Discussion Papers In Economics
And Business

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Discussion Paper 20-16

December 2020

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Abstract

This study examines the influence of import tariff policies on welfare in a two-country model with heterogeneous firms and variable markups. In this model, the market outcome under free trade provides too small (large) output level for more (less) productive firms and too many varieties due to markup pricing, which can be partially compensated by import tariffs. If countries cooperatively adopt a symmetric import tariff, the efficient tariff that maximizes the total welfare level of the two countries is positive when the introduction of the small symmetric import tariff sufficiently improves this within-sector misallocation.

Keywords: Variable markups, Misallocation, Firm heterogeneity

JEL Classification: F12, F13, L11

*I thank Koichi Futagami, Kazuhiro Yamamoto, Hayato Kato, and Koji Shintaku for helpful comments and suggestions. All remaining errors are my own.

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1 Introduction

In trade negotiations between WTO members, as the number of members increases, the coordination of interests between member countries tends to become more complicated, making it difficult to respond quickly to new challenges and rule-making. In such cases, bilateral or regional trade agreements still play an important role today, as they allow for relatively easy coordination of interests. When a country enters into a bilateral trade agreement, how should the agreement be design: should it be concluded between similar countries in terms of market size and technology, or should it be free trade agreement? Although much attention has been devoted to the study of international trade models that consider heterogeneous firms since the seminal work of Melitz (2003), little is known about the effects of bilateral trade policy in such models while these studies focus on the effect of unilateral trade policy.

The impact of bilateral trade agreement is deeply related to the model structure that deal with it. In particular, the canonical models of monopolistic competition with constant elasticity of substitution (CES) preferences, which generates constant markups, opposite externalities related to the inefficiency of the product variety offset each other. That is, the market equilibrium under free trade is efficient (when there is no other sector than the monopolistically competitive one). Therefore, there is no room for welfare improving policy intervention. As Dhingra and Morrow (2019) show, the market outcome is first best under CES preferences and demand-side elasticity determines how resources are misallocated.

In this study, I analyze the effects of uncooperative and cooperative trade policies on welfare with a focus on import tariff policies in a monopolistically competitive model with heterogeneous firms á la Melitz and Ottaviano (2008), who incorporate endogenous markups by introducing the linear demand system into the Melitz model. Incorporation of such a linear demand system makes the markups of firms endogenous. The presence of endogenous markups affects the efficiency of resource allocation. Based on the closed economy setting of the Melitz-Ottaviano model, Nocco et al. (2014) show that, in addition to the inefficiency of the product variety that arise under constant markups, endogenous markups create within-sector misallocation and market outcome becomes excessively inefficient.[1]

Then, the following question arises: how does bilateral import tariff policy affect this within-sector misallocation and welfare in the model of monopolistic competition with firm

[1] Using the multi-country setting of Melitz and Ottaviano (2008), Nocco et al. (2019) also show that free trade allocation of resources fails to be efficient due to this within-sector misallocation.

heterogeneity and variable markups? The present study shows that the market outcome under free trade creates this within-sector misallocation even in the model without firm entry, and that it can be partially improved by bilateral import tariff policy.

I modify the Melitz-Ottaviano model by incorporating ad valorem import tariffs without considering free entry in the differentiated goods sector.[2] In this model, there are two countries with two sectors: the firms in a sector produce differentiated varieties under monopolistic competition while firms in the other sector produce homogeneous good under perfect competition. In this setting, an increase in a country's import tariff affects its welfare through changes in cross-sector and within-sector allocations. With respect to the impact on cross-sector allocation, an increase in a country's import tariff increases income by increasing its tariff revenue and domestic profits and raising the average price of differentiated goods sold in that country, which shifts the production of goods sold in the country from the differentiated good sector to the homogeneous good sector. Moreover, it also affects within-sector allocation in the differentiated good sector through three channels: product variety, product selection, and product mix. As for the first channel, an increase in the import tariff decreases the number of varieties: it intensifies export competition among exporters in the country's trading partner and thereby forces some of exporters to stop exporting. Although it also increases the number of domestic firms in the country, the former effect (which decreases exporters) dominates the latter effect (which increases domestic firms). As for the second channel, by an increase in the import tariff, product selection becomes tougher for exporters in the country's trading partner and more relaxed for domestic firms in the country. This is because firms that stop (start) producing for the country by this tariff increase are the least productive exporters (domestic firms). The impact of an increase in the ad valorem import tariff on the last channel, product mix, can be divided into two effects. Firstly, an increase in the import tariff directly decreases the output level of exporters in the country's trading partner. This decrease is greater for less productive exporters, which in turn causes some of the least productive exporters to stop exporting. Secondly, this decrease in the number of exporters increases the output level of all surviving firms uniformly. As a result, an increase in the import tariff increases (decreases) the output level of more (less) productive exporters in the country's trading partner and increases the output level of domestic firms in the country. Thus, the welfare effect of a country's import tariff is determined by a combination of

[2] In the next section, I show that the number of firms changes with trade policies even without free entry.

these effects.

This study characterizes the uncooperative and cooperative import tariff policies, resulting in the following main findings. First, the Nash tariffs are positive and, when countries are symmetric, lower than the efficient import tariff that the countries adopt uniformly to maximize the total welfare level of the countries.[3] An increase in a country's import tariff increases its welfare mainly by increasing its income (and thereby the output level of homogeneous good sold in the country) when the import tariff is sufficiently small. By contrast, an increase in the country's import tariff harms its trading partner by decreasing its income owing to a decrease in export profits. As a result, the Nash tariffs are positive due to the effect of increasing income in both countries, and higher than the efficient tariff because, in a global welfare perspective, this effect is partially offset by the effect of the decreasing income in each country's trading partner.

Second, if countries cooperatively adopt a symmetric import tariff, the efficient tariff that maximizes the total welfare level of the two countries is positive under the following situation: the symmetric import tariff is introduced when global free trade prevails and its introduction sufficiently improves within-sector misallocation. Otherwise, global free trade is desirable. In this model, the free trade allocation of resources is inefficient in both cross-sector and within-sector allocation due to markup pricing. Compared to the first-best allocation set by a social planner to maximize the total welfare level of the countries, in the market allocation under free trade, too small total output level is produced in the differentiated good sector (cross-sector misallocation); too many varieties are sold (inefficient product variety); too low-productivity firms, both domestic firms and exporters, remain in the market (inefficient product selection); and too small (large) output level is produced by more (less) productive firms, both domestic firms and exporters (inefficient product mix), in both countries. Within-sector misallocation consists of inefficient product variety, product selection, and product mix. Thus, under free trade, markup pricing creates not only cross-sector misallocation due to undersupply of the total output level in the differentiated good sector, but also within-sector misallocation: more productive firms do not pass on their entire cost advantage to consumers by raising their markups and they end up selling less than first-best output level, which leaves room for less productive firms to end up being oversupplied and for the least productive firms to survive

[3]In this model, the optimal import tariff, which is set by a country to maximize its welfare with the import tariff set by the country's trading partner as given, is consistent with the Nash tariff. This is because the optimal import tariff is determined independently of the import tariff set by the country's trading partner.

inefficiently.[4] The introduction of the symmetric import tariff shifts the production from the differentiated good sector to the homogeneous good sector and thereby distorts cross-sector allocation.[5] Meanwhile, the introduction of the symmetric import tariff increases the total welfare level through partially improving within-sector misallocation.[6] As a result, the efficient tariff that maximizes the total welfare level of the two countries is positive if and only if the latter effect dominated the former. This result differs from that of Melitz and Ottaviano (2008), who treat trade liberalization as a decrease in transportation costs and show that bilateral trade liberalization leads to welfare gain for the two countries in both the long run and short run. Unlike transportation costs, changes in import tariffs have tariff-revenue impacts. The difference between transportation costs and import tariffs in these models results in a difference in the effects of trade policies on welfare.

Third, I analyze under what circumstances both countries can mutually benefit from the introduction of a symmetric import tariff from the initial situation of global free trade when the efficient tariff is positive. I find that the introduction of the symmetric import tariff improves welfare of both countries not only when countries are close to symmetric, but also when the degree of asymmetry across countries is large: when one country has a relatively larger population size and number of high-productivity firms than the other country.[7] In these cases, the introduction of the symmetric import tariff has little impact on the welfare level of such countries through changes in cross-sector allocation. Therefore, it improves the welfare level of the countries through improving within-sector misallocation when the efficient tariff is positive. This result indicates that it is crucial for countries that participate in a trade agreement to decide their import tariffs based on their relative size.

[4]Unlike this case, in the canonical models of monopolistic competition with CES preferences which includes the outside good sector, the number of varieties are inefficiently small because the market outcome under free trade provides inefficiently small total output level in the monopolistically competitive sector due to markup pricing but efficient output level of each firm in the sector due to their constant markups (Melitz and Redding, 2014 and 2015).

[5]It also distorts within-sector allocation through product selection and product mix of domestic firms by allowing the least productive domestic firms to produce and less productive domestic firms to increase the output level.

[6]It decreases the number of varieties in both countries by shutting out the least productive exporters, increases (decreases) the output level of more (less) productive exporters, and increases the output level of more productive domestic firms.

[7]If countries are symmetric and the efficient tariff is positive, then the welfare of both countries is maximized at the efficient import tariff.

1.1 Related literature

Some studies incorporate tariff policies into the Melitz-Ottaviano model. Bagwell and Lee (2020) incorporate import and export tariffs into the Melitz-Ottaviano model and study the impact of trade policy in a symmetric two-country economy. They show that starting at global free trade, the impact of introducing the total tariff (the sum of tariffs imposed when exporting from one country to the other) and its symmetric increase on joint welfare depends on a simple relationship among parameters. The present study obtains complementary results to this finding, even in a model allowing asymmetric countries and without free entry, and shows analytically the condition under which the introduction of a symmetric import tariff increases not only joint welfare, but also welfare in both countries. While Bagwell and Lee (2020) also show that, under some assumptions, symmetric Nash tariff is higher than the efficient tariff when the introduction of the symmetric tariff increases joint welfare, the present study can provide this result without requiring the assumptions they impose.[8]

Nocco et al. (2019) consider the efficiency properties of the market outcome in a multi-country setting of Melitz and Ottaviano (2008) and characterize the policy tools that national policy makers can use cooperatively to make the market achieve the efficient outcome. Under an unconstrained choice of tools, which include domestic and trade policies, and country-specific and firm-specific production subsidies/taxes, Nocco et al. (2019) show that the market can achieve the first-best outcome. When firm-specific production subsidies/taxes are unavailable, they consider a second-best scenario in which a per-unit production subsidy is offered to all firms and financed by a lump-sum tax on consumers. Relative to this work, the present study differs from theirs in policy instruments and model structure. In the present study, governments cooperatively choose the efficient ad valorem import tariff, which is uniform across countries and firms, without using domestic policy instruments. With respect the model structure, the model in this study, which does not consider free entry, has similar characteristics to the model in Nocco et al. (2019) in terms of how the free market outcome departs from the first-best outcome. In the present study, the market level of the number of varieties sold in each country is above that in the first-best outcome. This result

[8] To obtain this result, Bagwell and Lee (2020) assume that the symmetric Nash and efficient tariffs are interior solutions and that the joint-welfare function is quasi-concave in the symmetric tariff. By contrast, the present study can show that Nash tariffs are interior solutions; the efficient tariff is an interior solution when the introduction of the symmetric import tariff increases joint welfare; and joint welfare function is quasi-concave in the symmetric import tariff when the introduction of the symmetric import tariff increases joint welfare and the countries are symmetric.

implies that, even without free entry, the effect that Nocco et al. (2019) discuss as the entry externality created by endogenous markups is inherent in this model, where, by keeping price above marginal cost more than higher marginal cost rivals, lower marginal cost firms leave inefficiently larger room for a fringe of the highest marginal cost firms to produce.[9] Moreover, depending on whether per-unit subsidies or ad valorem taxes are used, the impact of the policy instrument on within-sector allocation varies: in the Melitz-Ottaviano model, while per-unit subsidies increase the output level of all firms uniformly, ad valorem taxes increase (decrease) the output level of more (less) productive firms.

Demidova (2017) removes the outside good from the Melitz-Ottaviano model and characterizes optimal unilateral import tariffs for small and large countries. She shows that the optimal tariffs are positive for both small and large countries. Compared to Demidova (2017), the present study analyzes the effect of tariff policies on cross-sector allocation as well as within-sector allocation and characterizes the efficient tariff which maximizes the total welfare level of two countries.

The present study is also related to the following studies. Demidova and Rodríguez-Clare (2009) and Haaland and Venables (2016) consider a small-country version of the Melitz model and characterize a unilateral trade policy that achieves first-best allocation. Felbermayr et al. (2013) extend Demidova and Rodriguez-Clare (2009) to the case of two large countries and characterize the optimal tariff. Campolmi et al. (2020) characterize the Nash equilibrium as consisting of first-best level labor subsidies that achieve production efficiency, and inefficient import subsidies and export taxes aimed at improving the domestic terms of trade when both domestic and trade policies are available. Costinot et al. (2020) consider the case of available domestic and trade policy instruments and characterize optimal unilateral tariffs both when tariffs are firm-specific and when they are uniform in a canonical model of intra-industry trade with monopolistic competition and firm-level heterogeneity. Bagwell and Lee (2018) incorporate a homogeneous good sector in the Melitz model in a symmetric two-country economy and show that, starting at global free trade, the introduction of a symmetric import tariff lowers joint welfare. These studies build on the model with CES preference, which generates constant markups. By contrast, the present study analyzes how tariff policies affect the within-sector misallocation created by endogenous markups and shows the case in

[9] In the model in Nocco et al. (2019), whether the number of varieties sold in each country is above or below that in the first-best outcome depends on relationship between this externality and other entry-related externalities.

which the introduction of a symmetric import tariff improves welfare in both countries.

1.2 Organization of the article

The rest of the paper is organized as follows. Section 2 describes our model. Section 3 shows that the Nash tariffs are positive, and conducts comparative statics. Section 4 characterizes the efficient tariff that maximizes the total welfare level of the two countries. Section 5 shows that both countries can simultaneously gain by imposing a symmetric import tariff compared to global free trade, even when the degree of asymmetry across countries is large. Section 6 concludes.

2 Model

In this study, it is assumed that there are two countries labeled H (home) and F (foreign), two sectors, and one production factor, which is labor in this model. Labor L_i $(i = H, F)$ is inelastically supplied by households in each country and is immobile between countries.

2.1 Households

The preferences of households are defined over a continuum of differentiated varieties of goods and a homogeneous good. The differentiated goods are indexed by $\omega \in \Omega_i$ and the homogeneous good is chosen as the numeraire. All households in country i share the same preference and each household maximizes the following utility function:

$$U_i = q_{0,i}^c + \alpha \int_{\Omega_i} q_i^c(\omega) d\omega - \frac{\gamma}{2} \int_{\Omega_i} q_i^c(\omega)^2 d\omega - \frac{\eta}{2} \left(\int_{\Omega_i} q_i^c(\omega) d\omega \right)^2, \tag{1}$$

subject to the budget constraint

$$q_{0,i}^c + \int_{\Omega_i} p_i(\omega) q_i^c(\omega) d\omega = I_i, \tag{2}$$

where Ω_i is the set of all available differentiated goods varieties in country i; $q_{0,i}^c$ and $q_i^c(\omega)$ are the individual consumption of the numeraire good and each variety ω in country i, respectively; $p_i(\omega)$ is the price of variety ω in country i; and I_i is the income of households in country i. Income consists of wage, profits, and the lump-sum transfer from a government. I assume that the households are stockholders of domestic firms. The parameters α, η, and γ are positive constants. A lower γ indicates that the differentiated varieties become closer substitutes and in the limit case of $\gamma = 0$, households care only about the total amount of differentiated goods they consume. η represents the degree of non-separability. When η equals zero, the utility function becomes separable across the differentiated varieties.

I assume that the households have positive demand for the numeraire good ($q^c_{0,i} > 0$).[10] Using the first-order conditions for utility maximization, the inverse demand for each variety ω is given by

$$p_i(\omega) = \alpha - \gamma q^c_i(\omega) - \eta Q_i, \quad \forall \omega \in \Omega^*_i, \tag{3}$$

where $\Omega^*_i \subset \Omega_i$ represents the subset of varieties in which $q^c_i(\omega) > 0$, and $Q_i \equiv \int_{\Omega^*_i} q^c_i(\omega) d\omega$ is the aggregate consumption of all differentiated goods. By integrating both sides of (3) over Ω^*_i, I obtain

$$Q_i = \frac{N_i}{\gamma + \eta N_i}(\alpha - \bar{p}_i), \tag{4}$$

where N_i is the number of consumed (domestic and imported) varieties, and $\bar{p}_i = (1/N_i) \int_{\Omega^*_i} p_i(\omega) d\omega$ is the average price of consumed varieties in country i. Using (3) and (4), I obtain the following market demand for variety ω in country i, $q_i(\omega)$:

$$q_i(\omega) = L_i q^c_i(\omega) = \frac{L_i}{\gamma}(p^{max}_i - p_i(\omega)), \tag{5}$$

where

$$p^{max}_i \equiv \frac{\gamma\alpha + \eta N_i \bar{p}_i}{\gamma + \eta N_i} \tag{6}$$

represents the threshold price in country i at which demand for a variety is driven to zero. Note that (3) implies $p^{max}_i \leq \alpha$.

2.2 Firms

Perfect competition prevails in the homogeneous good market. One unit of production of homogeneous good requires one unit of labor input. The homogeneous goods are freely traded between the countries. Thus, the wage becomes one in both countries.

[10] In Appendix 7.5, I show the sufficient condition for $q^c_{0,i} > 0$ in equilibrium.

In the differentiated goods sector, there is a continuum of K_i potential firms in country i, where K_i is assumed to be constant. They produce the differentiated goods under monopolistic competition. Each firm requires c units of labor to produce one unit of the differentiated good. I assume that the unit labor requirement c follows Pareto distribution:

$$c \sim G_i(c) = \left(\frac{c}{c_i^M}\right)^{\theta}, \quad c \in [0, c_i^M], \quad \theta \geq 1,$$

where $G_i(c)$ is cost distribution in country i, θ is an index of the dispersion of the cost, and c_i^M is the upper bound of the cost in country i. In addition, when firms in country i export their goods to country j, they face an iceberg trade cost τ_{ij} and an ad valorem import tariff t_{ij}, where $\tau_{ii} = t_{ii} = 1$, $\tau_{ij} \geq 1$, and $t_{ij} \geq 1$ for $i, j \in \{H, F\}$ and $i \neq j$.

Potential firms in country i determine whether they produce or shut down for domestic and foreign markets after governments in the countries set their tariffs. The firms produce for the country in which they can earn positive operating profits, and otherwise shut down. Then, the profit maximization problem for firms in country i with cost c that sells their goods to consumers in country j is given by

$$\max(\frac{p_{ij}}{t_{ij}} - \tau_{ij} c) q_{ij}, \qquad s.t. \ \ q_{ij} = \frac{L_j}{\gamma}(p_j^{max} - p_{ij}),$$

where p_{ij} and q_{ij} are the tariff inclusive price and the quantity sold in country j, respectively. Let $p_{ij}(c)$ and $q_{ij}(c)$ denote the profit-maximizing price and quantity set by country i's firms with cost c to sell their goods to country j. The profit-maximizing price and quantity are

$$\begin{aligned} p_{ij}(c) &= \frac{\tau_{ij} t_{ij}}{2}\left(\frac{p_j^{max}}{\tau_{ij} t_{ij}} + c\right), \\ q_{ij}(c) &= \frac{L_j \tau_{ij} t_{ij}}{2\gamma}\left(\frac{p_j^{max}}{\tau_{ij} t_{ij}} - c\right). \end{aligned}$$

Next, I define the cost cutoffs. Let c_{ij} be the upper bound of the cost for firms in country i that sell in the market of country j. Firms with cost c_{ij} are indifferent to producing for market j.

$$c_{ii} = \sup\{c : \pi_{ii}(c) > 0\} = p_i^{max}, \quad c_{ij} = \sup\{c : \pi_{ij}(c) > 0\} = \frac{p_j^{max}}{\tau_{ij} t_{ij}}. \tag{7}$$

As described in Melitz and Ottaviano (2008), the cost cutoff represents the toughness of competition in a market. In Appendix 7.2, I show that $c_{ii} > c_{ij}$ in equilibrium, which means that there are no firms that export but do not produce domestically. I call c_{ii} the domestic cost cutoff and c_{ij} for $i \neq j$ the export cost cutoff in country i, and assume that c_i^M is sufficiently high to be above c_{ii}.[11]

Using these cost cutoffs, I obtain the price, quantity, revenue, profit, and markup of firms in country i that sell their goods in country j:

$$
\begin{aligned}
p_{ij}(c) &= \frac{\tau_{ij}t_{ij}}{2}(c_{ij}+c)\left(=\frac{1}{2}(c_{jj}+\tau_{ij}t_{ij}c)\right), \quad (8)\\
q_{ij}(c) &= \frac{L_j\tau_{ij}t_{ij}}{2\gamma}(c_{ij}-c)\left(=\frac{L_j}{2\gamma}(c_{jj}-\tau_{ij}t_{ij}c)\right), \quad (9)\\
r_{ij}(c) &= \frac{L_j\tau_{ij}^2t_{ij}}{4\gamma}(c_{ij}^2-c^2),\\
\pi_{ij}(c) &= \frac{L_j\tau_{ij}^2t_{ij}}{4\gamma}(c_{ij}-c)^2,\\
\mu_{ij}(c) &= \frac{p_{ij}(c)}{\tau_{ij}c}=\frac{t_{ij}}{2}(\frac{c_{ij}}{c}+1).
\end{aligned}
$$

Lower-cost firms set lower prices but also higher markups. This generates the within-sector misallocation distortion, because more productive firms do not pass on their entire cost advantage to households by raising their markups and end up selling too little, while less productive firms end up being oversupplied.

2.3 Government

The government in country i imposes the tariff t_{ji} on exporters in country j and transfers the tariff revenue to households. A firm in country j produces for export if it can earn non-negative profits from sales in the foreign country. Among $K_jG_j(c_{jj})$ firms in country j, firms with cost c ($\leq c_{ji}$) export, so that the number of exporters is described by $K_jG_j(c_{ji})$. The budget constraint of the government is given by

$$
\begin{aligned}
T_i &= K_jG_j(c_{ji})(t_{ji}-1)\bar{r}_{ji}\\
&= \frac{L_ik_j\tau_{ji}{}^{-\theta}}{2\gamma(\theta+2)}(t_{ji}-1)t_{ji}{}^{-(\theta+1)}c_{ii}{}^{\theta+2}, \quad (10)
\end{aligned}
$$

[11]In Appendix 7.1, I show the sufficient condition for $c_{ii} < c_i^M$ in equilibrium

where $\bar{r}_{ji} = \frac{1}{G_j(c_{ji})}\int_0^{c_{ji}} r_{ji}(c)dG_j(c) = \frac{L_i}{G_j(c_{ji})}\frac{\tau_{ji}{}^{-\theta}t_{ji}{}^{-(\theta+1)}c_{ii}{}^{\theta+2}}{2\gamma(\theta+2)c_j^{M\theta}}$ is the average revenue of firms in country j from sales in country i and $k_j \equiv K_j/c_j^{M\theta}$. Let k_j be the productivity index of country j, which measures the number of productive firms in country j.[12]

2.4 Equilibrium

The number of sellers in country i, N_i, is composed of domestic producers and exporters in country j, that is,

$$
\begin{aligned}
N_i &= K_iG_i(c_{ii}) + K_jG_j(c_{ji}) \\
&= k_ic_{ii}{}^{\theta} + k_jc_{ji}{}^{\theta} \\
&= (k_i + k_j(\tau_{ji}t_{ji})^{-\theta})c_{ii}{}^{\theta}. \qquad (11)
\end{aligned}
$$

From (7), the domestic cost cutoff c_{ii}, is expressed as $c_{ii} = p_i^{max}$. Using this relationship, I rewrite the threshold price condition (6) as follows:

$$
\begin{aligned}
c_{ii} &= \frac{1}{\eta N_i + \gamma}(\gamma\alpha + \eta N_i\bar{p}_i) \\
\Leftrightarrow N_i &= \frac{2(\theta+1)}{A}\frac{\alpha - c_{ii}}{c_{ii}}, \qquad (12)
\end{aligned}
$$

where $A \equiv \eta/\gamma$ and

$$
\bar{p}_i = \frac{K_i\int_0^{c_{ii}} p_{ii}(c)dG_i(c) + K_j\int_0^{c_{ji}} p_{ji}(c)dG_j(c)}{N_i} = \frac{2\theta+1}{2(\theta+1)}c_{ii}. \qquad (13)
$$

Thus (11) and (12) determine the domestic cost cutoff c_{ii} and N_i. From (11), and (12), c_{ii} is determined by the following equation:

$$
\begin{aligned}
& k_i + k_j(\tau_{ji}t_{ji})^{-\theta} = \frac{2(\theta+1)}{A}\frac{\alpha - c_{ii}}{c_{ii}{}^{\theta+1}} \\
\Leftrightarrow \quad & A(k_i + k_j(\tau_{ji}t_{ji})^{-\theta})c_{ii}{}^{\theta+1} + 2(\theta+1)c_{ii} = 2(\theta+1)\alpha. \qquad (14)
\end{aligned}
$$

[12] For instance, the number of firms below cost c' (which satisfies $c' < c_{ji}$ and $c' < c_{ij}$) in both countries is $K_iG_i(c') = k_ic'^{\theta}$ and $K_jG_j(c') = k_jc'^{\theta}$. Then, the number of firms above cost c' is $k_i(c_{ii}^{\theta} - c'^{\theta})$ and $k_j(c_{jj}^{\theta} - c'^{\theta})$. As explained in Lemma 4, an increase in k_i or k_j decreases domestic cutoffs, c_{ii} and c_{jj}, so that it does not necessarily increase the number of firms above cost c' while increasing the number of firms below cost c'.

Note that the import tariff set by country j's government does not affect the domestic cost cutoff in country i: $dc_{ii}/dt_{ij} = 0$.

Totally differentiating (14), I show the following:

$$\frac{dc_{ii}}{dt_{ji}} = \frac{\theta A k_j {\tau_{ji}}^{-\theta}}{2(\theta+1)} \frac{{t_{ji}}^{-(\theta+1)} {c_{ii}}^{\theta+2}}{(\theta+1)\alpha - \theta c_{ii}} > 0. \tag{15}$$

Since the domestic cost cutoff is an increasing function of t_{ji}, the range of c_{ii} is expressed as

$$c_{ii}^{FT} \leq c_{ii} < c_{ii}^{AU} (< \alpha), \tag{16}$$

where $c_{ii}^{FT} \equiv \lim_{t_{ji} \to 1} c_{ii}$ and $c_{ii}^{AU} \equiv \lim_{t_{ji} \to \infty} c_{ii}$.

Next, I consider the impacts of an import tariff on the number of varieties and the export cost cutoff. From (12) and (15), I obtain

$$\frac{dN_i}{dt_{ji}} = -\frac{2(\theta+1)\alpha}{A{c_{ii}}^2} \frac{dc_{ii}}{dt_{ji}} < 0. \tag{17}$$

Noting that $c_{ji} = c_{ii}/(\tau_{ji} t_{ji})$ by (7), I can show that the export cost cutoff in country j, c_{ji}, is decreasing with t_{ji}:

$$\frac{dc_{ji}}{dt_{ji}} = -\frac{c_{ji}}{t_{ji}} \frac{\theta A k_i (\tau_{ji} t_{ji})^{\theta} {c_{ji}}^{\theta} + A(k_i (\tau_{ji} t_{ji})^{\theta} + k_j) {c_{ji}}^{\theta} + 2(\theta+1)}{(\theta+1) A (k_i (\tau_{ji} t_{ji})^{\theta} + k_j) {c_{ji}}^{\theta} + 2(\theta+1)} < 0.$$

Thus, from this equation and (15), I obtain the following lemma:

Lemma 1. *(Product selection) For countries i and j with $i, j \in \{H, F\}$ and $i \neq j$, an increase in country i's import tariff increases the domestic cost cutoff in country i and decreases the export cost cutoff in country j.*

As for the number of varieties, I obtain the following lemma:

Lemma 2. *(Product variety) For countries i and j with $i, j \in \{H, F\}$ and $i \neq j$, an increase in country i's import tariff results in an increase in the number of domestic firms in country i and a decrease in the number of exporters in country j, which leads to a decrease in the number of varieties sold in country i.*

An increase in country i's import tariff intensifies the export competition in country j and relaxes the domestic competition in country i, which makes the least exporters stop exporting and the least domestic firms start producing domestically (product selection effect). Since the former effect which decreases exporters dominates the latter effect which increases domestic firms, an increase in the import tariff decreases the number of varieties sold in country i (product variety effect).

In addition to product selection effect and product variety effect, an increase in country i's import tariff also affects prices and output levels set by exporters in country j. This impact differs depending on their productivity. Using (8) and (9), the effects of country i's import tariff on the price and output level set by country j's exporters with cost c are given by

$$\frac{dp_{ji}(c)}{dt_{ji}} = \frac{1}{2}\left(\frac{dc_{ii}}{dt_{ji}} + \tau_{ji}c\right) > 0, \quad \frac{dq_{ji}(c)}{dt_{ji}} = \frac{L_i}{2\gamma}\left(\frac{dc_{ii}}{dt_{ji}} - \tau_{ji}c\right) \begin{cases} > 0 & (0 \leq c < \frac{1}{\tau_{ji}}\frac{dc_{ii}}{dt_{ji}}) \\ < 0 & (\frac{1}{\tau_{ji}}\frac{dc_{ii}}{dt_{ji}} < c \leq c_{ji}) \end{cases}. \tag{18}$$

The impact of an increase in the ad valorem import tariff on price and output level can be divided into two effects. The second terms in (18) represent the direct effect of an increase in the import tariff. The impact of this effect depends on their productivity: more productive (lower c) exporters have a smaller increase in price and a smaller reduction in their exports. The first terms in (18) represent the effect of an increase in the domestic cutoff due to a decrease in the number of exporters, which causes all surviving firms to increase their price and output level uniformly.[13] As a result, an increase in the import tariff causes more productive exporters to increase their output level with smaller increase in their price, which decreases the output level of less productive exporters (see Appendix 7.3). Since these more productive exporters set lower price than less productive exporters (see (8)), an increase in the import tariff increases the output level of low-priced goods. The impact of an increase in the import tariff on product mix can be summarized as the following lemma:

Lemma 3. *(Product mix) For countries i and j with $i, j \in \{H, F\}$ and $i \neq j$, an increase in country i's import tariff increases (decreases) the output level of more (less) productive exporters in country j and increases the output level of domestic firms in country i.*

[13]The impact of an increase in country i's import tariff on the price and output level set by domestic firms in country i is determined only by this effect and do not depend on their productivity: $\frac{dp_{ii}(c)}{dt_{ji}} = \frac{1}{2}\frac{dc_{ii}}{dt_{ji}} > 0$ and $\frac{dq_{ii}(c)}{dt_{ji}} = \frac{L_i}{2\gamma}\frac{dc_{ii}}{dt_{ji}} > 0$.

Thus, an increase in the import tariff affects within-sector misallocation through three channels as explained in Lemmas 1–3, and these channel can be expressed by the domestic cost cutoffs: the domestic cost cutoffs determine the export cost cutoffs, the number of varieties, the output level, and thereby the welfare level. The domestic cost cutoffs vary with changes in the characteristics of goods, technology, and transportation costs. At the end of this section, I summarize the effects of these parameters on the domestic cost cutoff in the following lemma.

Lemma 4. *The domestic cost cutoff declines as varieties are closer substitutes (lower γ), the degree of non-separability is higher (higher η), the number of high-productivity firms in both countries is larger (higher k_i and k_j), and the transportation cost from country j to country i is lower (lower τ_{ji}).*

Proof. *Totally differentiating (14), I obtain*

$$
\begin{aligned}
\frac{dc_{ii}}{dA} &= -\frac{(k_i + k_j(\tau_{ji}t_{ji})^{-\theta})c_{ii}{}^{\theta+2}}{2(\theta+1)\left((\theta+1)\alpha - \theta c_{ii}\right)} < 0, \\
\frac{dc_{ii}}{dk_i} &= -\frac{Ac_{ii}{}^{\theta+2}}{2(\theta+1)\left((\theta+1)\alpha - \theta c_{ii}\right)} < 0, \\
\frac{dc_{ii}}{dk_j} &= -\frac{A(\tau_{ji}t_{ji})^{-\theta}c_{ii}{}^{\theta+2}}{2(\theta+1)\left((\theta+1)\alpha - \theta c_{ii}\right)} < 0, \\
\frac{dc_{ii}}{d\tau_{ji}} &= \frac{\theta A k_j \tau_{ji}{}^{-(\theta+1)} t_{ji}{}^{-\theta} c_{ii}{}^{\theta+2}}{2(\theta+1)\left((\theta+1)\alpha - \theta c_{ii}\right)} > 0,
\end{aligned}
$$

where $A = \eta/\gamma$.

3 Uncooperative tariff policy

In this section, I examine the Nash tariffs and characteristics of them.

3.1 Nash tariff

Substituting (3) into (2) yields

$$
q_{0,i}^c = I_i - \alpha \int_{\Omega_i^*} q_i^c(\omega) d\omega + \gamma \int_{\Omega_i^*} q_i^c(\omega)^2 d\omega + \eta \left(\int_{\Omega_i^*} q_i^c(\omega) d\omega \right)^2 . \tag{19}
$$

Then, plugging (18) into (1), I obtain the following welfare measure:

$$U_i = I_i + CS_i + AC_i, \tag{20}$$

where $CS_i \equiv \frac{\gamma}{2}\int_{\Omega_i^*} q_i^c(\omega)^2 d\omega$ and $AC_i \equiv \frac{\eta}{2}\left(\int_{\Omega_i^*} q_i^c(\omega)d\omega\right)^2 = \frac{\eta}{2}{Q_i}^2$. CS_i represents the sum of consumer surplus for each variety, because $\frac{\gamma}{2}q_i^c(\omega)^2$ corresponds to the triangular region under the demand curve for variety ω where $q_i^c(\omega) = (p_i^{max} - p_i(\omega))/\gamma$ by (5). AC_i represents consumer surplus for individual aggregate consumption of all differentiated goods, Q_i.[14] CS_i and AC_i are given by

$$\begin{aligned} CS_i &= \frac{\gamma}{2}\left[K_i\int_0^{c_{ii}}\left(\frac{c_{ii}-c}{2\gamma}\right)^2 dG_i(c) + K_j\int_0^{c_{ji}}\left(\frac{\tau_{ji}t_{ji}(c_{ji}-c)}{2\gamma}\right)^2 dG_j(c)\right] \\ &= \frac{(\alpha - c_{ii})c_{ii}}{2\eta(\theta+2)}, \end{aligned} \tag{21}$$

$$AC_i = \frac{\eta}{2}Q_i^2 = \frac{(\alpha - c_{ii})^2}{2\eta}, \tag{22}$$

where

$$Q_i = K_i\int_0^{c_{ii}}\frac{c_{ii}-c}{2\gamma}dG_i(c) + K_j\int_0^{c_{ji}}\frac{\tau_{ji}t_{ji}(c_{ji}-c)}{2\gamma}dG_j(c) = \frac{\alpha - c_{ii}}{\eta}. \tag{23}$$

Income consists of wage, profits from domestic and export sales, and the transfer. Income in country i is given by

$$\begin{aligned} I_i &= 1 + \frac{1}{L_i}\left[K_iG_i(c_{ii})\bar{\pi}_{ii} + K_iG_i(c_{ij})\bar{\pi}_{ij}\right] + \frac{T_i}{L_i} \\ &= 1 + \frac{k_i}{2\gamma(\theta+1)(\theta+2)}\left[{c_{ii}}^{\theta+2} + \frac{L_j}{L_i}{\tau_{ij}}^{-\theta}t_{ij}^{-(\theta+1)}{c_{jj}}^{\theta+2}\right] \\ &\quad + \frac{k_j{\tau_{ji}}^{-\theta}}{2\gamma(\theta+2)}(t_{ji}-1){t_{ji}}^{-(\theta+1)}{c_{ii}}^{\theta+2} \\ &= 1 + \frac{k_i + (\theta+1)k_j\tau_{ji}^{-\theta}(t_{ji}-1){t_{ji}}^{-(\theta+1)}}{2\gamma(\theta+1)(\theta+2)}{c_{ii}}^{\theta+2} + \frac{L_j}{L_i}\frac{k_i\tau_{ij}^{-\theta}t_{ij}^{-(\theta+1)}}{2\gamma(\theta+1)(\theta+2)}{c_{jj}}^{\theta+2} \end{aligned} \tag{24}$$

where $\bar{\pi}_{ij} = \frac{1}{G_i(c_{ij})}\int_0^{c_{ij}}\pi_{ij}(c)dG_i(c) = \frac{L_j}{G_i(c_{ij})}\frac{{\tau_{ij}}^{-\theta}{t_{ij}}^{-(\theta+1)}{c_{jj}}^{\theta+2}}{2\gamma(\theta+1)(\theta+2){c_i^M}^{\theta}}$ is the average profit of country

[14] When γ approaches 0 in the utility function (1), households care only about the aggregate consumption of differentiated goods, Q_i.

i's firms from sales in country j.

The welfare effect of its own tariff change can be decomposed into three terms:

$$\frac{dU_i}{dt_{ji}} = \frac{dI_i}{dt_{ji}} + \frac{dCS_i}{dt_{ji}} + \frac{dAC_i}{dt_{ji}}. \tag{25}$$

As for the first term in (25), I obtain following lemma.

Lemma 5. *The relationship between income and an import tariff in country i follows a hump-shaped curve.*

Proof. *See Appendix 7.4.*

Changes in country i's import tariff affect its income through changes in tariff revenue and profits from domestic sales. An increase in country i's import tariff increases the profits from domestic sales in country i. The relationship of tariff revenue in country i and its import tariff follows a hump-shaped curve due to a decrease in the number of exporters in country j and an increase in the average tariff revenue. Therefore, an increase in country i's import tariff increases its income when that import tariff is sufficiently small. This change in income reflect the consumption level of homogeneous good (see Appendix 7.5). An increase in country i's import tariff increases the consumption level of homogeneous good in that country when t_{ji} is sufficiently small.

Next, I state the effects of import tariffs on the second term and the third term in (25) in the following lemma.

Lemma 6. *An increase in the import tariff in country i lowers AC_i, and raises (lowers) CS_i when $\alpha \geq (<) 2c_{ii}$.*

Proof. *Differentiating (21) and (22) with respect to t_{ji}, I obtain*

$$\frac{dAC_i}{dt_{ji}} = -\frac{\alpha - c_{ii}}{\eta}\frac{dc_{ii}}{dt_{ji}} = -2B_i t_{ji}(\theta+2)(\alpha - c_{ii}) < 0 \tag{26}$$

$$\frac{dCS_i}{dt_{ji}} = \frac{\alpha - 2c_{ii}}{2\eta(\theta+2)}\frac{dc_{ii}}{dt_{ji}} = B_i t_{ji}(\alpha - 2c_{ii}) \tag{27}$$

where $B_i \equiv \frac{\theta k_j {\tau_{ji}}^{-\theta}}{4\gamma(\theta+1)(\theta+2)} \frac{{t_{ji}}^{-(\theta+2)} {c_{ii}}^{\theta+2}}{(\theta+1)\alpha - \theta c_{ii}} > 0.$

An increase in country i's import tariff increases the domestic cost cutoff, which increases the average price and decreases the aggregate output level in country i given by (13) and

(23), respectively. Thus, the consumer surplus for individual aggregate consumption, AC_i, decreases. However, an increase in the country's import tariff does not necessarily decrease the sum of consumer surplus for each variety, CS_i. Since the combination of differentiated goods consumed by households is determined by product selection, product variety, and product mix, the impact of an increase in country i's import tariff on CS_i is determined by changes in this within-sector allocation as in Lemmas 1–3. The net effect of the import tariff on CS_i is determined by these relationship and is positive when the domestic cost cutoff is sufficiently small: when $\alpha - 2c_{ii} > 0$ by (27).

Next, I show that the Nash tariffs are positive. From (26), (27), and (42) in Appendix 7.4, I obtain

$$\begin{aligned} \frac{dU_i}{dt_{ji}} &= \left(\frac{dI_i}{dt_{ji}} + \frac{dAC_i}{dt_{ji}}\right) + \frac{dCS_i}{dt_{ji}} \\ &= B_i\left[\left(\frac{\theta+1}{\theta} - t_{ji}\right)\psi_i(c_{ii}) + t_{ji}(\alpha - 2c_{ii})\right], \end{aligned} \tag{28}$$

where $\psi_i(c_{ii}) \equiv \frac{\theta(\theta+2)}{\theta+1}Ak_i{c_{ii}}^{\theta+1} + 2\theta c_{ii} + 2\alpha > 0$. The first term in brackets in (28) represents the effects of changes in the import tariff on welfare by changes in cross-sector allocation: increasing the import tariff increases income and the average price, which leads to an increase in the consumption level of homogeneous good and a decrease in the aggregate consumption level of differentiated goods.[15] It can be easily shown that this effect raises welfare for $t_{ji} < (\theta+1)/\theta$. The second term in brackets in (28) represents the effects of changes in the import tariff on welfare by changes in within-sector allocation. This effect raises welfare when $\alpha - 2c_{ii} > 0$. The welfare effect of the import tariff is determined by the sum of these two terms in (28). As a result, I obtain the following proposition.

Proposition 1. *In the two-country economy, the Nash tariffs, (t^n_{FH}, t^n_{HF}), are positive.*[16] *They are given by*

$$t^n_{ji} = \left(1 + \frac{1}{\theta}\right)\left[1 + \frac{\alpha - 2c_{ii}}{\frac{\theta(\theta+2)}{\theta+1}Ak_i{c_{ii}}^{\theta+1} + 2(\theta+1)c_{ii} + \alpha}\right] \quad i, j \in (H, F),\ i \neq j, \tag{29}$$

[15] From (41) and (52) in Appendix, I confirm that the impacts of the import tariff on income and the consumption level of homogeneous good are positive for $t_{ji} \in [1, \frac{\theta+1}{\theta}]$.

[16] In this model, since the import tariff in country i that maximizes its welfare with the import tariff set by country j as given is determined independently of the import tariff set by country j (see Appendix 7.6), this optimal import tariff in country i is consistent with the Nash tariff.

where c_{ii} is endogenously determined and $1 < t^n_{ji} < 2(1+1/\theta)$. The solution of (29) uniquely exists if $\theta \leq \bar{\theta}$ holds.

Proof. *See Appendix 7.6.*

Intuitively, the import tariff policy affects welfare through changes in cross-sector and within-sector allocation. The first component in (29) shows that the welfare effects of the import tariff through changes in cross-sector allocation are neutralized when $t_{ji} = 1 + 1/\theta$ (> 1). This is derived from the first term in brackets in (28). Then, the second component shows that the welfare effect of the import tariff through changes in within-sector allocation is positive (negative) when the second component is greater (less) than 1. As a result, even if the second component is less than 1, the first component dominates the second component and the Nash tariff is positive.

Although the Nash tariff is always positive, an increase in a country's import tariff decreases income in its trading partner owing to a decrease in its export profits:

Lemma 7. *In the two-country economy, an increase in a country's import tariff decreases income and does not affect the consumer surplus for individual aggregate consumption and the sum of consumer surplus for each variety in the country's trading partner.*

Proof. *As for income, see Appendix 7.4. As for AC_i and CS_i, differentiating (21) and (22) with respect to t_{ij}, I obtain*

$$\frac{dAC_i}{dt_{ij}} = \frac{dCS_i}{dt_{ij}} = 0 \tag{30}$$

where $B_i \equiv \frac{\theta k_j \tau_{ji}{}^{-\theta}}{4\gamma(\theta+1)(\theta+2)} \frac{t_{ji}{}^{-(\theta+2)} c_{ii}{}^{\theta+2}}{(\theta+1)\alpha - \theta c_{ii}} > 0$.

From this lemma, I obtain the following proposition.

Proposition 2. *An increase in country i's import tariff generates welfare loss in country j:*

$$\frac{dU_j}{dt_{ji}} = \frac{dI_j}{dt_{ji}} = -\frac{L_i}{L_j} B_i \frac{1}{\theta} \psi_i(c_{ii}) < 0. \tag{31}$$

Thus, an increase in country i's import tariff decreases income in country j, which decreases the consumption level of homogeneous good in country j (see Appendix 7.5), thereby decreasing the welfare level of country j.

3.2 Effects of a decrease in transportation costs on Nash tariffs

Next I examine how the Nash tariffs given by (29) is affected by changes in transportation costs: changes with τ_{ji} and τ_{ij}. In the rest of this section, I assume $\theta \leq \bar{\theta}$. Then, I establish the following lemma.

Lemma 8. *In the two-country economy, a decrease in transportation cost from country j to country i increases t^n_{ji}, but does not change by a decrease in transportation cost from country i to country j.*

Proof. *See Appendix 7.6.*

Intuitively, a decrease in τ_{ji} softens export competition in country j. Then, more productive exporters in country j increase their markups and decrease their output levels to maximize their export profits. Therefore, there is room for improving the welfare level of country i by increasing its import tariff, because it increases income (due to an increase in tariff revenue) and the output level of relatively low-priced goods produced by more productive firms.

4 Efficient trade policy

In this section, I consider the case in which both countries adopt a symmetric import tariff, $t_{ji} = t_{ij} \equiv t_w$, and examine how the total welfare level of two countries, $W \equiv L_H U_H + L_F U_F$, is affected by changes in the symmetric import tariff. The transportation costs are assumed to be zero, that is, $\tau_{ji} = \tau_{ij} = 1$ to focus on the effects of the symmetric import tariff.

To characterize the effect of the symmetric import tariff, I first compare the market allocation under free trade and the first best allocation set by a social planner to maximize the total welfare level of the countries. Then, I obtain the following lemma:

Lemma 9. *Compared to the first-best allocation, in the market allocation under free trade, (i) the total output level of differentiated goods sold in each country is below the first-best; (ii) the domestic and export cost cutoffs in each country are above the first-best; (iii) the number of varieties sold in each country is above the first-best; and (iv) the output level of more (less) productive firms, both domestic firms and exporters, sold in each country is below (above) the first-best.*

Proof. *See Appendix 7.7.*

Markup pricing causes the total output level in the differentiated good sector to be undersupplied, and thereby creates cross-sector misallocation. Moreover, it also creates within-sector misallocation. More productive firms do not pass on their entire cost advantage to consumers by raising their markups and they end up being undersupplied. This gives room for less productive firms to end up being oversupplied and for the least productive firms to survive inefficiently. Thus, with respect to within-sector allocation, the market outcome under free trade provides inefficiently high cost cutoffs (inefficient product selection), inefficiently large number of varieties (inefficient product variety), and inefficiently small (large) output level of more (less) productive firms (inefficient product mix).

Next, the impact of an increase in a symmetric import tariff on these allocations are summarized as follows.

Lemma 10. *An increase in a symmetric import tariff (i) decreases the total output level of differentiated goods sold in each country; (ii) increases (decreases) the domestic (export) cost cutoff in each country; (iii) decreases the number of varieties sold in each country; and (iv) increases (decreases) the output level of more (less) productive exporters and increases the output level of domestic firms sold in each country.*

Proof. *See Appendix 7.8.*

Therefore, from Lemmas 9 and 10, if countries start at global free trade and introduce a symmetric import tariff, its introduction distorts cross-sector allocation by shifting the production from the differentiated good sector to the homogeneous good sector. It also distorts within-sector allocation through product selection and product mix of domestic firms by allowing the least productive domestic firms to produce (product selection) and less productive domestic firms to increase the output level (product mix). Meanwhile, the introduction of the symmetric import tariff improves within-sector misallocation: it decreases the number of varieties sold in each country (product variety) by shutting out the least productive exporters (product selection), increases the output level of more productive domestic firms and exporters, and decreases the output level of less productive exporters (product mix). Thus, the introduction of the symmetric import tariff distorts cross-sector allocation and improves within-sector allocation through product variety effect. The impacts of its introduction on within-sector allocation through product selection and product mix effects have both improving and distorting effects. As a result, the net effect of the symmetric import tariff on the total welfare level is determined by the sum of these effects.

As shown in Appendix 7.9, differentiating W with respect to t_w yields

$$\begin{aligned} \frac{dW}{dt_w} &= \sum_{i\in\{H,F\}} L_i \left(\frac{dI_i}{dt_w} + \frac{dAC_i}{dt_w}\right) + \sum_{i\in\{H,F\}} L_i \frac{dCS_i}{dt_w} \\ &= (1-t_w) \sum_{i\in\{H,F\}} L_i B_i \psi_i(c_{ii}) + t_w \sum_{i\in\{H,F\}} L_i B_i (\alpha - 2c_{ii}) \\ &= \frac{\theta {t_w}^{-(\theta+2)}}{4\gamma(\theta+1)(\theta+2)} \left[(1-t_w) \sum_{i\in\{H,F\}} L_i b_i \psi_i(c_{ii}) + t_w \sum_{i\in\{H,F\}} L_i b_i (\alpha - 2c_{ii})\right], \end{aligned} \tag{32}$$

where $b_i \equiv \frac{k_j c_{ii}^{\theta+2}}{(\theta+1)\alpha - \theta c_{ii}} > 0$. The first term in brackets in (32) represents the total welfare effect of an increase in the symmetric import tariff by changes in cross-sector allocation in both countries. The second term in brackets in (32) represents the effect from changes in within-sector allocation in both countries. In contrast to (28), the first term in (32) is negative for $t_w > 1$ and is zero with global free trade, $t_w = 1$. Then, if countries start at global free trade, the second term in (32) determines whether the introduction of a small symmetric import tariff is desirable. Substituting $t_w = 1$ into (32), I obtain

$$\begin{aligned} \left.\frac{dW}{dt_w}\right|_{t_w=1} &= \frac{\theta}{4\gamma(\theta+1)(\theta+2)} \sum_{i\in\{H,F\}} L_i b_i (\alpha - 2c_{ii}^{FT}) \\ &= \frac{\theta(L_H b_H + L_F b_F)}{4\gamma(\theta+1)(\theta+2)} (\alpha - 2c_w^{FT}). \end{aligned} \tag{33}$$

Note that from (14), $c_{HH}^{FT} = c_{FF}^{FT} \equiv c_w^{FT}$, because $\tau_{FH} = \tau_{HF} = 1$. The introduction of a symmetric import tariff increases the total welfare level when it improves within-sector misallocation: when $\alpha - 2c_w^{FT} > 0$. As a result, I obtain the following proposition:

Proposition 3. *In the two-country economy, consider the case in which countries adopt a symmetric import tariff. The efficient tariff t_w^e that maximizes the total welfare level of two countries is positive if and only if $\alpha - 2c_w^{FT} > 0$. Otherwise, a global free trade policy is* *desirable:*

$$t_w^e = \begin{cases} 1 + \frac{\sum_{i\in\{H,F\}}[L_i b_i(\alpha-2c_{ii})]}{\sum_{i\in\{H,F\}}\left[L_i b_i\left(\frac{\theta(\theta+2)}{\theta+1}Ak_i {c_{ii}}^{\theta+1}+2(\theta+1)c_{ii}+\alpha\right)\right]} & \left(\alpha - 2c_w^{FT} > 0\right) \\ 1 & \left(\alpha - 2c_w^{FT} \leq 0\right) \end{cases}, \tag{34}$$

where c_{ii} is endogenously determined and $1 \leq t_w^e < 2$.

Proof. *See Appendix 7.9.*

Global free trade is inefficient when c_w^{FT} is low: when $\alpha - c_w^{FT} > 0$.[17] In other words, from Lemma 4, when there are several productive firms in both countries (large k_H and k_F) or varieties are close substitutes (large A). The intuition behind this result is as follows. An increase in the symmetric import tariff increases (decreases) the output level of more (less) productive exporters in both countries, which improves the within-sector misallocation through improving product mix. Then, the greater the number of more productive firms in both countries, the greater the number of exporters (varieties) that increase the output level by an increase in that tariff. These more productive exporters set lower price than less productive exporters, so that an increase in that tariff has a greater effect on increasing the total welfare level by improving this misallocation. Similarly, as varieties become closer substitutes, an increase in the symmetric import tariff works on increasing the total welfare level by improving this misallocation, because households come to prefer consuming more low-priced goods by increasing that tariff than consuming more varieties by decreasing that tariff.

To better understand the characteristics of the efficient tariff, consider the case in which countries are symmetric ($K_H = K_F$, $c_H^M = c_F^M$, $L_H = L_F$, and thus, $c_{HH} = c_{FF}$). Then, the efficient tariff (34) can be written as

$$t_w^e = \begin{cases} 1 + \frac{\alpha-2c_{ii}}{\frac{\theta(\theta+2)}{\theta+1}Ak_i {c_{ii}}^{\theta+1}+2(\theta+1)c_{ii}+\alpha} & \left(\alpha - 2c_w^{FT} > 0\right) \\ 1 & \left(\alpha - 2c_w^{FT} \leq 0\right) \end{cases}. \tag{35}$$

[17] This inequality is quite similar to the condition for too much entry at the market equilibrium in Bagwell and Lee (2020), where they show that this distortion can be corrected by the introduction of a symmetric tariff (see Propositions 4 and 8 in their paper). The model in the present study, which does not consider free entry, has similar characteristics to the model in Bagwell and Lee (2020) in that the inequality, $\alpha - c_w^{FT} > 0$, holds when there are too many potential firms in both countries, and that the introduction of a symmetric import tariff increases the total welfare level when this inequality holds.

The efficient tariff given by (35) has the same form as the second component of the Nash tariff given by (29).[18] In contrast to the Nash tariff, the efficient tariff is positive only when the introduction of an import tariff partially improves within-sector misallocation: $\alpha - 2c_w^{FT} > 0$. Comparing the Nash tariffs and the efficient tariff, I obtain the following proposition:

Proposition 4. *Assume $\theta \leq \bar{\theta}$. If countries are symmetric, the Nash tariffs are higher than the efficient tariff.*

Proof. *See Appendix 7.9.*

An increase in a country's import tariff increases its welfare mainly by increasing its income (and thereby the output level of homogeneous good sold in the country) when the import tariff is sufficiently small. By contrast, an increase in the country's import tariff harms its trading partner by decreasing its income (see Proposition 2). As a result, the Nash tariffs are positive due to the effect of increasing income in the countries, and higher than the efficient tariff because, in a global welfare perspective, this effect is partially offset by the effect of the decreasing income in each country's trading partner. Unlike the Nash tariffs, the efficient tariff is positive if and only if the introduction of a symmetric import tariff improves within-sector misallocation.

5 Mutual gains by symmetric tariff policy

In the previous section, I show that the efficient tariff is positive if and only if the introduction of a symmetric import tariff partially improves the within-sector misallocation: if and only if $\alpha - 2c_w^{FT} > 0$. In an asymmetric-country setting, however, there is a case in which the introduction of the symmetric import tariff decreases the welfare of a county while increasing the welfare of the other country even when the efficient tariff is positive.

Thus, in this section, I analyze what kind of asymmetric countries can mutually gain by the introduction of the symmetric import tariff from the initial situation of global free trade when the efficient tariff is positive. $L \equiv L_i/L_j$ and $k \equiv k_i/k_j$ denote the relative population size and the relative size of the productivity index for country i, respectively. In this section, I assume $\alpha - 2c_w^{FT} > 0$ to focus on the case in which the efficient tariff is positive. Since from

[18] Since the cost cutoffs are determined endogenously, the level of the efficient tariff (35) is different from that of the second component in (29).

(14) the level of c_w^{FT} depends on the aggregate size of productivity index, $k_i + k_j$, I assume that $k_i + k_j$ is constant and takes a value that ensures $\alpha - 2c_w^{FT} > 0$. Then, changes in the relative size k do not affect the aggregate size $k_i + k_j$ and thereby c_w^{FT}: $dc_w^{FT}/dk = 0$.

The welfare effect of the introduction of a symmetric import tariff in country i is given by

$$\begin{aligned} & \left.\frac{dL_iU_i}{dt_w}\right|_{t_w=1} > 0 \\ \Leftrightarrow \quad & L > \frac{2}{\theta+2}\frac{\beta_1 k + \beta_2}{\beta_3 k + \alpha}k \equiv \underline{L}(k), \end{aligned} \tag{36}$$

where $\beta_1 \equiv \alpha + \theta c_w^{FT}$, $\beta_2 \equiv (\theta+1)((\theta+1)\alpha - \theta c_w^{FT})$, and $\beta_3 \equiv (2\theta+1)\alpha - 2\theta c_w^{FT}$ are positive. Since $\underline{L}(k)$ is an increasing function with respect to k, it can be shown as depicted in Figure 1 (see Appendix 7.10).

From Figure 1, country i can gain by introducing a small symmetric import tariff when the pair of (k, L) is above $\underline{L}(k)$. In other words, the introduction of a symmetric import tariff improves the welfare of a country that has a larger population or a smaller productivity index. The intuition behind this finding is as follows. While the introduction of a symmetric import tariff increases income due to an increase in domestic profits and tariff revenue, it also decreases income due to a decrease in export profits in both countries. If a country has a larger population, firms in that country earn less profits from export sales than from domestic sales because the market size in its trading partner is small. Thus, the introduction of a symmetric import tariff improves welfare of the country with larger population by an increase in income through an increase in domestic profits. If a country has a smaller productivity index, there are fewer exporters in that country than exporters in its trading partner. In such a country, the share of tariff revenue in income is greater than that of export profits, so that the introduction of a symmetric import tariff improves the welfare of the country with a smaller productivity index by an increase in income through an increase in tariff revenue.

Next, I derive the welfare effect of introducing a symmetric import tariff in country j. In a similar way to (36), I obtain

$$\begin{aligned} & \left.\frac{dL_jU_j}{dt_w}\right|_{t_w=1} > 0 \\ \Leftrightarrow \quad & L < \frac{\theta+2}{2}\frac{\alpha k + \beta_3}{\beta_2 k + \beta_1}k \equiv \overline{L}(k). \end{aligned} \tag{37}$$

Since $\overline{L}(k)$ is an increasing function with respect to k and $\underline{L}(k) < \overline{L}(k)$ for $k > 0$ holds (see Appendix 7.10), $\overline{L}(k)$ can be shown as in Figure 1. Country j can gain by introducing a small symmetric import tariff when the pair of (k, L) is below $\overline{L}(k)$.

As a result, I obtain the following proposition.

Proposition 5. *Assume $\alpha - 2c_w^{FT} > 0$. In a two-country economy, if countries start at global free trade, the introduction of the symmetric import tariff improves welfare in both countries if and only if $\underline{L}(k) < L < \overline{L}(k)$.*

Proof. *See appendix 7.10.*

The introduction of a symmetric tariff improves welfare in both countries even when the degree of asymmetry across countries is large: one country has a larger population and productivity index than the other. The intuition behind this result is as follows. As mentioned above, imposing a symmetric tariff increases income in a country with a larger population owing to an increase in domestic profits. By contrast, it decreases income in a country with a larger productivity index owing to a decrease in export profits. Then, if a country has a larger population and productivity index, these opposite effects on income are offset.[19] As a result, the introduction of a symmetric import tariff increases the welfare of that country by improving the within-sector misallocation (because $\alpha - 2c_w^{FT} > 0$ holds). Similarly, it increases the welfare level of its trading partner by improving the within-sector misallocation. This is because following opposite effects are offset: the effect of imposing the symmetric tariff that decreases income in a country with a smaller population owing to a decrease in export profits and the effect that increases income in a country with a smaller productivity index owing to an increase in tariff revenue.

6 Conclusion

By incorporating ad valorem import tariffs and not considering free entry, I modify Melitz and Ottaviano (2008), who develop a trade model with heterogeneous firms and variable markups. In this model, the variable markups create within-sector misallocation: more productive firms do not pass on their entire cost advantage to consumers by raising their markups and they

[19] Specifically, changes in cross-sector allocation by the introduction of a symmetric import tariff have little impact on the welfare of that country compared to changes in within-sector allocation by the introduction of that tariff.

end up selling below the efficient output level, which causes less productive firms to end up being oversupplied and the least productive firms to survive inefficiently. Thus, the market level of the number of varieties is above that in the first-best outcome. This fact implies that, even without free entry, the effect that Nocco et al. (2019) discuss as the entry externality created by endogenous markups is inherent in this model.

I examine the influence of uncooperative and cooperative import tariff policies on welfare in a two-country model. The conclusions of this study are summarized as follows. First, the Nash tariffs are positive and, when countries are symmetric, lower than the efficient import tariff that the two countries adopt uniformly to maximize the total welfare level of the countries. Second, the efficient tariff is positive when, starting at global free trade, the introduction of the small symmetric import tariff sufficiently improves within-sector misallocation. This result is different from that of Melitz and Ottaviano (2008), who consider that trade liberalization decreases transportation costs and show that bilateral trade liberalization leads to welfare gain for both countries. The result in the present study is also different from that of Bagwell and Lee (2018), who adopt a CES preference setting that generates constant markups and show that the introduction of a small symmetric import tariff lowers the total welfare level of the two countries. Third, starting at global free trade, the introduction of a small symmetric import tariff improves the welfare level of both countries if and only if the countries are close to symmetric or one country has a larger population and number of high-productivity firms than the other. This result indicates that it is crucial for countries that enter a trade agreement to determine their import tariffs based on their relative size.

Finally, the present study focuses on the role of import tariff policies and therefore, does not use domestic policies and other trade policies as tools for bilateral trade policy. An interesting question is, by combining import tariffs with other policy instruments, whether the market can achieve the first best outcome without firm-specific production subsidies/taxes. I leave this question for consideration in future research.

7 Appendix

7.1 Sufficient condition for $c_{ii} < c_i^M$

From (15) and (16), $c_{ii} < c_i^M$ holds if $c_{ii}^{AU} < c_i^M$ holds. By using (14), c_{ii}^{AU} is determined by the following equation:

$$A k_i {c_{ii}}^{\theta+1} + 2(\theta+1)c_{ii} = 2(\theta+1)\alpha. \tag{38}$$

Since the left-hand side (LHS) of (38) is an increasing function of c_{ii}, the sufficient condition for $c_{ii} < c_i^M$ is

$$\begin{aligned} & A k_i {c_i^M}^{\theta+1} + 2(\theta+1)c_i^M > 2(\theta+1)\alpha \\ \Leftrightarrow \quad & \left(\frac{AK_i}{2(\theta+1)} + 1\right) c_i^M > \alpha, \end{aligned}$$

where $k_i \equiv K_i / {c_i^M}^\theta$ (see Figure 2).

7.2 Proof of $c_{ii} > c_{ij}$

c_{ii} is determined by (14). Similarly, noting that the relationship $c_{jj} = t_{ij}\tau_{ij}c_{ij}$, c_{ij} is determined by

$$\begin{aligned} & A(k_j + k_i(t_{ij}\tau_{ij})^{-\theta}){c_{jj}}^{\theta+1} - 2(\theta+1)c_{jj} = 2(\theta+1)\alpha \\ \Leftrightarrow \quad & A\left(k_i + k_j(t_{ij}\tau_{ij})^{\theta}\right) t_{ij}\tau_{ij}{c_{ij}}^{\theta+1} + 2(\theta+1)t_{ij}\tau_{ij}c_{ij} = 2(\theta+1)\alpha. \end{aligned} \tag{39}$$

Comparing coefficients of the LHS of (14) and (39),

$$\begin{aligned} & A\left(k_i + k_j(t_{ji}\tau_{ji})^{-\theta}\right) < A\left(k_i + k_j(t_{ij}\tau_{ij})^{\theta}\right) t_{ij}\tau_{ij} \\ & 2(\theta+1) < 2(\theta+1)t_{ij}\tau_{ij}. \end{aligned}$$

Thus $c_{ii} > c_{ij}$ (see Figure 3).

7.3 Product mix effect

From (9), the output level of country j's exporters with cost c is

$$q_{ji}(c) = \frac{L_i \tau_{ji} t_{ji}}{2\gamma}(c_{ji} - c). \tag{40}$$

The relationship between productivity and the output level of exporters can be depicted as the dotted line in Figure 4, where the intercepts along the vertical and horizontal axes are

$$\left(\frac{L_i \tau_{ji} t_{ji}}{2\gamma} c_{ji} =\right) \frac{L_i}{2\gamma} c_{ii}, \quad c_{ji}$$

respectively. Since from Lemma 1 an increase in t_{ji} increases c_{ii} and decreases c_{ji}, it shifts (40) from the dotted line to the solid line in Figure 4, which implies that an increase in country i's import tariff causes more (less) productive exporters in country j to increase (decrease) their output level.

7.4 Proof of Lemmas 5 and 7

Income is given by (24):

$$I_i = 1 + \frac{k_i + (\theta+1) k_j \tau_{ji}^{-\theta} (t_{ji} - 1) {t_{ji}}^{-(\theta+1)}}{2\gamma(\theta+1)(\theta+2)} {c_{ii}}^{\theta+2} + \frac{L_j}{L_i} \frac{k_i \tau_{ij}^{-\theta} t_{ij}^{-(\theta+1)}}{2\gamma(\theta+1)(\theta+2)} {c_{jj}}^{\theta+2}.$$

Differentiating both sides of this equation with respect to t_{ji}, I obtain

$$
\begin{aligned}
\frac{dI_i}{dt_{ji}} &= \frac{t_{ji}^{-(\theta+2)}{c_{ii}}^{\theta+2}}{2\gamma(\theta+1)(\theta+2)}\Big[(\theta+1)k_j\tau_{ji}^{-\theta}(-\theta t_{ji}+\theta+1) \\
&\quad + \left(k_i+(\theta+1)k_j\tau_{ji}^{-\theta}(t_{ji}-1){t_{ji}}^{-(\theta+1)}\right)(\theta+2)t_{ji}^{\theta+2}c_{ii}^{-1}\frac{dc_{ii}}{dt_{ji}}\Big] \\
&= \frac{\theta k_j{\tau_{ji}}^{-\theta}}{4\gamma(\theta+1)(\theta+2)}\frac{{t_{ji}}^{-(\theta+2)}{c_{ii}}^{\theta+2}}{(\theta+1)\alpha-\theta c_{ii}}\Big[2(\theta+1)\left(\frac{\theta+1}{\theta}-t_{ji}\right)((\theta+1)\alpha-\theta c_{ii}) \\
&\quad + \left(k_i+(\theta+1)k_j\tau_{ji}^{-\theta}(t_{ji}-1){t_{ji}}^{-(\theta+1)}\right)\frac{\theta+2}{\theta+1}At_{ji}c_{ii}^{\theta+1}\Big] \\
&= B_i\Big[2(\theta+1)\left(\frac{\theta+1}{\theta}-t_{ji}\right)((\theta+1)\alpha-\theta c_{ii}) \\
&\quad +\frac{\theta+2}{\theta+1}t_{ji}Ak_ic_{ii}^{\theta+1}+(\theta+2)(t_{ji}-1)Ak_j\tau_{ji}^{-\theta}t_{ji}^{-\theta}c_{ii}^{\theta+1}\Big] \qquad (41) \\
&= B_i\Big[2(\theta+1)\left(\frac{\theta+1}{\theta}-t_{ji}\right)((\theta+1)\alpha-\theta c_{ii}) \\
&\quad +\frac{\theta+2}{\theta+1}t_{ji}Ak_ic_{ii}^{\theta+1}+(\theta+2)(t_{ji}-1)\left(2(\theta+1)(\alpha-c_{ii})-Ak_ic_{ii}^{\theta+1}\right)\Big] \\
&= B_i\Big[(\theta+2)Ak_i{c_{ii}}^{\theta+1}+2(\theta+1)c_{ii}+2\frac{\theta+1}{\theta}\alpha-t_{ji}z_i(c_{ii})\Big], \qquad (42)
\end{aligned}
$$

where $B_i \equiv \frac{\theta k_j{\tau_{ji}}^{-\theta}}{4\gamma(\theta+1)(\theta+2)}\frac{{t_{ji}}^{-(\theta+2)}{c_{ii}}^{\theta+2}}{(\theta+1)\alpha-\theta c_{ii}} > 0$ and $z_i(c_{ii}) \equiv \frac{\theta(\theta+2)}{\theta+1}Ak_i{c_{ii}}^{\theta+1}+4(\theta+1)c_{ii}-2(\theta+1)\alpha$. I use the relationship $Ak_j\tau_{ji}^{-\theta}t_{ji}^{-\theta}c_{ii}^{\theta+1} = 2(\theta+1)(\alpha-c_{ii})-Ak_ic_{ii}^{\theta+1}$ from (14) at the fourth line. The characteristics of $z_i(c_{ii})$ is as follows:

$$
\begin{aligned}
\frac{dz_i(c_{ii})}{dt_{ji}} &= \left[\theta(\theta+2)Ak_ic_{ii}^{\theta}+4(\theta+1)\right]\frac{dc_{ii}}{dt_{ji}} > 0 \quad \text{for} \quad t_{ji} > 0 \qquad (43) \\
\lim_{t_{ji}\to 0} z_i(c_{ii}) &= z_i(0) = -2(\theta+1)\alpha < 0 \qquad (44) \\
\lim_{t_{ji}\to\infty} z_i(c_{ii}) &= 2\theta(\theta+2)(\alpha-c_{ii}^{AU})+4(\theta+1)c_{ii}^{AU}-2(\theta+1)\alpha \\
&= 2\left[\theta^2(\alpha-c_{ii}^{AU})+(\theta-1)\alpha+2c_{ii}^{AU}\right] > 0. \qquad (45)
\end{aligned}
$$

In deriving (45), I use the relationship $Ak_i{c_{ii}^{AU}}^{\theta+1} = 2(\theta+1)(\alpha-c_{ii}^{AU})$ from (38). Next, I define t_{z_i} which satisfies $z_i(c_{ii})|_{t_{ji}=t_{z_i}} = 0$. From (43) – (45) it is straightforward to show that $0 < t_{z_i} < \infty$ and the sign of (42) is positive if $t_{ji} \le t_{z_i}$:

$$
\frac{dI_i}{dt_{ji}} > 0 \quad \text{if } t_{ji} \le t_{z_i}. \qquad (46)
$$

If $t_{ji} > t_{z_i}$, the sign of (42) is described as

$$\begin{aligned}
\frac{dI_i}{dt_{ji}} \gtreqless 0 \Leftrightarrow t_{ji} &\lesseqgtr \frac{(\theta+2)Ak_i {c_{ii}}^{\theta+1} + 2(\theta+1)c_{ii} + 2\frac{\theta+1}{\theta}\alpha}{z_i(c_{ii})} \\
&= 1 + \frac{\frac{(\theta+2)}{\theta+1}Ak_i {c_{ii}}^{\theta+1} - 2(\theta+1)c_{ii} + 2\frac{(\theta+1)^2}{\theta}\alpha}{z_i(c_{ii})} \\
&= 1 + \frac{\frac{(\theta+2)}{\theta+1}Ak_i {c_{ii}}^{\theta+1} + 2(\theta+1)(\frac{\theta+1}{\theta}\alpha - c_{ii})}{z_i(c_{ii})} (> 1),
\end{aligned} \tag{47}$$

where the characteristics of the right hand side (RHS) of (47) are

$$RHS_{47} > 1, \quad \lim_{t_{ji} \to t_{z_i}} RHS_{47} = \infty \quad \text{for } t_{ji} > t_{z_i}. \tag{48}$$

In addition, differentiating the RHS of (47) with respect to t_{ji}, I obtain

$$\begin{aligned}
\frac{dRHS_{47}}{dt_{ji}} &= \frac{dRHS_{47}}{dc_{ii}}\frac{dc_{ii}}{dt_{ji}} \\
&= -\frac{\theta+2}{z_i(c_{ii})^2}\left[2(\theta+2)Ak_i c_{ii}^{\theta}\left((\theta+1)\alpha - \theta c_{ii}\right) + 4\frac{(\theta+1)^2}{\theta}\alpha\right]\frac{dc_{ii}}{dt_{ji}} < 0
\end{aligned} \tag{49}$$

for $t_{ji} > t_{z_i}$. From (46)–(49), the relationship between income and an import tariff in country i follows a hump-shaped curve (see Figures 5 and 6).

Next, I show the effects of an increase in the import tariff set by country j's government on income in country i. Differentiating (24) with respect to t_{ij}, I obtain

$$\begin{aligned}
\frac{dI_i}{dt_{ij}} &= \frac{L_j}{L_i}\frac{k_i\tau_{ij}^{-\theta}t_{ij}^{-(\theta+2)}c_{jj}^{\theta+2}}{2\gamma(\theta+1)(\theta+2)}\left[-(\theta+1) + (\theta+2)t_{ij}c_{jj}^{-1}\frac{dc_{jj}}{dt_{ij}}\right] \\
&= \frac{L_j}{L_i}\frac{\theta k_i\tau_{ij}^{-\theta}t_{ij}^{-(\theta+2)}c_{jj}^{\theta+2}}{4\gamma(\theta+1)(\theta+2)}\frac{1}{(\theta+1)\alpha - \theta c_{jj}}\left[-\frac{2(\theta+1)}{\theta}\left((\theta+1)\alpha - \theta c_{jj}\right) + \frac{\theta+2}{\theta+1}Ak_i\tau_{ij}^{-\theta}t_{ij}^{-\theta}c_{jj}^{\theta+1}\right] \\
&= \frac{L_j}{L_i}B_j\left[-\frac{2(\theta+1)}{\theta}\left((\theta+1)\alpha - \theta c_{jj}\right) + \frac{\theta+2}{\theta+1}\left(2(\theta+1)(\alpha - c_{jj}) - Ak_j c_{jj}^{\theta+1}\right)\right] \\
&= -\frac{L_j}{L_i}B_j\frac{1}{\theta}\left[\frac{\theta(\theta+2)}{\theta+1}Ak_j c_{jj}^{\theta+1} + 2\theta c_{jj} + 2\alpha\right] = -\frac{L_j}{L_i}B_j\frac{1}{\theta}\psi_j(c_{jj}) < 0,
\end{aligned} \tag{50}$$

where $B_j \equiv \frac{\theta k_i {\tau_{ij}}^{-\theta}}{4\gamma(\theta+1)(\theta+2)}\frac{{t_{ij}}^{-(\theta+2)}{c_{jj}}^{\theta+2}}{(\theta+1)\alpha - \theta c_{jj}} > 0$ and $\psi_j(c_{jj}) \equiv \frac{\theta(\theta+2)}{\theta+1}Ak_j {c_{jj}}^{\theta+1} + 2\theta c_{jj} + 2\alpha > 0$.

7.5 Homogeneous good

Here, I show the effects of an import tariff on the consumption of a homogeneous good in country i and the sufficient condition for $q^c_{0,i} > 0$. Plugging (8), (9), and (24) into (2), I obtain

$$\begin{aligned} q^c_{0,i} &= I_i - \int_{\Omega^*_i} p_i(\omega) q^c_i(\omega) d\omega \\ &= I_i - \frac{k_i + k_j (\tau_{ji} t_{ji})^{-\theta}}{2\gamma(\theta+2)} c_{ii}^{\theta+2} \\ &= 1 - \frac{\theta k_i + (\theta+1) k_j \tau_{ji}{}^{-\theta} t_{ji}{}^{-(\theta+1)}}{2\gamma(\theta+1)(\theta+2)} c_{ii}^{\theta+2} + \frac{L_j}{L_i} \frac{k_i \tau_{ij}{}^{-\theta} t_{ij}^{-(\theta+1)}}{2\gamma(\theta+1)(\theta+2)} c_{jj}{}^{\theta+2}, \end{aligned} \tag{51}$$

where $\int_{\Omega^*_i} p_i(\omega) q^c_i(\omega) d\omega = \frac{k_i + k_j(\tau_{ji} t_{ji})^{-\theta}}{2\gamma(\theta+2)} c_{ii}^{\theta+2}$. Differentiating (51) with respect to t_{ji}, I obtain

$$\begin{aligned} \frac{dq^c_{0,i}}{dt_{ji}} &= -\frac{1}{2\gamma(\theta+1)(\theta+2)} \Big[-(\theta+1)^2 k_j \tau_{ji}^{-\theta} t_{ji}^{-(\theta+2)} c_{ii}^{\theta+2} \\ &\quad + \left(\theta k_i + (\theta+1) k_j \tau_{ji}{}^{-\theta} t_{ji}{}^{-(\theta+1)}\right)(\theta+2) c_{ii}^{\theta+1} \frac{dc_{ii}}{dt_{ji}} \Big] \\ &= -\frac{k_j \tau_{ji}^{-\theta} t_{ji}^{-(\theta+2)} c_{ii}^{\theta+2}}{4\gamma(\theta+1)(\theta+2)\left((\theta+1)\alpha - \theta c_{ii}\right)} \Big[-2(\theta+1)^2 \left((\theta+1)\alpha - \theta c_{ii}\right) \\ &\quad + \frac{\theta^2(\theta+2)}{\theta+1} t_{ji} A k_i c_{ii}^{\theta+1} + \theta(\theta+2) A k_j \tau_{ji}^{-\theta} t_{ji}^{-\theta} c_{ii}^{\theta+1} \Big] \\ &= -\frac{k_j \tau_{ji}^{-\theta} t_{ji}^{-(\theta+2)} c_{ii}^{\theta+2}}{4\gamma(\theta+1)(\theta+2)\left((\theta+1)\alpha - \theta c_{ii}\right)} \Big[-2(\theta+1)^2 \left((\theta+1)\alpha - \theta c_{ii}\right) \\ &\quad + \frac{\theta^2(\theta+2)}{\theta+1} t_{ji} A k_i c_{ii}^{\theta+1} + \theta(\theta+2)\left(2(\theta+1)(\alpha - c_{ii}) - A k_i c_{ii}^{\theta+1}\right) \Big] \\ &= \frac{k_j \tau_{ji}^{-\theta} t_{ji}^{-(\theta+2)} c_{ii}^{\theta+2}}{4\gamma(\theta+1)(\theta+2)\left((\theta+1)\alpha - \theta c_{ii}\right)} \Big[2(\theta+1)\left(\alpha + \theta c_{ii}\right) + \theta(\theta+2) A k_i c_{ii}^{\theta+1} \\ &\quad - \frac{\theta^2(\theta+2)}{\theta+1} t_{ji} A k_i c_{ii}^{\theta+1} \Big] \\ &= \frac{\theta^2 A k_i k_j \tau_{ji}{}^{-\theta} t_{ji}^{-(\theta+2)} c_{ii}{}^{2\theta+3}}{4\gamma(\theta+1)^2\left((\theta+1)\alpha - \theta c_{ii}\right)} \left[\left(\frac{2(\theta+1)^2}{\theta^2(\theta+2) A k_i} \frac{\alpha + \theta c_{ii}}{c_{ii}^{\theta+1}} + \frac{\theta+1}{\theta} \right) - t_{ji} \right]. \end{aligned} \tag{52}$$

The sign of (52) is determined by

$$\frac{dq_{0,i}^{c}}{dt_{ji}} \gtreqless 0 \Leftrightarrow t_{ji} \lesseqgtr \frac{2(\theta+1)^2}{\theta^2(\theta+2)Ak_i}\frac{\alpha+\theta c_{ii}}{c_{ii}^{\theta+1}} + \frac{\theta+1}{\theta}(>1), \tag{53}$$

where the RHS of (53) is a decreasing function of t_{ji}:

$$\frac{dRHS_{53}}{dt_{ji}} = -\frac{2(\theta+1)^2}{\theta^2(\theta+2)Ak_i}\frac{(\theta+1)\alpha+\theta^2 c_{ii}}{c_{ii}^{\theta+2}}\frac{dc_{ii}}{dt_{ji}} < 0. \tag{54}$$

From (53) and (54), the relationship between the consumption level of homogeneous goods and the import tariff in country i follows a hump-shaped curve.

Meanwhile, differentiating (51) with respect to t_{ij}, I obtain

$$\frac{dq_{0,i}^{c}}{dt_{ij}} = -\frac{L_j}{L_i}B_j\left[\frac{\theta+2}{\theta+1}Ak_jc_{jj}^{\theta+1} + 2c_{jj} + \frac{2}{\theta}\alpha\right] = \frac{dI_i}{dt_{ij}} < 0. \tag{55}$$

Next, I show the sufficient condition for $q_{0,i}^{c} > 0$. Since the relationship between $q_{0,i}^{c}$ and country i's import tariff t_{ji} follows a hump-shaped curve and $q_{0,i}^{c}$ is a decreasing function of country j's import tariff t_{ij}, from (51) $q_{0,i}^{c} > 0$ holds if

$$1 - \frac{\theta k_i + (\theta+1)k_j{\tau_{ji}}^{-\theta}}{2\gamma(\theta+1)(\theta+2)}{c_{ii}^{FT}}^{\theta+2} > 0 \quad \text{and} \quad 1 - \frac{\theta k_i}{2\gamma(\theta+1)(\theta+2)}{c_{ii}^{AU}}^{\theta+2} > 0$$

$$\Leftrightarrow \quad c_{ii}^{FT} < \left(\frac{2\gamma(\theta+1)(\theta+2)}{\theta k_i + (\theta+1)k_j\tau_{ji}^{-\theta}}\right)^{\frac{1}{\theta+2}} \equiv \overline{c_{ii}^{FT}} \quad \text{and} \quad c_{ii}^{AU} < \left(\frac{2\gamma(\theta+1)(\theta+2)}{\theta k_i}\right)^{\frac{1}{\theta+2}} \equiv \overline{c_{ii}^{AU}}$$

hold. Therefore, by using (14), the sufficient condition for $q_{0,i}^{c} > 0$ is

$$\frac{A\left(k_i + k_j{\tau_{ji}}^{-\theta}\right)}{2(\theta+1)}\overline{c_{ii}^{FT}}^{\theta+1} + \overline{c_{ii}^{FT}} > \alpha \quad \text{and} \quad \frac{Ak_i}{2(\theta+1)}\overline{c_{ii}^{AU}}^{\theta+1} + \overline{c_{ii}^{AU}} > \alpha. \tag{56}$$

7.6 Proof of Proposition 1 and Lemma 8

7.6.1 Proof of Proposition 1

Putting (26), (27), and (42) into (25), I obtain

$$\begin{aligned} \frac{dU_i}{dt_{ji}} &= \frac{dI_i}{dt_{ji}} + \frac{dCS_i}{dt_{ji}} + \frac{dAC_i}{dt_{ji}} \\ &= B_i\Bigg[(\theta+2)Ak_i{c_{ii}}^{\theta+1} + 2(\theta+1)c_{ii} + 2\frac{\theta+1}{\theta}\alpha \\ &\quad -t_{ji}\left(\frac{\theta(\theta+2)}{\theta+1}Ak_i c_{ii}^{\theta+1} + 2(\theta+1)c_{ii} + \alpha\right)\Bigg]. \end{aligned} \tag{57}$$

The sign of (57) is determined by

$$\begin{aligned} \frac{dU_i}{dt_{ji}} \gtreqless 0 \Leftrightarrow t_{ji} &\lesseqgtr \frac{(\theta+2)Ak_i{c_{ii}}^{\theta+1} + 2(\theta+1)c_{ii} + 2\frac{\theta+1}{\theta}\alpha}{\frac{\theta(\theta+2)}{\theta+1}Ak_i c_{ii}^{\theta+1} + 2(\theta+1)c_{ii} + \alpha} \\ &= 1 + \frac{1}{\theta}\frac{\frac{\theta(\theta+2)}{\theta+1}Ak_i{c_{ii}}^{\theta+1} + (\theta+2)\alpha}{\frac{\theta(\theta+2)}{\theta+1}Ak_i{c_{ii}}^{\theta+1} + 2(\theta+1)c_{ii} + \alpha} \equiv \Phi_i(c_{ii})(>1). \end{aligned} \tag{58}$$

Since from (15) t_{ji} and c_{ii} have one-to-one correspondence, t_{ji} can be expressed as a function of c_{ii}. Rearranging (14), I obtain

$$t_{ji} = \frac{1}{\tau_{ji}}\left[\frac{Ak_j{c_{ii}}^{\theta+1}}{2(\theta+1)(\alpha - c_{ii}) - Ak_i{c_{ii}}^{\theta+1}}\right]^{1/\theta} \equiv t_{ji}(c_{ii}). \tag{59}$$

Substituting (59) into the LHS of (58), both sides of (58) are expressed as functions of c_{ii}:

$$\frac{dU_i}{dt_{ji}} \gtreqless 0 \Leftrightarrow t_{ji}(c_{ii}) \lesseqgtr \Phi_i(c_{ii}). \tag{60}$$

From (15) and (16), the characteristics of $t_{ji}(c_{ii})$ are expressed as follows.

$$\frac{dt_{ji}(c_{ii})}{dc_{ii}} > 0, \quad t_{ji}(c_{ii}^{FT}) = 1, \quad \lim_{c_{ii}\to c_{ii}^{AU}} t_{ji}(c_{ii}) = \infty \tag{61}$$

Next, I establish the shape of $\Phi_i(c_{ii})$. Differentiating $\Phi_i(c_{ii})$ with respect to c_{ii}, I obtain

$$\frac{d\Phi_i(c_{ii})}{dc_{ii}} = \frac{(\theta+2)\phi_i(c_{ii})}{\left(\frac{\theta(\theta+2)}{\theta+1}Ak_i{c_{ii}}^{\theta+1} + 2(\theta+1)c_{ii} + \alpha\right)^2}, \tag{62}$$

where $\phi_i(c_{ii}) \equiv (2\theta c_{ii} - (\theta+1)\alpha)\, Ak_i{c_{ii}}^{\theta} - 2\frac{\theta+1}{\theta}\alpha$. Then, the sign of $d\Phi_i(c_{ii})/dc_{ii}$ can be written as

$$\frac{d\Phi_i(c_{ii})}{dc_{ii}} \gtreqless 0 \Leftrightarrow \phi_i(c_{ii}) \gtreqless 0. \tag{63}$$

It is straightforward to show that $\phi_i(c_{ii}) < 0$ if $c_{ii} < \frac{\theta+1}{\theta}\frac{\alpha}{2}$. If $c_{ii} \geq \frac{\theta+1}{\theta}\frac{\alpha}{2}$, the characteristics of $\phi_i(c_{ii})$ are as follows:

$$\phi_i\left(\frac{\theta+1}{\theta}\frac{\alpha}{2}\right) = -2\frac{\theta+1}{\theta}\alpha < 0, \tag{64}$$

$$\frac{d\phi_i(c_{ii})}{dc_{ii}} = 2\theta Ak_i{c_{ii}}^{\theta} + (2\theta c_{ii} - (\theta+1)\alpha)\theta Ak_i{c_{ii}}^{\theta-1} > 0 \tag{65}$$

for $c_{ii} \geq \frac{\theta+1}{\theta}\frac{\alpha}{2}$. I define $\tilde{c_{ii}}$ which satisfies $\phi_i(\tilde{c_{ii}}) = 0$. Then, from (63) – (65), I obtain

$$\phi_i(c_{ii})\begin{cases} \leq 0 & (c_{ii} \leq \tilde{c_{ii}}) \\ > 0 & (c_{ii} > \tilde{c_{ii}}) \end{cases} \Leftrightarrow \frac{d\Phi_i(c_{ii})}{dc_{ii}}\begin{cases} \leq 0 & (c_{ii} \leq \tilde{c_{ii}}) \\ > 0 & (c_{ii} > \tilde{c_{ii}}) \end{cases} \tag{66}$$

Taking the limits of $\Phi_i(c_{ii})$ as $c_{ii} \to 0$ and $c_{ii} \to \infty$ gives

$$\lim_{c_{ii}\to 0}\Phi_i(c_{ii}) = 2(\frac{\theta+1}{\theta}), \quad \lim_{c_{ii}\to\infty}\Phi_i(c_{ii}) = 1 + \frac{1}{\theta} \tag{67}$$

respectively. From (66), (67), and $\Phi_i(c_{ii}) > 1$, I obtain

$$1 < \Phi_i(c_{ii}) < 2(\frac{\theta+1}{\theta}) \text{ for } c_{ii} \in (0, \infty). \tag{68}$$

Therefore, from (61) and (68), $\Phi_i(c_{ii})$ and $t_{ji}(c_{ii})$ intersect in $c_{ii} \in [c_{ii}^{FT}, c_{ii}^{AU})$ (see Figure 7). Since c_{ii} is an increasing function of t_{ji}, there is a welfare-maximizing import tariff that satisfies (58) with equality.

Next, I show the sufficient condition for the uniqueness of the solution of (29). From (61) and (66), the solution of (29) uniquely exists if $\Phi_i(c_{ii})$ is a decreasing function of c_{ii} for $c_{ii} \in [c_{ii}^{FT}, c_{ii}^{AU})$:

$$c_{ii}^{AU} \leq \tilde{c_{ii}}. \tag{69}$$

From (66), (69) holds if and only if

$$\begin{aligned}
&\phi_i(c_{ii}^{AU}) \leq 0 \\
\Leftrightarrow\ &\left(2\theta c_{ii}^{AU} - (\theta+1)\alpha\right) Ak_i {c_{ii}^{AU}}^{\theta} - 2\frac{\theta+1}{\theta}\alpha \leq 0 \\
\Leftrightarrow\ &\frac{2(\theta+1)}{\theta c_{ii}^{AU}}\left[2\theta^2 {c_{ii}^{AU}}^2 - (3\theta^2+\theta-1)\alpha {c_{ii}}^{AU} + \theta(\theta+1)\alpha^2\right] \geq 0 \\
\Leftrightarrow\ &2\theta^2 {c_{ii}^{AU}}^2 - (3\theta^2+\theta-1)\alpha {c_{ii}}^{AU} + \theta(\theta+1)\alpha^2 \geq 0 \\
\Leftrightarrow\ &2\theta^2\left(c_{ii}^{AU} - \frac{(3\theta^2+\theta-1)\alpha}{4\theta^2}\right)^2 - \frac{(3\theta^2+\theta-1)^2 - 8\theta^3(\theta+1)}{8\theta^2}\alpha^2 \geq 0.
\end{aligned} \tag{70}$$

In deriving (70), I use the relationship $Ak_i {c_{ii}^{AU}}^{\theta} = 2(\theta+1)\frac{\alpha - c_{ii}^{AU}}{c_{ii}^{AU}}$ from (38). Thus, (70) holds for all c_{ii}^{AU} if

$$\begin{aligned}
&(3\theta^2+\theta-1)^2 - 8\theta^3(\theta+1) \leq 0 \\
\Leftrightarrow\ &\theta \leq \sqrt{2} + \frac{1+\sqrt{5+4\sqrt{2}}}{2} \equiv \bar{\theta},
\end{aligned} \tag{71}$$

where $\bar{\theta} \approx 3.55$. Therefore, the sufficient condition for the uniqueness of the solution of (29) is (71).

7.6.2 Proof of Lemma 8

I assume $\theta \leq \bar{\theta}$ to ensure the uniqueness of the solution of (29). As shown above, t_{ji} is determined at the intersection of $t_{ji}(c_{ii})$ and $\Phi_i(c_{ii})$ in Figure 7. It is straightforward to show that a decrease in τ_{ji} affects only $t_{ji}(c_{ii})$ by shifting it up, as shown in Figure 8, which immediately proves that the Nash tariff t_{ji}^n increases and the domestic cost cutoff c_{ii} decreases as τ_{ji} falls. However, a decrease in τ_{ij} does not affect $t_{ji}(c_{ii})$ and $\Phi_i(c_{ii})$ so that the Nash tariff t_{ji}^n does not change.

7.7 Proof of Lemma 9

7.7.1 First-best outcome

To evaluate the efficiency of the market outcome under free trade, I consider the problem faced by a benevolent social planner who maximizes the total welfare of the countries, taking as given the endowment of labor (L_i) and potential firms (K_i), the production functions of the two goods, and the distribution of c ($c \sim G_i(c)$). Then, the planner chooses the quantity of the homogeneous good ($q_{0,i} = L_i q_{0,i}^c$) and the quantity of each differentiated good produced in country i and sold in country j ($q_{ij}(c) = L_j q_{ij}^c$).

Accordingly, given (1), the planner's problem is given by

$$\max_{q_{0,i}^c, q_{0,j}^c, q_{ii}(c), q_{ij}(c), q_{jj}(c), q_{ji}(c)} W = L_i U_i + L_j U_j, \tag{72}$$

subject to the resource constraint:

$$q_{0,i} + K_i \int_0^{c_i^M} c q_{ii}(c) dG_i(c) + K_i \int_0^{c_i^M} \tau_{ij} c q_{ij}(c) dG_i(c) = L_i, \tag{73}$$

$$q_{0,j} + K_j \int_0^{c_j^M} c q_{jj}(c) dG_j(c) + K_j \int_0^{c_j^M} \tau_{ji} c q_{ji}(c) dG_j(c) = L_j \tag{74}$$

for $i \neq j$, where

$$\begin{aligned}
L_iU_i = q_{0,i} &+ \alpha L_i \left(K_i \int_0^{c_i^M} \frac{q_{ii}(c)}{L_i} dG_i + K_j \int_0^{c_j^M} \frac{q_{ji}(c)}{L_i} dG_j \right) \\
&- \frac{\gamma}{2} L_i \left(K_i \int_0^{c_i^M} \left(\frac{q_{ii}(c)}{L_i} \right)^2 dG_i + K_j \int_0^{c_j^M} \left(\frac{q_{ji}(c)}{L_i} \right)^2 dG_j \right) \\
&- \frac{\eta}{2} L_i \left(K_i \int_0^{c_i^M} \frac{q_{ii}(c)}{L_i} dG_i + K_j \int_0^{c_j^M} \frac{q_{ji}(c)}{L_i} dG_j \right)^2 \\
= q_{0,i} &+ \alpha \left(K_i \int_0^{c_i^M} q_{ii}(c) dG_i + K_j \int_0^{c_j^M} q_{ji}(c) dG_j \right) \\
&- \frac{\gamma}{2L_i} \left(K_i \int_0^{c_i^M} q_{ii}(c)^2 dG_i + K_j \int_0^{c_j^M} q_{ji}(c)^2 dG_j \right) \\
&- \frac{\eta}{2L_i} \left(K_i \int_0^{c_i^M} q_{ii}(c) dG_i + K_j \int_0^{c_j^M} q_{ji}(c) dG_j \right)^2 .
\end{aligned} \tag{75}$$

Substituting (73) and (74) into (72), this problem can be rewritten as

$$\max_{q_{ii}(c), q_{ij}(c), q_{jj}(c), q_{ji}(c)} W = L_iU_i + L_jU_j, \tag{76}$$

where

$$\begin{aligned}
L_iU_i = L_i &- \left(K_i \int_0^{c_i^M} c q_{ii}(c) dG_i + K_i \int_0^{c_i^M} \tau_{ij} c q_{ij}(c) dG_i \right) \\
&+ \alpha \left(K_i \int_0^{c_i^M} q_{ii}(c) dG_i + K_j \int_0^{c_j^M} q_{ji}(c) dG_j \right) \\
&- \frac{\gamma}{2L_i} \left(K_i \int_0^{c_i^M} q_{ii}(c)^2 dG_i + K_j \int_0^{c_j^M} q_{ji}(c)^2 dG_j \right) \\
&- \frac{\eta}{2L_i} \left(K_i \int_0^{c_i^M} q_{ii}(c) dG_i + K_j \int_0^{c_j^M} q_{ji}(c) dG_j \right)^2 .
\end{aligned} \tag{77}$$

The first order conditions with respect to $q_{ii}(c)$ and q_{ji} give

$$q^s_{ii}(c) = \frac{L_i}{\gamma}(\alpha - \frac{\eta}{L_i}Q_i - c) \tag{78}$$
$$= \frac{L_i}{\gamma}(c^s_{ii} - c), \tag{79}$$

$$q^s_{ji}(c) = \frac{L_i}{\gamma}(\alpha - \frac{\eta}{L_i}Q_i - \tau_{ji}c) \tag{80}$$
$$= \frac{L_i\tau_{ji}}{\gamma}(c^s_{ji} - c), \tag{81}$$

respectively, where 's' labels first best optimum variables and

$$c^s_{ii} \equiv \alpha - \frac{\eta}{L_i}Q^s_i, \tag{82}$$
$$c^s_{ji} \equiv \frac{1}{\tau_{ji}}\left(\alpha - \frac{\eta}{L_i}Q^s_i\right) = \frac{c^s_{ii}}{\tau_{ji}}, \tag{83}$$
$$Q^s_i = \left(K_i\int_0^{c^M_i} q_{ii}(c)^s dG_i + K_j\int_0^{c^M_j} q_{ji}(c)^s dG_j\right). \tag{84}$$

c^s_{ii}, c^s_{ji} and Q^s_i represent domestic cost cutoff in country i, the export cost cutoff in country j, and the total output level of differentiated varieties sold in country i, respectively.

Integrating (78) and (80) gives

$$K_i\int_0^{c^s_{ii}} q^s_{ii}(c)dG_i = \frac{L_i}{\gamma}\left((\alpha - \frac{\eta}{L_i}Q^s_i)k_i {c^s_{ii}}^{\theta} - \frac{\theta}{\theta+1}k_i {c^s_{ii}}^{\theta+1}\right), \tag{85}$$

$$K_j\int_0^{c^s_{ji}} q^s_{ji}(c)dG_j = \frac{L_i}{\gamma}\left((\alpha - \frac{\eta}{L_i}Q^s_i)k_j {c^s_{ji}}^{\theta} - \tau_{ji}\frac{\theta}{\theta+1}k_j {c^s_{ji}}^{\theta+1}\right), \tag{86}$$

respectively, where $k_i = K_i/{c^M_i}^{\theta}$. Then, using (85), (86), and (83) to (84), I obtain

$$Q^s_i = \frac{L_i(k_i + k_j\tau_{ji}^{-\theta}){c^s_{ii}}^{\theta}(\alpha - \frac{\theta}{\theta+1}c^s_{ii})}{\gamma + \eta(k_i + k_j\tau_{ji}^{-\theta}){c^s_{ii}}^{\theta}} \tag{87}$$

Putting this into (82), the domestic cost cutoff is determined by the following equation:

$$A(k_i + k_j\tau_{ji}^{-\theta})c_{ii}^{s\,\theta+1} = (\theta+1)(\alpha - c_{ii}^s), \tag{88}$$

where $A = \eta/\gamma$. Then, using this equation, (87) can be written as

$$\begin{aligned} Q_i^s &= \frac{L_i}{\eta}\frac{A(k_i + k_j\tau_{ji}^{-\theta})c_{ii}^{s\,\theta+1}(\alpha - \frac{\theta}{\theta+1}c_{ii}^s)}{c_{ii}^s + A(k_i + k_j\tau_{ji}^{-\theta})c_{ii}^{s\,\theta+1}} \\ &= \frac{L_i}{\eta}\frac{(\theta+1)(\alpha - c_{ii}^s)(\alpha - \frac{\theta}{\theta+1}c_{ii}^s)}{c_{ii}^s + (\theta+1)(\alpha - c_{ii}^s)} \\ &= \frac{L_i}{\eta}\frac{(\theta+1)(\alpha - c_{ii}^s)(\alpha - \frac{\theta}{\theta+1}c_{ii}^s)}{(\theta+1)(\alpha - \frac{\theta}{\theta+1}c_{ii}^s)} \\ &= \frac{L_i(\alpha - c_{ii}^s)}{\eta}. \end{aligned} \tag{89}$$

The number of varieties in country i, N_i, is composed of domestic producers and exporters in country j, that is

$$\begin{aligned} N_i^s &= K_iG_i(c_{ii}^s) + K_jG_j(c_{ji}^s) \\ &= (k_i + k_j\tau_{ji}^{-\theta})c_{ii}^{s\,\theta}. \end{aligned} \tag{90}$$

Thus, I obtain the first best level of c_{ii}^s, c_{ji}^s, Q_i^s, N_i^s, $q_{ii}^s(c)$, and $q_{ji}^s(c)$ by (88), (83), (89), (90), (79), and (81), respectively.

7.7.2 Market outcome

Substituting $t_{ji} = t_{ij} = 1$ into (14), the domestic cost cutoff at the market equilibrium is determined by

$$A(k_i + k_j\tau_{ji}^{-\theta})c_{ii}^{m\,\theta+1} = 2(\theta+1)(\alpha - c_{ii}^m), \tag{91}$$

where 'm' labels the equilibrium values under free trade. Then, the export cost cutoff in country j is given by

$$c_{ji}^m = \frac{c_{ii}^m}{\tau_{ji}}. \tag{92}$$

From (23), (11), and (9), I obtain

$$Q_i^m = \frac{L_i(\alpha - c_{ii}^m)}{\eta}, \tag{93}$$

$$N_i^m = (k_i + k_j \tau_{ji}{}^{-\theta}) c_{ii}^{m\theta}, \tag{94}$$

$$q_{ii}^m(c) = \frac{L_i}{2\gamma}(c_{ii}^m - c), \tag{95}$$

$$q_{ji}^m(c) = \frac{L_i \tau_{ji}}{2\gamma}(c_{ji}^m - c). \tag{96}$$

7.7.3 Market failure

I examine how the free market outcome departs from the first-best outcome. Comparing the domestic cost cutoffs given by (88) and (91), it is straightforward to show that (see Figure 9)

$$c_{ii}^s < c_{ii}^m. \tag{97}$$

Then, from (83), (89), (90), (92), (93), and (94), the following relationships hold:

$$c_{ji}^s = \frac{c_{ii}^s}{\tau_{ji}} < \frac{c_{ii}^m}{\tau_{ji}} = c_{ji}^m, \tag{98}$$

$$Q_i^s = \frac{L_i(\alpha - c_{ii}^s)}{\eta} > \frac{L_i(\alpha - c_{ii}^m)}{\eta} = Q_i^m, \tag{99}$$

$$N_i^s = (k_i + k_j \tau_{ji}^{-\theta}) c_{ii}^{s\,\theta} < (k_i + k_j \tau_{ji}{}^{-\theta}) c_{ii}^{m\theta} = N_i^m. \tag{100}$$

From (79) and (95), the gap between $q_{ii}^s(c)$ and $q_{ii}^m(c)$ evaluates to

$$q_{ii}^s(c) - q_{ii}^m(c) = \frac{L_i}{2\gamma}(2c_{ii}^s - c_{ii}^m - c). \tag{101}$$

To confirm that the sign of $2c_{ii}^s - c_{ii}^m$ is positive, replace c_{ii}^m with $2c_{ii}^s$ on the LHS of (91) to obtain

$$A(k_i + k_j \tau_{ji}^{-\theta})(2c_{ii}^s)^{\theta+1} = 2^{\theta+1}(\theta+1)(\alpha - c_{ii}^s) \equiv LHS_{91}(2c_{ii}^s),$$

where I use (88). Similarly, replacing c_{ii}^m with $2c_{ii}^s$ on the RHS of (91) gives

$$2(\theta+1)(\alpha - 2c_{ii}^s) = 2(\theta+1)(\alpha - c_{ii}^s) - 2(\theta+1)c_{ii}^s \equiv RHS_{91}(2c_{ii}^s).$$

It is straightforward to show that

$$LHS_{91}(2c_{ii}^{s}) > RHS_{91}(2c_{ii}^{s}),$$

which implies $2c_{ii}^{s} - c_{ii}^{m} > 0$ (see Figure 10).

Therefore, the sign of (101) can be expressed as follows:

$$q_{ii}^{s}(c) - q_{ii}^{m}(c) \begin{cases} \geq 0 & (c \in [0, 2c_{ii}^{s} - c_{ii}^{m}]) \\ < 0 & (c \in (2c_{ii}^{s} - c_{ii}^{m}, c_{ii}^{m}]) \end{cases}. \tag{102}$$

From (81) and (96), the gap between $q_{ji}^{s}(c)$ and $q_{ji}^{m}(c)$ is given by

$$\begin{aligned} q_{ji}^{s}(c) - q_{ji}^{m}(c) &= \frac{L_i \tau_{ji}}{2\gamma}(2c_{ji}^{s} - c_{ji}^{m} - c) \\ &= \frac{L_i}{2\gamma}(2c_{ii}^{s} - c_{ii}^{m} - \tau_{ji} c). \end{aligned}$$

In the same way as (102), I obtain

$$q_{ji}^{s}(c) - q_{ji}^{m}(c) \begin{cases} \geq 0 & (\tau_{ji} c \in [0, 2c_{ii}^{s} - c_{ii}^{m}]) \\ < 0 & (\tau_{ji} c \in (2c_{ii}^{s} - c_{ii}^{m}, c_{ii}^{m}]) \end{cases}. \tag{103}$$

From (97)–(100), (102), and (103), I obtain Lemma 9.

7.8 Proof of Lemma 10

Consider the case of $t_{ji} = t_{ij} = t_w$. Since from (14) t_{ij} dose not affect c_{ii}, totally differentiating (14) gives

$$\frac{dc_{ii}}{dt_w} = \frac{dc_{ii}}{dt_{ji}} = \frac{\theta A k_j {\tau_{ji}}^{-\theta}}{2(\theta+1)} \frac{{t_w}^{-(\theta+1)} {c_{ii}}^{\theta+2}}{(\theta+1)\alpha - \theta c_{ii}} > 0. \tag{104}$$

Then, in the same way as Lemma 1, I obtain

$$\frac{dc_{ji}}{dt_w} = \frac{dc_{ji}}{dt_{ji}} < 0. \tag{105}$$

From (12), (23) and (104), it is straightforward to show that

$$\frac{dN_i}{dt_w} < 0, \quad \frac{dQ_i}{dt_w} < 0. \tag{106}$$

From (9), differentiating $q_{ii}(c)$ and $q_{ji}(c)$ with respect to t_w gives

$$\frac{dq_{ii}(c)}{dt_w} = \frac{L_i}{2\gamma}\frac{dc_{ii}}{dt_w} > 0, \tag{107}$$

$$\frac{dq_{ji}(c)}{dt_w} = \frac{L_i}{2\gamma}\left(\frac{dc_{ii}}{dt_w} - \tau_{ji}c\right)\begin{cases} > 0 & (0 \leq c < \frac{1}{\tau_{ji}}\frac{dc_{ii}}{dt_w}) \\ < 0 & (\frac{1}{\tau_{ji}}\frac{dc_{ii}}{dt_w} < c \leq c_{ji}) \end{cases}, \tag{108}$$

respectively (see Appendix 7.3).

From (104)–(108), I obtain Lemma 10.

7.9 Proof of Propositions 3 and 4

7.9.1 Proof of Proposition 3

Differentiating $W = L_H U_H + L_F U_F$ with respect to t_w, I obtain

$$\frac{dW}{dt_w} = \sum_{i\in\{H,F\}} L_i\left(\frac{dI_i}{dt_w} + \frac{dAC_i}{dt_w}\right) + \sum_{i\in\{H,F\}} L_i\frac{dCS_i}{dt_w}. \tag{109}$$

Since CS_i and AC_i depend only on t_{ji} from Lemmas 6 and 7, I obtain

$$\frac{dCS_i}{dt_w} = B_i\Big[t_w(\alpha - 2c_{ii})\Big] \tag{110}$$

$$\frac{dAC_i}{dt_w} = B_i\Big[-2t_w(\theta+2)(\alpha - c_{ii})\Big] \tag{111}$$

From (24), I_i is divided into terms that depend on t_{ji} and t_{ij}, respectively. Then, from (42) and (50), the derivative of I_i with respect to t_w is

$$\begin{aligned} \frac{dI_i}{dt_w} &= B_i\Bigg[(\theta+2)Ak_i{c_{ii}}^{\theta+1} + 2(\theta+1)c_{ii} + 2\frac{\theta+1}{\theta}\alpha \\ &\quad -t_w\left(\frac{\theta(\theta+2)}{\theta+1}Ak_i{c_{ii}}^{\theta+1} + 4(\theta+1)c_{ii} - 2(\theta+1)\alpha\right)\Bigg] \\ &\quad -\frac{L_j}{L_i}B_j\frac{1}{\theta}\psi_j(c_{jj}), \end{aligned} \tag{112}$$

where $\psi_j(c_{jj}) = \frac{\theta(\theta+2)}{\theta+1}Ak_j{c_{jj}}^{\theta+1} + 2\theta c_{jj} + 2\alpha > 0$. Using (111) and (112), I obtain

$$\begin{aligned} L_i\left(\frac{dI_i}{dt_w} + \frac{dAC_i}{dt_w}\right) &= L_iB_i\Bigg[(\theta+2)Ak_i{c_{ii}}^{\theta+1} + 2(\theta+1)c_{ii} + 2\frac{\theta+1}{\theta}\alpha \\ &\quad -t_w\left(\frac{\theta(\theta+2)}{\theta+1}Ak_i{c_{ii}}^{\theta+1} + 4(\theta+1)c_{ii} - 2(\theta+1)\alpha\right)\Bigg] \\ &\quad -L_jB_j\frac{1}{\theta}\psi_j(c_{jj}) - L_iB_i\left(2t_w(\theta+2)(\alpha - c_{ii})\right) \\ &= L_iB_i\left(\frac{\theta+1}{\theta} - t_w\right)\psi_i(c_{ii}) - L_jB_j\frac{1}{\theta}\psi_j(c_{jj}) \end{aligned} \tag{113}$$

Thus, the welfare effects of t_w through changes in cross-sector allocation in both countries are given by

$$\begin{aligned} \sum_{i\in\{H,F\}} L_i\left(\frac{dI_i}{dt_w} + \frac{dAC_i}{dt_w}\right) &= L_HB_H\left(\frac{\theta+1}{\theta} - t_w\right)\psi_H(c_{HH}) - L_FB_F\frac{1}{\theta}\psi_F(c_{FF}) \\ &\quad +L_FB_F\left(\frac{\theta+1}{\theta} - t_w\right)\psi_F(c_{FF}) - L_HB_H\frac{1}{\theta}\psi_H(c_{HH}) \\ &= L_HB_H(1-t_w)\psi_H(c_{HH}) + L_FB_F(1-t_w)\psi_F(c_{FF}) \\ &= (1-t_w)\sum_{i\in\{H,F\}} L_iB_i\psi_i(c_{ii}) \end{aligned} \tag{114}$$

Substituting (110) and (113) into (109), I obtain

$$\begin{aligned}
\frac{dW}{dt_w} &= \sum_{i\in\{H,F\}} L_i\left(\frac{dI_i}{dt_w}+\frac{dAC_i}{dt_w}\right)+\sum_{i\in\{H,F\}} L_i\frac{dCS_i}{dt_w} \\
&= (1-t_w)\sum_{i\in\{H,F\}} L_iB_i\psi_i(c_{ii})+t_w\sum_{i\in\{H,F\}} L_iB_i(\alpha-2c_{ii}) \\
&= \frac{\theta t_w^{-(\theta+2)}}{4\gamma(\theta+1)(\theta+2)}\left[(1-t_w)\sum_{i\in\{H,F\}} L_ib_i\psi_i(c_{ii})+t_w\sum_{i\in\{H,F\}} L_ib_i(\alpha-2c_{ii})\right],
\end{aligned} \tag{115}$$

where $b_i \equiv \frac{k_j c_{ii}^{\theta+2}}{(\theta+1)\alpha-\theta c_{ii}} > 0$. Substituting $t_w = 1$ into (115), I obtain

$$\left.\frac{dW}{dt_w}\right|_{t_w=1} = \frac{\theta\sum_{i\in\{H,F\}} L_ib_i(\alpha-2c_{ii}^{FT})}{4\gamma(\theta+1)(\theta+2)}. \tag{116}$$

Note that from (14) $c_{HH}^{FT} = c_{FF}^{FT} \equiv c_w^{FT}$ because $\tau_{ji} = \tau_{ij} = 1$. Then I can rewrite (116) as follows:

$$\left.\frac{dW}{dt_w}\right|_{t_w=1} = \frac{\theta(b_HL_H+b_FL_F)}{4\gamma(\theta+1)(\theta+2)}(\alpha-2c_w^{FT}). \tag{117}$$

If $\alpha - 2c_w^{FT} \le 0$, it is straightforward to show that $\alpha - 2c_{HH} \le 0$ and $\alpha - 2c_{FF} \le 0$ for $t_w \in [1,\infty)$ because the domestic cutoffs are increasing functions of t_w. Therefore, if $\alpha - 2c_w^{FT} \le 0$, then $dW/dt_w \le 0$ for $t_w \in [1,\infty)$:

$$\frac{dW}{dt_w} < 0 \quad \text{if} \quad \alpha - 2c_w^{FT} \le 0. \tag{118}$$

If $\alpha - 2c_w^{FT} > 0$, (117) is positive. In addition, rearranging (115), I obtain

$$\frac{dW}{dt_w} = \frac{\theta t_w^{-(\theta+2)}}{4\gamma(\theta+1)(\theta+2)}\sum_{i\in\{H,F\}} L_ib_i\left((1-t_w)\frac{\theta(\theta+2)}{\theta+1}Ak_ic_{ii}^{\theta+1}+2(\theta-t_w(\theta+1))c_{ii}+(2-t_w)\alpha\right), \tag{119}$$

where $dW/dt_w < 0$ for $t_w \ge 2$.

Therefore, if $\alpha - 2c_w^{FT} > 0$, there is an efficient tariff, t_w^e that maximizes the total welfare level of the two countries. From (119), the efficient tariff satisfies

$$\begin{aligned}
\frac{dW}{dt_w} = 0 \quad &\Leftrightarrow \quad \sum_{i\in\{H,F\}} L_i b_i \left((1-t_w^e)\frac{\theta(\theta+2)}{\theta+1} A k_i c_{ii}^{\theta+1} + 2(\theta - t_w^e(\theta+1))c_{ii} + (2-t_w^e)\alpha \right) = 0 \\
&\Leftrightarrow \quad t_w^e = \frac{\sum_{i\in\{H,F\}}\left[L_i b_i \left(\frac{\theta(\theta+2)}{\theta+1} A k_i {c_{ii}}^{\theta+1} + 2\theta c_{ii} + 2\alpha\right)\right]}{\sum_{i\in\{H,F\}}\left[L_i b_i \left(\frac{\theta(\theta+2)}{\theta+1} A k_i {c_{ii}}^{\theta+1} + 2(\theta+1) c_{ii} + \alpha\right)\right]} \\
&\Leftrightarrow \quad t_w^e = 1 + \frac{\sum_{i\in\{H,F\}}\left[L_i b_i(\alpha - 2c_{ii})\right]}{\sum_{i\in\{H,F\}}\left[L_i b_i \left(\frac{\theta(\theta+2)}{\theta+1} A k_i {c_{ii}}^{\theta+1} + 2(\theta+1) c_{ii} + \alpha\right)\right]}. \qquad (120)
\end{aligned}$$

7.10 Proof of Proposition 5

From (110)–(112), country i's welfare effect of a symmetric import tariff can be written as

$$\begin{aligned}
\frac{dL_iU_i}{dt_w} &= L_i\left(\frac{dI_i}{dt_w} + \frac{dCS_i}{dt_w} + \frac{dAC_i}{dt_w}\right) \\
&= \frac{\theta t_w^{-(\theta+2)}}{4\gamma(\theta+1)(\theta+2)}\Bigg[L_i b_i \Bigg\{(\theta+2)Ak_i c_{ii}^{\theta+1} + 2(\theta+1)c_{ii} + 2\frac{\theta+1}{\theta}\alpha \\
&\quad -t_w\left(\frac{\theta(\theta+2)}{\theta+1}Ak_i c_{ii}^{\theta+1} + 2(\theta+1)c_{ii} + \alpha\right)\Bigg\} - L_j b_j \frac{1}{\theta}\psi_j(c_{jj})\Bigg], \qquad (121)
\end{aligned}$$

where $b_i \equiv \frac{k_j c_{ii}^{\theta+2}}{(\theta+1)\alpha-\theta c_{ii}} > 0$ and $\psi_j(c_{jj}) \equiv \frac{\theta(\theta+2)}{\theta+1} A k_j {c_{jj}}^{\theta+1} + 2\theta c_{jj} + 2\alpha > 0$. Substituting $t_w = 1$ into (121), I obtain

$$\begin{aligned}
\left.\frac{dL_iU_i}{dt_w}\right|_{t_w=1} &= \frac{\theta}{4\gamma(\theta+1)(\theta+2)} \frac{{c_w^{FT}}^{\theta+2}}{(\theta+1)\alpha - \theta c_w^{FT}} \Bigg[(L_i - L_j)(\theta+2) k_i k_j \frac{A{c_w^{FT}}^{\theta+1}}{\theta+1} \\
&\quad + L_i k_j \frac{\theta+2}{\theta}\alpha - L_j k_i \left(2c_w^{FT} + \frac{2}{\theta}\alpha \right) \Bigg] \\
&= \frac{\theta}{4\gamma(\theta+1)(\theta+2)} \frac{L_j k_i {c_w^{FT}}^{\theta+2}}{(\theta+1)\alpha - \theta c_w^{FT}} \Bigg[2(\frac{L_i}{L_j} - 1)(\theta+2) \frac{k_j}{k_i + k_j} (\alpha - c_w^{FT}) \\
&\quad + \frac{L_i}{L_j}\frac{k_j}{k_i}\frac{\theta+2}{\theta}\alpha - 2c_w^{FT} - \frac{2}{\theta}\alpha \Bigg] \\
&= \frac{\theta}{4\gamma(\theta+1)(\theta+2)} \frac{L_j k_i {c_w^{FT}}^{\theta+2}}{(\theta+1)\alpha - \theta c_w^{FT}} \Bigg[2(L-1)(\theta+2)\frac{\alpha - c_w^{FT}}{k+1} \\
&\quad + \frac{L}{k}\frac{\theta+2}{\theta}\alpha - 2c_w^{FT} - \frac{2}{\theta}\alpha \Bigg] \\
&= \frac{\theta}{4\gamma(\theta+1)(\theta+2)} \frac{L_j k_i {c_w^{FT}}^{\theta+2}}{(\theta+1)\alpha - \theta c_w^{FT}} \frac{1}{\theta k(k+1)} \Bigg[2(L-1)\theta(\theta+2)(\alpha - c_w^{FT})k \\
&\quad + L(\theta+2)\alpha(k+1) - 2\theta c_w^{FT} k(k+1) - 2k(k+1)\alpha \Bigg],
\end{aligned} \tag{122}$$

where $L \equiv L_i/L_j$ and $k \equiv k_i/k_j$. I use the relationship $\frac{A{c_w^{FT}}^{\theta+1}}{\theta+1} = \frac{2(\alpha - c_w^{FT})}{k_i+k_j}$ from (14) at the second line. The sign of (122) is positive if the following inequality holds:

$$\begin{aligned}
&\left.\frac{dL_iU_i}{dt_w}\right|_{t_w=1} > 0 \\
\Leftrightarrow\ & 2(L-1)\theta(\theta+2)(\alpha - c_w^{FT})k + L(\theta+2)\alpha(k+1) - 2\theta c_w^{FT} k(k+1) - 2k(k+1)\alpha > 0 \\
\Leftrightarrow\ & (\theta+2)\left(\left((2\theta+1)\alpha - 2\theta c_w^{FT}\right)k + \alpha\right) L > 2(\alpha + \theta c_w^{FT})k^2 + 2(\theta+1)((\theta+1)\alpha - \theta c_w^{FT})k \\
\Leftrightarrow\ & L > \frac{(\alpha + \theta c_w^{FT})k + (\theta+1)((\theta+1)\alpha - \theta c_w^{FT})}{\left((2\theta+1)\alpha - 2\theta c_w^{FT}\right)k + \alpha} \frac{2k}{\theta+2} = \frac{2}{\theta+2}\frac{\beta_1 k + \beta_2}{\beta_3 k + \alpha} k \equiv \underline{L}(k),
\end{aligned} \tag{123}$$

where $\beta_1 \equiv \alpha + \theta c_w^{FT}$, $\beta_2 \equiv (\theta+1)((\theta+1)\alpha - \theta c_w^{FT})$, and $\beta_3 \equiv (2\theta+1)\alpha - 2\theta c_w^{FT}$ are positive.

In the same way, the introduction of a small symmetric import tariff increases the welfare of country j if the following inequality holds:

$$\begin{aligned} &\left.\frac{dL_jU_j}{dt_w}\right|_{t_w=1} > 0 \\ \Leftrightarrow\quad & L < \frac{\theta+2}{2}\frac{\alpha k+\beta_3}{\beta_2 k+\beta_1}k \equiv \overline{L}(k). \end{aligned} \tag{124}$$

I assume that $k_i + k_j$ is constant and takes a value that ensures $\alpha - 2c_w^{FT} > 0$. Then, changes in k do not affect $k_i + k_j$ and thereby c_w^{FT}: $dc_w^{FT}/dk = 0$. Differentiating $\underline{L}(k)$ and $\overline{L}(k)$ with respect to k, I obtain

$$\frac{d\underline{L}(k)}{dk} = \frac{2}{\theta+2}\frac{\beta_1\beta_3 k^2 + 2\alpha\beta_1 k + \alpha\beta_2}{(\beta_3 k+\alpha)^2} > 0 \tag{125}$$

$$\frac{d\overline{L}(k)}{dk} = \frac{\theta+2}{2}\frac{\alpha\beta_2 k^2 + 2\alpha\beta_1 k + \beta_1\beta_3}{(\beta_2 k+\beta_1)^2} > 0. \tag{126}$$

The difference between $\underline{L}(k)$ and $\overline{L}(k)$ is

$$\overline{L}(k) - \underline{L}(k) = \frac{\theta\left((2\theta^2+5\theta+4)\alpha - 2\theta(\theta+1)c_w^{FT}\right)(\alpha - 2c_w^{FT})}{2(\theta+2)(\beta_2 k+\beta_1)(\beta_3 k+\alpha)}(k+1)^2 k > 0. \tag{127}$$

Thus, $\overline{L}(k) > \underline{L}(k)$ for $k > 0$. From (123)–(127), I obtain

$$\left.\frac{dL_iU_i}{dt_w}\right|_{t_w=1} > 0 \quad \text{and} \quad \left.\frac{dL_jU_j}{dt_w}\right|_{t_w=1} > 0 \Leftrightarrow \underline{L}(k) < L < \overline{L}(k). \tag{128}$$

Finally, substituting $k = 1$ into $\underline{L}(k)$ and $\overline{L}(k)$, I obtain

$$\underline{L}(1) = \frac{2}{\theta+2}\frac{\beta_1+\beta_2}{\beta_3+\alpha} = 1 - \frac{\theta(\alpha - 2c_w^{FT})}{(\theta+2)\left((\theta+1)\alpha - \theta c_w^{FT}\right)} < 1 \tag{129}$$

$$\overline{L}(1) = \frac{\theta+2}{2}\frac{\alpha+\beta_3}{\beta_2+\beta_1} = 1 + \frac{\theta(\alpha - 2c_w^{FT})}{(\theta^2+2\theta+2)\alpha - \theta^2 c_w^{FT}} > 1. \tag{130}$$

Using (123)–(130), I depict Figure 1.

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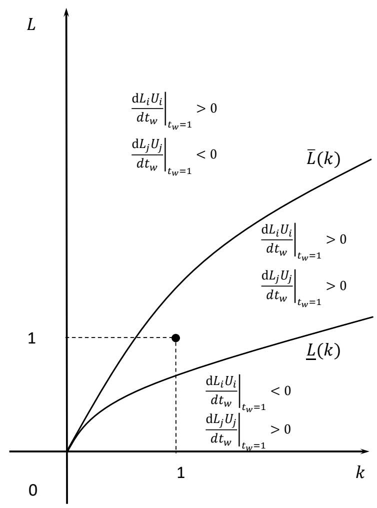

Figure 1: Welfare effect of the introduction of a small symmetric import tariff

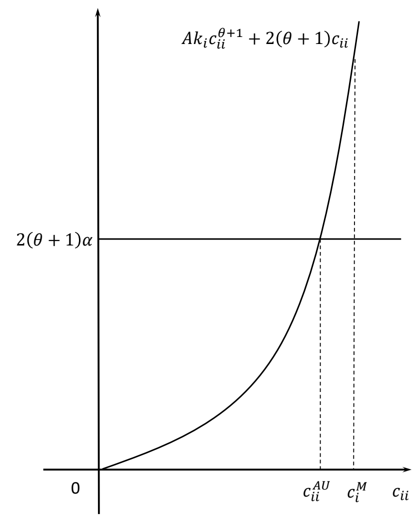

Figure 2: Sufficient condition for $c_{ii} < c_i^M$

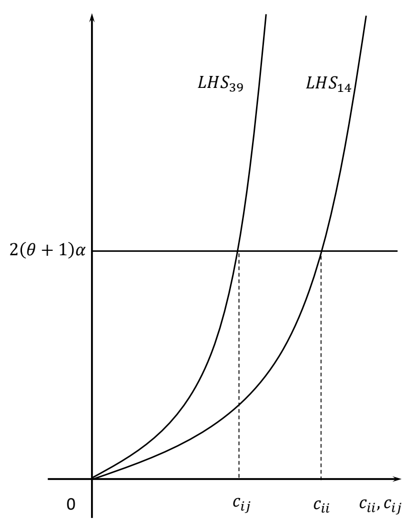

Figure 3: Proof of $c_{ij} < c_{ii}$

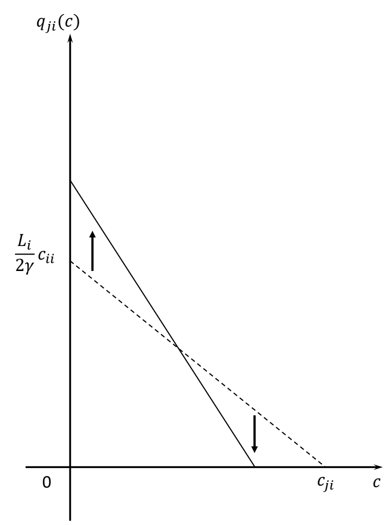

Figure 4: Effects of an increase in t_{ji} on $q_{ji}(c)$

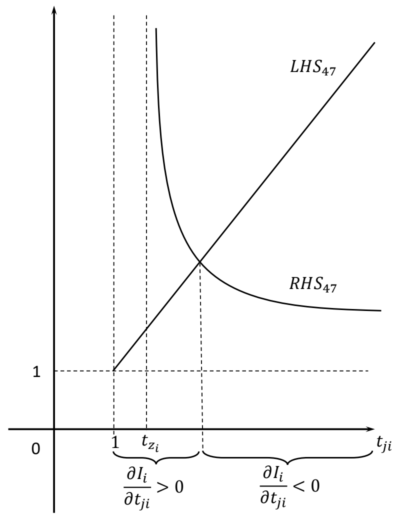

Figure 5: Proof of Lemma 3 when $t_{z_i} \leq 1$

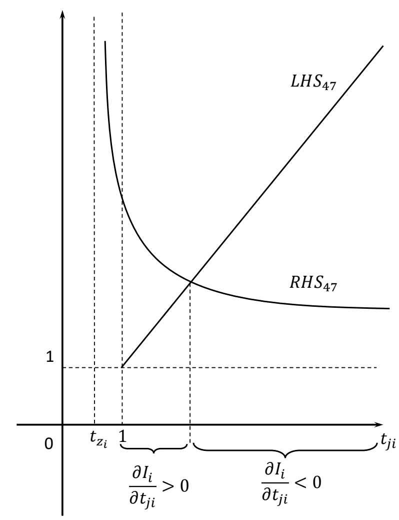

Figure 6: Proof of Lemma 3 when $t_{z_i} > 1$

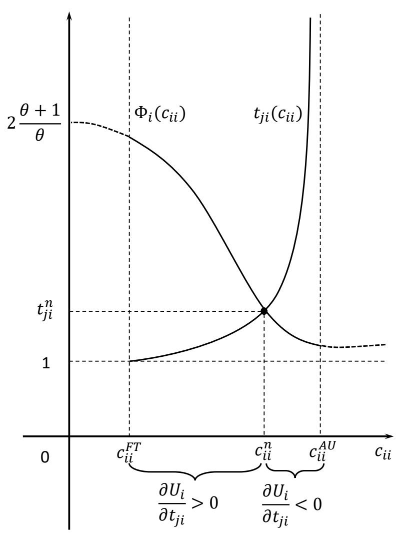

Figure 7: Proof of Proposition 1

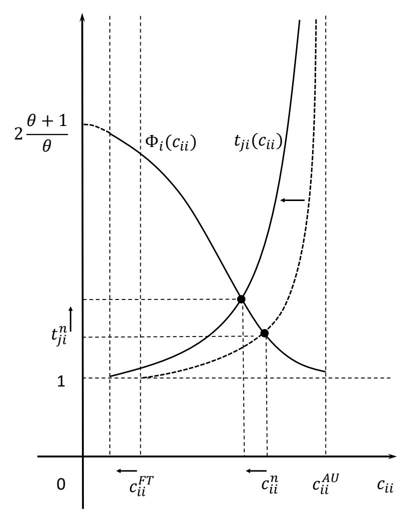

Figure 8: Proof of Lemma 8

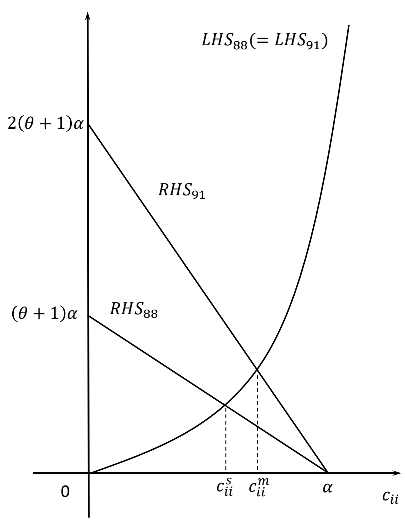

Figure 9: Proof of Lemma 9

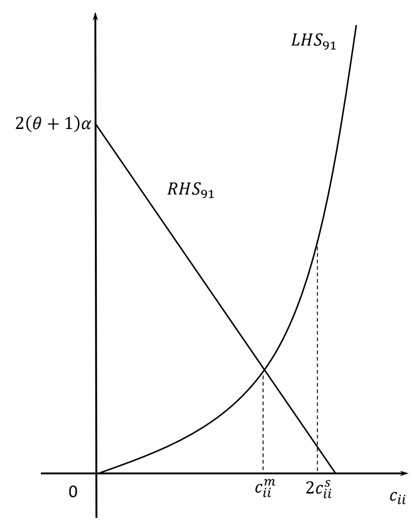

Figure 10: Proof of Lemma 9