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Michi NISHIHARA

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Graduate School of Economics
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

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Michi NISHIHARA†

Abstract

Many startups have recently opted for sellout to large firms with great market power and profit margins. This paper attempts to understand and guide such startup sellout decisions by developing a model in which the firm reacts to random approaches of high- and low-type acquirers. Optimally, the startup takes either a high-price strategy—accepting only a high-type acquirer in a good economic state— or a flexible strategy—accepting a high-type acquirer in an intermediate economic state and either type in a good economic state— based on a tradeoff between sellout pricing and timing efficiency. With asymmetric information, where the acquirer types are unobservable, the startup accepts a high-price acquisition more eagerly and a low-price acquisition more restrictively to reduce the acquirer's information rent. Then, asymmetric information increases the probability of a high-price sellout and delays the sellout. A model analysis shows that the market parameters as well as anticipation of acquirers (i.e., the acquirers' arrival rate, valuation, and transparency) greatly affect the sellout price, sellout timing, firm value, and stock price reaction.

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†Corresponding Author. Graduate School of Economics, Osaka University, 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, Japan, E-mail: nishihara@econ.osaka-u.ac.jp, Phone: 81-6-6850-5242, Fax: 81-6-6850-5277

1 Introduction

Over the last few decades, industry concentration has grown, increasing the gap between the profitability of giant firms and others (e.g., Grullon, Larkin, and Michaely (2019), Autor, Dorn, Katz, Patterson, and Reenen (2020)). For instance, Amazon, Apple, Google, Facebook, and Microsoft, the "tech giants" in the information technology industry, have greatly increased their market power and profit margins. In such circumstances, many startups would prefer to be acquired by large firms, rather than developing themselves as independent firms through initial public offerings and growth investments (see Poulsen and Stegemoller (2008), Gao, Ritter, and Zhu (2013)). For example, Instagram chose to be sold to Facebook in order to expand its business in 2012. This study tries to understand and guide such startup sellout decisions by developing a real options model as follows.[1]

Assume that a startup with stochastic cash flows has an option for sellout to a large acquirer. The acquirer can pay a higher price than the startup's standalone value because it can extract more cash flows from the startup's assets (e.g., synergies, market power, and economies of scope and scale). The startup times the sellout considering the irreversibility and sunk costs of the process. Moreover, the model assumes two novel features. The first is the illiquidity of sellout opportunities stemming from frictions in the merger and acquisition (M&A) searching and matching process. I assume that the sellout is feasible only at the acquirers' arrival time modeled by Poisson jump times.[2] The second feature is the heterogeneity of acquirers. Two acquirer types (hereafter called high and low types) are considered, where the high type extracts more cash flows from the acquired assets than the low type.

In the model, I derive the startup's sellout policy and firm value analytically. The results are presented in two cases. In the first case, the startup accepts only a high type when the state variable (i.e., the cash flow level) lies above a certain threshold. This policy is called the high-price strategy. The firm value and policy in the high-price strategy coincide with those in the benchmark model with only high types. The high-price strategy pursues the pricing rather than timing efficiency. In fact, the startup forgoes low-price acquisitions even when the state variable is high. In the second case, the sellout policy involves two thresholds (the high-price and low-price sellout thresholds). The startup accepts only a high type when the state variable lies between the two thresholds, and accepts either type when the state variable lies above the low-price sellout threshold. This policy is called the flexible strategy. The flexible strategy pursues the timing rather than pricing efficiency. In fact, the startup chooses sellout for the high state variable even when the sellout price is low. In the flexible strategy, both the sellout time and price are unpredictable; this is consistent with the

[1] The real options approach is now widely accepted as an alternative to the net present value method, especially in decisions involving uncertainty and irreversibility (e.g., see Dixit and Pindyck (1994) and Trigeorgis and Tsekrekos (2018)).

[2] The model considers the sellout process through private negotiations, where potential bidders approach the target sequentially. Firms are sold in their growth stage through private negotiations rather than auction (see Fidrmuc, Roosenboom, Paap, and Teunissen (2012)).

empirical evidence of high uncertainty in M&A deals (see Cornett, Tanyeri, and Tehranian (2011) and Wang (2018)).

The tradeoff between the sellout pricing and timing efficiency determines whether the startup takes the high-price or flexible strategies. Notably, I derive the explicit condition and show that the startup is more likely to take the high-price strategy when the high type's arrival rate and growth rate of the state process are relatively high. A numerical analysis of the model yields the following comparative statics results. With lower acquirers' arrival rate, the startup accepts an acquisition more eagerly, but this sellout policy does not fully offset the decrease in liquidity of sellout opportunities. In fact, a lower arrival rate delays the sellout and decreases the firm value. Contrary to the well-known effect of the growth rate, the low-price sellout threshold increases with higher growth rate. The startup restricts a low-price acquisition more severely because a higher growth rate increases the advantage of high-price sellout. The startup with higher growth rate and lower volatility is more likely to be acquired by a high type (i.e., firm with higher efficiency). To my knowledge, these are novel predictions that are yet to be tested in the empirical M&A literature.

I extend the baseline results to studies with asymmetric information, where startups are unaware of the acquirer types. The asymmetric information model is relevant when the acquirers are less transparent. The startup with asymmetric information faces the risk of sellout to a high type at a low price. Then, in contrast to the symmetric information case, the high-type acquirer gains information rent in acquisition. This result is consistent with the empirical evidence of low acquisition premiums of less transparent acquirers (e.g., Bargeron, Schlingemann, Stulz, and Zutter (2008), Battigalli, Chiarella, Gatti, and Orlando (2017), and Golubov and Xiong (2020)). The startup decreases the high-price sellout threshold and increases the low-price sellout threshold to reduce the information rent.[3] Compared to the symmetric information case, this distortion increases the high-price sellout probability but delays the sellout timing, which results in social inefficiency. This social loss due to asymmetric information tends to be highest when asymmetric information changes the startup strategy from the flexible strategy to the high-price strategy.

Furthermore, I examine the firm value jump at the sellout time. This can be interpreted as acquisition premium or target stock price reaction. A sellout leads to a positive jump in firm value from the unpredictability of sellout timing (and pricing). This finding is consistent with the empirical evidence of positive target stock price reactions (e.g., Huang and Walkling (1987), Boone and Mulherin (2007), and Hackbarth and Morellec (2008)). In the flexible strategy case, the jump size is not monotonic relative to the state variable because the startup accepts a low-price acquisition for a high state variable. Similarly, the jump size is not monotonic relative to the arrival rates because the sellout price can change with the arrival rates. These results show that the target's stock price reaction depends on the interactions between the sellout timing and price.

Finally, I explain the contribution of this study to the related literature. From the methodolog-

[3]This finding is consistent with the intertemporal price discrimination literature (e.g., Stokey (1979), Landsberger and Meilijson (1985), Nishihara and Shibata (2019)). In different settings, these studies show that a seller delays low-price sales to reduce the information rent to high types.

ical perspective, I solve the optimal stopping problem constrained to Poisson jump times. Dupuis and Wang (2002) first solves this type of constrained optimal stopping problem. By extending Dupuis and Wang (2002)'s model, Hugonnier, Malamud, and Morellec (2015a), Hugonnier, Malamud, and Morellec (2015b), and Morellec, Valta, and Zhdanov (2015) study the illiquidity effects of fund-raising on investment, while Nishihara and Shibata (2021) investigate the illiquidity effects of sellout opportunities on bankruptcy. However, all these studies assume the homogeneity of illiquid option exercise opportunities. To my knowledge, this is the first study to solve the problem with two option types arriving randomly at different arrival rates. I also clarify the relationship between the homogeneous and heterogeneous problems by deriving the explicit condition under which a two-type problem is reduced to a one-type problem. Note that no asymmetric information problem occurs for the homogeneous problem. This study first solves the asymmetric information problem.

The study also contributes to the real options literature on M&A. Lambrecht (2004) develops friendly merger and hostile takeover models, while Morellec and Zhdanov (2008) study the bidder competition effects on takeover and capital structure decisions. Lukas, Pereira, and Rodrigues (2019) show that serial acquisition is optimal in certain situations. As regards the M&A of startups, Nishihara (2017) examines the startup choice between sellout and initial public offerings, whereas Ferreira and Pereira (2021) examine the entry and exit (trade sale) decisions of venture capitalists. In these studies with no frictions in the M&A searching and matching process, the M&A timing is predictable, and hence, there is no shock in the target firm value at the M&A time. In contrast, this study shows that the illiquidity of sellout opportunities distorts the timing decision and leads to a shock at the sellout time. Nishihara and Shibata (2021) examine how illiquid sellout opportunities affect the capital structure and default decisions, although they assume homogeneous acquirers. By assuming the heterogeneity of acquirers, this study examines the timing as well as pricing decisions and the effects of asymmetric information.

The remainder of this paper is organized as follows. Section 2 introduces the model setup. Sections 3.1 shows the benchmark model results with homogeneous acquirers. Sections 3.2 and 3.3 present the main model results with heterogeneous acquirers under symmetric and asymmetric information, respectively. Section 4 analyzes the results numerically and explains the implications. Section 5 concludes the paper.

2 Model setup

Consider a startup (or an entrepreneur in a certain section, with technology, patent, etc.) operating and generating continuous streams of cash flows $X(t)$, where $X(t)$ follows a geometric Brownian motion,

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}B(t)\ (t > 0), \quad X(0) = x;$$

here, $B(t)$ denotes the standard Brownian motion defined in a filtered probability space, $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\})$, and $\mu, \sigma(> 0)$ and $x(> 0)$ are constants. The net present value (NPV) of cash flows at time 0 is equal to $\pi(x) = x/(r - \mu)$.

Infinitely many high- and low-type acquirers value the firm as $a_H\pi(X(t))-I_a$ and $a_L\pi(X(t))-I_a$, respectively, where $a_H > a_L > 1$ and $I_a \geq 0$.[4] Throughout this study, H and L stand for high and low types, respectively, and $\Delta a = a_H - a_L$. The assumption that $a_i > 1$ indicates that acquirers extract more cash flows from the business than the startup can. The synergies associated with acquisition can be included in the parameters a_i $(i = H, L)$, while I_a represents the sunk cost in acquisition. High and low types approach the startup at Poisson jump times with arrival rates λ_H and λ_L, respectively. For model tractability, I assume that the Poisson arrival processes and $X(t)$ are independent.[5]

The startup pays the transaction cost $I_b(> 0)$ when acquired. I denote the total costs by $I = I_a + I_b$. When the startup is sold at price $a_i\pi(X(t)) - I_a$, it gains $a_i\pi(X(t)) - I$ instead of losing the future cash flows' NPV $\pi(X(t))$. The total payoff then becomes $(a_i - 1)\pi(X(t)) - I$. I assume that the startup is risk-neutral and chooses between accepting and rejecting an acquirer to maximize the expected firm value. If the firm rejects an acquirer, it has to wait until another acquirer arrives.

This study examines two types of information structures. In Section 3.2, I explore the symmetric information case, where the startup can observe the acquirer types. In Section 3.3, I explore the asymmetric information case, where the startup cannot observe the acquirer types. In reality, asymmetric information matters more when the acquirers have fewer business relationships with the startup. It also matters when the acquirers are unlisted and have lower transparency. In case of symmetric information, the startup can be sold at the fair price $a_i\pi(X(t)) - I_a$ contingent on the acquirer type i. In contrast, in case of asymmetric information, the startup cannot post a price contingent on the acquirer type. For simplicity, I assume that the startup makes a take-it-or-leave-it price offer. An acquirer would purchase the startup if the offer price is not higher than its valuation, and would leave it if the price is higher than its valuation. In case of asymmetric information, a high type can obtain information rent by purchasing the startup at the low price $a_L\pi(X(t)) - I_a$ (for details, see Section 3.3).

The model is based on the following empirical observations. Startups tend to sell their growing businesses to large firms because large firms can extract more profits from them by virtue of their synergies, market power, economies of scale and scope, speed in bringing products to the market, and so on (e.g., Poulsen and Stegemoller (2008), Gao, Ritter, and Zhu (2013), Grullon, Larkin, and Michaely (2019)).[6] A startup in the growth stage is typically sold through private negotiations, with the potential bidders approaching the target firm sequentially, rather than through auctions

[4]I can solve the problem with more than two types in the same manner. In that case, the number of startup strategies become more than two, and hence, I cannot derive an explicit condition for each strategy. The main implications in this paper will remain unchanged.

[5]Hugonnier, Malamud, and Morellec (2015a), Hugonnier, Malamud, and Morellec (2015b), Morellec, Valta, and Zhdanov (2015), and Nishihara and Shibata (2021) also assume that the economic state and Poisson processes are independent. In fact, one cannot analytically derive any solution without independence.

[6]For a well-known example, Instagram was sold to Facebook in 2012. The sellout process is private negotiation rather than auction.

(see Fidrmuc, Roosenboom, Paap, and Teunissen (2012)). Business sellout transactions, unlike the sellout of financial assets, involve high transaction costs, irreversibility, illiquidity, and price uncertainty. The real options model captures the sunk cost and irreversibility in real asset trading. By adopting the Poisson arrival processes of acquirers, the model captures the illiquidity (i.e., searching and matching frictions) of the sellout being feasible only when the firm meets a counterparty.[7] The model incorporates the heterogeneity of acquirers and asymmetric information, to capture the transaction price uncertainty.

Finally, I explain how the model differs technically from previous M&A real options models. In most previous studies, the acquisition is predictable because the timing is endogenously determined by the acquirer, target firm, or both firms. For example, in the takeover models of Lambrecht (2004) and Morellec and Zhdanov (2008), the target can precisely predict the acquirer's takeover time and optimize the terms (i.e., the price). Hackbarth and Morellec (2008) investigate the asymmetric information between the firm insiders and outside investors, although the merger timing is predictable to the target firm and acquirer managers. Unlike these papers, this study assumes that the target startup optimizes the price but cannot predict the acquirers' arrival time, modeling the acquirers' arrival as an unpredictable and exogenous event. This assumption holds true when friction exists in the searching and matching process. In fact, some firms fail to reach an agreement in M&A negotiations. This assumption is also plausible when the acquirers' arrival timing is determined by factors unobservable to the target startup (e.g., acquirers' competitive strategy, business environment, and long-term plan). Although Nishihara and Shibata (2021) model the acquirers' arrival as an unpredictable and exogenous event, their model lacks acquirer heterogeneity (i.e., price uncertainty). Many papers (e.g., Boone and Mulherin (2007), Cornett, Tanyeri, and Tehranian (2011), and Wang (2018)) empirically show high uncertainty in M&A deals, and this study is consistent with them.

3 Model Solutions

3.1 Homogeneous acquirers

As benchmark, this subsection considers a one-type acquirer arrival setup. I fix $i \in \{H, L\}$, and assume that $\lambda_i > 0$ and $\lambda_j = 0$, where $j = \{H, L\} \setminus \{i\}$. Technically, the firm's optimal sellout timing problem can be regarded as an optimal stopping problem constrained to Poisson jump times, which is solved in Dupuis and Wang (2002). I denote the firm value when an acquirer is absent as $V_i(x)$. When an acquirer arrives, the firm value becomes $\max\{a_i\pi(X(t)) - I, V_i(X(t))\}$, because the firm optimizes the choice between accepting and forgoing the acquirer. As proved in Dupuis and Wang (2002), the optimal policy becomes a threshold policy, that is, $\inf\{t \geq 0 \mid X(t) \geq x_i\}$,

[7] Rhodes-Kropf and Robinson (2008) examine an M&A market with search frictions similar to those in this study, but in different contexts.

and $V_i(x)$ satisfies the ordinary differential equations (ODEs)

$$\mu x V_i'(x) + 0.5\sigma^2 x^2 V_i''(x) + x = rV_i(x) \quad (0 < x < x_i), \tag{1}$$

$$\mu x V_i'(x) + 0.5\sigma^2 x^2 V_i''(x) + \lambda_i(a_i\pi(x) - I - V_i(x)) + x = rV_i(x) \quad (x > x_i), \tag{2}$$

where $\lambda_i(a_i\pi(x) - I - V_i(x))$ in (2) corresponds to the fact that $V_i(x)$ changes to $a_i\pi(x) - I$ with probability $\lambda_i \mathrm{d}t$ in infinitesimal time interval $\mathrm{d}t$ in the sellout region $x > x_i$.[8] Region $x < x_i$ is the operation region, where the firm operates normally, whereas region $x \geq x^i$ is the sellout region, where the firm aims for sellout. State process $X(t)$ can cross the threshold x_i many times because the startup may not meet an acquirer in the sellout region.

As shown in Dupuis and Wang (2002), $V_i(x)$ is continuously differentiable at x_i[9] and satisfies boundary conditions

$$V_i(x_i) = a_i\pi(x_i) - I, \tag{3}$$

$$\lim_{x\to 0} V_i(x) = 0, \tag{4}$$

$$\lim_{x\to\infty} V_i(x)/x < \infty. \tag{5}$$

Condition (3) means that the startup is indifferent to whether to accept or reject a proposal at $X(t) = x_i$ (i.e., the optimality of x_i). Condition (4) is a standard condition arising from the fact that if x goes to zero, the stochastic process $X(t)$ will stay at zero, and hence, $V_i(x_i)$ goes to 0 (e.g., Dixit and Pindyck (1994)). Condition (5) occurs because the firm value is bounded by $a_i\pi(x)$.

Throughout the paper, I define the notations

$$\beta_y = 0.5 - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - 0.5\right)^2 + \frac{2(r+y)}{\sigma^2}} \ (> 1), \tag{6}$$

$$\gamma_y = 0.5 - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - 0.5\right)^2 + \frac{2(r+y)}{\sigma^2}} \ (< 0), \tag{7}$$

for an arbitrary parameter, $y > 0$, and simplify the notations by $\beta = \beta_0$ and $\gamma = \gamma_0$ for $y = 0$. I also define the zero-NPV thresholds $x_i^{NPV} = (r - \mu)I/(a_i - 1)$ for $i = H, L$. By solving (1) and (2) with boundary conditions (3)–(5), I have the following proposition. For proof of the proposition, see Appendix A.

Proposition 1 *The firm value $V_i(x)$ $(i = H, L)$ is given by*

$$V_i(x) = \begin{cases} \pi(x) + A_{1,i}x^{\beta} & (x < x_i), \\ \pi(x) + \dfrac{\lambda_i(a_i - 1)\pi(x)}{r + \lambda_i - \mu} - \dfrac{\lambda_i I}{r + \lambda_i} + A_{2,i}x^{\gamma_{\lambda_i}} & (x \geq x_i), \end{cases} \tag{8}$$

[8] ODEs (1) and (2) are similar to those in the model with operating costs and temporary suspension, where the firm can freely switch between operation and suspension (see Chapter 6.2 of Dixit and Pindyck (1994)).

[9] This is not the smooth pasting (i.e., optimality) condition in Dixit and Pindyck (1994), but similar to the piecewise C^2 property of the value function in Morellec, Valta, and Zhdanov (2015). This condition is also used in Chapter 6.2 of Dixit and Pindyck (1994). Technically, this property follows from Theorem 4.4.9 in Karatzas and Shreve (1998).

where the sellout threshold x_i *and coefficients* $A_{1,i}, A_{2,i}$ *are defined by*

$$x_i = \frac{(r+\lambda_i-\mu)((\beta-\gamma_{\lambda_i})r+\beta\lambda_i)x_i^{NPV}}{(r+\lambda_i)((\beta-\gamma_{\lambda_i})(r-\mu)+(\beta-1)\lambda_i)}, \tag{9}$$

$$A_{1,i} = \frac{(a_i-1)\pi(x_i)-I}{x_i^{\beta}}, \tag{10}$$

$$A_{2,i} = \frac{1}{x_i^{\gamma_{\lambda_i}}}\left(\frac{(a_i-1)x_i}{r+\lambda_i-\mu}-\frac{rI_i}{r+\lambda_i}\right). \tag{11}$$

The startup's optimal sellout policy is to accept an acquisition only if the offer arrives at time t satisfying $X(t) \geq x_i$. As in the standard real options argument, if sunk cost I is present, the firm waits for a sufficiently good economic environment for sellout. By (9), I can show that $\lim_{\lambda_i\to 0} x_i = x_i^{NPV}$ and $\lim_{\lambda_i\to\infty} x_i = x_i^{NPV}\beta/(\beta-1)$. In fact, the sellout threshold x_i monotonically increases in the arrival rate λ_i. Intuitively, this means that in case of lower acquisition offer frequency, the startup accepts an acquisition offer more eagerly in order to alleviate the illiquidity effect. However, as the numerical examples in Section 4 show, this earnest sellout policy does not fully offset the illiquidity effects; in fact, the sellout timing is delayed with lower λ_i. Note that the sellout time is not the same as, but comes after, the first hitting time to the sellout threshold x_i, because the firm has to wait for a Poisson jump.[10] Thus, a lower λ_i decreases x_i but delays the sellout time.

Next, I explain the firm value $V_i(x)$ in Proposition 1. From the problem definition, $V_i(x)$ monotonically increases in λ_i. By (8), I can show that $\lim_{\lambda_i\to 0} V_i(x) = \max\{\pi(x), a_i\pi(x)-I\}$ (i.e., the NPV) and that $\lim_{\lambda_i\to\infty} V_i(x)$ agrees with the real options value in the liquid model. That is, a lower acquisition offer frequency decreases the firm value by the embedded sellout option value. This is consistent with the empirical observations of Song and Walkling (2000) and Cornett, Tanyeri, and Tehranian (2011). Next, I explain expression (8) in more detail. For $x < x_i$, $\pi(x)$ is the NPV of perpetual cash flows, and $A_{1,i}x^{\beta}$ represents the value of the sellout option. In this region, where the firm does not accept an acquirer, $V_i(x) > a_i\pi(x) - I$ holds. For $x \geq x_i$, the first three terms represent the expected payoff of sellout as soon as an acquirer arrives, and the last term $A_{2,i}x^{\gamma_{\lambda_i}}$ represents the value of the option to postpone sellout when an acquirer arrives for $X(t) < x_i$. In this region, where the firm accepts an acquirer, $V_i(x) \leq a_i\pi(x) - I$ holds.

Proposition 1 shows how the illiquidity of sellout opportunities affects the startup's sellout policy and option value. However, owing to the homogeneity of acquirers, this proposition entails no implication about the startup's pricing policy (i.e., which type acquires the startup and how much the acquirer pays). In the next section, I incorporate acquirer heterogeneity into the model and explore the startup's timing and pricing decisions simultaneously.

3.2 Symmetric information case

This subsection assumes that $\lambda_i > 0$ ($i = H, L$), and that the acquirer types are observable to the startup. I denote the firm value when an acquirer is absent as $V(x)$. When an i-type acquirer

[10]Mathematically, the probability of a Poisson jump occurring at the first hitting time is zero.

arrives, the firm value becomes $\max\{a_i\pi(X(t)) - I, V(X(t))\}$, because the startup optimizes the choice between accepting and not accepting the i-type acquirer. As in Dupuis and Wang (2002), I can show that the optimal policy is a threshold policy. In other words, the acceptance region for an i-type acquisition becomes $\{x \geq x_i^*\}$, where $x_H^* < x_L^* \leq \infty$, and $V(x)$ satisfies ODEs:

$$\mu x V'(x) + 0.5\sigma^2 x^2 V''(x) + x = rV(x) \quad (0 < x < x_H^*), \tag{12}$$

$$\mu x V'(x) + 0.5\sigma^2 x^2 V''(x) + \lambda_H(a_H\pi(x) - I - V(x)) + x = rV(x) \quad (x_H^* < x < x_L^*), \tag{13}$$

$$\mu x V'(x) + 0.5\sigma^2 x^2 V''(x) + \bar{\lambda}(\bar{a}\pi(x) - I - V(x)) + x = rV(x) \quad (x > x_L^*), \tag{14}$$

where I define $\bar{\lambda} = \lambda_H + \lambda_L$ and $\bar{a} = (\lambda_H a_H + \lambda_L a_L)/\bar{\lambda}$. In (13), $\lambda_H(a_H\pi(x) - I - V(x))$ corresponds to the fact that $V(x)$ becomes $a_H\pi(x) - I$ with probability $\lambda_H \mathrm{d}t$ in infinitesimal time interval $\mathrm{d}t$, and in (14), $\bar{\lambda}(\bar{a}\pi(x) - I - V(x))$ corresponds to the fact that $V(x)$ becomes $a_H\pi(x) - I$ with probability $\lambda_H \mathrm{d}t$ and $a_L\pi(x) - I$ with probability $\lambda_L \mathrm{d}t$ in infinitesimal time interval $\mathrm{d}t$. As in $V_i(x)$ in Section 3.1, $V(x)$ is continuously differentiable at x_i^* by Theorem 4.4.9 in Karatzas and Shreve (1998), and $V(x)$ satisfies boundary conditions

$$V(x_i^*) = a_i\pi(x_i^*) - I \quad (i = H, L), \tag{15}$$

$$\lim_{x\to 0} V(x) = 0, \tag{16}$$

$$\lim_{x\to\infty} V(x)/x < \infty. \tag{17}$$

Condition (15) corresponds to condition (3) in Section 3.1, meaning that the startup is indifferent to whether to accept or not accept an i-type acquirer at $X(t) = x_i^*$ (i.e., the optimality of x_i^*). Conditions (16) and (17) correspond to conditions (4) and (5) in Section 3.1.

If

$$V_H(x) \geq a_L\pi(x) - I \quad (x \geq 0), \tag{18}$$

which is equivalent to condition (19) in the following proposition, the startup takes the high-price strategy, that is, $x_L^* = \infty$, where the startup always rejects a low type. This is because the firm cannot increase the firm value beyond $V_H(X(t))$ by gaining payoff $a_L\pi(X(t)) - I$ for any $X(t)$. In this case, the firm value $V(x)$ and sellout threshold x_H^* agree with $V_H(x)$ and x_H in Proposition 1. Otherwise, the startup would take the flexible strategy, that is, $x_L^* < \infty$, where the startup accepts even a low type for $X(t) \geq x_L^*$. This is because the firm can increase the firm value beyond $V_H(X(t))$ by accepting a low type. In this case, by solving ODEs (12)–(14) with boundary conditions (15)–(17), I have the following proposition. For proof of the proposition, see Appendix B.

Proposition 2 *If condition*

$$a_L - 1 \leq \frac{\lambda_H(a_H - 1)}{r + \lambda_H - \mu} \tag{19}$$

holds, the startup takes the high-price strategy, that is, $x_L^* = \infty$. *The firm value* $V(x)$ *and sellout threshold* x_H^* *agree with* $V_H(x)$ *and* x_H *in Proposition 1.*

If condition (19) does not hold, the startup takes the flexible strategy, that is, $x_L^* < \infty$. The firm value $V(x)$ is given by

$$V(x) = \begin{cases} \pi(x) + B_1 x^{\beta} & (x < x_H^*), \\ \pi(x) + \dfrac{\lambda_H (a_H - 1)\pi(x)}{r + \lambda_H - \mu} - \dfrac{\lambda_H I}{r + \lambda_H} + B_2 x^{\beta_{\lambda_H}} + B_3 x^{\gamma_{\lambda_H}} & (x \in [x_H^*, x_L^*)), \\ \pi(x) + \dfrac{\bar{\lambda}(\bar{a} - 1)\pi(x)}{r + \bar{\lambda} - \mu} - \dfrac{\bar{\lambda} I}{r + \bar{\lambda}} + B_4 x^{\gamma_{\bar{\lambda}}} & (x \geq x_L^*), \end{cases} \tag{20}$$

where coefficients B_i $(i = 1, 2, 3, 4)$ and sellout thresholds x_i^* $(i = H, L)$ are as defined in Appendix B.

In case of heterogeneity of acquirers, the startup faces the dilemma of choosing sellout at a high price as well as with good timing efficiency. By Proposition 2, the startup's policy can be classified into two cases, the high-price strategy and flexible strategy based on the tradeoff between sellout pricing and timing efficiency. In the high-price strategy, the startup forgoes low types and waits for a high type even in case of very good economic environment. In this sense, the high-price strategy pursues pricing efficiency rather than timing efficiency. In contrast, in the flexible strategy, the startup accepts even a low-price acquisition in a very good economic environment (i.e., $X(t) \geq x_L^*$). That is, the flexible strategy purses timing efficiency rather than pricing efficiency.

Note that Proposition 2 shows the explicit condition (19) under which the startup takes the high-price strategy. Condition (19) can be rewritten as

$$\frac{\bar{\lambda}(\bar{a} - 1)}{r + \bar{\lambda} - \mu} \leq \frac{\lambda_H (a_H - 1)}{r + \lambda_H - \mu}, \tag{21}$$

where the left- and right-hand sides of (21) correspond to the second terms of equation (20) for $x \geq x_L^*$ and equation (8) for $x \geq x_H$, respectively. Condition (21) means that the discounted present value of $a_H - 1$ at a high type's first arrival time is not lower than the discounted present value of $\bar{a} - 1$ at either type's first arrival time. From (19), higher a_H and λ_H as well as lower a_L and $r - \mu$ are likely to lead to the high-price strategy. The reason is as follows. Higher a_H and lower a_L increase the advantage of high-price sellout over low-price sellout. Lower $r - \mu$ also increases the advantage of high-price sellout, because $r - \mu$ is the denominator of $\pi(x)$. Higher λ_H decreases the timing inefficiency of waiting for only a high type. Thus, with higher a_H and λ_H as well as lower a_L and $r - \mu$, the startup purses the pricing efficiency stemming from the high-price strategy.

Next, I discuss the flexible strategy and its value, because the high-price strategy has already been explained after Proposition 1. In the flexible strategy case, the startup accepts only a high type in an intermediate economic environment, that is, $X(t) \in [x_H^*, x_L^*)$. In this region, the startup pursues the pricing efficiency stemming from high-price sellout. However, if $X(t)$ rises beyond x_L^*, the startup accepts even a low type because the timing efficiency from accepting either type dominates the pricing efficiency. In this region, the sellout timing as well as sellout price is unpredictable because it depends on the acquirer type. This result is consistent with the empirical evidence of high uncertainty in M&A deals (e.g., Cornett, Tanyeri, and Tehranian (2011) and Wang (2018)).

I cannot derive closed-form expressions of x_i^* $(i = H, L)$, but by the monotonic increase of $V(x)$ with respect to λ_i $(i = H, L)$ and $x_i^* = \inf\{x \geq 0 \mid V(x) \leq a_i\pi(x) - I\}$, I can readily show that x_i^* $(i = H, L)$ monotonically increases in λ_i $(i = H, L)$. This means that with lower frequency of acquisition offers, the startup is more eager to accept an acquisition and alleviate the illiquidity effect. However, as shown in the numerical examples in Section 4, this earnest sellout policy does not fully offset the illiquidity effects; in fact, the sellout timing is delayed with lower λ_i. These results are in line with Proposition 1.

Now, I explain the firm value (20) in Proposition 2. For $x < x_H^*$, term $B_1 x^\beta$ stands for the value of sellout option. In this case, where the firm rejects any acquirer, $V(x) > a_H\pi(x) - I$ holds. For $x \in [x_H^*, x_L^*)$, the first three terms represent the expected payoff of sellout to the high type that arrives first. In this case, where the firm accepts only a high type, $a_L\pi(x) - I < V(x) \leq a_H\pi(x) - I$ holds. The terms $B_2 x^{\beta_{\lambda_H}} + B_3 x^{\gamma_{\lambda_H}}$ represent the option value to change the policy when $X(t)$ passes either x_H^* or x_L^* before the arrival of a high type. For $x \geq x_L^*$, the first three terms represent the expected payoff of sellout to the acquirer that arrives first. In this case, where the firm accepts either type, $V(x) \leq a_L\pi(x) - I$ holds. The last term $B_4 x^{\gamma_{\lambda_H}}$ represents the option value to change the policy when $X(t)$ passes x_L^* before the acquirer's arrival.

From a technical viewpoint, Proposition 2 extends the results of Dupuis and Wang (2002) (or equivalently Proposition 1) to the case with heterogeneity in option payoffs. Proposition 2 also clarifies the relationship between heterogeneous and homogeneous problems by deriving the explicit condition (19) under which the heterogeneous problem reduces to the homogeneous problem. The solution in the flexible strategy case is novel, but is similar to that in Morellec, Valta, and Zhdanov (2015) and Nishihara and Shibata (2021). In the investment and financing model of Morellec, Valta, and Zhdanov (2015), for intermediate levels of the state variable, a firm invests only when a private financing opportunity is available, but invests with public debt financing when the state variable exceeds a certain threshold. In the bankruptcy model of Nishihara and Shibata (2021), for intermediate levels of the state variable, a firm exits only when sellout is feasible, but bankrupts when the state variable decreases to a certain threshold. Unlike this paper, previous studies consider two options, one (i.e., private debt finance, sellout) feasible only at Poisson jump times, and the other (i.e., public debt finance, default) feasible at any time.

3.3 Asymmetric information case

This subsection assumes that $\lambda_i > 0$ $(i = H, L)$ and the acquirer types are unobservable to the startup. I denote the firm value when an acquirer is absent as $U(x)$. When an acquirer arrives, the startup makes a take-it-or-leave-it price offer. The startup posts a price from among $a_L\pi(x) - I_a, a_H\pi(x) - I_a$, and ∞ (i.e., rejection), because it cannot increase the firm value by any other price. The acquirer then purchases the firm if the posted price is not higher than its evaluation. As in Sections 3.1 and 3.2, the optimal policy becomes a threshold policy. In other words, the i-type pricing region becomes $\{x \geq x_i^{**}\}$, where $x_H^{**} < x_L^{**} \leq \infty$, and $U(x)$ satisfies ODEs:

$$\mu x U'(x) + 0.5\sigma^2 x^2 U''(x) + x = rU(x) \quad (0 < x < x_H^{**}), \tag{22}$$

$$\mu x U'(x) + 0.5\sigma^2 x^2 U''(x) + \lambda_H(a_H\pi(x) - I - U(x)) + x = rU(x) \quad (x_H^{**} < x < x_L^{**}), \tag{23}$$

$$\mu x U'(x) + 0.5\sigma^2 x^2 U''(x) + \bar{\lambda}(a_L\pi(x) - I - U(x)) + x = rU(x) \quad (x > x_L^{**}). \tag{24}$$

ODEs (22) and (23) are the same as ODEs (12) and (13) in Section 3.2, respectively, but the third term in ODE (24) is different from that in ODE (14). Indeed, $\bar{\lambda}(a_L\pi(x) - I - U(x))$ indicates that $U(x)$ becomes $a_L\pi(x) - I$ with probability $\bar{\lambda}\mathrm{d}t$ in infinitesimal time interval $\mathrm{d}t$. In this region, unlike in the symmetric information case, the sellout price is low even for a high type because the startup cannot determine the price contingent on the acquirer type.

As in Sections 3.1 and 3.2, $U(x)$ is continuously differentiable at x_i^{**}, and $U(x)$ satisfies boundary conditions

$$U(x_H^{**}) = a_H\pi(x_H^{**}) - I, \tag{25}$$

$$U(x_L^{**}) = (a_L - \Delta a\lambda_H/\lambda_L)\pi(x_L^{**}) - I, \tag{26}$$

$$\lim_{x\to 0} U(x) = 0 \tag{27}$$

$$\lim_{x\to\infty} U(x)/x < \infty. \tag{28}$$

Boundary conditions (25), (27), and (28) are the same as (3), (4), and (5) in Section 3.2, respectively, but (26) is different from (3). The term $\Delta a\lambda_H/\lambda_L\pi(x_L^{**})$ in (26) stands for the opportunity cost of the low-price sale, that is, the loss from sellout to a high type at a low price. Here, condition (26) is derived as follows. Suppose that an acquirer arrives at the threshold x_L^{**}. By setting a low price, the firm value becomes $a_L\pi(x_L^{**}) - I$ because the acquirer would purchase the startup. By setting a high price, the firm value becomes $(a_H\pi(x_L^{**}) - I)\lambda_H/\bar{\lambda} + U(x_L^{**})\lambda_L/\bar{\lambda}$ because the acquirer would purchase the startup if and only if it is a high type. Note that the acquirer is a high type with probability $\lambda_H/\bar{\lambda}$. By solving equation

$$a_L\pi(x_L^{**}) - I = (a_H\pi(x_L^{**}) - I)\lambda_H/\bar{\lambda} + U(x_L^{**})\lambda_L/\bar{\lambda},$$

I obtain the boundary condition (26), which ensures that the startup is indifferent to whether to set a high or low price at $X(t) = x_L^{**}$ (i.e., the optimality of x_L^*).

If

$$(a_H\pi(x) - I)\lambda_H/\bar{\lambda} + V_H(x)\lambda_L/\bar{\lambda} \geq a_L\pi(x) - I \quad (x \geq 0) \tag{29}$$

holds, which is equivalent to condition (31) in the following proposition, the startup takes the high-price strategy, that is, $x_L^{**} = \infty$, where the startup posts only the high price. This is because the expected payoff by the high-price strategy (i.e., the left-hand side of (29)) dominates the low payoff (i.e., the right-hand side of (29)) for any $X(t)$. In this case, the firm value $U(x)$ and sellout threshold x_H^{**} agree with $V_H(x)$ and x_H in Proposition 1. Condition (29) can be rewritten as

$$V_H(x) \geq a_L\pi(x) - I - \Delta a\pi(x)\lambda_H/\lambda_L \quad (x \geq 0). \tag{30}$$

Compared to (18), (30) has an extra term $-\Delta a\pi(x)\lambda_H/\lambda_L$, which is the opportunity cost of the low-price sale, that is, the loss from sellout to a high type at a low price. If (29) (or equivalently (30) or (31)) does not hold, the startup takes the flexible strategy, that is, $x_L^{**} < \infty$, where the startup allows the low-price sale for $X(t) \geq x_L^{**}$. In fact, the firm can increase the firm value beyond $V_H(X(t))$ by setting the low price. In this case, by solving ODEs (22)–(24) with boundary conditions (25)–(28), I have the following proposition. For proof of the proposition, see Appendix C.

Proposition 3 *If condition*

$$a_L - 1 - \frac{\Delta a \lambda_H}{\lambda_L} \leq \frac{\lambda_H(a_H - 1)}{r + \lambda_H - \mu} \tag{31}$$

holds, the firm takes the high-price strategy, that is, $x_L^{**} = \infty$*. The firm value* $U(x)$ *and sellout threshold* x_H^{**} *agree with* $V_H(x)$ *and* x_H *in Proposition 1.*

If condition (31) does not hold, the firm takes the flexible strategy, that is, $x_L^{**} < \infty$*. The firm value* $U(x)$ *is given by*

$$U(x) = \begin{cases} \pi(x) + \tilde{B}_1 x^{\beta} & (x < x_H^{**}), \\ \pi(x) + \dfrac{\lambda_H(a_H - 1)\pi(x)}{r + \lambda_H - \mu} - \dfrac{\lambda_H I}{r + \lambda_H} + \tilde{B}_2 x^{\beta_{\lambda_H}} + \tilde{B}_3 x^{\gamma_{\lambda_H}} & (x \in [x_H^{**}, x_L^{**})), \\ \pi(x) + \dfrac{\bar{\lambda}(a_L - 1)\pi(x)}{r + \bar{\lambda} - \mu} - \dfrac{\bar{\lambda} I}{r + \bar{\lambda}} + \tilde{B}_4 x^{\gamma_{\bar{\lambda}}} & (x \geq x_L^{**}), \end{cases} \tag{32}$$

where the coefficients $\tilde{B}_i$ $(i = 1, 2, 3, 4)$ *and sellout thresholds* x_i^{**} $(i = H, L)$ *are as defined in Appendix C.*

As in Proposition 1 in Section 3.2, Proposition 2 shows that the startup chooses either the high-price strategy, abandoning the low-price sellout to pursue pricing efficiency, or the flexible strategy, adopting the low-price sellout to pursue timing efficiency. The choice is determined by whether condition (31) holds or not. Compared to condition (19), the left-hand side of (31) has an extra term $-\Delta a\lambda_H/\lambda_L$. As with the difference between (18) and (30), the extra term represents the opportunity cost of the low-price sellout, that is, the loss from sellout to a high type at a low price. In other words, with asymmetric information, the pricing inefficiency stemming from the flexible strategy increases, and the startup is more likely to choose the high-price strategy.

Condition (31) can be rewritten as

$$\frac{\bar{\lambda}(a_L - 1)}{r + \bar{\lambda} - \mu} \leq \frac{\lambda_H(a_H - 1)}{r + \lambda_H - \mu}, \tag{33}$$

where the left- and right-hand sides of (21) correspond to the second terms of equation (32) for $x \geq x_L^{**}$ and equation (8) for $x \geq x_H$, respectively. As in the symmetric information case, with higher a_H and λ_H as well as lower a_L and $r - \mu$, the startup is more likely to take the high-price strategy. Note that from condition (31), the firm tends to take the high-price strategy with higher Δa and lower λ_L. This is explained by the opportunity cost of the low-price sellout. The pricing inefficiency increases with higher Δa, and the probability of the acquirer type being high (i.e., $\lambda_H/\bar{\lambda}$) increases with lower λ_L. Hence, with higher Δa and lower λ_L, the startup tends to take the high-price strategy in order to remove the pricing inefficiency.

In the region $a_L - 1 \leq \lambda_H(a_H - 1)/(r + \lambda_H - \mu)$, the startup takes the high-price strategy, that is, the strategy with only high type (i.e., Proposition 1), regardless of the information structure. Here, asymmetric information does not affect the sellout policy and option value, i.e., $U(x) = V(x) = V_H(x)$. In the region $a_L - 1 - \Delta a \lambda_H/\lambda_L \leq \lambda_H(a_H - 1)/(r + \lambda_H - \mu) < a_L - 1$, the startup takes the high-price strategy under asymmetric information and flexible strategy under symmetric information. Thus, asymmetric information changes the strategy and reduces the firm value, i.e., $U(x) = V_H(x) < V(x)$. In the region $\lambda_H(a_H - 1)/(r + \lambda_H - \mu) < a_L - 1 - \Delta a \lambda_H/\lambda_L$, the startup takes the flexible strategy irrespective of the information structure. However, the sellout thresholds x_i^{**} $(i = H, L)$ are different from x_i^* $(i = H, L)$, and the firm value decreases with asymmetric information, i.e., $U(x) < V(x)$.

I cannot derive closed-form expressions of x_i^{**} $(i = H, L)$, but by the monotonic increase of $U(x)$ with respect to λ_i $(i = H, L)$, $x_H^{**} = \inf\{x \geq 0 \mid U(x) = a_H \pi(x) - I\}$, and $x_L^{**} = \inf\{x \geq 0 \mid U(x) = (a_L - \Delta a \lambda_H/\lambda_L)\pi(x) - I\}$, I can show that x_H^{**} monotonically increases in λ_i $(i = H, L)$ and x_L^{**} monotonically increases in λ_H. By $U(x) < V(x)$, I can also show that $x_H^{**} < x_H^*$. Although I cannot analytically prove this, I find that x_L^{**} monotonically decreases in λ_L, and $x_L^{**} > x_L^*$ holds in the numerical examples in Section 4.

From these results, in case of lower frequency of high type arrivals, the startup accepts an acquisition more eagerly to alleviate the illiquidity effect. These results agree with Propositions 1 and 2. More notably, in case of lower frequency of low type arrivals, the startup accepts a high-price acquisition more eagerly and a low-price acquisition more restrictively. This is because the opportunity cost of a low-price sellout increases with a lower λ_L. Then, despite decreased liquidity, the startup restricts the low-price sellout more severely. This is a key difference from the symmetric information case. In case of asymmetric information, the startup extends the gap between the high-price sellout threshold x_H^{**} and low-price sellout threshold x_L^{**} to reduce the high type's information rent. This result agrees with the previous literature on intertemporal price discrimination (e.g., Stokey (1979), Landsberger and Meilijson (1985), Nishihara and Shibata (2019)). In fact, Stokey (1979) and Landsberger and Meilijson (1985) show that a seller can delay sales at low prices to discriminate between buyer types, while Nishihara and Shibata (2019) show that a firm can delay sellout or the default timing for a low type to decrease the acquirer's information rent.

Proposition 3 technically contributes to the literature on the optimal stopping problem constrained to Poisson jump times. Indeed, previous papers (e.g., Dupuis and Wang (2002), Morellec, Valta, and Zhdanov (2015), Nishihara and Shibata (2021)) lack in heterogeneity and hence do not consider any problem under asymmetric information. To my knowledge, Proposition 3 first solves the problem involving heterogeneity of option payoffs and asymmetric information.

Although the acquirer's surplus is zero under symmetric information, a high type can obtain information rent by acquiring the firm at a low price under asymmetric information. The following corollary derives the expected acquirer payoff at time 0 (denoted by $W(x)$). For proof of the corollary, see Appendix D.

Corollary 1 *If condition (31) is satisfied, $W(x) = 0$. If condition (31) is not satisfied, $W(x)$ is given by*

$$W(x) = \begin{cases} D_1 x^{\beta} & (x < x_H^{**}), \\ D_2 x^{\beta_{\lambda_H}} + D_3 x^{\gamma_{\lambda_H}} & (x \in [x_H^{**}, x_L^{**})), \\ \dfrac{\lambda_H \Delta a \pi(x)}{r + \bar{\lambda} - \mu} + D_4 x^{\gamma_{\bar{\lambda}}} & (x \geq x_L^{**}), \end{cases} \tag{34}$$

where the sellout thresholds x_i^{**} $(i = H, L)$ and coefficients D_i $(i = 1, 2, 3, 4)$ are defined as in Appendices C and D.

The positive acquirer's information rent is consistent with empirical evidence. In fact, many studies such as Bargeron, Schlingemann, Stulz, and Zutter (2008), Battigalli, Chiarella, Gatti, and Orlando (2017), and Golubov and Xiong (2020) show that private and less transparent firms pay lower acquisition premiums. I define the social loss due to asymmetric information as $L(x) = V(x) - U(x) - W(x)$. In the next section, I examine $L(x)$ to study the welfare effects of asymmetric information.

Throughout this study, I assume that the acquirer pays cash to acquire the startup. Although the results under symmetric information (i.e., Propositions 1 and 2) hold true regardless of the acquirer's payment method, the results under asymmetric information (i.e., Proposition 3 and Corollary 1) depend on the payment method. In fact, using of stock and payment contingent on the post-merger performance (e.g., Lukas, Reuer, and Welling (2012)) decreases the acquirer's information rent. The startup would then prefer such a contract, whereas the acquirer would prefer cash payment to maximize its information rent. The conflict over payment method is beyond the scope of this study, but would be an interesting topic for a future research.

4 Numerical analysis

In this section, I carry out numerical analysis, including comparative statics of the arrival rates λ_i $(i = H, L)$, cash flow growth rate μ, and volatility σ. I set the baseline parameter values as in Table 1. The values of r, μ, and σ are typical to the real options literature (e.g., Dixit and Pindyck (1994)). In particular, I set μ and σ so as to satisfy $\mu - 0.5\sigma^2 > 0$, meaning that $X(t)$ has a growing trend, and focus on the startup in the growth stage.

Figure 1 shows how the startup's sellout strategy changes with the expected waiting time $1/\lambda_i$ $(i = H, L)$. With sufficiently frequent high type arrivals (i.e., $1/\lambda_H \leq 5$), the startup takes the high-price strategy regardless of the information structure. In the region between the black and blue dashed lines, the firm takes the high-price strategy only with asymmetric information, whereas, in the right-hand side region, the firm takes the flexible strategy regardless of the information structure. I set the baseline parameters $\lambda_H = 0.125$ and $\lambda_L = 0.375$ (see the baseline point in Figure 1) so as to satisfy conditions (19) and (31), and examine mainly the flexible strategy case and the effects of asymmetric information. These values mean that on average, an acquirer approaches the startup every two years (i.e., $1/\bar{\lambda} = 2$), and low types arrive three times more frequently than high types (i.e., $\lambda_L/\lambda_H = 3$).

For the baseline parameter values, I have $x_H^* = 1.504, x_L^* = 5.472$, and $V(x) = 22.382$ under symmetric information and $x_H^{**} = 1.5, x_L^{**} = 24.606$, and $U(x) = 22.374$ under asymmetric information. As explained after Proposition 3, with asymmetric information, the startup increases the gap between the high- and low-price sellout thresholds to decrease the acquirer's information rent. In particular, the firm takes x_L^{**} much higher than x_L^*. The difference between $V(x)$ and $U(x)$ may look very small because the NPV of cash flows with no sellout, that is, $\pi(x) = 20$, accounts for most of $V(x)$ and $U(x)$.

4.1 Impacts of high type arrival rate

Figure 2 plots the thresholds x_i^*, x_i^{**}, firm values $V(x), U(x)$, and social loss $L(x)$ with varying levels of expected waiting time for the high type, $1/\lambda_H$. The other parameter values are set as in Table 1. In addition, the figure plots the probability of high-price sellout (denoted by $P_H(x)$ and $Q_H(x)$ under symmetric and asymmetric information, respectively) and state price of sellout (denoted by $S(x)$ and $T(x)$ under symmetric and asymmetric information, respectively). The state prices, defined by $S(x) = \mathbb{E}[\mathrm{e}^{-r\tau}]$ and $T(x) = \mathbb{E}[\mathrm{e}^{-r\tilde{\tau}}]$, where τ and $\tilde{\tau}$ denote the sellout time under symmetric and asymmetric information, respectively, measure the time until sellout.

Condition (19) in Proposition 2 holds for $1/\lambda_H \leq 5$, whereas condition (31) in Proposition 3 holds for $1/\lambda_H \leq 7.6$. In these regions, the startup takes the high-price strategy, where x_L^* and x_L^{**} are infinity, and $P_H(x)$ and $Q_H(x)$ are equal to 1. As explained after Propositions 2 and 3, in the top panels, x_i^* and x_i^{**} $(i = H, L)$ decrease in $1/\lambda_H$. In particular, I find the effects of $1/\lambda_H$ on x_L^* and x_L^{**} very large, meaning that the firm's response to a low-price acquisition depends greatly on the frequency of high type arrivals. These results show that with lower frequency of high type arrivals, the startup accepts an acquisition more eagerly (especially of a low type) to alleviate the illiquidity effect.

The center-right panel shows that $P_H(x)$ and $Q_H(x)$ decrease in $1/\lambda_H$; this implies that low-price sellout is more likely to occur with lower frequency of high type arrivals. Note that the large decrease in x_L^* and x_L^{**} with higher $1/\lambda_H$ also contributes to this result. The bottom-left panel shows that $S(x)$ and $T(x)$ decrease in $1/\lambda_H$, meaning that lower frequency of high type arrivals lengthens the expected time until sellout. To summarize, with lower frequency of high type arrivals, the firm accepts an acquirer more eagerly, but this earnest sellout policy does not fully offset the decreased frequency effect. In fact, lower frequency of high type arrivals delays the sellout time and reduces the high-price sellout probability and firm value.

Next, I examine the differences between the results under symmetric and asymmetric information. In the top panels of Figure 2, as explained after Proposition 3, $x_H^{**} \leq x_H^*$ and $x_L^{**} > x_H^{**}$ hold. The center-right panel shows that $Q_H(x)$ is higher than $P_H(x)$, while the bottom-left panel shows that $T(x)$ is lower than $S(x)$. These results are interpreted as follows. With asymmetric information, the startup accepts a high-price sellout more eagerly and a low-price sellout more restrictively to reduce the acquirer's information rent. Thus, the startup can keep the sellout price high, but this delays the sellout timing.

The welfare effects of asymmetric information are not monotonic with respect to $1/\lambda_H$. In fact, the bottom-right panel shows that social loss $L(x)$ has an inverted U-shape in $1/\lambda_H$. The reason is that asymmetric information matters most for intermediate levels of $1/\lambda_H$. For $1/\lambda_H \leq 5$, there is no difference between the symmetric and asymmetric information cases, i.e., $x_H^{**} = x_H^* = x_H, x_L^{**} = x_L^* = \infty, U(x) = V(x) = V_H(x), P_H(x) = Q_H(x) = 1$, and $L(x) = 0$. For $1/\lambda_H \to \infty$, the firm does not meet a high type, and therefore asymmetric information does not matter, i.e., $x_L^{**} = x_L^* = x_L, U(x) = V(x) = V_L(x), P_H(x) = Q_H(x) = 0$, and $L(x) = 0$. For intermediate levels of $1/\lambda_H$, especially in the region of $5 < 1/\lambda_H \leq 7.6$, where the startup takes the high-price strategy only in the presence of asymmetric information, the effects of asymmetric information become relatively large, leading to high $L(x)$.

4.2 Impacts of low type arrival rate

Figure 3 plots $x_i^*, x_i^{**}, V(x), U(x), P_H(x), Q_H(x), S(x), T(x)$, and $L(x)$ with varying levels of expected waiting time for a low type, $1/\lambda_L$. The other parameter values are set as in Table 1. In all the depicted regions, the startup takes the flexible strategy under symmetric information, and the flexible strategy for $1/\lambda_L \leq 3.3$ and high-price strategy for $1/\lambda_L > 3.3$ under asymmetric information. As explained after Proposition 3, the opportunity cost of low-price sellout increases with higher $1/\lambda_L$, and the startup would then be more likely to choose the high-price strategy.

As explained after Proposition 2, with symmetric information, x_i^* $(i = H, L)$ decrease in $1/\lambda_L$ in the top panels and $S(x)$ decreases in $1/\lambda_L$ in the bottom-left panel. This can be interpreted as follows. With lower frequency of low type arrivals, the startup would accept an acquisition more eagerly to alleviate the decreased frequency effect. However, this earnest policy will not fully offset the decreased frequency effect and delay the sellout. These impacts of $1/\lambda_L$ under symmetric information are similar to but weaker than those of $1/\lambda_H$ in Section 4.1.

More interestingly, a higher $1/\lambda_L$ decreases x_H^{**} but increases x_L^{**} under asymmetric information. The effect of $1/\lambda_L$ on x_L^{**} is quite large and different from that of $1/\lambda_H$ or the symmetric information case. As explained after Proposition 3, this result stems from the opportunity cost of low-price sellout. In fact, the probability of high acquirer type increases with higher $1/\lambda_L$. Then, with higher $1/\lambda_L$, the startup would restrict low-price sellout more severely to mitigate the risk of sellout to a high type at a low price, although it would accept a high-price sellout more eagerly. By this sellout policy under asymmetric information, the high-price sellout probability $Q_H(x)$ increases more sharply than $P_H(x)$ in the center-right panel of Figure 3, but the state price $T(x)$ decreases more sharply than $S(x)$ in the bottom-left panel. In other words, the startup optimizes the sellout policy with a tradeoff between sellout pricing and timing efficiency.

As in $1/\lambda_H$ in Section 4.1, the welfare effects of asymmetric information are not monotonic with respect to $1/\lambda_L$. For $1/\lambda_L \to 0$, the firm meets a low type at a low-price sellout threshold, and hence asymmetric information does not matter, i.e., $x_L^{**} = x_L^*, U(x) = V(x),$, and $L(x) = 0$. For

$1/\lambda_L \to \infty$, a low type never arrives, and therefore asymmetric information does not matter, i.e., $x_H^{**} = x_H^* = x_H, U(x) = V(x) = V_H(x), P_H(x) = Q_H(x) = 1$, and $L(x) = 0$. For intermediate levels of $1/\lambda_L$, the effects of asymmetric information become relatively large. Note that $V(x) - U(x)$ and $L(x)$ are highest especially for $1/\lambda_L \approx 3.3$, where the firm takes the high-price strategy only with asymmetric information.

4.3 Impacts of cash flow growth rate

Figure 4 plots $x_i^*, x_i^{**}, V(x), U(x), P_H(x), Q_H(x), S(x), T(x)$, and $L(x)$ with varying cash flow growth rate levels, μ. The other parameter values are set as in Table 1. The startup takes the flexible strategy for $\mu < 0.033$ and high-price strategy for $\mu \geq 0.033$ under symmetric information, and the flexible strategy for $\mu < 0.049$ and high-price strategy for $\mu \geq 0.049$ under asymmetric information. As conditions (19) and (31) explain, a higher μ, which magnifies the gap between high and low valuations, intensifies the pricing inefficiency stemming from the flexible strategy, and hence the startup is more likely to take the high-price strategy.

In Figure 4, x_H^* and x_H^{**} decrease in μ, whereas $V(x), U(x), S(x)$, and $T(x)$ increase in μ.[11] These comparative static results are consistent with the standard results (e.g., Dixit and Pindyck (1994)). In fact, a higher μ is known to accelerate the option exercise timing and increases the project value. More notably, x_L^* and x_L^{**} increase in μ in the top-right panel, in contrast to the standard result. A higher μ expands the gap between the high and low sellout prices, and therefore the startup accepts a high-price sellout more eagerly and a low-price sellout more restrictively. Thus, the high-price sellout probabilities $P_H(x)$ and $Q_H(x)$ increase in μ. These results lead to the empirical prediction that startups with higher growth rates are more likely to be acquired by higher-efficiency firms. To my knowledge, this prediction has not been tested empirically.

The welfare effects of asymmetric information are not monotonic with respect to μ. For $\mu \geq 0.049$, where the startup takes the same high-price strategy regardless of information structures, asymmetric information makes no difference to welfare. As in the previous subsections, for intermediate levels of μ, especially $\mu \approx 0.033$, where the firm changes the flexible strategy to the high-price strategy under asymmetric information, $L(x)$ becomes relatively high.

4.4 Impacts of cash flow volatility

Figure 5 plots $x_i^*, x_i^{**}, V(x), U(x), P_H(x), Q_H(x), S(x), T(x)$, and $L(x)$ with varying levels of cash flow volatility, σ. The other parameter values are set as in Table 1. Neither condition (19) nor condition (31) depends on σ, and the startup therefore takes the flexible strategy in all areas.

In Figure 5, $x_i^*, x_i^{**}, V(x)$, and $U(x)$ increase in σ, whereas $S(x)$ and $T(x)$ decrease in σ. These comparative static results are consistent with the standard result (e.g., Dixit and Pindyck (1994))

[11] In the figure, $x_H^*, V(x)$, and $S(x)$ almost align with $x_H^*, U(x)$, and $T(x)$, respectively, because the impacts of μ on these values are much stronger than those of asymmetric information.

that a higher σ delays the option exercise timing and increases the value of waiting.[12] More notably, the high-price sellout probabilities $P_H(x)$ and $Q_H(x)$ decrease in σ, in the center-right panel. This is because a higher σ, which makes the dynamics of $X(t)$ more volatile, increases the probability that $X(t)$ passes through interval $[x_H^*, x_L^*)$ (or $[x_H^{**}, x_L^{**})$) without meeting a high type. In other words, higher volatility can rapidly improve the economic environment such that the startup can gain sufficiently high proceeds even from a low-price sellout. These results lead to the empirical prediction that startups with higher volatility are more likely to be acquired by lower-efficiency firms. To my knowledge, this prediction has not been tested empirically.

Figure 5 shows that the impacts of asymmetric information increase monotonically in σ. In fact, the differences, $x_H^* - x_H^{**}, x_L^{**} - x_L^*, V(x) - U(x), P_H(x) - Q_H(x)$, and $S(x) - T(x)$, as well as social loss $L(x)$, increase in σ. The reason seems to be the same as discussed above. That is, with higher σ, which has the potential to increase $X(t)$ rapidly, a low-price sellout matters more. This increases the inefficiency due to asymmetric information because a low-price sellout with asymmetric information leads to loss from sellout to a high type at a low price. To alleviate the increased inefficiency, the startup adjusts the sellout policy more, leading to $x_H^* - x_H^{**}, x_L^{**} - x_L^*, P_H(x) - Q_H(x)$, and $S(x)-T(x)$ increasing in σ. However, this adjustment does not fully offset the increased inefficiency, and $V(x) - U(x)$ and $L(x)$ therefore increase in σ.

4.5 Stock price reaction at sellout

Figure 6 shows the expected jump in startup value at the sellout time under symmetric and asymmetric information. With symmetric information, acquisition changes the firm value from $V(X(t))$ to $a_H\pi(X(t)) - I$ for $X(t) \in [x_H^*, x_L^*)$, and to $a_i\pi(X(t)) - I$ for $X(t) \geq x_L^*$. Then, the expected jump size becomes $(a_H\pi(X(t)) - I - V(X(t)))/V(X(t))$ for $X(t) \in [x_H^*, x_L^*)$, and $(\bar{a}\pi(X(t)) - I - V(X(t)))/V(X(t))$ for $X(t) \in [x_H^*, x_L^*)$. Under asymmetric information, the jump size becomes $(a_H\pi(X(t)) - I - U(X(t)))/U(X(t))$ for $X(t) \in [x_H^{**}, x_L^{**})$ and $(a_L\pi(X(t)) - I - U(X(t)))/U(X(t))$ for $X(t) \in [x_H^*, x_L^*)$. The jump size can be interpreted as the target stock price reaction or acquisition premium.

From Figure 6, the jump size for the flexible strategy is not monotonic with respect to $X(t)$. In fact, the jump size at acquisition with symmetric information increases monotonically in $X(t)$ for $X(t) \in [x_H^*, x_L^*)$, and $X(t) > x_L^*$, but falls at $X(t) = x_L^*$. This same result can be found under asymmetric information. This can be explained as follows. At the threshold x_L^* (or x_L^{**}), the startup changes its sellout strategies and accepts a low-price acquisition. By allowing a low-price acquisition, the sellout probability increases, whereas the sellout price decreases. Then, despite better economic environment, the jump size decreases. In particular, under asymmetric information, the jump size reduces more sharply at the threshold x_L^{**}. This result leads to the empirical prediction that asymmetric information intensifies stock price reactions at acquisition.

Figure 6 also shows that a higher $1/\lambda_H$ leads to larger jump sizes as long as the sellout region

[12] In contrast, Sarkar (2021) shows that the relationship between uncertainty and investment is not monotonic in the optimal investment timing and size problem.

does not change. This is because the firm values $V(X(t))$ and $U(X(t))$ decrease with higher $1/\lambda_H$. In other words, with higher $1/\lambda_H$, the firm value includes less sellout option values, and the sellout therefore brings a larger shock to the firm value. This result is consistent with the empirical findings of Cornett, Tanyeri, and Tehranian (2011) that a lower merger anticipation increases the target stock price reaction at merger announcement. More notably, the jump size is not monotonic when the sellout region changes with $1/\lambda_H$. For instance, see the reactions for $X(t) = 10$. Under symmetric information, the jump size increases with $1/\lambda_H = 8, 6$, and 10. Under asymmetric information, the jump size increases with $1/\lambda_H = 10, 8$, and 6. These results show the ambiguous relationship between merger anticipation and target stock price reaction. Indeed, a lower merger anticipation does not necessarily increase the stock price reaction because it may also decrease the price that the target firm aims for. I omit a figure with respect to $1/\lambda_L$ because the same results hold true.

Most of the M&A real option models (e.g., Lambrecht (2004), Morellec and Zhdanov (2008), and Lukas, Pereira, and Rodrigues (2019)) do not consider the friction in the M&A searching and matching process. Hence, those models can predict the M&A timing, and therefore no jump occurs in firm value at the time of M&A. Hackbarth and Morellec (2008) consider the asymmetric information between firm insiders and outside investors, and examine the jump in firm value, but the target and acquirer managers in their models precisely predict the merger timing and price. Nishihara and Shibata (2021) consider the illiquidity of sellout opportunities, but do not consider the heterogeneity in sellout prices. This study can better account for the empirical findings of high uncertainty in M&A deals (e.g., Cornett, Tanyeri, and Tehranian (2011) and Wang (2018)), positive target stock price reactions (e.g., Huang and Walkling (1987) and Boone and Mulherin (2007)), and lower acquisition premiums offered by less transparent acquirers (e.g., Bargeron, Schlingemann, Stulz, and Zutter (2008), Battigalli, Chiarella, Gatti, and Orlando (2017), and Golubov and Xiong (2020)).

5 Conclusion

This study develops a model in which a startup optimizes the sellout timing and price for high- and low-type acquirers approaching randomly and sequentially. Under symmetric and asymmetric information on acquirer type, I analytically derive the startup's sellout policy and firm value. In particular, the startup takes either the high-price strategy—accepting only a high type— or flexible strategy—accepting even a low type in a very good economic environment—based on a tradeoff between sellout pricing and timing efficiency. The model analysis yields the following results.

The startup tends to take the high-price strategy when the high type's arrival rate and cash flow growth rate are high. With lower acquirers' arrival rate, the startup accepts an acquisition more eagerly, but this policy does not fully offset the decrease in liquidity. In fact, lower arrival rates delay the sellout timing and decrease the firm value. A higher growth rate increases the low-price sellout threshold because it magnifies the gap between high and low valuations. Higher growth rate and lower volatility increase the high-price sellout probability and accelerate the sellout. With asymmetric information, the high-type acquirer can collect information rent by purchasing the startup at a low price. To reduce the acquirer's information rent, the startup decreases the high-price sellout threshold and increases the low-price sellout threshold, especially for low arrival rates of low types. This policy adjustment maintains the sellout price, but delays the sellout time, leading to social loss. This social loss due to asymmetric information tends to be high when asymmetric information changes the startup' strategy from flexible to high-price strategy. The model can also explain the positive target stock price reactions (or acquisition premiums) at acquisition. The jump size can be non-monotonic with respect to either the state variable or acquirer arrival rate due to the interaction between the sellout price and timing.

A Proof of Proposition 1

The general solution to ODE (1) with (4) is expressed as $V_i(x) = \pi(x) + A_{1,i}x^{\beta}$, where $A_{1,i}$ is an unknown coefficient. Then, by (5) and the continuity of $V_i(x)$, I have the expression (10). The general solution to ODE (2) with (5) is expressed as

$$V_i(x) = \pi(x) + \frac{\lambda_i(a_i-1)\pi(x)}{r+\lambda_i-\mu} - \frac{\lambda_i I}{r+\lambda_i} + A_{2,i}x^{\gamma_{\lambda_i}},$$

where $A_{2,i}$ is an unknown coefficient. Then, by (3) and the continuity of $V_i(x)$, I have the expression (11). By the continuous differentiability of $V_i(x)$ at x_i, I have

$$\beta A_1 x_i^{\beta} = \frac{\lambda_i(a_i-1)\pi(x_i)}{r+\lambda_i-\mu} + \gamma_{\lambda_i}A_2 x_i^{\gamma_{\lambda_i}}. \tag{35}$$

By substituting (10) and (11) into (35), I have

$$\begin{aligned}\beta((a_i-1)\pi(x_i)-I) &= \frac{\lambda_i(a_i-1)\pi(x_i)}{r+\lambda_i-\mu} + \gamma_{\lambda_i}\left(\frac{(a_i-1)x_i}{r+\lambda_i-\mu} - \frac{rI}{r+\lambda_i}\right)\\ x_i &= \frac{(r+\lambda_i-\mu)((\beta-\gamma_{\lambda_i})r+\beta\lambda_i)x_i^{NPV}}{(r+\lambda_i)((\beta-\gamma_{\lambda_i})(r-\mu)+(\beta-1)\lambda_i)},\end{aligned}$$

where $x_i^{NPV} = (r-\mu)I/(a_i-1)$.

B Proof of Proposition 2

First, assume that (19) holds. Now, I can show that $V_H(x) > a_L\pi(x) - I$ $(x \geq 0)$ as follows. For $x < x_H$, I have

$$V_H(x) > a_H\pi(x) - I \geq a_L\pi(x) - I. \tag{36}$$

By $A_{2,H} > 0$ in (8), I have, for $x \geq x_H$,

$$\begin{aligned}V_H(x) &> \pi(x) + \frac{\lambda_H(a_H-1)\pi(x)}{r+\lambda_H-\mu} - \frac{\lambda_H I}{r+\lambda_H}\\ &\geq a_L\pi(x) - \frac{\lambda_H I}{r+\lambda_H} &(37)\\ &> a_L\pi(x) - I, &(38)\end{aligned}$$

where (37) follows from (19). By (36) and (38), I have $V_H(x) > a_L\pi(x) - I$ for all $x \geq 0$. I also have $V(x) > a_L\pi(x) - I$ for all $x \geq 0$ because of $V(x) \geq V_H(x)$. Then, there is no acceptance region for a low type, and $V(x) = V_H(x)$ holds.

Next, assume that (19) does not hold. From (8), I have $V_H(x) < a_L\pi(x) - I$ when x is sufficiently large. Hence, I have $V(x) \neq V_H(x)$, meaning that the firm accepts a low type. Assume that the value function $V(x)$ exists. Then, I can straightforwardly show that an optimal policy exists among threshold policies specified by thresholds x_i^* $(i = H, L)$, where $x_H^* < x_H^*$, and that $V(x)$ is an increasing, convex, and continuously differentiable function satisfying ODEs (12)–(14) with boundary conditions (15)–(17).

The solution to ODE (12) with boundary conditions (15) for $i = H$ and (16) becomes (20) for $x < x_H^*$. Similarly, the solution to ODE (13) with boundary condition (15) for $i = H, L$ is (20) for $x \in (x_H^*, x_L^*)$, while the solution to ODE (14) with boundary conditions (15) for $i = L$ and (17) is (20) for $x > x_L^*$. Notations B_i $(i = 1, \ldots, 7)$ are defined by

$$\begin{aligned}
B_1 &= (x_H^*)^{-\beta}((a_H - 1)\pi(x_H^*) - I),\\
B_2 &= \frac{(x_H^*)^{\gamma_{\lambda_H}} B_6 - (x_L^*)^{\gamma_{\lambda_H}} B_5}{(x_L^*)^{\beta_{\lambda_H}}(x_H^*)^{\gamma_{\lambda_H}} - (x_H^*)^{\beta_{\lambda_H}}(x_L^*)^{\gamma_{\lambda_H}}},\\
B_3 &= \frac{(x_L^*)^{\beta_{\lambda_H}} B_5 - (x_H^*)^{\beta_{\lambda_H}} B_6}{(x_L^*)^{\beta_{\lambda_H}}(x_H^*)^{\gamma_{\lambda_H}} - (x_H^*)^{\beta_{\lambda_H}}(x_L^*)^{\gamma_{\lambda_H}}},\\
B_4 &= (x_L^*)^{-\gamma_{\bar{\lambda}}} B_7,\\
B_5 &= \frac{(a_H - 1)x_H^*}{r + \lambda_H - \mu} - \frac{rI}{r + \lambda_H},\\
B_6 &= (a_L - 1)\pi(x_L^*) - \frac{\lambda_H(a_H - 1)\pi(x_L^*)}{r + \lambda_H - \mu} - \frac{rI}{r + \lambda_H},\\
B_7 &= (a_L - 1)\pi(x_L^*) - \frac{\bar{\lambda}(\bar{a} - 1)\pi(x_L^*)}{r + \bar{\lambda} - \mu} - \frac{rI}{r + \bar{\lambda}}.
\end{aligned}$$

Because of the continuous differentiability of $V(x)$ at x_i^* $(i = H, L)$, (x_H^*, x_L^*) becomes a solution[13] to

$$\frac{\lambda_H(a_H - 1)\pi(x_H^*)}{r + \lambda_H - \mu} + \beta_{\lambda_H} B_2 (x_H^*)^{\beta_{\lambda_H}} + \gamma_{\lambda_H} B_3 (x_H^*)^{\gamma_{\lambda_H}} = \beta((a_H - 1)\pi(x_H^*) - I), \tag{39}$$

$$\frac{\lambda_H(a_H - 1)\pi(x_L^*)}{r + \lambda_H - \mu} + \beta_{\lambda_H} B_2 (x_L^*)^{\beta_{\lambda_H}} + \gamma_{\lambda_H} B_3 (x_L^*)^{\gamma_{\lambda_H}} = \frac{\bar{\lambda}(\bar{a} - 1)\pi(x_L^*)}{r + \lambda - \mu} + \gamma_{\bar{\lambda}} B_7. \tag{40}$$

C Proof of Proposition 3

First, assume that (31) holds. In this case, I can show that

$$(a_H\pi(x) - I)\lambda_H/\bar{\lambda} + V_H(x)\lambda_L/\bar{\lambda} \geq a_L\pi(x) - I \ (x \geq 0) \tag{41}$$

[13] I find a unique solution to the nonlinear system (39) and (40) in all numerical examples, although I cannot analytically prove its existence and uniqueness.

as follows. For $x < x_H$, I have $V_H(x) > a_H\pi(x) - I$, and hence

$$(a_H\pi(x) - I)\lambda_H/\bar{\lambda} + V_H(x)\lambda_L/\bar{\lambda} > a_H\pi(x) - I \\ \geq a_L\pi(x) - I. \tag{42}$$

By $A_{2,H} > 0$ in (8), I have, for $x \geq x_H$,

$$\begin{aligned}
&(a_H\pi(x) - I)\lambda_H/\bar{\lambda} + V_H(x)\lambda_L/\bar{\lambda} \\
>&(a_H\pi(x) - I)\lambda_H/\bar{\lambda} + \left(\pi(x) + \frac{\lambda_H(a_H-1)\pi(x)}{r+\lambda_H-\mu} - \frac{\lambda_H I}{r+\lambda_H}\right)\lambda_L/\bar{\lambda} \\
\geq&a_H\pi(x)\lambda_H/\bar{\lambda} + \left(a_L - \frac{\Delta a\lambda_H}{\lambda_L}\right)\lambda_L/\bar{\lambda} - \frac{\lambda_H(r+\bar{\lambda})I}{\bar{\lambda}(r+\lambda_H)} \qquad (43)\\
=&a_L\pi(x) - \frac{\lambda_H(r+\bar{\lambda})I}{\bar{\lambda}(r+\lambda_H)} \\
>&a_L\pi(x) - I, \qquad (44)
\end{aligned}$$

where (43) follows from (31). By (44) and (42), I have (41). I also have $(a_H\pi(x) - I)\lambda_H/\bar{\lambda} + U(x)\lambda_L/\bar{\lambda} \geq a_L\pi(x) - I$ for all $x \geq 0$ because $U(x) \geq V_H(x)$. Then, the firm value under the high-price strategy dominates the payoff by low-price sellout, and $U(x) = V_H(x)$ holds.

Next, assume that (31) does not hold. From (8), $(a_H\pi(x)-I)\lambda_H/\bar{\lambda}+V_H(x)\lambda_L/\bar{\lambda} < a_L\pi(x)-I$ when x is sufficiently large. Hence, I have $U(x) \neq V_H(x)$, meaning that the startup posts a low price. Assume that value function $U(x)$ exists. As in Appendix B, I can derive the value function $U(x)$ and thresholds x_i^{**} $(i = H, L)$, and hence omit the derivation details. By solving ODEs (22)–(24) with boundary conditions (25)–(28), I have Proposition 3, where coefficients $\tilde{B}_i$ $(i = 1, \ldots, 7)$ are defined by

$$\begin{aligned}
\tilde{B}_1 &= (x_H^{**})^{-\beta}((a_H-1)\pi(x_H^{**}) - I), \\
\tilde{B}_2 &= \frac{(x_H^{**})^{\gamma_{\lambda_H}}\tilde{B}_6 - (x_L^{**})^{\gamma_{\lambda_H}}\tilde{B}_5}{(x_L^{**})^{\beta_{\lambda_H}}(x_H^{**})^{\gamma_{\lambda_H}} - (x_H^{**})^{\beta_{\lambda_H}}(x_L^{**})^{\gamma_{\lambda_H}}}, \\
\tilde{B}_3 &= \frac{(x_L^{**})^{\beta_{\lambda_H}}\tilde{B}_5 - (x_H^{**})^{\beta_{\lambda_H}}\tilde{B}_6}{(x_L^{**})^{\beta_{\lambda_H}}(x_H^{**})^{\gamma_{\lambda_H}} - (x_H^{**})^{\beta_{\lambda_H}}(x_L^{**})^{\gamma_{\lambda_H}}}, \\
\tilde{B}_4 &= (x_L^{**})^{-\gamma_{\bar{\lambda}}}\tilde{B}_7, \\
\tilde{B}_5 &= \frac{(a_H-1)x_H^{**}}{r+\lambda_H-\mu} - \frac{rI}{r+\lambda_H}, \\
\tilde{B}_6 &= (a_L - 1 - \Delta a\lambda_H/\lambda_L)\pi(x_L^{**}) - \frac{\lambda_H(a_H-1)\pi(x_L^{**})}{r+\lambda_H-\mu} - \frac{rI}{r+\lambda_H}, \\
\tilde{B}_7 &= (a_L - 1 - \Delta a\lambda H/\lambda_L)\pi(x_L^{**}) - \frac{\bar{\lambda}(a_L-1)\pi(x_L^{**})}{r+\bar{\lambda}-\mu} - \frac{rI}{r+\bar{\lambda}},
\end{aligned}$$

and (x_H^{**}, x_L^{**}) becomes a solution[14] to

$$\frac{\lambda_H(a_H-1)\pi(x_H^{**})}{r+\lambda_H-\mu} + \beta_{\lambda_H}\tilde{B}_2(x_H^{**})^{\beta_{\lambda_H}} + \gamma_{\lambda_H}\tilde{B}_3(x_H^{**})^{\gamma_{\lambda_H}} = \beta((a_H-1)\pi(x_H^{**}) - I), \tag{45}$$

$$\frac{\lambda_H(a_H-1)\pi(x_L^{**})}{r+\lambda_H-\mu} + \beta_{\lambda_H}\tilde{B}_2(x_L^{**})^{\beta_{\lambda_H}} + \gamma_{\lambda_H}\tilde{B}_3(x_L^{**})^{\gamma_{\lambda_H}} = \frac{\bar{\lambda}(a_L-1)\pi(x_L^{**})}{r+\lambda-\mu} + \gamma_{\bar{\lambda}}\tilde{B}_7. \tag{46}$$

[14]I found a unique solution to the nonlinear system (45) and (46) in all numerical examples, although I cannot analytically prove its existence and uniqueness.

D Proof of Corollary 1

If condition (31) is satisfied, I readily have $W(x) = 0$. Assume that (31) does not hold. Now, $W(x)$ satisfies ODEs

$$\mu x W'(x) + 0.5\sigma^2 x^2 W''(x) = rW(x) \quad (0 < x < x_H^{**}), \tag{47}$$

$$\mu x W'(x) + 0.5\sigma^2 x^2 W''(x) + \lambda_H(-W(x)) = rW(x) \quad (x_H^{**} < x < x_L^{**}), \tag{48}$$

$$\mu x W'(x) + 0.5\sigma^2 x^2 W''(x) + \bar{\lambda}(\Delta a\pi(x)\lambda_H/\bar{\lambda} - W(x)) = rW(x) \quad (x > x_L^{**}), \tag{49}$$

with boundary conditions

$$\lim_{x\to 0} W(x) = 0, \tag{50}$$

$$\lim_{x\to\infty} W(x)/x < \infty, \tag{51}$$

and $W(x)$ continuously differentiable at thresholds x_i^{**} $(i = H, L)$. In ODE (48), $\lambda_H(-W(x))$ corresponds to $W(x)$ becoming 0 with probability $\lambda_H \mathrm{d}t$ in infinitesimal time interval $\mathrm{d}t$ in region $x_H^{**} < x < x_L^{**}$, whereas $\bar{\lambda}(\Delta a\pi(x)\lambda_H/\bar{\lambda} - W(x))$ in ODE (49) corresponds to $W(x)$ becoming $\Delta a\pi(x)\lambda_H/\bar{\lambda}$ (information rent) with probability $\bar{\lambda}\mathrm{d}t$ in infinitesimal time interval $\mathrm{d}t$ in region $x > x_L^{**}$. The general solutions to ODE (47) with boundary condition (50), and ODEs (48) and (49) with boundary condition (51) become (34) for $x < x_H^{**}$, $x \in (x_H^{**}, x_L^{**})$, and $x > x_L^{**}$, respectively.

By the continuous differentiability of $W(x)$ at thresholds x_i^{**} $(i = H, L)$, I have

$$D_1(x_H^{**})^{\beta} = D_2(x_H^{**})^{\beta_{\lambda_H}} + D_3(x_H^{**})^{\gamma_{\lambda_H}}, \tag{52}$$

$$D_1\beta(x_H^{**})^{\beta} = D_2\beta_{\lambda_H}(x_H^{**})^{\beta_{\lambda_H}} + D_3\gamma_{\lambda_H}(x_H^{**})^{\gamma_{\lambda_H}}, \tag{53}$$

$$D_2(x_L^{**})^{\beta_{\lambda_H}} + D_3(x_L^{**})^{\gamma_{\lambda_H}} = \frac{\lambda_H \Delta a\pi(x_L^{**})}{r + \bar{\lambda} - \mu} + D_4(x_L^{**})^{\gamma_{\bar{\lambda}}}, \tag{54}$$

$$D_2\beta_{\lambda_H}(x_L^{**})^{\beta_{\lambda_H}} + D_3\gamma_{\lambda_H}(x_L^{**})^{\gamma_{\lambda_H}} = \frac{\lambda_H \Delta a\pi(x_L^{**})}{r + \bar{\lambda} - \mu} + D_4\gamma_{\bar{\lambda}}(x_L^{**})^{\gamma_{\bar{\lambda}}}. \tag{55}$$

By (52) and (53), I have

$$D_2(\beta_{\lambda_H} - \beta)(x_H^{**})^{\beta_{\lambda_H}} + D_3(\gamma_{\lambda_H} - \beta)(x_H^{**})^{\gamma_{\lambda_H}} = 0. \tag{56}$$

By (54) and (55), I have

$$D_2(\beta_{\lambda_H} - \gamma_{\bar{\lambda}})(x_L^{**})^{\beta_{\lambda_H}} + D_3(\gamma_{\lambda_H} - \gamma_{\bar{\lambda}})(x_L^{**})^{\gamma_{\lambda_H}} = \frac{(1 - \gamma_{\bar{\lambda}})\lambda_H \Delta a\pi(x_L^{**})}{r + \bar{\lambda} - \mu}. \tag{57}$$

By (56) and (57), I have

$$D_2 = \frac{(\gamma_{\lambda_H} - \beta)(x_H^{**})^{\gamma_{\lambda_H}}(1 - \gamma_{\bar{\lambda}})\lambda_H \Delta a\pi(x_L^{**})}{D_5(r + \bar{\lambda} - \mu)} \tag{58}$$

$$D_3 = -\frac{(\beta_{\lambda_H} - \beta)(x_H^{**})^{\beta_{\lambda_H}}(1 - \gamma_{\bar{\lambda}})\lambda_H \Delta a\pi(x_L^{**})}{D_5(r + \bar{\lambda} - \mu)} \tag{59}$$

$$D_5 = (\beta_{\lambda_H} - \gamma_{\bar{\lambda}})(\gamma_{\lambda_H} - \beta)(x_L^{**})^{\beta_{\lambda_H}}(x_H^{**})^{\gamma_{\lambda_H}} - (\beta_{\lambda_H} - \beta)(\gamma_{\lambda_H} - \gamma_{\bar{\lambda}})(x_H^{**})^{\beta_{\lambda_H}}(x_L^{**})^{\gamma_{\lambda_H}}. \tag{60}$$

Coefficients D_1 and D_4 immediately follows from (52) and (54).

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Table 1: Baseline parameter values

r	μ	σ	λ_H	λ_L	a_H	a_L	I	x
0.08	0.03	0.18	0.125	0.375	1.5	1.4	10	1

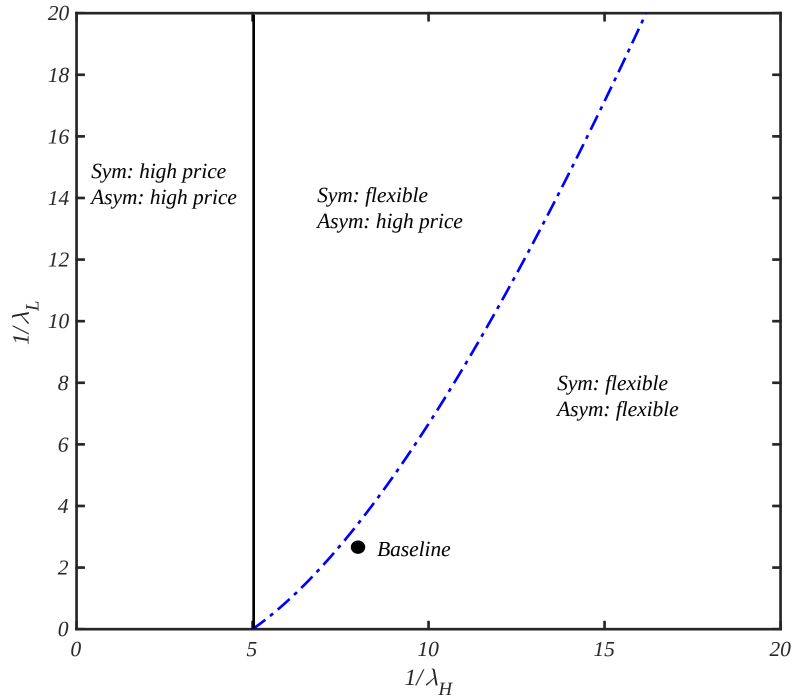

Figure 1: Strategy choice with respect to expected waiting time $1/\lambda_i$ $(i = H, L)$. The other parameter values are set as in Table 1. With symmetric information, the firm takes the high-price strategy for $1/\lambda_H \leq 5$, whereas with asymmetric information, the firm takes the high-price strategy in the area on the left-hand side of the blue dashed line. In the region between the black and blue dashed lines, asymmetric information changes the sellout strategy.

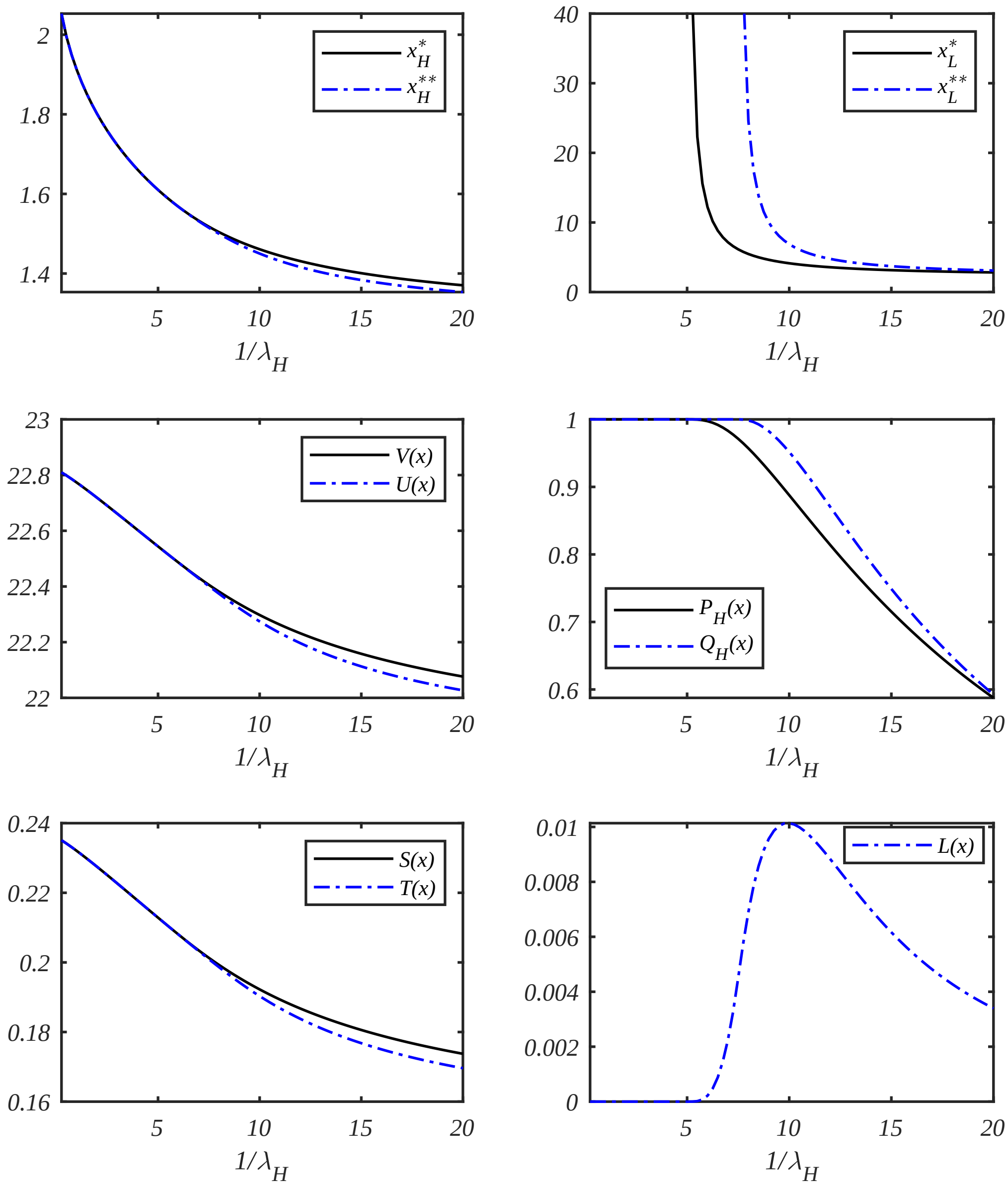

Figure 2: Comparative statics with respect to expected waiting time for high type, $1/\lambda_H$. The other parameter values are set as in Table 1. The figure plots the sellout thresholds x_i^*, x_i^{**}, firm values $V(x), U(x)$, high-price sellout probabilities $P_H(x), Q_H(x)$, state prices $S(x), T(x)$, and social loss $L(x)$.

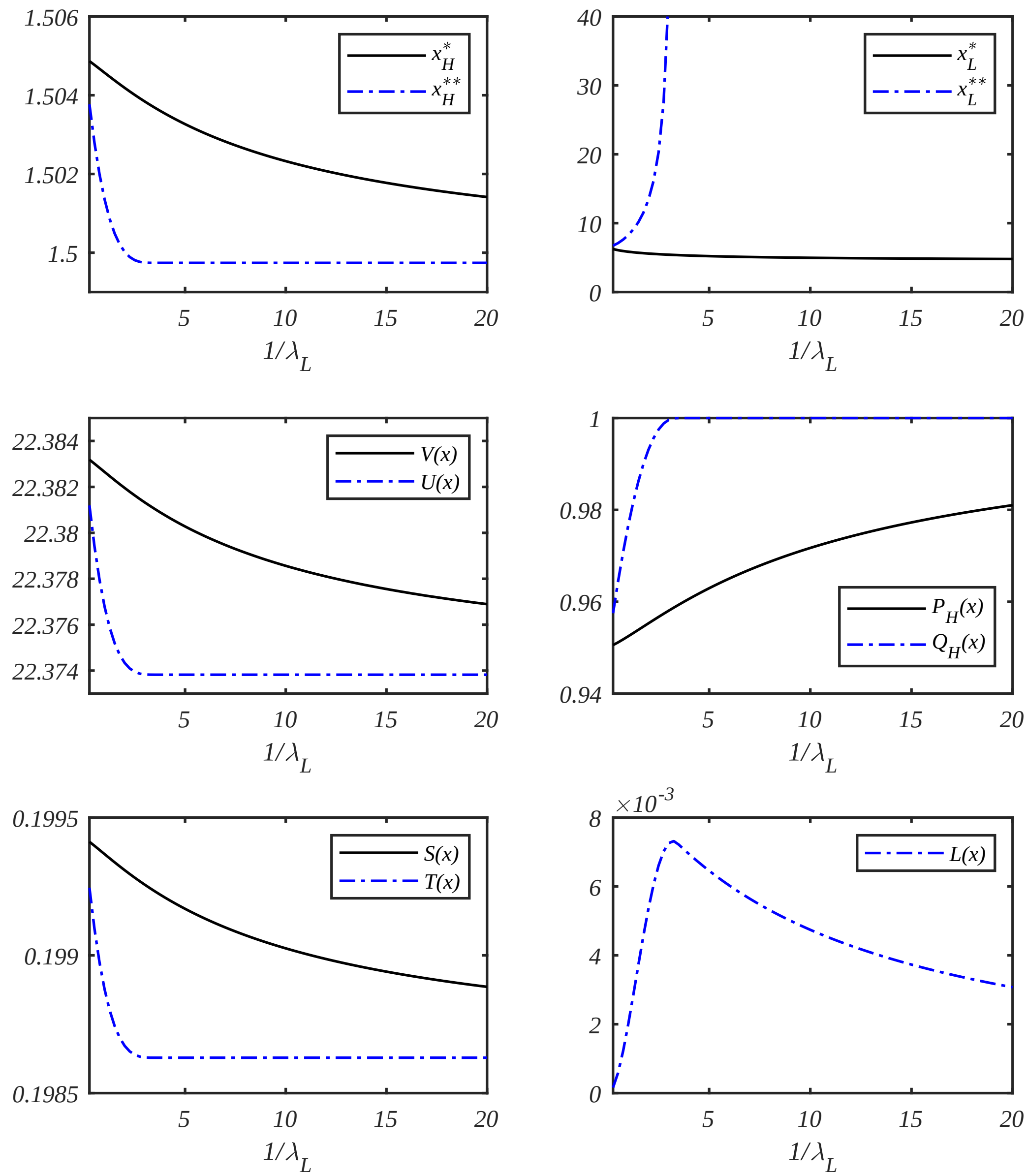

Figure 3: Comparative statics with respect to expected waiting time for low type, $1/\lambda_L$. The other parameter values are set as in Table 1. The figure plots the sellout thresholds x_i^*, x_i^{**}, firm values $V(x), U(x)$, high-price sellout probabilities $P_H(x), Q_H(x)$, state prices $S(x), T(x)$, and social loss $L(x)$.

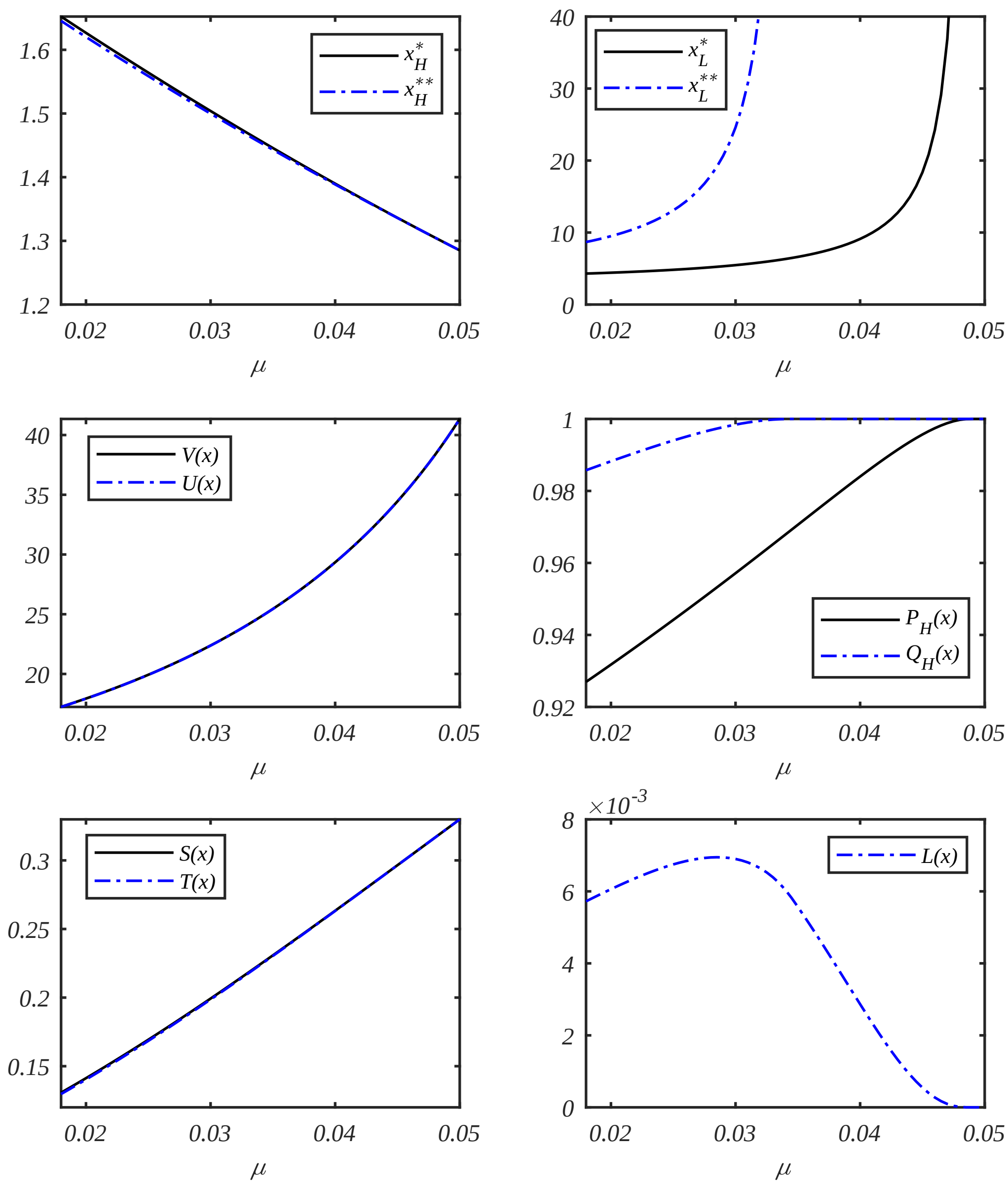

Figure 4: Comparative statics with respect to growth rate μ. The other parameter values are set as in Table 1. The figure plots the sellout thresholds x_i^*, x_i^{**}, firm values $V(x), U(x)$, high-price sellout probabilities $P_H(x), Q_H(x)$, state prices $S(x), T(x)$, and social loss $L(x)$.

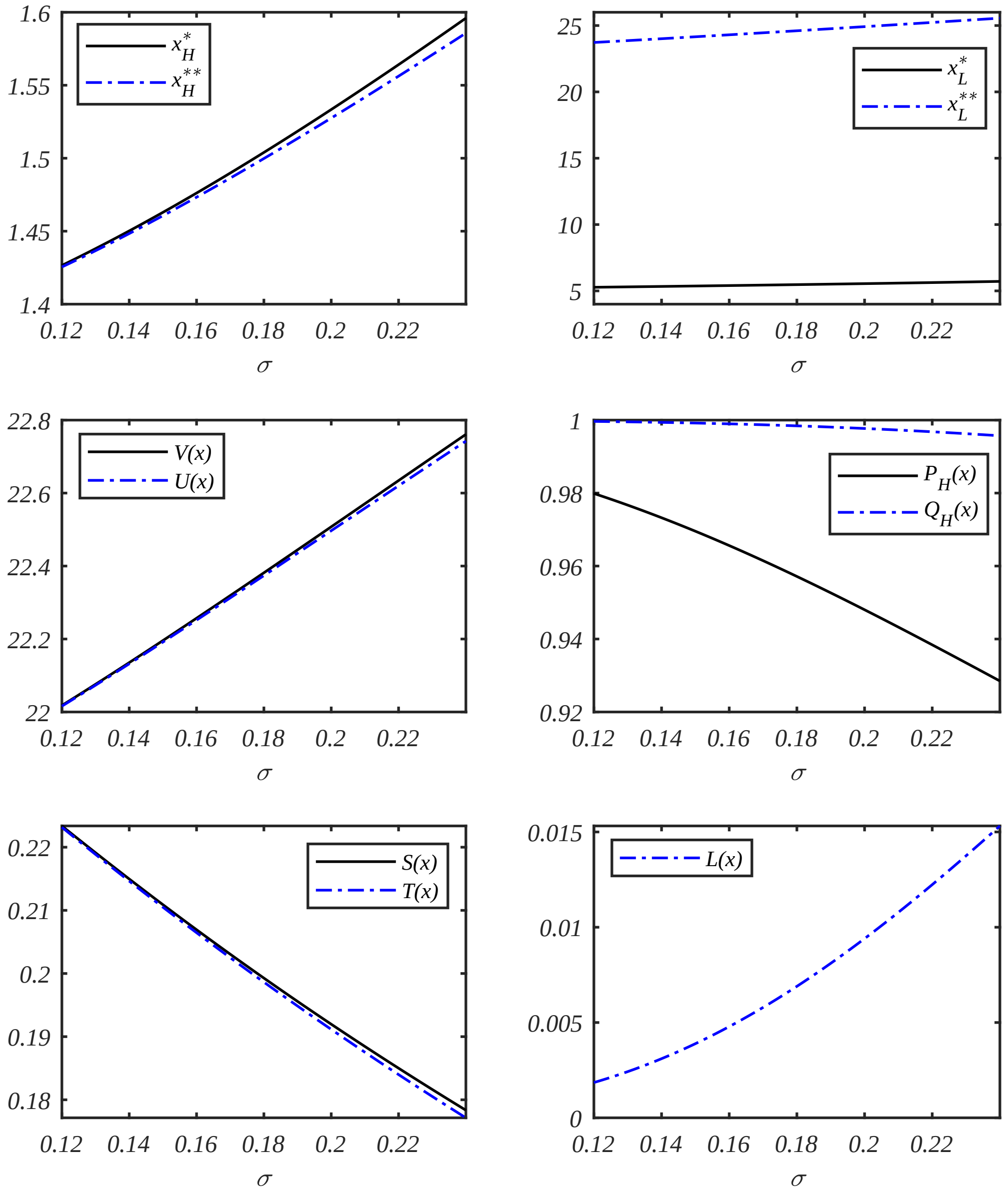

Figure 5: Comparative statics with respect to volatility σ. The other parameter values are set as in Table 1. The figure plots the sellout thresholds x_i^*, x_i^{**}, firm values $V(x), U(x)$, high-price sellout probabilities $P_H(x), Q_H(x)$, state prices $S(x), T(x)$, and social loss $L(x)$.

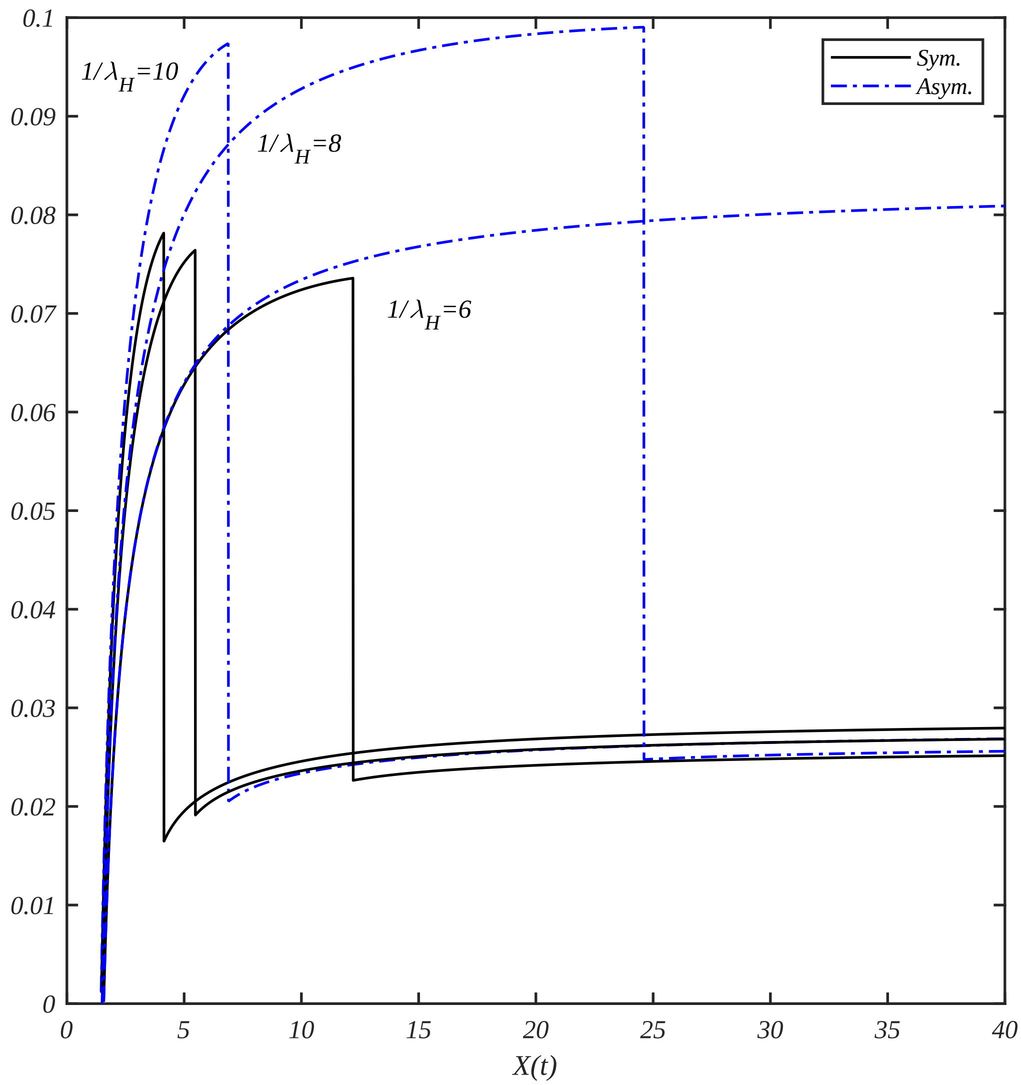

Figure 6: Jump sizes at the sellout time for $1/\lambda_H = 6, 8,$ and 10. The other parameter values are set as in Table 1. The black lines plot the symmetric information cases, i.e., $(a_H\pi(X(t)) - I - V(X(t)))/V(X(t))$ for $X(t) \in [x_H^*, x_L^*)$ and $(\bar{a}\pi(X(t)) - I - V(X(t)))/V(X(t))$ for $X(t) \in [x_H^*, x_L^*)$, while the blue dotted line plots the asymmetric information cases, i.e., $(a_H\pi(X(t)) - I - U(X(t)))/U(X(t))$ for $X(t) \in [x_H^{**}, x_L^{**})$ and $(a_L\pi(X(t)) - I - U(X(t)))/U(X(t))$ for $X(t) \in [x_H^*, x_L^*)$.