the principal's payoff and hence contradicts the optimality of $\mathbf{w}^*$.

First, we expand the set of agents offered IPE wage schemes while keeping their incentives unchanged. More precisely, those offered non-IPE wage schemes w_i^* under the original contract are now offered the new schemes $\tilde{w}_i$ defined by (18). While they incur lower risk under the modified scheme than under the original one $\mathbf{w}^*$, they still choose the same third-best actions a_i^*. Thus, such a modification of wage schemes can reduce the total wage from $\overline{W}$ as long as FTW remains satisfied. We denote by N^* the set of agents who are offered the new wage schemes $\tilde{w}_i$ defined above.

To satisfy FTW, we divide the remaining set of agents $N \setminus N^*$ into two disjoint subsets L^* and L^{**}. The agents in $N \setminus N^*$ are randomly selected into L^* and L^{**} by the principal after she observes the outputs of agents. Then, the agents assigned to L^* are paid according to some IPE wage schemes $w_i(y_i)$, while those assigned to L^{**} equally share the residual wage defined as the fixed budget $\overline{W}$ minus total wages of all other agents than them:

$$(1/\#L^{**}) \left\{ \overline{W} - \sum_{j \in N^* \cup L^*} w_j(y_j) \right\} \tag{20}$$

where $\#L^{**}$ denotes the cardinality of set L^*.

Thus, under the random wage scheme constructed above, agent $i \in N \setminus N^*$ faces the following wage:

$$w_i(y_i, y_{-i}) \equiv \begin{cases} w_i(y_i) & \text{with probability } \beta(N) \\ (1/\#L^{**}) \left\{ \overline{W} - \sum_{j \in N^* \cup L^*} w_j(y_j) \right\} & \text{with probability } 1 - \beta(N) \end{cases} \tag{21}$$

where $\beta(N) \equiv \#L^*/\#(N \setminus N^*)$ is the equal probability for each agent in $N \setminus N^*$ being selected into the set of L^*. The agents belonging to set L^{**} serve to fix total wages at $\overline{W}$ regardless of their realized outputs, which ensures that FTW holds.

Second, the next step is to find appropriate wage schemes $\{w_i\}_{i \in L^*}$ in the first line of (21) such that each agent in $N \setminus N^*$ obtains the same expected payoff as under the original contract w_i^* given all the others choose the third-best actions a_{-i}^*. This ensures that the agents in $N \setminus N^*$ choose the same third-best actions a_i^* as they do under the original contract w_i^*. We can show that such wage schemes actually exist (see the Appendix). Then the remaining problem is to check whether the agents offered the above random wage incur higher risk because their wages vary with the outputs of others y_{-i} when they are paid the residual wage (20). However, when the number of agents N is so large that $\#L^{**}$ is so, the probability that (20) affects an agent's payoff tends to be sufficiently small because of the Law of Large Numbers. Thus, they equally share only negligible risk when N is sufficiently large, which causes no additional cost to the principal under the modified contract.

In this way, the principal can increase her payoff by modifying the original contract $\mathbf{w}^*$ if only a limited proportion of agents are offered IPE wage schemes, even when the number of agents N is large. This establishes Proposition 4.

The important lesson of Proposition 4 is that the principal has "most" agents working under piece rate contracts contingent only on their individual outputs. In general, for FTW to be satisfied, no agents can work under piece rate contracts because then their total wages vary with their realized outputs. Hence, we use the random wage scheme defined in (21) to ensure that most agents are paid according to piece rates, while the remaining few agents are randomly paid an equal share of the residual wage (20). The existence of the latter agents always fixes total wages at a constant for any output realization. By taking the proportion of agents who face such a random scheme to be small enough as N rises, the principal can improve her payoff by having most agents work under piece rate contracts.

5.2 Asymptotic Efficiency

Next, we use Proposition 4 to show that the principal can approximate the second-best payoff, which is the outcome when FTW is dropped, when the number of agents N becomes sufficiently large. Thus, when many agents exist, the principal asymptotically incurs no loss from the target budget constraint.

As the objective for the principal to achieve, we focus on the average payoff per agent. When we vary the number of agents N, a direct scale effect arises such that N directly increases or decreases the principal's payoff. To abstract the pure incentive effect of large organizations from this direct scale effect and make the comparison with the case of a single agent transparent, we treat the average payoff as the target of the principal. We define the average payoff of the principal under the second-best contract $\{\hat{w}_i, \hat{a}_i\}_{i=1}^N$ as follows:

$$\hat{\pi} \equiv (1/N) \left\{ E_{\mathbf{y}}[R(\mathbf{y})|\hat{\mathbf{a}}] - \sum_{i=1}^{N} E_{y_i}[\hat{w}_i(y_i)|\hat{a}_i] \right\}$$

Then, we show the following result.

Proposition 5. *Suppose that Assumptions 2 and 4 are satisfied. Then, for any $\varepsilon > 0$, there exists some $\overline{N}$ such that for all $N \geq \overline{N}$ the average payoff of the principal is at least $\hat{\pi} - \varepsilon$; in other words, the principal can approximately attain the second-best payoff $\hat{\pi}$ on average as $N \to \infty$.*

Proof. See the Appendix.

Proposition 5 is understood from the viewpoint of Proposition 4. In the proof of Proposition 4, we set the wage schemes for most agents as IPE, while we consider the random scheme for the remaining agents as shown in (20). We modify this scheme by setting the wage schemes for most agents as the second-best ones $\hat{w}_i$ and those for the remaining others as that similar to the random wage (20). Because of the Law of Large Numbers, the agents offered the latter scheme face low income risk when the number of agents N is sufficiently large. In this way, most agents face almost the same wage schemes as the second-best ones $\hat{w}_i$ and hence choose the second-best actions $\hat{a}_i$. Thus, the principal can approximate the second-best payoff per agent as the number of agents is sufficiently large.

To show the above asymptotic efficiency result, we maintain the assumption that the utility function of each agent is additively separable between his income and action (Assumption 2). However, we can weaken this assumption when we focus on the case that action set A is finite.

We now return to the general case considered in Section 3 that the utility function of agent i is defined as $U_i : \mathbb{R} \times A \to \mathbb{R}$, where agent i's utility is given by $U(w_i, a_i)$ for his income w_i and action a_i. We also maintain Assumption 1: some IPE contract $\{\hat{w}_i, \hat{a}_i\}$ solves Problem SB for each agent i, where $\hat{w}_i$ depends only on his output y_i. Such contract satisfies both IC and IR in Problem SB for implementing the second-best action $\hat{a}_i \in A$ from agent

i:

$$E_{y_i}[U_i(w_i(y_i), \hat{a}_i)|\hat{a}_i] \geq E_{y_i}[U_i(w_i(y_i), a_i)|a_i], \ \ \forall \, a_i \neq \hat{a}_i \tag{IC}$$

$$E_{y_i}[U_i(w_i(y_i), \hat{a}_i)|\hat{a}_i] \geq \overline{U}_i \tag{IR}$$

We make the following assumption.

Assumption 5. *For any $\delta > 0$, there exists some contract $\{w_i^\delta, \hat{a}_i\}$ for each agent i such that $|w_i^\delta(y_i) - \hat{w}_i(y_i)| < \delta$ for all $y_i \in Y$ and both IC and IR above are satisfied with strict inequalities.*

Assumption 5 states that there exists a contract $\{w_i^\delta, \hat{a}_i\}$ in a neighborhood of the second–best contract $\{\hat{w}_i, \hat{a}_i\}$ such that agents have the strict incentive to choose the second-best actions and strictly prefer accepting the second best-contract to rejecting it. We provide the sufficient conditions for Assumption 5 below and discuss how this assumption is a weak restriction.

We use a contract $\{w_i^\delta, \hat{a}_i\}$ defined in Assumption 5 in order to approximate the second–best contract while keeping both IC and IR as strict inequalities in the limit as the number of agents N goes to infinity. The reason that we need strict IC and IR is as follows. As we have already discussed, in order to satisfy FTW, there must be some agents whose wage schemes are interdependent in the sense that their wages vary with outputs of others. Thus, these agents incur additional risks which may distort their action choices and the acceptance decisions of contracts from the second-best optimum. Since such additional risks cannot be exactly zero in the third-best case, we need to perturb wage schemes from the second-best ones in order to modify IC and IR corresponding to these additional risks. Strict IC and IR ensure that we can incorporate such additional (but small when N is large) risks into IC and IR without affecting incentives of agents.

In fact we show the following result.

Proposition 6. *Suppose that Assumptions 1 and 5 hold. Suppose also that agent i's utility is given by $U_i(w_i, a_i)$, where U_i is continuous and increasing in w_i. Then, for any $\varepsilon > 0$, there exists some $\overline{N}$ such that for all $N \geq \overline{N}$ the principal can obtain at least $\hat{\pi} - \varepsilon$.*

Proof. See the Appendix.

To show this result, we use the following simple sharing rule:

$$w_i(y_i, y_{-i}) \equiv w_i^\delta(y_i) + (1/N)\left\{\hat{W}_\delta - \sum_{n=1}^{N} w_n^\delta(y_n)\right\} \tag{22}$$

for the wage scheme $\{w_i^\delta, \hat{a}_i\}$ defined in Assumption 5. Here, $\hat{W}_\delta$ is the total expected wages of all agents under the schemes $\{w_n^\delta, \hat{a}_n\}_{i=1}^N$:

$$\hat{W}_\delta \equiv \sum_{n=1}^{N} E_{y_n}[w_n^\delta(y_n)|\hat{a}_n].$$

This contract is a special form of the simple sharing rule defined in the previous section by setting an equal share $\alpha_i = 1/N$ for all i. Then, the agents face virtually the second-best wage schemes $\{\hat{w}_i\}_{i=1}^N$ when N is sufficiently large and δ is taken to be small because, according to Assumption 5, w_i^δ can be sufficiently close to the second-best wage scheme $\hat{w}_i$ and the equal share of the residual wage $(1/N)\{\hat{W}_\delta - \sum_n w_n^\delta\}$ approaches zero in probability as $N \to \infty$ because of the Law of Large Numbers. Moreover, when δ is taken to be sufficiently small, $\hat{W}_\delta \to \hat{W}$ holds, where $\hat{W}$ is the total expected wage cost in the second best. Thus, the principal can approximately obtain the second-best payoff $\hat{\pi}$ on average when the number of agents N is sufficiently large. Green and Stokey (1983) and Malcomson (1986) show that rank-order contracts perform as well as piece rate contracts do when there are many agents. Green and Stokey (1983) assume that the set of agents is finite and obtain the asymptotic efficiency result as their number tends to infinity. Malcomson (1986) analyzes the case of a continuum of agents. When there are many agents, the rank ordering of each agent's relative performance almost exactly reflects only his own performance. Thus, rank-order contracts do not impose higher risk on agents than piece rate contracts do in the presence of infinitely many agents.

Our asymptotic result differs from these studies in several important aspects. First, the above studies restricted their analyses only to the case of identical agents and focused only on the symmetric equilibrium in which agents follow an identical effort strategy given the wage scheme. However, we allow agents to be heterogeneous and impose no such symmetric restrictions on their equilibrium behaviors. Although rank-order contracts may perform well in symmetric environments, they may be ineffective at mitigating the incentive problem in more general cases when agents are heterogeneous. For example, under a standard rank-order tournament, agents are paid different prizes based on the rankings of their relative outputs and these prizes do not depend on their identities (i.e., the same prize is given to the winner whoever he is). However, such prize structure may be an insufficient instrument to efficiently manage the different incentives of heterogeneous agents who do not necessarily behave in a symmetric manner. By contrast, we show that the simple sharing rule given in (22) is sufficient to achieve the second-best efficiency regardless of agents' heterogeneity as the number of agents goes to infinity.

Second, in finite environments in which the action set and output set are both finite, we can allow more general utility functions of agents than the additively separable form used in the studies cited above (see Proposition 6). Even when agents' preferences exhibit the income effect, our approximate efficiency result remains valid provided a weak finiteness condition, Assumption 5, is guaranteed.

Remark. Assumption 5 is satisfied under fairly weak restrictions. When we focus on a *finite* environment with finite action set A and finite output set Y, we can show that Assumption 5 is weaker than the standard assumption that the utility functions of agents are additively separable as long as a weak restriction is imposed on the probability distributions of outputs as made in Assumption 3. In fact, Assumption 5 is satisfied under Assumptions 2 and 3. Since Assumption 3 generically holds when the number of outputs is larger than that of actions ($\#Y > \#A$), Assumption 5 is generically weaker than assuming the additive separable utility functions.

From Assumption 3, we can find a vector $(v^\delta(y))_{y\in Y}$ such that for a given $\delta > 0$,

$$\sum_{y\in Y}(p_i(y|\hat{a}_i) - p_i(y|a))v_i^\delta(y) \geq \delta$$

holds for any $a \neq \hat{a}_i$ (Rockafellar (1970, p. 198, Proposition 22.1)).

Recall that $\hat{w}_i$ is the second-best wage scheme implementing the second-best action $\hat{a}_i$. Let $\hat{u}_i(y) \equiv u_i(\hat{w}_i(y))$ be the corresponding utility payments. Then, we define $u_i^\delta(y) \equiv \hat{u}_i(y) + v_i^\delta(y)$ for each $y \in Y$, which ensures that

$$\begin{aligned}\sum_y (p_i(y|\hat{a}_i) - p_i(y|a))u_i^\delta(y) &= \sum_y (p_i(y|\hat{a}_i) - p_i(y|a))\hat{u}_i(y) + \sum_y (p_i(y|\hat{a}_i) - p_i(y|a))v_i^\delta(y) \\ &\geq G_i(\hat{a}_i) - G_i(a) + \delta \\ &> G_i(\hat{a}_i) - G_i(a)\end{aligned}$$

for any $a \neq \hat{a}_i$. By adding a constant v_i to $u_i^\delta(y)$ for each $y \in Y$, we can also ensure that

$$\sum_y p_i(y|\hat{a}_i)(u_i^\delta(y) + v_i) - G_i(\hat{a}_i) > \overline{U}_i.$$

By letting $\delta \to 0$, we can take $v_i^\delta(y) \to 0$ and $v_i \to 0$. By defining $w_i^\delta(y_i) \equiv u_i^{-1}(u_i^\delta(y_i))$ for each $y_i \in Y$, wage scheme $\{w_i^\delta, \hat{a}_i\}$ satisfies IC and IR with strict inequalities. As long as action set A and output set Y are both finite with $\#Y > \#A$, Assumption 5 is generically satisfied under Assumptions 2.

6 Concluding Remarks

In this paper we investigated a wide class of principal–agent problems with moral hazard and target budgets. The latter requires that the principal fixes a budget for the total wages paid to all agents regardless of their outputs realized. We showed that the presence of such target budgets cause no loss of efficiency when there are at least two risk-neutral agents. We then discussed that our result contributes to the literature on relational contracts with multiple agents, which has not thus far fully addressed what the principal can achieve in the static benchmark. In contrast to most studies in that strand of the literature, we showed that subjective evaluations never constrain the principal, even in static environments with at least two risk-neutral agents. We characterized the third-best contracts as when there is at most one risk-neutral agent, which no longer guarantee that the second-best efficiency is achieved. Then, we showed that the optimal contract is given by the presented simple sharing rule if and only if agents have CARA preferences. In addition, beyond CARA utility functions, we showed that under the third-best contract, most agents are compensated according to almost their individual outputs when their number is sufficiently large. We then showed that the principal can approximate the second-best payoff on average as the number of agents is sufficiently large.

We conclude the paper by discussing some possible extensions of the model. First, we ruled out money burning from the model. When this is allowed, the principal can weaken the target budget constraint as $\overline{W} \geq \sum_{i=1}^{N} w_i(\mathbf{y})$. Thus, the principal can dispose of $\overline{W} - \sum_{i=1}^{N} w_i(\mathbf{y}) > 0$ if she wants. However, as we show in Lemma A2 in Appendix A, this never happens under the third-best contract. If $\overline{W} > \sum_{i=1}^{N} w_i(\mathbf{y}'')$ for some $\mathbf{y}''$, whereas $\overline{W} = \sum_{i=1}^{N} w_i(\mathbf{y}')$ for other $\mathbf{y}'$, then the principal can change the wages of some agent i for the different outputs of others y''_{-i} and y'_{-i} to make his IC and IR unchanged. Intuitively, since agent i's action a_i affects only his own output y_i, which is also not correlated with the outputs of others y_{-i}, it is still possible to keep agent i's expected utility unchanged by changing his wages for the different outputs of others y_{-i}. However, such variational changes in agent i's wage can reduce total wages further from the optimal contract. Thus, $\overline{W} = \sum_n w_n(\mathbf{y})$ must be satisfied at the optimum.

Second, we focused on the situation in which the outputs of agents are technologically independent of each other. Thus, we ruled out the case that the actions taken by one agent directly and technologically affect the outputs of others. However, considering how the principal designs teamwork and job structure so that agents help and cooperate with each other is an important issue (e.g., Itoh (1991), Ishihara (2017)). It would thus be interesting in future work to examine the extent to which the target budget constraint affects teamwork formation and job design in organizations.

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7 Appendix A: Omitted Proofs

7.1 Proof of Lemma 1

Suppose that $U_i(w_i, a_i) = H_i(a_i)u_i(w_i) - G_i(a_i)$ for each $i \in N$. Suppose also that the wage scheme of some agent i depends on the outputs of others y_{-i}, that is, $\hat{w}_i(y_i, y_{-i})$ under the second best contract. Then we define the new wage scheme $\tilde{w}_i$ as follows

$$u_i(\tilde{w}_i(y_i)) = E_{y_{-i}}[u_i(\hat{w}_i(y_i, y_{-i}))|\hat{a}_{-i}]$$

for each $y_i \in Y$. Then we have

$$\begin{aligned} U_i(\tilde{w}_i(y_i), a_i) - E_{y_{-i}}[U_i(\hat{w}_i(y_i, y_{-i}), a_i)|\hat{a}_{-i}] &= H_i(a_i)\{u_i(\tilde{w}_i(y_i)) - E_{y_{-i}}[u_i(\hat{w}_i(y_i))|\hat{a}_{-i}]\} \\ &= 0 \end{aligned}$$

for any $y_i \in Y$ and any $a_i \in A$. Thus agent i obtains the expected payoff under the above new scheme $\tilde{w}_i$ as follows

$$E_{y_i}[U_i(\tilde{w}_i(y_i), a_i)|a_i] = E_{\mathbf{y}}[U_i(\hat{w}_i(y_i, y_{-i}), a_i)|a_i, \hat{a}_{-i}]$$

which is maximized at the second-best action $a_i = \hat{a}_i$. Then all agents choose the second-best actions $\hat{\mathbf{a}}$.

The principal can however weakly reduce the wage cost by the new wage scheme because

$$u_i(\tilde{w}_i(y_i)) = E_{y_{-i}}[u_i(\hat{w}_i(y_i, y_{-i}))|\hat{a}_{-i}] \leq u_i(E_{y_{-i}}[\hat{w}_i(y_i, y_{-i})|\hat{a}_{-i}])$$

which implies that $\tilde{w}_i(y_i) \leq E_{y_{-i}}[\hat{w}_i(y_i, y_{-i})|\hat{a}_{-i}]$ and hence

$$E_{y_i}[\tilde{w}_i(y_i)|\hat{a}_i] \leq E_{\mathbf{y}}[\hat{w}_i(\mathbf{y})|\hat{\mathbf{a}}]$$

for every agent i. Q.E.D.

7.2 Proof of Lemma 2

We proceed to prove Lemma 2 by several steps. First, we consider the relaxed problem of the original problem, Problem TB, by replacing FTW by its weak inequality. Second, we show that the so called Slater condition is satisfied in such relaxed problem, which ensures that the optimal solution to the relaxed problem must satisfy the Kurash-Kuhn-Tucker (KKT) conditions. Third, we show that the optimal solution to the relaxed problem coincides with the one of the original problem, Problem TB. Finally, by using the KKT conditions, we prove Lemma 2.

By replacing FTW by the version of a weak inequality, we consider the relaxed problem of the original Problem TB as follows.

Problem RP

$$\text{Min } \overline{W}$$

subject to IC, IR and

$$\overline{W} \geq \sum_{i=1}^{N} w_i(\mathbf{y}) \quad \forall\, \mathbf{y} \in Y^N \tag{FTW$''$}$$

In what follows we denote by $P(y_{-i}|a_{-i}) \equiv \prod_{j \neq i} p_j(y_j|a_j)$ the joint probability of outputs of all other agents than agent i. Also we denote by $P(\mathbf{y}|\mathbf{a}) = \prod_{i=1}^{N} p_i(y_i|a_i)$ the joint probability of all agents' outputs $\mathbf{y} \in Y^N$ conditional on an action profile of them $\mathbf{a} \in A^N$.

We can use the change of variables by letting $u_i(\mathbf{y}) = u_i(w_i(\mathbf{y}))$ and $w_i(\mathbf{y}) = \phi_i(u_i(\mathbf{y}))$ for the inverse function ϕ_i of u_i. Then, the above problem RP is equivalent to the following.

Problem RP*

$$\text{Min}_{\{u_i(\mathbf{y})\}_{i \in I}, \overline{W}} \quad \overline{W}$$

subject to

$$\sum_{y_{-i}} P(y_{-i}|a^*_{-i}) \sum_{y_i} p_i(y_i|\hat{a}_i) u_i(\mathbf{y}) - G_i(\hat{a}_i) \geq \sum_{y_{-i}} P(y_{-i}|a^*_{-i}) \sum_{y_i} p_i(y_i|a) u_i(\mathbf{y}) - G_i(a) \quad \forall\, a \neq \hat{a}_i, \tag{IC}$$

$$\sum_{y_{-i}} P(y_{-i}|a^*_{-i}) \sum_{y_i} p_i(y_i|a^*_i) u_i(\mathbf{y}) - G_i(a^*_i) \geq \overline{U}_i \tag{IR}$$

$$\overline{W} \geq \sum_{i=1}^{N} \phi_i(u_i(\mathbf{y})) \quad \forall\, \mathbf{y} \in Y^N \tag{FTW$''$}$$

Then, since ϕ_i is a convex function, this can be a problem of convex programming. It is known that, when the Slater condition is satisfied, the necessary condition for the optimal solution to the above problem RP* satisfies the following KKT conditions (Takayama (Theorem 1.D.2, 1985, p.92): there exist some non-negative multipliers $\lambda_i \geq 0$ for $i \in N$, $\mu_i(a) \geq 0$ for $i \in N$ and $a \neq a^*_i$, and $\eta(\mathbf{y}) \geq 0$ for $\mathbf{y} \in Y^N$ such that

$$-1 + \sum_{\mathbf{y} \in Y^N} \eta(\mathbf{y}) = 0, \tag{A1}$$

$$P(\mathbf{y}|\mathbf{a}^*) u'_i(w^i(\mathbf{y})) \left\{ \lambda_i + \sum_{a \neq a^*_i} \mu_i(a) \left(1 - \frac{p_i(y_i|a)}{p_i(y_i|a^*_i)} \right) \right\} - \eta(\mathbf{y}) = 0 \tag{A2}$$

We first show the following.

Lemma A1. *The Slater condition is satisfied in Problem RP*.*

Proof. Since ϕ_i is a convex function as well as both IC and IR are linear functions of $u_i(\mathbf{y})$ in Problem RP*, it suffices to show that there exists an interior point $(\overline{W}, u_1(\mathbf{y}), ..., u_N(\mathbf{y}))$ in the constraint set. Under Assumption 3, by using the theorem on existence of solutions to a system of inequalities (Rockafellar (Proposition 22.1, 1970, p.198)), we can find a vector $(u_i(y_i))_{y_i \in Y}$ such that

$$\sum_{y_i \in Y} p_i(y_i|a_i^*)u_i(y_i) - G_i(a_i^*) > \sum_{y_i \in Y} p_i(y_i|a)u_i(y_i) - G_i(a).$$

Thus, IC of Problem RP* is satisfied as strict inequalities. By adding some constant v_i to these values of $(u_i(y_i))_{y_i \in Y}$, we can also ensure that IR of Problem RP* is satisfied as strict inequalities. Finally, by taking a large enough $\overline{W}$, FTW″ is satisfied as strict inequalities given the utility payments we have defined above. Q.E.D.

Lemma A2. $\eta(\mathbf{y}) > 0$ *for all* $\mathbf{y} \in Y^N$.

Proof. By (A1), $\eta(\mathbf{y}) > 0$ must hold for some $\mathbf{y} \in Y^N$. Let $\mathbf{y} = (y_1, ..., y_i, ..., y_N)$ for such $\mathbf{y}$.

Then, when we look at (A2) for agent i, we have

$$P(\mathbf{y}|\mathbf{a}^*)u_i'(w_i(\mathbf{y}))\left\{\lambda_i + \sum_{a \neq a_i^*} \mu_i(a)\left(1 - \frac{p_i(y_i|a)}{p_i(y_i|a_i^*)}\right)\right\} = \eta(y_1, ..., y_i, ..., y_N) > 0$$

so that

$$\lambda_i + \sum_{a \neq a_i^*} \mu_i(a)\left(1 - \frac{p_i(y_i|a)}{p_i(y_i|a_i^*)}\right) > 0. \tag{A3}$$

Now we take any output profile of $\mathbf{y}'' = (y_1'', ..., y_i, ..., y_N'')$ by fixing y_i of agent i's output but arbitrary outputs of others y_{-i}''. Then (A2) and (A3) imply that

$$0 < \lambda_i + \sum_{a \neq a_i^*} \mu_i(a)\left(1 - \frac{p_i(y_i|a)}{p_i(y_i|a_i^*)}\right) = \eta(y_1'', ..., y_i, ..., y_N'')/P(\mathbf{y}''|\mathbf{a}^*)u_i'(w_i(\mathbf{y}''))$$

so that $\eta(y_1'', ..., y_i, ..., y_N'') > 0$ for any $\mathbf{y}_{-i}'' \in Y^{N-1}$. Then we take any agent $j \neq i$ and obtain

$$0 < \eta(y_1'', ..., y_i, ..., y_j'', ..., y_N'') = P(\mathbf{y}''|\mathbf{a}^*)u_j'(w_j(\mathbf{y}''))\left\{\lambda_j + \sum_{a \neq a_j^*} \mu_j(a)\left(1 - \frac{p_j(y_j''|a)}{p_j(y_j''|a_j^*)}\right)\right\}$$

so that

$$\lambda_j + \sum_{a \neq a_j^*} \mu_j(a) \left(1 - \frac{p_j(y_j''|a)}{p_j(y_j''|a_j^*)} \right) > 0$$

for any $y_j'' \in Y$. Since this holds for any $y_j'' \in Y$ and any agent $j \neq i$, when we take any arbitrary $\tilde{\mathbf{y}} = (y_1, ..., y_i, ..., y_j'', ..., y_N) \in Y^N$, we obtain

$$0 < \lambda_j + \sum_{a \neq a_j^*} \mu_j(a) \left(1 - \frac{p_j(y_j''|a)}{p_j(y_j''|a_j^*)} \right) = \eta(y_1, ..., y_j'', ..., y_N)/P(\tilde{\mathbf{y}}|\mathbf{a}^*)u_j'(w_j(\tilde{\mathbf{y}}))$$

so that $\eta(y_1, ..., y_j'', ..., y_N) > 0$ for any $\tilde{\mathbf{y}} = (y_j'', y_{-j}) \in Y^N$. Q.E.D.

Lemma A2 implies that FTW$''$ must be binding at the optimum of Problem RP* or equivalently Problem RP. Thus the optimal solution to Problem RP is same as the original problem TB.

By Lemma A2, we have

$$\lambda_j + \sum_{a \neq a_j^*} \mu_j(a) \left(1 - \frac{p_j(y_j|a)}{p_j(y_j|a_j^*)} \right) > 0$$

for all $j \in N$ and all $y_j \in Y$. Then we can rearrange (A2) as follows

$$\frac{u_i'(w_i(\mathbf{y}))}{u_j'(w_j(\mathbf{y}))} = \frac{\lambda_j + \sum_{a \neq a_j^*} \mu_j(a) \left(1 - \frac{p_j(y_j|a)}{p_j(y_j|a_j^*)} \right)}{\lambda_i + \sum_{a \neq a_i^*} \mu_j(a) \left(1 - \frac{p_i(y_i|a)}{p_i(y_i|a_i^*)} \right)} = e^{\xi_j(y_j) - \xi_i(y_i)} \tag{A4}$$

for any $i \neq j$.

First, suppose that all agents are risk averse, that is, $u_i'' < 0$ for all i. Then, we show Lemma 2. From (A4), by letting g_i be the inverse function of u_i', we obtain

$$w_j(\mathbf{y}) = g_j \left(u_i'(w_i(\mathbf{y})) e^{\xi_i(y_i) - \xi_j(y_j)} \right) \tag{A5}$$

for any $j \neq i$. Summing both sides of this over all $j \neq i$, we obtain

$$\sum_{j \neq i} w_j(\mathbf{y}) = \sum_{j \neq i} g_j \left(u_i'(w_i(\mathbf{y})) e^{\xi_i(y_i) - \xi_j(y_j)} \right)$$

which, by using $\overline{W} = \sum_{j \neq i} w_j(\mathbf{y}) + w_i(\mathbf{y})$ (FTW), can be written by

$$\overline{W} = w_i(\mathbf{y}) + \sum_{j \neq i} g_j \left(u_i'(w_i(\mathbf{y})) e^{\xi_i(y_i) - \xi_j(y_j)} \right). \tag{A6}$$

Here u_i' is strictly decreasing and g_j is strictly decreasing for all $j \neq i$. Thus, $\sum_{j\neq i} g_j(u_i'(\cdot))$ is strictly increasing. Then, the right hand side of (A6) is strictly increasing in $w_i(\mathbf{y})$. Also, the right hand side goes to $+\infty$ $(-\infty)$ as $w_i(\mathbf{y}) \to \infty$ $(-\infty)$. Thus, there exists a unique solution of $w_i(\mathbf{y})$ to equation (A6). We denote it by

$$w_i(\mathbf{y}) = Q_i(\xi_i(y_i) - \xi_1(y_1), ..., \xi_i(y_i) - \xi_N(y_N))$$

which depends on differences in piece rate wages $(\xi_i(y_i) - \xi_j(y_j))_{j\neq i}$ between agent i and others $j \neq i$.

Furthermore, we can verify from (A6) that its right hand side is strictly decreasing in ξ_i because $\sum_{j\neq i} g_j(\cdot)$ is strictly decreasing. Thus Q_i is strictly increasing function of ξ_i.[16] Furthermore, $g_j(\cdot)$ is strictly decreasing for all $j \neq i$. Thus, Q_i is strictly deceasing in ξ_j for $j \neq i$.

Next suppose that one agent is risk neutral bu all others are risk averse. Let agent 1 be risk neutral so that his utility is given by $w_1 - G_1(a_1)$. Then, (A4) implies that $1/u_i'(w_i(\mathbf{y})) = e^{\xi_i(y_i)-\xi_1(y_1)}$ for any $i \neq 1$ from which we have $w_i(\mathbf{y}) = g_i(e^{\xi_1(y_1)-\xi_i(y_i)})$ which is strictly increasing in ξ_i but strictly decreasing in ξ_1. By summing this over all $i \neq 1$ and using FTW, we obtain $w_1(\mathbf{y}) = \overline{W} - \sum_{i\neq 1} g_i(e^{\xi_1(y_1)-\xi_i(y_i)})$ which is also strictly increasing in ξ_1 but decreasing in ξ_i. Q.E.D.

7.3 Proof of Proposition 2

Under CARA utility functions we have assumed in the main text, (A4) can be written by

$$r_i w_i(\mathbf{y}) - r_j w_j(\mathbf{y}) = \xi_i(y_i) - \xi_j(y_j)$$

where

$$\xi_i(y_i) \equiv \ln\left(\lambda_i + \sum_{a\neq a_i^*} \mu_i(a)\left(1 - \frac{p_i(y_i|a)}{p_i(y_i|a_i^*)}\right)\right). \tag{A7}$$

[16] Suppose that some agents are induced to choose the least costly action, $a_i^* = 0$. For such agent i, we drop his IC from the Problem RP and obtain the KKT condition as follows:

$$\frac{u_i'(w_i(\mathbf{y}))}{u_j'(w_j(\mathbf{y}))} = \frac{\lambda_i}{\lambda_j + \sum_{a\neq a_j^*} \mu_j(a)\left(1 - \frac{p_j(y_j|a)}{p_j(y_j|a_j^*)}\right)}$$

for any $j \neq i$. Then, we can set $\xi_i \equiv \ln \lambda_i$ which is independent of agent i's output y_i and derive the optimal contract for agent i as $w_i(\mathbf{y}) = Q_i(\xi_i - \xi_1(y_1), ..., \xi_i - \xi_N(y_N))$. Since Q_i does not change with y_i, agent i is actually induced to choose the least costly action $a_i = 0$, which trivially satisfies his IC. Thus, without loss of generality, we can set $\mu_i(a) = 0$ for all $a \neq a_i^*$ so that $\xi_i = \ln \lambda_i$ when agent i is induced to choose the least costly action $a_i = 0$.

Note that, since λ_i and $\mu_i(a)$ are independent of outputs, $\xi_i(y_i)$ depends only on agent i's output y_i through the likelihood ratio $p_i(y_i|a)/p_i(y_i|a_i^*)$.

We sum both sides of (A7) over all $j \neq i$ to obtain

$$\sum_{j\neq i}(1/r_j)r_i w_i(\mathbf{y}) - \sum_{j\neq i} w_j(\mathbf{y}) = \sum_{j\neq i}(1/r_j)\xi_i(y_i) - \sum_{j\neq i}(1/r_j)\xi_j(y_j)$$

which is further arranged as

$$\sum_{j\neq i}(1/r_j)r_i w_i(\mathbf{y}) - \overline{W} + w_i(\mathbf{y}) = \sum_{j\neq i}(1/r_j)\xi_i(y_i) - \sum_{j\neq i}(1/r_j)\xi_j(y_j)$$

due to FTW in Problem TB. Then we can derive

$$\left(\sum_{j\neq i}(1/r_j)r_i + 1\right) w_i(\mathbf{y}) = \overline{W} + \sum_{j\neq i}(1/r_j)\xi_i(y_i) - \sum_{j\neq i}(1/r_j)\xi_j(y_j)$$

which yields the following desired result

$$w_i(\mathbf{y}) = (1/r_i)\xi_i(y_i) + \frac{1/r_i}{\sum_{l=1}^N (1/r_l)}\left\{\overline{W} - \sum_{l=1}^N (1/r_l)\xi_l(y_l)\right\}.$$

Q.E.D.

7.4 Proof of Proposition 3

From (A4), we obtain

$$\ln u_j'(w_j(\mathbf{y})) - \ln u_i'(w_i(\mathbf{y})) = \xi_i(y_i) - \xi_j(y_j) \tag{A8}$$

for each $i \neq j$.

We define the following function

$$h_i(w_i(\mathbf{y})) \equiv \ln u_i'(w_i(\mathbf{y})) + \gamma_i w_i(\mathbf{y}) \tag{A9}$$

for some positive constant $\gamma_i > 0$. Then, the above optimality condition (A8) is written by

$$-\gamma_j w_j(\mathbf{y}) + \gamma_i w_i(\mathbf{y}) = \xi_i(y_i) - \xi_j(y_j) + h_i(w_i(\mathbf{y})) - h_j(w_j(\mathbf{y})).$$

By dividing both sides of (A9) by γ_j and summing them over all $j \neq i$, we obtain

$$-\sum_{j\neq i} w_j(\mathbf{y}) + \sum_{j\neq i}(1/\gamma_j)\gamma_i w_i(\mathbf{y}) = \sum_{j\neq i}(1/\gamma_j)\left\{\xi_i(y_i) - \xi_j(y_j) + h_i(w_i(\mathbf{y})) - h_j(w_j(\mathbf{y}))\right\}.$$

Since $\overline{W} = \sum_n w_n(\mathbf{y})$, this can be written by

$$-\overline{W} + \gamma_i \left(1 + \sum_{j \neq i}(1/\gamma_j)\right) w_i(\mathbf{y}) = \sum_{j \neq i}(1/\gamma_j) \left\{\xi_i(y_i) - \xi_j(y_j) + h_i(w_i(\mathbf{y})) - h_j(w_j(\mathbf{y}))\right\}.$$

Thus we derive the third best contract for agent i in the following implicit form:

$$w_i(\mathbf{y}) = H_i(\xi_i, \xi_{-i}) \equiv \frac{1/\gamma_i}{\sum_n (1/\gamma_n)} \sum_{j \neq i}(1/\gamma_j) \left\{\xi_i(y_i) - \xi_j(y_j) + h_i(w_i(\mathbf{y})) - h_j(w_j(\mathbf{y}))\right\}. \tag{A10}$$

Now suppose that the third best contract is given by the simple sharing rule as follows

$$w_i(\mathbf{y}) = \beta_i \xi_i(y_i) + \alpha_i \left\{\overline{W} - \sum_{l=1}^{N} \beta_l \xi_l(y_l)\right\}$$

where $\sum_n \alpha_n = 1$ and $\beta_i \neq 0$ for any $i \in N$. Note that $\beta_i \neq 0$ holds for all i due to Lemma 2 (Q_i is strictly increasing in ξ_i).

This is a linear function of $\{\xi_n(y_n)\}_{n=1}^N$. Keeping this in mind, we differentiate H_i twice with respect to $\xi_i(y_i)$ in order to obtain

$$\frac{\partial^2 H_i}{\partial \xi_i^2} = \sum_{j \neq i}(1/\gamma_j)(1-\alpha_i)^2 \beta_i^2 h_i''(w_i(\mathbf{y})) - \sum_{j \neq i}(1/\gamma_j)\alpha_j^2 \beta_i^2 h_j''(w_j(\mathbf{y}))$$

which must be zero because w_i is linear with respect to ξ_i:

$$\sum_{j \neq i}(1/\gamma_j)(1-\alpha_i)^2 \beta_i^2 h_i''(w_i(\mathbf{y})) - \sum_{j \neq i}(1/\gamma_j)\alpha_j^2 \beta_i^2 h_j''(w_j(\mathbf{y})) = 0. \tag{A11}$$

From this, it must be that for any $i \neq m$,

$$\sum_{j \neq i}(1/\gamma_j)(1-\alpha_i)^2 \beta_i^2 h_i''(w_i(\mathbf{y})) - (1/\gamma_m)\alpha_m^2 \beta_i^2 h_m''(w_m(\mathbf{y})) - \sum_{j \neq i,m}(1/\gamma_j)\alpha_j^2 \beta_i^2 h_j''(w_j(\mathbf{y})) = 0. \tag{A12}$$

By changing the roles of i and m, we obtain the similar equation:

$$\sum_{j \neq m}(1/\gamma_j)(1-\alpha_m)^2 \beta_m^2 h_m''(w_m(\mathbf{y})) - (1/\gamma_i)\alpha_i^2 \beta_m^2 h_i''(w_i(\mathbf{y})) - \sum_{j \neq i,m}(1/\gamma_j)\alpha_j^2 \beta_m^2 h_j''(w_j(\mathbf{y})) = 0. \tag{A13}$$

From (A12), (A13) and $\beta_n^2 \neq 0$ for all n, we have

$$\left[\sum_{j \neq i}(1/\gamma_j)(1-\alpha_i)^2 + (1/\gamma_i)\alpha_i^2\right] h_i''(w_i(\mathbf{y})) = \left[\sum_{j \neq m}(1/\gamma_j)(1-\alpha_m)^2 + (1/\gamma_m)\alpha_m^2\right] h_m''(w_m(\mathbf{y})). \tag{A14}$$

This implies that, if $h_i''(w_i(\mathbf{y})) > (<)0$, then $h_m''(w_m(\mathbf{y})) > (<)0$ must hold for any $m \neq i$.

Also, we differentiate H_i twice with respect to ξ_j for any $j \neq i$ to obtain

$$\frac{\partial^2 H_i}{\partial \xi_j^2} = \sum_{j\neq i}(1/\gamma_j)\alpha_i^2\beta_m^2 h_i''(w_i(\mathbf{y})) - (1/\gamma_m)(1-\alpha_m)^2\beta_m^2 h_m''(w_m(\mathbf{y})) - \sum_{l\neq i,m}(1/\gamma_l)\alpha_l^2\beta_m^2 h_l''(w_l(\mathbf{y}))$$

which must be zero again:

$$\sum_{j\neq i}(1/\gamma_j)\alpha_i^2\beta_m^2 h_i''(w_i(\mathbf{y})) - (1/\gamma_m)(1-\alpha_m)^2\beta_m^2 h_m''(w_m(\mathbf{y})) - \sum_{l\neq i,m}(1/\gamma_l)\alpha_l^2\beta_m^2 h_l''(w_l(\mathbf{y})) = 0.$$

By using (A12) and (A14), we obtain

$$\sum_{j\neq i}(1/\gamma_j)(2\alpha_i - 1)h_i''(w_i(\mathbf{y})) = (1/\gamma_m)(1-2\alpha_m)h_m''(w_m(\mathbf{y})) \tag{A15}$$

for any $i \neq m$.

Now suppose that $h_i''(w_i(\mathbf{y})) > 0$ holds for some i. Then, we know that $h_m''(w_m(\mathbf{y})) > 0$ for all $m \neq i$. Then, from (A15) we have $\alpha_i > 1/2$ and $\alpha_m < 1/2$ for any $m \neq i$. Take $j \neq m, i$ (recall that $N \geq 3$). Then $\alpha_j < 1/2$. However, by replacing i by m and m by $j \neq i$ in the above equation (A15) respectively, we obtain $\alpha_j > 1/2$ which is a contradiction to $\alpha_j < 1/2$ for $j \neq i$. Similarly, if $\alpha_i < 1/2$, we have a contradiction as well.

Next suppose that $h_i''(w_i(\mathbf{y})) < 0$. Then, $h_m''(w_m(\mathbf{y})) < 0$ for any $m \neq i$. By multiplying -1 both sides of (A15), we can use the same argument as above.

Finally, suppose that $\alpha_i = 1/2$. Then, $\alpha_m = 1/2$ must hold for any $m \neq i$ by (A16). However, this implies that $1 = \sum_n \alpha_n = N/2$, contradicting to $N \geq 3$.

Thus we have established the result that

$$\begin{aligned} h_i''(w_i(\mathbf{y})) &= h_i''(z_i) \\ &\equiv h_i''\left((1-\alpha_i)\beta_i\xi_i + \alpha_i\left(\overline{W} - \sum_{j\neq i}\beta_j\xi_j\right)\right) \\ &= 0 \end{aligned}$$

must hold for all $(\xi_1, ..., \xi_N) \in \mathbb{R}^N$ and all $i \in N$ when the optimal contract becomes the simple sharing rule (which is the linear function of $(\xi_1, ..., \xi_N)$). Since ξ_n varies over $\mathbb{R}$ for each $n = 1, 2, ..., N$, z_i does so. Then, since $h_i''(z_i) = 0$ for all $z_i \in \mathbb{R}$, it must be that

$$h_i''(z_i) = \left(\frac{u_i''(z_i)}{u_i'(z_i)}\right)' = 0$$

for all $z_i \in \mathbb{R}$. This implies that the degree of the absolute risk aversion of agent i,

$$-\frac{u_i''(z)}{u_i'(z)},$$

must be constant for any i.[17] Q.E.D.

7.5 Proof of Proposition 4

Suppose contrary to the claim that some $\alpha'' \in (0,1)$ and $\varepsilon'' > 0$ exist such that for any N_0 there exists $N \geq N_0$ which ensures that more than $N - \lfloor \alpha'' N \rfloor$ agents are offered the wage schemes w_i^* at the third-best contract, which solves Problem TB, as follows

$$E_{y_{-i}}[w_i^*(y_i, y_{-i})|a_{-i}^*] - \tilde{w}_i(y_i) \geq \varepsilon''$$

for some $y_i \in Y$. Recall that we called the wage scheme satisfying the above inequality *non-IPE wage scheme*. Let $\tilde{N}$ denote the set of all risk averse agents who are offered non-IPE wage schemes under the original contract $\{w_i^*, a_i^*\}_{i=1}^N$.[18]

By definition of $\tilde{w}_i(y_i)$,

$$u_i(\tilde{w}_i(y_i)) = E_{y_{-i}}[u_i(w_i^*(y_i, y_{-i}))|a_{-i}^*].$$

Then $\tilde{w}_i(y_i) \leq E_{y_{-i}}[w_i^*(y_i, y_{-i})|a_{-i}^*]$ for each $y_i \in Y$ because u_i is a concave function. For any risk averse agent who is offered the non-IPE wage scheme $w_i^*(y_i, y_{-i})$ under the original contract w^*, we must have that

$$\tilde{w}_i(y_i) < E_{y_{-i}}[w_i^*(y_i, y_{-i})|a_{-i}^*]$$

for some $y_i \in Y$, which in turn implies that

$$E_{y_i}[\tilde{w}_i(y_i)|a_i^*] < E_{\mathbf{y}}[w_i^*(y_i, y_{-i})|a_i^*, a_{-i}^*].$$

We define

$$\rho_i(N) \equiv E_{\mathbf{y}}[w_i^*(y_i, y_{-i})|a_i^*, a_{-i}^*] - E_{y_i}[\tilde{w}_i(y_i)|a_i^*]$$

for agent $i \in \tilde{N}$ who is offered a non-IPE wage scheme under $\{w_i^*, a_i^*\}$. Thus it must be that $\rho_i(N) \geq \varepsilon'' > 0$ for all large N for any $i \in \tilde{N}$. Then, by our supposition it must be that for

[17] If there is (at most) one risk neutral agent i, $u_i'' = 0$ holds so that $(u_i''/u_i')' = 0$ as well.

[18] Let N'' denote the set of all agents who are offered non-IPE wage schemes under the original contract. Let r denote a unique risk neutral agent if exists. We define $\tilde{N} \equiv N'' \setminus \{r\}$ if $r \in N''$ and $\tilde{N} = N''$ if $r \notin N''$ respectively.

any $i \in \tilde{N}$ we have $\lim_{N\to\infty} \rho_i(N) \geq \underline{\rho} > 0$ for some $\underline{\rho} > 0$. Then we can take some $\rho \in (0, \underline{\rho})$ such that

$$E_{y_i}[\tilde{w}_i(y_i)|a_i^*] + \rho < E_{\mathbf{y}}[w_i^*(y_i, y_{-i})|a_i^*, a_{-i}^*] \tag{A16}$$

for any $i \in \tilde{N}$.

Since $\tilde{w}_i$ yields the same expected payoff to agent i as that under the original wage scheme w_n^*, i.e.,

$$E_{y_i}[u_i(\tilde{w}_i(y_i))|a_i] = E_{y_i,y_{-i}}[u_i(w_i^*(y_i, y_{-i}))|a_i, a_{-i}^*]$$

for any action $a_i \in A$, agent i still has the incentive to choose the third-best action a_i^* and obtains the same equilibrium payoff as that under the original third-best contract w_i^*. This means that the modified contract $\tilde{w}_i$ satisfies IC and IR in Problem M-SB for implementing the third-best action a_i^*. Thus, $\tilde{w}_i$ is feasible in Problem M-SB for implementing the third-best action a_i^*. Recall that $\hat{w}_n(\cdot; a_n^*)$ is the optimal wage scheme solving Problem M-SB for implementing the third-best action a_n^* (by replacing a_n by a_n^* in Problem M-SB). In what follows we will use the shorthand notation $\hat{w}_n^*(\cdot) \equiv \hat{w}_i(\cdot; a_i^*)$ by suppressing its dependency on a_n^*. By the definition of the second-best wage scheme $\hat{w}_i^*$ implementing the third-best action a_i^* in Problem M-SB, we must then have

$$E_{y_i}[\hat{w}_i^*(y_i)|a_i^*] \leq E_{y_i}[\tilde{w}_i(y_i)|a_i^*]$$

so that due to (A16)

$$E_{\mathbf{y}}[w_n^*(\mathbf{y})|\mathbf{a}^*] > E_{y_i}[\hat{w}_n^*(y_n)|a_n^*] + \rho \tag{A17}$$

for any $i \in \tilde{N}$, that is, for any risk averse agent i who is offered the non-IPE wage scheme $w_i^*(y_i, y_{-i})$ under the supposed third best contract.

Now we modify the original contract $\{w_i^*, a_i^*\}_{i=1}^N$ as follows:

- Let N^* be the set of agents with $\#N^* = \lfloor \alpha N \rfloor$ for some $\alpha \in (\alpha'', 1)$. Let N^* include all the agents $i \notin \tilde{N}$ who are *not* offered non-IPE wage schemes at the original contract $\{w_i^*, a_i^*\}$ and one risk neutral agent (if such agent exists). Thus all agents in $N \setminus N^*$ are risk averse and offered non-IPE schemes which depend on the outputs of others under the supposed wage scheme $w_i^*(y_i, y_{-i})$. We take a large enough N to ensure that $\lfloor \alpha N \rfloor > \lfloor \alpha'' N \rfloor$.

- For agent $n \in N^*$, the principal offers the second best wage scheme $\hat{w}_n$ which implements the third best action a_n^*. Thus $\hat{w}_n$ satisfies IC and IR in Problem M-SB for implementing the action a_n^*. As we have explained above, such $\hat{w}_n$ exists because $\tilde{w}_n^*$ is feasible in Problem M-SB.

- $N \setminus N^*$ is divided into the two disjoint sets L^* and L^{**} where $L^* \cup L^{**} = N \setminus N^*$ and $L^* \cap L^{**} = \emptyset$. Let $\#L^* = \lfloor \beta(N - \#N^*) \rfloor$ and $\#L^{**} = N - \#N^* - \#L^*$ for some $\beta \in (0,1)$. Here note that $\#L^{**} = (N - \#N^*) - \lfloor \beta(N - \#N^*) \rfloor \geq (1-\beta)(N - \#N^*) = (1-\beta)(N - \lfloor \alpha N \rfloor) \geq (1-\beta)(N - \alpha N) = (1-\beta)(1-\alpha)N$. Thus $\#L^{**} \to \infty$ when $N \to \infty$.

- Define
$$\beta(N) \equiv \#L^*/\#(N \setminus N^*)$$
which is the equal probability for each agent in $N \setminus N^*$ to be assigned to the set L^*. Then, for agent $i \in N \setminus N^*$, the new wage scheme is defined as
$$\tilde{w}_i(y_i, y_{-i}) = \begin{cases} w_i(y_i) & \text{w.p. } \beta(N) \\ (1/\#L^{**}) \left\{ \overline{W} - \sum_{n \in N^*} \hat{w}_n^*(y_n) - \sum_{i \in L^*} w_i(y_i) \right\} & \text{w.p. } 1 - \beta(N) \end{cases}$$
where
$$\overline{W} \equiv \sum_{n=1}^{N} (E_{y_n}[\hat{w}_n^*(y_n)|a_n^*] + \rho_n), \tag{A18}$$
and $\rho_n \equiv \rho > 0$ satisfies (A17) for any $n \in L^*$, and $\rho_n = 0$ for any $n \notin L^*$ respectively.

  Note that the total wages of all agents becomes constant at $\overline{W}$ for any realization of the outputs $\mathbf{y} \in Y^N$.

  The new contract consists of the following: After the principal observes the realized output profile $\mathbf{y} \in Y^N$, she divides the set of agents in $N \setminus N^*$ randomly into the two subsets L^* and L^{**}. Each agent in $N \setminus N^*$ is equally likely to belong to either L^* or L^{**}, although agents in N^* do not face such randomization.

Under the above random scheme, each agent $i \in N \setminus N^*$ is selected into L^* and paid $w_i(y_i)$ with probability $\beta(N)$ while he is selected into the remaining set L^{**} and paid $(1/\#L^{**})\{\overline{W} - \sum_{n \in N^*} \hat{w}_n^*(y_n) - \sum_{j \in L^*} w_j(y_j)\}$ with probability $1 - \beta(N)$ respectively. Also, by construction, the above contract satisfies FTW: the principal always pays $\overline{W}$ whatever outputs are realized. Moreover, the principal is willing to assign the agents in $N \setminus N^*$ randomly into L^* and L^{**} because her total payment does not depend on whoever agents are selected from $N \setminus N^*$ into L^*.

Step 1. Agent n in N^* faces the following expected payoff:
$$E_{y_n}[u_n(\hat{w}_n(y_n))|a_n] - G_n(a_n),$$
which is maximized at $a_n = a_n^*$ because $\hat{w}_n$ implements the action a_n^* by its definition. Each agent $n \in N^*$ also obtains at least the reservation utility because $\hat{w}_n^*$ satisfies IR in Problem

M-SB with the implementation of $a_i = a_i^*$.

Step 2. Now consider any agent $i \in N \setminus N^*$. Note that

$$\frac{\beta(N - \#N^*)}{N - \#N^*} = \beta \geq \beta(N) \geq \frac{\beta(N - \#N^*) - 1}{N - \#N^*} \geq \beta - \frac{1}{(1-\alpha)N}$$

so that $\beta(N) \to \beta$ as $N \to \infty$. When we fix an output of agent $i \in N \setminus N^*$ at each $y_i \in Y$ and take expectation over $y_{-i} \in Y^{N-1}$, agent i's expected payoff is given by

$$\beta(N)u_i(w_i(y_i)) + (1-\beta(N))E_{y_{-i}}\left[u_i\left(\frac{1}{\#L^{**}}\left(\overline{W} - \sum_{n \in N^*}\hat{w}_n(y_n) - \sum_{j \in L^*} w_j(y_j)\right)\right)\Big| a_{-i}^*\right] - G_i(a_i)$$

conditional on a_{-i}^*. Recall that $\#Y = K$ and $y^k \in Y$ for $k = 1, 2, ..., K$.

Our next step is to find a $K \times \#L^*$ dimensional vector $\mathbf{w} \equiv ((w_i(y^1), ..., w_i(y^K))_{i \in L^*}$ to satisfy

$$\beta(N)u_i(w_i(y^k)) + (1-\beta(N))E_{y_{-i}}\left[u_i\left(\frac{1}{\#L^{**}}\left(\overline{W} - \sum_{n \in N^*}\hat{w}_n(y_n) - \sum_{j \in L^*} w_j(y_j)\right)\right)\Big| a_{-i}^*\right]$$
$$= u_i(\hat{w}_i^*(y^k)) \qquad \text{(A19)}$$

for each $k = 1, 2, ..., K$ and each $i \in L^*$. We will show the existence of $(w_i(y^k))_{i \in L^*, y^k \in Y}$ solving (A19) by sequence of several steps below.

Step 2-1. We take $\beta \in (0,1)$ to be close to 1 and a large N so that $\beta(N)$ is close to β. Then, since u_i is increasing with $u_i(\infty) = \overline{u}$ and $\lim_{w \to w_{\min}} u_i(w) = -\infty$ together with the fact that the second best contract $\hat{w}_i$ is bounded[19] so that $u_i(\hat{w}_i^*(y^k)) < \overline{u}$ for each $y^k \in Y$, there exists a unique $w_i(y^k)$ satisfying the above equation (A19) for each output $y_i = y^k$ ($k = 1, 2, ..., K$) for given $\{w_j(y^k)\}_{j \neq i, j \in L^*, y^k \in Y}$. We denote by $\phi_i^k(\mathbf{w}_{-i})$ such solution where $\mathbf{w}_{-i} \equiv (w_j(y^1), ..., w_j(y^K))_{j \neq i, j \in L^*}$. Note that ϕ_i^k is increasing in each argument and continuous.

Step 2-2. We find lower bounds for ϕ_i^k, which we denote by $\underline{w}_i(y^k)$, as follows. We set $u_i(\underline{w}_i) = \underline{u}$ for $i \in L^*$ and $\underline{w}_i(y^k) \equiv \underline{w}_i$ for all $y^k \in Y$. Then we show that $\phi_i^k(\mathbf{w}_{-i}) \geq \underline{w}_i$ for

[19]The constraint set $\Gamma^{SB}(a_n^*)$ in Problem M-SB to implement the action a_n^* is non-empty because by our supposition $\tilde{w}_n$ satisfies IC and IR in Problem M-SB for implementing a_i^*. The compactness of $\Gamma^{SB}(a_n^*)$ was shown by Grossman and Hart (1983). Then the second best contract $\hat{w}_n$ exists (Grossman and Hart (1983)), which implies that the set of the second best contracts is bounded.

all $\mathbf{w}_{-i}$ and all $i \in L^*$, that is,

$$\beta(N)u_i(\underline{w}_i) + (1-\beta(N))E_{y_{-i}}\left[u_i\left(\frac{1}{\#L^{**}}\left(\overline{W} - \sum_{n\in N^*}\hat{w}_n^*(y_n) - \sum_{j\in L^*} w_j(y_j)\right)\right)\Big|a_{-i}^*\right]$$
$$\leq\; u_i(\hat{w}_i^*(y^k)).$$

for all $y^k \in Y$. Since the left hand side is decreasing in $w_j(y_j)$ for each $j \in L^*$, the above inequality holds for any $w_j(y_j) \geq \underline{w}_j$ if

$$\beta(N)u_i(\underline{w}_i) + (1-\beta(N))E_{y_{-i}}\left[u_i\left(\frac{1}{\#L^{**}}\left(\overline{W} - \sum_{n\in N^*}\hat{w}_n^*(y_n) - \sum_{j\in L^*} \underline{w}_j\right)\right)\Big|a_{-i}^*\right]$$
$$\leq\; u_i(\hat{w}_i^*(y^k)).$$

By the definition of $\underline{w}_i$, the above inequality is equivalent to

$$F_i(\underline{u}) \equiv \beta(N)\underline{u} + (1-\beta(N))E_{y_{-i}}\left[u_i\left(\frac{1}{\#L^{**}}\left(\overline{W} - \sum_{n\in N^*}\hat{w}_n^*(y_n) - \sum_{j\in L^*} u_j^{-1}(\underline{u})\right)\right)\Big|a_{-i}^*\right]$$
$$\leq\; u_i(\hat{w}_i^*(y^k))$$

for all $y^k \in Y$.

We then show that

$$\begin{aligned} F_i'(\underline{u}) \;&=\; \beta(N) - \frac{1-\beta(N)}{\#L^{**}}E_{y_{-i}}\left[u_i'\left(\frac{1}{\#L^{**}}\left(\overline{W} - \sum_{n\in N^*}\hat{w}_n^*(y_n) - \sum_{j\in L^*} u_j^{-1}(\underline{u})\right)\right)\Big|a_{-i}^*\right] \\ &\qquad \times \sum_{j\in L^*}\frac{1}{u_j'(u_j^{-1}(\underline{u}))} \end{aligned}$$

so that, since $u_i'(\infty) < \infty$, $u_j^{-1}(-\infty) = -\infty$ and $u_j'(w_{\min}) = +\infty$, by letting $\underline{u} \to -\infty$, we obtain

$$\begin{aligned} F_i'(\underline{u}) \;&\geq\; \beta(N) - \frac{(1-\beta(N))\#L^*}{\#L^{**}} \times \frac{1}{\min_j u_j'(u_j^{-1}(\underline{u}))} \\ &\qquad \times E_{y_{-i}}\left[u_i'\left(\frac{N}{\#L^{**}}\left(\min_n E[\hat{w}_n(y_n)|a_n^*] - \max_{n,y_n}\hat{w}_n(y_n) - \max_j u_j^{-1}(\underline{u})\right)\right)\Big|a_{-i}^*\right] \\ &\to\; \beta(N) \end{aligned}$$

where $N/\#L^{**} \to 1/(1-\alpha)(1-\beta)$ and $(1-\beta(N))\#L^*/\#L^{**} \to \beta$ when $N \to \infty$.

We can also show that $F_i''(\underline{u}) < 0$ so that F_i is a concave function. This implies that

$$F_i(\underline{u}) \leq F_i(b) + F_i'(b)(\underline{u} - b)$$

for any $b \in (-\infty, \infty)$. By the above result, taking a small enough b, we can ensure that $F_i'(b) \geq \beta(N) > 0$. Then, given a sufficiently small b such that $F_i'(b) \geq \beta(N) > 0$, we show that

$$F_i(\underline{u}) \leq F_i(b) + F_i'(b)(\underline{u} - b) \to -\infty$$

when $\underline{u} \to -\infty$. This is the desired result.

Step 2-3. We next find an upper bound for ϕ_i^k, which we denote by $\overline{w}_i(y^k)$, that is, $\phi_i^k(\mathbf{w}_{-i}) \leq \overline{w}_i(y^k)$ for each $k = 1, 2, ..., K$.

We set

$$\overline{w}_i(y^k) \equiv \hat{w}_i^*(y^k) + \rho$$

for each $y^k \in Y$ and each $i \in L^*$. Here recall the definition of $\rho > 0$ given by (A16).

We want to show that

$$\beta(N)u_i(\overline{w}_i(y^k)) + (1-\beta(N))E_{y_{-i}}\left[u_i\left(\frac{1}{\#L^{**}}\left(\overline{W} - \sum_{n\in N^*}\hat{w}_n^*(y_n) - \sum_{j\in L^*} w_j(y_j)\right)\right)\Big| a_{-i}^*\right]$$
$$> \ u_i(\hat{w}_i^*(y^k))$$

for all $w_j(y_j) \leq \overline{w}_j(y_j)$ for $j \neq i$, $j \in L^*$ and all $y_j \in Y$. Since the left hand side is decreasing in $w_j(y_j)$ for $j \in L^*$, the above inequality holds if

$$\beta(N)u_i(\overline{w}_i(y^k)) + (1-\beta(N))E_{y_{-i}}\left[u_i\left(\frac{1}{\#L^{**}}\left(\overline{W} - \sum_{n\in N^*}\hat{w}_n^*(y_n) - \sum_{j\in L^*} \overline{w}_j(y_j)\right)\right)\Big| a_{-i}^*\right]$$
$$> \ u_i(\hat{w}_i^*(y^k)).$$

Consider the second term in the above left hand side:

$$E_{y_{-i}}\left[u_i\left(\frac{1}{\#L^{**}}\left(\overline{W} - \sum_{n\in N^*}\hat{w}_n^*(y_n) - \sum_{j\in L^*} \overline{w}_j(y_j)\right)\right)\Big| a_{-i}^*\right].$$

Let

$$Z(\mathbf{y}) \equiv \frac{1}{\#L^{**}}\left(\overline{W} - \sum_{n\in N^*}\hat{w}_n^*(y_n) - \sum_{j\in L^*} \overline{w}_j(y_j)\right)$$

for $\mathbf{y} = (y_j)_{j\in N^*\cup L^*}$. Here, since (A18)

$$\begin{aligned}\overline{W} &= \sum_{n=1}^{N}(E_{y_n}[\hat{w}_n^*(y_n)|a_n^*] + \rho_n) \\ &= \sum_{n\in N^*} E_{y_n}[\hat{w}_n^*(y_n)|a_n^*] + \sum_{n\in L^*}(E_{y_n}[\hat{w}_n(y_n)|a_n^*] + \rho) + \sum_{n\in L^{**}} E_{y_n}[\hat{w}_n^*(y_n)|a_n^*],\end{aligned}$$

and $\overline{w}_j(y_j) \equiv \hat{w}_j^*(y_j) + \rho$ for $j \in L^{**}$, we obtain

$$\begin{aligned} Z(\mathbf{y}) &= \frac{1}{\#L^{**}} \left\{ \sum_{n \in N^* \cup L^*} (E_{y_n}[\hat{w}_n^*(y_n)|a_n^*] - \hat{w}_n(y_n)) + \sum_{n \in L^{**}} E_{y_n}[\hat{w}_n^*(y_n)|a_n^*] \right\} \\ &\equiv \tilde{Z}(\mathbf{y}) + \frac{1}{\#L^{**}} \sum_{n \in L^{**}} E_{y_n}[\hat{w}_n^*(y_n)|a_n^*] \end{aligned}$$

where

$$\tilde{Z}(\mathbf{y}) \equiv \frac{\#(N^* \cup L^*)}{\#L^{**}} \left(\frac{1}{\#(N^* \cup L^*)} \right) \left(\sum_{n \in N^* \cup L^*} (E_{y_n}[\hat{w}_n^*(y_n)|a_n^*] - \hat{w}_n^*(y_n)) \right).$$

Here note that

$$\begin{aligned} \frac{\#(N^* \cup L^*)}{\#L^{**}} &= \frac{\#N^* + \#L^*}{\#L^{**}} \\ &= \frac{\lfloor \alpha N \rfloor + \lfloor \beta(N - \lfloor \alpha N \rfloor) \rfloor}{N - \lfloor \alpha N \rfloor - \lfloor \beta(N - \lfloor \alpha N \rfloor) \rfloor} \\ &\leq \frac{\lfloor \alpha N \rfloor + \beta(N - \lfloor \alpha N \rfloor)}{N - \lfloor \alpha N \rfloor - \beta(N - \lfloor \alpha N \rfloor)} \\ &= \frac{\beta N + (1-\beta)\lfloor \alpha N \rfloor}{(1-\beta)(N - \lfloor \alpha N \rfloor)} \\ &\leq \frac{\beta N + (1-\beta)\alpha N}{(1-\beta)(N - \alpha N)} \\ &= \frac{\beta + (1-\beta)\alpha}{(1-\alpha)(1-\beta)}. \end{aligned}$$

We then obtain

$$E_{y_{-i}}[u_i(Z(\mathbf{y}))|a_{-i}^*] = E_{y_{-i}} \left[u_i \left(\tilde{Z}(\mathbf{y}) + \frac{1}{\#L^{**}} \sum_{n \in L^{**}} E_{y_n}[\hat{w}_n^*(y)|a_n^*] \right) \middle| a_{-i}^* \right]. \qquad \text{(A20)}$$

Let

$$h \equiv \frac{1}{\#L^{**}} \sum_{n \in L^{**}} E_{y_n}[\hat{w}_n^*(y)|a_n^*],$$

$$h^* \equiv \min_{n \in N} E_{y_n}[\hat{w}_n^*(y_n)|a_n^*]$$

and

$$h^{**} \equiv \max_{n \in N, y \in Y} \hat{w}_n^*(y_n)$$

respectively. Since the second best contract $\hat{w}_n$ is bounded, h^* and h^{**} are bounded as well.[20] We denote by $\Pr(z; a_{-i}^*)$ the probability that an event z occurs conditional on the

[20]See footnote 19.

action profile a^*_{-i} of others than i. Then (A20) can be further written by

$$\begin{aligned}
& E_{\mathbf{y}}[u_i(Z(\mathbf{y}))|a^*_{-i}] \\
= & \Pr\left(|\tilde{Z}(\mathbf{y})| < \varepsilon; a^*_{-i}\right) E_{y_{-i}}[u_i(\tilde{Z}(\mathbf{y})+h)|a^*_{-i}, |\tilde{Z}(\mathbf{y})| < \varepsilon] \\
& + \Pr\left(|\tilde{Z}(\mathbf{y})| \geq \varepsilon; a^*_{-i}\right) E_{y_{-i}}[u_i(\tilde{Z}(\mathbf{y})+h)|a^*_{-i}, |\tilde{Z}(\mathbf{y})| \geq \varepsilon] \\
\geq & \Pr\left(|\tilde{Z}(\mathbf{y})| < \varepsilon; a^*_{-i}\right) E_{y_{-i}}[u_i(\tilde{Z}(\mathbf{y})+h^*)|a^*_{-i}, |\tilde{Z}(\mathbf{y})| < \varepsilon] \\
& + \Pr\left(|\tilde{Z}(\mathbf{y})| \geq \varepsilon; a^*_{-i}\right) E_{y_{-i}}[u_i(\tilde{Z}(\mathbf{y})+h^*)|a^*_{-i}, |\tilde{Z}(\mathbf{y})| \geq \varepsilon] \\
\geq & \Pr\left(|\tilde{Z}(\mathbf{y})| < \varepsilon; a^*_{-i}\right) u_i(-\varepsilon+h^*) \\
& + \Pr\left(|\tilde{Z}(\mathbf{y})| \geq \varepsilon; a^*_{-i}\right) u_i\left(\frac{\alpha+(1-\alpha)\beta}{(1-\alpha)(1-\beta)}(h^*-h^{**})+h^*\right)
\end{aligned}$$

When N is sufficiently large, $\#(N^* \cup L^*) = \lfloor \alpha N \rfloor + \lfloor \beta(N - \lfloor \alpha N \rfloor)\rfloor \to \infty$. By using $\#(N^* \cup L^*)/\#L^{**} \leq (\alpha + (1-\beta)\alpha)/(1-\alpha)(1-\beta)$ for all N, the Law of Large Numbers implies that

$$\lim_{N\to\infty} \Pr\left(\left|\tilde{Z}(\mathbf{y})\right| \leq \varepsilon; a^*_{-i}\right) \to 1$$

To see this, note that the variance of $\hat{w}^*_i(y_i)$ conditional on a^*_i, denoted by $\mathrm{Var}(\hat{w}^*_i(y_i)|a^*_i)$, is finite because the optimal solution $\hat{w}^*_i$ to Problem (M-SB) is bounded as we have noted above. Thus $\lim_{N\to\infty}\sum_{n=1}^{N}(1/n^2)\mathrm{Var}(\hat{w}^*_n(y_n)|a^*_n) < +\infty$ so that the Law of Large Numbers is applied (see Sen and Singer (1993, p.67, Proposition 2.3.10)).

Thus we have

$$\begin{aligned}
& \lim_{N\to\infty}\left\{\Pr\left(|\tilde{Z}(\mathbf{y})| < \varepsilon; a^*_{-i}\right) u_i(-\varepsilon+h^*)\right. \\
& \qquad \left. + \Pr\left(|\tilde{Z}(\mathbf{y})| \geq \varepsilon; a^*_{-i}\right) u_i\left(\frac{\alpha+(1-\alpha)\beta}{(1-\alpha)(1-\beta)}(h^*-h^{**})+h^*\right)\right\} \\
= & \ u_i(-\varepsilon+h^*) > -\infty.
\end{aligned}$$

We take $\beta \in (0,1)$ and fix it to satisfy

$$\beta u_i(\hat{w}^*_i(y^k)+\rho) + (1-\beta)u_i(-\varepsilon+h^*) > u_i(\hat{w}^*_i(y^k))$$

for all $y^k \in Y$ and all $i \in L^*$. Such $\beta \in (0,1)$ exists because $\rho > 0$ and u_i is strictly increasing.

For such a given $\beta \in (0,1)$, we can take a sufficiently large N so that $\beta(N) \to \beta$ and

$$\begin{aligned}
& \beta(N)u_i(\overline{w}_i(y^k)) + (1-\beta(N))E_{y_{-i}}\left[u_i\left(\frac{1}{\#L^{**}}\left(\overline{W} - \sum_{n\in N^*}\hat{w}^*_n(y_n) - \sum_{j\in L^*}\overline{w}_j(y_j)\right)\right)\Big| a^*_{-i}\right] \\
> & \ u_i(\hat{w}^*_i(y^k))
\end{aligned}$$

for each $y^k \in Y$ and each $i \in L^*$. This completes Step 2-3.

Step 3. If necessary, we can take a small enough $\underline{w}_i$ for $i \in L^*$ such that $\underline{w}_i < \overline{w}_i(y^k)$ for each y^k and each $i \in L^*$. We then consider $w_i(y^k) \in [\underline{w}_i, \overline{w}_i(y^k)]$ for each $y^k \in Y$ $(k = 1, 2, ..., K)$ and each $i \in L^*$.

By Step 2 above, we can ensure that the continuous mapping $\phi_i^k(\mathbf{w}_{-i})$ which we have obtained in Step 1 lies in the compact set $[\underline{w}_i, \overline{w}_i(y^k)]$ for each $k = 1, 2, ..., K$ and $i \in L^*$. We then define a continuous mapping

$$\phi \equiv (\phi_i^1, ..., \phi_i^K)_{i \in L^*}$$

from the compact set $\prod_{i \in L^*} \prod_{k=1}^K [\underline{w}_i, \overline{w}_i(y^k)]$ to itself. Then there exists a fixed point of ϕ by the Brouwer's fixed point Theorem

$$\mathbf{w} = \phi(\mathbf{w})$$

which can be written by

$$\begin{aligned}
&\beta(N)u_i(w_i(y^k)) + (1-\beta(N))E_{y_{-i}}\left[u_i\left(\frac{1}{\#L^{**}}\left(\overline{W} - \sum_{n \in N^*}\hat{w}_n^*(y_n) - \sum_{j \in L^*} w_j(y_j)\right)\right)\middle| a_{-i}^*\right] \\
&= u_i(\hat{w}_i^*(y^k))
\end{aligned}$$

for each $y^k \in Y$ $(k = 1, 2, ..., K)$ and $i \in L^*$. This is the desired result (A19). Each agent $i \in L^*$ thus obtains the same expected payoff as that under the original contract w_i^*:

$$\begin{aligned}
&\beta(N)E_{y_i}[u_i(w_i(y_i))|a_i] \\
&+ (1-\beta(N))E_{y_{-i}}\left[u_i\left(\frac{1}{\#L^{**}}\left(\overline{W} - \sum_{n \in N^*}\hat{w}_n^*(y_n) - \sum_{j \in L^*} w_j(y_j)\right)\right)|a_{-i}^*\right] - G_i(a_i) \\
&= E_{y_i}[u_i(\hat{w}_i^*(y_i))|a_i, a_{-i}^*] - G_i(a_i)
\end{aligned}$$

for each action $a_i \in A$. Thus agent $i \in L^*$ chooses the original third best action a_i^* (because $\hat{w}_i^*$ implements a_i^* by its definition) and obtains the same expected payoff as that attained under the original third best contract $\{w_i^*, a_i^*\}_{i \in L^*}$. Thus the principal can implement the original actions $\mathbf{a}^* \in A^N$ from all agents.

Finally, by construction the total wage is given by (A18)

$$\begin{aligned}
\overline{W} &= \sum_{n \notin L^*} E_{y_n}[\hat{w}_n^*(y_n)|a_n^*] + \sum_{n \in L^*}(E_{y_n}[\hat{w}_n^*(y_n)|a_n^*] + \rho) \\
&< \overline{W}(\mathbf{a}^*) \\
&= \sum_{n \notin L^*} E_{\mathbf{y}}[w_n^*(\mathbf{y})|\mathbf{a}^*] + \sum_{n \in L^*} E_{\mathbf{y}}[w_n^*(\mathbf{y})|\mathbf{a}^*]
\end{aligned}$$

because $E_{y_n}[\hat{w}_n^*(y_n)|a_n^*] \leq E_{\mathbf{y}}[w_n^*(\mathbf{y})|\mathbf{a}^*]$ for all $n \in N$ and, in particular, $E_{y_n}[\hat{w}_n^*(y)|a_n^*]+\rho < E_{\mathbf{y}}[w_n^*(\mathbf{y})|\mathbf{a}^*]$ holds for any $n \in L^*$ due to the definition of $\rho > 0$ (see (A17)). Thus the principal can reduce the total wage payments, which shows that the original contract $\{w_i^*, a_i^*\}_{i=1}^N$ is not optimal to solve Problem TB, a contradiction. Q.E.D.

7.6 Proof of Proposition 5

Recall that $\hat{\mathbf{a}} = (\hat{a}_1, ..., \hat{a}_N)$ denotes the second-best action profile and $\hat{\mathbf{w}} = (\hat{w}_1, ..., \hat{w}_N)$ the second-best wage profile, which solve Problem SB. As shown in Lemma 1, the second-best wage scheme can be an IPE under the supposed utility functions of agents. Thus, without loss of generality, we can suppose that $\hat{w}_i$ depends only on agent i's output y_i, that is, $\hat{w}_i(y_i)$.

We modify the proof of Proposition 4 by replacing the action a_i^* by the second-best one $\hat{a}_i$. We also replace the wage scheme w_i^* in Proposition 4 by the second-best one $\hat{w}_n$ which solves Problem M-SB for implementing the second-best action $\hat{a}_i$ from agent i. All agents in N^* are offered the second-best wage schemes $\hat{w}_n$ implementing the second-best action $\hat{a}_n$ while all others in $N \setminus N^*$ are offered the same random wage scheme as used in the proof of Proposition 4:

$$w_i(y_i, y_{-i}) \equiv \begin{cases} w_i(y_i) & \text{w.p. } \beta(N) \\ (1/\#L^{**})\left\{\overline{W} - \sum_{n\in N^*} \hat{w}_n(y_n) - \sum_{n\in L^*} w_n(y_n)\right\} & \text{w.p. } 1-\beta(N) \end{cases}$$

Here we also modify the total wage $\overline{W}$ as follows

$$\overline{W} \equiv \sum_{n\in N} (E_{y_n}[\hat{w}_n(y_n)|\hat{a}_n] + \rho_n)$$

where $\rho_n \equiv \rho > 0$ for $n \in L^*$ and $\rho_n = 0$ for $n \notin L^*$ respectively.

Then the same argument as the proof of Proposition 4 shows that the principal can attain the total wage $\overline{W}$ for implementing the second-best action profile $\hat{\mathbf{a}}$. To see this, we need to find the wage schemes appeared in the first line of the above expression $\{w_i\}_{i\in L^*}$ which satisfy

$$\beta(N)u_i(w_i(y^k)) + (1-\beta(N))E_{y_{-i}}\left[u_i\left(\frac{1}{\#L^{**}}\left(\overline{W} - \sum_{j\in L^*} w_j(y_j) - \sum_{n\in N^*} \hat{w}_n(y_n)\right)\right) \Big| \hat{a}_{-i}\right]$$
$$= \; u_i(\hat{w}_i(y^k)) \qquad \text{(A21)}$$

for each output $y^k \in Y$ $(k = 1, 2, ..., K)$. The proof of Proposition 4 is modified as follows: as in Step 2-1 we find a continuous function $\phi_i^k(\mathbf{w}_{-i})$ which solves (A21) for $w_i(y^k)$, for each output $y^k \in Y$. Step 2-2 can be also adapted as well. To show Step 2-3 we set

$\overline{w}_i(y^k) \equiv \hat{w}_i(y^k) + \rho$ for agent $i \in L^*$ and each y^k $(k = 1, 2, ..., K)$ where $\rho > 0$ is a small positive constant. Then, when N is large enough and $\beta(N)$ is taken to be close to $\beta \simeq 1$, we can show that the left hand side of (A21) is larger than its right hand side evaluated at $w_i(y^k) = \overline{w}_i(y^k) + \rho_n$ for $k = 1, 2, ..., K$ and $i \in L^*$, completing Step 2-2. Finally, letting $\phi_i \equiv (\phi_i^k(\mathbf{w}_{-i}))_{k=1,2,...,K}$, the continuous mapping $\phi = (\phi_1(\mathbf{w}_{-1}), ..., \phi_N(\mathbf{w}_{-N}))$ defined on $\prod_{i \in L^*} \prod_{k=1,2,...,K} [\underline{w}_i, \overline{w}_i(y^k)]$ to itself has a fixed point which corresponds to the desired wage schemes. Here $\rho > 0$ can be small enough when N is sufficiently large.

Then the average payoff of the principal per agent is given by

$$\begin{aligned}(1/N)\{E_{\mathbf{y}}[R(\mathbf{y})|\hat{\mathbf{a}}] - \overline{W}\} &= (1/N)\{E_{\mathbf{y}}[R(\mathbf{y})|\hat{\mathbf{a}}] - \hat{W}\} - (1/N)\sum_{n \in L^*} \rho \\ &\geq (1/N)\{E_{\mathbf{y}}[R(\mathbf{y})|\hat{\mathbf{a}}] - \hat{W}\} - \beta(1-\alpha)\rho \\ &\equiv \hat{\pi} - \varepsilon\end{aligned}$$

where $\varepsilon \equiv \beta(1-\alpha)\rho > 0$. We can take $\varepsilon > 0$ to be small enough when $N \to \infty$ so that $\rho > 0$ is small enough. Q.E.D.

7.7 Proof of Proposition 6

We take the wage schemes $\{w_i^\delta\}_{i=1}^N$ which satisfy Assumption 5 and then define the total wage as

$$\hat{W}_\delta \equiv \sum_{i=1}^N E_{y_i}[w_i^\delta(y_i)|\hat{a}_i]$$

where $\hat{W}_\delta \to \hat{W}$ as $\delta \to 0$.

Consider the following simple sharing rule for agent i: for each output profile $(y_i, y_{-i}) \in Y^N$,

$$\tilde{w}_i(y_i, y_{-i}) \equiv w_i^\delta(y_i) + (1/N)\left\{\hat{W}_\delta - \sum_{n=1}^N w_n^\delta(y_n)\right\}.$$

Under the wage scheme $\tilde{w}_i$ defined above, agent i obtains the following expected payoff:

$$E_{\mathbf{y}}[U_i(\tilde{w}_i(y_i, y_{-i}), a_i)|a_i, a_{-i}]$$

where $E_{\mathbf{y}}[\cdot|a_i, a_{-i}]$ denotes the expectation over output profiles $(y_i, y_{-i}) \in Y^N$ conditional on an action profile (a_i, a_{-i}).

Our target is to implement the second best action profile $\hat{\mathbf{a}} = (\hat{a}_1, ..., \hat{a}_N)$ at the total cost $\hat{W}_\delta$ by using the wage scheme $(\tilde{w}_1, ..., \tilde{w}_N)$ defined above. Thus we need to show that

$$E[U_i(\tilde{w}_i(y_i, y_{-i}), \hat{a}_i)|\hat{a}_i, \hat{a}_{-i}] \geq E[U_i(\tilde{w}_i(y_i, y_{-i}), a_i)|a_i, \hat{a}_{-i}] \quad \text{(IC*)}$$

for any $a_i \neq \hat{a}_i$, and

$$E[U_i(\tilde{w}_i(y_i, y_{-i}), \hat{a}_i)|\hat{a}_i, \hat{a}_{-i}] \geq \overline{U}_i \tag{IR*}$$

When N is large enough, IC* is almost identical to the original IC in the second-best problem, Problem SB, given in the main text. This is because the second term of $\tilde{w}_i$ has only negligible impacts on the agent i's expected payoff when $N \to \infty$ due to the Law of Large Numbers. We also need to check that, given $(a_i, \hat{a}_{-i})$ for $a_i \neq \hat{a}_i$ (only agent i deviates to choose $a_i \neq \hat{a}_i$), the same limit argument holds. Also, when N is large enough, agent i faces the almost same expected payoff as what he obtains under the second-best wage scheme so that IR* is satisfied for a sufficiently large N. We will show these results below.

Since $|w_i^\delta(y_i) - \hat{w}_i(y_i)| < \delta$ for all $y_i \in Y$ and the second best wage scheme $\hat{w}_i$ is bounded, w_i^δ is bounded as well. Thus, its variance $\mathrm{Var}(w_i^\delta(y_i)|\hat{a}_i)$ conditional on $\hat{a}_i$ is finite. This implies that

$$\lim_{N\to\infty} \sum_{n=1}^{N} (1/n^2)\mathrm{Var}(w_i^\delta(y_i)|\hat{a}_i) < +\infty$$

because $\sum_{n=1}^{\infty}(1/n^2) < 2$. Then, the Law of Large Numbers shows that

$$(1/N)\left\{\overline{W}_\delta - \sum_{n=1}^{N} w_i^\delta(y_i)\right\} \to 0$$

in probability when $N \to \infty$ (see Sen and Singer (1993, p.67, Proposition 2.3.10)).

Define

$$Z(\mathbf{y}) \equiv (1/N)\left\{\overline{W}_\delta - \sum_{n=1}^{N} w_n^\delta(y_n)\right\}.$$

By the above argument the probability that the event of $|Z(\mathbf{y})| \geq \varepsilon$ occurs conditional on the second best action profile $(\hat{a}_i, \hat{a}_{-i})$ must be

$$\Pr(|Z(\mathbf{y})| \geq \varepsilon; \hat{a}_i, \hat{a}_{-i}) \to 0$$

as $N \to \infty$, given $\varepsilon > 0$.

We also denote by

$$E_{\mathbf{y}}[\,\cdot\,|a_i, \hat{a}_{-i}, |Z(\mathbf{y})| < \varepsilon]$$

the expectation over $\mathbf{y} \in Y^N$ conditional on an action profile $(a_i, \hat{a}_{-i})$ and the event that $|Z(\mathbf{y})| < \varepsilon$ occurs.

Under the wage scheme defined above, agent i obtains the following expected payoff:

$$\begin{aligned}
&E_{\mathbf{y}}[U_i(\tilde{w}_i(y_i, y_{-i}))|a_i, \hat{a}_{-i}] \\
&= E_{\mathbf{y}}\left[U_i\left(w_i^\delta(y_i) + Z(\mathbf{y}), a_i\right)|a_i, \hat{a}_{-i}\right] \\
&= \Pr(|Z(\mathbf{y}| \geq \varepsilon; a_i, \hat{a}_{-i})E_{\mathbf{y}}[U_i(w_i^\delta(y_i) + Z(\mathbf{y}), a_i)|a_i, \hat{a}_{-i}, |Z(\mathbf{y})| \geq \varepsilon] \\
&\quad + \Pr(|Z(\mathbf{y}| < \varepsilon; a_i, \hat{a}_{-i})E_{\mathbf{y}}[U_i(w_i^\delta(y_i) + Z(\mathbf{y}), a_i)|a_i, \hat{a}_{-i}, |Z(\mathbf{y})| < \varepsilon]
\end{aligned}$$

Then we can show that the above limit argument still holds even when we replace $(\hat{a}_i, \hat{a}_{-i})$ by $(a_i, \hat{a}_{-i})$ (thus only agent i's action is changed from $\hat{a}_i$ to any $a_i \in A$). That is,

$$\lim_{N\to\infty} \Pr\left(|Z(\mathbf{y})| \geq \varepsilon; a_i, \hat{a}_{-i}\right) \to 0$$

for any $a_i \in A$. To see this, define

$$\Delta_i(a_i) \equiv E_{y_i}[w_i^\delta(y_i)|\hat{a}_i] - E_{y_i}[w_i^\delta(y_i)|a_i]$$

for any $a_i \in A$ and let

$$|\max_{a_i\in A} \Delta_i(a_i)|/N < \varepsilon/2$$

by taking a large N. Then, for an arbitrary $\eta > 0$, we can find a large enough N to ensure that

$$\begin{aligned}
&\Pr\left(|Z(\mathbf{y})| \geq \varepsilon; a_i, \hat{a}_{-i}\right) \\
&= \Pr\left(\frac{1}{N}\left|\sum_{n=1}^{N} E_{y_n}[\hat{w}_n^\delta(y_n)|\hat{a}_n] - \sum_{n=1}^{N} w_n^\delta(y_n)\right| \geq \varepsilon; a_i, \hat{a}_{-i}\right) \\
&= \Pr\left(\frac{1}{N}\left|\sum_{n\neq i}^{N} E_{y_n}[\hat{w}_n^\delta(y_n)|\hat{a}_n] + E_{y_i}[w_i^\delta(y_i)|a_i] + \Delta_i(a_i) - \sum_{n=1}^{N} w_n^\delta(y_n)\right| \geq \varepsilon; a_i, \hat{a}_{-i}\right) \\
&\leq \Pr\left(\frac{1}{N}\left|\sum_{n\neq i} E_{y_n}[w_n^\delta(y_n)|\hat{a}_n] + E_{y_i}[w_i^\delta(y_i)|a_i] - \sum_{n=1}^{N} w_n^\delta(y_n)\right| + \frac{1}{N}\left|\Delta_i(a_i)\right| \geq \varepsilon; a_i, \hat{a}_{-i}\right) \\
&\leq \Pr\left(\frac{1}{N}\left|\sum_{n\neq i} E_{y_n}[w_n^\delta(y_n)|\hat{a}_n] + E_{y_i}[w_i^\delta(y_i)|a_i] - \sum_{n=1}^{N} w_n^\delta(y_n)\right| \geq \varepsilon/2; a_i, \hat{a}_{-i}\right) \\
&\leq \eta
\end{aligned}$$

where the last inequality follows from the Law of Large Numbers.

Define

$$\overline{w} \equiv \max_{i\in N, y\in Y} w_i^\delta(y), \quad \underline{w} \equiv \min_{i\in N, y\in Y} w_i^\delta(y).$$

Then we have

$$\overline{w} - \underline{w} \geq Z(\mathbf{y}) \equiv (1/N)\left\{\hat{W}_\delta - \sum_{n=1}^{N} w_n^\delta(y_n)\right\} \geq \underline{w} - \overline{w}.$$

We take $\Gamma > 0$ and $\gamma > 0$ such that

$$\min_{i\in N, a\in A} U_i(2\underline{w} - \overline{w}, a) \geq -\gamma$$

and

$$\Gamma > \max_{i\in N, a\in A} U_i(2\overline{w} - \underline{w}, a)$$

Then we can show that

$$\begin{aligned}
&\Pr(|Z(\mathbf{y})| \geq \varepsilon; a_i, \hat{a}_{-i}) E_{\mathbf{y}}[U_i(w_i^\delta(y_i) + Z(\mathbf{y}), a_i)|a_i, \hat{a}_{-i}, |Z(\mathbf{y})| \geq \varepsilon] \\
\geq\ & \Pr(|Z(\mathbf{y})| \geq \varepsilon; a_i, \hat{a}_{-i}) U_i(\underline{w} + (\underline{w} - \overline{w}), a_i) \\
\geq\ & -\eta\gamma
\end{aligned}$$

when $N \to \infty$.

Thus, when N is large enough, agent i obtains at least the following expected payoff by choosing the second best action $\hat{a}_i$:

$$\begin{aligned}
&\Pr(|Z(\mathbf{y})| \geq \varepsilon; \hat{a}_i, \hat{a}_{-i}) E_{\mathbf{y}}[U_i(w_i^\delta(y_i) + Z(\mathbf{y}), \hat{a}_i)|\hat{a}_i, \hat{a}_{-i}, |Z(\mathbf{y})| \geq \varepsilon] \\
&\quad + \Pr(|Z(\mathbf{y})| < \varepsilon; \hat{a}_i, \hat{a}_{-i}) E_{y_i}[U_i(w_i^\delta(y_i) - \varepsilon, a_i)|\hat{a}_i] \\
\geq\ & -\eta\gamma + \Pr(|Z(\mathbf{y})| < \varepsilon; \hat{a}_i, \hat{a}_{-i}) E_{y_i}[U_i(w_i^\delta(y_i) - \varepsilon, \hat{a}_i)|\hat{a}_i] \\
=\ & E_{y_i}[U_i(w_i^\delta(y_i), \hat{a}_i)|\hat{a}_i] + U_i^*
\end{aligned}$$

where

$$U_i^* \equiv -\eta\gamma + \Pr(|Z(\mathbf{y})| < \varepsilon; \hat{a}_i, \hat{a}_{-i}) E_{y_i}[U_i(w_i^\delta(y_i) - \varepsilon, \hat{a}_i)|\hat{a}_i] - E_{y_i}[U_i(w_i^\delta(y_i), \hat{a}_i)|\hat{a}_i]$$

which converges to zero as $N \to \infty$ so that $\eta \to 0$ and $\varepsilon \to 0$.

On the other hand, when agent i deviates to choose $a_i \neq \hat{a}_i$, he can obtain at most

$$\begin{aligned}
&\Pr(|Z(\mathbf{y})| \geq \varepsilon; a_i, \hat{a}_{-i}) E_{\mathbf{y}}[U_i(w_i^\delta(y_i) + Z(\mathbf{y}), a_i)|a_i, \hat{a}_{-i}, |Z(\mathbf{y})| \geq \varepsilon] \\
&\quad + \Pr(|Z(\mathbf{y})| < \varepsilon; a_i, \hat{a}_{-i}) E_{y_i}[U_i(w_i^\delta(y_i) + Z(\mathbf{y}), a_i)|a_i] \\
\leq\ & \Pr(|Z(\mathbf{y})| \geq \varepsilon; a_i, \hat{a}_{-i}) U_i(2\overline{w} - \underline{w}, a_i) \\
&\quad + \Pr(|Z(\mathbf{y})| < \varepsilon; a_i, \hat{a}_{-i}) E_{y_i}[U_i(w_i^\delta(y_i) + \varepsilon, a_i)|a_i] \\
\leq\ & \Pr(|Z(\mathbf{y})| \geq \varepsilon; a_i, \hat{a}_{-i})\Gamma + \Pr(|Z(\mathbf{y})| < \varepsilon; a_i, \hat{a}_{-i}) E_{y_i}[U_i(w_i^\delta(y_i) + \varepsilon, a_i)|a_i] \\
\leq\ & \eta\Gamma + \Pr(|Z(\mathbf{y})| < \varepsilon; a_i, \hat{a}_{-i}) E_{y_i}[U_i(w_i^\delta(y_i) + \varepsilon, a_i)|a_i] \\
\leq\ & E_{y_i}[U_i(w_i^\delta(y_i), a_i)|a_i] + U_i^{**}
\end{aligned}$$

where

$$U_i^{**} \equiv \eta\Gamma + \max_{a \in A} \left\{ \Pr(|Z(\mathbf{y})| < \varepsilon; a_i, \hat{a}_{-i}) E_{y_i}[U_i(w_i^\delta(y_i) + \varepsilon, a_i)|a_i] - E_{y_i}[U_i(w_i^\delta(y_i), a_i)|a_i] \right\}$$

which converges to zero when $N \to \infty$ because

$$\lim_{N \to \infty} \Pr(|Z(\mathbf{y})| < \varepsilon; a_i, \hat{a}_{-i}) \to 1$$

together with $\varepsilon \to 0$ and $\eta \to 0$.

Now we take $\tilde{\delta} > 0$ such that

$$E_{y_i}[U_i(\hat{w}_i^\delta(y_i), \hat{a}_i)|\hat{a}_i] \geq E_{y_i}[U_i(w_i^\delta(y_i), a_i)|a_i] + \tilde{\delta}, \quad \text{for any } a_i \neq \hat{a}_i$$

and

$$E_{y_i}[U_i(\hat{w}_i^\delta(y_i), \hat{a}_i)|\hat{a}_i] \geq \overline{U}_i + \tilde{\delta}$$

Such $\tilde{\delta} > 0$ exists due to the strict inequalities of IC and IR under w_i^δ.

Then, when N is large enough so that $U_i^{**} - U_i^* \leq \tilde{\delta}$ for all i, we can ensure that

$$\begin{aligned} E_{y_i}[U_i(w_i^\delta(y_i), \hat{a}_i)|\hat{a}_i] &\geq E_{y_i}[U_i(w_i^\delta(y_i), a_i)|a_i] + \tilde{\delta} \\ &\geq E_{y_i}[U_i(w_i^\delta(y_i), a_i)|a_i] + U_i^{**} - U_i^* \end{aligned}$$

for all $a_i \neq \hat{a}_i$. Then IC* is satisfied.

Also agent i obtains at least the following expected payoff by choosing $\hat{a}_i$:

$$E_{y_i}[U_i(w_i^\delta(y_i), \hat{a}_i)|\hat{a}_i] + U_i^*.$$

When N is so large that $|U_i^*| \leq \tilde{\delta}$ for all i, we have

$$\begin{aligned} E_{y_i}[U_i(w_i^\delta(y_i), \hat{a}_i)|\hat{a}_i] + U_i^* &\geq E_{y_i}[U_i(w_i^\delta(y_i), \hat{a}_i)|\hat{a}_i] - \tilde{\delta} \\ &\geq \overline{U}_i \end{aligned}$$

showing that the individual rationality constraint IR is satisfied as well.

Thus we have established the result that the principal can approximately attain the following total wage for implementing the second best actions $\hat{\mathbf{a}}$ from all agents:

$$\begin{aligned} (1/N)W_\delta &\equiv (1/N)\sum_{n=1}^{N} E_y[w_i^\delta(y)|\hat{a}_n] \\ &\leq (1/N)\sum_{n=1}^{N} E_y[\hat{w}_n(y)|\hat{a}_n] + \delta \\ &= (1/N)\hat{W} + \delta \\ &\to (1/N)\hat{W} \end{aligned}$$

where $\delta \to 0$ as $N \to \infty$. Q.E.D.

8 Appendix B: Extensions

8.1 Monotonicity of the Third-Best Contract

First, we show that the third-best contract $w_i(\mathbf{y})$ shown in Lemma 2 is non-decreasing in y_i but non-increasing in y_{-i} when MLRP and CDFC are satisfied.

Proposition B1. *Suppose that MLRP and CDFC are satisfied. Then the third best contract* $w_i(\mathbf{y}) = Q_i(\xi_i(y_i) - \xi_1(y_1), ..., \xi_i(y_i) - \xi_N(y_N))$ *shown in Lemma 2 is non-decreasing in* y_i *and non-increasing in* y_j *for* $j \neq i$.

Proof. It is readily verified that w_i is non-decreasing in y_i and non-increasing in y_{-i} under MLRP when $\mu_l(a) > 0$ holds only for $a < \hat{a}_l$ for all $l \in N$ because ξ_i is non-decreasing in y_i under MLRP. It thus suffices to show that the third best contract solving Problem RP without upward IC constraints actually satisfies upward IC constraints, that is, IC is never binding at any $a > \hat{a}_i$ for each agent i.

We can re-write the expected payoff of agent i as

$$EU_i(a_i) \equiv \sum_{y_{-i}} P(y_{-i}|\hat{\mathbf{a}}_{-i})\Delta u_i(y^{k+1}, \mathbf{y}_{-i})(1 - F_i(y^k|a_i)) - G_i(a_i)$$

where, since w_i is non-decreasing in y_i, we have

$$\Delta u_i(y^{k+1}, y_{-i}) \equiv u_i(w_i(y^{k+1}, y_{-i})) - u_i(w_i(y^k, y_{-i})) > 0$$

for each output y^k, $k = 1, 2, ..., K$. Since IC must be binding for some $a' < \hat{a}_i$ (otherwise, w_i is non-increasing in y_i under MLRP so that agent i never chooses $\hat{a}_i > 0$), if IC is also binding at some $a'' > \hat{a}_i$, then we have $EU_i(\hat{a}_i) = EU_i(a'') = EU_i(a')$. However, since some $\alpha \in (0, 1)$ exists such that $\hat{a}_i = \alpha a'' + (1 - \alpha)a'$, CDFC and strict convexity of G_i imply that $EU_i(\hat{a}_i) > \alpha EU_i(a'') + (1 - \alpha)EU_i(a') = EU_i(\hat{a}_i)$, a contradiction. Q.E.D.

8.2 Third-Best Contract with CARA Utility Functions

In the main text we have assumed that all agents are risk averse and have the CARA utility function as $u_i(w) = (1/r_i)(1 - e^{-r_i w})$ for $r_i > 0$. In this subsection we modify the third-best contract when there is one risk neutral agent while all others are risk averse (recall that we are assuming that there is at most one risk neutral agent in Section 4).

Suppose that agent 1 is risk neutral and his utility function is given by $w - G_1(a)$ while any other $j \neq 1$ has the CARA utility function. Then equation (A4) is modified for agent

1 and any other $j \neq 1$ as $-r_j w_j(\mathbf{y}) = \xi_1(y_1) - \xi_j(y_j)$. By dividing both sides of this by r_j, summing this over all $j \neq 1$ and using FTW, we obtain

$$w_1(\mathbf{y}) = \overline{W} + \sum_{j \neq 1} (1/r_j)(\xi_1(y_1) - \xi_j(y_j)).$$

When $N = 2$ so that $i = 1, 2$, we have $w_2(\mathbf{y}) = \overline{W} - w_1(\mathbf{y}) = (1/r_2)(\xi_2(y_2) - \xi_1(y_1))$.

Now suppose that $N \geq 3$. For any pair of risk averse agents $i \neq j$ where $i, j \neq 1$, we still have (A4) from which we can derive

$$w_i(\mathbf{y}) = \frac{1/r_i}{\sum_{l \neq 1}(1/r_l)} \left\{ \overline{W} - w_1(\mathbf{y}) + \sum_{j \neq i,1} (\xi_i(y_i) - \xi_j(y_j)) \right\}$$

by summing (A4) over all $j \neq i, 1$ and using FTW: $\overline{W} = \sum_{l \neq 1} w_l(\mathbf{y}) + w_1(\mathbf{y})$. Then, we can substitute the above $w_1(\mathbf{y})$ into this expression to obtain

$$w_i(\mathbf{y}) = (1/r_i)(\xi_i(y_i) - \xi_1(y_1))$$

for each $i \neq 1$. Thus, all but risk neutral agent should be compensated according to only the difference between his own piece rate $\xi_i(y_i)$ and the risk neutral agent's one $\xi_1(y_1)$.

These optimal contracts can be also obtained by letting $r_1 \to 0$ while keeping $r_i > 0$ for any $i \neq 1$ in the expression of the simple sharing rule $w_i(\mathbf{y})$ stated in Proposition 2 as well.

Next, we extend the result about the third-best contract with CARA utility functions (Proposition 2) to allow agents to choose continuous actions.

We suppose that agent's action is given by $A = [0, \infty)$ and maintain the assumption that Y is finite. We slightly change notations which we have used in the main text by letting $p^i(y_i|a_i)$ denote the probability of agent i's output being realized as y_i. Here superscript i denotes an index of agent whereas we have used subscript i to denote it in the main text. We assume that the action cost $G_i(a)$ and the probability of output $p^i(y_i|a)$ are all continuously differentiable functions of action $a_i \in A$. In particular we assume that G_i is increasing and strictly convex, $G' > 0$ and $G'' > 0$ for all $a > 0$. We also denote by $p^i_a(y|a) \equiv \partial p^i(y|a)/\partial a$ the partial derivative of p^i with respect to agent i's action a_i.

We make the following conditions which are continuous versions of MLRP and CDFC.

Condition (MLRP*). *$p^i_a(y|i|a_i)/p^i(y_i|a_i)$ is increasing in y_i for each agent i and action $a_i \in A$.*

Condition (CDFC*). *Let $F^i(y|a) = \sum_{z \leq y} p^i(z|a)$ denote the CDF of $p^i(y|a)$. Then $F^i(y|a)$ is a convex function of $a \in A$, that is, $F^i_{aa} \geq 0$..*

We then show the following result.

Proposition B2. *Consider the continuous action model given above and suppose that MLRP* and CDFC* are satisfied. Suppose also that agents' preferences are represented by CARA utility functions as in the main text. Then the third-best contract is given as follows*

$$w_i(\mathbf{y}) = (1/r_i)\psi_i(y_i) + \frac{1/r_i}{\sum_{l=1}^N (1/r_l)} \left\{ \overline{W} - \sum_{l=1}^N (1/r_l)\psi_l(y_l) \right\}$$

where

$$\psi_i(y_i) \equiv \ln \left(\lambda_i + \mu_i \left(1 - \frac{p^i_a(y_i|a^*_i)}{p^i(y_i|a^*_i)} \right) \right).$$

Proof. We replace the incentive compatibility constraint for implementing the third best action a^*_i

$$\sum_{y_{-i}} P(y_{-i}|a^*_{-i}) \sum_{y_i} p^i(y_i|\hat{a}_i) u_i(w_i(\mathbf{y})) - G_i(a^*_i) \geq \sum_{y_{-i}} P(y_{-i}|a^*_{-i}) \sum_{y_i} p^i(y_i|a_i) u_i(w_i(\mathbf{y})) - G_i(a_i) \tag{IC}$$

by the corresponding first order condition:

$$\sum_{y_{-i}} P(y_{-i}|a^*_{-i}) \sum_{y_i} p^i_a(y_i|a^*_i) u_i(w_i(\mathbf{y})) - G'_i(a^*_i) = 0 \tag{FIC}$$

whenever $a^*_i > 0$. In what follows we assume that $a^*_i > 0$ for all i and discuss the case of $a^*_i = 0$ later.

We further replace the above FIC by the weaker one:

$$\sum_{y_{-i}} P(y_{-i}|a^*_{-i}) \sum_{y_i} p^i_a(y_i|a^*_i) u_i(w_i(\mathbf{y})) - G'_i(a^*_i) \geq 0 \tag{FIC$''$}$$

Then we consider the following doubly relaxed problem:

Problem D-RP

$$\min \quad \overline{W}$$

subject to FIC$''$, IR and FTW$''$:

$$\overline{W} \geq \sum_{i=1}^N w_i(\mathbf{y}) \quad \forall\, \mathbf{y} \in Y^N.$$

Here, the above problem is relaxed from the original third best problem, Problem TB, in two respects: one is that IC is doubly relaxed by FIC$''$ and the other is that FTW is replaced by a weaker constraint FTW$''$.

Lemma B1. *The Slater condition is satisfied in Problem D-RP.*

Proof. We can change variables as $u_i(\mathbf{y}) \equiv u_i(w_i(\mathbf{y}))$ for each $\mathbf{y} \in Y^N$ and $w_i(\mathbf{y}) = \phi_i(u_i(\mathbf{y}))$ for $\phi_i \equiv u_i^{-1}$ in the above problem, Problem D-RP. Since MLRP holds, there exists some $\hat{y}_i$ such that $p_a^i(y|a_i^*) > (<) p_a^i(y|a_i^*)$ for all $y > (<) \hat{y}_i$. Then we can consider an individual utility scheme $u_i(y_i)$ such that $u_i(y_i)$ takes a large positive value for any $y_i > \hat{y}_i$ while it takes a large negative value for any $y_i < \hat{y}_i$. Then we can ensure that

$$\sum_{y_i > \hat{y}_i} p_a^i(y_i|a_i^*)u_i(y_i) + \sum_{y_i < \hat{y}_i} p_a^i(y_i|a_i^*)u_i(y_i) > G_i'(a_i^*).$$

Thus FIC$''$ is satisfied as a strict inequality. By adding a large constant v to $u_i(y_i)$, $(u_i(y_i) + v)_{y_i \in Y}$ can satisfy IR as a strict inequality as well. Finally, by taking a large enough $\overline{W}$, FTW$''$ is strictly satisfied given the above wage scheme. Q.E.D.

By Lemma B1, the necessary condition for the optimal solution to Problem D-RP must satisfy the following KKT conditions: there exist some non-negative $\lambda_i \geq 0$, $\mu_i \geq 0$ and $\eta(\mathbf{y}) \geq 0$ such that

$$-1 + \sum_{\mathbf{y} \in Y^N} \eta(\mathbf{y}) = 0, \tag{B1}$$

$$P(\mathbf{y}|\mathbf{a}^*)u_i'(w_i(\mathbf{y})) \left\{ \lambda_i + \mu_i \left(1 - \frac{p_a^i(y_i|a_i^*)}{p^i(y_i|a_i^*)} \right) \right\} - \eta(\mathbf{y}) = 0. \tag{B2}$$

Then we can show

Lemma B2. $\eta(\mathbf{y}) > 0$ *for all* $\mathbf{y} \in Y^N$.

Proof. This is shown by a similar argument to Lemma A2. Q.E.D.

By Lemma B2, we obtain the following formula for the optimal contract which solves Problem D-RP:

$$w_i(\mathbf{y}) = (1/r_i)\psi_i(y_i) + \frac{1/r_i}{\sum_{l=1}^N (1/r_l)} \left\{ \overline{W} - \sum_{l=1}^N (1/r_l)\psi_l(y_l) \right\}$$

where
$$\psi_i(y_i) \equiv \ln\left(\lambda_i + \mu_i\left(1 - \frac{p_a^i(y_i|a_i^*)}{p^i(y_i|a_i^*)}\right)\right).$$
Here, ψ_i is non-decreasing in y_i under MLRP.

Finally, we can check that FIC$''$ holds as equality and agent's expected payoff function is a concave function of his action. Thus the relaxed problem D-RP is exactly same as the original problem, Problem TB. Note first that, if FIC$''$ holds as a strict inequality, $\mu_i = 0$ must be satisfied. This yields $\psi_i(y_i) = \ln \lambda_i$ so that $w_i(\mathbf{y})$ is independent of y_i, that is, $w_i(\mathbf{y}) = w_i(y_{-i})$. Then, since $\sum_{y_i \in Y} p_a^i(y|a) = 0$ holds, agent i would choose $a_i = 0$ and hence
$$\sum_{y_{-i}} P(y_{-i}|a_{-i}^*)u_i(w_i(y_{-i}))\sum_{y_i} p_a^i(y_i|a_i^*) - G_i'(a_i^*) = -G_i'(a_i^*) \leq 0,$$
contradicting to our supposition that FIC$''$ holds as a strict inequality. Second, note that agent i's expected payoff is given by
$$\begin{aligned}
&\sum_{y_{-i}} P(y_{-i}|a_{-i}^*)\sum_{y_i} p^i(y_i|a)u_i(w_i(y_i, \mathbf{y}_{-i})) - G_i(a) \\
&= \sum_{y_{-i}} P(y_{-i}|a_{-i}^*)\Delta u_i(y^k, y_{-i})(1 - F^i(y^k|a)) - G_i(a)
\end{aligned}$$
where $\Delta u_i(y^k, y_{-i}) \equiv u_i(w_i(y^k, y_{-i})) - u_i(w_i(y^{k-1}, y_{-i})) \geq 0$ for each $k = 1, 2, ..., K$ because w_i and ψ_i are non-decreasing in y_i. Since F^i is convex with respect to action a_i due to CDFC*, the above expected payoff is a concave function of a_i.

Finally, we treat the case that $a_i^* = 0$ for some agent i. This is stated as the following IC constraint:
$$\sum_{y_{-i}} P(y_{-i}|a_{-i}^*)p_a^i(y_i|0)u_i(w_i(\mathbf{y})) - G_i'(0) \leq 0.$$
Letting $\mu_i^0 \geq 0$ be the Lagrange multiplier for this constraint, we can modify the piece rate wage appeared in the optimal contract for agent i as follows
$$\psi_i^0(y_i) \equiv \ln\left(\lambda_i - \mu_i^0 \frac{p_a^i(y_i|0)}{p^i(y_i|0)}\right)$$
which is non-increasing in y_i under MLRP. Thus, w_i is non-increasing in y_i so that agent i's expected payoff is always non-increasing in his action a_i. This implies that agent i optimally chooses the least costly action $a_i = 0$. Q.E.D.