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Abstract

An axiomatic characterization of price or market mechanisms is one of the most important problems in general equilibrium theory. Based on the general equilibrium framework and the characterization of the price mechanism, this paper provides a new perspective or a unified viewpoint on some axioms in social choice theory and a setting for the informational efficiency problem of the allocation mechanisms. Our arguments focus on a contemporary reconsideration and generalization of the category theoretic method in Sonnenschein (1974) and the replica stability arguments in social choice theory like Thomson (1988) and Nagahisa (1994). Sonnenschein's axiomatic characterization of the price mechanism is extended to an *economy-dependent welfare form* of a *universal implementability theorem*. The framework provides new methods and general settings in treating mechanisms with messages or information, and many social choice axioms. Our result also has an important economic interpretation that the price mechanism can be characterized as a *universal* rule that is *stable* in assuring sufficiently high utility levels for each member of a small economy relative to its large expansions.

Keywords: Price Mechanism, Axiomatic Characterization, Category Theory, Informational Efficiency, Universal Implementability, Message Mechanism

JEL classification: C60, D50, D71

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1 Introduction

An axiomatic characterization of price or market mechanisms is one of the most important problems in general equilibrium theory. Based on the general equilibrium framework and the characterization of the price mechanism, this paper provides a new perspective or a unified viewpoint on some axioms in social choice theory and a setting for the informational efficiency problem on allocation mechanisms with messages like Hurwicz (1960), Mount and Reiter (1974), Sonnenschein (1974), and so forth. Our arguments are particularly concerned with a contemporary reconsideration and generalization of the *category theoretical axiomatic method* in Sonnenschein (1974), its stability axiom for messages with respect to economic expansion, and the *replica stability axiom* in such social choice arguments as Thomson (1988) and Nagahisa (1994). These stability axioms and characterizations of the price mechanism have closely related theoretical structures. Such a unified view enables us to obtain a generalization of Sonnenschein's axiomatic characterization of the price mechanism to extend his *dictionary property theorem* based directly on *agent-characteristics* to a *universal implementability theorem* based on economies and *utility levels* or welfare conditions.

In this paper, we characterize the price mechanism as a unique mechanism that *implements* the resource allocation process of all other message mechanisms and satisfies several important axioms: core property, replica stability of responses, Sonnenschein's expansion possibility, etc.[1] The replica stability social choice axiom is reformulated into an axiom on equilibrium responses to an allocation mechanism with messages. Sonnenschein's axiom is generalized to incorporate economy-dependent responses and arguments based on utility or welfare levels.

Based on a standard general equilibrium setting, our framework provides a new method to treat the relation between arguments on such informational efficiency problems as Hurwicz and axiomatic characterizations through fundamental social choice axioms like individual rationality, Pareto-optimality, local independency, monotonicity, and incentive compatibility.[2] Moreover, it provides a contemporary game-theoretic framework of a mechanism to treat a general class of messages, allowing for the number of agents and an equilibrium concept in an economy as variables.[3] Sonnenschein's arguments are reconstructed and formulated here as a *universal implementability* theorem of the price system, insisting that the price mechanism is a unique mechanism that can implement all other *message mechanisms* having equilibrium concepts compatible with the core (Axiom C_2) and satisfying a certain stability condition on equilibrium messages with respect to economic expansion (Axiom C_3').[4]

Our result also characterizes an interesting feature of the price mechanism. By rigorously treating the role of Sonnenschein's expansion possibility axiom while separating from it the replica stability property of responses, we determined that the condition that uniquely and universally characterizes the price mechanism or a price message is nothing but the following feature: a message that assures a sufficiently

[1] A ground-breaking work on this axiomatic characterization problem was given in Sonnenschein (1974) as a corollary to the Debreu-Scarf core limit theorem. His approach is important to relate the classical informational efficiency problem on allocation mechanisms like Hurwicz (1960) and Mount and Reiter (1974) to a game-theoretic treatment of the mechanism. Sonnenschein's category theoretic characterization enables us to treat a more general class of messages and contemporary model settings than classical informational efficiency arguments. Unfortunately, the meaning and significance of Sonnenschein's approach are not fully understood.

[2] See Concluding Remark 2. Hence, the approach is meaningful to relate the classical informational efficiency problem on allocation mechanisms with many social choice arguments.

[3] A contemporary game theoretic interpretation of a mechanism (like Mas-Colell et al. 1995; Chapter 23) for our approach will be treated in the Appendix.

[4] See Concluding Remarks 1 and 3. We also note that the category theoretic argument or the universal implementability is different from the simple uniqueness about an informational size or isomorphism argument in classical studies like Osana (1978), Jordan (1982), etc. Universal implementability is a method to define a certain concept as a special object that can be *uniquely referenced* in a wide class of objects that is axiomatically determined.

satisfactory allocation possibility for members of each small economy relative to its large expansions. More precisely, price can be characterized as a message that assures each agent in small economy $\mathcal{E}$ a sufficiently good allocation in *equilibrium* in the following sense: with respect to the message, if it is an equilibrium message for $\mathcal{E}$ under a certain (e.g., a game theoretic) equilibrium concept, one has to anticipate the *possibility* that the utility level of each member for the equilibrium of a *large* expansion of $\mathcal{E}$ is at most the level that she obtained in $\mathcal{E}$.[5] Although it is true that the price mechanism is a universal rule for a global economy, the property that ensures its universality is its *stability* feature as a good message for each small economy.

2 The Basic Model

In this paper, $\mathcal{E}$, $\mathcal{E}'$, $\cdots$ and I, I', $\cdots$ are used to denote economies and index sets of agents, respectively. An *economy* $\mathcal{E}$ with index set of finite *agents* I consists of the *preference preordering* and the *initial endowment* for $i \in I$, denoted respectively by $\precsim_i$ and ω_i. For each $i \in I$, we define *feasible consumption set* X_i as $X_i = R^\ell_+$. The preference preordering of i in economy $\mathcal{E}$, $\precsim_i$, is a subset of $X_i \times X_i$ and the initial endowment of i in $\mathcal{E}$, ω_i, is an element of R^ℓ_{++}.

We can write economy $\mathcal{E}$ as $\mathcal{E} = (\precsim_i, \omega_i)_{i\in I}$. Suppose that the preference $\precsim_i$ is represented by a utility function of each individual, $u_i : X_i \to R$, and each u_i satisfies continuity, strict monotonicity, and strict quasi-concavity (strict convexity in the sense of Debreu 1959). Moreover, for the sake of simplicity, we restrict the class of all economies to those including at least one agent whose utility function is differentiable.[6] We write the set of economies as $\mathcal{E}\mathit{con}$.

For each $\mathcal{E} = (\precsim_i, \omega_i)_{i\in I}$ in $\mathcal{E}\mathit{con}$, sequence $(x_i \in X_i)_{i\in I}$ is called an *allocation* for $\mathcal{E}$. Allocation $x = (x_i \in X_i)_{i\in I}$ is said to be *feasible* if

$$\sum_{i\in I} x_i = \sum_{i\in I} \omega_i. \tag{1}$$

A *coalition* in economy $\mathcal{E} = (\succsim_i, \omega_i)_{i\in I}$ is a non-empty set of agents $S \subset I$. Feasible allocation x is said to be a *core allocation* if there are no coalition S and no $y = (y_i)_{i\in S}$, satisfying (a) $\sum_{i\in S} y_i = \sum_{i\in S} \omega_i$ and (b) $y_i \succsim x_i$ for all $i \in S$ and $y_i \succ x_i$ for at least one $i \in S$. We call the set of all core allocations the *core* of economy $\mathcal{E}$ and denote it by $\mathcal{C}\mathit{ore}(\mathcal{E})$. Allocation x is said to be *blocked* by coalition S if conditions (a) and (b) hold.

Next, we define a message mechanism on an economy. Let A be a set. Given a *message*, $a \in A$, we assume that for each economy $\mathcal{E} = (\succsim_i, \omega_i)_{i\in I}$, allocation $f(a, \mathcal{E}) = (f_i(a, \mathcal{E}))_{i\in I} \in \prod_{i\in I} X_i$ is defined. We call f on $A \times \mathcal{E}\mathit{con}$ a *response function*. It is also possible to consider the response function of each agent i, f_i, as a composition of an constraint correspondence in X_i for individual choices under message a and a profit maximization process of agent i. In this sense, our economy-dependent form of Sonnenschein's response function can be identified with a generalized form of the contemporary game theoretic mechanism concept (see Concluding Remark 1 and Appendix). As in Sonnenschein (1974), we consider an *equilibrium correspondence*, $\mu : \mathcal{E}\mathit{con} \ni \mathcal{E} \mapsto \mu(\mathcal{E}) \subset A$, and a correspondence, g, that defines for each economy $\mathcal{E} = (\succsim_i, \omega_i)_{i\in I}$ a subset of its feasible allocations. We call the triple, (A, μ, f), as a *message mechanism*

[5] See Axiom (C_3') in Section 3. Note that in our framework, the concept of equilibrium, μ, is not confined to any restricted senses. Concept μ is treated axiomatically in the relation among other concepts like message space A, response function f, and so forth.

[6] For our result, the conditions that imply the Debreu-Scarf core limit theorem will be sufficient. We use the differentiability condition, however, since it simplifies many arguments and the proof of the main theorem.

(an resource allocation mechanism with messages) based on a social choice correspondence, g, if

$$g(\mathcal{E}) = \{(f_i(a, \mathcal{E}))_{i \in I} \mid a \in \mu(\mathcal{E})\}. \tag{2}$$

For economies, $\mathcal{E} = (\succsim_i, \omega_i)_{i \in I}$ and $\mathcal{E}' = (\succsim'_i, \omega'_i)_{i \in I'}$, we write $\mathcal{E} \subset \mathcal{E}'$ to mean that (i) $I \subset I'$, (ii) for each $i \in I$, the preference of i, $\precsim_i$ is equal to $\precsim'_i$, and (iii) for each $i \in I$, $\omega_i = \omega'_i$.

Now we reconstruct the category theoretic argument in Sonnenschein (1974) while explicitly treating his several implicit settings and assumptions as independent axioms. Let us consider the following conditions on f and μ.

Axiom (C_1): Responses are invariant for the extension of the economy. (Messages are not economy-dependent.) That is, $\forall a \in A$, $\forall \mathcal{E} \in \mathcal{E}con$, $\forall \mathcal{E}' \in \mathcal{E}con$, $\mathcal{E} \subset \mathcal{E}'$,

$$f(a, \mathcal{E}) \text{ is a restriction of } f(a, \mathcal{E}') \text{ on members of } \mathcal{E}. \tag{3}$$

Axiom (C_2): Equilibrium responses are core compatible. That is, $\forall a \in A$, $\mathcal{E} \in \mathcal{E}con$,

$$a \in \mu(\mathcal{E}) \Longrightarrow f(a, \mathcal{E}) \in \mathcal{C}ore(\mathcal{E}). \tag{4}$$

Axiom (C_3): Mechanism satisfies Sonnenschein's Axiom S. That is, for each economy $\mathcal{E}$ and each message $a \in A$, there exists an economy $\mathcal{E}' \supset \mathcal{E}$ such that a is an equilibrium message for $\mathcal{E}'$.

Define the set of price vectors, P, as $P = \{(p_1, \cdots, p_\ell) \in R^\ell_+ \mid \sum_{k=1}^{\ell} p_k = 1\}$. The *price mechanism* is an allocation mechanism with messages, (P, π, e), where for each $\mathcal{E} = (\precsim_i, \omega_i)_{i \in I} \in \mathcal{E}con$, $\pi(\mathcal{E}) \subset P$ is the set of all competitive equilibrium prices for $\mathcal{E}$, and for each $p \in P$ and $\mathcal{E}$, $e(p, \mathcal{E}) = (\xi_i(p))_{i \in I} \in \prod_{i \in I} X_i$ is the list of the value of each agent's demand function.[7]

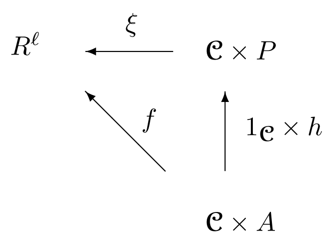

Figure 1: Commutative Diagram for the Universal Mapping Problem in Sonnenschein (1974)

The commutative diagram in Figure 1 with respect to the class of agents' characteristics $\mathfrak{C}$, information sets, demand structure and any equilibrium structures satisfying Axioms (C_1), (C_2) and (C_3) was proved in Sonnenschein (1974). His results can be restated as follows.

Proposition (Sonnenschein 1974; Propositions 1 and 7): If (A, μ, f) is a message mechanism based on a social choice correspondence, and if (A, μ, f) satisfies Axioms (C_1), (C_2) and (C_3), then we have the following characterization of the price mechanism:

[7] The demand function exists since each agent's utility function is strictly quasi-concave.

(i) There exists a unique function $h : A \to P$ such that the triangle in Figure 1 commutes (Dictionary Property).

(ii) Function h can be taken as continuous (or differentiable) when the domain of message spaces are restricted on topological spaces (resp. differentiable manifolds) with continuous (resp. differentiable) response functions.

(iii) Moreover, the mechanism that can play the above dictionary property isunique up to isomorphism (Universal Mapping Property).

3 Generalized Axioms and Theorems

Now, we consider more general conditions to characterize the price mechanism. In the following, Axioms (C_1), (C_2) and (C_3) are reformulated and weakened. The economy-independent axiom of messages, (C_1), will be replaced with the following replica stability axiom for responses, (C_1').[8] Moreover, Sonnenschein's Axiom S is generalized by incorporating an economy-dependent response function together with a utility form characterization as the following, (C_3').

Axiom (C_1'): $\forall \mathcal{E} \in \mathcal{E}con$, $\forall a \in \mu(\mathcal{E})$, $\forall n$, $f(a, \mathcal{E}^n) = (f(a, \mathcal{E}))^n$.[9]

Axiom (C_3'): For each message $a \in A$, there exists $\mathcal{E}^* \in \mathcal{E}con$ such that $a \in \mu(\mathcal{E}^*)$. Moreover, for each $\mathcal{E} \in \mathcal{E}con$ and $a \in \mu(\mathcal{E})$, and for each $\mathcal{E}' \in \mathcal{E}con$ such that $\mathcal{E} \subset \mathcal{E}'$, there exists $\mathcal{E}^* = (\precsim_i^*, \omega_i^*)_{i \in I^*}$, $\mathcal{E}' \subset \mathcal{E}^*$, such that a is an equilibrium message for $\mathcal{E}^*$ and $u_i(f_i(a, \mathcal{E})) \geqq u_i(f_i(a, \mathcal{E}^*))$ for each agent i in $\mathcal{E}$.

In the previous section, Axiom (C_1) indicates that the responses of agents do not depend on the scale of the economy. The above extended axioms, (C_1') and (C_3'), generalize the settings in Sonnenschein (1974) in the sense that responses are restricted to equilibrium messages and allowed to be economy-dependent except for the replica extension. Note also that Axiom (C_3') is given in the utility form. The framework would enable us to discuss problems such as relation between the stability of price mechanism and an expansion possibility of economies.

To obtain an extension theorem of Sonnenschein (1974; Propositions 1 and 7), we use an alternative of the commutative diagram in Figure 1 by considering the dependence of responses on economy $\mathcal{E}$. Now we have a main theorem that is an extension of the proposition in the previous section. Moreover, for this purpose, we focus our attention on equilibrium messages.[10]

Main Theorem: *If a message mechanism based on a social choice correspondence, (A, μ, f), satisfies Axioms (C_1'), (C_2) and (C_3'), we have the following characterization of the price mechanism:*

(i) *There exists a unique function $h : A \to P$ such that the triangle in Figure 2 commutes for each $\mathcal{E} \in \mathcal{E}con$ and $a \in \mu(\mathcal{E})$* (Universal Implementability).

[8] See Thomson (1988), Nagahisa (1994), etc. In those frameworks the replica stability concept was applied to a social choice result or an equilibrium. Here, we use the same concept on the structure of agents' responses to the messages.

[9] For each allocation x in economy $\mathcal{E}$, we denote by x^n the n-fold replica allocation in the n-fold replica economy, $\mathcal{E}^n$, for $n = 1, 2, , \cdots$.

[10] Our approach does not restrict the domain of messages, A, when for every message $a \in A$, there exists at least one economy $\mathcal{E}$ such that $a \in \mu(\mathcal{E})$ under Axiom (C_3'). The convention therefore is sufficient for our purpose to characterize the universal implementability of the price mechanism since condition (C_3') as well as (C_3) request that every message will be an equilibrium message.

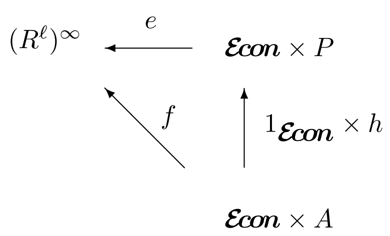

Figure 2: A Commutative Diagram for Economy Dependent Message Mechanisms

(ii) *Function h can be taken as continuous (or differentiable) when the domain of message spaces are restricted on topological spaces (resp. differentiable manifolds) with continuous (resp. differentiable) response functions.*

(iii) *Moreover, the mechanism that can play the above universal implementability property is unique up to isomorphism* (Universal Mapping Property) *among the class of mechanisms satisfying Axioms* (C_1'), (C_2) *and* (C_3').

Proof: [**The first and second assertions: Universal Implementability**] Let a and $\mathcal{E}$ be an arbitrary pair of a message and an economy satisfying $\mathcal{E} = (\precsim_i, \omega_i)_{i \in I} \in \mathcal{E}con$ and $a \in \mu(\mathcal{E})$. Since a is an equilibrium message for $\mathcal{E}$, $f(a, \mathcal{E})$ is Pareto-optimal. By considering that at least one member of $\mathcal{E}$ is differentiable, we obtain a unique supporting price p for allocation $f(a, \mathcal{E})$.

Next, we show that under p, $f_i(a, \mathcal{E})$ satisfies the budget constraint for all $i \in I$. Assume not. Then, the allocation $f(a, \mathcal{E})$ is not a Walras allocation. By Debreu-Scarf core limit theorem (Debreu and Scarf 1963; Theorem 3), there exist a positive n such that the n-fold replica allocation of $f(a, \mathcal{E})$ cannot be in the core of the n-fold replica economy of $\mathcal{E}$, $\mathcal{E}^n$. Hence, we have a set G of the agents in $\mathcal{E}^n$ who can block the n-fold replica allocation of $f(a, \mathcal{E})$. Under (C_1'), we have $(f(a, \mathcal{E}))^n = f(a, \mathcal{E}^n)$. Furthermore, by applying (C_3') on $\mathcal{E}^n$, there exists $\mathcal{E}^*$ such that $a \in \mu(\mathcal{E}^*)$, $\mathcal{E}^n \subset \mathcal{E}^*$ and $u_i(f_i(a, \mathcal{E}^*)) \leqq u_i(f_i(a, \mathcal{E}^n))$ for all i of $\mathcal{E}^n$. It follows that, by (C_2), $f(a, \mathcal{E}^*)$ is an element of $\mathcal{C}ore(\mathcal{E}^*)$ satisfying $u_i(f_i(a, \mathcal{E}^*)) \leqq u_i(f_i(a, \mathcal{E}^n))$ for all i of $\mathcal{E}^n$. But this is impossible because G in $\mathcal{E}^n$ blocks the utility allocation under $(f(a, \mathcal{E}))^n$. Therefore, $p \cdot (f_i(a, \mathcal{E}) - \omega_i) = 0$ for all $i \in I$. Define h as $h(a) = p$, then $p \cdot (f_i(a, \mathcal{E}) - \omega_i) = 0$ and the strict convexity of preferences necessarily implies that $f_i(a, \mathcal{E}) = \xi_i(h(a))$.

Function h is necessarily continuous (or differentiable) when we restrict the class of message spaces on topological spaces (resp. differentiable manifolds) with continuous (resp. differentiable) response functions. Indeed, for each $a \in A$, it is possible (e.g., by considering an economy with Cobb-Douglas utility agents,) to choose an economy such that near $h(a)$, a coordinate of the list of demand functions, ξ_i, provides a local homeomorphism (resp. diffeomorphism). The composition of such local homeomorphism (resp. diffeomorphism) and the coordinate response function f_i of f is locally equal to h. Therefore h is continuous (resp. differentiable).

[**The third assertion: Universal Mapping Property**] Though it would be possible to obtain this result as a corollary to the fundamental mathematical theorem on the universal mapping (see, e.g., Bourbaki 1970; Chapter IV, Section 3), we present a direct proof since (i) the completeness of this paper is also desirable and (ii) the restriction of our commutative diagram argument to equilibrium messages requires some special treatments for a direct application of the universal mapping theorem.

Note that Axiom (C_3') ensures that every message $a \in A$ can be an equilibrium message for at least one economy in $\mathcal{E}con$. Suppose that (P, π, e) and (P', π', e') are two message mechanisms satisfying Axioms (C_1'), (C_2) and (C_3') and having the above universal implementability property. Then, by the first assertion of this theorem, we obtain two mappings $h : P' \to P$ and $h' : P \to P'$ such that the diagram in Figure 2 commutes for each equilibrium messages. Moreover, the uniqueness of such mappings also implies that both $h \circ h' : P \to P$ and $h' \circ h : P' \to P'$ are identity mappings. It follows that h and h' must be bijections such that $h^{-1} = h'$. The requests on h and h' to be continuous (or differentiable) imply that both h and h' are homeomorphisms (resp. diffeomorphisms). Bijection (homeomorphism, diffeomorphism, resp.) h also gives the bijection (homeomorphism, diffeomorphism, resp.) between the values $\pi(\mathcal{E})$ and $\pi'(\mathcal{E})$ for each $\mathcal{E}$ as well as the restrictions of graphs of $e(\cdot, \mathcal{E})$ on the graph of π and $e'(\cdot, \mathcal{E})$ on the graph of π' for each $\mathcal{E}$. Hence, the mechanism satisfying the universal implementability (restricted on the graph of its equilibrium correspondence) is unique up to isomorphism. ■

4 Concluding Remarks

1. We have thus obtained a generalization of Sonnenschein (1974) theorem on the price mechanism in an economy-dependent manner based on each agent's utility level. The condition has a meaning that the price mechanism can be characterized as a representative mechanism that can implement all economy-dependent message mechanisms ensuring core property and sufficiently high utility levels or stable actions for members of each small economy. The concept of mechanism in our paper can be interpreted in a contemporary game theoretic context that can treat a wide class of economies with messages by identifying its message space with a component of the strategy sets (see Appendix).

2. The setting in this paper suggests a new method or a perspective to treat arguments on the allocation mechanism with messages and axiomatic characterizations through fundamental social choice axioms like individual rationality, Pareto-optimality, local independency, monotonicity, incentive compatibility and so forth. Based on our papers on replica core limit arguments (Urai and Murakami 2016a and Murakami and Urai 2017a), we are preparing other papers on overlapping-generations price-money mechanism, price-dividend mechanism for satiation economies, and characterization based on monotonicity and incentive compatibility, etc. (Urai and Murakami 2016b, Murakami and Urai 2017b and Urai and Murakami 2017). For each cases, the utility form (welfare) characterization of this paper will provide other extension possibilities and interesting arguments.

3. In this paper, Sonnenschein's arguments are reconstructed as a pure category theoretic *universal implementability* theorem. By treating the number of agents as a variable, we can incorporate various topological or dimensional structures of message spaces. For this purpose, the equilibrium correspondence $\mu : \mathcal{E}con \ni \mathcal{E} \mapsto \mu(\mathcal{E}) \subset A$ is used in the triple (A, μ, f) of the message mechanism. The idea to utilize the equilibrium correspondence μ to characterize the role of messages in economies consisting of different number of participants has an advantage in applying the same replication method to various social choice and information problems. As mentioned in the remark 2, the local independency, monotonicity, incentive compatibility, etc., can also be reconsidered from this general viewpoint to characterize the price mechanism.

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Appendix

A Contemporary Game Theoretic Interpretation of the Message Mechanism

We can identify our economy-dependent response function form of the message mechanism with a generalized sense of the contemporary game theoretic mechanism as follows. (For convenience sake, in the following, the number of agents of an economy will be fixed as n.)

Let S_0, S_1, ..., S_n be $n+1$ strategy sets of $n+1$ players. The 0-th player will be considered a special participant, and its strategic space S_0 will be identified with the space of messages. The spaces $S_1, \cdots, S_n$ will be strategic spaces of consumer players.

With each strategy profile, $(s_0, s_1, \ldots, s_n) \in S = S_0 \times S_1 \times \cdots \times S_n$, is associated an outcome, $\mathfrak{g}(s_0, s_1, \ldots, s_n) \in X$, where function $\mathfrak{g}$ is called an *outcome function* and X is called an *outcome space*. The list $(S_0, S_1, \ldots, S_n, \mathfrak{g})$ is called a *mechanism* (Mas-Colell et al. 1995; Chapter 23, p. 866). If we add a *constraint correspondence* $K : S \to S$ and a condition that a strategy profile $s = (s_0, s_1, \ldots, s_n)$ is an element of $K(s)$, we obtain a *generalized sense* of mechanism, $(S_0, S_1, \ldots, S_n, K, \mathfrak{g})$.

Now we see that an economy-dependent message mechanism (A, μ, f) with respect to economies with n agents in $\boldsymbol{\mathcal{E}con}$ can be identified with a generalized sense of mechanism.

Consider a space of messages A as S_0, and feasible consumption sets $X_1, \ldots, X_n$ of n agents as $S_1, \ldots, S_n$, respectively. For constraint correspondence K, we consider a special case that value $K(a, x_1, \ldots, x_n)$ of $(a, x_1, \ldots, x_n) \in A \times X_1 \times \cdots \times X_n$ does not depend on x_1, ..., x_n. The domain and the range of outcome function $\mathfrak{g}$ are fixed to be $A \times X_1 \times \cdots \times X_n$ and $X_1 \times \cdots \times X_n$, respectively. Moreover, $\mathfrak{g}$ is defined as $\mathfrak{g} = \mathrm{pr}_{X_1} \times \cdots \times \mathrm{pr}_{X_n}$ where pr_{X_i} is the projection on X_i for each $i = 1, \ldots, n$. We call such a mechanism, $(A, X_1, \ldots, X_n, K, \mathfrak{g})$, a *message mechanism*. A special case that A is the price space and K is the budget constraint correspondence, is called the *price mechanism*.

When an economy $\boldsymbol{\mathcal{E}} \in \boldsymbol{\mathcal{E}con}$ consisting of n agents is given, the utility functions and initial endowments of agents in $\boldsymbol{\mathcal{E}}$ on X_1, ..., X_n are specified and we obtain a generalized game setting on the above message mechanism, $(A, X_1, \ldots, X_n, K, \mathfrak{g})$. The list of best response functions of agents, 1, ..., n, will be identified with our economy-dependent form of the response function.