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Abstract

This study presents an overlapping-generations model with physical and human capital accumulation and considers probabilistic voting over capital and labor taxes and public debt to finance public education expenditure. Our analysis shows that the greater political power of the old induces the government to raise the labor tax on the young and lower the capital tax on the old as well as issue debt. The analysis also shows that the introduction of a debt ceiling rule calls for a rise in the labor tax and thus lowers the welfare of the currently working generation. However, it increases the growth rate, and this growth effect raises the welfare of future generations. These benefits last for a long period even if the rule is imposed only for a limited time.

- Keywords: Capital taxation, Public debt, Economic growth, Probabilistic voting, Overlapping-generations model
- JEL Classification: D70, E24, H63

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1 Introduction

How the government should distribute the tax burden between capital and labor income has been a long-standing issue in the public finance literature since the seminal paper of Ramsey (1927). A large part of the literature has tackled this issue by assuming a benevolent planner who can commit to future policies. For example, Judd (1985), Chamley (1986), and Atkeson, Chari, and Kehoe (1999) show that the optimal tax rate on capital income is zero in the long run; expenditure should thus be financed solely by labor income taxation. This conclusion is robust to whether the government can issue debt or must run a balanced budget in each period (Ljungqvist and Sargent, 2012, Chapter 16); it is also robust in overlapping-generations models in which all households have finite lifespans, whereas the planner has an infinite horizon (Erosa and Gervais, 2002; Garriga, 2017).

However, this so-called Ramsey approach has the following two limitations. Firstly, politicians, as representatives of the current population, are short-lived and unable to commit to complete sequences of future policies in practice (Acemoglu, Golosov, and Tsyvinski , 2011). In particular, these policymakers try to meet the conflicting preferences of different generations (Hassler et al., 2005; Bassetto, 2008). Secondly, older generations are likely to pass the burden onto future generations by issuing public debt, a key source of government revenue (Song, Storesletten, and Zilibotti, 2012; Müller, Storesletten, and Zilibotti, 2016). To control excessive debt issues, many countries have formulated national or supranational fiscal rules that ensure the sustainability of public finance (Schaechter et al., 2012).

Given this background, the following questions arise: (i) how does the conflict between generations influence fiscal policymaking, (ii) how does policymaking in turn affect growth and welfare across generations, and (iii) does the introduction of a debt rule increase growth and welfare? To answer these questions, we present an overlapping-generations model with physical and human capital accumulation (e.g., Lambrecht, Michel, and Vidal, 2005; Kunze, 2014; Ono and Uchida, 2016, 2018b). Each generation comprises many identical individuals who live over three periods: young, middle, and old ages. Middle-aged individuals (called parents) care about their children's wage income. Public education spending and parental human capital are inputs in the human capital production process, thereby contributing to children's human capital formation and economic growth. Governments, as elected representatives, finance public education spending through taxes on capital and labor income and public debt issues.

Under this framework, we consider the politics of fiscal policy formation. In particular, we assume probabilistic voting à la Lindbeck and Weibull (1987) to demonstrate the extent to which generations face conflict over such policies. In each period, middle-aged and old individuals vote.[1] The government in power maximizes the political objective function of the weighted sum of the utilities of the middle-aged and old populations. In this voting environment, the current policy choice affects the decision on future policy via physical and human capital accumulation. To demonstrate this intertemporal effect, we employ the concept of a Markov-perfect equilibrium under which fiscal policy today depends on the current payoff-relevant state variables, namely physical and human capital and public debt. Given this process, we characterize the political equilibrium in the presence or absence of a debt rule, which yields the following findings.

Firstly, the relative political power of the old is important for shaping tax rates. In particular, the greater political power of the old incentivizes the government to shift the tax burden from the old to the young. In addition, in the absence of any debt rule, the education expenditure-to-GDP ratio and debt-to-GDP ratio decrease with the greater political power of the old. The effect on the education expenditure-to-GDP ratio is intuitive since the old do not benefit from education expenditure. However, the effect on the debt-to-GDP ratio, which is the opposite to that suggested by Song, Storesletten, and Zilibotti (2012), is somewhat counterintuitive since the old may have an incentive to pass the fiscal burden onto future generations by issuing public debt. This counterintuitive result arises because the old want to lower the capital tax burden and public education expenditure as their political weight increases. A decrease in capital tax revenue is compensated for by an increase in debt issues, while a decrease in education expenditure is associated with a decrease in public debt issues. The analysis shows that the latter negative effect on debt outweighs the former positive effect in the present framework.

Secondly, we consider an alternative scenario, called tax financing, in which the government is prohibited from issuing public debt; hence, its expenditure is financed solely through taxation. This scenario, while is an extreme one, enables us to investigate in a tractable way the effect of controlling debt issues. We show that the labor (capital) tax rate increases (decreases) if the government changes its instrument from debt financing to tax financing. We then analyze the welfare consequences of this change and show a trade-off in terms of utility across generations. The initial generation is worse off since its labor tax burden increases to compensate for the loss of revenue from debt issues. However, tax financing removes the crowding-out effect of public debt on physical capital and thus increases the growth rate. The initial generation cannot therefore benefit from this positive effect of increased growth, whereas future generations do benefit as it outweighs the negative effect of the increased tax burden.

[1] The young may also have an incentive to vote since they benefit from public education in the future. However, for the tractability of the analysis, we assume that politicians do not care about the young's preferences following Saint-Paul and Verdier (1993), Bernasconi and Profeta (2012), and Lancia and Russo (2016). This assumption is supported in part by the fact that a large number of the young are below the voting age.

Thirdly, we consider a more realistic fiscal rule that sets an explicit ceiling for public debt as a percentage of GDP, which has been widely introduced in developed countries (Schaechter et al., 2012). We investigate the effect of such a fiscal tightening rule (i.e., lowering the ceiling) based on a numerical analysis and obtain qualitatively similar growth and welfare consequences as those obtained in the second analysis; in other words, this fiscal tightening rule is growth-enhancing, but not Pareto-improving. We also find that even if the rule is imposed only for limited periods, it has a long-lasting effect on utility across generations since increased human capital is bequeathed from generation to generation. This long-lasting effect was absent from the study by Barseghyan and Battaglini (2016), who consider a temporary austerity program in the absence of human capital. Our result suggests the importance of connecting seemingly unrelated subjects, namely public education and public debt, to consider the impact of debt rules over time and across generations.

The present study contributes to the following two strands of the literature. The first is the literature on the politics of capital taxation in the overlapping-generations framework. Earlier studies assumed no production sector and said nothing about the effect of capital income (i.e., interest rate income) taxation on economic growth (Renström, 1996; Huffman, 1996; Dolmas and Huffman, 1997). In addition, these studies dropped the conflict of interest between generations from their analyses by assuming that either the young or the old generation is decisive in voting. Notable exceptions are Razin, Sadka, and Swagel (2004) and Mateos-Planas (2010), who assume majority voting, and Bassetto (2008) and Ono and Uchida (2018a), who assume bargaining or competition between generations. All these studies assume a balanced government budget and thus no budget deficit. However, the current generation may have an incentive to pass the fiscal burden onto future, unborn generations by issuing public debt. To address this issue, the present study allows for deficit finance and investigates its impact on economic growth and welfare across generations.

The second strand includes studies of the political economy of public debt (Battaglini and Coate, 2008; Barseghyan, Battaglini, and Coate, 2013; Debortoli and Nunes, 2013; Azzimonti, Battaglini, and Coate, 2016; Barseghyan and Battaglini, 2016; Battaglini and Coate, 2016; Cunha and Ornelas, 2018). In particular, the present study is closely related to Song, Storesletten, and Zilibotti (2012), Röhrs (2016), Müller, Storesletten, and Zilibotti (2016), Song, Storesletten, and Zilibotti (2016), and Arawatari and Ono (2017), who demonstrate the conflict of interest between generations over public debt policy. Arai, Naito, and Ono (2018), Ono (2018), and Ono and Uchida (2018b) extend these analyses by including physical and/or human capital and present the effects of the politics of public debt on economic growth and welfare across generations. However, all these studies have ignored capital taxation, which is an important policy instrument, and said nothing about the conflict of interest between generations over the distribution of tax burdens. The present study overcomes this limitation by introducing capital taxation.

The organization of the remainder of this paper is as follows. Section 2 presents the model. Section 3 describes the political equilibrium and Section 4 characterizes the steady-state equilibrium. Section 5 compares the debt- and tax-financing political equilibria; it also investigates the effects of the debt ceiling rule. Section 6 presents concluding remarks.

2 Model

The discrete time economy starts in period 0 and consists of overlapping generations. Individuals are identical within a generation and live for three periods: youth, middle, and old ages. Each middle-aged individual gives birth to $1+n$ children. The middle-aged population for period t is N_t and the population grows at a constant rate of $n(> -1)$: $N_{t+1} = (1+n)N_t$.

2.1 Individuals

Individuals display the following economic behavior over their lifecycles. During youth, they make no economic decisions and receive public education financed by the government. In middle age, individuals work, receive market wages, and make tax payments. They use after-tax income for consumption and savings. Individuals retire in their elderly years and receive and consume returns from savings.

Consider an individual born in period $t-1$. In period t, the individual is middle-aged and endowed with h_t units of human capital inherited from his or her parents. The individual supplies them inelastically in the labor market and obtains labor income $w_t h_t$, where w_t is the wage rate per efficient unit of labor in period t. After paying tax $\tau_t w_t h_t$, where $\tau_t \in (0,1)$ is the period t labor income tax rate, the individual distributes the after-tax income between consumption c_t and savings invested in physical capital s_t. Therefore, the period t budget constraint for the middle age becomes

$$c_t + s_t \leq (1-\tau_t) w_t h_t.$$

The period $t+1$ budget constraint in elderly age is

$$d_{t+1} \leq \left(1-\tau_{t+1}^k\right) R_{t+1} s_t,$$

where d_{t+1} is consumption, τ_{t+1}^k is the period $t+1$ capital income tax rate, $R_{t+1}(> 0)$ is the gross return from investment in capital, and $R_{t+1} s_t$ is the return from savings. The results are qualitatively unchanged if capital income tax is on the net return from saving rather than the gross return from saving.

Period t middle-aged individuals care about their children's income , $w_{t+1}h_{t+1}$. Children's human capital in period $t+1$, h_{t+1}, is a function of government spending on public education, x_t, and parents' human capital, h_t. In particular, h_{t+1} is formulated using the following equation:

$$h_{t+1} = D\left(x_t\right)^{\eta}\left(h_t\right)^{1-\eta}, \tag{1}$$

where $D(>0)$ is a scale factor and $\eta \in (0,1)$ denotes the elasticity of education technology with respect to education spending.[2]

We assume that parents are altruistic toward their children and concerned about their income in middle age, $w_{t+1}h_{t+1}$. The preferences of an individual born in period $t-1$ are specified by the following expected utility function in the logarithmic form:

$$U_t = \ln c_t + \beta\left[\ln d_{t+1} + \gamma \ln w_{t+1}h_{t+1}\right],$$

where $\beta \in (0,1)$ is a discount factor and $\gamma(>0)$ denotes the intergenerational degree of altruism. We substitute the budget constraints and human capital production function into the utility function to form the following unconstrained maximization problem:

$$\max_{\{s_t\}} \ln\left[(1-\tau_t)w_t h_t - s_t\right] + \beta\left[\ln R_{t+1}s_t + \gamma \ln w_{t+1}D\left(x_t\right)^{\eta}\left(h_t\right)^{1-\eta}\right].$$

By solving this problem, we obtain the following savings and consumption functions:

$$s_t = \frac{\beta}{1+\beta}\cdot(1-\tau_t)w_t h_t, \tag{2}$$

$$c_t = \frac{1}{1+\beta}\cdot(1-\tau_t)w_t h_t \text{ and } d_{t+1} = \frac{\beta R_{t+1}}{1+\beta}\cdot(1-\tau_t)w_t h_t.$$

2.2 Firms

Each period contains a continuum of identical firms that are perfectly competitive profit maximizers. According to Cobb–Douglas technology, they produce a final good Y_t using two inputs: aggregate physical capital K_t and aggregate human capital $H_t \equiv N_t h_t$. Aggregate output is given by

$$Y_t = A\left(K_t\right)^{\alpha}\left(H_t\right)^{1-\alpha},$$

where $A(>0)$ is a scale parameter and $\alpha \in (0,1)$ denotes the capital share.

[2]Private investment in education may also contribute to human capital formation. For example, parents' time (Glomm and Ravikumar, 1995, 2001, 2003; Glomm and Kaganovich, 2008) or spending (Glomm, 2004; Lambrecht, Michel, and Vidal, 2005; Kunze, 2014) devoted to education may complement public education. In the present study, we abstract private education from the main analysis to simplify the presentation of the model.

Let $k_t \equiv K_t/H_t$ denote the ratio of physical to human capital. The first-order conditions for profit maximization with respect to H_t and K_t are

$$w_t = (1-\alpha)A\left(k_t\right)^{\alpha}, \tag{3}$$

$$\rho_t = \alpha A\left(k_t\right)^{\alpha-1}, \tag{4}$$

where w_t and ρ_t are labor wages and the rental price of capital, respectively. The conditions state that firms hire human and physical capital until the marginal products are equal to the factor prices. Capital is assumed to depreciate fully within each period.

2.3 Government Budget Constraint

Public education expenditure is financed by both taxes on capital and labor income and public bond issues. Let B_t denote aggregate inherited debt. The government budget constraint in period t is

$$B_{t+1} + \tau_t^k R_t s_{t-1} N_{t-1} + \tau_t w_t h_t N_t = N_{t+1} x_t + R_t B_t,$$

where B_{t+1} is newly issued public bonds, $\tau_t^k R_t s_{t-1} N_{t-1}$ is aggregate capital tax revenue, $\tau_t w_t h_t N_t$ is aggregate labor tax revenue, $N_{t+1}x_t$ is aggregate expenditure on public education, and $R_t B_t$ is debt repayment. We assume a one-period debt structure to derive analytical solutions from the model as well as that the government in each period is committed to not repudiating the debt.

By dividing both sides of the above expression, we obtain a per-capita form of the constraint:

$$(1+n)\hat{b}_{t+1} + \tau_t^k R_t \frac{s_{t-1}}{1+n} + \tau_t w_t h_t = (1+n)x_t + R_t \hat{b}_t, \tag{5}$$

where $\hat{b}_t \equiv B_t/N_t$ is per-capita public debt. We use the notation $\hat{b}_t$, rather than b_t, to distinguish per-capita public debt, $\hat{b}_t \equiv B_t/N_t$, from public debt per human capital, $b_t \equiv B_t/H_t$.

2.4 Economic Equilibrium

Public bonds are traded in the domestic capital market. The market-clearing condition for capital is $B_{t+1} + K_{t+1} = N_t s_t$, which expresses the equality of total savings by the middle-aged population in period t, $N_t s_t$, to the sum of the stocks of aggregate public debt and aggregate physical capital at the beginning of period $t+1$, $B_{t+1}+K_{t+1}$. By using $k_{t+1} \equiv K_{t+1}/H_{t+1}$, $h_{t+1} = H_{t+1}/N_{t+1}$, and the savings function in (2), we can rewrite the condition as

$$(1+n)\left(k_{t+1}h_{t+1} + \hat{b}_{t+1}\right) = \frac{\beta}{1+\beta}(1-\tau_t)w_t h_t. \tag{6}$$

The following defines the economic equilibrium in the present model.

Definition 1. Given a sequence of policies, $\left\{\tau_t^k, \tau_t, x_t\right\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations $\left\{c_t, d_t, s_t, k_{t+1}, \hat{b}_{t+1}, h_{t+1}\right\}_{t=0}^{\infty}$ and prices $\{\rho_t, w_t, R_t\}_{t=0}^{\infty}$ with the initial conditions $k_0(>0), \hat{b}_0(\geq 0)$ and $h_0(>0)$, such that (i) given $\left(w_t, R_{t+1}, \tau_t^k, \tau_t, x_t\right)$, $\left(c_t^y, c_{t+1}^o, s_t\right)$ solves the utility maximization problem; (ii) given (w_t, ρ_t), k_t solves a firm's profit maximization problem; (iii) given $\left(w_t, h_t, R_t, \hat{b}_t\right)$, $\left(\tau_t^k, \tau_t, x_t, \hat{b}_{t+1}\right)$ satisfies the government budget constraint; (iv) an arbitrage condition $\rho_t = R_t$ holds; and (v) the capital market clears: $(1+n)\cdot\left(k_{t+1}h_{t+1} + \hat{b}_{t+1}\right) = s_t$.

In the economic equilibrium, the indirect utility of the middle-aged population in period t, V_t^M, and that of the old population in period t, V_t^o, can be expressed as functions of fiscal policy, physical and human capital, and public debt as follows:

$$
\begin{aligned}
V_t^M &= (1+\beta)\ln(1-\alpha)A\left(k_t\right)^{\alpha} h_t\left(1-\tau_t\right) + \beta\left[(\alpha-1)+\gamma\alpha\right]\ln k_{t+1} \\
&\quad + \beta\ln\left(1-\tau_{t+1}^k\right) + \beta\gamma\eta\ln x_t + \phi^M\left(h_t\right), \qquad (7)\\
V_t^o &= \ln\left(1-\tau_t^k\right) + \phi^O\left(k_t, h_t, \hat{b}_t\right), \qquad (8)
\end{aligned}
$$

where $\phi^M\left(h_t\right)$ and $\phi^O\left(k_t, h_t, \hat{b}_t\right)$ include policy-irrelevant and constant terms, and they are defined by

$$
\begin{aligned}
\phi^M\left(h_t\right) &\equiv \beta\gamma\ln D\left(h_t\right)^{1-\eta} + \left(\ln\frac{1}{1+\beta} + \beta\ln\frac{\beta}{1+\beta}\right) + \beta\left(\ln\alpha A + \gamma\ln\left(1-\alpha\right)A\right),\\
\phi^O\left(k_t, h_t, \hat{b}_t\right) &\equiv \ln\alpha A\left(k_t\right)^{\alpha-1}\left(\hat{b}_t + k_t h_t\right)(1+n) + \gamma\ln(1-\alpha)A\left(k_t\right)^{\alpha} h_t,
\end{aligned}
$$

respectively.

3 Political Equilibrium

In this section, we consider voting on fiscal policy. In particular, we employ probabilistic voting à la Lindbeck and Weibull (1987). In this voting scheme, there is electoral competition between two office-seeking candidates. Each candidate announces a set of fiscal policies subject to the government budget constraint. As demonstrated in Persson and Tabellini (2000), the two candidates' platforms converge in the equilibrium to the same fiscal policy that maximizes the weighted average utility of voters.

In the present framework, the young, middle-aged, and elderly have an incentive to vote. While the young may benefit from current public education expenditure through human capital accumulation, we assume that their preferences are not taken into account by politicians. We impose this assumption, which is often used in the literature (e.g., Saint-Paul and Verdier, 1993; Bernasconi and Profeta, 2012; Lancia and Russo, 2016), for tractability reasons. However, the assumption could be supported in part by the fact that a large number of the young are below the voting age.

Thus, the political objective is defined as the weighted sum of the utility of the middle-aged and old, given by $\tilde{\Omega}_t \equiv \omega V_t^o + (1+n)(1-\omega)V_t^M$, where $\omega \in [0,1]$ and $1-\omega$ are the political weights placed on the old and middle-aged in period t, respectively. The weight on the middle-aged is adjusted by the gross population growth rate, $(1+n)$, to reflect their share of the population. To gain the intuition, we divide $\tilde{\Omega}_t$ by $(1+n)(1-\omega)$ and redefine the objective function as follows:

$$\Omega_t = \frac{\omega}{(1+n)(1-\omega)} V_t^o + V_t^M,$$

where the coefficient $\omega/(1+n)(1-\omega)$ of V_t^o represents the relative political weight on the old.

We substitute V_t^M in (7) and V_t^o in (8) into Ω_t. By rearranging the terms, we obtain

$$\begin{aligned}\Omega_t \simeq & \frac{\omega}{(1+n)(1-\omega)} \ln\left(1-\tau_t^k\right) + (1+\beta)\ln(1-\alpha) A\left(k_t\right)^\alpha h_t\left(1-\tau_t\right) \\ & + \beta\left[(\alpha-1)+\gamma\alpha\right]\ln k_{t+1} + \beta\ln\left(1-\tau_{t+1}^k\right) + \beta\gamma\eta\ln x_t. \end{aligned} \tag{9}$$

We use the notation $\simeq$ because irrelevant terms are omitted from the expression of Ω_t.

We introduce the following notations to define the optimization problem with choices from compact sets:

$$\tilde{x}_t \equiv \frac{x_t}{A\left(k_t\right)^\alpha h_t},\ \tilde{b}_{t+1} \equiv \frac{\hat{b}_{t+1}}{A\left(k_t\right)^\alpha h_t},$$

where $\tilde{x}_t$ and $\tilde{b}_{t+1}$ denote the education expenditure-to-GDP ratio and government debt-to-GDP ratio, respectively. The state and policy variables are assumed to be in compact sets:

$$\begin{aligned}&\frac{\hat{b}_t}{k_t h_t} \in S \equiv \left[0, \left(\hat{b}/kh\right)_{upper}\right],\ \tilde{x} \in S_x \equiv \left[0, \tilde{x}_{upper}\right],\ \tilde{b} \in S_b \equiv \left[0, \tilde{b}_{upper}\right], \\ &\tau^k \in S_{\tau^k} \equiv \left[-\tau_{lower}^k, 1\right],\ \text{and } \tau \in S_\tau \equiv \left[-\tau_{lower}, 1\right].\end{aligned}$$

Note that $\left(\hat{b}/kh\right)_{upper}$, $\tilde{x}_{upper}$, and $\tilde{b}_{upper}$ are the upper limits of the corresponding variables, while $-\tau_{lower}^k (<0)$ and $-\tau_{lower} (<0)$ are the lower limits of the corresponding variables. These are exogenous and set sufficiently high or low to ensure interior solutions. We allow for the possibility of negative tax rates, but will later derive the conditions for $\tau^k \geq 0$ and $\tau \geq 0$.

With the use of (3)–(6), we can reformulate the expression in (9) as follows:

$$\Omega_t \simeq \underbrace{\frac{\omega}{(1+n)(1-\omega)}}_{\text{(p.1)}} \ln\left(1-\tau_t^k\right)$$
$$+ \underbrace{(1+\beta)}_{\text{(p.2)}} \ln\left[1-\left(1-\tau_t^k\right)\alpha\left(\frac{\hat{b}_t}{k_t h_t}+1\right)-(1+n)\tilde{x}_t+(1+n)\tilde{b}_{t+1}\right]$$
$$+ \underbrace{\beta\left[(\alpha-1)+\gamma\alpha\right]}_{\text{(p.3)}} \ln\left[1-\left(1-\tau_t^k\right)\alpha\left(\frac{\hat{b}_t}{k_t h_t}+1\right)-(1+n)\tilde{x}_t-\frac{1}{\beta}(1+n)\tilde{b}_{t+1}\right]$$
$$+ \underbrace{\beta}_{\text{(p.4)}} \ln\left(1-\tau_{t+1}^k\right) + \underbrace{\beta\eta\left(1-\alpha\right)\left(1+\gamma\right)}_{\text{(p.5)}} \ln\tilde{x}_t. \tag{10}$$

The terms (p.1)–(p.5) in (10) present the relative political weight on the old, the weight on the young's utility of lifetime consumption, the weight on the utility of interest rate income plus the weight on the utility of children's wage income, the weight on the marginal cost of capital income taxation in terms of utility, and the weight on the utility of children's human capital, respectively. Appendix A.1 shows the derivation of (10).

The political objective function in (10) suggests that the current policy choice affects the decision on future policy via physical and human capital accumulation. In particular, the period t choices of τ_t^k, $\tilde{x}_t$, and $\tilde{b}_{t+1}$ (equivalently, τ_t^k, x_t, and $\hat{b}_{t+1}$) affect the formation of physical and human capital in period $t+1$. This in turn influences the decision making on period-$t+1$ fiscal policy. To demonstrate such an intertemporal effect, we employ the concept of a Markov-perfect equilibrium under which fiscal policy today depends on the current payoff-relevant state variables.

In the present framework, the payoff-relevant state variables are the ratio of physical to human capital, k_t, human capital, h_t, and public debt, $\hat{b}_t$. These are summarized as $\hat{b}/kh$ as observed in the political objective function in (10). Thus, the expected rate of capital income tax for the next period, τ_{t+1}^k, is given by the function of the next period stock of government debt and physical and human capital, $\tau_{t+1}^k = T^k\left(\hat{b}_{t+1}/k_{t+1}h_{t+1}\right)$. By using recursive notation with z' denoting the next period z, we can now define a Markov-perfect political equilibrium in the present framework as follows.

Definition 2. A *Markov-perfect political equilibrium* is a set of functions, $\langle T, T^k, X, B\rangle$, where $T: S \to S_\tau$ is a labor income tax rule, $\tau = T(\hat{b}/kh)$, $T^k: S \to S_{\tau^k}$ is a capital income tax rule, $\tau^k = T^k(\hat{b}/kh)$, $X: S \to S_x$ is a public education expenditure rule, $\tilde{x} = X(\hat{b}/kh)$, and $B: S \to S_b$ is a public debt rule, $\tilde{b}' = B(\hat{b}/kh)$, such that the following conditions are satisfied:

(i) The capital market clears,

$$(1+n)\left(\tilde{b}' + \frac{k'h'}{A\left(k\right)^{\alpha} h}\right) = \frac{\beta}{1+\beta}\left(1-\tau\right)\left(1-\alpha\right); \tag{11}$$

(ii) Given $\hat{b}/kh$, $\left\langle T(\hat{b}/kh), T^k(\hat{b}/kh), X(\hat{b}/kh), B(\hat{b}/kh)\right\rangle = \arg\max \Omega$ subject to $\tau^{k\prime} = T^k\left(\hat{b}'/k'h'\right)$, the capital market-clearing condition in (11), the government budget constraint,

$$(1+n)\tilde{b}' + \tau(1-\alpha) + \tau^k\alpha\left(1+\frac{\hat{b}}{kh}\right) = (1+n)\tilde{x} + \alpha\frac{\hat{b}}{kh}, \tag{12}$$

and the human capital formation function, $h' = D(h)^{1-\eta}\left(X(\hat{b}/kh)\right)^{\eta}$, where Ω is defined by (10).

3.1 Characterization of the Political Equilibrium

To obtain the set of policy functions in Definition 2, we conjecture the following capital tax rate in the next period:

$$\tau^{k\prime} = 1 - T^k_{un}\frac{1}{1+\frac{\hat{b}'}{k'h'}}, \tag{13}$$

where $T^k_{un}(>0)$ is constant. The subscript "un" means that public bond issuance is "unconstrained." In the next section, we consider a case where public bond issuance is "constrained" by a constitutional rule and compare it with the unconstrained case.

The conjecture in (13) is reformulated by using (11) and (12) as follows:

$$\tau^{k\prime} = 1 - T^k_{un}\frac{1-\left(1-\tau^k\right)\alpha\left(1+\frac{\hat{b}}{kh}\right) - (1+n)\tilde{x} - \frac{1}{\beta}(1+n)\tilde{b}'}{1-\left(1-\tau^k\right)\alpha\left(1+\frac{\hat{b}}{kh}\right) - (1+n)\tilde{x} + (1+n)\tilde{b}'}. \tag{14}$$

The substitution of (14) into the political objective function in (10) leads to

$$\Omega \simeq \frac{\omega}{(1+n)(1-\omega)}\ln\left(1-\tau^k\right) + \ln Z^1 + \alpha\beta(1+\gamma)\ln Z^2 + \beta\eta(1-\alpha)(1+\gamma)\ln\tilde{x}, \tag{15}$$

where

$$Z^1 \equiv 1-\left(1-\tau^k\right)\alpha\left(1+\frac{\hat{b}}{kh}\right) - (1+n)\tilde{x} + (1+n)\tilde{b}',$$

$$Z^2 \equiv 1-\left(1-\tau^k\right)\alpha\left(1+\frac{\hat{b}}{kh}\right) - (1+n)\tilde{x} - \frac{1}{\beta}(1+n)\tilde{b}'.$$

The first-order conditions with respect to τ^k, $\tilde{b}'$, and $\tilde{x}$ are

$$\tau^k: -\frac{\omega}{(1+n)(1-\omega)}\frac{1}{1-\tau^k} + \frac{\alpha\left(1+\frac{\hat{b}}{kh}\right)}{Z^1} + \frac{\alpha\beta(1+\gamma)\alpha\left(1+\frac{\hat{b}}{kh}\right)}{Z^2} = 0, \tag{16}$$

$$\tilde{b}': \frac{1+n}{Z^1} + \frac{\alpha(1+\gamma)(1+n)}{Z^2} \leq 0, \tag{17}$$

$$\tilde{x}: \frac{\beta\eta(1-\alpha)(1+\gamma)}{\tilde{x}} - \frac{1+n}{Z^1} - \frac{\alpha\beta(1+\gamma)(1+n)}{Z^2} = 0. \tag{18}$$

A strict inequality holds in (17) if $\tilde{b}' = 0$. By using these conditions, we can verify the conjecture in (13) and obtain the following result.

Proposition 1. *There is a Markov-perfect political equilibrium distinguished by $\hat{b}' = 0$ if $(1-\alpha)/\alpha \le \gamma$, and $\hat{b}' > 0$ otherwise. The corresponding policy functions of τ^k, x, τ, and $\hat{b}'$ are as follows:*

$$\tau^k = 1 - T^k_{un}\frac{1}{1+\frac{\hat{b}}{kh}},$$
$$(1+n)x = X_{un}A(k)^{\alpha}h,$$
$$\tau = \begin{cases} 1 - \frac{1}{1-\alpha}\frac{1+\alpha\beta(1+\gamma)}{\frac{\omega}{(1+n)(1-\omega)}+\psi} & \text{if } \frac{1-\alpha}{\alpha} \le \gamma, \\ 1 - \frac{1+\beta}{1-\alpha}\frac{1}{\frac{\omega}{(1+n)(1-\omega)}+\psi} & \text{if } \frac{1-\alpha}{\alpha} > \gamma, \end{cases}$$
$$(1+n)\hat{b}' = \begin{cases} 0 & \text{if } \frac{1-\alpha}{\alpha} \le \gamma, \\ B_{un}A(k)^{\alpha}h & \text{if } \frac{1-\alpha}{\alpha} > \gamma, \end{cases}$$

where ψ, T^k_{un}, X_{un}, and B_{un} are defined by

$$\psi \equiv 1 + \beta(1+\gamma)(\alpha + \eta(1-\alpha)),$$
$$T^k_{un} \equiv \frac{1}{\alpha}\frac{\frac{\omega}{(1+n)(1-\omega)}}{\frac{\omega}{(1+n)(1-\omega)}+\psi},$$
$$X_{un} \equiv \frac{\beta\eta(1-\alpha)(1+\gamma)}{\frac{\omega}{(1+n)(1-\omega)}+\psi},$$
$$B_{un} \equiv \frac{\beta[1-\alpha(1+\gamma)]}{\frac{\omega}{(1+n)(1-\omega)}+\psi}.$$

Proof. See Appendix A.2 .

The result in Proposition 1 suggests that the tax rates, τ^k and τ, and public education expenditure, x, are affected by the weights, (p.1)–(p.3) and (p.5), of the political objective function in (10). To understand the effects, we reformulate the policy functions of τ^k, τ, and x in Proposition 1 to clearly specify the effects of the weights on the policy functions as follows:

$$\tau^k = 1 - \frac{1}{\alpha}\left\{1 + \frac{(1+\beta)+[\beta(\alpha-1)+\beta\gamma\alpha]+\beta\eta(1-\alpha)(1+\gamma)}{\frac{\omega}{(1+n)(1-\omega)}}\right\}^{-1} \times \frac{1}{1+\frac{\hat{b}}{kh}},$$
$$\tau = \begin{cases} 1 - \frac{1}{1-\alpha}\left\{1 + \frac{\frac{\omega}{(1+n)(1-\omega)}+\beta\eta(1-\alpha)(1+\gamma)}{(1+\beta)+[\beta(\alpha-1)+\beta\gamma\alpha]}\right\}^{-1} & \text{if } \frac{1-\alpha}{\alpha} \le \gamma, \\ 1 - \frac{1}{1-\alpha}\left\{1 + \frac{\frac{\omega}{(1+n)(1-\omega)}+[\beta(\alpha-1)+\beta\gamma\alpha]+\beta\eta(1-\alpha)(1+\gamma)}{(1+\beta)}\right\}^{-1} & \text{if } \frac{1-\alpha}{\alpha} > \gamma, \end{cases}$$
$$\frac{(1+n)x}{A(k)^{\alpha}h} = \left\{1 + \frac{\frac{\omega}{(1+n)(1-\omega)}+(1+\beta)+[\beta(\alpha-1)+\beta\gamma\alpha]}{\beta\eta(1-\alpha)(1+\gamma)}\right\}^{-1}.$$

The terms (p.1)–(p.3) and (p.5) in (10), which appear in the above equations, affect the tax rates and public education expenditure in the following ways. First, the term (p.1), representing the political weight on the old, implies that the greater political power of the old leads to a larger weight on the utility of their consumption. This incentivizes the government to shift the tax burden from the old to the young as well as to reduce public education expenditure. Second, the term (p.2), representing the weight on the young's utility of lifetime consumption, implies that a larger weight on this utility provides the government with an incentive to shift the tax burden from the young to the old as well as to reduce the tax burden on the young by cutting public education expenditure. Third, the term (p.5), representing the weight on the utility of children's human capital, indicates that a larger weight on this utility implies greater altruism toward children. This in turn provides the government with an incentive to increase public education expenditure by raising the capital and labor tax rates.

The term (p.3) includes the weight on the utility of the interest rate, represented by $\beta(\alpha - 1)$, and the weight on the utility of children's wage income, represented by $\beta\gamma\alpha$. These two weights have opposing effects on the tax rate and public education expenditure. The first weight induces the government to reduce saving and capital to raise interest rate income. For this purpose, the government shifts the tax burden from the old to the young; it also increases public education expenditure to raise the tax burden on the young. The second weight has the opposite effect since the young want to increase capital and thus the wage income of their children. The net effect depends on their relative magnitudes.

The policy function of $\hat{b}'$ when $\hat{b}' > 0$ is reformulated as follows:

$$\frac{(1+n)\hat{b}'}{A(k)^{\alpha} h} = \frac{\overbrace{\beta}^{\text{(p.2)}} - \overbrace{[\beta(\alpha - 1) + \beta\gamma\alpha]}^{\text{(p.3)}} - \overbrace{\beta}^{\text{(p.4)}}}{\frac{\omega}{(1+n)(1-\omega)} + \psi}.$$

This indicates that the weights (p.2)–(p.4) are crucial to the government's financial stance. The term (p.2) implies that a greater weight on the young's utility of consumption incentivizes the government to issue more public debt to finance the young's increased consumption. The term (p.3) implies that an increase in public bond issues crowds out physical capital accumulation and thus increases the interest rate. However, at the same time, it decreases children's wage income, thereby giving the government an incentive to cut public bond issues. The term (p.4) indicates that an increase in public bond issues raises the capital tax burden on the young in their old-age period. This also incentivizes the government to cut public bond issues. In sum, the government chooses $\hat{b}' > 0$ and, therefore, borrows in the capital market if the effect of (p.2) outweighs the sum of the effects of (p.3) and (p.4), that is, if $\gamma < (1 - \alpha)/\alpha$.

3.2 Education Expenditure and Government Debt

The result established in Proposition 1 indicates that public education expenditure and government debt are affected by the two population factors, ω and n. To investigate these effects appropriately, we focus on the education expenditure-to-GDP ratio, $N_{t+1}x_t/Y_t$, and debt-to-GDP ratio, B_{t+1}/Y_t, and analyze the effects of an increase in ω and a decrease in n on these ratios.

Proposition 2. *Consider the Markov-perfect political equilibrium demonstrated in Proposition 1.*

(i) *The education expenditure-to-GDP ratio, $N_{t+1}x_t/Y_t$, decreases with the greater political power of the old and a lower population growth rate: $\partial\left(N_{t+1}x_t/Y_t\right)/\partial\omega < 0$ and $\partial\left(N_{t+1}x_t/Y_t\right)/\partial n > 0$.*

(ii) *The government debt-to-GDP ratio, B_{t+1}/Y_t, decreases with the greater political power of the old and a lower population growth rate: $\partial\left(B_{t+1}/Y_t\right)/\partial\omega < 0$ and $\partial\left(B_{t+1}/Y_t\right)/\partial n > 0$.*

To confirm the statement in Proposition 2, we compute the public education expenditure-to-GDP ratio and debt-to-GDP ratio as follows:

$$\begin{aligned}\frac{N_{t+1}x_t}{Y_t} &= \frac{\beta\eta\left(1-\alpha\right)\left(1+\gamma\right)}{\frac{\omega}{(1+n)(1-\omega)}+\psi};\\ \frac{B_{t+1}}{Y_t} &= \frac{\beta\left[1-\alpha\left(1+\gamma\right)\right]}{\frac{\omega}{(1+n)(1-\omega)}+\psi}.\end{aligned}$$

These ratios depend on the two population factors, ω and n, through the term $\omega/(1+n)(1-\omega)$, representing the relative political weight on the old. Higher ω and lower n imply a larger political weight on the old in the political objective function. A larger weight on the old in turn provides the government with an incentive to reduce public education expenditure since the old benefit nothing from it. Therefore, the education expenditure-to-GDP ratio decreases as ω increases and n decreases.

The debt-to-GDP ratio also decreases as ω increases and n decreases. This is somewhat counterintuitive since the old may have an incentive to pass the fiscal burden onto future generations. This incentive arises in the framework of Song, Storesletten, and Zilibotti (2012) in which the old are assumed to owe no tax burden but benefit from public good provision. However, in the present framework, the old receive no benefit from public expenditure, but owe a capital tax burden. This implies that the old want to lower the capital tax rate and public education expenditure as their political weight increases. A decrease in the capital tax rate is associated with increased public debt, whereas a decrease in public education expenditure is associated with decreased public debt. That is, there are two opposing effects on public debt, but the negative effect outweighs the positive one in the present framework. Thus, the debt-to-GDP ratio decreases as the political weight on the old increases.

4 Steady State

Having established the policy functions, we are now ready to demonstrate the accumulation of physical and human capital. We substitute the policy functions in Proposition 1 into the capital market-clearing condition in (11) and human capital formation function in (1), and obtain

$$k' = \Psi_K \left[A\left(k\right)^{\alpha}\right]^{1-\eta}, \tag{19}$$

$$\frac{h'}{h} = D\Psi_H \left[A\left(k\right)^{\alpha}\right]^{\eta}, \tag{20}$$

where Ψ_K and Ψ_H are defined by

$$\Psi_K \equiv \begin{cases} \Psi_{K,b'=0} \equiv \frac{\beta}{1+\beta} \frac{1+\alpha\beta(1+\gamma)}{\frac{\omega}{(1+n)(1-\omega)}+\psi} \left\{(1+n)D\left[\frac{X_{un}}{1+n}\right]^{\eta}\right\}^{-1} & \text{if } \frac{1-\alpha}{\alpha} \leq \gamma, \\ \Psi_{K,b'>0} \equiv \frac{\alpha\beta(1+\gamma)}{\frac{\omega}{(1+n)(1-\omega)}+\psi} \left\{(1+n)D\left[\frac{X_{un}}{1+n}\right]^{\eta}\right\}^{-1} & \text{if } \frac{1-\alpha}{\alpha} > \gamma, \end{cases} \tag{21}$$

and

$$\Psi_H \equiv \left[\frac{X_{un}}{1+n}\right]^{\eta}, \tag{22}$$

respectively. Appendix A.3 shows the derivation of (19) and (20).

Given $\{k_0, h_0\}$, the sequence $\{k_t, h_t\}$ is distinguished by the above two equations in (19) and (20). A *steady state* is defined as a political equilibrium with $k_t = k_{t+1}$. In other words, the ratio of physical to human capital is constant in a steady state. Equation (19) implies that there is a unique, stable steady state. Human capital increases along the steady-state path, as suggested in (20). The following proposition summarizes the argument thus far and identifies conditions for which tax rates are set within a range $[0, 1)$.

Proposition 3. *There is a unique, stable steady-state equilibrium distinguished by (i) $b' = 0$, $\tau^k \in [0, 1)$, and $\tau \in [0, 1)$ if $(1-\alpha)/\alpha \leq \gamma$ and*

$$\frac{\alpha}{1-\alpha}\psi - \frac{\beta\eta}{1-\alpha}\left(1+\gamma\right)\left(1-\alpha\right) \leq \frac{\omega}{(1+n)(1-\omega)} \leq \frac{\alpha}{1-\alpha}\psi, \tag{23}$$

and (ii) $b' > 0$, $\tau^k \in [0, 1)$, and $\tau \in [0, 1)$ if $\gamma < (1-\alpha)/\alpha$ and

$$\frac{1+\beta}{1-\alpha} - \psi \leq \frac{\omega}{(1+n)(1-\omega)} \leq \frac{\psi}{\gamma}. \tag{24}$$

Proof. See Appendix A.4.

The result in Proposition 3 suggests that the relative political weight on the old, represented by $\omega/(1+n)(1-\omega)$, plays a crucial role in shaping the tax rates. When $\omega/(1+n)(1-\omega)$ is above the upper bound (below the lower bound) in (23) and (24), the relative political weight on the old is too high (low) to incentivize the government to tax the old (young). The government would rather subsidize the old (young) by choosing $\tau^k < 0$ ($\tau < 0$). We rule out this possibility by imposing the upper (lower) bound of $\omega/(1+n)(1-\omega)$.

Based on the result established thus far, we can define the growth rate of the economy and investigate how it is affected by the two population factors, ω and n. Consider the output-to-human capital ratio defined by $y \equiv Y/H = A\,(k)^{\alpha}\,h$. Then, the growth rate of y is

$$\frac{y'}{y} = \frac{A\,(k')^{\alpha}\,h'}{A\,(k)^{\alpha}\,h},$$

where x' denotes the next period $x(=k,h,y)$. In the steady state with $k'=k$, the growth rate of y is equal to the growth rate of human capital, h'/h. Therefore, in what follows, we focus on the steady-state growth rate of human capital.

Recall the growth rate of human capital given by (20). From (19), the term $(A\,(k)^{\alpha})^{\eta}$ in the steady state becomes $(A\,(k)^{\alpha})^{\eta} = (\Psi_K)^{\frac{\alpha\eta}{1-\alpha(1-\eta)}}\,(A)^{\frac{\eta}{1-\alpha(1-\eta)}}$. Thus, the steady-state growth rate becomes

$$\frac{h'}{h} = D\Psi_H\,(\Psi_K)^{\frac{\alpha\eta}{1-\alpha(1-\eta)}}\,(A)^{\frac{\eta}{1-\alpha(1-\eta)}}. \tag{25}$$

The following proposition establishes the effects of changes in ω and n on the growth rate via the terms Ψ_H and Ψ_K.

Proposition 4. *The steady-state growth rate increases as the population growth rate decreases, but it decreases as the political weight on the old increases:* $\partial\,(h'/h)\,/\partial n < 0$ *and* $\partial\,(h'/h)\,/\partial\omega < 0$.

The proof and interpretation of Proposition 4 is as follows. The steady-state growth rate in (25) is reformulated as follows:

$$\frac{h'}{h} = \left\{\frac{1}{1+n}\frac{1}{\frac{\omega}{(1+n)(1-\omega)}+\psi}\right\}^{\frac{\eta}{1-\alpha(1-\eta)}} \times UR,$$

where UR includes the terms unrelated to n and ω. A decrease in ω leads to a larger relative political weight on the old, as observed by the term $\omega/(1+n)(1-\omega)$. This provides the government with an incentive to cut public education expenditure, which in turn reduces human capital accumulation. Thus, a greater political weight on the old results in a lower steady-state growth rate.

A decrease in n has two opposing effects on the steady-state growth rate. On the one hand, it leads to a larger political weight on the old, as observed by the term $\omega/(1+n)(1-\omega)$. As in the case of increased ω, this provides the government with an incentive to cut public education expenditure, which negatively affects human capital accumulation. On the other hand, a decrease in n results in an increase in per-capita public education expenditure, as observed by the term $1/(1+n)$. This works to enhance human capital accumulation, and this positive effect outweighs the negative effect. Therefore, a lower population growth rate results in a higher steady-state growth rate.

5 Fiscal Rules

In the previous section, we considered fiscal policy and economic growth in the absence of constraints on public bond issues except for the flow budget constraint. In other words, we assumed no rule on public bond issues. However, in the real world, many countries have introduced fiscal rules that control public bond issuance (Schaechter et al., 2012). This brings us to the question of how fiscal rules shape the choice of fiscal policy and affect economic growth over time.

To answer this question, in Section 5.1 we first consider the following alternative scenario in which the government is prohibited from issuing public bonds and thus its expenditure is financed solely through taxation. In this tax-financing case, we assume $\gamma < \alpha/(1-\alpha)$, which implies that in the absence of the tax-financing rule, the government borrows in the capital market and issues public bonds. In other words, the government wants to issue public bonds to finance its expenditure, but their issuance is prohibited when the tax-financing rule is introduced. We then compare the tax rates, expenditure, and economic growth in the debt-financing case in the previous section with the tax-financing case. We also investigate the welfare consequences of shifting from debt financing to tax financing.

The requirement for tax financing is somewhat extreme because in reality the government is allowed to issue public bonds as long as their issuance is below some debt ceiling. Hence, in Section 5.2, we overcome this shortcoming by considering an alternative fiscal rule for managing the debt issuance-to-GDP ratio widely introduced in developed countries.

5.1 Tax Financing versus Debt Financing

The policy functions in the tax-financing case are those associated with $\hat{b}' = 0$ in Proposition 1. The corresponding steady-state growth rate is computed by setting $\hat{b}' = 0$ in (25). To investigate the differences between the tax-financing and debt-financing cases, we compare their tax rates, τ^k and τ, public education expenditure-to-GDP ratio, $(1+n)x/y$, and economic growth, h'/h. the variables in the tax-financing and debt-financing cases are denoted by the subscripts "tax" and "debt,"respectively.

Proposition 5. *Suppose that $\gamma < \alpha/(1-\alpha)$ holds. Given the initial conditions k_0 and b_0, tax financing and debt financing are compared as follows:*

$$\tau_0^k\big|_{tax} = \tau_0^k\big|_{debt}; \;\; \tau_t^k\big|_{tax} < \tau_t^k\big|_{debt} \; \text{for } t \geq 1; \;\; \tau|_{tax} > \tau|_{debt};$$
$$(1+n)x/y|_{tax} = (1+n)x/y|_{debt}; \; \text{and} \;\; h'/h|_{tax} > h'/h|_{debt}.$$

Proof. See Appendix A.5.

In the initial period, the government needs to finance the repayment of outstanding public debt, b_0, regardless of the financing method. Thus, the capital tax rates are equal in the two financing cases in the initial period. However, from period 1 onward, the government incurs no repayment costs in the tax-financing case, while it still incurs such costs in the debt-financing case. Because of this difference, the capital tax rate is lower in the tax-financing case than in the debt-financing case from period 1.

By contrast, the labor tax rate is higher in the tax-financing case than in the debt-financing case. When the tax-financing rule is introduced, the government needs to compensate for the loss of revenue from bond issues by raising the labor income tax rate. An increase in revenue from the labor tax is offset by a decrease in revenue from the capital tax and public bond issues. Thus, the education expenditure-to-GDP ratio remains unchanged. However, the introduction of tax financing removes the crowding-out effect of public bonds. This positive effect on physical capital enhances human capital accumulation and economic growth.

The result in Proposition 6 suggests that the shift from debt financing to tax financing increases the growth rate and benefits future generations, but may worsen the current middle-aged population because of the increased labor tax burden. We investigate this welfare implication and obtain the following result.

Proposition 6. *Suppose that $\gamma < \alpha/(1-\alpha)$ holds.*

(i) *The welfare of the initial old population is unaffected; generation 0 is made worse off by shifting from debt financing to tax financing.*

(ii) *There is a critical period, denoted by $\hat{t}(>1)$, such that generation $t \leq \hat{t}$ is made worse off, whereas generation $t > \hat{t}$ is made better off by shifting from debt financing to tax financing.*

Proof. See Appendix A.6.

The welfare of the initial old population is unaffected by shifting to tax financing since their tax burden is unchanged. However, the choice of tax financing has two opposing effects on current and future generations. Tax financing raises the tax burden of the middle-aged population as demonstrated in Proposition 5. This lowers the lifetime income of the middle-aged and thus lowers their lifetime utility of consumption. This is the negative effect of tax financing. However, tax financing removes the crowding-out effect of public bonds on capital and thus enhances human capital accumulation. This positive effect appears from generation 1 onward and accumulates over time, but generation 0 cannot enjoy this benefit. Therefore, generation 0 suffers from a negative effect, whereas distant future generations benefit from a positive effect that outweighs the negative one.

5.2 Debt Rule

This section extends the analysis of the previous section by considering the following debt rule:

$$\frac{B_{t+1}}{Y_t} \leq \bar{u}.$$

This rule resembles the debt rule that sets an explicit ceiling for public debt as a percentage of GDP (Schaechter et al., 2012). This is reformulated as

$$(1+n)\hat{b}_{t+1} \leq \bar{u}A\left(k_t\right)^{\alpha} h_t \Leftrightarrow (1+n)\tilde{b}' \leq \bar{u}, \tag{26}$$

where $\bar{u}$ is defined by

$$\bar{u} \equiv \varepsilon B_{un}, \quad \varepsilon \in [0,1)\,.$$

The rule resembles the tax-financing case in Section 5.1 when $\varepsilon \to 0$ and the unconstrained debt-financing case in Section 3 when $\varepsilon \to 1$.

We retain the assumption of $\gamma < (1-\alpha)/\alpha$, implying that the government chooses debt financing in the absence of any debt rule. Debt issuance in the absence of the rule in (26) is given by $(1+n)\hat{b}' = B_{un}A\left(k\right)^{\alpha} h$ as demonstrated in Proposition 1. When the debt rule in (26) is introduced, it is always binding since $\varepsilon < 1$. Thus, the issue of public bonds in the absence of the rule in (26) is $(1+n)\hat{b}' = \varepsilon B_{un}A\left(k\right)^{\alpha} h$, or

$$(1+n)\tilde{b}' = \varepsilon B_{un}A.$$

With the use of $(1+n)\tilde{b}' = \varepsilon B_{un}A$, the government budget constraint in (12) is reformulated as

$$\varepsilon B_{un} + \tau\left(1-\alpha\right) + \tau^{k}\alpha\left(1+\frac{\hat{b}}{kh}\right) = (1+n)\tilde{x} + \alpha\frac{\hat{b}}{kh}, \tag{27}$$

and the capital market-clearing condition in (11) is rewritten as

$$\varepsilon B_{un} + (1+n)\frac{k'h'}{A\left(k\right)^{\alpha} h} = \frac{\beta}{1+\beta}\left(1-\tau\right)\left(1-\alpha\right). \tag{28}$$

The political objective function in (10) is reformulated as

$$\begin{aligned}
\Omega &\simeq \frac{\omega}{(1+n)(1-\omega)} \ln\left(1-\tau^k\right) \\
&+ (1+\beta)\ln\left[1-\left(1-\tau^k\right)\alpha\left(\frac{\hat{b}}{kh}+1\right)-(1+n)\tilde{x}+\varepsilon B_{un}\right] \\
&+ \beta\left[(\alpha-1)+\gamma\alpha\right]\ln\left[1-\left(1-\tau^k\right)\alpha\left(\frac{\hat{b}}{kh}+1\right)-(1+n)\tilde{x}-\frac{1}{\beta}\varepsilon B_{un}\right] \\
&+ \beta\ln\left(1-\tau^{k\prime}\right)+\beta\eta\left(1-\alpha\right)\left(1+\gamma\right)\ln\tilde{x}.
\end{aligned} \tag{29}$$

Following the procedure described in Section 3, we conjecture the following capital tax rate in the next period:

$$\tau^{k\prime} = 1 - T^k_{con}\frac{1}{1+\hat{b}'/k'h'},$$

where the subscript "*con*" of T^k_{con} implies that public bond issuance is "constrained" by the rule in (26). From (26), (27), and (28), we can rewrite the above conjecture as follows:

$$\tau^{k\prime} = 1 - T^k_{con}\frac{\varepsilon B_{un}+(1-\alpha)+\tau^k\alpha\left(1+\frac{\hat{b}}{kh}\right)-(1+n)\tilde{x}-\alpha\frac{\hat{b}}{kh}-\frac{1+\beta}{\beta}\varepsilon B_{un}}{\varepsilon B_{un}+(1-\alpha)+\tau^k\alpha\left(1+\frac{\hat{b}}{kh}\right)-(1+n)\tilde{x}-\alpha\frac{\hat{b}}{kh}}. \tag{30}$$

Appendix A.7 shows the derivation of (30).

We substitute (30) into the political objective function in (29) and rearrange the terms to obtain

$$\Omega \simeq \frac{\omega}{(1+n)(1-\omega)}\ln\left(1-\tau^k\right)+\ln\tilde{Z}^1+\alpha\beta\left(1+\gamma\right)\ln\tilde{Z}^2+\beta\eta\left(1-\alpha\right)\left(1+\gamma\right)\ln\tilde{x}, \tag{31}$$

where

$$\tilde{Z}^1 \equiv 1-\left(1-\tau^k\right)\alpha\left(1+\frac{\hat{b}}{kh}\right)-(1+n)\tilde{x}+\varepsilon B_{un}, \tag{32}$$

$$\tilde{Z}^2 \equiv 1-\left(1-\tau^k\right)\alpha\left(1+\frac{\hat{b}}{kh}\right)-(1+n)\tilde{x}-\frac{1}{\beta}\varepsilon B_{un}. \tag{33}$$

The first-order conditions with respect to τ^k and $\tilde{x}$ are

$$\tau^k : -\frac{\omega}{(1+n)(1-\omega)}\frac{1}{1-\tau^k}+\alpha\left(1+\frac{\hat{b}}{kh}\right)\left[\frac{1}{\tilde{Z}^1}+\frac{\alpha\beta\left(1+\gamma\right)}{\tilde{Z}^2}\right]=0, \tag{34}$$

$$\tilde{x} : \frac{\beta\eta\left(1-\alpha\right)\left(1+\gamma\right)}{(1+n)\tilde{x}}-\left[\frac{1}{\tilde{Z}^1}+\frac{\alpha\beta\left(1+\gamma\right)}{\tilde{Z}^2}\right]=0. \tag{35}$$

By solving (34) and (35) for τ^k and $\tilde{x}$, we obtain the following result.

Proposition 7. *Suppose that $\gamma < (1-\alpha)/\alpha$ holds. In the presence of the debt rule in (26), a Markov-perfect political equilibrium is distinguished by the following policy functions:*

$$\tau^k = 1 - T^k_{con} \frac{1}{1+\hat{b}/kh},$$

$$(1+n)x = X_{con} A (k)^\alpha h,$$

$$\tau = 1 - \frac{1}{1-\alpha}\left[(1+\varepsilon B_{un}) - \left(1 + \frac{\omega}{(1+n)(1-\omega)} \frac{1}{\beta\eta(1-\alpha)(1+\gamma)}\right) X_{con}\right],$$

where T^k_{con} and X_{con} are defined as

$$T^k_{con} \equiv \frac{1}{\alpha} \frac{\omega}{(1+n)(1-\omega)} \frac{1}{\beta\eta(1-\alpha)(1+\gamma)} \frac{H - \sqrt{(H)^2 - 4GI}}{2G},$$

$$X_{con} \equiv \frac{H - \sqrt{(H)^2 - 4GI}}{2G},$$

$$G \equiv \left[1 + \frac{\omega}{(1+n)(1-\omega)} \frac{1}{\beta\eta(1-\alpha)(1+\gamma)}\right] \left[\frac{\omega}{(1+n)(1-\omega)} + \psi\right] > 0,$$

$$H \equiv \beta\eta(1-\alpha)(1+\gamma)\left[1 + \frac{\omega}{(1+n)(1-\omega)} \frac{1}{\beta\eta(1-\alpha)(1+\gamma)}\right]\left[(1+\varepsilon B_{un}) + \left(1 - \frac{\varepsilon B_{un}}{\beta}\right)\right]$$
$$+ \alpha\beta(1+\gamma)(1+\varepsilon B_{un}) + \left(1 - \frac{\varepsilon B_{un}}{\beta}\right)$$
$$> 0,$$

$$I \equiv \beta\eta(1-\alpha)(1+\gamma)(1+\varepsilon B_{un})\left(1 - \frac{\varepsilon B_{un}}{\beta}\right) > 0.$$

Proof. See Appendix A.8.

Following the procedure described in Section 3, we show the existence and uniqueness of steady-state capital. Recall the capital market-clearing condition in (28), which is rewritten as follows:

$$\begin{aligned} k' &= \frac{\frac{\beta}{1+\beta}(1-\tau)(1-\alpha) - \varepsilon B_{un}}{(1+n)D(h)^{1-\eta}(x)^\eta} A(k)^\alpha h \\ &= \frac{\frac{\beta}{1+\beta}(1-\tau)(1-\alpha) - \varepsilon B_{un}}{(1+n)D\left(\frac{X_{con}}{1+n}\right)^\eta} \left[A(k)^\alpha\right]^{1-\eta}, \end{aligned} \tag{36}$$

where the first equality comes from the human capital formation function given by $h' = D(h)^{1-\eta}(x)^\eta$ and the second equality comes from the policy function of x presented in Proposition 7. Equation (36) indicates that there is unique, stable steady-state capital. In the next section, we focus on steady states and compare cases in the presence and absence of the debt rule in (26) in terms of the education expenditure-to-GDP ratio, capital and labor taxes, and growth rates. We also compare the cases in terms of utility across generations.

5.3 Numerical Analysis

Our task here is to compare cases in the absence and presence of the debt rule in (26) based on numerical methods. Our strategy is to calibrate the model economy such that the steady-state equilibrium with $b > 0$ matches some key statistics of average OECD countries during 1995–2014.[3] We fix the share of capital at $\alpha = 1/3$ following Song, Storesletten, and Zilibotti (2012) and Lancia and Russo (2016). Each period lasts 30 years; this assumption is standard in quantitative analyses of the two- or three-period overlapping-generations model (e.g., Gonzalez-Eiras and Niepelt, 2008; Lancia and Russo, 2016). Our selection of β is 0.99 per quarter, which is also standard in the literature (e.g., Kydland and Prescott, 1982; De la Croix and Doepke, 2002). Since agents in the present model plan over generations that span 30 years, we discount the future by $(0.99)^{120}$.

We assume an annual population growth rate of 1.0059, which was the OECD average during 1995–2014. This assumption implies that the net population growth rate for 30 years is $(1.0059)^{30} - 1$. For η, the estimate in Cardak (2004) implies that the elasticity of school quality is in the range of $0.1 - -0.3$. Based on this estimation, we set $\eta = 0.2$. We also set ω to 0.45 to attain interior solutions for the capital and labor taxes.

For γ, we focus on the education expenditure-to-GDP ratio in the steady state:

$$\frac{N_{t+1}x_t}{Y_t} = X_{un} = \frac{\beta\eta\,(1-\alpha)\,(1+\gamma)}{\frac{\omega}{(1+n)(1-\omega)} + \psi}.$$

Given $\alpha = 1/3$, $\beta = (0.99)^{120}$, $1 + n = (1.0059)^{30}$, $\eta = 0.2$, and $\omega = 0.45$, we can solve this expression for γ by using the average ratio observed in OECD countries of 0.051 and obtain $\gamma = 1.62$.

To determine the two productivity parameters, A and D, we normalize the steady-state wage, w, to unity. Thus, we have $w = (1-\alpha)A(k)^{\alpha} = 1$, or

$$(1-\alpha)\left[\frac{\frac{\beta}{1+\beta}\,(1-\tau)\,(1-\alpha) - B_{un}}{(1+n)\left(\frac{X_{un}}{1+n}\right)^{\eta}}\right]^{\alpha/(1-\alpha(1-\eta))} (D)^{-\alpha/(1-\alpha(1-\eta))}\,(A)^{1/(1-\alpha(1-\eta))} = 1. \quad (37)$$

We also use the data on the per-capita GDP gross growth rate of 1.02, which was the OECD average during 1995–2014. We substitute these data and the values of α, β, n, η, γ, and ω into the following equation expressing the per-capita GDP gross growth rate:

$$\frac{h'}{h} = \left(\frac{X_{un}}{1+n}\right)^{\eta}\left[\frac{\frac{\beta}{1+\beta}\,(1-\tau)\,(1-\alpha) - B_{un}}{(1+n)\left(\frac{X_{un}}{1+n}\right)^{\eta}}\right]^{\alpha\eta/(1-\alpha(1-\eta))} (D)^{(1-\alpha)/(1-\alpha(1-\eta))}\,(A)^{\eta/(1-\alpha(1-\eta))}$$
$$= (1.02)^{30}. \quad (38)$$

[3]Source: World Development Indicators, https://data.worldbank.org/products/wdi (Accessed on March 3, 2018).

We solve the two equations, (37) and (38), for A and D, and obtain $A = 3.37$ and $D = 3.14$.

The economy is assumed to be in a steady state in period 0. The initial capital k_0 is computed by solving equation (36) for k. The initial value of human capital, h_0, is set at $h_0 = 2$ to allow logarithmic utility to be positive for the tractability of the analysis. From the result in Section 3, in the absence of any fiscal rule, the ratio $\hat{b}/kh$ in the steady state is given by $\hat{b}/kh = [1-\alpha(1+\gamma)]/\alpha(1+\gamma)$. Thus, we set $\hat{b}_0$ at $\hat{b}_0 = \{[1-\alpha(1+\gamma)]/\alpha(1+\gamma)\} k_0 h_0$ and compare the cases with and without a debt rule for the same initial conditions.

5.3.1 Comparative Statics

We study how the steady-state equilibrium responds to changes in the debt rule in (26). In particular, we focus on ε. When $\varepsilon = 1$, the equilibrium policy functions and corresponding economic growth rate coincide with those in the absence of the debt rule as in Section 3. When $\varepsilon = 0$, they coincide with those in the tax-financing case as in Section 5.1. In the following, we consider a decrease in ε that aims to tighten fiscal discipline and investigate its impact on fiscal policy, economic growth, and welfare across generations. Figure 1 plots the education expenditure-to-GDP ratio, capital and labor tax rates, ratio of physical to human capital, and per-capita growth rate in the steady state, taking ε on the horizontal axis from 0 to 1.

[Figure 1 here.]

Fiscal tightening (i.e., a decrease in ε) has the following effects on the education expenditure-to-GDP ratio. Firstly, the government raises the labor tax rate to compensate for the loss of revenue from public bond issues, as depicted in Panel (a). This lowers the disposable income of the middle-aged population and thus their lifetime consumption, which in turn raises the marginal costs of public education expenditure in terms of utility. This is a negative effect of fiscal discipline that works through the term $\tilde{Z}^1$ in (35). Secondly, a decrease in the disposable income of the middle-aged population leads to less saving and thus a lower ratio of physical to human capital. However, at the same time, fiscal tightening reduces the crowding-out effect of public debt. The net effect on the ratio of physical to human capital is positive as shown in Panel (b). This effect works through the term $\tilde{Z}^2$ in (35). In sum, the two opposing effects on the choice of public education expenditure produce an initial increase in the education expenditure-to-GDP ratio followed by a decrease, as depicted in Panel (c).

Next, consider the effect of fiscal tightening on the choice of the capital tax rate. Firstly, this affects the marginal benefit of capital taxation as shown by the term $1/\tilde{Z}^1 +$

$\alpha\beta(1+\gamma)/\tilde{Z}^2$. This effect parallels the effect on the choice of public education expenditure as described above. However, an additional effect on the marginal benefit arises through the term $\hat{b}/kh$ in (34). Fiscal tightening lowers the ratio $\hat{b}/kh$ and thus produces a negative effect on the marginal benefit. This negative effect outweighs the effect via the term $1/\tilde{Z}^1 + \alpha\beta(1+\gamma)/\tilde{Z}^2$. Therefore, the government chooses a lower capital tax rate to balance the marginal cost and benefit as ε decreases, as depicted in Panel (d). Finally, fiscal tightening raises the per-capita growth rate as shown in Panel (e) because the positive effects through the increased ratio of physical to human capital and decreased capital tax rate outweigh the negative effect through the increased labor tax and non-monotone effect through public education expenditure.

5.3.2 Comparative Dynamics

The comparative static analysis shows that the physical-to-human capital ratio and steady-state growth rate increase as ε decreases. This finding suggests that future generations benefit from increased physical and human capital. However, are all generations made better off by fiscal tightening? To answer this question, Figure 2 plots the utility of generations from the initial old population for three scenarios, $\varepsilon = 0.2$, 0.5, and 0.8. Table 1 summarizes the results of the first four generations, indicating that the initial old population as well as generation 1 onward are made better off by the introduction of the debt rule in (26), whereas generation 0 is made worse off (see also Figure 2). Thus, fiscal tightening is not Pareto-improving.

[Figure 2 here.]

[Table 1 here.]

The mechanism behind the result is straightforward. Under the present assumption, the government's optimal choice of the debt-to-GDP ratio is B_{un}; however, its choice is limited up to $\varepsilon B_{un} (< B_{un})$ by the rule in (26). Because of this constraint, the government in period 0 is unable to attain an "interior optimum." In particular, the constraint hits the middle-aged population in generation 0. Governments from period 1 are also constrained by the rule, but they benefit from the higher levels of physical and human capital bequeathed from past generations. This benefit outweighs the cost of the constraint in (26). Therefore, the introduction of the debt rule creates a trade-off between generations in terms of utility.

The effects of decreased ε on utility is monotone from generation 0 onward, as shown in Figure 2 and Table 1. However, the effect is non-monotone as for the initial old population. In particular, a decrease in ε from 1.0 to 0.5 improves their utility, but a further decrease worsens it. This non-monotone effect stems from the initial decrease followed by an increase in the period 0 capital tax rate as depicted in Figure 3. The U-shaped pattern of the period 0 capital tax rate parallels the hump-shaped pattern of public education expenditure described above.

[Figure 3 here.]

Finally, we consider the case where the debt rule is imposed only for limited periods. In particular, the debt rule in (26) is introduced in period 2, but terminated at the end of period 2, 3, or 4, meaning that successive governments from the termination period onward are free to choose policies with no rule. Figure 4 illustrates the effect of this temporary implementation of the fiscal rule on fiscal policies, the ratio of physical to human capital, the growth rate over time, and utility across generations when $\varepsilon = 0.5$. As might be expected, the rule produces a temporary effect on fiscal policies, the physical-to-human capital ratio, and economic growth. However, it has a long-lasting effect on utility across generations owing to the increased human capital bequeathed from generation to generation. This long-lasting effect of the temporary rule, which was not shown in Barseghyan and Battaglini (2016), is caused by human capital accumulation stimulated by debt-financed public education expenditure. Therefore, this result suggests the importance of connecting seemingly unrelated subjects, public education and public debt, to consider the impact of a debt rule over time and across generations.

[Figure 4 here.]

6 Conclusion

In this study, we developed an overlapping-generations model with physical and human capital accumulation. In the model, public education and parental human capital were used as inputs in the process of human capital formation. Public education spending was financed through taxes on capital and labor income and public debt issues. Under this framework, we demonstrated the endogenous policy formation and showed that the greater political power of the old incentivizes the government to shift the tax burden from the old to the young, thereby resulting in a lower capital tax rate and a higher labor tax rate. We also showed that the greater political power of the old leads to a lower debt-to-GDP ratio. Moreover, we considered the role of a debt ceiling rule and found that fiscal tightening increases the growth rate, but lowers the welfare of the currently working generation. However, the growth effect of the rule lasts for long periods via human capital accumulation and benefits future generations even if it is imposed only for a limited time.

The presented model can be extended in several directions. For instance, one could allow for the participation of the young in voting. Given that the young prefer more public education expenditure to allow their human capital to form, it would be reasonable to expect that their participation incentivizes the government to spend more on public education by raising tax rates. One could also consider public good provision as an alternative mode of spending and investigate the conflict over the composition of expenditure. One could go further and study alternative fiscal rules such as balanced budget rules, expenditure rules, and revenue rules. Such extensions are left to future research.

A Proofs

A.1 Derivation of (10)

Recall equation (9) in the main text:

$$\Omega_t \simeq \frac{\omega}{(1+n)(1-\omega)} \ln\left(1-\tau_t^k\right) + (1+\beta)\ln \underbrace{(1-\alpha)A(k_t)^{\alpha} h_t (1-\tau_t)}_{(*1)}$$
$$+\beta\left[(\alpha-1)+\gamma\alpha\right]\ln \underbrace{k_{t+1}}_{(*2)} + \beta \ln\left(1-\tau_{t+1}^k\right) + \beta\gamma\eta \ln x_t. \tag{39}$$

The term (*1) is reformulated as follows:

$$\begin{aligned}(1-\alpha)A(k_t)^{\alpha} h_t(1-\tau_t) &= (1-\alpha)A(k_t)^{\alpha} h_t - \left[(1+n)x_t + R_t\hat{b}_t - (1+n)\hat{b}_{t+1} - \tau_t^k R_t \frac{s_{t-1}}{1+n}\right] \\ &= (1-\alpha)A(k_t)^{\alpha} h_t - \left[(1+n)x_t + \alpha A(k_t)^{\alpha-1}\hat{b}_t - (1+n)\hat{b}_{t+1}\right. \\ &\quad \left. -\tau_t^k \alpha A(k_t)^{\alpha-1}\left(\hat{b}_t + k_t h_t\right)\right], \end{aligned} \tag{40}$$

where the first equality comes from (5) and the second equality comes from (3), (4), and (6). The term (*2) is reformulated as follows:

$$\begin{aligned} k_{t+1} &= \frac{1}{(1+n)h_{t+1}}\left[\frac{\beta}{1+\beta}(1-\tau_t)w_t h_t - (1+n)\hat{b}_{t+1}\right] \\ &= \frac{1}{(1+n)D(h_t)^{1-\eta}(x_t)^{\eta}}\left\{\frac{\beta}{1+\beta}\left[A(k_t)^{\alpha} h_t - \left(1-\tau_t^k\right)\alpha A(k_t)^{\alpha-1}\left(\hat{b}_t + k_t h_t\right)\right.\right. \\ &\quad \left.\left. -(1+n)x_t + (1+n)\hat{b}_{t+1}\right] - (1+n)\hat{b}_{t+1}\right\} \\ &= \frac{\frac{\beta}{1+\beta}}{(1+n)D(h_t)^{1-\eta}(x_t)^{\eta}}\left[A(k_t)^{\alpha} h_t - \left(1-\tau_t^k\right)\alpha A(k_t)^{\alpha-1}\left(\hat{b}_t + k_t h_t\right) - (1+n)x_t - \frac{1}{\beta}(1+n)\hat{b}_{t+1}\right] \end{aligned} \tag{41}$$

where the first equality comes from (6) and the second equality comes from (3), (4), and(6). From (40) and (41), Ω_t in (39) is reformulated as

$$\begin{aligned}\Omega_t &\simeq \frac{\omega}{(1+n)(1-\omega)}\ln\left(1-\tau_t^k\right) \\ &\quad + (1+\beta)\ln\left[A(k_t)^{\alpha} h_t - \left(1-\tau_t^k\right)\alpha A(k_t)^{\alpha-1}\left(\hat{b}_t + k_t h_t\right) - (1+n)x_t + (1+n)\hat{b}_{t+1}\right] \\ &\quad + \beta\left[(\alpha-1)+\gamma\alpha\right]\ln\left[A(k_t)^{\alpha} h_t - \left(1-\tau_t^k\right)\alpha A(k_t)^{\alpha-1}\left(\hat{b}_t + k_t h_t\right) - (1+n)x_t - \frac{1}{\beta}(1+n)\hat{b}_{t+1}\right] \\ &\quad + \beta\ln\left(1-\tau_{t+1}^k\right) + \beta\eta(1-\alpha)(1+\gamma)\ln x_t. \end{aligned} \tag{42}$$

With the use of $\tilde{x}_t$ and $\tilde{b}_{t+1}$, we can further reformulate Ω_t in (42) as in (10).

■

A.2 Proof of Proposition 1

Suppose that $\tilde{b}' = 0$ holds. This leads to $Z^1 = Z^2$. Thus, (17) implies that $\tilde{b}' = 0$ if $(1+n) - \alpha(1+\gamma)(1+n) \leq 0$, that is, if $(1-\alpha)/\alpha \leq \gamma$.

When $\tilde{b}' = 0$ holds, (16) and (18) are rewritten as

$$\frac{\omega}{(1+n)(1-\omega)} \frac{1}{1-\tau^k} = \frac{1}{Z^1} \alpha \left[1 + \alpha\beta(1+\gamma)\right] \left(1 + \frac{\hat{b}}{kh}\right), \tag{43}$$

and

$$\beta\eta(1-\alpha)(1+\gamma) \frac{1}{\tilde{x}} = \frac{1}{Z^1}(1+n)\left[1 + \alpha\beta(1+\gamma)\right], \tag{44}$$

respectively. From (43) and (44), we obtain

$$(1+n)\tilde{x} = \frac{(1+n)(1-\omega)}{\omega} \beta\eta(1-\alpha)(1+\gamma)\alpha \left(1 + \frac{\hat{b}}{kh}\right)\left(1-\tau^k\right). \tag{45}$$

We substitute (45) into (43) and rearrange the terms to obtain

$$1-\tau^k = \frac{\frac{\omega}{(1+n)(1-\omega)}}{\frac{\omega}{(1+n)(1-\omega)} + \psi} \frac{1}{\alpha\left(1 + \frac{\hat{b}}{kh}\right)}. \tag{46}$$

Thus, the guess in (13) is verified if T_{un}^k is specified by

$$T_{un}^k \equiv \frac{1}{\alpha} \frac{\frac{\omega}{(1+n)(1-\omega)}}{\frac{\omega}{(1+n)(1-\omega)} + \psi}.$$

By plugging (46) into (45), we obtain the policy function of X as in Proposition 1. We can obtain the labor tax rate τ by substituting the policy functions of τ^k, x, and $\tilde{b}' = 0$ into the government budget constraint in (12).

Next, suppose that $\tilde{b}' > 0$ holds; the first-order condition with respect to $\tilde{b}'$ in (17) holds with an equality. From (17) and (18), we obtain

$$\frac{(1+\beta)(1+n)}{Z^1} = \frac{\beta\eta(1-\alpha)(1+\gamma)}{\tilde{x}}. \tag{47}$$

From (16) and (18), we have

$$\alpha\left(1 + \frac{\hat{b}}{kh}\right)\left(1-\tau^k\right) = \frac{\frac{\omega}{(1+n)(1-\omega)}}{\beta\eta(1-\alpha)(1+\gamma)}(1+n)\tilde{x}. \tag{48}$$

We substitute (48) into the first-order condition with respect to $\tilde{b}'$ in (17). After rearranging the terms, we obtain the relation between $\tilde{x}$ and $\tilde{b}'$:

$$\left[1-\alpha(1+\gamma)\right]\left[1 - \left(\frac{\frac{\omega}{(1+n)(1-\omega)}}{\beta\eta(1-\alpha)(1+\gamma)} + 1\right)(1+n)\tilde{x}\right] = \left(\alpha(1+\gamma) + \frac{1}{\beta}\right)(1+n)\tilde{b}'. \tag{49}$$

We can obtain another relation between $\tilde{x}$ and $\tilde{b}'$ by substituting (48) into (47):

$$(1+n)\tilde{b}' = \left[\frac{1+\beta}{\beta\eta(1-\alpha)(1+\gamma)} + 1 + \frac{\frac{\omega}{(1+n)(1-\omega)}}{\beta\eta(1-\alpha)(1+\gamma)} \right] (1+n)\tilde{x} - 1. \quad (50)$$

By solving (49) and (50) for $\tilde{x}$ and $\tilde{b}'$, we obtain the policy functions of x and $\hat{b}'$ as in Proposition 1. Note that $\hat{b}' > 0$ holds if and only if $\gamma < (1-\alpha)/\alpha$.

We substitute the obtained policy function, X, into (48) to derive

$$1 - \tau^k = \frac{\frac{\omega}{(1+n)(1-\omega)}}{\frac{\omega}{(1+n)(1-\omega)} + \psi} \frac{1}{\alpha\left(1 + \frac{\hat{b}}{kh}\right)}.$$

Finally, we can compute the labor tax rate by substituting the obtained τ^k, x, and $\hat{b}'$ into the government budget constraint in (12).

■

A.3 Derivation of (19) and (20)

Recall the human capital formation function, $h' = D(x)^\eta (h)^{1-\eta}$. With the use of the policy function of x in Proposition 1, this function is rewritten as

$$\frac{h'}{h} = D\left[\frac{1}{1+n}\frac{\beta\eta(1-\alpha)(1+\gamma)}{\frac{\omega}{(1+n)(1-\omega)} + \psi}\right]^\eta [A(k)^\alpha]^\eta, \quad (51)$$

or,

$$\frac{h'}{h} = D\Psi_H [A(k)^\alpha]^\eta,$$

where Ψ_H is defined as in (22). The expression in (51) holds for both $\hat{b}' = 0$ and $\hat{b}' > 0$ because the policy functions of x are the same in these two cases.

Next, consider the capital market-clearing condition in (11). Suppose that $\hat{b}' = 0$ (i.e., $\frac{1-\alpha}{\alpha} \leq \gamma$) holds. With the use of the policy functions in Proposition 1, (11) is rewritten as

$$(1+n)k'h' = \frac{\beta}{1+\beta}\frac{1+\alpha\beta(1+\gamma)}{\frac{\omega}{(1+n)(1-\omega)} + \psi} A(k)^\alpha h. \quad (52)$$

By substituting (51) into (52) and rearranging the terms, we obtain

$$k' = \frac{\beta}{1+\beta}\frac{1+\alpha\beta(1+\gamma)}{\frac{\omega}{(1+n)(1-\omega)} + \psi} \left\{(1+n)D\left[\frac{X_{un}}{1+n}\right]^\eta\right\}^{-1} [A(k)^\alpha]^{1-\eta}, \quad (53)$$

where X_{un} is defined in Proposition 1.

Alternatively, suppose that $\hat{b}' > 0$ (i.e., $\frac{1-\alpha}{\alpha} > \gamma$) holds. With the use of the policy function of τ in Proposition 1, (11) is rewritten as

$$(1+n)k'h' = \frac{\alpha\beta(1+\gamma)}{\frac{\omega}{(1+n)(1-\omega)} + \psi} A(k)^\alpha h. \quad (54)$$

By substituting (51) into (54) and rearranging the terms, we obtain

$$k' = \frac{\alpha\beta(1+\gamma)}{\frac{\omega}{(1+n)(1-\omega)} + \psi} \left\{ (1+n)D \left[\frac{X_{un}}{1+n} \right]^{\eta} \right\}^{-1} \left[A(k)^{\alpha} \right]^{1-\eta}. \tag{55}$$

(53) and (55) are summarized as in (19).

■

A.4 Proof of Proposition 3

Equation (19) suggests that there is a unique, stable steady-state equilibrium level of the physical-to-human capital ratio, k, for both $(1-\alpha)/\alpha \leq \gamma$ and $(1-\alpha)/\alpha > \gamma$.

Suppose that $(1-\alpha)/\alpha \leq \gamma$ (i.e., $b' = 0$) holds. The tax rates τ^k and τ in Proposition 1 are

$$\tau^k = 1 - \frac{\frac{\omega}{(1+n)(1-\omega)}}{\frac{\omega}{(1+n)(1-\omega)} + \psi} \frac{1}{\alpha} < 1,$$
$$\tau = 1 - \frac{1}{1-\alpha} \frac{1 + \alpha\beta(1+\gamma)}{\frac{\omega}{(1+n)(1-\omega)} + \psi} < 1.$$

Direct calculation leads to

$$\tau^k \geq 0 \Leftrightarrow \frac{\omega}{(1+n)(1-\omega)} \leq \frac{\alpha}{1-\alpha}\psi,$$
$$\tau \geq 0 \Leftrightarrow \frac{\alpha}{1-\alpha}\psi - \frac{\beta\eta}{1-\alpha}(1+\gamma)(1-\alpha) \leq \frac{\omega}{(1+n)(1-\omega)}.$$

These are summarized in (23).

Next, suppose that $(1-\alpha)/\alpha > \gamma$ (i.e., $b' > 0$) holds. In the steady state, $\hat{b}/kh = \hat{b}'/k'h'$ holds, so

$$\frac{\hat{b}}{kh} = \frac{\frac{\beta[1-\alpha(1+\gamma)]}{\frac{\omega}{(1+n)(1-\omega)}+\psi} A(k)^{\alpha} h}{\frac{\alpha\beta(1+\gamma)}{\frac{\omega}{(1+n)(1-\omega)}+\psi} A(k)^{\alpha} h} = \frac{1-\alpha(1+\gamma)}{\alpha(1+\gamma)}, \tag{56}$$

where the first equality comes from the policy function of $\hat{b}'$ in Proposition 1 and the capital market-clearing condition in (54).

From (56), the tax rate on capital, τ^k, when $b' > 0$ becomes

$$\tau^k = 1 - \frac{\frac{\omega}{(1+n)(1-\omega)}}{\frac{\omega}{(1+n)(1-\omega)} + \psi} \frac{1}{\alpha\left(1 + \frac{1-\alpha(1+\gamma)}{\alpha(1+\gamma)}\right)},$$

or

$$\tau^k = 1 - \frac{\frac{\omega}{(1+n)(1-\omega)}(1+\gamma)}{\frac{\omega}{(1+n)(1-\omega)} + \psi} < 1.$$

Direct calculation leads to

$$\tau^k \geq 0 \Leftrightarrow \frac{\omega}{(1+n)(1-\omega)} \leq \frac{\psi}{\gamma}. \tag{57}$$

The tax rate on labor, τ, is given by

$$\tau = 1 - \frac{1+\beta}{1-\alpha} \frac{1}{\frac{\omega}{(1+n)(1-\omega)} + \psi} < 1.$$

We obtain

$$\tau \geq 0 \Leftrightarrow \frac{1+\beta}{1-\alpha} - \psi \leq \frac{\omega}{(1+n)(1-\omega)}. \tag{58}$$

(57) and (58) are summarized in (24).

■

A.5 Proof of Proposition 5

First, compare the capital tax rates. From Proposition 1, given k_0 and b_0, we immediately find that $\tau_0^k\big|_{tax} = \tau_0^k\big|_{debt}$ holds. For $t \geq 1$, $\tau_t^k\big|_{tax} < \tau_t^k\big|_{debt}$ holds because $\hat{b}/kh = 0 (> 0)$ holds in the tax-financing (debt-financing) case.

Next, compare the labor tax rates and growth rates. Direct comparison leads to

$$\tau|_{tax} > \tau|_{debt} \Leftrightarrow \gamma < \frac{1-\alpha}{\alpha},$$

and

$$h'/h|_{tax} > h'/h|_{debt} \Leftrightarrow \gamma < \frac{1-\alpha}{\alpha}.$$

The condition $\gamma < \alpha/(1-\alpha)$ holds by assumption. Finally, we immediately obtain $(1+n)x/y|_{tax} = (1+n)x/y|_{debt}$ from Proposition 1.

■

A.6 Proof of Proposition 6

(i) Recall the indirect utility function of the old given by (8). Because $\tau_0^k\big|_{tax} = \tau_0^k\big|_{debt}$ holds as demonstrated in Proposition 5, we immediately obtain $V_0^o|_{tax} = V_0^o|_{debt}$.

To consider the effect on the utility of generation 0, V_0^M, recall the political objective function in period 0:

$$\Omega_0 = \frac{\omega}{(1+n)(1-\omega)} V_0^o + V_0^M.$$

When $\gamma < \alpha/(1-\alpha)$ holds, Ω_0 is maximized by choosing $\hat{b}' > 0$ (Proposition 1). However, the government choice is constrained when it is forced to finance its expenditure solely by taxes. That is, the government attains a lower value of its objective under tax financing than under debt financing: $\Omega_0|_{tax} < \Omega_0|_{debt}$. This implies $V_0^M\big|_{tax} < V_0^M\big|_{debt}$ since $V_0^o|_{tax} = V_0^o|_{debt}$ holds.

(ii) Recall the indirect utility function of the middle-aged in (7). Suppose that from some period $t_0(\geq 1)$ onward, the economy is in a steady state regardless of the government's financing method. The indirect utility function in (7) becomes

$$\begin{aligned} V_t^M\big|_j &= (1+\beta)\ln(1-\alpha)A\left(k|_j\right)^{\alpha}\left(h_t|_j\right)\left(1-\tau|_j\right)+\beta\left[(\alpha-1)+\gamma\alpha\right]\ln k|_j \\ &\quad +\beta\ln\left(1-\tau_{t+1}^k|_j\right)+\beta\gamma\eta\ln x_t|_j+\phi^M\left(h_t\right)|_j, \\ &\simeq (1+\alpha(\beta+\gamma))\ln k|_j+(1+\beta)\ln\left(1-\tau|_j\right)+\beta\ln\left(1-\tau_{t+1}^k|_j\right) \\ &\quad +\beta\gamma\eta\ln x_t|_j+\left[(1+\beta)+\beta\gamma(1-\eta)\right]\ln h_t|_j,\ j=tax,debt \end{aligned} \tag{59}$$

where the constant terms are omitted from the expression.

With the use of the policy functions in Proposition 1, (59) is rewritten as follows:

$$\begin{aligned} V_t^M\big|_j &\simeq (1+\alpha(\beta+\gamma))\ln k|_j+(1+\beta)\ln\left(1-\tau|_j\right)+\beta\ln\frac{1}{1+\left(\hat{b}_{t+1}/k_{t+1}h_{t+1}\right)\big|_j} \\ &\quad +\beta\gamma\eta\ln A\left(k|_j\right)^{\alpha}\left(h_t|_j\right)+\left[(1+\beta)+\beta\gamma(1-\eta)\right]\ln h_t|_j, \end{aligned}$$

or

$$\begin{aligned} V_t^M\big|_j &\simeq \left[1+\alpha(\beta+\gamma+\beta\gamma\eta)\right]\ln k|_j+(1+\beta)\ln\left(1-\tau|_j\right) \\ &\quad +\left[1+\beta(1+\gamma)\right]\ln\left(h'/h|_j\right)^{t-t_0}h_{t_0}|_j,\quad j=tax,debt. \end{aligned}$$

The direct comparison of $V_1^M\big|_{tax}$ and $V_1^M\big|_{debt}$ leads to

$$\begin{aligned} V_1^M\big|_{tax} \gtrless V_1^M\big|_{debt} &\Leftrightarrow (t-t_0)\left[1+\beta(1+\gamma)\right]\ln\frac{h'/h|_{tax}}{h'/h|_{debt}} \\ &\gtrless \left[1+\alpha(\beta+\gamma+\beta\gamma\eta)\right]\ln\frac{k|_{debt}}{k|_{tax}}+(1+\beta)\ln\frac{1-\tau|_{debt}}{1-\tau|_{tax}} \\ &\quad +\left[1+\beta(1+\gamma)\right]\ln\frac{h_{t_0}|_{debt}}{h_{t_0}|_{tax}}, \end{aligned} \tag{60}$$

where the left-hand and right-hand sides of (60) are denoted by LHS and RHS, respectively. These satisfy the following properties:

$$\partial LHS/\partial t>0,\ LHS|_{t=t_0}=0,\ \lim_{t\to\infty}LHS=\infty,\ \text{and}\ \partial RHS/\partial t=0.$$

Therefore, there is a positive integer, denoted by $\hat{t}$, such that $LHS \gtrless RHS \Leftrightarrow V_t^M\big|_{tax} \gtrless V_t^M\big|_{debt}$ for $t \gtrless \hat{t}$.

■

A.7 Derivation of (30)

With the use of (26) and (28), we can rewrite $\hat{b}'/k'h'$ as

$$\begin{aligned}\frac{\hat{b}'}{k'h'} &= \frac{\frac{1}{1+n}\varepsilon B_{un} A (k)^{\alpha} h}{\frac{1}{1+n}\frac{\beta}{1+\beta}\left[(1-\tau)(1-\alpha) - \frac{1+\beta}{\beta}\varepsilon B_{un}\right] A (k)^{\alpha} h} \\ &= \frac{\varepsilon B_{un}}{\frac{\beta}{1+\beta}\left[(1-\tau)(1-\alpha) - \frac{1+\beta}{\beta}\varepsilon B_{un}\right]}.\end{aligned}$$

This leads to

$$1 + \frac{\hat{b}'}{k'h'} = \frac{(1-\tau)(1-\alpha)}{(1-\tau)(1-\alpha) - \frac{1+\beta}{\beta}\varepsilon B_{un}}. \tag{61}$$

From (61), the conjecture of $\tau^{k\prime} = 1 - T^k_{con}\frac{1}{1+\hat{b}'/k'h'}$ is rewritten as

$$\tau^{k\prime} = 1 - T^k_{con}\frac{(1-\tau)(1-\alpha) - \frac{1+\beta}{\beta}\varepsilon B_{un}}{(1-\tau)(1-\alpha)}. \tag{62}$$

The substitution of the government budget constraint in (27) into (62) leads to (30).

■

A.8 Proof of Proposition 7

The first-order conditions in (34) and (35) are summarized as

$$1 - \tau^k = \frac{\omega}{(1+n)(1-\omega)}\frac{(1+n)\tilde{x}}{\beta\eta(1-\alpha)(1+\gamma)}\frac{1}{\alpha\left(1+\hat{b}/kh\right)}. \tag{63}$$

With the use of (63), $\tilde{Z}^1$ and $\tilde{Z}^2$ in (32) and (33) are rewritten as follows:

$$\begin{cases} \tilde{Z}^1 = (1+\varepsilon B_{un}) - \left[1 + \frac{\omega}{(1+n)(1-\omega)}\frac{1}{\beta\eta(1-\alpha)(1+\gamma)}\right](1+n)\tilde{x}, \\ \tilde{Z}^2 = (1+\varepsilon B_{un}) - \left[1 + \frac{\omega}{(1+n)(1-\omega)}\frac{1}{\beta\eta(1-\alpha)(1+\gamma)}\right](1+n)\tilde{x} - \frac{1+\beta}{\beta}\varepsilon B_{un}. \end{cases} \tag{64}$$

The substitution of (64) into the first-order condition with respect to $\tilde{x}$ in (35) leads to

$$\frac{\beta\eta(1-\alpha)(1+\gamma)}{(1+n)\tilde{x}} = \frac{1}{\tilde{Z}^1} + \frac{\alpha\beta(1+\gamma)}{\tilde{Z}^1 - \frac{1+\beta}{\beta}\varepsilon B_{un}}. \tag{65}$$

Figure A.1 illustrates the graph of (65), taking $(1+n)\tilde{x}$ on the horizontal axis. The figure indicates that there are two candidates for a solution to (65). However, the larger one is not feasible since $\tilde{Z}^1 < 0$ holds. Therefore, the smaller one is the solution to (65).

[Figure A.1 here.]

To derive the solution to (65), we reformulate (65) as follows:

$$\frac{\beta\eta(1-\alpha)(1+\gamma)}{(1+n)\tilde{x}} = \frac{\tilde{Z}^1 - \frac{1+\beta}{\beta}\varepsilon B_{un} + \alpha\beta(1+\gamma)\tilde{Z}^1}{\tilde{Z}^1\left(\tilde{Z}^1 - \frac{1+\beta}{\beta}\varepsilon B_{un}\right)},$$

or

$$f((1+n)\tilde{x}) \equiv G((1+n)\tilde{x})^2 - H(1+n)\tilde{x} + I = 0, \tag{66}$$

where G, H, and I are defined in Proposition 7. Note that $H > 0$ and $I > 0$ hold because $\varepsilon B_{un} < \beta$ holds.

By solving (66) for $(1+n)\tilde{x}$ and taking the smaller solution, we obtain

$$(1+n)\tilde{x} = X_{con} \equiv \frac{H - \sqrt{(H)^2 - 4GI}}{2G}. \tag{67}$$

The substitution of (67) into (63) yields

$$1 - \tau^k = \frac{\omega}{(1+n)(1-\omega)} \frac{\frac{H-\sqrt{(H)^2-4GI}}{2G}}{\beta\eta(1-\alpha)(1+\gamma)} \frac{1}{\alpha\left(1+\hat{b}/kh\right)}, \tag{68}$$

which verifies the initial guess. Finally, the labor tax rate is derived by substituting (67) and (68) into the government budget constraint in (27).

■

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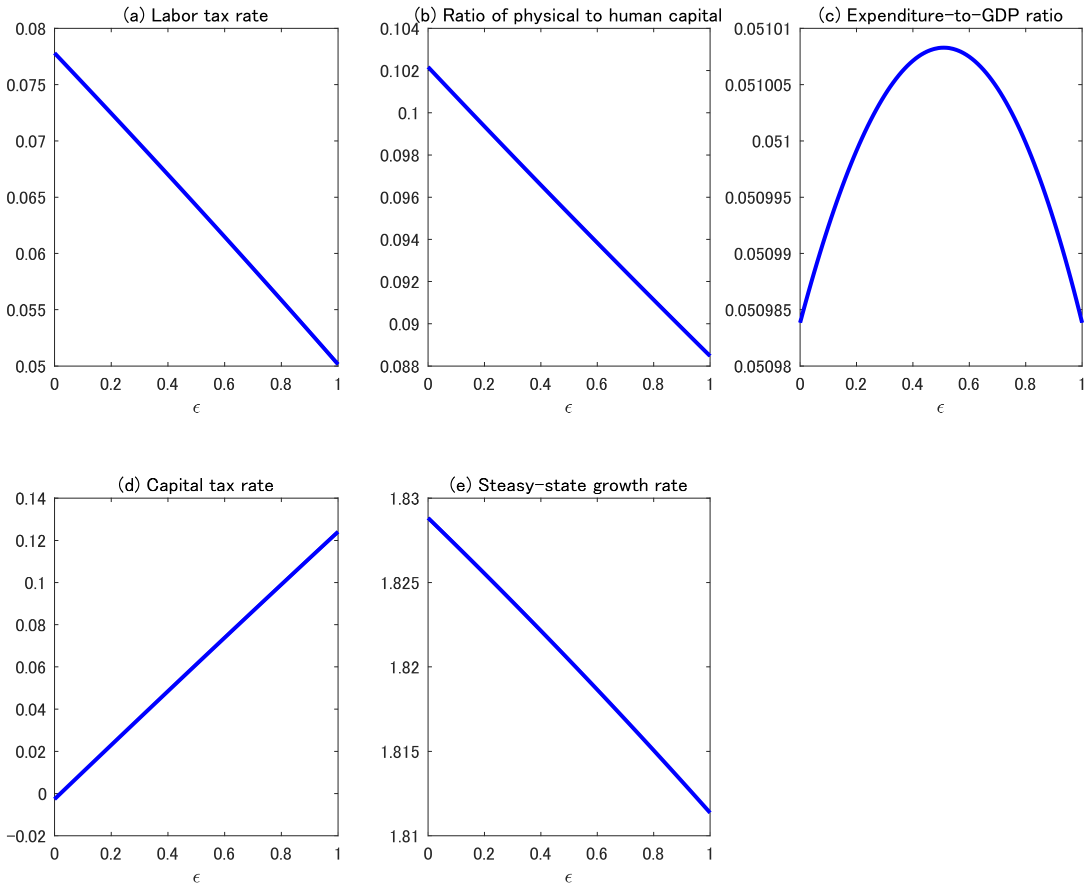

Figure 1: Effects of decreased ε on the labor tax rate (Panel (a)), ratio of physical to human capital (Panel (b)), education expenditure-to-GDP ratio (Panel (c)), capital tax rate (Panel (d)), and steady-state growth rate (Panel (e)).

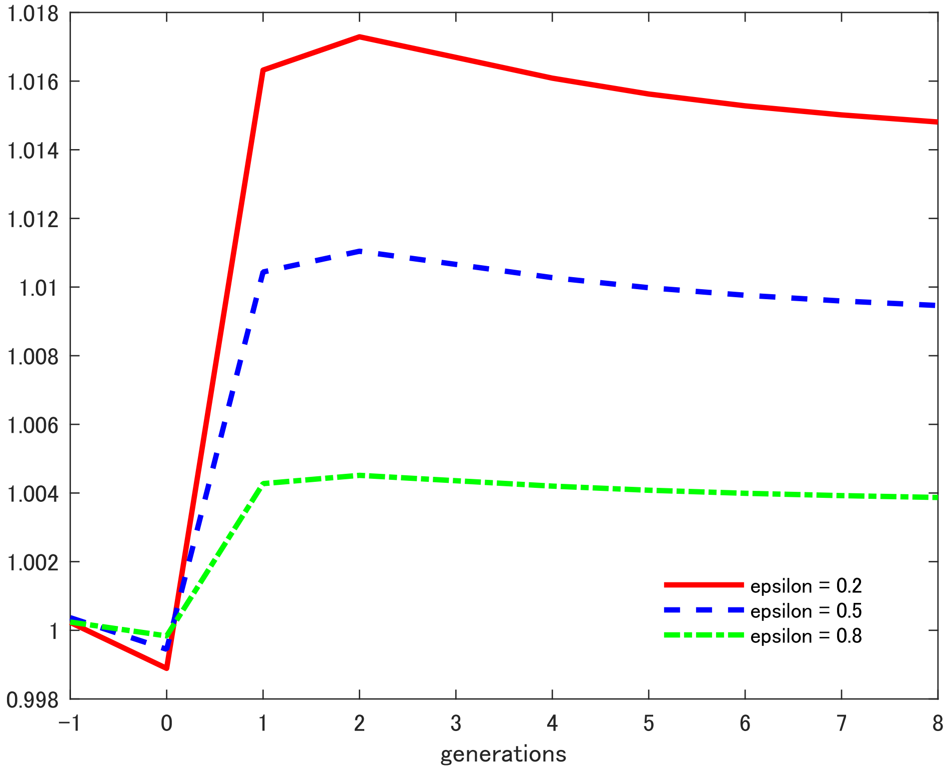

Figure 2: Utility of generations from the initial old population (denoted by -1) to generation 9.

	$\epsilon = 0.2$	$\epsilon = 0.5$	$\epsilon = 0.8$	$\epsilon = 1.0$
Initial old population	1.3029	1.3031	1.3029	1.3026
Generation 0	1.2346	1.2352	1.2357	1.2359
Generation 1	2.3336	2.3201	2.3059	2.2961
Generation 2	3.4144	3.3934	3.3715	3.3563

Table 1: Utility of the first four generations

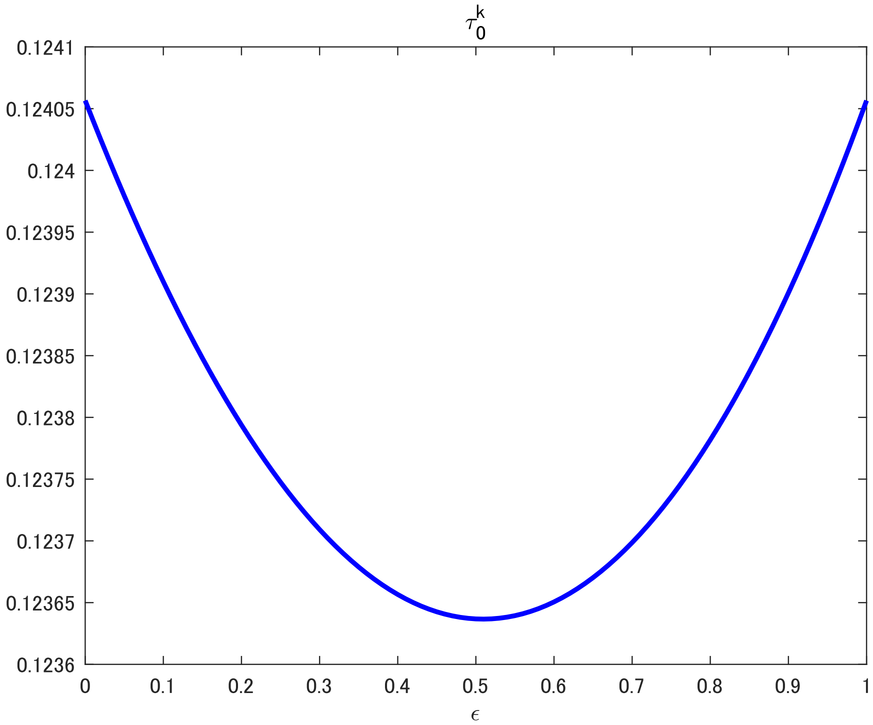

Figure 3: Period 0 capital tax rate.

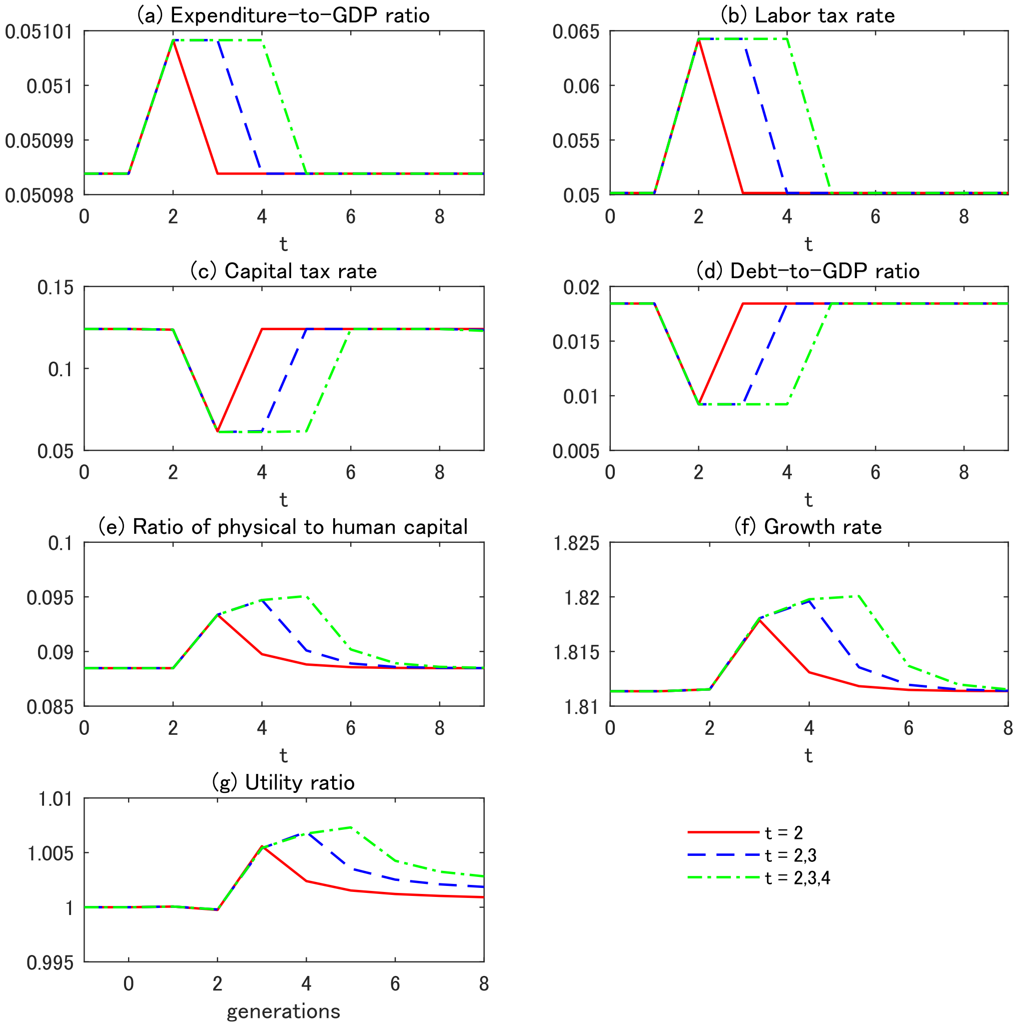

Figure 4: Effects of the temporary implementation of the fiscal rule in period 2. The solid, dashed, and chain lines depict the cases where the rule is terminated at the end of periods 2, 3, and 4, respectively. Note: Panel (g) plots the ratio of utility in the presence of the temporary implementation to that in the absence of the implementation.

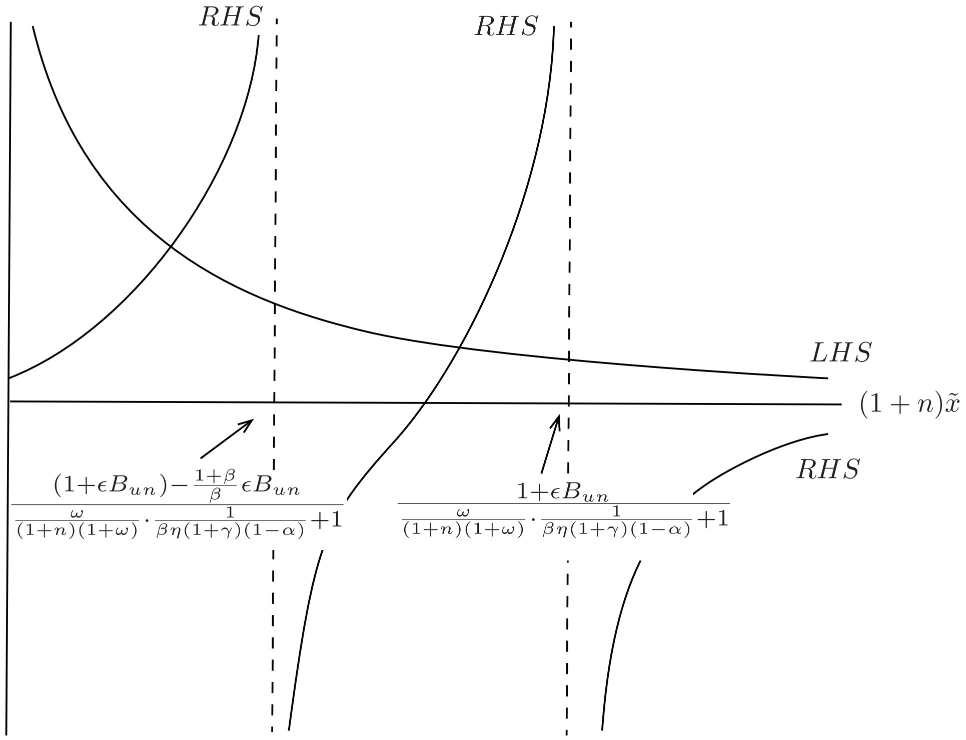

Figure A.1: Illustration of the left-hand side and right-hand side of equation (65).