Discussion Papers In Economics And Business

Credit and Bankruptcy in a Temporary Equilibrium Model

Weiye Cheny

Discussion Paper 18-23

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Credit and Bankruptcy in a Temporary Equilibrium Model

Weiye Cheny

Discussion Paper 18-23

July 2018

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Credit and Bankruptcy in a Temporary Equilibrium Model*

Weiye Chen†

Abstract

A temporary general equilibrium in bankruptcy model with finite periods was analyzed in this paper, which (i) every consumer only issues one type of bond to financial market in each period; (ii) the bank has right to circulate currency, and never face bankruptcy. The model was an extesion of the Bankruptcy model in Eichberger(1989), based on the assumptions that the occurrence of moral hazard is prevented by the credit scheme law, which depends on the current information and forecast function. The main result of this paper enables as to develop the liquidation rule without penalties. This rule can also be used to interpret liquidating distribution in Bankruptcy Act. In addition, the bankruptcy mechanism plays an effective role even if the chain-reaction bankruptcy occurred. Moreover, we can prove that the economy will never collapse in an overlapping model which has some newborn in every period.

JEL classicication:D52; D53; D59
Keywords: Temporary equilibrium; Bankruptcy; Credit scheme; Liquidation rule; Money

*I am deeply indebted to Ken Urai for the careful guidance and construction criticism. I am gratefull that received many useful commands from Shingo Ishiguro and Yousuke Yasuda.

†The Graduate School of Economics, Osaka Unicersity, 1-7, Machikaneyama, Toyonaka, Osaka 560-0043, Japan. E-mail : chenweiye198966@yahoo.co.jp

1 Introduction

It is a well-known fact that the bankruptcy of state-owned companies is one of the serious problems of China during the 1990s-2000s. The impact on society of such bankruptcy is that some imperfect liquidation rules which led to financial market chaos such as non-performing loans. The most difficult thing is to explain the accommodation of government or private funds, if the liquidating distribution should be provided as per the Bankruptcy Act which is legislated by the government of China or Japan. All of those liquidation rules are provided likewise for bankruptcy law, and the bankrupt's property that should be distributed must obey the provision as "A bankruptcy trustee, shall evaluate the value of any and all property that belongs to the bankruptcy estate as of the time of commencement of bankruptcy proceedings"[1]. Moreover, the provision, "a bankruptcy creditor, with regard to a banruptcy claim that is determined. may enforce compulsory execution against the bankrupt based on the entries in the schedule of bankruptcy creditors"[2], shows that the liquidation rule is not chosen by any agent. In this paper, I will propose an equilibrium model with some conditions to achieve the provision of the Bankruptcy Act.

In general equilibrium model with incomplete markets, there has many frameworks that introduces the general equilibrium theory of bankruptcy. In these frameworks, Modica, Rustichini and Tallo[1998] proves that general equilibrium does not exist in the bankruptcy model with Arrow-Debreu security if the liquidation rule is denoted as the provision which provides for the Bankruptcy Act. However, Grandmond and Laroque[1975] or Eichberger[1989] addressed the accommodation of funds between government by using the temporary general equilibrium model. And there also leaves a space to discuss the liquidating distribution when consumers face bankruptcy. I build a model that extends the financial market which is introduces by Eichberger[1989], and discuss the liquidation rule for monetary policy. In this paper, I denote the financial market with private bonds, and permit the accommodation of private funds. This argument demonstrates that the chain reaction bankruptcy has occurred. This financial market is not easy to consider as Arrow-Debreu security if the agent is denoted as the price-taker. Since the financial net wealth may lead the budget set to be non-convexity, as the example introduced by

[1]The Article 153(1), section 1 Investigation of the Status of the Bankrupt's Property, chapter VI Administration of the Bankruptcy Estate, Bankruptcy Act of Japan, Act No. 75 of June 2, 2004

[2]The Ariticle 221(2), chapter IX Close of Bankruptcy Proceedings, Bankruptcy Act of Japan, Act No. 75 of June 2, 2004

Rothschild and Stiglitz[1976]. In my model, I propose a financial market different from Eichberger[1989], and extend the credit scheme to lead the budget set to be convex. The conclusion of this paper is different form Eichberger[1989] that there exists a temporary equilibrium even if the chain reaction has occurred. Moreover, I demonstrate that temporary equilibriums is also existed in overlapping model with some necessary conditions.

In this model ,there have two points different from the frameworks such as the general equilibrium model with bankruptcy. First, I considering the bankruptcy clearing mechanism without penalties. The default rate of assets are chosen by the consumer themselves under the direct utility penalty which is addressed by Dubey, Geanakoplos and Shubik[2000, 2005].Furthermore, in some literatures, there is no penalty, but have to taken some security deposits on shork position to make sure every consumer has positive net wealth for the next period as introduced by Araujo and Páscoa[2002]. The fact is, by the provision in the Bankruptcy Act, the liquidation rule is legally provided and has enforcement potency. Furthermore, the direct utility penalty is also to lead to the welfare lost more than the general part. The second point is, I talk about the accommodation of funds and the way to valuate the credit of every consumer, which is never discussed by the general equilibrium model of bankruptcy. This argument is useful for making the monitory policy and extending the general equilibrium model into the dynamic model.

In section 2, the standard model was created in finite periods, which contains the consumers and the bank(e.g. central bank or government). In a temporary sense, all agents should make consumption decisions on period 1 under the price given. Also this decision is under the expectation that depends on the signal s_1, and the incpmplete expectation causes to the bankruptcy occurring. In section 3, there will discuss the structure of financial market by example. Under this structure, the credit scheme law will be introduced and the bankruptcy rule should be defined. In section 4, the main theorem will be proved under necessary condintions, which shows there exists the temporary equilibrium even if the chain reaction bankruptcy has occurred. In section 5, there will introduce the extension about overlapping model, and show that the economy exists a temporary equilibrium in every period under some necessary conditions.

2 The Economy

Considering an economy with a finite set $L = \{1, \ldots, L\}$ for non-storable commodities, bonds and money can be traded during a sequential period $T = \{1, \ldots, T\}$. There are two types of agents in the economy, one is named as consumer with finite set $I = \{1, \ldots, I\}$, and the other is a bank(e.g. central bank or government which has abiliry to suply the money) with singleton $B = \{b\}$. The commodity l will be traded only for consumers with price p_t^l in the period $t \in T$. There is no necessary to consider any commodity price will being 0 since I forgive the free disposal. So that the price vector assumes as $P_t \in R_+^L$. The bond a_t^i, in the proper sense, means the individual $i \in I$'s credit that is calculated from others. Note the agent i maybe has different credit from different agent. The agent i can sell one unit bond a_t^i to others at the price q_t^i in the period t and require one unite money repayment in the next period. Denote the bond supply as $a_t^{i^-} \in R_-$ and the bond demand $a_t^{i^+} \in R_+$. To simplify, I assume that the money price always equal to 1 in every period. So the interest rate of the bond a_t^i equals to $q_t^{i^{-1}} - 1$. In general, the price of the bond should be non-negative, hence $Q_t \in [0, 1]_+^I$ in each period t.

Each agent will make consumption decisions by focusing on the price of commodities and bonds. So that both of them should have some future price expectations that depend on their own knowledge and the current information, even if they do not know the real future price that comes out under uncertainty economy. Also, they would make an individual consumption plan after receiving a signal $s_1^i \in S_1 \equiv (P_1, Q_1, 1)$ of the price system in the current period. Furthermore, all agents must receive the same signal about the current period, since the history is centainty. Therefore, the forecast function of agent i is continuity and denoted as:

$$\psi^i : s_1 \in S_1 \mapsto (s_2^i, \ldots, s_T) \in \times_{t=2}^T S_t. \tag{1}$$

Note that the future price maybe different on the basis of individually forecast. Also it is not necessary to consider that all the entire agent has a same forecast. The consumer i has initial endowment $e_t^i \in R^L$ in each period $t \in T^i \subset T$ when he is alive. The consumer i will make the consumption decision $a^i = (x^i, A^i, m^i)$ during the individual life under utility function $U^i : (x_1^i, \ldots, x_{T^i}^i) \in R^{L \times T^i} \mapsto R$ which is represented his perference. In the economy, every consumer should has non-zero endowment $e_t^i \neq 0$ during his life, and hence $\sum_{i\in I} e_t^i \gg 0$.

In this model, the bond means the credit of consumers, and the trade of bond is more likely promising. However, it also can be considered as nominal assets, which can be classified into two types of the financial sense. The bond supplies a_t^{i-} from the agent i can be considered as short-position or assets, and the bond demand a_t^{i+} is similar to long-position or liabilities. In fact, any agent cannot use the credit of others to make a promise, hence the difference between the normal asset and the bond in this model is that every agent must supply only one type bond in each period. Using this argument, the consumer i's budget constraint on the period t should be written as:

$$P_t \cdot x_t^i + Q_t \cdot A_t^i + m_t^i \leq P_t \cdot e_t^i + \mathbf{1} \cdot A_{t-1}^i + m_{t-1} \tag{2}$$

$$x_t^i \geq 0, \qquad m_t^i \geq 0$$

The equation (2) shows that the right-hand is consumer's net worth at beginning of the period t and the left-hand is consumption ,financial obligation and money holding at the end of the period t. Moreover, $q_t^i(a_t^{i-} + a_t^{i+}) = q_t^i a_r^{i*}$ said that every consumer will either demand or supply the same bond.

Next, the bank shoul be introduced as spacial agent without initial endowment, consumption(e.g. $e^b = 0$ and $x^b = 0$) and commodity preference during every period. However, the privilege of that produce the currency allows the bank that has ability to trade bonds and money as consumer and claim for the unlimited credit from other agents. Then the bank's action is assumed as $a^b \equiv (0, A^b, M^b)$. In general ,the short-position of bank can be thought of as saving, and the long-position would be loan. Those argument shows that the bank's budget constraint should composed with bonds and money:

$$M_t + Q_t \cdot A_t^b = M_{t-1} + 1 \cdot A_{t-1}^b, \tag{3}$$

$$M_t^b \geq 0.$$

Note the new currency supply $M_t - M_{t-1}$ not necessary to be non-negative, since the money demand of economy may be zero in period t in some situation. In this model, the money just means the social credit using, neither paper money nor commodity money which has the real value. Hence the negative currency supply does not mean the money disposal.

3 Credit Scheme

In this section, I will introduce the rule of credit scheme ban bankruptcy. Before to give the definition, there has a few examples to explain some serious problem such that a general equilibrium cannot exist in some bankruptcy model. Considering the two-period model as $T = \{1, 2\}$, consumer 1(resp. 2) has innitial endowment $e^i \equiv (e_1^1, e_2^1)$(resp. $e^2 \equiv (e_1^2, e_2^2)$). The initial asset of two consumer is assumed as $(a_0^{1,2^-}, 0)$ and $(0, a_0^{1,2^+})$. Simply, the initial asset market is clear, that is $a_0^{1,2^-} + a_0^{1,2^+} = 0$, and money holding is zero. The bank is denoted as what was explained in section 2 in this paper. For simplicity, the credit of consumers in two periods has lower-bound $(K, 0) \in R_-^2$. Without loss of generality, each consumer could hold short-position and long-position simultaneously, as $A_t^i \neq 0$. Suppose that every consumer can sell two different types of bond $A_1^{i^-} \in R_-^2$, one type of bond, a_1^{i,j^-}, is demanded for other consumers, and the other one a_1^{i,b^-} can only be sold to the bank. Assuming all the agents are price-takers, and the price of period 1 given as $(P_1, Q_1, \mathbf{1})$, where $q_1^{1,2} \neq q_1^{1,b}$. In uncertainty economy, consumer 1 may face bankruptcy when he cannot pay back all of his debt, as $P_1 \cdot e_1^1 - Q_1 \cdot A_1^{1^{-*}} + a_0^{1^-} < 0$, where $a_1^{1,2^{-*}} + a_1^{1,b^{-*}} = K$. Then the budget set of consumer 1 maight be non-convexity under some conditions.

Consider the budget constraint of consumer 1 is:

$$\begin{cases} P_1 \cdot x_1^1 + Q_1 \cdot A_1^1 + m_1^1 = P_1 \cdot e_1^1 + a_0^{1,2^-} & \text{if consumer 1 is not facing to bankruptcy;} \\ P_1 \cdot x_1^1 = \max\{P_1 \cdot e_1^1 + a_0^{1,2^-} - Q_1 \cdot A_1^{1^{-*}}, 0\} & \text{otherwise} \end{cases}$$

in period 1; and in period 2 is:

$$\begin{cases} P_2 \cdot x_2^1 = \max\{P_2 \cdot e_2^1 + \mathbf{1} \cdot A_1^1, 0\} & \text{if consumer 1 is not face to bankruptcy in period 1;} \\ P_2 \cdot x_2^1 = \max\{P_2 \cdot e_2^1 + K, 0\} & \text{otherwise.} \end{cases}$$

Similary, the budget constraint of consumer 2 must be considered in the same logic as consumer 1. Finally, the budget constraint of bank is assumed as equation (3) in section 2.

Now, the first problem accrues when consumer 1 faces bankruptcy in the period 1 with some components of short-position subjecting to the condition $a_1^{1,2-}+a_1^{1,b-}=K$. The most important argument is that the consumer 1's action of bond must depend on the other's choice. Considering the case of $P_1\cdot e_1^1+\mathbf{1}\cdot A_0^1-q_1^{1,b}K<0$ and $P_1\cdot e_1^1+\mathbf{1}\cdot A_0^1-q_1^{1,2}K>0$ where $q_1^{1,2}\gg q_1^{1,b}$. Then, there exists a component of bond, $(a_1^{1,2-^*},0)$, that satisfies $P_1\cdot e_1^1+\mathbf{1}\cdot A_0^1-q_1^{1,2}a_1^{1,2-^*}=0$, and existes an other component of bond, $(a_1^{1,2-^{'}},a_1^{1,b-^{'}})$, which implies $P_1\cdot e_1^1+\mathbf{1}\cdot A_0^1-q_1^{1,2}a_1^{1,2-^{'}}-q_1^{1,b}a_1^{1,b-^{'}}=0$. So, consumer 1 must declare bankruptcy in the period 1 when the consumer 2 chooses the action as $A_1^{2+}\equiv\{(a_1^{1,2*},0,0,0,0|a_1^{1,2*}<-a_1^{1,2-^{'}}\}$. From the definition of the consumer's action of bankruptcy, the feasible action set of consumer 1 in the period 1 can be draw as figure (1). The figure (1) shows that the budget set can be separated out into two parts as bankruptcy and nornal. The normal part, as the triangular pyramid fulling with slant line, is convexity and continuity. The bankruptcy part, the line of $(0,0,-K)$ and $(0,a_1^{1,2-^{'}},a_1^{1,b-^{'}})$, also is convexity and continuity. However, the budget set is no longer a convex set since the component of two point that is picked up from the different part may not be feasible.

The second problem is the non-convexity of comsumption set in all periods. This problem is easy to regard as moral hazard. The independence of consumer's forecast function allows the consumer to take more credit in the current period when he has ability to pay back all the next period under the debtor's expectation. Consider the consumer 1 will face bankruptcy only in period 2. Then the creditor can choose some credit as $a_1^{1,2-}\in(a_1^{1,2-^{''}},K]$ while the consumption becomes to $x_2^1=0$ and $x_1^1\geq 0$, where $a_1^{1,2-^{''}}\equiv\{a_1^{1,2-^{''}}\in[K,0]|P_2\cdot e_2^1+m_1^1+a_1^{1,2-^{''}}=0\}$. In figure (2), the point on the line of $(\frac{P_1\cdot e_1^1+a_0^{1,2-}}{P_1},0)$ and $(\frac{P_1\cdot e_1^1+a_0^{1,2-}-q_1^{1,2}K}{P_1},0)$ is a feasible allocation, but the component of any allocations may not be feasible. Hence the budget set becomes non-convexity.

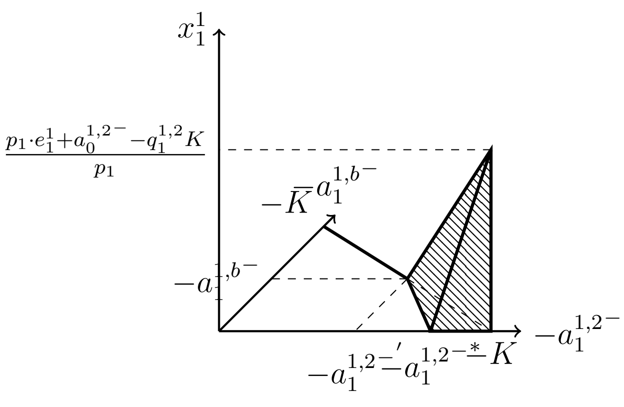

Figure (1)

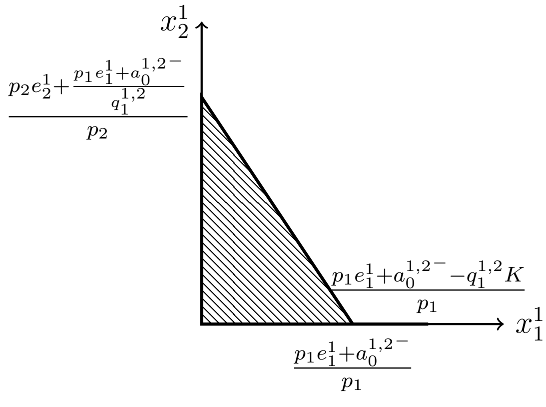

Figure (2)

The figure (2) was already discussed by Eichberger[1989], and shows that moral hazard arises in incomplete market. The figure (1) shows that the moral hazard also occurs for a given price when the type of bound supplied from any consumer of bond is more than one, and shows that the liquidation rule is not clear in some situations. The most important argument is that every consumer may become bankrupt on the basis of the opponents' choice. Hence the budget set is non-covexity, and the economy cannot exist equilibrium.

Hereafter, some methods used in the bankrutcy models will be explained. It is easy to show that the economy always exist general equilibrium if the unlimited credit condition $K \to \infty$, called Ponzi-strategy, holds. The unlimited credit condition implies that every consumer has the ability to repays all debts, and hence the problem in the figure (1) will never occur. A more complex problem is to consider the economy with limited credit consumer. In the general equilibrium bankruptcy model without penalty, Araujo and Páscoa[2002] shows that there exists a general equilibrium in the economy without creditor, that is, the credit limited K is non-negative. Modica, Rustichini and Tallo[1998]n addressed that an equilibrium also be existed when the epectation of state of nature is same for all the consumers. On the other hand , the well-known method is to consider the bankruptcy with direct utility penalty like Zame[1993] and Dubey, Geanakoplos and Shubik[2005]. The repayment could be chosen less than the promised if the consumer accepts the direct penalty, and this argument guarantees that the initial wealth of any consumer will never be negative. However, this method implies the debtor always be the weakly position in the financial market and the direct utility penalty proves a more welfare loss.

In this paper, I will introduce a method about credit scheme to cover the problem of moral hazard. The problem is how to measure the credit of agent i which means the aviable bond supply set. It is easy to consider that each agent has different credit limited in different situation. Moreover, every agent has independent expectation of future from his own cognition that leads the calculation method of the credit limited is not nesserary to be same. The fact is that no one could be able to calculate the real credit of any agent from such little information about future. But the forecast function makes sense in which the agent has an ability to examine the credit expectation depending on a signal s_1 observing in particular. The interesting thing is that no one prefers to take a debt when a less repayment recieving is realized. This argument shows that the agent i's credit constraint not only depends on the signal of the first period but also be influenced by the net wealth of each period. Thus, it is hard to calculate the net wealth of each agent in the future since it contains bonds and money holding. On the other hand, every agent has ability to repay his debt if the repayment amount is less than the endowment in the next period. Moreover, an agent also never faces bankruptcy if the market can ensure his credit in future that can cover the current debt. Under these arguments, the credit limited of the agent in the current period must be less than the summation of initial endowment wealth and his credit wealth in the next period. Following, the definition of consumer i's credit constraint is assumed as:

Definition 3.1. *The credit scheme of the consumer $i \in I$ assumes as:*

(1) the credit constraint is a continuous function $\rho^i : s_1 \in S_1 \mapsto \rho^i(s_1, \ldots, s_T) \in R^T_-$;

(2) the credit constraint is a continuous function from the agent j under the forecasting price system $\phi^j(s_1)$ and consistent Q_t in period t:

$$\rho\_t^{i,j}(s_1) + \phi^j_{P,t+1}(s_1) \cdot e^i_t - \phi^j_{Q,t+1}(s_1) \cdot \rho^{i,j}_{t+1} \geq 0.$$

Note every consumer must supply one type of bond to the financial market. In addtion, the continuity of price forecast function $\phi^i(s_1)$ and the signal set S_1 imply that the credit constraint $\rho^{i,j}_t(s_1)$ is also continuity. The second part of definition 3.1 shows that the credit limited of the agent i maybe different from the expected between himself and the others.

The definition of credit constraint also leaves a position of moral hazard, explained in the example, even though it guarantees that consumer has right to make promise in the current period. The question is, how the bank guarantee the credit limit of consumer. The privilege of issuing the currency limit requires that the bank should make higher accuracy of price expectation than others in each period. Hence given the consumers price expectation and signal $s_1 \in S_1$, the price expectation of the bank must cover the entire situation which is forecasted by every consumers. The bank credit constraint is defined as following:

Definition 3.2. *The credit scheme of the bank $b \in B$ is assumed as:*

(1) the price expectation of bank satisfies: $\psi^b_{P,t}(s_1) \leq \phi^i_{P,t}(s_1)$ for all $i \in I$;

(2) the credit constraint of bank defines as a continuous function $\rho^b : s_1 \in S_1 \mapsto \rho^b(s_1, \ldots s_T) \in R^T_-$*, furthermore,* $\rho^{b,i}_t(s_1) = -\infty$ *for all* $t \in T$*;*

(3) the credit constraint of consumer $i \in I$ *from the bank* b *with consistent* Q_t *in period* t *is assumed as:*

$$\rho^{i,b}_t(s_1) + \psi^b_{P,t+1}(s_1) \cdot e^i_{t+1} - \psi^b_{Q,t+1}(s_1) \cdot \rho^{i,b}_{t+1} \geq 0.$$

From the same argument with Definition 3.1, the credit constraint $\rho^{i,b}_t(s_1)$ of consumer i is also continuity. With these following definitions of credit constraint, the next lemma can be proved:

Proposition 3.1. $0 \geq \rho^{i,b}_t(s_1) \geq \rho^{i,j}_t$ *for all* $i, j \in I$ *and* $t \in T$ *implies*

$$\rho^{i,b}_t(s_1) + \psi^i_{P,t+1}(s_1) \cdot e^i_{t+1} \geq \rho^{i,j}_t(s_1) + \psi^i_{P,t+1}(s_1) \cdot e^i_{t+1}.$$

The condition in Proposition 3.1 always matches with the consistent price $\psi^i_{Q,t+1}(s_1)$. The bank takes the lesser value of expectation price from all of these consumers to make sure the moral hazard will not occure in all the price system which is expected by every consumer, even though some of those system is unexpected by some consumers. Note the incomplete prediction leads the bank hardly to forecast the real future price.

In an uncertainty economy, an unexpected price system may occurs, which means that some agents take a mistake forecasting function. So, the asymmetric information should be considered in the economy as adverse selection or moral hazard. The forecasting function of an agent i is identical with others since it is characterized by individual knowledge and signal. The agent i cannot demand assets more than the credit that expected over himself. Furthermore, an agent cannot pay back his debts in the next period if the future credit limit is over-valuation on the real. Following this argument, an agent may face bankruptcy in some periods. In this model, the bank covers some situation of unepected price but cannot cover all of the state of nature. This argument leads us to consider the problem of bankruptcy under some price system. In some economy languages, an agent has some credit from others and he may not face bankruptcy even if his net wealth is negative in the current period. The Definition 3.1 and 3.2 guarantee that the consumer i has different credit constraint on different agent. However, Proposition 3.1 shows that moral hazard will not arise if the credit constraint on consumer is calculated by the bank. So, the following law of credit scheme should be hold.

Assumption 1. *The law of credit scheme of consumer i satisfies* $\rho_t^{i,b}(s_1) = \rho_t^{i,b}(s_1) \in R_-$.

Following the Assumption 1, the bankruptcy law of agent i in period t becomes to:

$$-q_t^i \rho_t^i(s_1) + P_t \cdot e_t^i + \mathbf{1} \cdot A_{t-1}^i + m_{t-1}^i < 0. \tag{4}$$

From the definition of bond, the consumer i has no behaver to demand a long-position when he faces bankruptcy in the current period. Let the credit scheme be a vector $\rho_t^i \equiv (0, \ldots, \rho_t^{i^-}, \ldots, 0) \in R^I$ such as $\rho_t^{i^-} = \rho_t^i(s_1)$, the equation (4) can be rewritten as:

$$-Q_t^i \cdot \rho_t^i + P_t \cdot e_t^i + \mathbf{1} \cdot A_{t-1}^i + m_{t-1}^i < 0. \tag{5}$$

The equation (5) shows, the consumer who faces bankruptcy has non-positive wealth and never consume any commodity excepting the case of commodity price equivalent to 0. Furthermore, let consumption set of consumer i be $x_t^i \in R_+^L$, the bankrupt i can take an action as $\alpha_t^i(0, \rho_t^i, 0)$ in period t where $x_t^i = 0$ means the minimum consumption normalizing to zero.

Since the consumer has no such wealth to clear up the debts when he declares bankruptcy in some period, the return of bonds maybe less than the promising. Also, the liability loss is only shared with debtors who has the same creditor that declares bankruptcy. This paper assumes that every bankrupt must do his best to pay back his debts, so the aviable action of bankrupt equals to $\alpha_t^i(0, \rho_t^i, 0)$. This assumption shows that the debtor always have some non-negative return that is no greater than the promissing in the period t. Following, the real return function of debtor i in the period t is assumed as:

$$G_{t-1}^i = K_{t-1} \cdot A_{t-1}^{i^+}. \tag{6}$$

$K_{t-1} \equiv (\ldots, \kappa_{t-1}^i, \ldots)'$ is the discounted return rate matrix where

$$\kappa_{t-1}^i = \begin{cases} \frac{\min\{P_t \cdot e_t^i + m_{t-1}^i + G_{t-1}^i - Q_t \cdot \rho_t^i, \mathbf{1} \cdot A_{t-1}^{i^-}\}}{\mathbf{1} \cdot A_{t-1}^{i^-}} & \text{is } i \text{ is bankruptcy} \\ [0, 1] & \text{others.} \end{cases} \tag{7}$$

The definition of credit constraint shows that the positive future endowment will inevitably increasing the credit limit of the current period. Moreover, the discounted return rate must be positive, as $K_{t-1} \in [0,1]^{I+1}$, since the positive endowment and the non-negative credit. Notably, the discounted return rate only relatives to creditors.

4 Existence of Temporary Equilibrium

In this setion, I will explain that there exists temporary equilibrium in the bankruptcy model. The consumer i will choose the action $\alpha_t^i = (x_t^i, A_t^i, m_t^i)$ in every period $t \in T^i$ that maximize his utility function $u^i(x_1^i, \dots, x_{T^i}^i)$ subjects to the budget constraint $P_t \cdot x_t^i + Q_t \cdot A_t^i + m_t^i \leq \max\{P_t \cdot e_t + \mathbf{1} \cdot ({A_{t-1}^i}^- + G_{t-1}^i) + m_{t-1}^i, Q_t \cdot \rho_t^i\}$. It is easy to check that the future action has an impact on the current budget constraint. Also, it is easy to varify that the consumer i's current budget constraint relatives to an initial financial wealth (A_0^i, m_0^i) and a signal s_1. Actually, the incompeted prediction lead any consumer to choose an unaccurate action. Using the price prediction method and the credit scheme, agents can make a decision on the current period in the temporary sense. So, the budget set of consumer i is:

$$B^i = \left\{ (x^i, A^i, m^i \left| \begin{array}{l} Q_1 \cdot A_1^i + P_1 \cdot x_1^i + m_1^i \leq \max\{m_0^i + P_1 \cdot e_1^i + \mathbf{1} \cdot (G_0^i + {A_0^i}^-), Q_1 \cdot \rho_1^i\}, \\ \psi_{Q,t}^i(s_1) \cdot A_t^i + \psi_{P,t}^i(s_1) \cdot x_t^i + m_t^i \leq m_{t-1}^i + \psi_{P,t-1}^i(s_1) \cdot e_t^i + \mathbf{1} \cdot A_{t-1}^i \\ \text{for all } s_1 \in S_1 \text{ and } t \in T^i \setminus \{1\} \end{array} \right. \right\} \tag{8}$$

Note Proposition 3.1 and Assumption 1 show that the bankruptcy never arise in the part of the budget set when $t > 1$. To varify the aviable action set, I have following assumption:

Assumption 2. *The preference $\preceq$ of consumer i can be represented by utility function $u^i : R^{L \cdot T^i} \mapsto R$ which is continuity, monotonicity and strictly concavity.*

Let the set of optimal action be $\alpha^{i*}(\alpha_0^i.s_1) \equiv \{(x_t^{i*}, A_t^{i*}, m_t^{i*}) | t \in T^i\}$ in which α_t^{i*} is assumed as the projection of $\alpha^{i*}(\alpha_0^i.s_1)$, the following lemma can be proved:

Proposition 4.1. *The properties of consumer i's action must be hold as:*

(1) the demand correspondence of consumer i:

$$\xi^i = \prod_{t \in T^i} \arg\max\{u^i(x_1^i, \ldots, x_{T^i}^i) | \alpha_t^i \in \Pr_t \alpha^i(\alpha_0^i, s_1) \subset B^i\}$$

is nonempty, u.h.c, compact- and convex-value;

(2) the optimal choice of creditor satisfies:

$$\alpha_t^{i^*} \equiv \{(x_t^{i^*}, A_t^i, m_t^i) | A_t^{i^-} = A_t^{i^{-*}} + \epsilon,\ m_t^i = m_t^{i^*} - \epsilon,\ \epsilon \in R\} \in \xi_t^i,$$

if $q_t^i = 1$*;*

(3) the optimal choice of debtor satisfies:

$$\alpha_t^{i^*} \equiv \{(x_t^{i^*}, A_t^i, m_t^i) | A_t^{j^+} = A_t^{j^{+*}} + \epsilon,\ m_t^i = m_t^{i^*} - \epsilon,\ \epsilon \in R\} \in \xi_t^i,$$

if $q_t^j = 1$*;*

(4) $p_t^l \to 0$ *implies* $x_t^{i,l} \to \infty$ *for some* $l \in L$*.*

Proof. Appendix. □

The Proposition 4.1 shows that the bond holding is identical with the money holding when the interest rate of bond equivalent to 0. Because all the bonds have a risk that the repaument maybe less than promise, the action in period t may not satisfy the property (2) and (3) when bankruptcy occure. In this paper, the identical market signal and the credit scheme law guarantee that every consumer makes the decision on price predicting method and credit constraints. Then every projection of consumer's action must almost satisfy all the properties under the forecasting price system. The property (4) shows that the feasible consumption set becomes infinite set when the commodity price goes to zero.

Let the financial policy of bank, A_t^b, and the monetary policy, $-M_t^b$, the bank, as central bank or government, has ability to issue currency and administrant public assistance. Every consumer could issue some bonds for the bank, require to withdraw some money, even the asset market has no position for bank. In some general model, the bank's action maybe affects the consumer's choice such as supply some bonds like saving. However, there is safficient to consider long-position of the bank because the bank will never go to bakruptcy and the credit scheme law implies that the financial policy supplied from the bank has upper-bound $\sum_{i\in I} a_t^{i+} \leq -\sum_{i\in I} \rho_t^i(s_1)$. This argument shows that consumers never choose to hold money when the interest rate of the bank bond $q_t^{b^{-1}} - 1$ is greater than zero. More importantly, it prevents some consumers (speculator in abuse language), who will get more financial wealth without sell any endowment under the bond price $q_t^i > q_t^b$, from occurence. In equilibrium, it easy to varify that the interest rate of bank bond always be the lowest in every period because the lowest interest rate of bank's bond is identical with money holding in this model.

The bank budget set, which is not identical with the consumer's, only contains the financial wealth in every period. So the current monetary policy depends on the long-position choice, the repayment of long-position and the money holding of all the consumers at the end of the past period. In the sense of bankruptcy, the repayment of long-position from consumers maybe less than the promissing. Let the bank action in the period $t \in T$ be $\alpha^b = (0, A^b, -M^b)$ in which the projection defined as $\alpha_t^b = (0, A_t^b, -M_t^b)$ and the return function G_{t-1}, the bank would choose the optimal monetary policy subject to the budget set:

$$B_t^b = \{(0, A_t^b, -M_t^b)|M_t^b - M_{t-1}^b = \mathbf{1} \cdot G_{t-1}^b - Q_t \cdot A_t^b, \sum_{i\in I} a_t^{i+} \leq -\sum_{i\in I} \rho_t^i(s_1)\}. \quad (9)$$

Let the utility function of bank be $u_t^b : -M_t^b \in R_- \mapsto R$, the bank must choose the best monetary polisy to satisfy the budget set $\prod_{t\in T} B_t^b$. From the credit scheme law, the budget set of bank is bounded:

$$B_t^b \subset \{(0, A_t^b, -M_t^b)|a_t^{i+} \in [0, -\rho_t^i(s_1)],\ M_t^b \in [0, -\sum_{i\in I} \rho_t^i(s_1)],\ for\ all\ t \in T\ and\ i \in I\}.$$

The negative money supply $M_t^b - M_{t-1}^b < 0$ indicates that the quantity of currency should be decreased in current period. In this paper, this problem will not be dicussed in detail since the bank has ability to dispose money as she want.

On the basis of all the assumption, the temporary general equilibrium is defined as follow:

Definition 4.1. *There exists a temprary general equilibrium in current period if and only if for all $i \in I \cup B$ the optimal action $\alpha_1^{i^*}$ must satisfy:*

(1) $\alpha_1^{i^*} \in \Pr_1 B^i$*;*

(2) $sum_{i \in I} x_1^{i^*} = \sum_{i \in I} e_1^i$*;*

(3) $-a_1^{i^{-*}} = \sum_{j \neq i} a_1^{j^{+*}}$.

Before to introduce the main theory, the initial financial wealth (A_0^i, m_0^i) of consumer is necessary to be charactized. Let the consumer set $\bar{I} = \{i \in I | A_0^{i^-} + m_0^i \geq 0\}$, I have following assumption.

Assumption 3. *There exists* $i \in \bar{I}$ *such that* $\rho_t^i < 0$*, and exists* $j \in I$ *such that* $e_t^j \gg 0$ *for all* $t \geq 2$.

Assumption 3 shows that the economy exists at least one consumer who never faces bankruptcy at the beginning of current period. Moreover, since for such a consumer should make a decision upon over all periods, each consumer benefits from the increasing commodity demand in the current period by decreasing the future wealth when the commodity price tends to zero. By the discussion in the past , the next theorem can be proved:

Theorem 4.1. *There exists a temporary general equilibrium as in definition 4.1 under the Assumption A, B and C.*

Proof. Appendix. □

Note this temporary equilibrium is a consideration that the market is clearing in the period 1, even though the demand correspondence depends on the future expected. The Assumption 1 guarantees that moral hazard will be never happen in this model, i.e. the budget set of all agent must be convex set. The Assumption 3 shows that all price must never goes to zero that causes infinity demand.

5 Equilibrium in Overlapping Model

In this section, the temporary general equilibrium definded as Definition 4.1 will be extented in every period. The argument of temporary general equilibrium as in theorem 4.1 shows that the market is only clearing in the current period, so it is easy to check that the economy has equilibrium in each period if the future becomes as some agents' expected. However, the following example shows that the incomplete prediction leads the market collapse in some situations.

Let the temporary general equilibrium allocationin the period 1 be $({x_1^i}^*, {A_1^i}^*, {m_1^i}^*)$ for all $i \in I \cup B$, the signal $s_2 \in S_2$ in period 2 and the financial equilibrium allocation (A_1^*, m_1^*) respect to the initial financial wealth. The temporary general equilibrium may not exist in the period 2, since the financila equilibrium allocation (A_1^*, m_1^*) may not satisfy the Assumption 3. Consider the siduation of ${A_1^i}^* \in R_-^I$ and ${m_1^i}^* = 0$ for all $i \in I$, the financial market is clearing in the period 1. Recall the forecasting function $\psi^i(s_2)$ and credit scheme $\rho_2^i(s_2)$ respected to s_2, the value of $\psi_{P,t}^i(s_2)$, $\psi_{Q,t}^i(s_2)$ and e^i in the compact set implies that $\rho_2^i(s_2)$ is finite value. Moreover, let $\mathbf{1} \cdot {A_1^i}^* + P_2 \cdot e_2^i \ll 0$ for all commodity price. Thus, all comsumers must go to bankruptcy and commodity price $P_2 \to 0$ if $Q_2 \to 0$. On the other hand, the financial market is always clearing since the bank receives all the debt request from consumers. But the commodity market no more clearing if any price is positive. Therefore, the temporary equilibrium could be existed on the period 2 only if $(P_2, Q_2) = 0$, and consequently the economy is collapse.

Now, considering the overlapping model with infinite periods, and denote the consumer set as I_t in every period. The simple case is that every consumer only lives for two period, as Grandmond and Laroque[1975] model. It is easy to check that the old consumer has no credit to take any debt by the notation of credit scheme law and never participate in the financial market. It means that the old consumer cannot take any accommodation of funds from bank or private when they face bankruptcy. On the other hand, the young consumer will never face bankruptcy since he have no initial financial wealth. Then the bank cannot carry out the tasks clear as in past section. Therefore, the overlapping model will be extented as every consumer will live for finite periods, and assume $T^{i_t} \neq T^{j_t}$ for i_t, $j_t \in I_t$ in general. Let $I_t^{'} = \{i_t \in I_t | A_t^{i_t} = 0, m_t^{i_t} = 0\}$ be the subset of I_t, which contains the newborn consumers without initial financial wealth, the overlapping model will be contributed using the method of the forecasting function represented to every signal.

Before proving the existence of temporary general equilibrium, there has a problem that a temporary equilibrium does not be existed even though $I^{'} \neq \emptyset$. When all the consumers will only live for the period t, the notation of credit scheme law implies $\rho_t^{i_t} = 0$ for all $i \in I_t$. On the other word, all the financial market in the period t will never open, i.e., $(A_t^{i_t}, m_t^{i_t}) = (0, 0)$ for all consumers. Hence the financial market clearing implies

$P_t^* \cdot \sum_{i_t \in I_t^+} ({x_t^{i_t}}^* - e_t^{i_t}) = -P_t^* \cdot \sum_{i_t \in I_t^-} e_t^{i_t} \ll 0$, and the commodity market will be cleared only if $P_t^* = 0$. Then the next assumption should be stated:

Assumption 4. *There exists* $\bar{I}_t = \{i_t \in I_t | {A_t^{i_t}}^- + m_t^{i_t} \leq 0,\ \rho_t^{i_t} < 0\}$*, and* $I_t^{'} \cap \bar{I}_t \neq \emptyset$.

Note Assumption 4 implies that there exists at least one consumer who has non-zero credit limited and never face bankruptcy at the beginning of period t. Moreover, the set $I_t^{'} \cap \bar{I}_t$ may be empty, i.e., the newborn maybe not occured in period t. Using the Assumption 4, it is easy to check the Assumption 3 always hold in each period, and the next proposition can be proved:

Proposition 5.1. *There exists a temporary general equilibrium as definition 4.1 in every period under the assumption A, B and D.*

Proof. This proof should use the same logic with Theorem 4.1. Let $({A_{t-1}^{I_{t-1}}}^*, {m_{t-1}^{I_{t-1}}}^*)$ be the equilibrium allocation of period $t-1$, then the Assumption 4 implies Assumption 3 respected to $s_t \in S_t$. By Proposition 4.1 and lemma 6.4, 6.5, the financial market clearing implies the commodity market clearing with $(P_t^*, Q_t^*) \gg (0, 0)$. Hence this proof is completed. □

6 Conclusion

In prior sections, I argue the expected future in a simple way without probabolity. But the argument of monetary policy guaranteed that the moral hazard will never occur in the expectation of consumers. Let there be some subject probability by future state expected of consumer i. There also exists a temporary general equilibrium under the monetary policy when the Assumption 2 holds.

In this paper, I presume an bankruptcy model without the penalty. The key points of my model are: (i) extend then financial market for private assets that the chain reaction occures among consumers; (ii) propose a bankruptcy clearing mechanism in dynamic model using temporary sense. This paper discusses the existence of equilibirium in which the liquidation rule is un-chosen by any consumer. The returen rate of bond also relative to the default in some previous analysis of some general equilibrium models. The important argument is that the monetary policy of bank can be considered as the penalty for bankrupt severely.

References

[1] Mário R. Páscoa Aloisio P. Araujo. Bancruptcy in a model of unsecured claims. *Economic Theory*, 20(3):455–481, 2002.

[2] Pradeep Dubey, John Geanakoplos, and Martin Shubik. Default in a general equilibrium model with incomplete markets. (1247), Jan 2000.

[3] Jürgen Eichberger. A note on bankruptcy rules and credit constraints in temporary equilibrium. *Econometrica*, 57(3):707–715, 1989.

[4] Guy Laroque Jean-Michel Grandmont. On money and banking. *The Review of Economic Studies*, 42(2):207–236, 1975.

[5] Joseph Stiglitz Michael Rothschild. Equilibrium in competitive insurance markets: An essay on the economics of imperfect information. *The Quarterly Journal of Economics*, 90(4):629–649, 1976.

[6] Martin Shubik Pradeep Dubey, John Geanakoplos. Default and punishment in general equilibrium. *Econometrica*, 73(1):1–37, 2005.

[7] J.-Marc Tallon Salvatore Modica, Aldo Rustichini. Unawareness and bankruptcy: A general equilibrium model. *Economic Theory*, 12(2):259–292, 1998.

[8] William R. Zame. Efficiency and the role of default when security markets are incomplete. *The American Economic Review*, 83(5):1142–1164, 1993.

APPENDIX

Denote the price set:

$$\Delta_1 = \{(P_1, Q_1, \mathbf{1}) | \sum_{l \in L} p_1^l = 1,\ q_1^i \in [0,1],\ for\ all\ i \in I\};$$

$$\Delta^i(s_1) = \{(\psi^i_{P,t}(s_1), \psi^i_{Q,t}(s_1)\mathbf{1}) | \sum_{l \in L} \psi^{i,l}_{P,t}(s_1) = 1,\ \psi^i_{Q_t} \in [0,1]^I,\ for\ all\ i \in I\ and\ t \in T^i \backslash \{1\}\},$$

and a compact and convex set $C \in R^L$ such that $\sum_{i \in I} e^i_t \in \inf C$. So, $x^i_t \in \inf C$.

Denote the budget correspondence as:

$$B^i : \Delta_1 \times \Delta^i(s_1) \times [0,1]^I \times R^I \times R_+ \mapsto B^i(\Delta_1, \Delta^i(s_1), K_0, A_0^i, m_0^i),$$

and the $t \in T^i \setminus \{1\}$ projection budget correspondence:

$$B_t^i : \Delta^i(s_1) \times R^I \times R_+ \mapsto B_t^i(\Delta^i(s_1), A_{t-1}^i, m_{t-1}^i).$$

Lemma 6.1. *The budget correspondence $B_t^i : \Delta^i(s_1) \times R^I \times R_+ \mapsto B_t^i(\Delta^i(s_1), A_{t-1}^i, m_{t-1}^i)$ is continuous on $\Delta^i(s_1) \times R^I \times R_+$.*

Proof. Let $\{\Delta^i(s_1)\}^\nu \to \{\Delta^i(s_1)\}^*$, $\{(x_t^i, A_t^i, m_t^i)\}^\nu \to (x_t^i, A_t^i, m_t^i)^*$. First, I will show the correspondence is closed. Let $\{(x_t^i, A_t^i, m_t^i)\}^\nu \in B_t^i$ and $0 < \psi_{Q,t}^i(s_1) \cdot A_t^{i\nu} + \psi_{P,t}^i(s_1) \cdot x_t^{i\nu} + m_t^{i\nu} \leq m_{t-1}^i + \psi_{P,t-1}^i(s_1) \cdot e_t^i + \mathbf{1} \cdot A_{t-1}^i$. It is easy to varify $(x_t^i, A_t^i, m_t^i)^* \in B_t^i$ if $\nu \to \infty$. If $-\psi_{Q,t}^i \cdot \rho_t^i + m_{t-1}^i + \psi_{P,t}^i(s_1) \cdot e_t^i + \mathbf{1} \cdot A_{t-1}^i = 0$, then $\{(x_t^i, A_t^i, m_t^i)\}^\nu \to (0, \rho_t^i, 0) \in B_t^i$. Therefore the correspondence is closed. Second, the budget correspondence is l.h.c if the wealth of endowment and initial financial is positive. From the proposition 4.1, $-\psi_{Q,t}^i \cdot \rho_t^i + m_{t-1}^i + \psi_{P,t}^i(s_1) \cdot e_t^i + \mathbf{1} \cdot A_{t-1}^i \geq -\psi_{Q,t}^i \cdot \rho_t^i + \psi_{P,t}^i(s_1) \cdot e_t^i + \mathbf{1} \cdot \rho_{t-1}^i \geq 0$. As in Werner lemma 1(iii), the budget correspondence is l.h.c if $-\psi_{Q,t}^i \cdot \rho_t^i + \psi_{P,t}^i(s_1) \cdot e_t^i + \mathbf{1} \cdot \rho_{t-1}^i > 0$. If the net wealth equals to 0, then $\{(x_t^i, A_t^i, m_t^i)\}^\nu \to (0, \rho_t^i, 0)$. Hence, the correspondence $\mathrm{Pr}_{t=2}^{T^i} B_t^i(\Delta^i(s_1), A_{t-1}^i, m_{t-1}^i)$ is continuous. □

Then turn to the budget set in the current period, consider the correspondence as:

$$B_1^i = \{(x_1^i, A_1^i, m_t^i) | Q_1 \cdot A_1^i + P_1 \cdot x_1^i + m_1^i \geq \max\{m_0^i + P_1 \cdot e_1^i + \mathbf{1} \cdot (G_0^i + A_0^{i\,-}), Q_1 \cdot \rho_1^i\}.$$

Lemma 6.2. *The correspondence $B_1^i : \Delta_1 \times R^I \times [0,1]^I \times R_+ \mapsto B_1^i(\Delta_1, K_0, A_0^i, m_0^i)$ is continuous when $(P_1, Q_1) \gg (0,0)$.*

Proof. The conclusion follows as the proof of $B_t^i(\Delta^i(s_1), A_{t-1}^i, m_{t-1}^i)$. If $\max\{m_0^i + P_1 \cdot e_1^i + \mathbf{1} \cdot (G_0^i + A_0^{i\,-}), Q_1 \cdot \rho_1^i\} = m_0^i + P_1 \cdot e_1^i + \mathbf{1} \cdot (G_0^i + A_0^{i\,-})$ for all $(P_1, Q_1, \mathbf{1}, K_0, A_0^i, m_0^i) \in \Delta_1 \times R^I \times [0,1]^I \times R_+$, then the correspondenc $B_1^i(\Delta_1, K_0, A_0^i, m_0^i)$ is closed and l.h.c. Hence it is necessary to varify that the budget correspondence always satisfy this lemma when the bankruptcy occure.

(i) Given the price vector and the dicounted return rate matrix $(P_1, Q_1) \gg (0,0)$, it is easy to varify that the budget set of bankrupt is singleton $(0, \rho_1^i, 0)$ by the definition of bankruptcy law. Let $\{(\Delta_1, K_0)\}^\nu \to \{(\Delta_1, K_0)\}^*$, $\{(x_1^i, A_1^i, m_1^i)\}^\nu \to \{(x_1^i, A_1^i, m_1^i)\}^*$. If the net income $m_0^i + P_1 \cdot e_1^i + \mathbf{1} \cdot (G_0^i + A_0^{i-}) - Q_1 \cdot \rho_1^i \leq 0$, then there exists a large enough ν such that $P_1^\nu \cdot x_1^{i\,\nu} \to 0$ and $m_1^{i\,\nu} \to 0$ where $P_1^* \cdot x_1^{i\,*} + m_1^{i\,*} = \lim P_1^\nu \cdot x_1^{i\,\nu} + m_1^{i\,\nu} = 0$, i.e $(0, \rho_1^i, 0) \in B_1^i$. If the net income is positive, it is easy to check that $a_1^{i-} \in [\rho_1^i(s_1), 0]$ and $Q_1 \cdot A_1^i + P_1 \cdot x_1^i + m_1^i = m_0^i + P_1 \cdot e_1^i + \mathbf{1} \cdot (G_0^i + A_0^{i-})$, and hence B_1^i is closed from the ρ_1^i.

(ii) To show the correspondence $B_1^i(\Delta_1, K_0, A_0^i, m_0^i)$ is l.h.c, it only needs to consider the situation of bankruptcy. However, for all $(P_1, Q_1) \gg 0$ implies $(0, \rho_1^i, 0) \in B_1^i(\Delta_1, K_0, A_0^i, m_0^i)$. This proof is completed if taking a sequence $\{(0, \rho_1^i, 0)\}^\nu$ and it acturely tends to $(0, \rho_1^i, 0)$.

Then this proof is completed. □

Lemma 6.3. *The budget correspondence $B^i(\Delta_1, \Delta^i(s_1), K_0, A_0^i, m_0^i)$ is compact- and convex-value whenever $(P_1, Q_1.\psi_{P,t}^i(s_1), \psi_{Q,t}^i(s_1)) \gg (0,0,0,0)$ for all $i \in I$ and $t \in T$.*

Proof. This correspondence is clearly compact-value, since the available commodity space is the subset of a large compact and convex set $C \in R^L$. From the Proposition 3.1, it is sufficient to check B_t^i is convex-value in the siduation of bankruptcy. Turn to the projection 1 of correspondence $B^i(\Delta_1, \Delta^i(s_1), K_0, A_0^i, m_0^i)$. If the net income is positive, then the convexity of $B^i(\Delta_1, \Delta^i(s_1), K_0, A_0^i, m_0^i)$ can be proved as the standard way. If the net income is non-positive, the value set of $B_1^i(\Delta_1, K_0, A_0^i, m_0^i)$ must equivalent to the singleton $(0, \rho_1^i, 0)$, and hence it is convex-value. So that the budget correspondence $B^i(\Delta_1, \Delta^i(s_1), K_0, A_0^i, m_0^i)$ is compact- and convex-value follows from Debrue P15 (7) and P23 (11). □

Proof of Proposition 4.1

Proof. (1) Lemma 6.1, 6.2 and 6.3 show that the budget correspondence is continuous, compact- and convex-value. From the maximum theory, the demand correspondence ξ^i is non-empty, u.h.c and compact-value on $\Delta_1 \times \Delta^i(s_1) \times [0,1]^I \times R^i \times R_+$. The Lemma 6.1, 6.2 and 6.3 are guarantee that all the assumption in Grandmond (A.4.1) are satisfy. Hence the correspondence ξ^i is convex-value, and this proof is completed.

(2),(3) The proof of property (2) and (3) is the same logic. Let $\epsilon \in R$, $q_1^i = 1$ and consumption choice $({x_1^i}^*, {A_1^i}^*, {m_1^i}^*) \in \xi_1^i$ which belongs to the projection 1 of demand correspondence ξ^i. In the period 1, it easy to check that the condition $q_1^i({a_1^i}^{-*} + \epsilon) + ({m_1^i}^* - \epsilon) = q_1^i {a_1^i}^{-*} + {m_1^i}^* + q_1^i \epsilon - \epsilon = q_1^i {a_1^i}^{-*} + {m_1^i}^*$ holds for all $\epsilon \in R$. If ${a_1^i}^{-*} + \epsilon \leq \rho_1^i(s_1)$, this equation always hold. Moreover, there has the same financial effect of $({A_1^i}^*, {m_1^i}^*)$ and $({A_1^i}^* + \epsilon, {m_1^i}^* - \epsilon)$, since the budget set of consumer i in the period $t \in T^i \setminus \{1\}$ is only consider as standard way.

(4) This property is following as standard argument bu Hildenbrand (1.2, Corollary 1).

□

The property (4) of Proposition 4.1 shows that the excess demand goes to infinity when the price $p_1^l \to \infty$. However, the commodity space is considered in the compact and convex set C, and Lemma 6.3 shows the price p_1^l is positive if $x_1^l < \infty$. Moreover, any consumer has no incentive to take debts if the interest rate tends to infinity. Hence, the price in equilibrium economy only should consider as positive.

$$\bar{\Delta}_1 = \{(P_1, Q_1) \in \Delta_1 | p_1^l \geq \epsilon,\ q_1^i \geq \epsilon,\ for\ all\ l \in L,\ and\ i \in I\}$$

Let the excess demand correspondence:

$$z_t^i(\Delta_1, \Delta^i(s_1), K_1, A_0^i, m_0^i) \equiv \xi_t^i - (e_t^i, 0, 0),$$

I will show these correspondence has fixed-point.

Before to show these correspondence has fixed-point, there has neccesary to show the correspondence $\Delta_1 \times K_0 \to K_0$ and $K_0 \to \Delta_1$ is u.h.c, compact- and convex-value. Let the function $K_0^i : \Delta_1 \times K_0 \mapsto \kappa_0^i$ as the discount return rate in $\kappa_0^i \in [0, 1]$. It is clear that this function is compact- and convex-value. Also, this function is continuous if ${A_0^i}^- < 0$. Otherwise ${A_0^i}^- = 0$ means $\kappa_0^i \in [0, 1]$ and hence u.h.c.

Lemma 6.4. *For every $\{(A_0^i, m_0^i)\}_{i \in I}$, the correspondence $K_0 \to \bar{\Delta}_1$ is u.h.c, compact- and convex-value.*

Proof. The price vector $\bar{\Delta}_1$ is compact- and convex-value that following as the definition of Δ_1, hence it only to prove the closedness of correspondence. Denote the correspondence by $\mathbb{K}$, and let $\{(P_1, Q_1, K_0)\}^\nu \to (P_1, Q_1, K_0)^*$ with $\{P_1, Q_1\}^\nu \in \bar{\Delta}_1$. The correspondence $\mathbb{K}$ is closedness if $(P_1, Q_1, K_0)^* \in \mathbb{K}(K_0^*)$ when $\{(P_1, Q_1)\}^\nu \in \mathbb{K}(K_0^\nu)$. Denote $B_\epsilon(K_0^*)$ as the ϵ-ball of K_0^* for all $\epsilon \ll 0$, then there can exist subsequence $K_0^\nu \in B_\epsilon(K_0^*)$. Moreover, there can find ν^* such that $K_0^* \in B_{\frac{\epsilon}{\nu^*}}(K_0^{\nu^*}) \subset B_\epsilon(K_0^*)$. Let $\mathbb{K}^{-1}$ be the inverse correspondence, and $B_{\frac{\epsilon}{\nu^*}}(P_1^{\nu^*}, Q_1^{\nu^*}) \subset B_\epsilon(P_1^*, Q_1^*)$. It easy to varify that $B_{\frac{\epsilon}{\nu^*}}(K_0^{\nu^*}) \subset \mathbb{K}^{-1}(B_\epsilon(P_1^*, Q_1^*))$, and hence $K_0^* \in \mathbb{K}^{-1}(B_\epsilon(P_1^*, Q_1^*))$. The closedness of correspondence $\mathbb{K}$ is completed(Proposition 1, Hildenbrand P22). □

Denote the price vector:

$$\mu(x_1, A_1, m_1) = \arg\max\{P_1 \cdot \sum_{i \in I}(x_1^i - e_1^i) + Q_1 \cdot A_1 + m_1 | (P_!, Q_1) \in \bar{\Delta}_1\}$$

maximize the value of every excess demand $(x_1, A_1, m_!) \in \prod_{i \in I} z_1^i$.

Lemma 6.5. *The correspondence* $\mu : \prod_{i \in I} z_1^i \mapsto \bar{\Delta}_1$ *is non-empty, compact-, convex-value and u.h.c.*

Proof. Since $P_1 \cdot \sum_{i \in I}(x_1^i - e_1^i) + Q_1 \cdot A_1 + m_1$ is continuous function and $\bar{\Delta}_1$ is actually convex and compact, then $\mu(x_1, A_1, m_1)$ is not empty. By the maximum theorem, $\mu : \prod_{i \in I} z_1^i \mapsto \bar{\Delta}_1$ is compact-, convex-value and u.h.c. □

Proof of Theorem 4.1

Proof. Let $\Xi_1 \subset R^L \times R^I \times R_+$ be a convex and compact set with the range of $\prod_{i \in I} \xi_1^i$, continuous on the compact domain $\bar{\Delta}_1 \times \Delta^i \times [0, 1]^I$. Also, $\mu : \Xi_1 \setminus \prod_{i \in I}(e_1^i, 0, 0) \mapsto \bar{\Delta}_1$ is u.h.c respected to $\prod_{i \in I} z_1^i$. The product-correspondence $\prod_{i \in I} \xi_1^i \times \mu \times \mathbb{K}$ is mapping on the convex and compact set $\Delta_1 \times \Delta^i \times [0, 1]^I$ to itself. By Proposition 4.1 and Lemma 6.4, 6.5, this correspondence is non-empty, compact-, convex-value and u.h.c. Hence, there exists a fixed point by Kakutani's fixed point theorem. Moreover, the consumer problem has a solution which let the price vector in $\bar{\Delta}_1$, if given the initial financial wealth and signal $\text{Pr}_{i \in I}(s_1, A_0^i, m_0^i) \in S_1 \times R^{I \times I} \times R_+^I$.

The final question is to varify the Walras laws. Remain the condition in the temporary equilibrium:

(1) $K_0^* = K_0(\Delta_1, K_0^*)$;

(2) $(x_1^i, A_1^i, m_1^i) \in \xi_1^i(P_1^*, Q_1^*, (\psi_{P,t}^i(s_1), \psi_{Q,t}^i(s_1))_{t \in T^i \setminus \{1\}}, K_0^i)$;

(3) $\sum_{i\in I} x_1^i = \sum_{i\in I} e_1^i$, $\sum_{i\in I\cup B} A_1^i = 0$ and $\sum_{i\in I} m_1^i - M_1^b = 0$.

The equilibrium allocation must satisfy the following condition.

Give the initial financial wealth and signal $\Pr_{i\in I}(s_1, A_0^i, m_0^i)$, let $I^+ = \{i \in I | ({x_1^i}^*, {A_1^i}^*, {m_1^i}^*) \neq (0, \rho_1^i, 0)\}$ and $I^- = I \setminus I^+$, the optimal action of consumers and bank must satisfy the budget constraint:

$$P_1^* \cdot \sum_{i\in I^+} ({x_1^i}^* - e_1^i) + Q_1^* \cdot \sum_{i\in I^+} {A_1^i}^* + \sum_{i\in I^+} {m_1^i}^* = \mathbf{1} \cdot \sum_{i\in I^*} (G_0^i + {A_0^i}^-) + \sum_{i\in I^+} m_0^i,$$

$$-Q_1^* \cdot {A_1^b}^* - {M_1^b}^* = -M_0^b$$

From the definition of the real return function,

$$\begin{aligned}\sum_{i\in I^+} (G_0^i + {A_0^i}^-) &= P_1^* \cdot \sum_{i\in I^-} e_1^i - Q_1^* \cdot \sum_{i\in I^-} \rho_1^i + \sum_{i\in I^-} m_0^i + \sum_{i\in I^+} {A_0^i}^+ + \sum_{i\in I^+} {A_0^i}^- \\ &= P_1^* \cdot \sum_{i\in I^-} e_1^i - Q_1^* \cdot \sum_{i\in I^-} \rho_1^i + \sum_{i\in I^-} m_0^i.\end{aligned}$$

Summing up all agents' equilibrium constraint:

$$\begin{aligned}&P_1^* \cdot \sum_{i\in I^+} ({x_1^i}^* - e_1^i) + Q_1^* \cdot \sum_{i\in I^+} {A_1^i}^* + \sum_{i\in I^+} {m_1^i}^* - Q_1^* \cdot {A_1^b}^* - {M_q^b}^* \\ =&\mathbf{1} \cdot \sum_{i\in I^+} (G_0^i + {A_0^i}^-) + \sum_{i\in I^+} m_0^i - M_0^b \\ =&P_1^* \cdot \sum_{i\in I^-} e_1^i - Q_1^i \cdot \sum_{i\in I^-} \rho_1^i + \sum_{i\in I} m_0^i - M_0^b,\end{aligned}$$

hence

$$P_1^* \cdot \sum_{i\in I} ({x_1^i}^* - e_1^i) + Q_1^* \cdot (\sum_{i\in I^+} {A_1^i}^* + \sum_{i\in I^-} \rho_1^i) + \sum_{i\in I^+} {m_1^i}^* - Q_1^* \cdot {A_1^b}^* - {M_q^b}^* = 0.$$

Then the price vector in $\bar{\Delta}_1$ implies $\sum_{i\in I}({x_1^i}^* - e_1^i) = 0$, $\sum_{i\in I\cup B} {A_1^i}^* = 0$ and $\sum_{i\in I} {m_1^i}^* - M_1^b = 0$, and the proof of Theorem 5.1 is completed. □