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The effects of asset liquidity on dynamic bankruptcy decisions *

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Abstract

We develop a dynamic bankruptcy model with asset illiquidity. In the model, a distressed firm chooses between sell-out and default, as well as its timing under the assumption that sell-out is feasible only at Poisson jump times, where the arrival rate of acquirers stands for asset liquidity. With lower asset liquidity, the firm increases the sell-out region to mitigate the risk of not finding an acquirer until bankruptcy. Despite the larger sell-out region, lower asset liquidity increases the default probability and decreases the equity, debt, and firm values. In the optimal capital structure, with lower asset liquidity, the firm reduces leverage, but the cautious capital structure does not fully offset the increased default risk. The stock price reaction caused by sell-out depends on the sell-out timing. When the firm's asset value is not sufficiently high, the stock price jump size is an inverted U-shape with the economic state variable. Lower asset liquidity increases the jump size due to greater surprise. These results fit empirical observations.

JEL Classifications Code: G13; G32; G33.

Keywords: liquidation; illiquidity; real option; M&A.

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1 Introduction

Asset liquidity (or asset deployability) affects firms' operating and financing decisions (e.g., Shleifer and Vishny (1992), Schlingemann, Stulz, and Walkling (2002), and Ortiz-Molina and Phillips (2014)). For firms in financial distress, asset illiquidity is crucial because they can sell assets to avoid bankruptcy. As shown in Hotchkiss and Mooradian (1998) and Maksimovic and Phillips (1998), some firms are successfully acquired by other firms as a form of exit strategy, but others fail. For instance, the Japanese electronics manufacturer Sharp Corporation was acquired by the Taiwanese manufacturer Foxconn to get through financial distress, while the American retail company Circuit City filed for bankruptcy after it failed to be acquired by the American video company Blockbuster.

The objective of this paper is to explore the effects of asset illiquidity on a distressed firm's exit decisions. We build a model of a firm with console debt as in the standard literature (e.g., Black and Cox (1976) and Leland (1994)). As in Mella-Barral and Perraudin (1997), Nishihara and Shibata (2017), and Nishihara and Shibata (2018), the firm can sell out (i.e., be acquired by another firm) to avoid bankruptcy. On sell-out, shareholders of the firm obtain the residual value (i.e., the sales price minus the face value of debt), while debt holders are repaid the face value of debt. On default, shareholders stop coupon payments of debt, and the former debt holders take over the firm, where a fraction of the firm value is lost to bankruptcy costs. Shareholders (or equivalently, managers acting on behalf of shareholders' interests) optimize the exit choice between sell-out and default as well as its timing. A key difference from previous literature is an assumption that sell-out is feasible only at Poisson jump times. This assumption captures the illiquidity associated with searching for an acquirer.[1] Throughout the paper, we regard the arrival rate of acquirers as the firm's asset liquidity.

We analytically derive model solutions, which are classified into the following cases. When asset value over debt value and asset liquidity are high, the firm does not default but sells out for a sufficiently low state variable. When asset value over debt value is low, the firm does not sell out but defaults at the default threshold. Notably, for the intermediate parameter values, the firm has both sell-out and default regions. If an acquirer fortunately appears in the sell-out region until the state variable hits the default threshold, the firm succeeds in selling out. Otherwise, the firm defaults. For relatively high asset values, the sell-out region is below a certain threshold (i.e., threshold policy of sell-out), whereas for

[1] Cornett, Tanyeri, and Tehranian (2011) and Wang (2018) empirically show the low predictability of M&As. Rhodes-Kropf and Robinson (2008) also examine a merger market with search frictions.

relatively low asset values, the sell-out region is between an interval strictly higher than the default threshold (i.e., interval policy of sell-out). Compared with the deterministic choice between sell-out and default in Mella-Barral and Perraudin (1997), Nishihara and Shibata (2017), and Nishihara and Shibata (2018), our result of the stochastic choice fits empirical observations well.

In the model, we reveal the effects of asset liquidity on the firm's exit policy, default probability, equity, debt, and firm values. With lower asset liquidity, the firm increases the sell-out region to mitigate the risk of not finding an acquirer until bankruptcy takes place. Notably, this earnest asset sales policy cannot fully offset the decreased liquidity effect. Lower asset liquidity, despite the larger sell-out region, increases the default probability and decreases the equity, debt, and firm values. These results are consistent with empirical evidence (e.g., Ortiz-Molina and Phillips (2014), Song and Walkling (2000), and Cornett, Tanyeri, and Tehranian (2011)). In fact, Ortiz-Molina and Phillips (2014) show that lower asset liquidity reduces a firm's operating flexibility and increases the cost of capital and the bankruptcy risk. Song and Walkling (2000) and Cornett, Tanyeri, and Tehranian (2011) show that higher acquisition probabilities (which can be interpreted as higher asset liquidity in our model) tend to increase target stock prices.

In the optimal capital structure, with lower asset liquidity, the firm reduces its leverage to mitigate the increased bankruptcy risk. Intriguingly, the cautious capital structure and earnest asset sales policy cannot fully offset the decreased liquidity effect. Lower asset liquidity thus increases the default probability and decreases the debt and firm values. The positive relationship between asset liquidity and leverage is consistent with the empirical observations in Sibilkov (2009) and Campello and Giambon (2013).

The model also yields an upward jump of the equity value at the sell-out time. The upward jump can be interpreted as a stock price reaction to the acquisition. Our model shows that the stock price reaction greatly depends on the sell-out timing (i.e., the economic state variable when the firm is acquired). If asset value is sufficiently high, the jump size decreases with the state variable due to the threshold policy of sell-out. This implies that being acquired is more beneficial to shareholders as the firm is closer to default. More notably, if asset value is not sufficiently high, the jump size becomes an inverted U-shape with the state variable due to the interval-type policy of sell-out. This suggests that being acquired is not very beneficial to shareholders when the distance to default is either too short or too long. These results highlight the dependence of acquisition timing on stock price reactions and help explain the dispersion of target stock price reactions.

We also show that lower asset liquidity leads to a more positive stock price reaction because of greater surprise. This result aligns with Cornett, Tanyeri, and Tehranian (2011)'s empirical finding that stock price reactions increase with lower merger anticipation.

The remainder of this paper is organized as follows. Section 2 briefly reviews the related literature. Section 3 introduces the model setup. After Section 4.1 shows the model solutions for the unlevered firm, Section 4.2 shows the model solutions for the levered firm. Section 5 demonstrates numerical examples and economic implications. After Section 6 discusses the model extensions, Section 7 concludes the paper.

2 Literature review

This paper explores the interactions of two real options: sell-out and default. Although many real options models focus on investment timing, some papers investigate exit options (for a recent review of the real options literature, see Trigeorgis and Tsekrekos (2018)). For instance, Hagspiel, Huisman, and Nunes (2016) examine a firm's choice between investment and exit, whereas Guerra, Kort, Nunes, and Oliveira (2018) investigate a firm's mothballing option before permanent exit. The above papers do not focus on a levered firm's default decision. Regarding a default option, Black and Cox (1976) first derive default timing, which is endogenously determined in shareholders' interests, and Leland (1994) derives the optimal capital structure determined by the tradeoff between the tax benefits of debt and bankruptcy cost. As in most papers, we borrow these standard frameworks of endogenous default and optimal capital structure.

By extending the standard frameworks, many papers investigate joint investment and financing decisions. For instance, Mauer and Sarkar (2005), Shibata and Nishihara (2012), and Sundaresan, Wang, and Yang (2015) derive optimal investment timing and capital structure. Morellec, Valta, and Zhdanov (2015) and Shibata and Nishihara (2015) examine optimal investment and financing choices between market debt and private debt. Hackbarth, Mathews, and Robinson (2014) examine optimal investment, financing, and organizational form. Charalambides and Koussis (2018) investigate optimal investment and operation with financing by debt allowing future coupon reduction. Lukas and Thiergart (2019) explore the effects of debt financing on a real option held jointly by a firm and a government. These papers examine the interactions of default and investment options.

This paper considers not an investment option but a sell-out option in addition to a default option. In this sense, Mella-Barral and Perraudin (1997), Lambrecht and Myers

(2008), Nishihara and Shibata (2017), and Nishihara and Shibata (2018), who consider the choice between sell-out (or scrap) and default, are most closely related to this paper. Mella-Barral and Perraudin (1997) show that debt renegotiation prevents shareholders from early default and leads to efficient liquidation timing. Lambrecht and Myers (2008) show that debt can reduce manager-shareholder conflicts and efficiently speed up liquidation. Nishihara and Shibata (2017) and Nishihara and Shibata (2018) show that information asymmetries can change a firm's exit policy from sell-out to default. However, they do not focus on the illiquidity risk of not finding an acquirer; in their models, sell-out is feasible at any time.

Although the above papers consider full liquidation with debt repayment rather than partial liquidation, Morellec (2001) develops the model in which a distressed firm can liquidate assets partially and sequentially until bankruptcy. He shows that for unsecured debt, lower liquidation costs play negative roles in increasing bankruptcy risk and decreasing debt value through the channel of managerial asset disposition. Nishihara and Shibata (2016) also examine partial liquidation with debt restructuring and show that partial asset sales decrease equity value. These models do not include the risk of not finding an asset acquirer.

He and Xiong (2012), He and Milbradt (2014), and Chen, Cui, He, and Milbradt (2018) explore the effects of debt market illiquidity on bankruptcy decisions. They show that debt market illiquidity hastens bankruptcy timing and increases default risk via the channel of debt rollover risk. They focus on the illiquidity of the secondary debt market rather than real asset illiquidity; in fact, these models do not include real asset sales.

Technically, this paper treats the optimal stopping (sell-out timing) problem constrained within Poisson jump times. The unlevered firm's sell-out timing problem in Section 4.1 is essentially the same as the problem solved in Dupuis and Wang (2002). By extending this mathematical framework, Hugonnier, Malamud, and Morellec (2015a), Hugonnier, Malamud, and Morellec (2015b), and Morellec, Valta, and Zhdanov (2015) reveal the effects of the illiquidity of fund-raising on investment, cash holdings, and capital structure. They focus on capital and credit supply illiquidity rather than real asset illiquidity.

Despite the growing literature on dynamic corporate finance issues, to our knowledge, no paper investigates the effects of asset illiquidity (i.e., the risk of not finding an acquirer) on the dynamic bankruptcy procedure. The main contribution of this paper is to reveal how asset illiquidity distorts a firm's exit choice between sell-out and default, exit timing, bankruptcy risk, and equity, debt, and firm values, as well as optimal capital structure. In addition, this paper also yields new implications about stock market reactions in the acquisition period.

3 Model Setup

3.1 Firm until exit

The model builds on the standard setup (e.g., Mella-Barral and Perraudin (1997), Goldstein, Ju, and Leland (2001), and Lambrecht and Myers (2008)). Consider a firm with console debt with coupon C, i.e., the firm continues to pay coupon C to debt holders until bankruptcy. First, we assume an exogenous coupon C, although in Section 5 we also examine the optimal capital structure. The firm receives continuous streams of earnings before interest and taxes (EBIT) $X(t)$, where $X(t)$ follows a geometric Brownian motion

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}B(t) \ (t > 0), \quad X(0) = x,$$

where $B(t)$ denotes the standard Brownian motion defined in a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\})$ and $\mu, \sigma(> 0)$ and $x(> 0)$ are constants. Assume that the initial value, $X(0) = x$, is sufficiently high to exclude the firm's exit at the initial time. For convergence, we assume that $r > \mu$, where a positive constant r denotes the risk-free interest rate. We assume that managers operate the firm on behalf of shareholders' interests, and hence we do not distinguish between shareholders and managers. Shareholders continue to receive cash flows $(1 - \tau)(X(t) - C)$ until exit, where $\tau \in (0, 1)$ denotes a corporate tax rate.

3.2 Asset sales opportunities

Following a Poisson process with the arrival rate λ, potential acquirers arrive and bid

$$P(X(t)) = \frac{aX(t)}{r - \mu} + \theta \tag{1}$$

for the firm, where $a \in [0, 1)$ and $\theta > 0$. We assume the linear function (1) following Lambrecht (2001), Lambrecht and Myers (2008), Nishihara and Shibata (2016), and Nishihara and Shibata (2018). Equation (1) means that after partial assets are scrapped at a fixed price θ, an acquirer perpetually receives cash flows $aX(t)$ from the remaining assets, where $a \in [0, 1)$ represents that the revenues contract due to the partial liquidation. Transaction costs and the acquirer's gains can also be included in the parameters a and θ. Section 6.2 discusses a case in which an acquirer's bid price is contingent on the target stock price.

For model tractability, we assume that the Poisson process is independent of $X(t)$.[2] If the firm rejects an acquirer's proposal, the firm has to wait until another acquirer arrives.

The novelty of the model is the Poisson arrival of asset sales opportunities. We regard the arrival rate λ as the degree of asset liquidity. A lower λ means lower asset liquidity because $1/\lambda$ equals the expected duration between acquirers' arrivals. In reality, distressed firms do not always find acquirers that offer favorable bid prices, although previous papers (e.g., Lambrecht and Myers (2008), Nishihara and Shibata (2016), and Nishihara and Shibata (2018)) consider liquid models, which correspond to $\lambda = \infty$ in our model. Our illiquid model can capture the effects of asset illiquidity on the bankruptcy process. Hugonnier, Malamud, and Morellec (2015a), Hugonnier, Malamud, and Morellec (2015b), and Morellec, Valta, and Zhdanov (2015) propose dynamic investment models with illiquidity of fund-raising, which is modeled by a Poisson process, although our model focuses on the illiquidity of real asset sales rather than fund-raising.

3.3 Exit choice between sell-out and default

As in Mella-Barral and Perraudin (1997), Lambrecht and Myers (2008), and Nishihara and Shibata (2018), we consider two types of the firm's exit. A successful exit without formal bankruptcy is called sell-out. On sell-out, shareholders of the distressed firm sell all assets to an acquirer at price $P(X(t))$. According to the absolute priority rule (APR) of debt, debt holders are repaid the face value of debt, which equals C/r for the console debt. Then, shareholders receive the residual value, i.e., $(1-\tau)P(X(t)) - C/r$. Note that the asset redeployment also increases social welfare. Indeed, $P(X(t)) > X(t)/(r-\mu)$ holds because the residual value is nonnegative when shareholders prefers to sell out.

The other exit is called default, which leads to a formal bankruptcy. On default, shareholders stop coupon payments to debt holders. According to the APR, debt holders take over the firm, and shareholders receive nothing. As in the standard literature (e.g., Leland (1994), Mella-Barral and Perraudin (1997), and Goldstein, Ju, and Leland (2001)), the former debt holders can operate the firm as a going concern, but a fraction $\alpha \in (0,1)$ of the firm's asset value is lost to the bankruptcy costs (filing fees, attorney fees, etc.). Contrasted to sell-out, the deadweight costs of bankruptcy lead to social loss.

[2]Hugonnier, Malamud, and Morellec (2015a), Hugonnier, Malamud, and Morellec (2015b), and Morellec, Valta, and Zhdanov (2015) also assume independence between the economic state process and the Poisson process representing illiquidity of fund-raising. In fact, without the independence, we cannot analytically derive any solution.

For simplicity, as in Lambrecht and Myers (2008), and Nishihara and Shibata (2018), we assume no opportunity of debt renegotiation and restructuring.[3] As in the standard literature, shareholders of the distressed firm makes the exit decision in their own interests. They do not take into account of debt in place, which leads to agency conflicts between shareholders and debt holders, and debt is priced at the initial time under the rational expectation of shareholders' exit policy.

4 Model Solutions

4.1 Unlevered firm

As a benchmark, this subsection examines the all-equity firm's exit policy. Suppose that there is no coupon payment (i.e., $C = 0$). Because there is no coupon payment, there is no possibility of default. Instead, the firm can sell out when an offer is favorable. Technically, the firm's optimal sell-out timing problem is regarded as an optimal stopping problem constrained to Poisson jump times, which was solved in Dupuis and Wang (2002).

We use $E_U(x)$ to denote the firm value when an asset sale opportunity is not available, and we use $\tilde{E_U}(x)$ to denote the firm value when an acquirer arrives. We have $\tilde{E_U}(x) = \max\{(1-\tau)P(x), E_U(x)\}$ because the firm value $\tilde{E_U}(x)$ turns to $E_U(x)$ by forgoing an asset sale opportunity. As proved in Dupuis and Wang (2002), the optimal sell-out policy becomes the threshold policy, i.e., $\inf\{t \geq 0 \mid X(t) \leq x_U^s\}$, and $E_U(x)$ satisfies ordinary differential equations (ODEs):

$$\mu x E_U'(x) + 0.5\sigma^2 x^2 E_U''(x) + \lambda((1-\tau)P(x) - E_U(x)) + (1-\tau)x = rE_U(x) \ (x < x_U^s), \tag{2}$$

$$\mu x E_U'(x) + 0.5\sigma^2 x^2 E_U''(x) + (1-\tau)x = rE_U(x) \ (x > x_U^s), \tag{3}$$

where $\lambda((1-\tau)P(x) - E_U(x))$ in (2) corresponds to the fact that $E_U(x)$ changes to $(1-\tau)P(x)$ with probability $\lambda \mathrm{d}t$ in infinitesimal time interval $\mathrm{d}t$ in the sell-out region $x < x_U^s$. As shown in Dupuis and Wang (2002), $E_U^s(x)$ is continuously differentiable at x_U^s[4] and also satisfies boundary conditions $E_U^s(x_U^s) = (1-\tau)P(x_U^s), \lim_{x\to 0} E_U^s(x) < \infty$,

[3]Several papers (e.g., Morellec, Valta, and Zhdanov (2015), and Shibata and Nishihara (2015)) distinguish between private debt, which can be renegotiated, and market debt, which cannot be renegotiated. In this context, our model is more relevant to firms funded with market debt than firms funded with private debt.

[4]This is not the smooth pasting (i.e., optimality) condition in Dixit and Pindyck (1994), but it is the same condition as the piecewise C^2 property of the value function in Morellec, Valta, and Zhdanov (2015). Technically, this property follows from Theorem 4.4.9 in Karatzas and Shreve (1998).

and $\lim_{x\to\infty} E_U^s(x)/x < \infty$. The boundary condition $E_U^s(x_U^s) = (1-\tau)P(x_U^s)$ means that the firm is indifferent to whether to accept or reject an acquirer's offer at x_U^s (i.e., the optimality of x_U^s). The other boundary conditions are trivial. By solving (2) and (3) with these boundary conditions, we have the following proposition, where we define, for $y > 0$,

$$\beta_y = 0.5 - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - 0.5\right)^2 + \frac{2(r+y)}{\sigma^2}} \ (> 1), \tag{4}$$

$$\gamma_y = 0.5 - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - 0.5\right)^2 + \frac{2(r+y)}{\sigma^2}} \ (< 0), \tag{5}$$

and, we simplify the notations by $\beta = \beta_0$ and $\gamma = \gamma_0$. For the proof, refer to Appendix A.

Proposition 1

$$E_U(x) = \begin{cases} (1-\tau)\left\{\dfrac{(r+a\lambda-\mu)x}{(r-\mu)(r+\lambda-\mu)} + \dfrac{\lambda\theta}{r+\lambda} + \left(\dfrac{x}{x_U^s}\right)^{\beta_\lambda}\left(\dfrac{(a-1)x_U^s}{r+\lambda-\mu} + \dfrac{r\theta}{r+\lambda}\right)\right\} & (x \le x_U^s), \\ (1-\tau)\left\{\dfrac{x}{r-\mu} + \left(\dfrac{x}{x_U^s}\right)^{\gamma}\left(\dfrac{(a-1)x_U^s}{r-\mu} + \theta\right)\right\} & (x > x_U^s), \end{cases} \tag{6}$$

where the sell-out threshold x_U^s *is defined by*

$$x_U^s = \frac{(r+\lambda-\mu)((\gamma-\beta_\lambda)r + \gamma\lambda)}{(r+\lambda)((\gamma-\beta_\lambda)(r-\mu) + (\gamma-1)\lambda)}\frac{\theta(r-\mu)}{1-a}. \tag{7}$$

The firm's optimal policy is to sell out only if an acquirer arrives at time t satisfying $X(t) \le x_U^s$. For $X(t) \le x_U^s$, the firm is better off selling out because of a positive constant θ embedded in $P(X(t))$. The lower equation in (6) stands for the firm value when the firm value is higher than the sell-out payoff, i.e, $E_U(x) > (1-\tau)P(x)$. The upper equation in (6) stands for the firm value when the firm value is lower than the sell-out payoff, i.e, $E_U(x) \le (1-\tau)P(x)$. Then, the firm wishes to immediately sell out, but it may take time because of asset illiquidity. To be more precise, the upper equation in (6) can be decomposed as follows. The first and second terms of the equation stand for the expected payoff of selling out as soon as an acquirer appears. The last term of the equation stands for the value of the option to forgo an offer when it arrives at time t satisfying $X(t) > x_U^s$.

It should be noted that the sell-out threshold x_U^s is derived in the closed-form expression (7). By virtue of the closed-form solution, we can easily show that x_U^s monotonically decreases to the solution of the liquid model, i.e., $\gamma\theta(r-\mu)/(\gamma-1)(1-a)$ as $\lambda \to \infty$. We can also show that x_U^s monotonically increases to the solution of the net present value model, i.e., $\theta(r-\mu)/(1-a)$ as $\lambda \to 0$. Similarly, we can easily show that the firm value $E_U(x)$ converges to the corresponding solutions in the liquid and net present value models. To summarize, lower asset liquidity λ decreases the value of waiting and increases the sell-out threshold. With lower asset liquidity, the firm accepts an acquirer's offer more eagerly because the expected waiting time for another offer increases.

4.2 Levered firm

This subsection examines the levered firm's exit policy. Suppose that the firm has issued console debt with coupon $C(>0)$ at the initial time. We use $E(x), D(x)$, and $F(x) = E(x)+D(x)$ to denote the equity, debt, and firm values when sell-out is infeasible, while we use $\tilde{E}(x), \tilde{D}(x)$, and $\tilde{F}(x) = \tilde{E}(x)+\tilde{D}(x)$ to denote the equity, debt, and firm values when sell-out is feasible. We have $\tilde{E}(x) = \max\{(1-\tau)P(x) - C/r, E(x)\}$[5] because shareholders optimize the choice between sell-out and forgoing an acquirer. Recall that shareholders receive the residual value $(1-\tau)P(x) - C/r$ on sell-out.

Clearly, $E(x)$ is a nonnegative and nondecreasing function. $E(x)$ is also higher than $(1-\tau)x/(r-\mu) - C/r$, which is the payoff of perpetually operating the firm. Throughout the paper, we suppose that $E(x)$ is a convex function.[6] Then, the two functions $E(x)$ and $(1-\tau)P(x) - C/r$ intersect twice at most. We can classify the functions into Cases (S), (DS), (DOS), and (D) based on the relationships between the two functions (see Figure 1). In the upper right panel of Figure 1, we have default, sell-out, and operation regions from left (i.e., $x = 0$), and we call this Case (DS). In the lower left panel, we have default, operation, sell-out, and operation regions from left, and we call this Case (DOS). Intuitively, we can see from Figure 1 that Case (S) holds for the highest residual value, followed in order by Case (DS), Case (DOS), and Case (D).

In Case (S), (i.e., when the residual value is highest) the two functions intersect once, and $E(x)$ is strictly positive for $x > 0$. There is no possibility of default, and the intersection point x^s is the sell-out threshold. The firm accepts an acquirer's offer for $X(t) \leq x^s$. As in Section 4.1, we can derive $E(x)$ by solving ODEs

$$\mu x E'(x) + 0.5\sigma^2 x^2 E''(x) + \lambda((1-\tau)P(x) - C/r - E(x)) + (1-\tau)(x-C) = rE(x) \ (x \in \mathcal{S}), \tag{8}$$

$$\mu x E'(x) + 0.5\sigma^2 x^2 E''(x) + (1-\tau)(x-C) = rE(x) \ (x \in \mathcal{O}), \tag{9}$$

[5]There is no tax shield on repayment of the principal C/r.

[6]We cannot mathematically prove it, but it always holds in numerical examples.

where $\mathcal{S} = (0, x^s)$ and $\mathcal{O} = (x^s, \infty)$ are the sell-out and operation region, along with the boundary conditions $E(x^s) = (1-\tau)P(x^s) - C/r, \lim_{x\to 0} E(x) < \infty, \lim_{x\to\infty} E(x)/x < \infty$, and continuous differentiability of $E(x)$ at x^s. In ODE (8), $\lambda((1-\tau)P(x) - C/r - E(x))$ corresponds to the fact that $E(x)$ changes to $(1-\tau)P(x) - C/r$ with probability $\lambda \mathrm{d}t$ in time interval $\mathrm{d}t$ in the sell-out region. Because of no default risk, the debt value $D(x)$ equals the riskless debt value C/r.

In Case (DS), the two functions intersect once, and $E(x)$ equals zero for $x \in (0, x^d]$. The intersection point x^s is the sell-out threshold. The firm accepts an acquirer's offer for $X(t) \leq x^s$. However, if no acquirer appears in the sell-out region until $X(t)$ hits the default threshold x^d, the firm defaults. We can derive $E(x)$ by solving ODEs (8) and (9) with the sell-out region $\mathcal{S} = (x^d, x^s)$ and the operation region $\mathcal{O} = (x^s, \infty)$. The boundary conditions are $E(x^d) = 0, E(x^s) = (1-\tau)P(x^s) - C/r$, and $\lim_{x\to\infty} E(x)/x < \infty$, The condition $E(x^d) = 0$ follows from the assumption that shareholders gain nothing on default, and the other conditions are the same as in Section 4.1. By the smooth pasting (optimality) condition, $E(x)$ is continuously differentiable at x^d, and as in Section 4.1, $E(x)$ is also continuously differentiable at x^s.

In this case, there is default risk, and the debt value $D(x)$ follows

$$\mu x D'(x) + 0.5\sigma^2 x^2 D''(x) + \lambda(C/r - D(x)) + C = rD(x) \ (x \in \mathcal{S}), \tag{10}$$

$$\mu x D'(x) + 0.5\sigma^2 x^2 D''(x) + C = rD(x) \ (x \in \mathcal{O}), \tag{11}$$

where $\mathcal{S} = (x^d, x^s)$ and $\mathcal{O} = (x^s, \infty)$. In ODE (10), $\lambda(C/r - D(x))$ corresponds to the fact that $D(x)$ changes to the repayment value C/r with probability $\lambda \mathrm{d}t$ in time interval $\mathrm{d}t$ in the sell-out region. We can derive $D(x)$ by solving ODEs (10) and (11). The boundary conditions are $D(x^d) = (1-\alpha)E_U(x)$ and $\lim_{x\to\infty} D(x) = C/r$. The former means that debt holders receive the unlevered firm value minus bankruptcy costs at the default threshold, whereas the latter means that the debt value approaches the riskless value for an infinitely large state variable. As in $E(x)$, $D(x)$ is continuously differentiable at x^s, while $D(x)$ is continuous at x^d. Note that $D(x)$ is not continuously differentiable at x^d because the default threshold x^d does not maximize $D(x)$ but $E(x)$.

In Case (DOS), the two functions intersect twice, and $E(x)$ equals zero for $x \in (0, x^d]$. Notably, the sell-out region, unlike in Cases (DS) and (S), becomes the interval $[x^{s1}, x^{s2}]$. The firm accepts an acquirer's offer for $X(t) \in [x^{s1}, x^{s2}]$. However, if no acquirer appears in the sell-out region until $X(t)$ hits the default threshold x^d, the firm defaults. We can derive $E(x)$ by solving ODEs (8) and (9) with $\mathcal{S} = (x^{s1}, x^{s2})$ and $\mathcal{O} = (x^d, x^{s1}) \cup (x^{s2}, \infty)$. As in Case (DS), the boundary conditions are $E(x^d) = 0, E(x^{s1}) = (1-\tau)P(x^{s1}) - C/r, E(x^{s2}) =$

$(1-\tau)P(x^{s2}) - C/r$, and $\lim_{x\to\infty} E(x)/x < \infty$, and $E(x)$ are continuously differentiable at x^d, x^{s1}, and x^{s2}. We can also derive the debt value $D(x)$ by solving ODEs (10) and (11) with $\mathcal{S} = (x^{s1}, x^{s2})$ and $\mathcal{O} = (x^d, x^{s1}) \cup (x^{s2}, \infty)$. The boundary conditions are $D(x^d) = (1-\alpha)E_U(x)$ and $\lim_{x\to\infty} D(x) = C/r$, and we have continuous differentiability of $D(x)$ at x^{s1} and x^{s2}, and continuity of $D(x)$ at x^d.

In Case (D), (i.e., when the residual value is lowest) the two functions do not intersect, and $E(x)$ equals zero for $x \in (0, x^d]$. There is no possibility of sell-out because $E(x)$ dominates $(1-\tau)P(x) - C/r$. The firm defaults when $X(t)$ hits the default threshold x^d. We can derive $E(x)$ by solving ODE (9) with $\mathcal{O} = (x^d, \infty)$. Note the boundary conditions $E(x^d) = 0, \lim_{x\to\infty} E(x)/x < \infty$, and continuous differentiability of $E(x)$ at x^d. In this default only case, $E(x)$ is essentially the same as the equity value of Leland (1994).

By choosing the maximal equity value in all cases, we have the following proposition, where we define

$$\epsilon = (1-a)^{\frac{-\gamma}{1-\gamma}} \in (0, 1]. \tag{12}$$

For the proof, refer to Appendix B.

Proposition 2

Case (S):

$$\frac{C}{r\theta} \le \frac{(1-\tau)\lambda}{\tau\lambda + (1-\tau)(r+\lambda)}. \tag{13}$$

The firm prefers to sell out.

$$E(x) = \begin{cases} (1-\tau)\left\{\dfrac{(r+a\lambda-\mu)x}{(r-\mu)(r+\lambda-\mu)} - \dfrac{C}{r} + \dfrac{\lambda}{r+\lambda}\left(\theta - \dfrac{\tau C}{(1-\tau)r}\right)\right. \\ \left. + \left(\dfrac{x}{x^s}\right)^{\beta_\lambda}\left(\dfrac{(a-1)x^s}{r+\lambda-\mu} + \dfrac{r}{r+\lambda}\left(\theta - \dfrac{\tau C}{(1-\tau)r}\right)\right)\right\} & (x \le x^s), \\ (1-\tau)\left\{\dfrac{x}{r-\mu} - \dfrac{C}{r} + \left(\dfrac{x}{x^s}\right)^{\gamma}\left(\dfrac{(a-1)x^s}{r-\mu} + \theta - \dfrac{\tau C}{(1-\tau)r}\right)\right\} & (x > x^s), \end{cases} \tag{14}$$

$$D(x) = \frac{C}{r},$$

where the sell-out threshold x^s is defined by

$$x^s = \frac{(r+\lambda-\mu)((\gamma-\beta_\lambda)r + \gamma\lambda)}{(r+\lambda)((\gamma-\beta_\lambda)(r-\mu) + (\gamma-1)\lambda)}\frac{(r-\mu)}{1-a}\left(\theta - \frac{\tau C}{(1-\tau)r}\right). \tag{15}$$

Cases (DS) and (DOS) :

$$\frac{(1-\tau)\lambda}{\tau\lambda + (1-\tau)(r+\lambda)} < \frac{C}{r\theta} \le \frac{1-\tau}{\tau + (1-\tau)\epsilon}. \tag{16}$$

The firm can either default or sell out.

Case (DS):

$$E(x) = \begin{cases} 0 & (x \leq x^d), \\ (1-\tau)\left\{\dfrac{(r+a\lambda-\mu)x}{(r-\mu)(r+\lambda-\mu)} - \dfrac{C}{r} + \dfrac{\lambda}{r+\lambda}\left(\theta - \dfrac{\tau C}{(1-\tau)r}\right) + A_1 x^{\beta_\lambda} + A_2 x^{\gamma_\lambda}\right\} & (x \in (x^d, x^s]), \\ (1-\tau)\left\{\dfrac{x}{r-\mu} - \dfrac{C}{r} + \left(\dfrac{x}{x^s}\right)^\gamma \left(\dfrac{(a-1)x^s}{r-\mu} + \theta - \dfrac{\tau C}{(1-\tau)r}\right)\right\} & (x > x^s), \end{cases} \tag{17}$$

$$D(x) = \begin{cases} (1-\alpha)E_U(x) & (x \leq x^d), \\ \dfrac{C}{r} - B_1 x^{\beta_\lambda} - B_2 x^{\gamma_\lambda} & (x \in (x^d, x^s]), \\ \dfrac{C}{r} - B_3\left(\dfrac{x}{x^s}\right)^\gamma & (x > x^s), \end{cases} \tag{18}$$

where we denote

$$\begin{aligned} A_1 &= \frac{x^{s\gamma_\lambda} F_1 - x^{d\gamma_\lambda} F_2}{x^{d\beta_\lambda} x^{s\gamma_\lambda} - x^{s\beta_\lambda} x^{d\gamma_\lambda}}, \\ A_2 &= \frac{x^{d\beta_\lambda} F_2 - x^{s\beta_\lambda} F_1}{x^{d\beta_\lambda} x^{s\gamma_\lambda} - x^{s\beta_\lambda} x^{d\gamma_\lambda}}, \\ F_1 &= -\frac{(r+a\lambda-\mu)x^d}{(r-\mu)(r+\lambda-\mu)} + \frac{C}{r} - \frac{\lambda}{r+\lambda}\left(\theta - \frac{\tau C}{(1-\tau)r}\right), \\ F_2 &= \frac{(a-1)x^s}{r+\lambda-\mu} + \frac{r}{r+\lambda}\left(\theta - \frac{\tau C}{(1-\tau)r}\right), \\ B_1 &= \frac{(\gamma-\gamma_\lambda)x^{s\gamma_\lambda}(C/r - (1-\alpha)E_U(x^d))}{(\beta_\lambda-\gamma)x^{s\beta_\lambda}x^{d\gamma_\lambda} - (\gamma_\lambda-\gamma)x^{d\beta_\lambda}x^{s\gamma_\lambda}}, \\ B_2 &= \frac{(\beta_\lambda-\gamma)x^{s\beta_\lambda}(C/r - (1-\alpha)E_U(x^d))}{(\beta_\lambda-\gamma)x^{s\beta_\lambda}x^{d\gamma_\lambda} - (\gamma_\lambda-\gamma)x^{d\beta_\lambda}x^{s\gamma_\lambda}}, \\ B_3 &= B_1 x^{s\beta_\lambda} + B_2 x^{s\gamma_\lambda} \end{aligned}$$

and the default and sell-out thresholds x^d, x^s *are the solutions to*

$$\frac{r+a\lambda-\mu}{(r-\mu)(r+\lambda-\mu)} + \beta_\lambda A_1 x^{d\beta_\lambda - 1} + \gamma_\lambda A_2 x^{d\gamma_\lambda - 1} = 0, \tag{19}$$

$$\frac{r+a\lambda-\mu}{(r-\mu)(r+\lambda-\mu)} + \beta_\lambda A_1 x^{s\beta_\lambda - 1} + \gamma_\lambda A_2 x^{s\gamma_\lambda - 1} = \frac{1+\gamma(a-1)}{r-\mu} + \frac{\gamma}{x^s}\left(\theta - \frac{\tau C}{(1-\tau)r}\right). \tag{20}$$

Case (DOS):

$$E(x) = \begin{cases} 0 & (x \le x^d), \\ (1-\tau)\left\{\dfrac{x}{r-\mu} - \dfrac{C}{r} + A_1 x^{\beta} + A_2 x^{\gamma}\right\} & (x \in (x^d, x^{s1})), \\ (1-\tau)\left\{\dfrac{(r+a\lambda-\mu)x}{(r-\mu)(r+\lambda-\mu)} - \dfrac{C}{r} + \dfrac{\lambda}{r+\lambda}\left(\theta - \dfrac{\tau C}{(1-\tau)r}\right) + A_3 x^{\beta_\lambda} + A_4 x^{\gamma_\lambda}\right\} & (x \in [x^{s1}, x^{s2}]), \\ (1-\tau)\left\{\dfrac{x}{r-\mu} - \dfrac{C}{r} + \left(\dfrac{x}{x^{s2}}\right)^{\gamma}\left(\dfrac{(a-1)x^{s2}}{r-\mu} + \theta - \dfrac{\tau C}{(1-\tau)r}\right)\right\} & (x > x^{s2}), \end{cases} \tag{21}$$

$$D(x) = \begin{cases} (1-\alpha)E_U(x) & (x \le x^d), \\ \dfrac{C}{r} - B_1 x^{\beta} - B_2 x^{\gamma} & (x \in (x^d, x^{s1})), \\ \dfrac{C}{r} - B_3 x^{\beta_\lambda} - B_4 x^{\gamma_\lambda} & (x \in [x^{s1}, x^{s2}]), \\ \dfrac{C}{r} - B_5\left(\dfrac{x}{x^{s2}}\right)^{\gamma} & (x > x^{s2}), \end{cases} \tag{22}$$

where we denote

$$\begin{aligned} A_1 &= \frac{x^{s\gamma}F_1 - x^{d\gamma}F_2}{x^{d\beta}x^{s\gamma} - x^{s\beta}x^{d\gamma}}, \\ A_2 &= \frac{x^{d\beta}F_2 - x^{s\beta}F_1}{x^{d\beta}x^{s\gamma} - x^{s\beta}x^{d\gamma}}, \\ A_3 &= \frac{x^{s2\gamma_\lambda}F_3 - x^{s1\gamma_\lambda}F_4}{x^{s1\beta_\lambda}x^{s2\gamma_\lambda} - x^{s2\beta_\lambda}x^{s1\gamma_\lambda}}, \\ A_4 &= \frac{x^{s1\beta_\lambda}F_4 - x^{s2\beta_\lambda}F_3}{x^{s1\beta_\lambda}x^{s2\gamma_\lambda} - x^{s2\beta_\lambda}x^{s1\gamma_\lambda}}, \\ F_1 &= -\frac{x^d}{r-\mu} + \frac{C}{r}, \\ F_2 &= \frac{(a-1)x^{s1}}{r-\mu} + \theta - \frac{\tau C}{(1-\tau)r}, \\ F_3 &= \frac{(a-1)x^{s1}}{r+\lambda-\mu} + \frac{\lambda}{r+\lambda}\left(\theta - \frac{\tau C}{(1-\tau)r}\right), \\ F_4 &= \frac{(a-1)x^{s2}}{r+\lambda-\mu} + \frac{r}{r+\lambda}\left(\theta - \frac{\tau C}{(1-\tau)r}\right), \end{aligned}$$

and the default and sell-out thresholds x^d, x^{s1}, x^{s2} *are the solutions to*

$$\frac{x^d}{r-\mu} + \beta A_1 x^{d\beta} + \gamma A_2 x^{d\gamma} = 0, \tag{23}$$

$$\frac{x^{s1}}{r-\mu} + \beta A_1 x^{s1\beta} + \gamma A_2 x^{s1\gamma} = \frac{(r+a\lambda-\mu)x^{s1}}{(r-\mu)(r+\lambda-\mu)} + \beta_\lambda A_3 x^{s1\beta_\lambda} + \gamma_\lambda A_4 x^{s1\gamma_\lambda}, \tag{24}$$

$$\frac{(r+a\lambda-\mu)x^{s2}}{(r-\mu)(r+\lambda-\mu)} + \beta_\lambda A_3 x^{s2\beta_\lambda} + \gamma_\lambda A_4 x^{s2\gamma_\lambda} = \frac{(1+\gamma(a-1))x^{s2}}{r-\mu} + \gamma\left(\theta - \frac{\tau C}{(1-\tau)r}\right). \tag{25}$$

The coefficients B_i $(i = 1, 2, \ldots, 5)$ are the solution of the linear system:

$$x^{d\beta}B_1 + x^{d\gamma}B_2 = \frac{C}{r} - (1-\alpha)E_U(x^d), \tag{26}$$

$$x^{s1\beta}B_1 + x^{s1\gamma}B_2 - x^{s1\beta_\lambda}B_3 - x^{s1\gamma_\lambda}B_4 = 0, \tag{27}$$

$$\beta x^{s1\beta}B_1 + \gamma x^{s1\gamma}B_2 - \beta_\lambda x^{s1\beta_\lambda}B_3 - \gamma_\lambda x^{s1\gamma_\lambda}B_4 = 0, \tag{28}$$

$$x^{s2\beta_\lambda}B_3 + x^{s2\gamma_\lambda}B_4 - B_5 = 0, \tag{29}$$

$$\beta_\lambda x^{s2\beta_\lambda}B_3 + \gamma_\lambda x^{s2\gamma_\lambda}B_4 = 0. \tag{30}$$

Case (D):

$$\frac{C}{r\theta} > \frac{1-\tau}{\tau + (1-\tau)\epsilon}. \tag{31}$$

The firm prefers to default.

$$E(x) = \begin{cases} 0 & (x \le x^d), \\ (1-\tau)\left\{\dfrac{x}{r-\mu} - \dfrac{C}{r} + \left(\dfrac{x}{x^d}\right)^\gamma \left(-\dfrac{x^d}{r-\mu} + \dfrac{C}{r}\right)\right\} & (x > x^d), \end{cases} \tag{32}$$

$$D(x) = \begin{cases} (1-\alpha)E_U(x) & (x \le x^d), \\ \dfrac{C}{r} - \left(\dfrac{x}{x^d}\right)^\gamma \left(\dfrac{C}{r} - (1-\alpha)E_U(x^d)\right) & (x > x^d), \end{cases}$$

where the default threshold x^d is defined by

$$x^d = \frac{\gamma(r-\mu)C}{(\gamma-1)r}. \tag{33}$$

In each case, $\tilde{E}(x)$ and $\tilde{D}(x)$ equal $(1-\tau)P(x) - C/r$ and C/r, respectively, in the sell-out region, whereas $\tilde{E}(x)$ and $\tilde{D}(x)$ equal $E(x)$ and $D(x)$, respectively, in the operation region. The firm values $F(x)$ and $\tilde{F}(x)$ equal $E(x)+D(x)$ and $\tilde{E}(x)+\tilde{D}(x)$, respectively.

We can see from (13) that Case (S) (i.e., the sell-out only case), tends to hold for lower coupon, higher scrap value, and higher asset liquidity. In Case (S), the firm's optimal policy is to sell out only if an acquirer arrives at time t satisfying $X(t) \le x^s$. Debt is riskless, and $E(x)$ is similar to $E_U(x)$ in Proposition 1. The sell-out threshold x^s is lower than x_U^s because of the term $\tau C/(1-\tau)r$ (cf. (15) and (7)). This is because the tax benefits of coupon payments are lost after sell-out.

On the other hand, we can see from (31) that Case (D), i.e., the default only case, tends to hold for higher coupon and lower asset value. Because of $\partial\epsilon/\partial\sigma \ge 0$, a higher σ also increases the region of Case (D). The impact of σ is the same as shown by Nishihara and Shibata (2018).[7] In Case (D), the firm's optimal policy is to default once $X(t)$ hits the default threshold x^d. On bankruptcy, debt holders take over the firm, but a fraction α of the firm value is lost as bankruptcy costs. There is no possibility of sell-out, and hence, $E(x)$ and $D(x)$ are essentially the same as shown by Leland (1994). In this case, the default threshold x^d is derived in the closed form (33).

Cases (DS) and (DOS) are novel. These intermediate cases do not appear in any previous literature. Indeed, in the previous models allowing asset sales at any time (e.g., Mella-Barral and Perraudin (1997), Nishihara and Shibata (2017), and Nishihara and Shibata (2018)), the firm sells out if (31) does not hold. We can see from (16) that the novel cases tend to appear for intermediate coupon and asset value, as well as for lower market volatility. In these cases, the firm's exit choice between sell-out and default is not determined at the initial time but depends on the realized paths of $X(t)$ and the Poisson process. The firm succeeds in sell-out only if an acquirer appears in the sell-out region until $X(t)$ hits the default threshold x^d. Otherwise, the firm defaults.

The stochastic exit choice is contrasted with the deterministic choice in Mella-Barral and Perraudin (1997), Nishihara and Shibata (2017), and Nishihara and Shibata (2018). This result fits empirical observations well. In fact, before bankruptcy, many firms search for acquirers that will save them. Some firms (e.g., Sharp Corporation) succeed in being acquired, while other firms (e.g., Circuit City) fail and become bankrupt. More generally, M&A literature, such as Cornett, Tanyeri, and Tehranian (2011) and Wang (2018), empirically shows the low predictability of targets of acquisitions.

We cannot analytically prove but can numerically show (also intuitively from Figure 1) that Case (DS) holds for relatively lower coupon and higher asset value, while Case (DOS) holds for relatively higher coupon and lower asset value. According to our computations for a wide range of parameter values, we always have a unique solution either in Case (DS) or in Case (DOS).

Now, we see $E(x)$ and $D(x)$ in Case (DS) more closely. The lower equation in (17) stands for the equity value in the operation region, i.e, $E(x) > (1-\tau)P(x) - C/r$. The center equation in (17) stands for the equity value in the sell-out region, i.e, $E(x) \leq (1-\tau)P(x) - C/r$. The first three terms of the center equation represent the expected payoff of selling out as soon as an acquirer appears. The last two terms, i.e., $A_1 x^{\beta_\lambda} + A_2 x^{\gamma_\lambda}$

[7] Intuitively, a higher σ increases the default option value than the sell-out option value because the convexity of the default option is stronger than that of the sell-out option. By the similar logic, Kort, Murto, and Pawlina (2010) show that higher volatility is more likely to lead to lumpy rather than stepwise investment.

stand for the value of the options to default and to delay sell-out. The upper equation in (17) means that the equity value is zero in the default region.

Debt holders are repaid the face value on sell-out, while they cannot fully recover the face value on bankruptcy. Hence, debt is risky. The lower equation in (18) represents the debt value in the operation region. The term C/r stands for the value of perpetually receiving coupons, whereas the second term measures the possibility of $X(t)$ hitting the sell-out threshold x^s. The center equation in (18) stands for the debt value in the sell-out region. The term C/r stands for the value of perpetually receiving coupons, whereas the last two terms, i.e., $B_1 x^{\beta_\lambda} + B_2 x^{\gamma_\lambda}$ measure the possibility of $X(t)$ hitting either x^d or x^s. The upper equation in (18) means that debt holders receive the unlevered firm value subtracted by bankruptcy costs at the default time.

Next, we turn to $E(x)$ and $D(x)$ in Case (DOS). The main difference from Case (DS) is that the sell-out region is the interval $[x^{s1}, x^{s2}]$. Because of the lower residual value from sell-out in Case (DOS), unlike in Case (DS), shareholders are better off forgoing an acquirer's offer for $X(t) < x^{s1}$. Then, we have two disconnected operation regions (x^d, x^{s1}) and (x^{s2}, ∞) (cf. Figure 1). This is why $E(x)$ and $D(x)$ are expressed as the four equations in (21) and (22). All the equations can be explained as in Case (DS); we omit the detailed explanations. The existence of the intermediate operation region (x^d, x^{s1}) is similar to the result of Guerra, Kort, Nunes, and Oliveira (2018) who examine a mothballing option before permanent exit. Although their model considers neither debt financing nor asset illiquidity, they show that the operation region can exist between the mothballing region and the exit threshold.

In Cases (DS) and (DOS), the firm's exit choice between sell-out and default is stochastic. The following proposition analytically derives the default probability, denoted by $P^d(x)$.

Proposition 3 *Suppose that* $\mu - 0.5\sigma^2 < 0$.
Case (S):

$$P^d(x) = 0.$$

Case (DS):

$$P^d(x) = \begin{cases} 1 & (x \leq x^d), \\ G_1 x^{\beta_{\lambda-r}} + G_2 x^{\gamma_{\lambda-r}} & (x \in (x^d, x^s]), \\ G_3 & (x > x^s), \end{cases} \tag{34}$$

where x^d *and* x^s *are the default and sell-out thresholds in Case (DS) of Proposition 2,* *and the coefficients G_1, G_2, G_3 are defined by*

$$G_1 = -\frac{\gamma_{\lambda-r} x^{s\gamma_{\lambda-r}}}{\beta_{\lambda-r} x^{s\beta_{\lambda-r}} x^{d\gamma_{\lambda-r}} - \gamma_{\lambda-r} x^{d\beta_{\lambda-r}} x^{s\gamma_{\lambda-r}}},$$

$$G_2 = \frac{\beta_{\lambda-r} x^{s\beta_{\lambda-r}}}{\beta_{\lambda-r} x^{s\beta_{\lambda-r}} x^{d\gamma_{\lambda-r}} - \gamma_{\lambda-r} x^{d\beta_{\lambda-r}} x^{s\gamma_{\lambda-r}}},$$

$$G_3 = G_1 x^{s\beta_{\lambda-r}} + G_2 x^{s\gamma_{\lambda-r}}.$$

Case (DOS):

$$P^d(x) = \begin{cases} 1 & (x \leq x^d), \\ G_1 x^{1-2\mu/\sigma^2} + G_2 & (x \in (x^d, x^{s1})), \\ G_3 x^{\beta_{\lambda-r}} + G_4 x^{\gamma_{\lambda-r}} & (x \in [x^{s1}, x^{s2}]), \\ G_5 & (x > x^{s2}), \end{cases} \tag{35}$$

where x^d, x^{s1}, and x^{s2} are the default and sell-out thresholds in Case (DOS) of Proposition 2, and the coefficients G_i $(i = 1, 2, \ldots, 5)$ are the solution to the linear system:

$$\begin{aligned} x^{d1-2\mu/\sigma^2} G_1 + G_2 &= 1, \\ x^{s11-2\mu/\sigma^2} G_1 + G_2 - x^{s1\beta_{\lambda-r}} G_3 - x^{s1\gamma_{\lambda-r}} G_4 &= 0, \\ (1 - 2\mu/\sigma^2) x^{s11-2\mu/\sigma^2} G_1 + G_2 - \beta_{\lambda-r} x^{s1\beta_{\lambda-r}} G_3 - \gamma_{\lambda-r} x^{s1\gamma_{\lambda-r}} G_4 &= 0, \\ x^{s2\beta_{\lambda-r}} G_3 + x^{s2\gamma_{\lambda-r}} G_4 - G_5 &= 0, \\ \beta_{\lambda-r} x^{s2\beta_{\lambda-r}} G_3 + \gamma_{\lambda-r} x^{s2\gamma_{\lambda-r}} G_4 &= 0. \end{aligned}$$

Case (D):

$$P^d(x) = 1.$$

Similarly, we can derive $P^d(x)$ for $\mu - 0.5\sigma^2 \geq 0$, but we omit the expressions to avoid redundancy.

Note that $P^d(x)$ is constant for $x \geq x^s$ in Case (DS) and for $x \geq x^{s2}$ in Case (DOS) because $X(t)$ hits x^s and x^{s2} almost surely under the assumption of $\mu - 0.5\sigma^2 < 0$. $X(t)$ also hits the default threshold x^d almost surely, but a Poisson jump may occur in the sell-out region before $X(t)$ hits the default threshold x^d. Hence, $P^d(x)$ decreases with x for $x \in (x^d, x^s]$ in Case (DS) and for $x \in (x^d, x^{s2}]$ in Case (DOS). Under the assumption of $\mu - 0.5\sigma^2 < 0$, the sell-out probability is $1 - P^d(x)$.

As explained after Proposition 2, contrasted with the deterministic choice (i.e., $P^d(X(t)) = 0$ or 1) in the previous models (e.g., Mella-Barral and Perraudin (1997), Nishihara and Shibata (2017), and Nishihara and Shibata (2018)), the stochastic dynamics of $P^d(X(t))$

is more realistic. In the next section, we will examine the effects of asset liquidity on the firm's default and sell-out policies, equity, debt, and firm values, and default probability in full detail.

5 Numerical analysis and implications

In this section, we conduct numerical analysis, including comparative statics with respect to asset liquidity λ and volatility σ. We set the baseline parameter values in Table 1. Asset liquidity λ is set at 2, which means that an asset sales opportunity is expected to arrive every half year. The other parameter values are standard and similar to those of Nishihara and Shibata (2018).

For the baseline parameter values, Case (DOS) holds true, and we have the default and sell-out thresholds $x^d = 0.432, x^{s1} = 0.472$, and $x^{s2} = 0.796$, as well as the equity and debt values $E(x) = 5.308$ and $D(x) = 16.647$. Debt is risky, i.e., $D(x) = 16.647 < C/r = 16.667$, and the default probability is $P^d(x) = 0.0065$. Figure 2 shows a sample path of the equity value $E(X(t))$ with no offer in the sell-out region. We depict the dynamics of $E(X(t))$ rather than $X(t)$ because it can be interpreted as the stock price dynamics until bankruptcy. Note that the stock price equals $E(X(t))/N$, where N denotes the number of shares. In the figure, the sell-out region is between $E(x^{s1}) = 2.846$ and $E(x^{s2}) = 0.0953$. The firm attempts to sell out in the region, but in this example, no acquirer appears in the region. Then, the firm defaults when the equity value falls zero, i.e., $E(x^d) = 0$. If an acquirer arrives in the sell-out region, the firm sells out, and then the equity value $E(X(t))$ jumps upward to $(1-\tau)P(X(t)) - C/r$.

5.1 Impacts of asset liquidity

Figure 3 depicts the default and sell-out thresholds, the equity, debt, and firm values, and the default probability for $1/\lambda \in (0, 2]$, where $1/\lambda$ equals the expected duration (year) between acquirers' arrivals. The other parameter values are set in Table 1. In Figure 3, we also depict the sell-out threshold of the liquid model (i.e., $\lambda = \infty$), denoted by x^{sb}, for comparison. For all $1/\lambda \in (0, 2]$, Case (DOS) holds true. The right-hand side of (13) monotonically increases up to $1-\tau$ as λ increases. For the baseline parameter values, we have $1-\tau < C/r\theta \leq (1-\tau)/(\tau + (1-\tau)\epsilon)$, and hence, Case (DOS) always holds. If $C/r\theta < 1-\tau$ holds, a higher λ leads to Case (S).

In the top-left panel, the sell-out threshold x^{s2} is higher than x^{sb} of the liquid model, and the sell-out region $[x^{s1}, x^{s2}]$ expands with higher $1/\lambda$. In other words, with lower liquidity, the firm accepts an acquirer's offer more eagerly to mitigate the risk of not finding an acquirer until bankruptcy. In the top-right and bottom-right panels, the default threshold x^d and the default probability $P^d(x)$ increase in $1/\lambda$. This implies that the firm's earnest asset sales policy does not fully offset the decreased liquidity, and lower asset liquidity increases the default risk.

These results are similar to those of Hugonnier, Malamud, and Morellec (2015b) who consider the model in which a firm can increase debt to receive more tax benefits only at Poisson jump times. They show that with a lower arrival rate of creditors, the firm decreases the debt restructuring threshold to mitigate the risk of not finding creditors; despite this effort, the restructuring probability decreases. This means that the firm's earnest debt restructuring policy does not fully offset the decreased credit supply. Although they focus on debt expansion in economic expansion, we focus on asset sales in economic downturn. He and Xiong (2012), He and Milbradt (2014), and Chen, Cui, He, and Milbradt (2018) show that the illiquidity of the secondary debt market hastens bankruptcy timing and increases the default probability via the channel of rollover risk. Unlike in their papers, we study the effects of real asset illiquidity. Morellec (2001) shows that for unsecured debt, lower asset liquidation cost, which leads to managerial asset disposition, increases the bankruptcy probability and decreases the debt value. Unlike Morellec (2001), we focus on sell-out, where debt holders are repaid the face value of debt; hence, our model does not have the possibility for asset disposition to decrease the debt value.

Finally, the center panels of Figure 3 show that both the debt value $D(x)$ and the equity value $E(x)$ decrease in $1/\lambda$. The decrease in $D(x)$ is consistent with the stylized fact (e.g., Shleifer and Vishny (1992) and Sibilkov (2009)). Shleifer and Vishny (1992) argue that lower asset liquidity decreases the post-default value debt holders receive, and our model captures the effect because the post-default value $(1-\alpha)E_U(x)$ decreases in $1/\lambda$. In addition, our model suggests that lower asset liquidity also decreases $D(x)$ through the increased default probability channel. Furthermore, $E(x)$ decreases in $1/\lambda$ because lower asset liquidity decreases shareholders' sell-out option value. This result is consistent with empirical evidence in Ortiz-Molina and Phillips (2014), Song and Walkling (2000), and Cornett, Tanyeri, and Tehranian (2011). Ortiz-Molina and Phillips (2014) show that real asset illiquidity reduces a firm's operating flexibility and through this channel increases the cost of capital. Song and Walkling (2000) and Cornett, Tanyeri, and Tehranian (2011) show that a stock price increases as the probability of being acquired increases. Note that the arrival rate λ can be interpreted as the probability of being acquired.

So far, we have examined the effects of asset liquidity with a fixed initial coupon C. How do the results change if the firm can precisely anticipate the degree of asset liquidity and optimize the capital structure at the initial time? As in the standard literature (e.g., Leland (1994)), the firm chooses the optimal capital structure to maximize the firm value $F(x)$, taking account of the trade-off between the tax benefits of debt and bankruptcy costs.

Figure 4 shows the optimal coupon C, leverage $LV(= D(x)/F(x))$, and credit spread $CS(= C/D(x) - r)$ for $1/\lambda \in (0, 2]$. In the figure, the optimal C and LV decreases in $1/\lambda$, although CS increases in $1/\lambda$. These results are interpreted as follows. Lower asset liquidity increases the bankruptcy risk, and as a result, the firm reduces debt financing. Despite the firm's more cautious capital structure, the default risk increases (see CS in Figure 4 and $P^d(x)$ in Figure 5) with decreased asset liquidity because the direct impact of decreased asset liquidity dominates the impact from the reduced leverage. These results are consistent with the empirical findings of Sibilkov (2009), Campello and Giambon (2013), and Ortiz-Molina and Phillips (2014), who show that leverage is positively related to asset liquidity.

Figure 5 depicts the default and sell-out thresholds, the equity, debt, and firm values, and the default probability for $1/\lambda \in (0, 2]$ under the optimal capital structure. Case (DOS) holds for $1/\lambda \in (0, 1.25]$, while Case (DS) holds for $1/\lambda \in (1.25, 2]$. This is because a higher $1/\lambda$ decreases the optimal coupon, which changes from Case (DOS) to Case (DS).

Aside from the default threshold x^d and the equity value $E(x)$, Figure 5 is similar to Figure 3. In the top-left panel of Figure 5, the sell-out region expands with $1/\lambda$. With lower liquidity, in addition to using a more cautious capital structure, as in a case with a fixed coupon, the firm accepts an acquirer's offer more eagerly to mitigate the risk of not finding an acquirer. In the bottom-right panel, the default probability $P^d(x)$ increases in $1/\lambda$, although the increase is mitigated from that of a case with a fixed coupon (cf. $P^d(x)$ in Figure 3). In other words, the cautious capital structure and earnest asset sales policy do not fully offset the decreased liquidity, and then lower asset liquidity increases the default probability. Due to the increased bankruptcy risk, in the center-right and bottom-left panels, the debt and firm values monotonically decrease in $1/\lambda$. These results align with the empirical evidence (e.g., Ortiz-Molina and Phillips (2014), Song and Walkling (2000), and Cornett, Tanyeri, and Tehranian (2011)).

In the top-right panel, the default threshold x^d decreases in $1/\lambda$, and in the center-left panel, the equity value $E(x)$ decreases in $1/\lambda$. These results are contrary to those of a case with a fixed coupon (cf. x^d and $E(x)$ in Figure 3). The explanation follows. A higher $1/\lambda$ decreases the optimal C, which plays roles in decreasing x^d and increasing $E(x)$.

5.2 Impacts of cash flow volatility

Figure 6 depicts the default and sell-out thresholds, the equity, debt, and firm values, and the default probability for $\sigma \in [0.15, 0.35]$. The other parameter values are set in Table 1. Case (DOS) holds for $\sigma \in [0.15, 0.289)$, while Case (D) holds for $\sigma \in [0.289, 0.35]$. Recall that by $\partial\epsilon/\partial\sigma \geq 0$ and (31), a higher σ leads to Case (D).

In the top-left panel, a higher σ narrows the sell-out region, whereas in the top-right panel, a higher σ decreases the default threshold x^d. These results are consistent with the standard real options theory (e.g., Dixit and Pindyck (1994)) that a higher σ increases the option value of waiting.

In the center panels, a higher σ increases the equity value $E(x)$ and decreases the debt value $D(x)$. The wealth transfer from debt holders to shareholders is consistent with the standard result (e.g., Jensen and Meckling (1976) and Eisdorfer (2008)). In the bottom-left panel, the sensitivity of the firm value $F(x)$ to σ depends on this trade-off. Most notably, the change from Case (DOS) to Case (D) at $\sigma = 0.289$ greatly decreases $F(x)$ because the change greatly reduces $D(x)$. The sharp decrease in $D(x)$ stems from the sharp increase in the default probability $P^d(x)$. In fact, in the bottom-right panel, $P^d(x)$ sharply increases in $\sigma \approx 0.289$. These results lead to empirical implications that a slight increase in volatility can greatly increase default risk and decrease debt and firm values.

Now, we examine the impacts of σ under the optimal capital structure. Figure 7 shows the optimal coupon C, leverage LV, and credit spread CS for $\sigma \in [0.15, 0.35]$. In the figure, C and LV decrease in σ, whereas CS forms an inverted U-shape. The negative correlation between σ and LV is consistent with the standard theory (e.g., Harris and Raviv (1991) and Leland (1994)). A higher σ can reduce CS due to the lower LV, but it can increase CS due to more volatile cash flows. The ambiguous sensitivity of CS stems from this tradeoff.

Figure 8 depicts the default and sell-out thresholds, the equity, debt, and firm values, and the default probability for $\sigma \in [0.15, 0.35]$ under the optimal capital structure. Unlike in a case with a fixed coupon, Case (DOS) holds for all $\sigma \in [0.15, 0.35]$. This is because

(16) is satisfied, even for a higher σ, due to the reduced coupon. In the top panels, the sell-out and default thresholds x^{s1}, x^{s2}, and x^d decrease in σ. As in the case with a fixed coupon, this result aligns with the standard real options result.

In the center panels, as in the case with a fixed coupon, a higher σ leads to a wealth transfer from debt holders to shareholders. There are no sharp decreases in $D(x)$ and $F(x)$ because the firm can adjust the optimal capital structure in accordance with volatility changes. In the bottom-left panel, unlike in the case with a fixed coupon, $F(x)$ monotonically increases in σ. This is because the firm chooses the capital structure to maximize $F(x)$ and a higher σ increases the sell-out and default option values embedded in $F(x)$. In the bottom-right panel, as in CS in Figure 7, the default probability $P^d(x)$ is an inverted U-shape based on the trade-off between the reduced leverage and more volatile cash flows.

5.3 Stock price reactions at the sell-out time

In this subsection, we explore the market reaction when the firm sells out. In the model, an acquirer pays $P(X(t))$ at acquisition, and then, the equity value jumps from $E(X(t))$ to $(1-\tau)P(X(t)) - C/r$, which can be interpreted as the stock price reaction or the acquisition premium. Recall that $E(X(t)) \le (1-\tau)P(X(t)) - C/r$ always hold for $X(t)$ in the sell-out region. The positive stock price reaction is consistent with well-known facts. In fact, targets gain abnormal returns (around 20%) in M&As (e.g., Huang and Walkling (1987), Hotchkiss and Mooradian (1998), and Boone and Mulherin (2007)). Below, we explore the key drivers of the positive stock price reaction.

Figure 9 depicts the jump size, defined by $((1-\tau)P(X(t)) - C/r - E(X(t)))/E(X(t))$, with respect to $X(t)$ for varying levels of $1/\lambda$ and σ. In the left panels, the other parameter values are set in Table 1, and Case (DOS) holds true. In the right panels, we replace $C = 1$ with $C = 0.9$ to see the results in Case (DS). There is no jump for $x \notin [x^{s1}, x^{s2}]$ in Case (DOS) and for $x \notin [x^d, x^{s2}]$ in Case (DS) because the firm forgoes an acquirer's offer. In the left panels, i.e., Case (DOS), the jump size is an inverted U-shape with respect to $X(t)$. By Proposition 2, we have $(1-\tau)P(X(t)) - C/r - E(X(t)) = 0$ for the sell-out thresholds x^{s1} and x^{s2}, whereas we have $(1-\tau)P(X(t)) - C/r - E(X(t)) > 0$ for $X(t) \in (x^{s1}, x^{s2})$. In other words, the interval policy of sell-out in Case (DOS) leads to the inverted U-shaped jump size with respect to $X(t)$. On the other hand, in the right panel, i.e., Case (DS),[8] the jump size increases infinitely as $X(t)$ approaches x^d because the denominator $E(x^d)$ equals zero. According to our computations for a wide range of

[8]Only for $\sigma = 0.25$, Case (DS) holds, and the jump size is an inverted U-shape with $X(t)$.

parameter values, in Cases (DS) and (S), the jump size monotonically decreases in $X(t)$. We see these results more closely below.

The jump size is rewritten as $(1-\tau)P(X(t))/E(X(t)) - C/rE(X(t)) - 1$. The first term $(1-\tau)P(X(t))/E(X(t))$ decreases in $X(t)$. Indeed, the firm has a stronger incentive to sell out to the acquirer for lower $X(t)$ because sell-out involves partial liquidation with the fixed scrap price θ (called the sell-out effect). The second term $-C/rE(X(t))$ increases in $X(t)$ because the denominator $E(X(t))$ increases in $X(t)$ (called the debt repayment effect). The impact of $X(t)$ on the jump size is determined by the tradeoff between the two effects. Recall that increased asset value (or decreased coupon) changes the region from Case (DOS) to Cases (DS) and (S) (cf. Figure 1). When the target asset value is sufficiently high compared to debt value (i.e., Cases (DS) and (S)), the sell-out effect dominates the debt repayment effect. Hence, the jump size decreases in $X(t)$. When the target asset value is not sufficiently high compared to debt value (i.e., Case (DOS)), the sell-out effect is stronger for relatively high $X(t)$, but the debt repayment effect is stronger for relatively low $X(t)$. Hence, the jump size is an inverted U-shape with respect to $X(t)$.

Our results highlight the significance of acquisition timing on stock price reaction. The jump size greatly depends on the acquisition timing, even if we consider a transaction between the same two firms. To put it roughly, we have the following predictions. If the target asset value is relatively low, the stock price jump size is an inverted U-shape with the distance to default. If the target asset value is relatively high, the stock price jump size decreases in the distance to default. Although these have not been empirically tested to date, our model can potentially account for the dispersion of target stock price reactions.

The debt value also jumps upward from $D(X(t))$ to the riskless debt value C/r at the sell-out time in Cases (DS) and (DOS), whereas the debt value remains C/r in Case (S). With higher $X(t)$, $D(X(t))$ approaches C/r (see (18) and (22)), and then the jump size $(C/r - D(X(t)))/D(X(t))$ decreases. In other words, the debt value jump size increases as the economic state variable deteriorates. Because acquisition saves debt holders from bankruptcy risk in our model, this result is straightforward. Hence, we omit depicting a figure of the debt value jump.

Figure 9 also shows that a higher $1/\lambda$ and a lower σ increase the equity value jump size. Because $E(X(t))$ decreases in $1/\lambda$ (cf. E(x) in Figure 3), a higher $1/\lambda$ increases the difference between $E(X(t))$ and the sell-out payoff $(1-\tau)P(X(t)) - C/r$. As a result, the jump size increases in $1/\lambda$. Intuitively, with lower asset liquidity, the acquisition announcement brings a greater surprise to the market, leading to a more positive stock price reaction. This result is consistent with empirical evidence of Cornett, Tanyeri, and Tehranian (2011), who show that lower merger anticipation increases market reactions at the announcement. Because $E(X(t))$ increases in σ (cf. E(x) in Figure 6), a higher σ decreases the difference between $E(X(t))$ and $(1-\tau)P(X(t)) - C/r$. Hence, the jump size decreases in σ.

Lastly, Figure 10 depicts the jump size $((1-\tau)P(X(t)) - C/r - E(X(t)))/E(X(t))$ for varying levels of $1/\lambda$ and σ under the optimal capital structure, where the initial coupon C is chosen to maximize the initial firm value. The other parameter values are set in Table 1. Case (DS) holds for $1/\lambda = 1.5$, whereas Case (DOS) holds for the other parameter values. Note that the optimal C decreases in $1/\lambda$ and σ (cf. C in Figures 4 and 7); hence, the sell-out payoff $(1-\tau)P(X(t)) - C/r$ increases in $1/\lambda$ and σ. The comparative statics result with respect to $1/\lambda$ under the optimal capital structure is similar to that of the case with a fixed coupon. Indeed, the jump size increases in $1/\lambda$. Unlike in the case with a fixed coupon, the jump size slightly increases in σ. This is because a higher σ, which decreases the optimal C, increases $(1-\tau)P(X(t)) - C/r$ more than $E(X(t))$.

6 Extensions

6.1 Search cost

In the baseline model, we assume there is no cost for searching for an acquirer. In this subsection, we suppose that the firm has to pay flows of cost K while searching for an acquirer. ODE (8) is replaced with

$$\mu x E'(x) + 0.5\sigma^2 x^2 E''(x) + \lambda((1-\tau)P(x) - C/r - E(x)) + (1-\tau)(x - C - K) = rE(x) \ (x \in \mathcal{S}),$$

where $\mathcal{S}$ denotes the sell-out region. In the operation region, the firm does not have to search for an acquirer, meaning that ODE (9) remains unchanged. The problem can be solved as in Section 4. Clearly, with a higher K, the firm is more likely to default than in the baseline model. The main results in the previous sections remain unchanged in the presence of search cost.

6.2 Offer price contingent on the stock price

In the baseline model, an acquirer's offer price $P(X(t))$ (see (1)) depends on the state variable $X(t)$, but it is not contingent on the current equity value (or stock price) $E(X(t))$.

In this subsection, we suppose that an acquirer pays $\xi P(x)+(1-\xi)E(x)$, where $\xi \in (0,1]$ can be interpreted as the bargaining power of the distressed firm. Note that the offer price agrees with $P(x)$ for $\xi = 1$. ODE (8) is replaced with

$$\begin{aligned}&\mu x E'(x) + 0.5\sigma^2 x^2 E''(x) + \lambda((1-\tau)(\xi P(x) + (1-\xi)E(x)) - C/r - E(x)) \\ &+ (1-\tau)(x - C) = rE(x) \ (x \in \mathcal{S}) \end{aligned} \tag{36}$$

because shareholders gain $(1-\tau)(\xi P(x)+(1-\xi)E(x))-C/r$ at the sell-out time, whereas ODE (9) remains unchanged. We can rewrite (36) as

$$\mu x E'(x)+0.5\sigma^2 x^2 E''(x)+\tilde{\lambda}((1-\tau)\tilde{P}(x)-C/r-E(x))+(1-\tau)(x-\tilde{C}) = rE(x) \ (x \in \mathcal{S}), \tag{37}$$

where we define $\tilde{\lambda} = \lambda(\tau+(1-\tau)\xi)$, $\tilde{P}(x) = \xi P(x)/(\tau+(1-\tau)\xi)$, and $\tilde{C} = C/(\tau+(1-\tau)\xi)$. Note that $\tilde{\lambda} = \lambda, \tilde{P}(x) = P(x)$, and $\tilde{C} = C$ hold for $\xi = 1$.

We can solve the problem as in Section 4 because ODE (37) is of the same form as ODE (8). We have $\partial\tilde{\lambda}/\partial\xi > 0, \partial\tilde{P}(x)/\partial\xi > 0$, and $\partial\tilde{C}/\partial\xi < 0$. Then, lower bargaining power ξ plays the same role as lower asset liquidity and asset value, as well as higher coupon. This implies that the firm is more likely to default than in the baseline model. The main results in the previous sections remain unchanged in this case.

7 Conclusion

In this paper, we developed the dynamic bankruptcy model with illiquidity of sell-out opportunities. In the model, we analytically derived the firm's exit policy, its timing, the default probability, and the equity, debt, and firm values. The main implications are summarized below.

With lower asset liquidity, the firm accepts an acquirer's offer more eagerly to mitigate the risk of not finding an acquirer until bankruptcy takes place, but this earnest asset sales policy does not fully offset the decreased liquidity effect. Lower asset liquidity then increases the default probability and decreases the equity, debt, and firm values. In the optimal capital structure, the firm reduces leverage with lower asset liquidity, but this cautious capital structure does not fully offset the decreased illiquidity effect either. The stock price reaction caused by sell-out greatly depends on the sell-out timing. Intriguingly, when the target asset value is not sufficiently high, the stock price jump size can be an inverted U-shape with the state variable. Lower asset liquidity leads to a more positive stock price reaction due to greater shock. Our results fit empirical findings about asset illiquidity well.

A Proof of Proposition 1

A nonnegative solution to ODE (3) with $\lim_{x\to\infty} E_U^s(x)/x < \infty$ is expressed as $(1-\tau)x/(r-\mu)+U_1x^{\gamma}$, where U_1 is an unknown coefficient. Then, by $E_U^s(x_U^s) = (1-\tau)P(x_U^s)$, we have

$$U_1 = (1-\tau)\left(\frac{1}{x_U^s}\right)^{\gamma}\left(\frac{(a-1)x_U^s}{r-\mu}+\theta\right),$$

which leads to equation (6) for $x > x_U^s$.

A nonnegative solution to ODE (2) with $\lim_{x\to 0} E_U^s(x) < \infty$ is expressed as

$$\frac{(1-\tau)(r+a\lambda-\mu)x}{(r-\mu)(r+\lambda-\mu)} + \frac{(1-\tau)\lambda\theta}{r+\lambda} + U_2x^{\beta_\lambda},$$

where U_2 is an unknown coefficient. Then, by $E_U^s(x_U^s) = (1-\tau)P(x_U^s)$, we have

$$U_2 = (1-\tau)\left(\frac{1}{x_U^s}\right)^{\beta_\lambda}\left(\frac{(a-1)x_U^s}{r+\lambda-\mu}+\frac{r\theta}{r+\lambda}\right),$$

which leads to equation (6) for $x \le x_U^s$.

Finally, by continuous differentiability of $E_U^s(x)$ at x_U^s, we have

$$\frac{x_U^s}{r-\mu}+\gamma\left(\frac{(a-1)x_U^s}{r-\mu}+\theta\right) = \frac{(1-\tau)(r+a\lambda-\mu)x_U^s}{(r-\mu)(r+\lambda-\mu)} + \beta_\lambda\left(\frac{(a-1)x_U^s}{r+\lambda-\mu}+\frac{r\theta}{r+\lambda}\right),$$

by which we can derive x_U^s as (7).

B Proof of Proposition 2

First, we derive the solution in each case, and we will choose the appropriate solution later. In Case (S), $E(x)$ can be derived as in Appendix A, and hence, we omit the derivation of $E(x)$. Clearly, the debt value equals D/r because of no default risk. In Case (D), $E(x)$ and $D(x)$ are essentially the same as the well-known solutions in Goldstein, Ju, and Leland (2001), and hence, we omit the derivation of $E(x)$ and $D(x)$.

Consider Case (DS). The solution to ODE (8) with boundary conditions $E(x^d) = 0$ and $E(x^s) = (1-\tau)P(x^s)-C/r$ is expressed as (17) for $x \in (x^d, x^s]$. The solution to ODE (9) with boundary conditions $E(x^s) = (1-\tau)P(x^s) - C/r$ and $\lim_{x\to\infty} E(x)/x < \infty$ is expressed as (17) for $x > x^s$. Continuous differentiability of $E(x)$ at x^d and x^s leads to (19) and (20), respectively. A general solution to ODE (10) is expressed as (18) for $x \in (x^d, x^s]$. A general solution to ODE (11) with boundary condition $\lim_{x\to\infty} D(x) = C/r$ is expressed as (18) for $x > x^s$. Continuity and continuous differentiability at x^s lead to

$$x^{s\beta_\lambda}B_1 + x^{s\gamma_\lambda}B_2 = B_3, \tag{38}$$

$$\beta_\lambda x^{s\beta_\lambda}B_1 + \gamma_\lambda x^{s\gamma_\lambda}B_2 = \gamma B_3, \tag{39}$$

respectively. By substituting (38) into (39), we have

$$(\beta_\lambda - \gamma) x^{s\beta_\lambda} B_1 + (\gamma_\lambda - \gamma) x^{s\gamma_\lambda} B_2 = 0. \tag{40}$$

By (40) and boundary condition $D(x^d) = (1-\alpha)E_U(x^d)$, we have the expressions of B_1 and B_2 in Proposition 2. The expression of B_3 equals (38).

Next, consider Case (DOS). The solution to ODE (9) with boundary conditions $E(x^d) = 0$ and $E(x^{s1}) = (1-\tau)P(x^{s1}) - C/r$ is expressed as (21) for $x \in (x^d, x^{s1})$. The solution to ODE (8) with boundary conditions $E(x^{s1}) = (1-\tau)P(x^{s1}) - C/r$ and $E(x^{s2}) = (1-\tau)P(x^{s2}) - C/r$ is expressed as (21) for $x \in [x^{s1}, x^{s2}]$. The solution to ODE (9) with boundary conditions $E(x^{s2}) = (1-\tau)P(x^{s2}) - C/r$ and $\lim_{x\to\infty} E(x)/x < \infty$ is expressed as (21) for $x > x^{s2}$. Continuous differentiability of $E(x)$ at x^d, x^{s1}, and x^{s2} leads to (23), (24), and (25), respectively. A general solution to ODE (11) is expressed as (22) for $x \in (x^d, x^{s1})$. A general solution to ODE (10) is expressed as (22) for $x \in [x^{s1}, x^{s2}]$. A general solution to ODE (11) with boundary condition $\lim_{x\to\infty} D(x) = C/r$ is expressed as (22) for $x > x^{s2}$. Boundary condition $D(x^d) = (1-\alpha)E_U(x^d)$, continuity and continuous differentiability at x^{s1}, and continuity and continuous differentiability at x^{s2} lead to (26)–(30), respectively.

Now, we consider which case holds. Note that (14) is nonnegative if and only if (13) holds. Indeed, as $x \to 0$, (14) decreases to

$$(1-\tau)\left\{-\frac{C}{r} + \frac{\lambda}{r+\lambda}\left(\theta - \frac{\tau C}{(1-\tau)r}\right)\right\},$$

which is nonnegative if and only if (13) holds. In this case, default is not optimal for any $X(t)$ because (14) dominates the default payoff 0. This removes the possibility of Cases (DS), (DOS), and (D), and hence, we have Case (S) under condition (13).

Suppose that (13) does not hold. By the above argument, (14) is negative for a sufficiently low x, which means that Case (S) does not hold. The remaining candidates are Case (DS), (DOS), and (D). Note that (32) dominates the sell-out payoff $(1-\tau)P(x) - C/r$ for any $x(>0)$ if and only if (31) holds. In fact, we define

$$\begin{aligned} f(x) &= (1-\tau)\left\{\frac{x}{r-\mu} - \frac{C}{r} + \left(\frac{x}{x^d}\right)^\gamma \left(-\frac{x^d}{r-\mu} + \frac{C}{r}\right)\right\} - ((1-\tau)P(x) - C/r) \\ &= \frac{(1-\tau)(1-a)x}{r-\mu} + \left(\frac{x}{x^d}\right)^\gamma \frac{(1-\tau)C}{(1-\gamma)r} + \frac{\tau C}{r} - (1-\tau)\theta \end{aligned}$$

and have

$$
\begin{aligned}
&\min_{x \geq x^d} f(x) \\
=& f(x^d(1-a)^{1/(\gamma-1)}) \qquad (41)\\
=& \frac{(1-\tau)(1-a)x^d(1-a)^{1/(\gamma-1)}}{r-\mu} + (1-a)^{\gamma/(\gamma-1)}\frac{(1-\tau)C}{(1-\gamma)r} + \frac{\tau C}{r} - (1-\tau)\theta \\
=& \frac{((1-\tau)(1-a)^{\gamma/(\gamma-1)} + \tau)C}{r} - (1-\tau)\theta,
\end{aligned}
$$

which is positive if and only if (31) holds. We obtained (41) by solving the first order condition (i.e., $f'(x) = 0$) because of convexity of $f(x)$. Under condition (31), sell-out is not optimal for any $X(t)$ because (32) dominates the sell-out payoff. This removes the possibility of Cases (DS) and (DOS), and hence, we have Case (D) under (31). Note that

$$
\frac{(1-\tau)\lambda}{\tau\lambda + (1-\tau)(r+\lambda)} < \frac{1-\tau}{\tau + (1-\tau)\epsilon}.
$$

follows from $\epsilon \in (0, 1]$.

We now assume (16). By the above argument, (32) does not dominate the sell-out payoff, which removes the possibility of Case (D). Thus, the remaining candidates are Case (DS) and (DOS) under condition (16).

C Proof of Proposition 3

Clearly, we have $P^d(x) = 0$ in Case (S), and $P^d(x) = 1$ follows from $\mu - 0.5\sigma^2 < 0$ in Case (D). Suppose that (16) holds. The default probability $P^d(x)$ satisfies the following ODEs:

$$
\mu x P^{d\prime}(x) + 0.5\sigma^2 x^2 P^{d\prime\prime}(x) + \lambda(-P^d(x)) = 0 \ (x \in \mathcal{S}), \qquad (42)
$$

$$
\mu x P^{d\prime}(x) + 0.5\sigma^2 x^2 P^{d\prime\prime}(x) = 0 \ (x \in \mathcal{O}), \qquad (43)
$$

where $\mathcal{S}$ and $\mathcal{O}$ stand for the sell-out and operation regions, respectively. In ODE (42), $\lambda(-P^d(x))$ corresponds to the fact that $P^d(x)$ changes to 0 with probability $\lambda \mathrm{d}t$ in time interval $\mathrm{d}t$ in the sell-out region.

Now, consider Case (DS). A general solution to ODE (42) is expressed as (34) for $x \in (x^d, x^s]$. A general solution to ODE (43) with boundary condition $\lim_{x\to\infty} P^d(x) < \infty$ is expressed as (34) for $x > x^s$. As in derivation of $D(x)$ in Case (DS) in Appendix B, by continuity at x^d and x^s and continuous differentiability at x^s, we have G_1, G_2, and G_3 in Proposition 3.

Next, consider Case (DOS). A general solution to ODE (43) is expressed as (35) for $x \in (x^d, x^{s1})$. A general solution to ODE (42) is expressed as (35) for $x \in [x^{s1}, x^{s2}]$. A general solution to ODE (43) with boundary condition $\lim_{x\to\infty} P^d(x) < \infty$ is expressed as (35) for $x > x^{s2}$. As in derivation of $D(x)$ in Case (DOS) in Appendix B, by continuity at x^d, x^{s1}, and x^{s2}, as well as continuous differentiability at x^{s1} and x^{s2}, we have the linear equations of G_i $(i = 1, 2, \ldots, 5)$ in Proposition 3.

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Table 1: Baseline parameter values.

r	μ	σ	τ	α	a	θ	C	λ	x
0.06	0.01	0.2	0.15	0.3	0.5	15	1	2	1

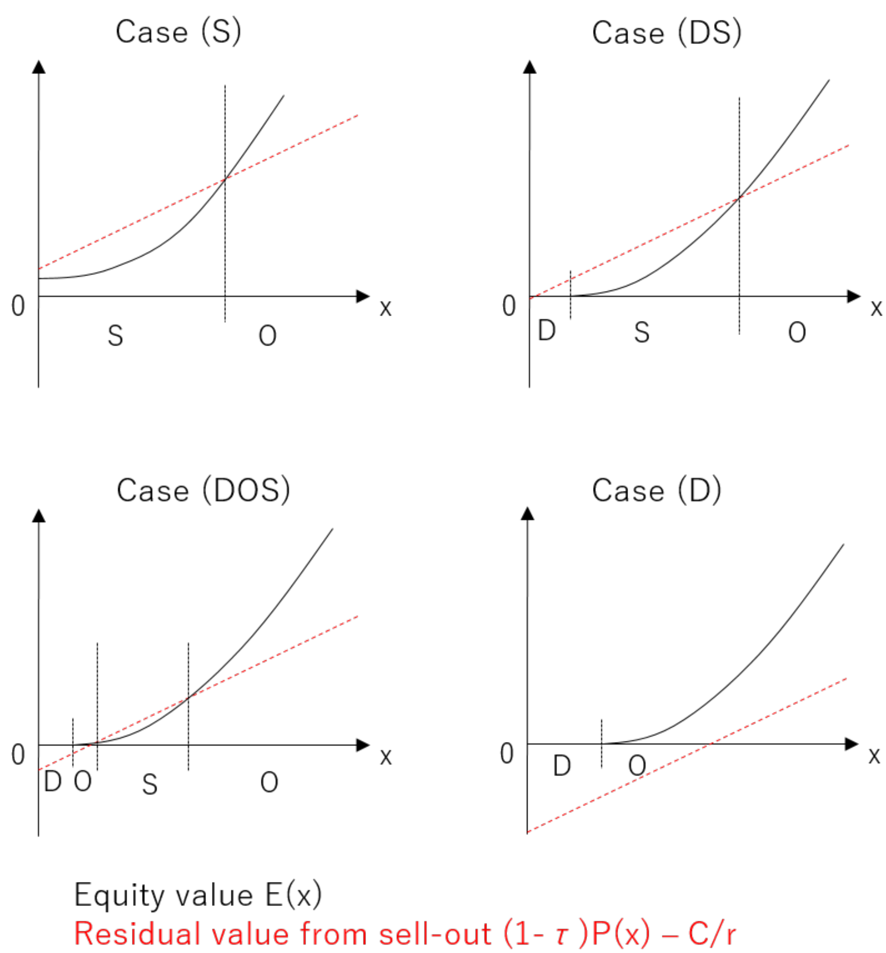

Figure 1: Four cases depending on $E(x)$ and $(1-\tau)P(x) - C/r$. The notations O, S, and D stand for the operation, sell-out, and default regions, respectively.

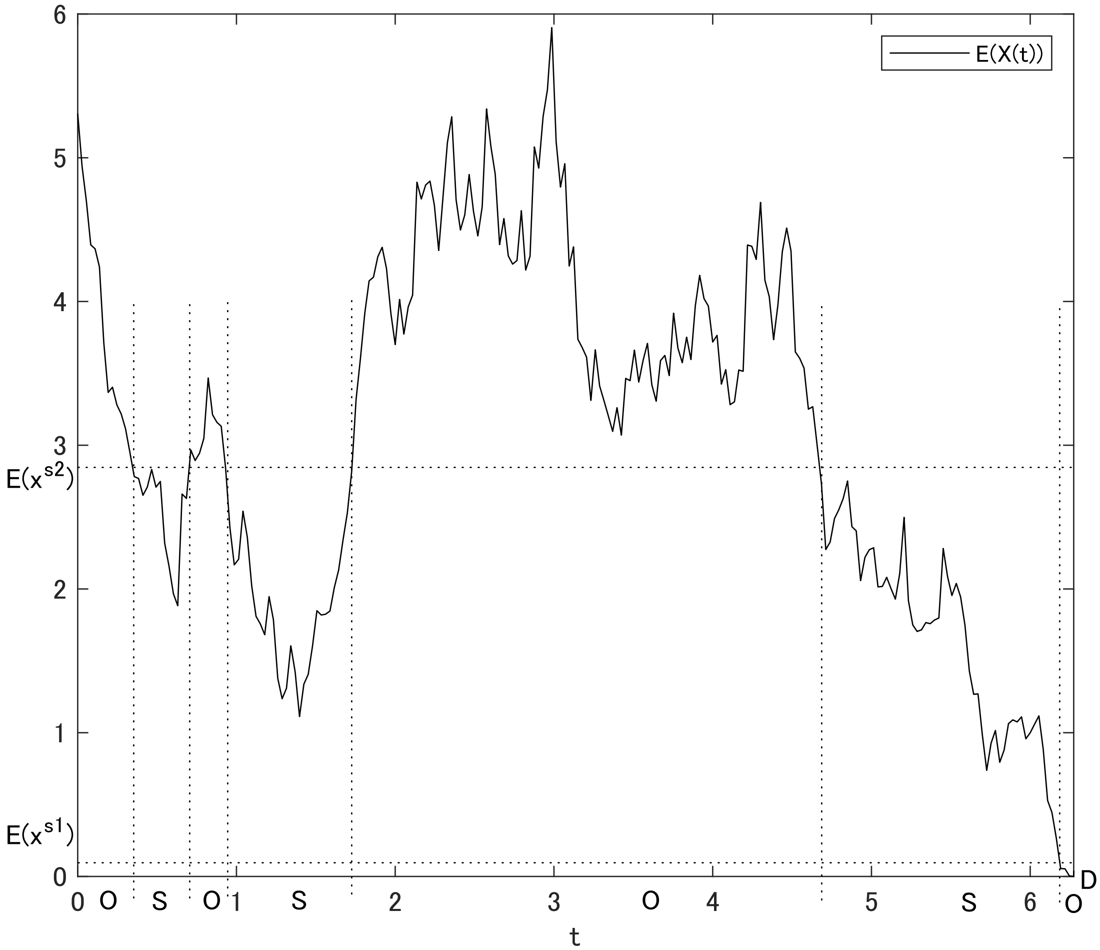

Figure 2: A sample path of the equity value $E(X(t))$. The notations O, S, and D stand for the operation, sell-out, and default periods, respectively.

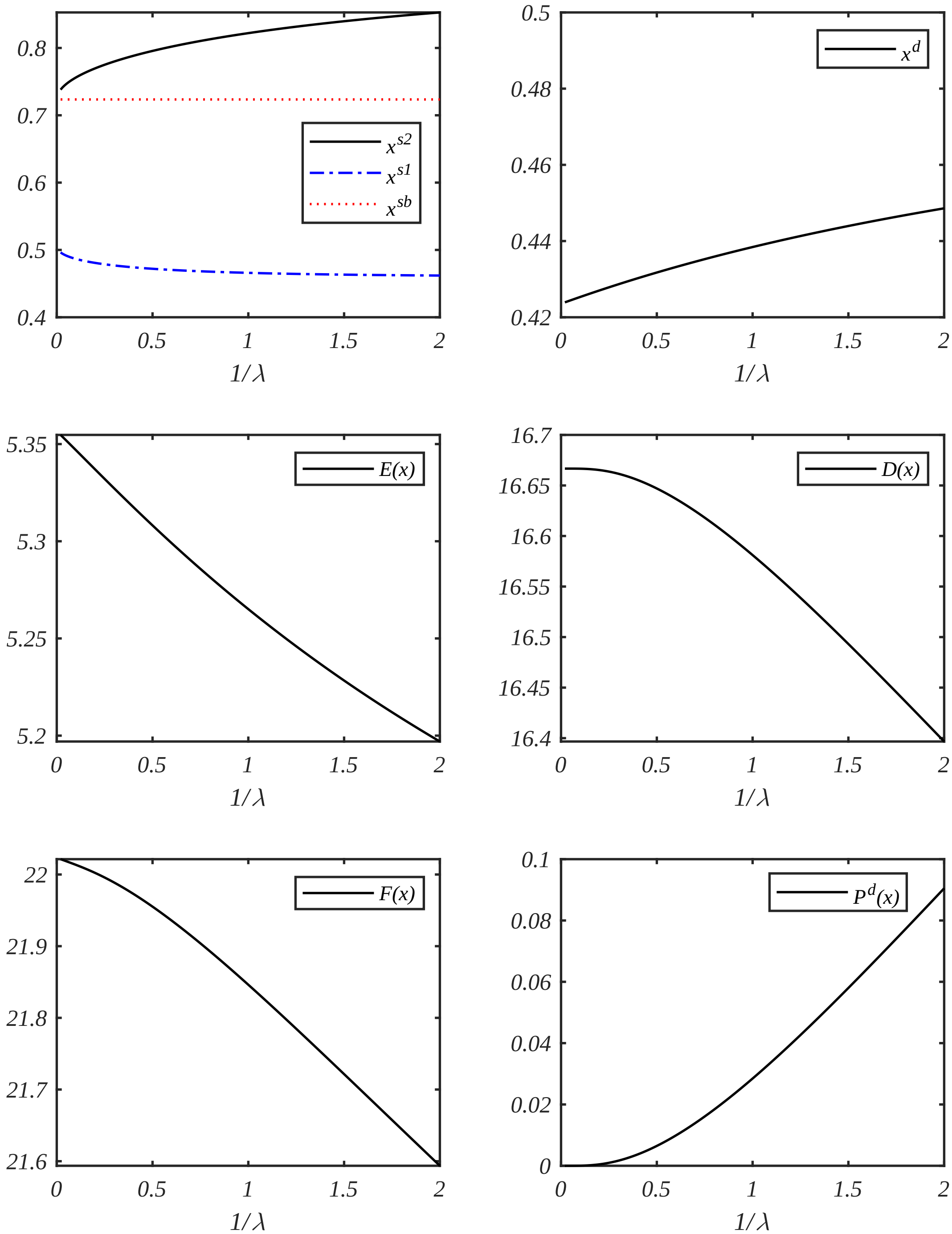

Figure 3: Comparative statics with respect to $1/\lambda$. The other parameter values are set in Table 1. Case (DOS) holds for all $1/\lambda \in (0, 2]$.

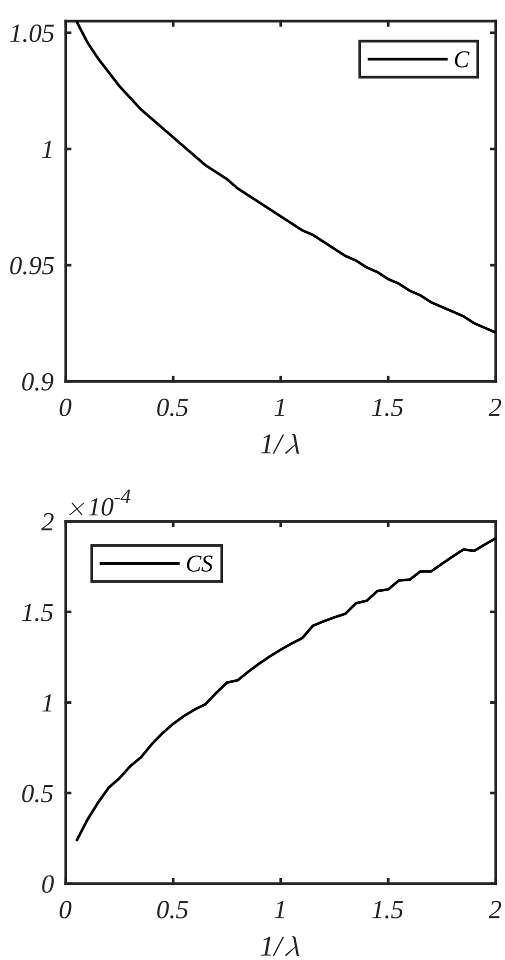

Figure 4: Optimal capital structure with respect to $1/\lambda$. Case (DOS) holds for $1/\lambda \in (0, 1.25]$, while Case (DS) holds for $1/\lambda \in (1.25, 2]$. The other parameter values are set in Table 1.

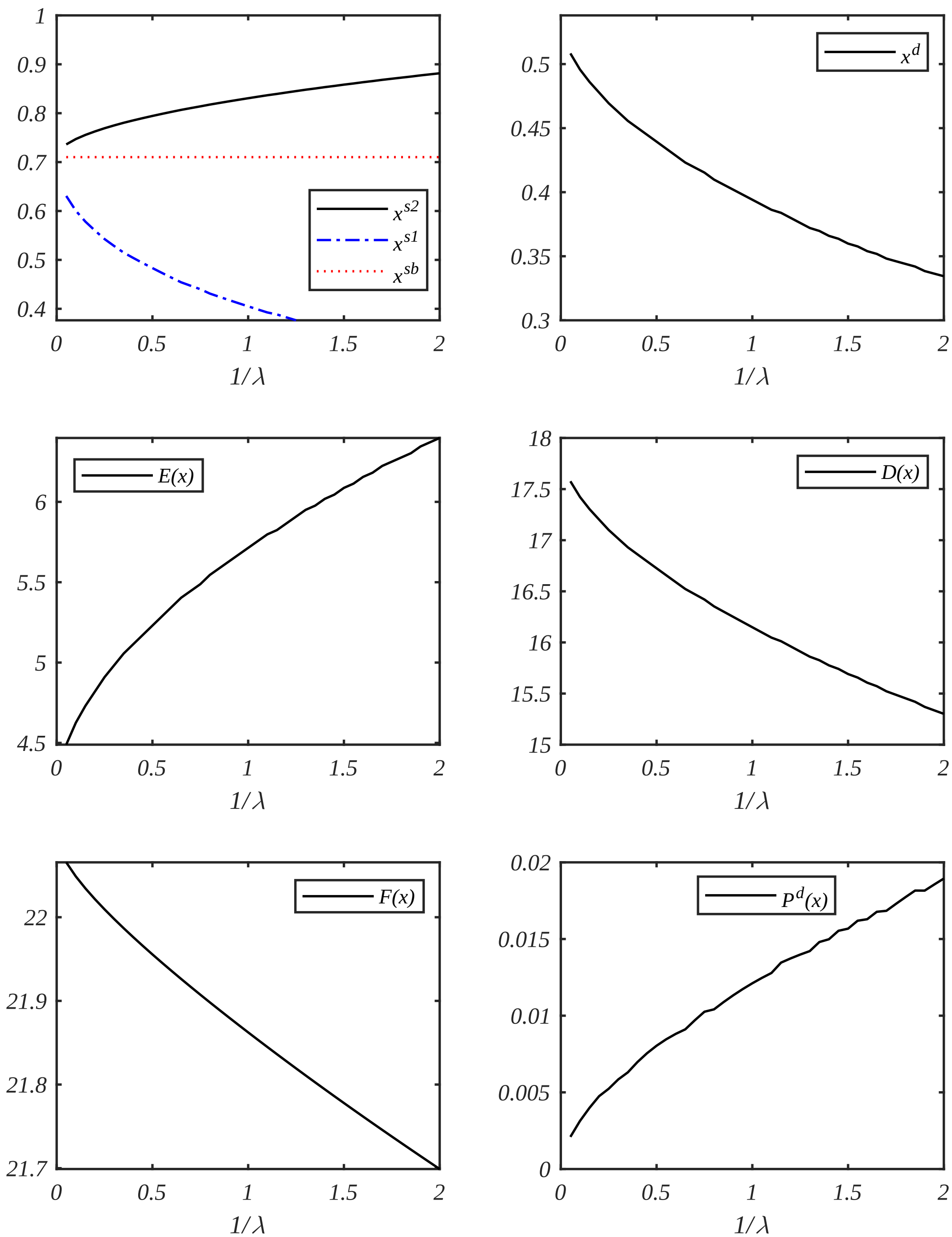

Figure 5: Comparative statics with respect to $1/\lambda$ under the optimal capital structure. Case (DOS) holds for $1/\lambda \in (0, 1.25]$, while Case (DS) holds for $1/\lambda \in (1.25, 2]$. The other parameter values are set in Table 1.

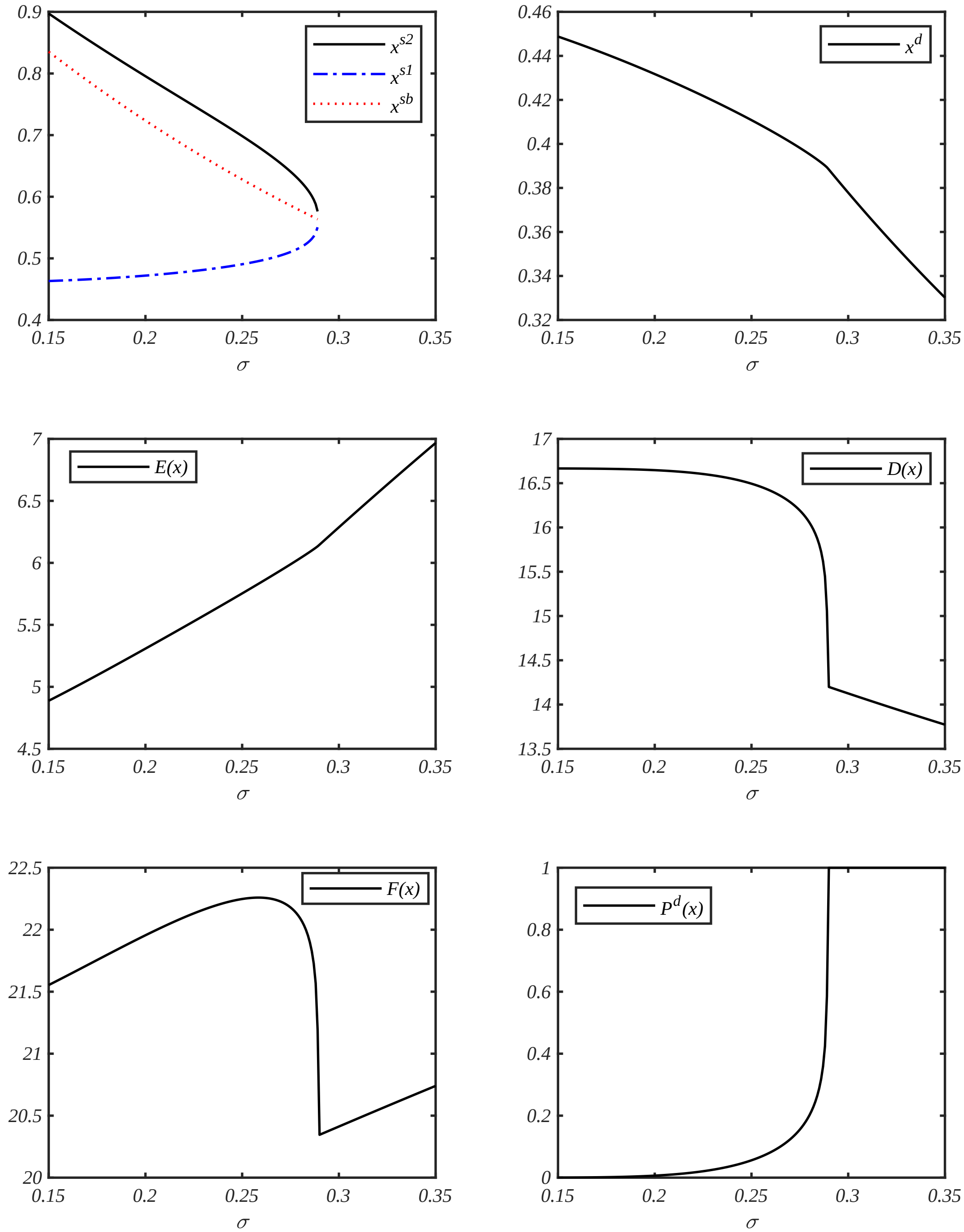

Figure 6: Comparative statics with respect to σ. The other parameter values are set in Table 1. Case (DOS) holds for $\sigma \in [0.15, 0.289)$, while Case (D) holds for $\sigma \in [0.289, 0.35]$.

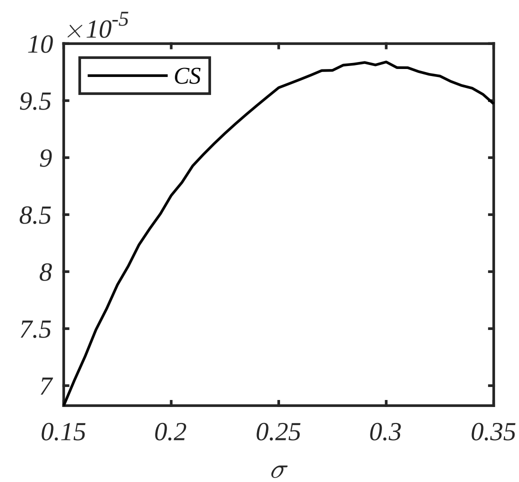

Figure 7: Optimal capital structure with respect to σ. The other parameter values are set in Table 1. Case (DOS) holds for all $\sigma \in [0.15, 0.35]$.

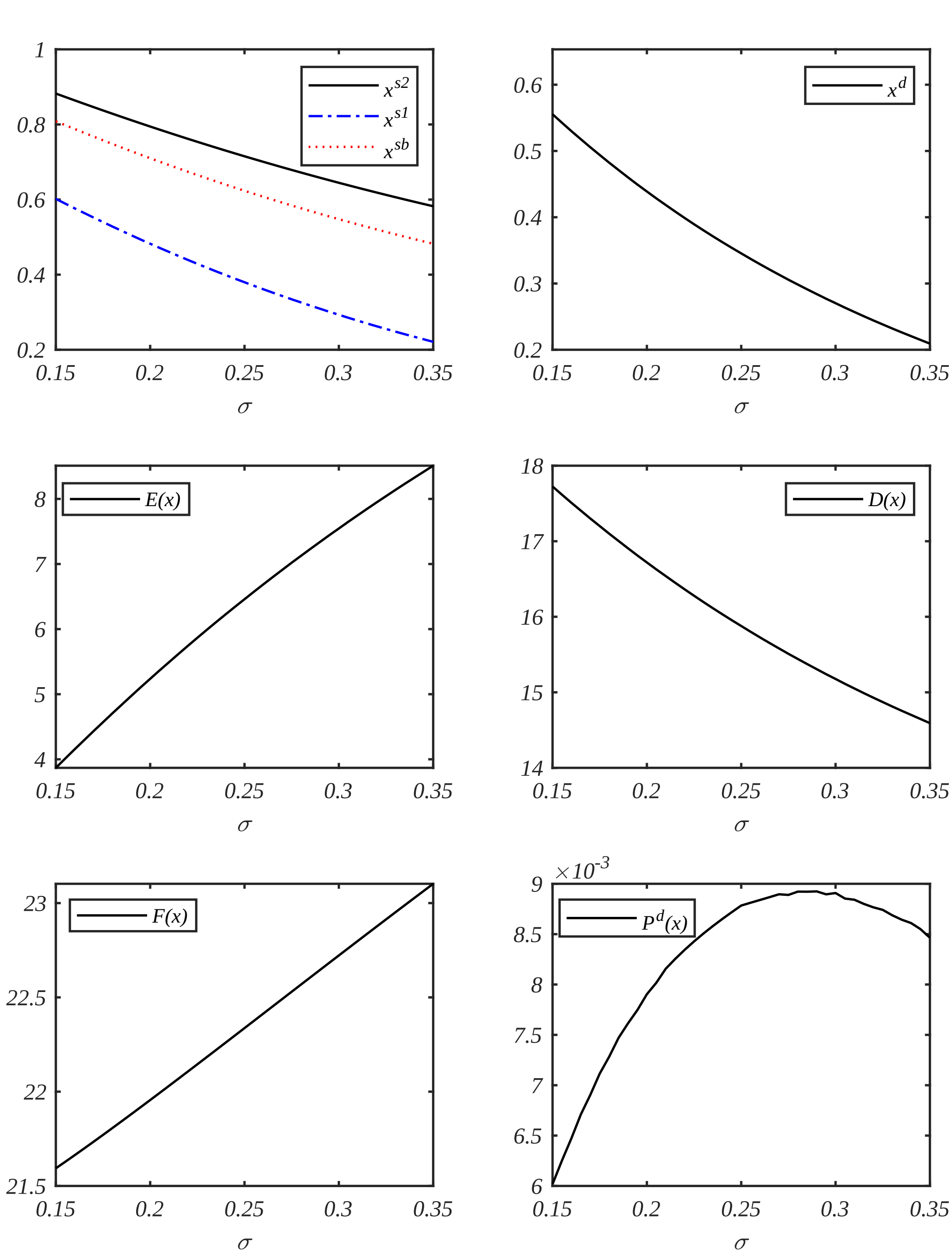

Figure 8: Comparative statics with respect to σ under the optimal capital structure. The other parameter values are set in Table 1. Case (DOS) holds for all $\sigma \in [0.15, 0.35]$.

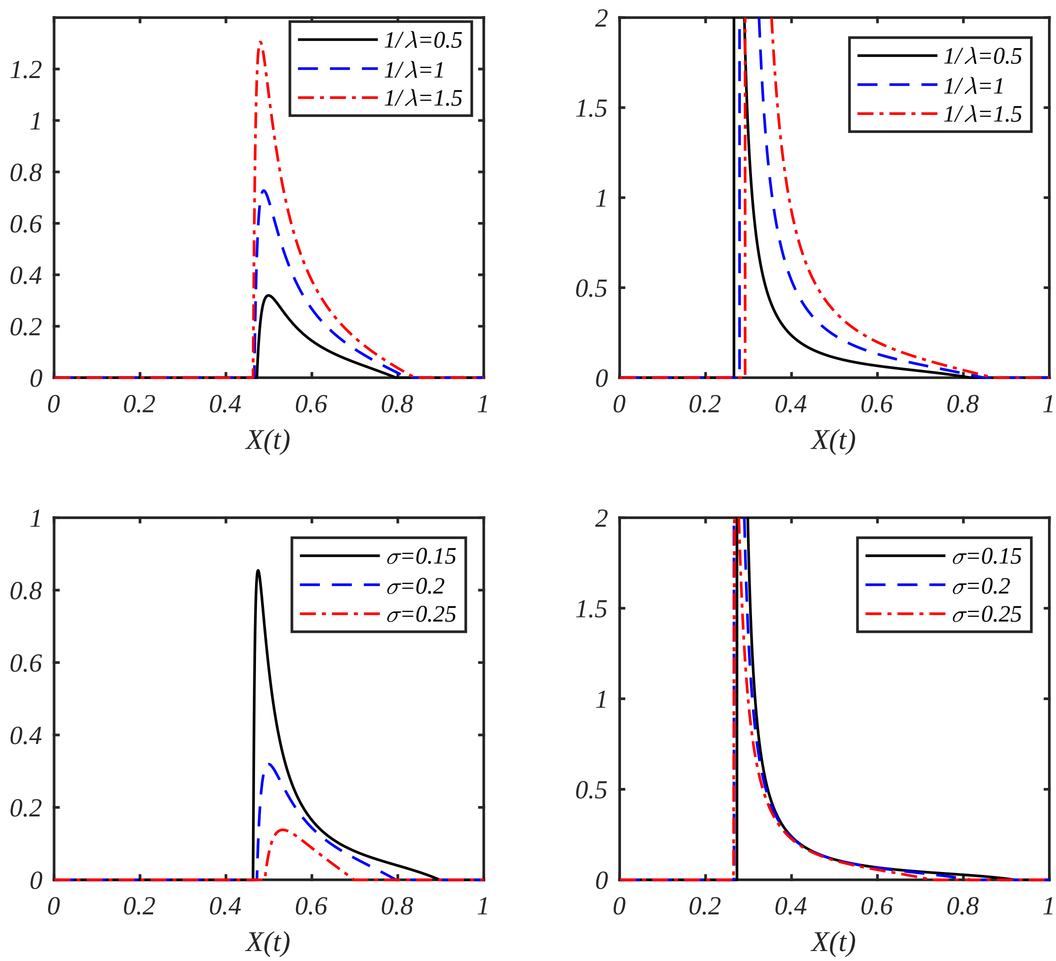

Figure 9: Stock price reaction. This figure depicts the jump size $(E(x) - (1-\tau)P(x) - C/r)/E(x)$. In the left panels, the other parameter values are set in Table 1, and Case (DOS) holds. In the right panels, we set $C = 0.9$, where Case (DOS) holds only for $\sigma = 0.25$, and Case (DS) holds for the other parameter values.

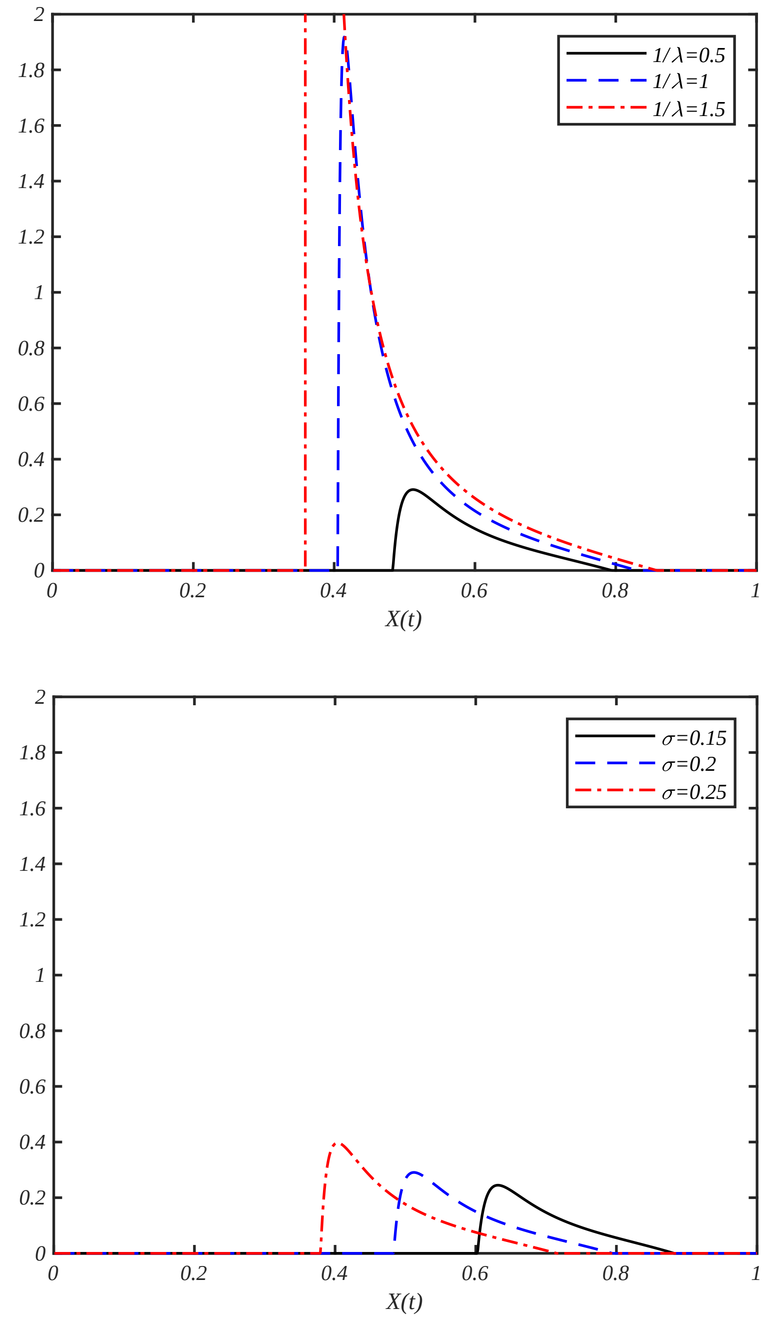

Figure 10: Stock price reaction under the optimal capital structure. This figure depicts the jump size $((1-\tau)P(X(t))-C/r-E(X(t)))/E(X(t))$ under the optimal capital structure. The other parameter values are set in Table 1. Case (DS) holds for $1/\lambda = 1.5$, while Case (DOS) holds for the other parameter values.