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# Efficient policy with firm heterogeneity and variable markups 

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markups*

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#### Abstract

This study analyzes policy instruments to make the market outcome achieve the efficient outcome in a model of monopolistic competition with firm heterogeneity and variable markups. In this model, firm heterogeneity and the markup pricing create distortions in the market equilibrium. Ad valorem and per-unit production taxes/subsidies work differently to improve these distortions. I show that by combining an ad valorem production tax and a per-unit production subsidy, these distortions are removed without adopting a firm-specific tax/subsidy.


Keywords: Variable markups, Misallocation, Firm heterogeneity, Monopolistic competition
JEL Classification: D4, D6, F1, L0, L1

[^0]
## 1 Introduction

In the canonical models of monopolistic competition with constant elasticity of substitution (CES) preferences, which generate constant markups, opposing externalities related to the inefficiency of product variety offset each other. That is, the market equilibrium is efficient when there is no other sector than the monopolistically competitive one. Therefore, there is no room for welfare-improving policy intervention, which reduces opportunities to address the many distortion-related problems originally associated with monopolistic competition. ${ }^{1}$

In this study, I examine the effects of policy intervention on welfare based on the model of Melitz and Ottaviano (2008), who incorporate endogenous markups by introducing the linear demand system into the model of Melitz (2003). The presence of endogenous markups affects the efficiency of resource allocation. Based on the closed economy setting of the MelitzOttaviano model, Nocco et al. (2014) show that, due to firm heterogeneity and variable markups, the market outcome fails to be efficient in terms of product selection, product mix, and product variety. ${ }^{2}$ The purpose of the present study is to provide policy instruments to correct these distortions and make the market outcome achieve the efficient outcome.

Regarding the effectiveness of policy instruments, Nocco et al. (2014) characterize the policy tools that policymakers can use to make the market outcome achieve the efficient outcome. Under an unconstrained choice of tools, Nocco et al. (2014) show that the market can achieve the first-best outcome through a firm-specific per-unit production tax/subsidy, a lump-sum entry tax, and a lump-sum tax on consumers. When the firm-specific production subsidy/tax is unavailable, they consider a second-best scenario in which a per-unit production subsidy is offered to all firms and financed by a lump-sum tax on consumers. Therefore, according to Nocco et al. (2014), in order to achieve the first-best outcome, policymakers need to determine taxes/subsidies based on the level of productivity (marginal cost) of each firm.

The present study shows, however, policymakers do not need to have any way of observing the level of productivity of each firm to make the market outcome achieve the first-best outcome: the first-best outcome can be decentralized through an uniform ad valorem production tax, a uniform per-unit production subsidy, and a lump-sum tax on consumers. Ad valorem

[^1]and per-unit taxes/subsidies play different roles on each distortion. As a result, combining an ad valorem production tax and a per-unit production subsidy can create a tax/subsidy system similar to a firm-specific production tax/subsidy, which enables the first-best resource allocation to be achieved without a firm-specific tax/subsidy.

The rest of the paper is organized as follows. Section 2 briefly presents the model. Section 3 derives the market and the efficient outcomes, and compares the outcomes. Section 4 characterizes the efficient policy that can be implemented to attain efficiency at the market equilibrium. Section 5 concludes.

## 2 Model

Following the closed economy of Melitz and Ottaviano (2008), consider an economy with two sectors and one production factor, which is labor. Labor $L$ is inelastically supplied by consumers. The preferences of consumers are defined over a continuum of differentiated varieties of goods and a homogeneous good. The differentiated goods are indexed by $\omega \in$ $\Omega$ and the homogeneous good is chosen as the numeraire. All consumers share the same preference and each consumer maximizes the following utility function:

$$
\begin{equation*}
U=q_{0}^{c}+\alpha \int_{\Omega} q^{c}(\omega) d \omega-\frac{\gamma}{2} \int_{\Omega} q^{c}(\omega)^{2} d \omega-\frac{\eta}{2}\left(\int_{\Omega} q^{c}(\omega) d \omega\right)^{2}, \tag{1}
\end{equation*}
$$

subject to the budget constraint

$$
q_{0}^{c}+\int_{\Omega} p(\omega) q^{c}(\omega) d \omega=I
$$

where $\Omega$ is the set of all available differentiated good varieties; $q_{0}^{c}$ and $q^{c}(\omega)$ are the individual consumption levels of the numeraire good and each variety $\omega$, respectively; $p(\omega)$ is the price of variety $\omega$; and $I$ is the income. The parameters $\alpha, \eta$, and $\gamma$ are positive constants. A lower $\gamma$ indicates that the differentiated varieties become closer substitutes and in the limit case of $\gamma=0$, consumers care only about the total amount of differentiated goods they consume. $\eta$ represents the degree of non-separability. When $\eta$ equals zero, the utility function becomes separable across the differentiated varieties. I assume that the consumers have positive demand for the numeraire good $\left(q_{0}^{c}>0\right)$.

Perfect competition prevails in the homogeneous good market. Production of one unit of a homogeneous good requires one unit of labor input. In the differentiated goods sector, each variety is produced by a monopolistically competitive firms. To enter the market, a firm pays a fixed cost $f_{e}>0$-a sunk labor requirement to design a new variety - and draw its marginal cost $c$ - the unit labor requirement. I assume that the unit labor requirement $c$ follows Pareto distribution:

$$
\begin{equation*}
c \sim G(c)=\left(\frac{c}{c_{M}}\right)^{\theta}, \quad c \in\left[0, c_{M}\right], \quad \theta \geq 1 \tag{2}
\end{equation*}
$$

where $G(c)$ is the cost distribution, $\theta$ is an index of the dispersion of the cost, and $c_{M}$ is the upper bound of the cost.

## 3 Market and efficient outcomes

### 3.1 Market outcome

The labor market and the market of the homogeneous good are perfectly competitive, and the homogeneous good is chosen as the numeraire. Thus, the wage becomes one.

Using the first-order conditions for utility maximization, the inverse demand for each variety $\omega$ is given by

$$
\begin{equation*}
p(\omega)=\alpha-\gamma q^{c}(\omega)-\eta Q^{c} \quad \forall \omega \in \Omega^{*}, \tag{3}
\end{equation*}
$$

where $\Omega^{*} \subset \Omega$ represents the subset of varieties in which $q^{c}(\omega)>0$, and $Q^{c} \equiv \int_{\Omega^{*}} q^{c}(\omega) d \omega$ is the total consumption of all differentiated goods by individuals. By integrating both sides of Eq. (3) over $\Omega^{*}$, I obtain

$$
\begin{equation*}
Q^{c}=\frac{N}{\gamma+\eta N}(\alpha-\bar{p}), \tag{4}
\end{equation*}
$$

where $N$ is the number of consumed varieties, and $\bar{p}=(1 / N) \int_{\Omega^{*}} p(\omega) d \omega$ is the average price of consumed varieties. Using Eqs. (3) and (4), I obtain the following market demand for variety $\omega, q(\omega)$ :

$$
q(\omega)=L q^{c}(\omega)=\frac{L}{\gamma}\left(p^{\max }-p(\omega)\right)
$$

where

$$
\begin{equation*}
p^{\max } \equiv \frac{\gamma \alpha+\eta N \bar{p}}{\gamma+\eta N} \tag{5}
\end{equation*}
$$

represents the threshold price at which demand for a variety is driven to zero. Note that Eq. (3) implies $p_{i}^{m a x} \leq \alpha$.

In the differentiated good sector, when a variety is produced by a firm with unit labor requirement $c$, the profit maximization problem for the firm is given by

$$
\max (p(c)-c) q(c), \quad \text { s.t. } q(c)=\frac{L}{\gamma}\left(p^{\max }-p(c)\right),
$$

where $p(c)$ and $q(c)$ denote the price and quantity set by a firm with marginal cost $c$, respectively. The profit-maximizing price and quantity are

$$
\begin{aligned}
p^{m}(c) & =\frac{1}{2}\left(p^{\max }+c\right), \\
q^{m}(c) & =\frac{L}{2 \gamma}\left(p^{\max }-c\right),
\end{aligned}
$$

where " $m$ " labels the equilibrium variables.
Next, I define the cost cutoff. Let $c^{m}$ be the upper bound of the cost for firms to earn positive operating profits at the market equilibrium:

$$
\begin{equation*}
c^{m}=\sup \{c: \pi(c)>0\}=p^{\max } \tag{6}
\end{equation*}
$$

As described in Melitz and Ottaviano (2008), the cost cutoff represents the toughness of competition in the market. Assume that $c_{M}$ is sufficiently high to be greater than $c^{m}$.

Using the cost cutoff, I obtain the price, quantity, profit, and markup of a firm with cost
c.

$$
\begin{align*}
p^{m}(c) & =\frac{1}{2}\left(c^{m}+c\right) \\
q^{m}(c) & =\frac{L}{2 \gamma}\left(c^{m}-c\right),  \tag{7}\\
\pi^{m}(c) & =\frac{L}{4 \gamma}\left(c^{m}-c\right)^{2}  \tag{8}\\
\mu^{m}(c) & =\frac{p^{m}(c)}{c}=\frac{1}{2}\left(\frac{c^{m}}{c}+1\right) .
\end{align*}
$$

Lower-cost firms set lower prices but also higher markups. This generates distortions because more productive firms do not pass on their entire cost advantage to households by raising their markups and end up selling too little, while less productive firms end up oversupplying the market.

Prior to entry, the expected firm profit is $\int_{0}^{c^{m}} \pi^{m}(c) d G(c)-f_{e}$. Due to free entry condition, an entrant's expected profit is offset by the sunk entry cost so that $\int_{0}^{c^{m}} \pi^{m}(c) d G(c)=f_{e}$. Given Eqs. (2) and (8), this free entry condition determines the cost cutoff:

$$
\begin{align*}
& \int_{0}^{c^{m}} \pi^{m}(c) d G(c)=f_{e} \\
\Leftrightarrow & c^{m}=\left[\frac{2 \gamma(\theta+1)(\theta+2) c_{M}^{\theta} f_{e}}{L}\right]^{\frac{1}{\theta+2}} . \tag{9}
\end{align*}
$$

Substituting Eq. (6) into Eq. (5), the number of varieties can be expressed in terms of the cost cutoff.

$$
\begin{align*}
c^{m} & =\frac{\gamma \alpha+\eta N \bar{p}}{\gamma+\eta N} \\
\Leftrightarrow N^{m} & =\frac{2 \gamma(\theta+1)}{\eta} \frac{\alpha-c^{m}}{c^{m}}, \tag{10}
\end{align*}
$$

where

$$
\begin{equation*}
\bar{p}=\frac{1}{G(c)} \int_{0}^{c^{m}} p(c) d G(c)=\frac{2 \theta+1}{2(\theta+1)} c^{m} . \tag{11}
\end{equation*}
$$

The number of entrants is then given by $N_{e}^{m}=N^{m} / G\left(c^{m}\right)$.
Substituting Eqs. (10) and (11) into Eq. (4), the total consumption level of the differen-
tiated varieties, $Q^{m}$, can be written by

$$
\begin{equation*}
Q^{m} \equiv L Q^{c}=\frac{L}{\eta}\left(\alpha-c^{m}\right) \tag{12}
\end{equation*}
$$

Finally, substituting Eqs. (7) and (10) into (1), the welfare level at the market equilibrium can be expressed in terms of the cost cutoff.

$$
W^{m}=L U=L+\frac{L}{2 \eta}\left(\alpha-c^{m}\right)\left(\alpha-\frac{\theta+1}{\theta+2} c^{m}\right) .
$$

### 3.2 Efficient outcome

A benevolent social planner maximizes welfare, taking as given the endowment of labor $(L)$; the production functions of the two goods; and the mechanism determining each variety's unit labor requirement $c$-as a random draw from the distribution $G(c)$ after $f_{e}$ units of labor have been allocated to the design of a variety.

The planner's problem is given by

$$
\begin{equation*}
\max _{q_{0}, q(c), N_{e}} W=L U \tag{13}
\end{equation*}
$$

subject to the resource constraint:

$$
\begin{equation*}
q_{0}+N_{e} \int_{0}^{c_{M}} c q(c) d G(c)+f_{e} N_{e}=L, \tag{14}
\end{equation*}
$$

where $q_{0}=L q_{0}^{c}$. Substituting Eq. (14) into Eq. (13), the planner's problem can be rewritten by

$$
\begin{align*}
\max _{q(c), N_{e}} W= & L-f_{e} N_{e}-N_{e} \int_{0}^{c_{M}} c q(c) d G(c)+\alpha N_{e} \int_{0}^{c_{M}} q(c) d G(c) \\
& -\frac{\gamma}{2 L} N_{e} \int_{0}^{c_{M}} q(c)^{2} d G(c)-\frac{\eta}{2 L}\left(N_{e} \int_{0}^{c_{M}} q(c) d G(c)\right)^{2} \tag{15}
\end{align*}
$$

The first order conditions are given by

$$
\begin{align*}
& \begin{aligned}
\frac{\partial W}{\partial q(c)}=0 & \Leftrightarrow q^{o}(c)=\frac{L}{\gamma}\left(\alpha-\frac{\eta}{L} Q^{o}-c\right) \\
& \Leftrightarrow q^{o}(c)=\frac{L}{\gamma}\left(c^{o}-c\right)
\end{aligned}  \tag{16}\\
& \frac{\partial W}{\partial N_{e}}=0  \tag{17}\\
& \Leftrightarrow-f_{e}-\int_{0}^{c_{M}} c q(c) d G(c)+\alpha \int_{0}^{c_{M}} q(c) d G(c)-\frac{\gamma}{2 L} \int_{0}^{c_{M}} q(c)^{2} d G(c)-\frac{\eta}{L} N_{e}\left(\int_{0}^{c_{M}} q(c) d G(c)\right)^{2}=0
\end{align*}
$$

where " $o$ " labels first-best optimum variables, $Q^{o} \equiv N_{e}^{o} \int_{0}^{c_{M}} q^{o}(c) d G(c)$ is the optimal total output level of differentiated varieties, and

$$
\begin{equation*}
c^{o} \equiv \alpha-\frac{\eta}{L} Q^{o} \tag{19}
\end{equation*}
$$

represents the optimal cost cutoff. Integrating Eq. (16), I obtain

$$
\begin{equation*}
Q^{o}=\frac{L N^{o}}{\gamma+\eta N^{o}}\left(\alpha-\frac{\theta}{\theta+1} c^{o}\right) \tag{20}
\end{equation*}
$$

Substituting Eq. (20) into Eq. (19) gives

$$
\begin{equation*}
N^{o}=\frac{\gamma(\theta+1)}{\eta} \frac{\alpha-c^{o}}{c^{o}} . \tag{21}
\end{equation*}
$$

The number of entrants is then

$$
\begin{equation*}
N_{e}^{o}=\frac{N^{o}}{G\left(c^{o}\right)}=\frac{\gamma(\theta+1) c_{M}{ }^{\theta}}{\eta} \frac{\alpha-c^{o}}{c^{o \theta+1}} . \tag{22}
\end{equation*}
$$

Substituting Eq. (21) into Eq. (20), I obtain

$$
\begin{equation*}
Q^{o}=\frac{L}{\eta}\left(\alpha-c^{o}\right) \tag{23}
\end{equation*}
$$

Therefore, substituting Eqs. (17) and (22) into Eq. (18), the optimal cost cutoff is determined by

$$
\begin{equation*}
c^{o}=\left[\frac{\gamma(\theta+1)(\theta+2) c_{M}^{\theta} f_{e}}{L}\right]^{\frac{1}{\theta+2}} \tag{24}
\end{equation*}
$$

Finally, substituting Eqs. (17) and (22) into (15), the welfare level at the first-best optimum can be expressed in terms of the cost cutoff.

$$
W^{o}=L+\frac{L}{2 \eta}\left(\alpha-c^{o}\right)^{2}
$$

### 3.3 Market failure

I compare the market outcome with the efficient outcome. As Nocco et al. (2014) show, the market outcome departs from the efficient outcome in terms of product selection, product mix, and product range.

The inefficiency of product selection is characterized by the gap between the cost cutoffs in Eqs. (9) and (24):

$$
c^{o}-c^{m}=-\left(2^{\frac{1}{\theta+2}}-1\right) c^{o}<0,
$$

which implies $c^{o}<c^{m}$. Thus, varieties with $c \in\left[c^{o}<c^{m}\right]$ is inefficiently supplied. Intuitively, more productive firms set higher markups in the market equilibrium, which leaves room for less productive firms to survive, albeit inefficiently.

The inefficiency of product mix is characterized by the gap between the output level of each firm with marginal cost $c$ in Eqs. (7) and (17):

$$
q^{o}(c)-q^{m}(c)=\frac{L}{2 \gamma}\left(\left(2-2^{\frac{1}{\theta+2}}\right) c^{o}-c\right) \begin{cases}\geq 0 & c \in\left[0,\left(2-2^{\frac{1}{\theta+2}}\right) c^{o}\right] \\ <0 & c \in\left(\left(2-2^{\frac{1}{\theta+2}}\right) c^{o}, c^{m}\right]\end{cases}
$$

which implies $q^{o}(c) \geq(<) q^{m}(c)$ for a firm with lower (higher) $c$. Accordingly, in the market equilibrium, more productive firms end up selling too little, while less productive firms end up oversupplying the market. More productive firms do not pass on their entire cost advantage to consumers by raising their markups, and they end up selling below the efficient output level, which causes less productive firms to oversupply the market.

The inefficiency of product variety is characterized by the gap between the number of varieties in Eqs. (10) and (21):

$$
N^{o}-N^{m}=\frac{\gamma(\theta+1)}{\eta}\left[1-\left(2^{\frac{\theta+1}{\theta+2}}-1\right) \frac{\alpha}{c^{o}}\right] \begin{cases}\geq 0 & c^{o} \geq\left(2^{\frac{\theta+1}{\theta+2}}-1\right) \alpha \\ <0 & c^{o}<\left(2^{\frac{\theta+1}{\theta+2}}-1\right) \alpha\end{cases}
$$

which implies that the number of varieties at the market equilibrium is smaller (larger) than the first-best when $\alpha$ is small (large) or $c^{o}$ is large (small). Although the direction of departure from the efficient number of varieties in the market outcome depends on relationship between parameters, the total output level of differentiated varieties in the market equilibrium is lower than that in the first-best. From Eqs. (12) and (23), the gap between the total output level is

$$
Q^{o}-Q^{m}=\frac{L}{\eta}\left(c^{m}-c^{o}\right)>0 .
$$

At the end of this section, I summarize in the following lemma how the market outcome departs from the efficient outcome.

Lemma 1. Compared to the first-best outcome, in the market outcome, the cost cutoff is above that in the first-best-inefficient product selection; the output level of more (less) productive firms is below (above) that in the first-best-inefficient product mix; the number of varieties is below (above) that in the first-best if and only if $c^{o} \geq(<)\left(2^{\frac{\theta+1}{\theta+2}}-1\right) \alpha$-inefficient product variety.

## 4 Efficient policy

In this section, I characterize the policy tools through which the market outcome can achieve the efficient outcome without the use of firm-specific taxes/subsidies. The efficient outcome can be decentralized through an ad valorem production tax and a per-unit production subsidy - these are uniform across firms-accompanied by a lump-sum tax on consumers.

### 4.1 Policy tools

Each consumer maximizes the utility function Eq. (1) subject to budget constraint

$$
q_{0}^{c}+\int_{\Omega} p(\omega) q^{c}(\omega) d \omega=1-T,
$$

where $T$ is a lump-sum tax (subsidy) on consumers if positive (negative) and assume that $1-T>0$. In the same way as in Section 3.1, the market demand for variety $\omega$ is given by Eq. (3).

In the differentiated good sector, the profit maximization problem for a firm with unit labor requirement $c$ is given by

$$
\max \pi(c)=\left(\frac{p(c)}{1+t_{a}}-\left(t_{u}+c\right)\right) q(c) \quad \text { s.t. } \quad q(c)=\frac{L}{\gamma}\left(p^{\max }-p(c)\right),
$$

where $p(c)$ is the taxes/subsidies-inclusive price, and $t_{a}>-1$ and $t_{u}$ are ad valorem and perunit production taxes (subsidies) if positive (negative), respectively. The profit maximizing price and quantity are

$$
\begin{gather*}
p^{g}(c)=\frac{1+t_{a}}{2}\left(\frac{p^{\max }}{1+t_{a}}+t_{u}+c\right), \\
q^{g}(c)=\frac{L\left(1+t_{a}\right)}{2 \gamma}\left(\frac{p^{\max }}{1+t_{a}}-t_{u}-c\right), \tag{25}
\end{gather*}
$$

where " $g$ " labels equilibrium variables under the introduction of policy tools by a government. From Eq. (25), the cost cutoff $c^{g}$ is

$$
\begin{equation*}
c^{g}=\frac{p^{\max }}{1+t_{a}}-t_{u} . \tag{26}
\end{equation*}
$$

Using the cost cutoff, I obtain the price, quantity, profit, and markup for a firm with cost $c$ :

$$
\begin{align*}
& p^{g}(c)=\frac{1+t_{a}}{2}\left(c^{g}+2 t_{u}+c\right), \\
& q^{g}(c)=\frac{L\left(1+t_{a}\right)}{2 \gamma}\left(c^{g}-c\right),  \tag{27}\\
& \pi^{g}(c)=\frac{L\left(1+t_{a}\right)}{4 \gamma}\left(c^{g}-c\right)^{2}, \\
& \mu^{g}(c)=\frac{p^{g}(c)}{c}=\frac{1+t_{a}}{2}\left(\frac{c^{g}}{c}+\frac{2 t_{u}}{c}+1\right) .
\end{align*}
$$

Due to the free entry condition, the expected profit is given as

$$
\begin{align*}
& \int_{0}^{c^{g}} \pi^{g}(c) d G(c)=f_{e} \\
\Leftrightarrow & c^{g}=\left[\frac{2 \gamma(\theta+1)(\theta+2) c_{M}^{\theta} f_{e}}{L\left(1+t_{a}\right)}\right]^{\frac{1}{\theta+2}} . \tag{28}
\end{align*}
$$

Note that from Eqs. (27) and (28) a per-unit production tax dose not affect the cost cutoff and the output level of each firm.

Using Eq. (26), rewrite Eq. (5) to obtain the number of varieties as a function of the cost cutoff:

$$
\begin{equation*}
N^{g}=\frac{2 \gamma(\theta+1)}{\eta} \frac{\alpha-\left(1+t_{a}\right)\left(c^{g}+t_{u}\right)}{\left(1+t_{a}\right) c^{g}} \tag{29}
\end{equation*}
$$

where

$$
\begin{equation*}
\bar{p}=\frac{1}{G\left(c^{g}\right)} \int_{0}^{c^{g}} p^{g}(c) d G(c)=\left(1+t_{a}\right)\left(\frac{2 \theta+1}{2(\theta+1)} c^{g}+t_{u}\right) . \tag{30}
\end{equation*}
$$

Thus, the number of entrant is given $N_{e}^{g}=N^{g} / G\left(c^{g}\right)$.
From Eqs. (4), (29), and (30), the total consumption level of the differentiated varieties, $Q^{g}$, can be written by

$$
Q^{g}=\frac{L}{\eta}\left(\alpha-\left(1+t_{a}\right)\left(c^{g}+t_{u}\right)\right) .
$$

Finally, substituting Eqs. (27) and (29) into (1), the welfare level can be expressed in terms of the cost cutoff.

$$
W^{g}=L U=L-L T+\frac{L}{2 \eta}\left[\alpha-\left(1+t_{a}\right)\left(c^{g}+t_{u}\right)\right]\left[\alpha-\left(1+t_{a}\right)\left(\frac{\theta+1}{\theta+2} c^{g}+t_{u}\right)\right] .
$$

### 4.2 Decentralization

Using these policy instruments-an ad valorem production tax, a per-unit production tax, and a lump-sum tax on consumers-I prove the following proposition.

Proposition 1. The efficient outcome can be achieved though the following policies:

- an ad valorem production tax $t_{a}^{*}=1$;
- a per-unit subsidy $t_{u}^{*}=-\frac{c^{o}}{2}$;
- a limp-sum tax on consumers $T^{*}=\frac{\left(\alpha-c^{\circ} c^{o}\right.}{\eta(\theta+2)}$,
where $t_{a}^{*}, t_{u}^{*}$, and $T^{*}$ are the efficient ad valorem tax, per-unit tax, and lump-sum tax on consumers, respectively.

By combining an ad valorem production tax and a per-unit production subsidy, the firstbest outcome can be achieved without adopting a firm-specific tax/subsidy. This is because a combination of these policy instruments can create a tax/subsidy system similar to a firmspecific per-unit production tax/subsidy shown by Nocco et al. (2014). ${ }^{3}$ In fact, in the present study, the net production tax imposed on one unit of good produced by a firm with $\operatorname{cost} c$ when $t_{a}=t_{a}^{*}$ and $t_{u}=t_{u}^{*}$ is

$$
\begin{equation*}
\frac{t_{a}^{*}}{1+t_{a}^{*}} p^{g}(c)+t_{u}^{*}=\frac{1}{2}\left(c-c^{o}\right), \tag{31}
\end{equation*}
$$

which implies that the net production tax for one unit of good is increasing in the cost $c$, being negative (non positive) for more productive firms with $c \in\left[0, c^{o}\right]$ and positive for less productive firms with $c \in\left(c^{o}, c_{M}\right]$. Eq. (31) is similar in form to the firm-specific per-unit production tax used by Nocco et al. (2014) to achieve the first-best outcome: $t_{u}^{N O S}(c)=c-c^{o}$.

### 4.3 The role of the ad velorem and per-unit taxes

To better understand the characteristics of the efficient policy indicated by Proposition 1, I examine the role of ad valorem and per-unit production taxes on distortions in terms of product selection, product mix, and product variety.

An ad valorem production tax is needed to improve the product selection and the product mix distortions. Differentiating Eqs. (28) and (27) with respect to $t_{a}$, I obtain

$$
\frac{d c^{g}}{d t_{a}}=-\frac{c^{g}}{(\theta+2)\left(1+t_{a}\right)}<0, \quad \frac{d q^{g}(c)}{d t_{a}}=\frac{L}{2 \gamma}\left(\frac{\theta+1}{\theta+2} c^{g}-c\right) \begin{cases}\geq 0 & c \in\left[0, \frac{\theta+1}{\theta+2} c^{g}\right] \\ <0 & c \in\left(\frac{\theta+1}{\theta+2} c^{g}, c^{g}\right]\end{cases}
$$

which indicates that an increase in $t_{a}$ decreases the cost cutoff and increases (decreases) the output level of more (less) productive firms. Accordingly, from Lemma 1, an increase in the ad valorem production tax improve the product selection and the product mix distortions

[^2]when $t_{a}$ is sufficiently low. It is straightforward to show that these distortions are removed by the efficient ad valorem tax: $c^{g}=c^{o}$ and $q^{g}(c)=q^{o}(c)$ when $t_{a}=t_{a}^{*}$. An increase in an ad valorem production tax also affects the number of varieties. Differentiating Eq. (29) with respect to $t_{a}$, I obtain
$$
\frac{d N^{g}}{d t_{a}}=-\frac{2 \gamma(\theta+1)\left((\theta+1) \alpha+\left(1+t_{a}\right) t_{u}\right)}{\eta(\theta+2)\left(1+t_{a}\right)^{2} c^{g}}<0
$$

An increase in $t_{a}$ decreases the number of varieties. When the efficient ad valorem tax is imposed, the number of varieties is below that in the first-best: $N^{g}<N^{o}$ when $t_{a}=t_{a}^{*}$ and $t_{u}=0$. Thus, while the efficient ad valorem tax can remove the product selection and the product mix distortions, it causes the product variety distortion due to an excessive reduction in firm entry.

A per-unit production subsidy is then needed to to improve the product variety distortion. An increase in a per-unit production tax affects the product variety distortion without changing the cost cutoff and the output level: $d c^{g} / d t_{u}=0$ and $d q^{g}(c) / d t_{u}=0 .{ }^{4}$ Differentiating Eq. (29) with respect to $t_{u}$, I obtain

$$
\frac{d N^{g}}{d t_{u}}=-\frac{2 \gamma(\theta+1)}{\eta c^{g}}<0
$$

A decrease in $t_{u}$ increases the number of varieties. Thus, the efficient per-unit subsidy removes the product variety distortion when $t_{a}=t_{a}^{*}: N^{g}=N^{o}$ when $t_{u}=t_{u}^{*}$ and $t_{a}=t_{a}^{*}$.

## 5 Conclusion

This study analyzes the efficient policy based on the closed economy setting of Melitz and Ottaviano (2008), a model of monopolistic competition with firm heterogeneity and variable markups. In this model, the market outcome departs from the first-best outcome in terms of product selection, product mix, and product variety due to markup pricing.

The conclusions of this study are summarized as follows. The first-best outcome can be decentralized through an ad valorem production tax and a per-unit production subsidy-

[^3]these are uniform across firms-accompanied by a lump-sum tax on consumers. This result is different from that of Nocco et al. (2014), who characterize the first-best policy as a firm-specific per-unit production tax/subsidy, a lump-sum entry tax, and a lump-sum tax on consumers. When the per-unit production subsidy cannot be differentiated across firms, they consider the second-best scenario. In the present study, I show that policymakers do not need to have any way of observing the level of productivity of each firm to make the market outcome achieve the first-best outcome. Ad valorem and per-unit taxes/subsidies play different roles on each distortion. As a result, combining an ad valorem production tax and a per-unit production subsidy can create a tax/subsidy system similar to a firm-specific per-unit production tax/subsidy, which enables efficient resource allocation to be achieved without a firm-specific tax/subsidy.

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[^1]:    ${ }^{1}$ As Dhingra and Morrow (2019) show, the market outcome is first-best under CES preferences, with demand-side elasticity determining how resources are misallocated.
    ${ }^{2}$ Using the multi-country setting of Melitz and Ottaviano (2008), Nocco et al. (2019) also show that free trade allocation of resources fails to be efficient.

[^2]:    ${ }^{3}$ See Section 5 of their paper

[^3]:    ${ }^{4}$ In a version of this model that regulates the entry of firms, both ad valorem and per-unit taxes/subsidies affect the cost cutoff, the output level, and the number of varieties. Even in the case, first-best outcome can be achieved by combining these taxes/subsidies without adopting a firm-specific tax/subsidy.

