Discussion Papers In Economics And Business

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Fiscal Rules in a Monetary Economy: Implications for Growth and Welfare*

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Abstract

This study considers two fiscal rules, a debt rule that controls the debt-to-GDP ratio, and an expenditure rule that controls the expenditure-to-GDP ratio, in a monetary growth model with financial intermediation. Tightening fiscal rules promotes economic growth and thus benefits future generations. However, there could be two equilibria of the nominal interest rates, and the welfare effects of the rules on the current generation are different between the two equilibria. In particular, the effects of a decreased debt-to-GDP ratio depend on its initial ratio; a low (high) ratio country has an incentive (no incentive) to reduce the ratio further from the viewpoint of the current generation's welfare. This result offers a reason for difficulties with fiscal reform in countries with already high debt-to-GDP ratios.

Keywords: Fiscal Rule; Government Debt; Economic Growth.

JEL Classifications: E62, E63, H63, O42.

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*This work was supported by the Japan Society for the Promotion of Science (JSPS) Grant-in-Aid for Scientific Research (C) Grant Number 18K01650.

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1 Introduction

The emergence and persistence of large fiscal deficits and public debt in many industrial and developing countries in the last few decades have raised concerns about fiscal sustainability and led to calls for appropriate adjustment with the use of fiscal rules (International Monetary Fund, 2009; Schaechter et al., 2012). The fiscal rules control public spending and/or public debt issuance, which in turn affects allocation of resources across generations (Heijdra and Ligthart, 2000; Yakita, 2008; Fernandez-Huertas Moraga and Vidal, 2010). In particular, recent studies suggest that fiscal austerity programs create trade-offs between a negative short-turn effect and a positive long-run effect in output (Bom and Ligthart, 2014), and the corresponding trade-offs between current generations' loss and future generations' benefit in terms of welfare (Glomm, Jung, and Tran, 2018).

The studies mentioned above are based on non-monetary growth models and thus, pay little attention to monetary factors. However, fiscal rules may influence monetary variables through the financial markets. For example, increased government debt is associated with increased money supply through open market operations. This influences the inflation and nominal interest rates, and thus, may affect the real activity and welfare across generations through allocative functions of financial markets (Bhattacharya et al., 1997; Schreft and Smith, 1997, 1998). Therefore, consideration of monetary factors is necessary for analyzing the fiscal rule effects on current and future generations.

Schreft and Smith (1997, 1998) introduce a role for private banks that provide liquidity following Diamond and Dybvig (1983) into Diamond's (1965) growth model with overlapping generations. In this framework, agents are subject to stochastic relocations, and only currency can be transported between locations. Agents seek to liquidate their holdings of bonds and capital to obtain currency once they have relocated. Under this environment, Schreft and Smith (1997, 1998) consider the effects of monetary policy on growth and welfare across generations via financial markets. However, they assume no direct government expenditures and taxes, and say nothing about the effect of fiscal rules on growth and welfare. Bhattacharya et al. (1997), Espinosa-Vega and Yip (1999, 2002), and Hung (2005) partly overcome this limitation by investigating the fiscal spending rule effects on economic growth, but these studies assume no tax or debt rule (Bhattacharya et al., 1997), or no fiscal deficit (Espinosa-Vega and Yip, 1999, 2002; Hung, 2005).[1]

[1]Further extensions are undertaken by assuming variations in structural parameters (Gomis-Porqueras, 2000), various types of private banking systems (Matsuoka, 2011; Paal, Smith, and Wang, 2013), the presence of banking crisis (Antinolfi and Keister, 2006), multiple production sectors (Ghossoub and

A notable exception is Schreft and Smith (2002), who consider the consequence of a declining stock of public debt. In their model, there are two separate entities: the treasury and central bank. The treasury finances public expenditures by labor income taxation and public debt issues. The central bank creates money, acquires public bonds in the capital market, and rebates all interest earned on its holding of public debt to the treasury, as in the cases of Japan and the United States. In particular, the central bank's balance sheet constraint requires that the value of the bank's outstanding liabilities (i.e., the monetary base) does not exceed the value of its holding of government bonds. Within this framework, Schreft and Smith (2002) show that there could be two equilibria of the nominal interest rates, because the earned interest prevails over the Laffer curve property with respect to the nominal interest rate, and that the equilibria are Pareto ranked. Their analysis provides the welfare implications of a debt rule (i.e., a declining stock of public debt), but the analysis is static in nature, because physical capital accumulation is abstracted away from their model. Thus, their model provides no growth implications of the debt rule and its impact on welfare.

To address these issues, this study extends the model of Schreft and Smith (2002) by assuming Barro (1990)-type public production services as an engine of economic growth. This assumption leads to AK technology as in Espinosa-Vega and Yip (1999, 2002), Hung (2005), and Bhattacharya, Haslag, and Martin (2009). In addition, we assume two fiscal rules that are widely used in industrial and developing countries: a debt rule, which keeps the debt-to-GDP ratio at a constant rate, and an expenditure rule, which keeps the expenditure-to-GDP ratio constant (Budina et al., 2012). Within this extended framework, the present study demonstrates how the two fiscal rules affect growth and nominal interest and inflation rates, and how they in turn affect welfare across generations.

The main results are as follows. First, there are two nominal interest rates that satisfy the treasury's budget constraint, but the lower one may violate the central bank's balance sheet constraint. In particular, the lower nominal interest rate is less likely to satisfy the balance sheet as the two fiscal rules are strengthened because the tightened fiscal discipline lowers the government bond issues, and thereby the central bank's holding of government bonds. Thus, the choice of fiscal rule determines the number of equilibria of the nominal interest rates. However, the growth path of physical capital is always unique regardless of the number of equilibria of the nominal interest rates, since the debt rule yields a one-

Reed, 2014), changing demand for cash (Schreft and Smith, 2000; Ghossoub and Reed, 2010), the US dollar as a choice of storage value (Antinolfi, Landeo, and Nikitin, 2007), and allocation of capital and workers (Ghossoub and Reed, 2017).

to-one relationship between government debt and physical capital. This result is in sharp contrast to those of previous studies that show multiple equilibrium paths of physical capital in the absence of a debt rule (Schreft and Smith, 1997, 1998; Espinosa-Vega and Yip, 1999, 2002).

Second, when there are two equilibria, the welfare effects of the two fiscal rules are different between the two equilibria. A decrease in the expenditure-to-GDP ratio lowers (raises) the nominal interest rate and thus, raises (lowers) the expected utility of each generation in a low (high) nominal interest rate equilibrium. A decrease in the debt-to-GDP ratio also has opposing effects on the two equilibria through the nominal interest rate, but its effect depends on the initial debt-to-GDP ratio. Furthermore, the decrease has an additional effect in that it reduces the crowding-out effect of government debt, promotes capital accumulation, and thereby benefits future generations.

The analysis shows that when the debt-to-GDP ratio is initially high, a decrease in the ratio leads to (a) a Pareto improvement in a low equilibrium, and (b) a loss to the current generation and a gain to future generations in a high equilibrium. The result is reversed when the ratio is initially low. A corollary of this result is that when the equilibrium is unique and is distinguished by the higher nominal interest rate, the current generation has no incentive (an incentive) to cut the debt-to-GDP ratio when the ratio is initially high (low). This result provides a possible explanation for the observed polarization of debt-to-GDP ratios among developed countries (OECD, 2018). The result also offers a reason for difficulties with fiscal reform in countries with already high debt-to-GDP ratios, such as Greece, Italy, and Japan.

The results described above are obtained under the assumption that the degree of relative risk aversion is below one, as in Bhattacharya et al. (1997) and Schreft and Smith (2002). The later part of this study makes an alternative assumption that the degree of relative risk aversion is above one, and shows that the equilibrium is unique. This analysis also shows that a decrease in the expenditure-to-GDP ratio raises the utility of the current generation; and that a decrease in the debt-to-GDP ratio raises (lowers) the utility of the current generation when the ratio is initially high (low). Thus, the welfare effects are entirely different in the two cases of low and high relative risk aversion. This suggests that agents' attitude toward risk is key to evaluating the welfare effects of the fiscal rules across generations.[2]

[2]The role of relative risk aversion is also investigated by Espinosa-Vega and Yip (1999, 2002), Gomis-Porqueras (2000), and Bhattacharya, Haslag, and Martin (2009), who assume a balanced government budget and no deficit. The present study differs from their studies in that it analyzes the role of relative

The organization of the rest of this paper is as follows. Section 2 presents the model. Section 3 characterizes a competitive equilibrium and Section 4 analyzes the welfare effects of fiscal rules when the degree of relative risk aversion is below one. Section 5 considers the case in which the degree of relative risk aversion is above one. Section 6 provides concluding remarks.

2 Model

The model is based on that developed by Schreft and Smith (1997, 1998, 2002). Consider a discrete-time, infinite-horizon economy that starts from period $t = 0$. The economy consists of two identical islands, each inhabited by an infinite sequence of overlapping generations. Agents in each generation live for two periods, youth and old age. Each location contains a continuum of ex ante identical young agents with a unit mass. In addition, in period $t = 0$, there is an initial old generation in each location.

There are three assets in this economy: capital, money, and government bonds. It is assumed that one unit of current consumption invested at time t becomes one unit of capital at time $t + 1$. The capital obtained in period $t + 1$, denoted by K_{t+1}, is then used for production in period $t + 1$. It is also assumed that capital fully depreciates in the production process.

The nominal supplies of money and government bonds in each location in period t are denoted by M_t and B_t, respectively. Bonds mature in one period, and each bond issued in period t is a sure claim to I_t units of currency in period $t + 1$. Thus, I_t is the gross nominal rate of interest. We let p_t denote the period-t price level, and $m_t \equiv M_t/p_t$ and $b_t \equiv B_t/p_t$ denote the period-t supplies of money and government bonds in real terms, respectively. The initial old agents at each location are endowed with the initial per capita capital stock, $k_0 > 0$, and the initial per capita money supply, $M_{-1} > 0$.

2.1 Private Agents

Each agent is endowed with one unit of labor in youth and nothing in old age. Young agents supply their labor force inelastically to firms, earn real wage w, and pay labor income tax at the rate $\tau \in (0, 1)$. Agents are assumed to value only second-period consumption, denoted by c. Thus, all of their after-tax income is saved for future consumption. Let $u(c)$ denote the common utility function of all agents. Specifically, we

risk aversion in the presence of government deficit and investigates how the fiscal rule effects depend on relative risk aversion.

assume the following form:

$$u(c) = \frac{(c)^{1-\sigma} - 1}{1-\sigma},$$

where $\sigma(> 0)$ denotes the degree of relative risk aversion. A higher σ is associated with a higher degree of relative risk aversion. We assume $\sigma \in (0, 1)$ in the main analysis, and leave the case of $\sigma > 1$ to the analysis in Section 5.

At the beginning of each period, agents cannot move between or communicate across locations. Goods can never be transported between locations. Thus, goods, labor, and asset market transactions occur within each location at the beginning of each period. After this trade is conducted, some randomly selected fraction $\pi \in (0, 1)$ of young agents is chosen to be moved to the other location.

It is assumed that only currency can be transported between locations and that limited communication prevents the cross-location exchange of privately issued liabilities. Therefore, relocated agents seek to liquidate their holdings of bonds and capital in order to obtain currency. The relocation plays the role of liquidity preference shocks, as in Diamond and Dybig (1983). Banks emerge to insure agents against these shocks. The fraction π is constant across periods and known by all agents, and the probability of being relocated is independently and identically distributed across young agents. Thus, there is no aggregate uncertainty.

2.2 Government

Government bonds and money are issued by the government sector, which comprises two separate entities: the treasury and central bank. The treasury finances its expenditure by levying labor income tax on the young agents and by issuing government bonds with a nominal value of B_t in period t. The real value of government bonds at the end of period t is B_t/p_t. Government bonds can be held by either private agents or the central bank. The real value of government bonds demanded by the private agents and by the central bank is denoted by b_t^p and b_t^c, respectively. The stock of government bonds demanded in period t is

$$b_t \equiv b_t^p + b_t^c. \tag{1}$$

The central bank issues fiat currency. In period t, the per capita value of the monetary base outstanding is M_t in nominal terms and $m_t \equiv M_t/p_t$ in real terms. The central bank's balance sheet constraint requires

$$m_t \le b_t^c, \tag{2}$$

implying that the value of the central bank's outstanding liabilities does not exceed the value of its holdings of government bonds. In addition, the central bank's holdings of government bonds are limited by the stock of government bonds outstanding:

$$0 \leq b_t^c \leq \frac{B_t}{p_t}.$$

As in the United States, the central bank rebates all interest earned on its holdings of government bonds to the treasury, but retains the bond's principal (Schreft and Smith, 2002). A similar policy is in place in Japan.[3] Let T_t and $R_t \equiv I_t p_t / p_{t+1}$ denote the nominal value of the rebate in period t and the gross real rate of interest paid on government bonds between periods t and $t+1$, respectively. Then, T_t is

$$\begin{aligned} T_t &= b_{t-1}^c R_{t-1} p_t - b_{t-1}^c p_{t-1} \\ &= b_{t-1}^c p_t \left(R_{t-1} - \frac{p_{t-1}}{p_t} \right). \end{aligned}$$

Thus, the real rebate is

$$\frac{T_t}{p_t} = b_{t-1}^c \left(R_{t-1} - \frac{p_{t-1}}{p_t} \right).$$

Given the central bank's behavior, the treasury's budget constraint becomes

$$G_t = \tau L_t w_t + \frac{B_t}{p_t} - R_{t-1} \frac{B_{t-1}}{p_{t-1}} + \frac{T_t}{p_t},$$

where G_t is government expenditure devoted to public services to private producers, $\tau L_t w_t$ is labor tax revenue, B_t/p_t is the revenue from issuing new bonds, $R_{t-1}B_{t-1}/p_{t-1}$ is the debt repayment costs, and T_t/p_t is the rebate from the central bank to the treasury. In particular, the government expenditure is assumed to follow the rule that the expenditure is a constant fraction of GDP, $G_t = \xi Y_t$, where $\xi \in (0, 1)$. The parameter ξ presents the first fiscal rule, on which we focus in the following analysis. Given $L_t = 1$ and $b_t \equiv B_t/p_t$, the treasury's budget constraint is rewritten as follows:

$$\xi y_t = \tau w_t + b_t - R_{t-1} b_{t-1} + b_{t-1}^c \left(R_{t-1} - \frac{p_{t-1}}{p_t} \right). \tag{3}$$

2.3 Firms

There is a continuum of identical firms with a unit mass in each location. They are perfectly competitive profit maximizers that produce output, Y_t, by using private capital, K_t, labor (i.e., the size of period-t young agents), L_t, and government services to private

[3]Source: Bank of Japan Act, Article 53. http://www.japaneselawtranslation.go.jp/law/detail/?id=92&vm=04&re=01 (Accessed on August 12, 2008).

producers, G_t, as in Barro (1990). Specifically, the aggregate production function in each location is given by

$$Y_t = \tilde{A} \left(G_t\right)^{1-\alpha} \left(K_t\right)^{\alpha} \left(L_t\right)^{1-\alpha},$$

where $\tilde{A} > 0$ and $0 < \alpha < 1$. For simplicity, capital is assumed to depreciate fully after production.

Each firm chooses capital and labor in order to maximize its profit, $\tilde{A} \left(G_t\right)^{1-\alpha} \left(K_t\right)^{\alpha} \left(L_t\right)^{1-\alpha} - r_t K_t - w_t L_t$, where r_t is the rental price of capital and w_t is the real wage rate. Because of the assumption of competitive markets, each firm takes r_t and w_t as given. The first-order conditions with respect to K_t and L_t are

$$r_t = \alpha \tilde{A} \left(G_t\right)^{1-\alpha} \left(k_t\right)^{\alpha-1} = \alpha y_t / k_y,$$
$$w_t = (1-\alpha) \tilde{A} \left(G_t\right)^{1-\alpha} \left(k_t\right)^{\alpha} = (1-\alpha) y_t,$$

where $k_t \equiv K_t/L_t$ and $y_t \equiv Y_t/L_t$ denote per capita capital and output, respectively. Because the size of each generation is unity, that is, $L_t = 1$, we have $k_t = K_t$ and $y_t = Y_t$ for all t.

Given the assumption of the expenditure rule, $G_t = \xi Y_t = \xi y_t$, the output y_t becomes $y_t = \tilde{A} \left(\xi y_t\right)^{1-\alpha} \left(k_t\right)^{\alpha}$, or,

$$y_t = \left(\tilde{A}\right)^{1/\alpha} \left(\xi\right)^{(1-\alpha)/\alpha} k_t.$$

By setting $A = \left(\tilde{A}\right)^{1/\alpha} \left(\xi\right)^{(1-\alpha)/\alpha}$, we obtain the AK technology of the production function, $y_t = Ak_t$. The first-order conditions with respect to capital and labor are reduced to

$$r_t = \alpha A \text{ and } w_t = (1-\alpha) A k_t, \tag{4}$$

respectively. Because of the assumption of full depreciation of capital, the following no-arbitrage condition holds in each period:

$$R_t = r_{t+1} = r \equiv \alpha A. \tag{5}$$

2.4 Private Banks

In each period, young agents deposit the entire value of their after-tax income, $(1-\tau)w_t$, in a bank. Banks use deposits to acquire money, capital, and government bonds. In addition, banks promise to pay agents who have (have not) relocated, that is, "movers" ("non-movers"), a gross real return of d_t^m (d_t^n) per unit on their deposits. We assume that there is free entry into banking and that banks are competitive in the sense that they take the real return on assets as given.[4] On the deposit side, we assume that intermediaries are Nash competitors; that is, banks announce deposit return schedules (d_t^m, d_t^n), taking the announced return schedules of other banks as given.

The return schedules must satisfy the following balance-sheet constraint:

$$m_t + b_t^p + i_t \leq (1-\tau)\, w_t,\ t \geq 0. \tag{6}$$

In period t, a young agent deposits his or her entire savings, $(1-\tau)\, w_t$, with a bank. The bank acquires m_t units of real balances and b_t^p units of real bond holdings, and makes an investment in capital of i_t.

The announced return schedules must also satisfy the following two constraints. First, banks promise to deliver the gross real return d_t^m to all of their depositors who have relocated in period t. By the law of large numbers, a fraction π of the bank's depositors have relocated; thus, the banks promise a payment per depositor of $\pi d_t^m (1-\tau)\, w_t$ to agents who have relocated in period t. Relocated agents must be given currency. Thus, banks face the constraint $p_{t+1}\pi d_t^m (1-\tau)\, w_t \leq M_t$, where $p_{t+1}\pi d_t^m (1-\tau)\, w_t$ represents the nominal deposit returns to relocated agents, and M_t the bank's currency holdings at the beginning of period $t+1$. Dividing both sides by p_t, the constraint is rewritten as

$$\pi d_t^m (1-\tau)\, w_t \leq m_t \frac{p_t}{p_{t+1}}.$$

Second, for the fraction $1-\pi$ of depositors who have not relocated in period t, banks promise a gross return of d_t^n per unit deposited. Thus banks owe $(1-\pi)d_t^n (1-\tau)\, w_t$ to non-movers, which they pay in period $t+1$. We assume $I_t > 1$ throughout the analysis; money is dominated in rate of return. Thus, banks do not carry cash reserves between periods. Non-movers are repaid from the return on the bank's bond holdings and capital investments. Thus, d_t^n must satisfy

$$(1-\pi)d_t^n (1-\tau)\, w_t \leq r_{t+1} i_t + R_t b_t^p.$$

Given that (5) holds, banks are indifferent between investing in bonds and physical capital.

Let $\gamma_t \equiv m_t / (1-\tau)\, w_t$ denote the bank's ratio of reserves to deposits in period t. This implies that $1-\gamma_t$ is the ratio of capital investments and bond holdings. Then the

[4]Alternatively, we can assume imperfect banking systems, as in Matsuoka (2011), Paal, Smith, and Wang (2013), and Ghossoub and Reed (2017). However, the present study keeps the standard assumption of competitive banking systems, since our focus here is on fiscal rules rather than banking systems.

above two constraints can be rewritten as

$$d_t^m \leq \frac{\gamma_t}{\pi} \frac{p_t}{p_{t+1}}, \tag{7}$$

$$d_t^n \leq \frac{r\left(1-\gamma_t\right)}{1-\pi}, \tag{8}$$

respectively. The constraint in (8) is derived by using (5).

Competition among banks for depositors forces banks to choose return schedules and portfolio allocations to maximize the expected utility of a representative depositor, subject to the constraints, (6), (7), (8), and $\gamma_t \in [0,1]$. In other words, banks choose γ_t to maximize

$$\pi \frac{\left(\frac{\gamma_t}{\pi}\left(1-\tau\right) w_t \frac{p_t}{p_{t+1}}\right)^{1-\sigma} - 1}{1-\sigma} + \left(1-\pi\right) \frac{\left(\frac{r(1-\gamma_t)}{1-\pi}\left(1-\tau\right) w_t\right)^{1-\sigma} - 1}{1-\sigma}$$

subject to $\gamma_t \in [0,1]$. The solution to the problem is

$$\gamma_t = \gamma\left(I_t\right) \equiv \left[\frac{1-\pi}{\pi}\left(I_t\right)^{(1-\sigma)/\sigma} + 1\right]^{-1}, \tag{9}$$

where the gross nominal interest rate is defined by

$$I_t = R_t p_{t+1}/p_t = r p_{t+1}/p_t. \tag{10}$$

Thus, a higher nominal interest rate implies a higher inflation rate.

The function $\gamma\left(\cdot\right)$, summarizing the behavior of the bank, gives the reserve-deposit ratio as a function of the nominal interest rate, I_t. The function $\gamma\left(\cdot\right)$ has the following properties: $\gamma'(\cdot) < 0$, $\gamma(0) = 1$, $\gamma(1) = \pi$, $\lim_{I \to \infty} \gamma\left(\cdot\right) = 0$ if $\sigma \in (0,1)$. In particular, $\gamma'(\cdot) < 0$ implies that higher nominal interest rates create higher opportunity costs of holding reserves and thus, a lower reserve-deposit ratio for the bank. Given the bank's optimal reserve-deposit ratio, equations (7) and (8) imply that

$$d_t^m = \frac{\gamma\left(I_t\right)}{\pi} \frac{p_t}{r p_{t+1}} r = \frac{\gamma\left(I_t\right)}{\pi} \frac{r}{I_t}, \tag{11}$$

$$d_t^n = \frac{1-\gamma\left(I_t\right)}{1-\pi} r, \tag{12}$$

where $d_t^m \leq d_t^n$ holds whenever $I_t > 1$ is satisfied.

We can now write the maximized expected utility of a representative depositor in period t as a function of the nominal interest rate, I_t, and the capital stock, k_t, as

$$V(I_t, k_t) = \frac{\pi}{1-\sigma}\left[\frac{\gamma\left(I_t\right)}{\pi I_t} r\left(1-\tau\right)\left(1-\alpha\right) A k_t\right]^{1-\sigma} + \frac{1-\pi}{1-\sigma}\left[\frac{1-\gamma\left(I_t\right)}{1-\pi} r\left(1-\tau\right)\left(1-\alpha\right) A k_t\right]^{1-\sigma},$$

or,

$$V(I_t, k_t) = \frac{[r(1-\tau)(1-\alpha)Ak_t]^{1-\sigma}}{1-\sigma}\left\{(\pi)^{\sigma}\left[\frac{\gamma(I_t)}{I_t}\right]^{1-\sigma} + (1-\pi)^{\sigma}[1-\gamma(I_t)]^{1-\sigma}\right\}. \tag{13}$$

Movers are made worse off whereas non-movers are made better off by increased nominal interest rates. Thus, the function in (13) indicates that the nominal interest rate has two opposing effects on the expected utility, but it turns out that the net effect is negative; the expected utility decreases as the nominal interest rate increases: $\partial V(I_t, k_t)/\partial I_t < 0$ (see Appendix A.1 for the proof). This result is used in the welfare analysis in Section 4.

3 Equilibrium

In equilibrium, the factor pricing relationships in (4) and the no-arbitrage condition in (5) hold. In addition, the demand for reserves, $\gamma(I_t)(1-\tau)w_t$, must be equal to the monetary base, m_t, in every period:

$$m_t = \gamma(I_t)(1-\tau)(1-\alpha)Ak_t, \ t \geq 0. \tag{14}$$

This is the money market equilibrium condition. Similarly, period-$t+1$ capital stock, k_{t+1}, must be equal to investment, $i_t : k_{t+1} = i_t$. From (1), (6), and (14), k_{t+1} is equivalent to

$$k_{t+1} = (1-\tau)(1-\alpha)Ak_t - b_t. \tag{15}$$

This is the capital market equilibrium condition.

The treasury's budget constraint in (3) is reformulated by using the factor pricing relationships in (4), the no-arbitrage condition in (5), the definition of the nominal interest rate in (10), and the money market equilibrium condition in (14), as follows:

$$\xi Ak_t = \tau(1-\alpha)Ak_t + b_t - rb_{t-1} + \begin{cases} b^c_{t-1}r\left(1-\frac{1}{I_{t-1}}\right), & t=0, \\ \gamma(I_{t-1})(1-\tau)(1-\alpha)Ak_{t-1}r\left(1-\frac{1}{I_{t-1}}\right), & t \geq 1, \end{cases} \tag{16}$$

where $\xi \in (0,1)$ denotes a constant fraction of government expenditure to GDP, $G_t = \xi Y_t$, which is aimed at controlling government expenditure.

The present study also assumes the following debt rule:

$$b_t = \mu y_t, \tag{17}$$

where $\mu \in (0,1)$. This rule is aimed at controlling government bond issues. Thus, in the present framework, there are two fiscal rules, represented by the parameters, ξ and

μ. Finally, the central bank's holdings of government bonds are limited by the stock of government bonds outstanding:

$$b_t^c \leq b_t. \tag{18}$$

We can now define a competitive equilibrium in the present model as follows.

Definition 1. A *competitive equilibrium* is a sequence of allocations, $\{b_t^c, b_t^p, b_t, k_t, d_t^m, d_t^n, i_t, m_t, \gamma_t\}$, and prices, $\{w_t, r_t, R_t, I_t\}$, such that given the initial conditions, $k_0 > 0$, $M_{-1} > 0$, $b_{-1} \geq 0$, $b_{-1}^c \geq 0$, and $b_{-1}^p \geq 0$, and the fiscal variables and rules, $\tau \in (0,1)$, $\mu \in (0,1)$, and $\xi \in (0,1)$, (i) γ_t solves the private bank's optimization problem to satisfy (9), and the bank's payments to movers and non-movers, $\{d_t^m, d_t^n\}$, satisfy (11) and (12), respectively; (ii) w_t and r_t satisfy the factor pricing relationships in (4), and the no-arbitrage condition in (5) holds; (iii) the central bank's balance sheet in (2) holds; (iv) the private bank's balance sheet in (6) holds; (v) the money market equilibrium condition in (14) holds; (vi) the capital market equilibrium condition in (15) holds; (vii) the treasury's budget constraint in (16) holds; (viii) the restriction on the central bank's holdings of government bonds in (18) holds; and (ix) the treasury authority follows the fiscal rules, $G_t = \xi y_t$ and $b_t = \mu y_t$.

Summarizing the conditions described in Definition 1, a competitive equilibrium path is characterized by a sequence of capital, monetary base, government bonds, and nominal interest rate, $\{k_{t+1}, m_t, b_t, I_t\}_{t=-1}^{\infty}$. Given $k_0(>0)$, $b_{-1}\,(\geq 0)$, and $m_{-1}(>0)$, the sequence satisfies the money market equilibrium condition in (14), the capital market equilibrium condition in (15), the treasury's budget constraint in (16), and the debt rule in (17) with a restriction on the central bank's holding of government bonds in (18).

The capital market-clearing condition in (15) with the debt rule in (14) determines the evolution of capital as

$$\frac{k_{t+1}}{k_t} = \left[(1-\tau)(1-\alpha) - \mu\right] A. \tag{19}$$

The growth rate of capital is constant across periods, since the production function exhibits a constant interest rate. In particular, a higher debt-to-GDP ratio lowers the growth rate, since government debt crowds out investment in capital. Given k_t, the debt level b_t is determined by the debt rule, $b_t = \mu A k_t$.

The treasury's budget constraint in (16) is reformulated by using the equation of capital in (19) and the debt rule in (17), as follows:

$$\begin{cases} [\xi - \tau(1-\alpha) - \mu]\, k_0 + \alpha b_{-1} = m_{-1}\alpha\,(1 - 1/I_{-1}), \\ \phi(\cdot)/\gamma(I_t) = \alpha(1-\tau)(1-\alpha)(1-1/I_t), \qquad t \geq 0, \end{cases} \tag{20}$$

where $\phi(\cdot)$ is defined by

$$\phi(\cdot) \equiv [\xi - \tau(1-\alpha) - \mu]\,[(1-\tau)(1-\alpha) - \mu] + \alpha\mu.$$

The nominal interest rate is constant across periods, except for the initial rate, I_{-1}. Given k_t and I_t, the monetary base, m_t, is determined by the money market-clearing condition in (14).

To determine the equilibrium nominal interest rate for $t \geq 0$, recall the second expression in (20), which is rewritten as follows:

$$\underbrace{\alpha(1-\tau)(1-\alpha)(1-1/I)}_{LHS} = \underbrace{\phi(\cdot)\left[\frac{1-\pi}{\pi}(I)^{(1-\sigma)/\sigma} + 1\right]}_{RHS}. \tag{21}$$

We assume $\phi(\cdot) > 0$ to find a nominal interest rate that is greater than one, $I > 1$. Under this assumption, the left-hand side and the right-hand side of (21), denoted by LHS and RHS, respectively, have the following properties:

$$\begin{aligned}
&\frac{\partial LHS}{\partial I} > 0,\ \frac{\partial^2 LHS}{\partial I^2} < 0,\ \ LHS|_{I=1} = 0,\ \ \lim_{I\to\infty} LHS = \alpha(1-\tau)(1-\alpha),\\
&\frac{\partial LHS}{\partial I} = \phi(\cdot)\frac{1-\sigma}{\sigma}\frac{1-\pi}{\pi}(I)^{(1-2\sigma)/\sigma} > 0,\\
&\frac{\partial^2 LHS}{\partial I^2}\phi(\cdot)\frac{1-\sigma}{\sigma}\frac{1-\pi}{\pi}\frac{1-2\sigma}{\sigma}(I)^{(1-3\sigma)/\sigma}\begin{cases} > 0 & \text{if } \sigma \in (0, 1/2), \\ = 0 & \text{if } \sigma = 1/2, \\ < 0 & \text{if } \sigma > 1/2, \end{cases}\\
&RHS|_{I=1} = \phi(\cdot)\frac{1}{\pi},\\
&\lim_{I\to\infty} RHS = \begin{cases} \infty & \text{if } \sigma \in (0,1), \\ \phi(\cdot)/\pi & \text{if } \sigma = 1, \\ \phi(\cdot) & \text{if } \sigma > 1. \end{cases}
\end{aligned}$$

Figure 1 graphically illustrates (21), using I on the horizontal axis. As illustrated in Figure 1, there could be two solutions to (21). However, either or both of them might not constitute an equilibrium, because the central bank's holdings of government bonds are limited by the stock of government bonds outstanding, $b_t^c \leq b_t$, or:

$$\gamma(I_t)(1-\tau)(1-\alpha)Ak_t \leq b_t.$$

With the debt rule in (17), this is rewritten as follows:

$$\gamma(I_t) \leq \frac{\theta\mu}{(1-\tau)(1-\alpha)}. \tag{22}$$

Thus, the equilibrium nominal interest rate must satisfy (22).

[Figure 1 is here.]

Let $\hat{I}$ denote the nominal interest rate that satisfies (22) with an equality. Since $\gamma(\cdot)$ is decreasing in its argument, the nominal interest rate that satisfies (21) constitutes an equilibrium if it is greater than or equal to $\hat{I}$. Using the definition of $\gamma(I_t)$ in (9), we derive $\hat{I}$ as follows:

$$\hat{I} = \left(\frac{\pi}{1-\pi}\right)^{\sigma/(1-\sigma)} \left[\frac{(1-\tau)(1-\alpha)}{\theta\mu} - 1\right]^{\sigma/(1-\sigma)}. \tag{23}$$

Thus, the restriction in (22) is equivalent to

$$\hat{I} \leq I. \tag{24}$$

The following proposition establishes the conditions for the existence of an equilibrium nominal interest rate.

Proposition 1.

(i) *There are two equilibria of the nominal interest rates if*

$$\frac{\phi(\cdot)}{\alpha(1-\tau)(1-\alpha)} + \frac{1}{1-\sigma}\left[\frac{\phi(\cdot)\,\frac{1-\sigma}{\sigma}\frac{1-\pi}{\pi}}{\alpha(1-\tau)(1-\alpha)}\right]^{\sigma} < 1 \tag{25}$$

holds and either of the following is satisfied: (a) $0 < \mu < \pi(1-\tau)(1-\alpha)$ *and*

$$\alpha\mu\left(1-\frac{1}{\hat{I}}\right) < \phi(\cdot) < \frac{\alpha(1-\tau)(1-\alpha)}{\frac{1-\sigma}{\sigma}\frac{1-\pi}{\pi}} \frac{1}{\left(\hat{I}\right)^{1/\sigma}}, \tag{26}$$

or (b) $\pi(1-\tau)(1-\alpha) \leq \mu < (1-\tau)(1-\alpha)$ *and*

$$0 \leq \phi(\cdot) < \frac{\alpha(1-\tau)(1-\alpha)}{\frac{1-\sigma}{\sigma}\frac{1-\pi}{\pi}}. \tag{27}$$

(ii) *There is a unique equilibrium of the nominal interest rate if the following conditions hold:*

$$0 < \mu < \pi(1-\tau)(1-\alpha) \ \textit{and}\ 0 \leq \phi(\cdot) < \alpha\mu\left(1-\frac{1}{\hat{I}}\right). \tag{28}$$

Proof. See Appendix A.2.

The conditions presented in Proposition 1 are interpreted as follows. First, condition (25) implies that there is some I, denoted by $\tilde{I} \in (1,\infty)$, such that $\alpha(1-\tau)(1-\alpha)\gamma\left(\tilde{I}\right)\left(1-1/\tilde{I}\right) > \phi(\cdot)$ holds. In other words, for some range of the nominal interest rate around $\tilde{I}$, the rebate from the central bank to the treasury outweighs the net expenditure of the treasury (i.e., government expenditure plus debt repayment minus tax revenue and revenue from issuing new government bonds). Therefore, given the Laffer curve property of the rebate, there are two nominal interest rates—one is below $\tilde{I}$, and the other is above $\tilde{I}$—that equate the rebate from the central bank and the net expenditure of the treasury, and thereby balance the treasury's budget constraint.

Given condition (25), the characterization of two equilibria are classified into two cases: (a) $0 < \mu < \pi(1-\tau)(1-\alpha)$ and (b) $\pi(1-\tau)(1-\alpha) \leq \mu < (1-\tau)(1-\alpha)$. Case (a) corresponds to $\hat{I} \geq 1$, as depicted in Panel (a) of Figure 1, and case (b) corresponds to $\hat{I} < 1$, as depicted in Panel (b) of Figure 1. In the latter case, μ must be bounded above from $(1-\tau)(1-\alpha)$ to ensure $k_{t+1}/k_t > 0$.

Based on the classification above, we next consider the condition in (26). The first inequality is equivalent to $LHS(\hat{I}) < RHS(\hat{I})$, implying that at $I = \hat{I}$, the rebate of interest from the central bank to the treasury is outweighed by the net expenditure of the treasury at $I = \hat{I}$. In other words, revenue shortage occurs. The second inequality is equivalent to $\partial LHS/\partial I|_{I=\hat{I}} > \partial RHS/\partial I|_{I=\hat{I}}$. This implies that a marginal increase in the rebate of interest outweighs a marginal increase in the net expenditure at $I = \hat{I}$. Thus, these two conditions together imply that there are some nominal interest rates, satisfying the restriction in (22), that balance the treasury's budget constraint, as illustrated in Panel (a) of Figure 1.

The condition in (27) is relevant to the case depicted in Panel (b) of Figure 1. The first inequality in (27) implies that the nominal interest rate that satisfies the treasury's budget in (21) is greater than one, and the second inequality in (27) implies that $1 < \tilde{I}$ holds. Thus, if either of them fails to hold, at least one of the equilibrium interest rates falls below one. This result implies that non-movers withdraw their deposits from the private bank before maturity, and thus, there is no investment in capital. Condition (27) prevents such an undesirable situation.

Finally, consider the condition in (28). The implications of $0 < \mu < \pi(1-\tau)(1-\alpha)$ and $0 \leq \phi(\cdot)$ are already described above. The condition $\phi(\cdot) < \alpha\mu\left(1 - 1/\hat{I}\right)$ is opposite to the first inequality in (26). Thus, the revenue stemming from the rebate of interest rate is sufficient to cover the net expenditure of the treasury at $I = \hat{I}$. The treasury's budget constraint could be balanced by lowering or raising the nominal interest rate from $I = \hat{I}$. In other words, there are two candidates for the equilibrium nominal interest rates, but the lower one fails to satisfy the restriction of the central bank's holdings of government bonds in (22). The higher one remains as an equilibrium nominal interest rate, as illustrated in Panel (c) of Figure 1.

Figure 2 illustrates the conditions presented in Proposition 1, using μ on the horizontal axis. Panels (a) and (b) depict the cases of $\xi = 0.2$ and 0.3, respectively. Other parameter values are set as $\theta = 0.99$, $\sigma = 0.85$, $\pi = 0.4$, $\alpha = 0.3$, and $\tau = 0.3$ When $\xi = 0.2$ in Panel (a), the model displays the two equilibria case for a certain range of μ, but not the unique equilibrium case for any value of μ. However, when ξ is slightly higher and is set at $\xi = 0.3$, as in Panel (b), the model displays two equilibria for a high range of μ whereas it displays a unique equilibrium for a low range of μ. Thus, the figure indicates that the model economy is more likely to attain two equilibria when the government sets higher ratios of μ and ξ.

[Figure 2 is here.]

In order to understand the abovementioned result, recall that there are two solutions of the nominal interest rates I that satisfy the treasury's budget constraint in (21), but the lower one may violate the restriction of the central bank's holding of government bonds in (22). Given this feature, consider an increase in the ratio of government expenditure to GDP, ξ, and an increase in the ratio of debt to GDP, μ. These increases imply that the government issues more public bonds to finance increased expenditure. This in turn makes the restriction in (22) less severe, resulting in the lower solution of (21) being more likely to satisfy the restriction in (22). Thus, the economy is more likely to attain two equilibria as fiscal discipline is weakened. This is a noteworthy feature of the present model.

Another noteworthy feature is that the equilibrium nominal interest rates could be two, whereas the growth rate of physical capital is always unique, as shown in equation (19). This result is in sharp contrast to those of previous studies that show multiple equilibrium paths of physical capital (Schreft and Smith, 1997, 1998; Espinosa-Vega and Yip, 1999, 2002). Given this multiplicity of equilibrium paths, Schreft and Smith (1997, 1998) argue that low capital equilibrium represents a development trap. In the present framework, such a trap never arises in equilibrium; any two economies attain the same growth rate. Nevertheless, two economies could be ranked in terms of expected utility owing to difference of the nominal interest rates. Given that the nominal interest rates depend on the two fiscal rules, μ and ξ, changes in these rules may have different implications for the two equilibria. This point is investigated further in the next section.

4 Welfare Analysis

The aim of this section is first to evaluate the two equilibria in terms of utility. Hereafter, an equilibrium with a lower (higher) nominal interest rate is simply called a low (high) equilibrium.

To achieve this aim, first recall the expected utility function of generation t in (13),

$$V_t = V(I_t, k_t) = \underbrace{\frac{[r(1-\tau)(1-\alpha)Ak_t]^{1-\sigma}}{1-\sigma}}_{(*1)}\underbrace{\Phi(I_t)}_{(*2)}, \tag{29}$$

where $\Phi(I_t)$ is defined as

$$\Phi(I_t) \equiv \left\{ (\pi)^\sigma \left[\frac{\gamma(I_t)}{I_t}\right]^{1-\sigma} + (1-\pi)^\sigma [1-\gamma(I_t)]^{1-\sigma} \right\}.$$

The function $\Phi(I)$ is decreasing in $I : \Phi'(I) < 0$ (see Appendix A.1).

Term (*1) in (29) is associated with capital accumulation, and term (*2) in (29) is associated with nominal interest rates. Because the growth rates are the same in both equilibria, the difference in utility between the two equilibria arises solely from the difference in the nominal interest rates. Given the property of $\partial V(I_t, k_t)/\partial I_t < 0$, we conclude that a low equilibrium is superior to a high equilibrium.

Our next aim is to analyze the effects of the two fiscal rules, represented by ξ and μ, on expected utility. In particular, we consider an unanticipated and permanent decrease in ξ and μ from period 0, which is aimed at tightening fiscal discipline, and its impact on the expected utility of generation $t(\geq 0)$. First, let us focus on the fiscal rule, ξ, which controls the expenditure-to-GDP ratio. This rule has an effect on the expected utility only via term (*2) in (29), since the growth rate of capital in (19) is independent of ξ. This feature leads to the following proposition.

Proposition 2. *Suppose that there are two equilibria of the nominal interest rates in period $t \geq 0$ as in Proposition 1(i). An unanticipated, permanent decrease in the expenditure-to-GDP ratio, ξ, from period 0 lowers (raises) the nominal interest rate and thereby raises (lowers) the expected utility of each generation $t(\geq 0)$ in a low (high) equilibrium.*

[Figure 3 here.]

Proof. See Appendix A.3.

Figure 3 illustrates the effect of a decrease in ξ on the nominal interest rates. The treasury's budget constraint in (16) suggests that a decrease in the ratio of expenditure to GDP, ξ, affects the determination of the nominal interest rate via term $\gamma(I)(1-\tau)(1-\alpha)Ar(1-1/I)$ representing the rebate from the central bank to the treasury. In other words, the rebate must be reduced through changes in the nominal interest rates to respond to decreased government expenditure. In particular, the nominal interest rates have two opposing effects on the rebate, described as follows.

First, lower nominal interest rates imply a lower opportunity cost of holding money and thus, a higher reserve-deposit ratio for the private bank. The central bank's balance-sheet constraint requires that the value of the central bank's outstanding liabilities, m, equals the value of its holding of government debt, b^c. Thus, a lower nominal interest rate leads to a larger value of the central bank's holding of government debt and thereby to a larger value of the rebate to the treasury. This is a positive effect of the nominal interest rate on the rebate.

Second, lower nominal interest rates imply a lower inflation rate. This in turn raises the real value of the bond's principal, $b^c_{t-1}/(p_t/p_{t-1})$, and thus, decreases the gross return minus the principal, which equals the rebate. This is a negative effect of the nominal interest rates on the rebate. In a low (high) equilibrium, the first positive effect is smaller (larger) than the second negative effect. Therefore, a decrease in ξ leads to a decrease (an increase) in the nominal interest rate and thus, raises (lowers) the expected utility of each generation in a low (high) equilibrium, since $\partial V/\partial I < 0$.

Next, consider the fiscal rule, μ, which controls the government debt-to-GDP ratio. This rule has an effect on the expected utility via the term (*1) in (29), since the growth rate of physical capital in (19) increases as μ decreases. The rule μ also has an effect on the expected utility via term (*2) in (29), since the determination of the equilibrium nominal interest rate depends on this rule, as observed in equation (21). The following proposition summarizes the net effect of decreased μ on the expected utility of generation $t \geq 0$.

Proposition 3. *Suppose that there are two equilibria of the nominal interest rates in period $t \geq 0$ as in Proposition 1(i). Consider an unanticipated, permanent decrease in the government debt-to-GDP ratio, μ, from period 0, and its impact on the expected utility across generations.*

(i) *If $2\mu < (1-\xi)(1-\alpha) + \xi - \tau(1-\alpha) - \alpha$ holds, then a decrease in μ leads to* (a) *a trade-off between generations in terms of utility in a low equilibrium; and* (b) *a Pareto improvement in a high equilibrium.*

(ii) *If* $2\mu > (1-\xi)(1-\alpha)+\xi-\tau(1-\alpha)-\alpha$ *holds, then a decrease in* μ *leads to* (a) *a Pareto improvement in a low equilibrium; and* (b) *a trade-off between generations in terms of utility in a high equilibrium.*

The proof and interpretation of the result in Proposition 3 are as follows. Recall the indirect utility function in (29). With the capital accumulation equation in (19), equation (29) is reformulated as follows:

$$V_t = \frac{\left[r\left(1-\tau\right)\left(1-\alpha\right)Ak_0\right]^{1-\sigma}}{1-\sigma}\underbrace{\left\{\left[\left(1-\tau\right)\left(1-\alpha\right)-\mu\right]A\right\}^{t(1-\sigma)}}_{(*3)}\underbrace{\Phi\left(I_t\right)}_{(*2)}. \tag{30}$$

Equation (30) indicates that a decrease in μ has effects on the expected utility of generation t via term (*3) representing the capital accumulation effect as well as term (*2) representing the nominal interest rate effect.

Consider first term (*2), showing the effect of a decrease in μ through the nominal interest rate. As we observe below, this effect is either positive or negative depending on the initial value of μ as well as the state of the equilibrium (i.e., low or high). To investigate the effect precisely, recall the treasury's budget constraint in (21), which determines the equilibrium nominal interest rate I. This is rewritten as

$$\left[\xi-\tau\left(1-\alpha\right)-\underbrace{\mu}_{(*4)}\right]\left[\left(1-\tau\right)\left(1-\alpha\right)-\underbrace{\mu}_{(*5)}\right]A+\underbrace{\alpha\mu}_{(*6)}=\gamma\left(I\right)\left(1-\tau\right)\left(1-\alpha\right)A(1-1/I), \tag{31}$$

where the right-hand side presents the rebate from the central bank to the treasury.

Equation (31) suggests that a decrease in μ affects the nominal interest rate and the rebate in the following three ways. First, revenue from new bond issuance, represented by term (*4), decreases as μ is reduced. Second, the crowding-out effect on capital, represented by term (*5), also decreases as μ is reduced, which in turn increases government expenditure. Finally, the debt outstanding, represented by term (*6), decreases as μ is reduced.

The first two effects imply that the rebate must increase to balance the treasury's budget, whereas the last effect implies the opposite. The net effect depends on the initial value of μ. Differentiation of the left-hand side of (31), denoted by LHS, with respect to

μ yields

$$\frac{\partial LHS}{\partial \mu} \gtrless 0 \Leftrightarrow \mu \gtrless \tilde{\mu} \equiv \frac{1}{2}\left[(1-\tau)(1-\alpha)+\xi-\tau(1-\alpha)-\alpha\right].$$

If μ is high such that $\mu > \tilde{\mu}$ holds, a decrease in μ results in a decrease in LHS. This implies there is a decrease in the rebate, which is realized by a decrease (an increase) in the nominal interest rate in a low (high) equilibrium, as illustrated in Figure 4. Therefore, a decrease in μ works to raise (lower) the expected utility through term (*2) in (30) in a low (high) equilibrium if $\mu < \tilde{\mu}$ holds. The opposite result holds if μ is high such that $\mu > \tilde{\mu}$.

[Figure 4 is here.]

Next, consider term (*3) in (30), showing a positive effect on expected utility through capital accumulation. A decrease in μ lowers the debt-to-GDP ratio, reduces the crowding-out effect of government debt, and thus, enhances capital accumulation. When the effect via term (*2) is positive, as in a high equilibrium in Proposition 3(i) and a low equilibrium in Proposition 3(ii), a decrease in μ definitely improves the expected utility of all generations from generation 0.

When the effect via term (*2) is negative, as in a low equilibrium in Proposition 3(i) and a high equilibrium in Proposition 3(ii), generation 0 becomes worse off, because there is no positive effect via term (*3) in period 0. However, all generations from some generation, say generation T, onward benefit from a decrease in μ, because the positive effect via term (*3) accumulates over time and starts to dominate the negative effect via term (*2) in some future period T. Therefore, there is a trade-off in terms of the expected utility between current and future generations.

Thus far, we have assumed two equilibria of the nominal interest rates. However, as presented in Proposition 1(ii), the low equilibrium may fail to satisfy the restriction on the central bank's holding of government bonds. When this is the case, we obtain the following corollary of Proposition 3.

Corollary 1. *Suppose that there is a unique equilibrium of the nominal interest rate, as in Proposition 1(ii). Consider an unanticipated, permanent change in fiscal rules from period 0.*

(i) *A decrease in ξ raises the nominal interest rate and thus, lowers the expected utility of generation 0.*

(ii) *If* $2\mu < (>)(1-\tau)(1-\alpha)+\xi-\tau(1-\alpha)-\alpha$ *holds, a decrease in* μ *lowers (raises) the nominal interest rate and thus, raises (lowers) the expected utility of generation 0.*

The first result shows that the current generation has little incentive to cut the expenditure-to-GDP ratio, and thus, suggests the difficulty of fiscal reform that is aimed at cutting public expenditure. The second result implies that the effect of a decrease in the debt-to-GDP ratio, μ, depends on the initial value of μ. When μ is low such that $2\mu < (1-\xi)(1-\alpha)+\xi-\tau(1-\alpha)-\alpha$, a decrease in μ benefits the current generation, and thus, incentivizes the current generation to decrease the ratio further. However, when μ, is high such that $2\mu > (1-\xi)(1-\alpha)+\xi-\tau(1-\alpha)-\alpha$, the current generation opposes a reduction of μ, because it lowers their expected utility. Thus, the result in Corollary 1 provides a possible explanation for an observed difference in debt-to-GDP ratios among developed countries sharing similar economic characteristics (OECD, 2018). The result also explains why fiscal reform is difficult in countries with already high debt-to-GDP ratios, such as Greece, Italy, and Japan.

5 Case of High Risk Aversion

Thus far, we have conducted the analysis by assuming that σ of the utility function $(c)^{1-\sigma}/(1-\sigma)$ is set within the range, $(0,1)$, as in Schreft and Smith (2002). Under this assumption, higher nominal interest rates imply higher opportunity costs of holding reserves, and thereby lead to a lower reserve-deposit ratio for the private bank. However, in their companion paper, Schreft and Smith (1998) argued that the demand for reserves is independent of the nominal rate of interest in their framework, and assumed $\sigma > 1$.

Given these conflicting arguments in the literature, this section provides the characterization of the competitive equilibrium when $\sigma > 1$. This assumption implies that agents are highly risk averse. We show that when $\sigma > 1$, the equilibrium, if it exists, is always unique, and that the welfare effects of changes in fiscal rules are totally different from the previous welfare effects.

Recall equation (21), which characterizes the equilibrium nominal interest rate for $t \geq 0$. When $\sigma > 1$, LHS and RHS in (21) have the following properties:

$$\frac{\partial LHS}{\partial I} > 0, \quad LHS|_{I=1} = 0, \quad \lim_{I\to\infty} LHS = \alpha(1-\tau)(1-\alpha),$$
$$\frac{\partial RHS}{\partial I} < 0, \; RHS|_{I=1} = \phi(\cdot)\frac{1}{\pi}, \quad \lim_{I\to\infty} RHS = \phi(\cdot).$$

These properties imply that the solution of (21), if it exists, is unique as illustrated in Figure 5. In other words, there is no multiplicity of equilibria when $\sigma > 1$. Thus, when $\sigma > 1$, there is a unique I that satisfies (21) if the following condition holds:

$$\lim_{I \to \infty} LHS > \lim_{I \to \infty} RHS \Leftrightarrow \alpha (1 - \tau)(1 - \alpha) > \phi(\cdot). \tag{32}$$

[Figure 5 is here.]

We reformulate the condition in (32) in terms of μ, as follows:

$$f(\mu) < 0,$$

where $f(\cdot)$ is defined by

$$\begin{aligned} f(\mu) \equiv (\mu)^2 - 2\left[(1-\tau)(1-\alpha) - \frac{1}{2}(1-\xi)\right]\mu \\ + (1-\tau)(1-\alpha)\left[(1-\tau)(1-\alpha) - (1-\xi)\right]. \end{aligned}$$

$f(\mu) = 0$ has two solutions, $\mu = (1-\tau)(1-\alpha)$ and $(1-\tau)(1-\alpha) - (1-\xi)$, and $f(\mu) < 0$ holds for $\mu \in ((1-\tau)(1-\alpha) - (1-\xi), (1-\tau)(1-\alpha))$. With the non-negative constraint of μ, we reformulate the condition in (32) as follows:

$$\max\{0, (1-\tau)(1-\alpha) - (1-\xi)\} < \mu < (1-\tau)(1-\alpha). \tag{33}$$

Next, consider the restriction on the central bank's holding of government bonds in (22). Using the definition of $\gamma(\cdot)$ in (9), we reformulate (22) as follows:

$$\frac{(1-\tau)(1-\alpha) - \theta\mu}{\theta\mu} \leq \frac{1-\pi}{\pi}(I)^{(1-\sigma)/\sigma}, \tag{34}$$

or

$$I \leq \hat{I}, \tag{35}$$

where $\hat{I}$ is defined in (23). Thus, when $\sigma > 1$, the restriction is satisfied if the nominal interest rate I is below the critical level $\hat{I}$. This result is opposite to (24) for the case of $\sigma \in (0, 1)$.

The analysis thus far suggests that there is a unique equilibrium nominal interest rate $I(> 1)$ if (33) and (35) hold. These conditions are summarized in the following proposition.

Proposition 4. *Suppose that $\sigma > 1$ holds. There is a unique equilibrium of the nominal interest rate if the following condition holds:*

$$\max\{0,(1-\tau)(1-\alpha)-(1-\xi)\}<\mu<\frac{\pi}{\theta}(1-\tau)(1-\alpha) \text{ and } \phi(\cdot)\leq\alpha\theta\mu\left(1-1/\hat{I}\right). \tag{36}$$

Proof. See Appendix A.4.

The lower bound of μ corresponds to the condition in (32). The upper bound of μ, $\pi(1-\tau)(1-\alpha)/\theta$, implies that the critical value of I, $\hat{I}$, is greater than 1 if $\mu < \pi(1-\tau)(1-\alpha)/\theta$. Otherwise, $\hat{I}$ is below 1, so that there is no equilibrium nominal interest rate $I(>1)$ that satisfies the restriction in (35). The condition $\phi(\cdot) \leq \alpha\theta\mu\left(1-1/\hat{I}\right)$ ensures that the solution of (21) satisfies the restriction in (35).

Given the characterization of the equilibrium in Proposition 4, we investigate the effects of fiscal rules, ξ and μ, on the equilibrium nominal interest rate as well as the expected utility of agents.

Proposition 5. *Suppose that $\sigma > 1$ and (36) hold. Consider an unanticipated, permanent decrease in ξ and μ from period 0.*

(i) *A decrease in ξ lowers the nominal interest rate and thereby raises the expected utility of generation 0.*

(ii) *A decrease in μ raises (lowers) the nominal interest rate and thereby lowers (raises) the expected utility of generation 0 if $2\mu < (>)\,(1-\tau)(1-\alpha)+\xi-\tau(1-\alpha)-\alpha$.*

Proof. See Appendix A.5.

Given that $\partial V/\partial I < 0$, the welfare of generation 0 is raised (lowered) by decreases in ξ and μ if the nominal interest rate decreases (increases). The result in Proposition 5 resembles (opposes) that obtained in a low (high) equilibrium when $\sigma \in (0,1)$ (see Propositions 2 and 3). Thus, if the low equilibrium when $\sigma \in (0,1)$ fails to satisfy the restriction on the central bank's holding of government bonds, as in Corollary 1, the effects on the expected utility are different in the two cases, $\sigma \in (0,1)$ and $\sigma > 1$. In other words, the welfare effects of changes in the fiscal rules depend heavily on the curvature of the utility function related to risk aversion. This result suggests that agents' attitudes toward risks shape the political feasibility of fiscal reforms.

6 Concluding Remarks

This study presented a monetary growth model with financial intermediation, in which the monetary base of the central bank is constrained by its holding of government debt, and introduced two fiscal rules into the model, that is, the debt rule, which keeps the debt-to-GDP ratio constant, and the expenditure rule, which keeps the expenditure-to-GDP ratio constant. Within this framework, the study investigated the effects of these two fiscal rules on the nominal interest rate, growth rate of physical capital, and welfare across generations. The analysis showed that strengthening the fiscal rules reduces the crowding-out effects of government debt, promotes physical capital accumulation, and benefits future generations. However, there could be multiple equilibria of the nominal interest rates, and thus, the welfare effects of the fiscal restraints through the nominal interest rates are different between the two equilibria. Thus, the net welfare consequences are also different between the two equilibria.

The present study focused on the debt rule to provide a mechanism for explaining the cross-country difference in government debt accumulation. It was shown that under a certain condition, the equilibrium nominal interest rate is unique and distinguished by the higher nominal interest rate, because the lower rate fails to satisfy the central bank's balance sheet constraint. In this situation, a reduction in the debt-to-GDP ratio raises (lowers) the expected utility of the current generation when the ratio is initially low (high). This result suggests that the current generation supports (opposes) a reduction of the ratio from the viewpoint of its utility when the ratio is initially low (high). Thus, low-ratio countries have an incentive to reduce the ratios further, while high-ratio countries have no such incentive. This result provides a mechanism for the cross-country differences in government debt accumulation. In addition, this mechanism could be viewed as an alternative to that proposed by Song, Storesletten, and Zilibotti (2012), who showed that the variation in countries' preferences for public goods is key to explaining the cross-country differences in government debt accumulation.

As a caveat to the analysis, it should be noted that the present analysis is based on the assumption that the treasury acts first and the central banks follow, because the present study focuses on the fiscal rules effects. An alternative assumption is that the central bank acts first and sets a sequence of the nominal interest rates, as in Schreft and Smith (2002, Section 3). In addition, regarding the fiscal rules, there are alternatives, such as the balanced-budget rule (Azzimonti, Battaglini, and Coate, 2016), the golden rule of public finance (Greiner and Semmler, 2000; Ghosh and Mourmouras, 2004a, 2004b; Minea and Villieu, 2009; Ueshina, 2018); and a fixed ratio of primary surplus to GDP (Bohn, 1995, 1998; Greiner, 2008). These alternatives are expected to provide interesting implications, and are left for future study.

A Proofs

A.1 Proof of $\partial V(I_t, k_t)/\partial I_t < 0$

Define $\tilde{V}(I_t)$ as follows:

$$\begin{aligned}
\tilde{V}\left(I_t\right) &\equiv \frac{V(I_t, k_t)}{\left[r\left(1-\tau\right)\left(1-\alpha\right)Ak_t\right]^{1-\sigma}} \\
&= \frac{1}{1-\sigma}\left\{\left(\pi\right)^{\sigma}\left[\frac{\gamma\left(I_t\right)}{I_t}\right]^{1-\sigma} + \left(1-\pi\right)^{\sigma}\left[1-\gamma\left(I_t\right)\right]^{1-\sigma}\right\}.
\end{aligned}$$

Differentiation of $\tilde{V}\left(I_t\right)$ with respect to I_t leads to

$$\begin{aligned}
\frac{\partial\tilde{V}\left(I_t\right)}{\partial I_t} =& -\left(\pi\right)^{\sigma}\left[\frac{\gamma\left(I_t\right)}{I_t}\right]^{-\sigma}\left(\frac{1}{I_t}\right)^{2}\frac{\frac{1-\pi}{\pi}\frac{1}{\sigma}\left(I_t\right)^{(1-\sigma)/\sigma}+1}{\left[\frac{1-\pi}{\pi}\left(I_t\right)^{(1-\sigma)/\sigma}+1\right]^{2}} \\
&+\left(1-\pi\right)^{\sigma}\left[1-\gamma\left(I_t\right)\right]^{-\sigma}\frac{\frac{1-\pi}{\pi}\frac{1-\sigma}{\sigma}\left(I_t\right)^{[(1-\sigma)/\sigma]-1}}{\left[\frac{1-\pi}{\pi}\left(I_t\right)^{(1-\sigma)/\sigma}+1\right]^{2}}.
\end{aligned}$$

The first term on the right-hand side is negative; the second term on the right-hand side is positive as long as $\sigma \in (0, 1)$. To determine the net effect, we reformulate the above expression as follows:

$$\begin{aligned}
\frac{\partial\tilde{V}\left(I_t\right)}{\partial I_t} =& \left(I_t\right)^{\sigma-2}\left[\frac{1-\pi}{\pi}\left(I_t\right)^{(1-\sigma)/\sigma}+1\right]^{\sigma-2} \\
&\times\left\{-\left(\pi\right)^{\sigma}\left[\frac{1-\pi}{\pi}\frac{1}{\sigma}\left(I_t\right)^{(1-\sigma)/\sigma}+1\right]+\left(\pi\right)^{\sigma}\frac{1-\pi}{\pi}\frac{1-\sigma}{\sigma}\left(I_t\right)^{(1-\sigma)/\sigma}\right\} \\
=& \left(I_t\right)^{\sigma-2}\left[\frac{1-\pi}{\pi}\left(I_t\right)^{(1-\sigma)/\sigma}+1\right]^{\sigma-2}\left(\pi\right)^{\sigma}\left[-\frac{1-\pi}{\pi}\left(I_t\right)^{(1-\sigma)/\sigma}-1\right] \\
<& \ 0,
\end{aligned}$$

where the last inequality holds because the sign of term $-\frac{1-\pi}{\pi}\left(I_t\right)^{(1-\sigma)/\sigma}-1$ is negative.

■

A.2 Proof of Proposition 1

To show the existence of an equilibrium nominal interest rate, recall equation (21), which determines the equilibrium nominal interest rate. Figure 1 graphically illustrates equation (21), using I on the horizontal axis. Let $\tilde{I}$ denote the value of I that satisfies $\partial LHS/\partial I = \partial RHS/\partial I$. Solving $\partial LHS/\partial I = \partial RHS/\partial I$ for I, we obtain

$$\tilde{I} \equiv \left[\frac{\alpha\left(1-\tau\right)\left(1-\alpha\right)}{\phi\left(\cdot\right)\frac{1-\sigma}{\sigma}\frac{1-\pi}{\pi}}\right]^{\sigma}.$$

As observed in Figure 1, there are two solutions of equation (21) distinguished by $I > 1$ if the following conditions hold:

$$\tilde{I} > 1 \text{ and } LHS(\tilde{I}) > RHS(\tilde{I}). \tag{37}$$

The two conditions in (37) are reformulated as follows:

$$\tilde{I} > 1 \Leftrightarrow \phi(\cdot) < \frac{\alpha(1-\tau)(1-\alpha)}{\frac{1-\sigma}{\sigma}\frac{1-\pi}{\pi}}, \tag{38}$$

and

$$LHS(\tilde{I}) > RHS(\tilde{I}) \Leftrightarrow \frac{\phi(\cdot)}{\alpha(1-\tau)(1-\alpha)} + \frac{1}{1-\sigma}\left[\frac{\phi(\cdot)\frac{1-\sigma}{\sigma}\frac{1-\pi}{\pi}}{\alpha(1-\tau)(1-\alpha)}\right]^{\sigma} < 1. \tag{39}$$

Given conditions in (38) and (39), either of the following two cases occur: (i) both solutions satisfy the restriction of the central bank's holdings of government bonds in (22); and (ii) only one of the solutions satisfies the restriction in (22). In what follows, we consider these two cases in turn.

(i) The two solutions satisfy the restriction of the central bank's holdings of government bonds in (22) if either of the following holds:

$$\hat{I} \leq 1, \tag{40}$$

or

$$\hat{I} > 1, \quad \left.\frac{\partial LHS}{\partial I}\right|_{I=\hat{I}} > \left.\frac{\partial RHS}{\partial I}\right|_{I=\hat{I}}, \text{ and } LHS(\hat{I}) < RHS(\hat{I}). \tag{41}$$

Panels (a) and (b) of Figure 1 illustrate cases satisfying (40) and (41), respectively.

The condition in (40) is reformulated as follows:

$$\hat{I} \leq 1 \Leftrightarrow \pi(1-\tau)(1-\alpha) \leq \mu. \tag{42}$$

The three conditions in (41) are reformulated as follows:

$$\hat{I} > 1 \Leftrightarrow \pi(1-\tau)(1-\alpha) > \mu, \tag{43}$$

$$\left.\frac{\partial LHS}{\partial I}\right|_{I=\hat{I}} > \left.\frac{\partial RHS}{\partial I}\right|_{I=\hat{I}} \Leftrightarrow \frac{\phi(\cdot)\frac{1-\sigma}{\sigma}\frac{1-\pi}{\pi}}{\alpha(1-\tau)(1-\alpha)} < \frac{1}{\left(\frac{\pi}{1-\pi}\right)^{1/(1-\sigma)}\left[\frac{(1-\tau)(1-\alpha)}{\mu} - 1\right]^{1/(1-\sigma)}}, \tag{44}$$

$$LHS(\hat{I}) < RHS(\hat{I}) \Leftrightarrow \alpha\mu\left(1 - \frac{1}{\hat{I}}\right) < \phi(\cdot) \tag{45}$$

Thus, there are multiple equilibria of nominal interest rates if (38), (39), and either (42) or the combination of (43), (44), and (45) hold. These conditions are summarized in the first part of Proposition 1.

(ii) As illustrated in Panel (c) of Figure 1, a lower nominal interest rate fails to satisfy the restriction in (22) if the following conditions hold:

$$\hat{I} > 1 \text{ and } LHS(\hat{I}) > RHS(\hat{I}).$$

These conditions are reformulated as follows:

$$\hat{I} > 1 \Leftrightarrow \pi (1-\tau)(1-\alpha) > \mu, \tag{46}$$

$$LHS(\hat{I}) > RHS(\hat{I}) \Leftrightarrow \phi(\cdot) < \alpha\mu \left(1 - \frac{1}{\hat{I}}\right). \tag{47}$$

Thus, there is a unique equilibrium of the nominal interest rate if (38), (39), (46), and (47) hold. These are summarized in the second part of Proposition 1.

■

A.3 Proof of Proposition 2

Recall (21), which determines the equilibrium nominal interest rates. LHS of (21) is independent of ξ, whereas RHS of (21) decreases as ξ decreases. Thus, as illustrated in Figure 3, the nominal interest rate decreases (increases) in a low (high) equilibrium as ξ decreases.

Recall equation (29), which presents the expected utility of generation t, V_t. The equation indicates that V_t depends on k_t and I_t, but k_t is independent of ξ, as demonstrated in (19). Thus, given the property of $\partial V/\partial I < 0$, we obtain the result in Proposition 2.

■

A.4 Proof of Proposition 4

If $\mu \geq (1-\tau)(1-\alpha)/\theta$, (34) always holds for $I > 1$ because the left-hand side of (34) is non-positive. Thus, the solution of equation (21) always satisfies the restriction in (35). If $\mu < (1-\tau)(1-\alpha)/\theta$, the solution of equation (21) must be below $\hat{I}$. Since the equilibrium I must be also greater than 1, the critical value $\hat{I}$ must be greater than 1:

$$\hat{I} > 1 \Leftrightarrow \frac{\pi}{\theta}(1-\tau)(1-\alpha) > \mu.$$

As depicted in Figure 5, the solution of (21) satisfies the restriction in (35) if $LHS|_{I=\hat{I}} \geq RHS|_{I=\hat{I}}$ holds, that is, if $\alpha\theta\mu\left(1 - 1/\hat{I}\right) \geq \phi(\cdot)$ holds. Thus, the solution of equation (21) satisfies the restriction in (35) if either of the following conditions holds:

$$\mu \geq \frac{1}{\theta}(1-\tau)(1-\alpha), \tag{48}$$

or

$$\mu < \frac{\pi}{\theta}\left(1-\tau\right)\left(1-\alpha\right) \text{ and } \phi\left(\cdot\right) \leq \alpha\theta\mu\left(1-1/\hat{I}\right). \tag{49}$$

There is a nominal interest rate $I\,(\geq 1)$ that satisfies (21) and (35) if either (i) (33) and (48) hold, or (ii) (33) and (49) hold. There is no $\mu(>0)$ that satisfies the first sufficient condition. The second sufficient condition is summarized as follows:

$$\max\left\{0, \left(1-\tau\right)\left(1-\alpha\right)-\left(1-\xi\right)\right\} < \mu < \frac{\pi}{\theta}\left(1-\tau\right)\left(1-\alpha\right) \text{ and } \phi\left(\cdot\right) \leq \alpha\theta\mu\left(1-1/\hat{I}\right). \tag{50}$$

Thus, there is a unique equilibrium of the nominal interest rate $I\,(\geq 1)$ if (50) holds.

■

A.5 Proof of Proposition 5

Recall equation (21), which determines the equilibrium nominal interest rate for $t \geq 0$, which is rewritten as

$$\frac{\alpha\left(1-\tau\right)\left(1-\alpha\right)\left(1-1/I\right)}{\frac{1-\pi}{\pi}\left(I\right)^{(1-\sigma)/\sigma}+1} = \phi\left(\cdot\right). \tag{51}$$

The left-hand side of (51) is increasing in I when $\sigma > 1$ and is independent of ξ and μ. The right-hand side is independent of I but is affected by ξ and μ in the following way:

$$\partial\phi\left(\cdot\right)/\partial\xi > 0,$$
$$\partial\phi\left(\cdot\right)/\partial\mu \gtrless 0 \Leftrightarrow 2\mu \gtrless \left(1-\tau\right)\left(1-\alpha\right)+\xi-\tau(1-\alpha)-\alpha.$$

Thus, we obtain

$$\partial I/\partial\xi > 0,$$
$$\partial I/\partial\mu \gtrless 0 \Leftrightarrow 2\mu \gtrless \left(1-\tau\right)\left(1-\alpha\right)+\xi-\tau(1-\alpha)-\alpha.$$

From (29), the expected utility of generation 0 is

$$V_0 = V(I_0, k_0) = \frac{\left[r\left(1-\tau\right)\left(1-\alpha\right)Ak_0\right]^{1-\sigma}}{1-\sigma}\Phi\left(I_0\right),$$

where $\Phi\left(I_0\right)$ is decreasing in I_0: $\Phi'\left(I_0\right) < 0$. Since k_0 is a given initial condition, a higher (lower) I_0 is associated with a lower (higher) V_0.

■

References

[1] Antinolfi, G., and Keister, T., 2006. Discount window policy, banking crises, and indeterminacy of equilibrium. *Macroeconomic Dynamics* 10, 1–19.

[2] Antinolfi, G., Landeo, C.M., and Nikitin, M., 2007. Dollarization and the inflation threshold. *Canadian Journal of Economics* 40, 628–649.

[3] Azzimonti, M., Battaglini, M., and Coate, S., 2016. The costs and benefits of balanced budget rules: Lessons from a political economy model of fiscal policy. *Journal of Public Economics* 136, 45–61.

[4] Barro, R.J., 1990. Government spending in a simple model of endogenous growth. *Journal of Political Economy* 98, S103–S125.

[5] Bhattacharya, J., Guzman, M.G., Huybens, E., and Smith, B.D., 1997. Monetary, fiscal, and reserve requirement policy in a simple monetary growth model. *International Economic Review* 38, 321–350.

[6] Bhattacharya, J., Haslag, J., and Martin, A., 2009. Optimal monetary policy and economic growth. *European Economic Review* 53, 210–221.

[7] Bohn, H., 1995. The sustainability of budget deficit in a stochastic economy. *Journal of Money, Credit and Banking* 27, 257–271.

[8] Bohn, H., 1998. The behavior of US public debt and deficits. *Quarterly Journal of Economics* 113, 949–963.

[9] Bom, P.R.D, and Ligthart, J.E., 2014. Public infrastructure investment, output dynamics, and balanced budget fiscal rules. *Journal of Economic Dynamics and Control* 40, 334–354.

[10] Budina, N., Kinda, T., Schaechter, A., and Weber, A., 2012. Fiscal rules at a glance: Country details from a new dataset. IMF Working Paper 12/273, International Monetary Fund.

[11] Diamond, D.W., and Dybvig, P.H., 1983. Bank runs, deposit insurance, and liquidity. *Journal of Political Economy* 91, 401–419.

[12] Diamond, P.A., 1965. National debt in a neoclassical growth model. *American Economic Review* 55, 1126–1150.

[13] Espinosa-Vega, M.A., and Yip, C.K., 1999. Fiscal and monetary policy interactions in an endogenous growth model with financial intermediaries. *International Economic Review* 40, 595–616.

[14] Espinosa-Vega, M.A., and Yip, C.K., 2002. Government financing in an endogenous growth model with financial market restrictions. *Economic Theory* 20, 237–257.

[15] Fernandez-Huertas Moraga, J., and Vidal, J.-P., 2010. Fiscal sustainability and public debt in an endogenous growth model. *Journal of Pension and Economic Finance* 9, 277–302.

[16] Ghosh, S., and Mourmouras, I.A., 2004a. Endogenous growth, welfare and budgetary regimes. *Journal of Macroeconomics* 26, 623–635.

[17] Ghosh, S., and Mourmouras, I.A., 2004b. Debt, growth and budgetary regimes. *Bulletin of Economic Research* 56, 241–250.

[18] Ghossoub, E.A., and Reed, R.R., 2010. Liquidity risk, economic development, and the effects of monetary policy. *European Economic Review* 54, 252–268.

[19] Ghossoub, E.A., and Reed, R.R., 2014. The cost of capital, asset prices, and the effects of monetary policy. *Journal of Macroeconomics* 42, 211–228.

[20] Ghossoub, E.A., and Reed, R.R., 2017. Financial development, income inequality, and the redistributive effects of monetary policy. *Journal of Development Economics* 126, 167–189.

[21] Glomm, G., Jung, J., and Tran, C., 2018. Fiscal austerity measures: Spending cuts vs. tax increases. *Macroeconomic Dynamics* 22, 501–540.

[22] Gomis-Porqueras, P., 2000. Money, banks and endogenous volatility. *Economic Theory* 15, 735–745.

[23] Greiner, A., 2008. Human capital formation, public debt and economic growth. *Journal of Macroeconomics* 30, 415–427.

[24] Greiner, A., and Semmler, W., 2000. Endogenous growth, government debt and budgetary regimes. *Journal of Macroeconomics* 22, 363–384.

[25] Heijdra, B.J., and Ligthart, J.E., 2000. The dynamic macroeconomic effects of tax policy in an overlapping generations model. *Oxford Economic Papers* 52, 677–701.

[26] Hung, F.-S., 2005. Optimal composition of government public capital financing. *Journal of Macroeconomics* 27, 704–723.

[27] International Monetary Fund, 2009. Fiscal rules—Anchoring expectations for sustainable public finances. Staff Board Paper, December 16. https://www.imf.org/external/np/pp/eng/2009/121609.pdf (Accessed on July 21, 2018).

[28] Matsuoka, T., 2011. Monetary policy and banking structure. *Journal of Money, Credit, and Banking* 43, 1109–1120.

[29] Minea, A., and Villieu, P., 2009. Borrowing to finance public investment? The 'golden rule of public finance' reconsidered in an endogenous growth setting. *Fiscal Studies* 30, 103–133.

[30] OECD, 2018. General government debt (indicator). doi: 10.1787/a0528cc2-en (Accessed on August 2, 2018).

[31] Paal, B., Smith, B.D, and Wang, K., 2013. Monopoly versus competition in banking: Some implications for growth and welfare. *Annals of Economics and Finance* 14, 853–892.

[32] Schaechter, A., Kinda, T., Budina, N., and Weber, A., 2012. Fiscal rules in response to the crisis—Toward the "next generation" rules. A new dataset. IMF Working Paper 12/187, International Monetary Fund.

[33] Schreft, S.L., and Smith, B.D., 1997. Money, banking, and capital formation. *Journal of Economic Theory* 73, 157–182.

[34] Schreft, S.L., and Smith, B.D., 1998. The effects of open market operations in a model of intermediation and growth. *Review of Economic Studies* 65, 519–550.

[35] Schreft, S.L., and Smith, B.D., 2000. The evolution of cash transactions: Some implications for monetary policy. *Journal of Monetary Economics* 46, 97–120.

[36] Schreft, S.L., and Smith, B.D., 2002. The conduct of monetary policy with a shrinking stock of government debt. *Journal of Money, Credit, and Banking* 34, 848–882.

[37] Song, Z., Storesletten, K., and Zilibotti, F., 2012. Rotten parents and disciplined children: A politico-economic theory of public expenditure and debt. *Econometrica* 80, 2785–2803.

[38] Ueshina, M., 2018. The effect of public debt on growth and welfare under the golden rule of public finance. *Journal of Macroeconomics* 55, 1–11.

[39] Yakita, A., 2008. Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting. *Journal of Public Economics* 92, 897–914.

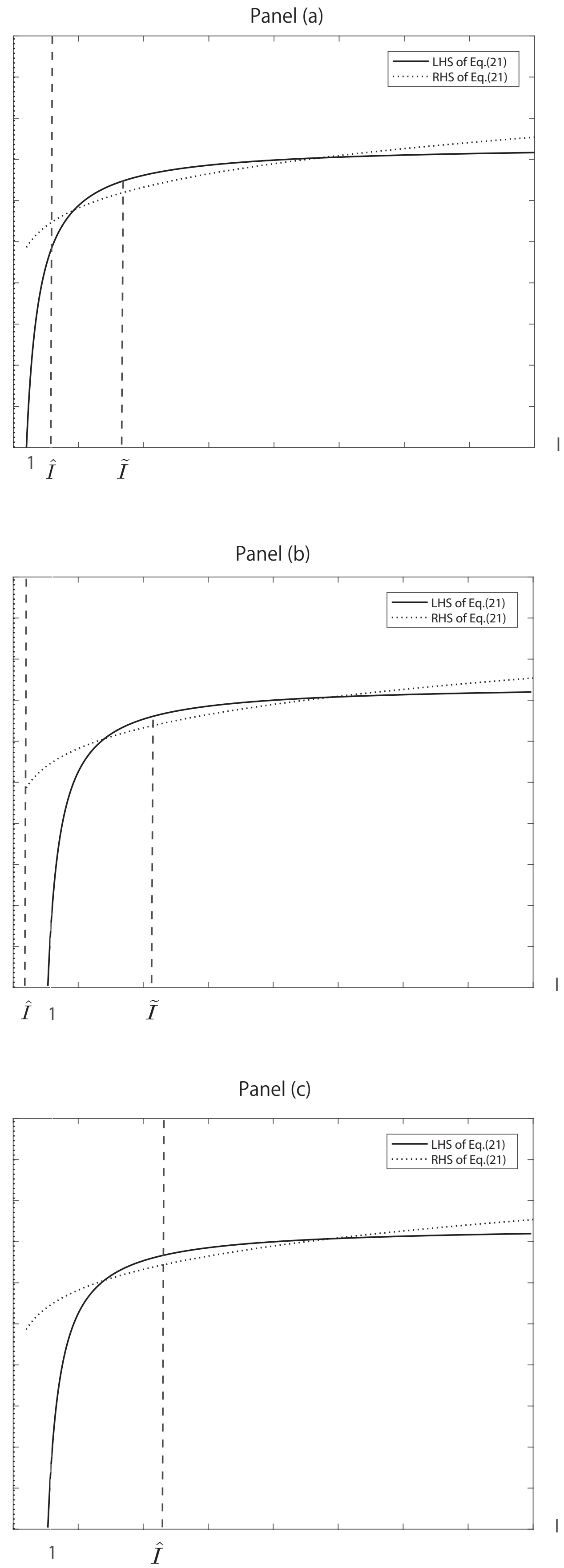

Figure 1: Illustration of LHS and RHS of Eq. (21), using I on the horizontal axis.

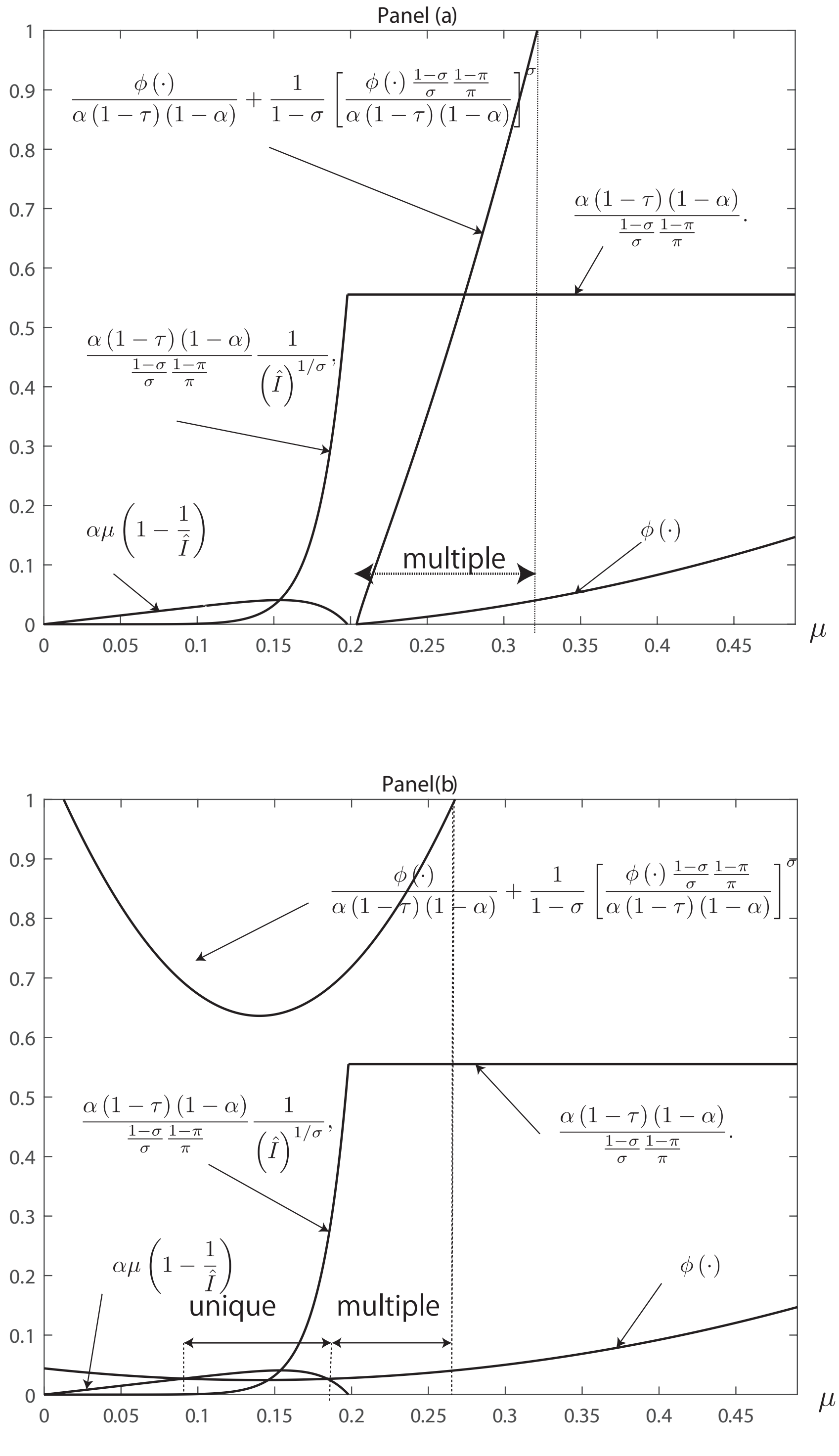

Figure 2: Conditions (25)–(28) in Proposition 1 when $\xi = 0.2$ (Panel (a)) and $\xi = 0.3$ (Panel (b)). The horizontal axis is μ. Other parameter values are set as $\theta = 0.99$, $\sigma = 0.85$, $\pi = 0.4$, $\alpha = 0.3$, and $\tau = 0.3$; these values are used in Figures 3 and 4.

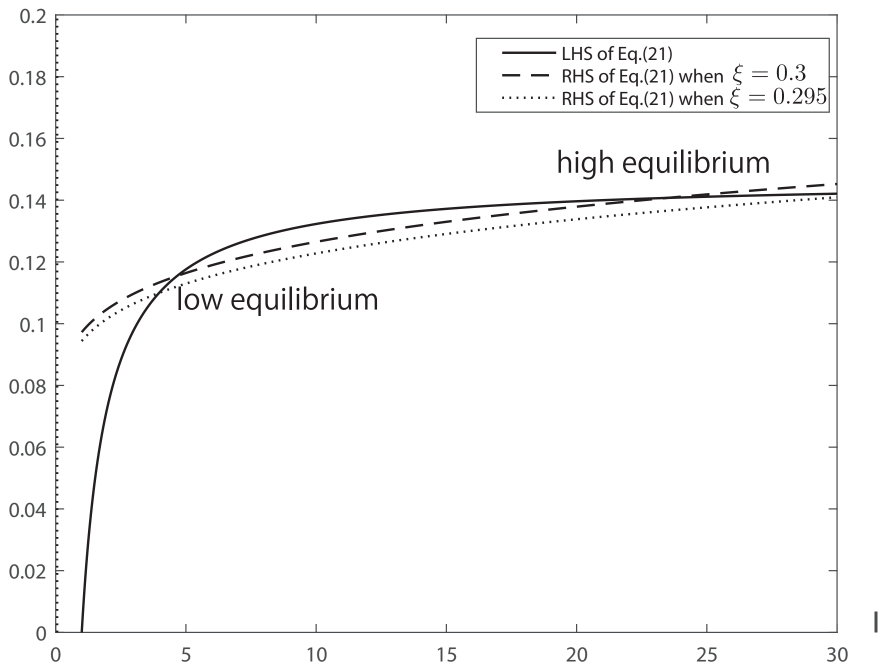

Figure 3: Illustration of Eq. (21) when $\mu = 0.26$ and $\xi = 0.3$ or 0.295, using I on the horizontal axis.

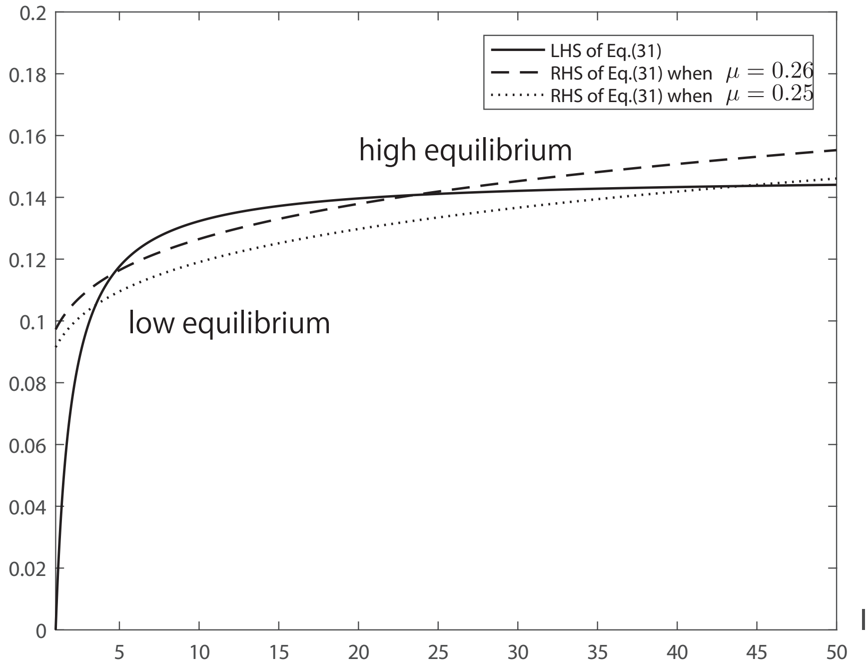

Figure 4: Illustration of Eq. (31) when $\xi = 0.3$ and $\mu = 0.26$ or 0.25, using I on the horizontal axis.

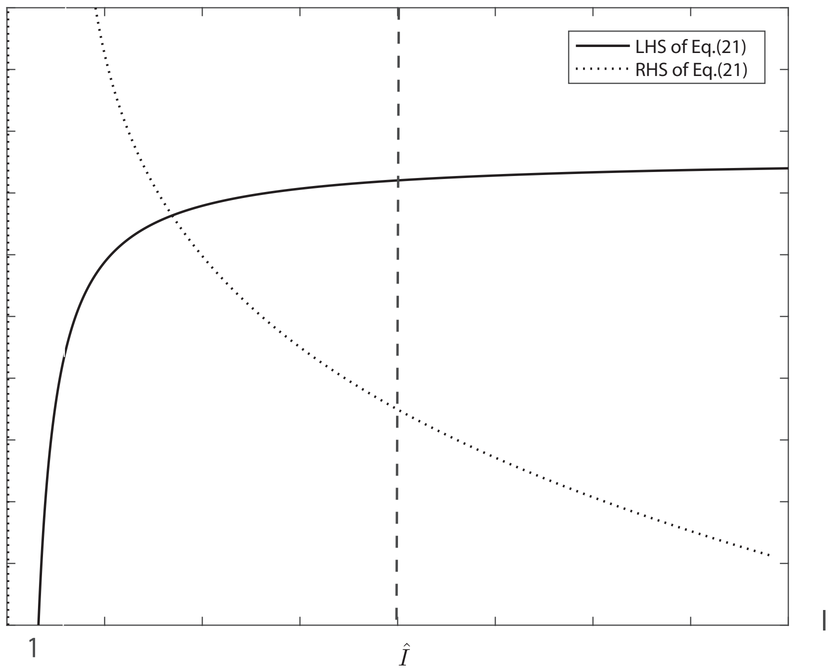

Figure 5: Illustration of Eq. (21) when $\sigma > 1$, using I on the horizontal axis.