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for monopolistic firms

Takahiro Ishii

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Graduate School of Economics
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

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Takahiro Ishii†

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Abstract

The present study examines the effects of free technology sharing by a monopolistic final-good firm with other final-good firms. To this end, we consider two cases—first, where there exists one final-good firm in the final-good market and second, where there exist two final-good firms in the final-good market. Considering the free entry into the differentiated intermediate-goods market, the results of this study show that, when another firm enters the final-good market and transforms it into a two-firm oligopoly, cost efficiency improves because of an increase in the number of intermediate-goods firms. Furthermore, there is a possibility that the incumbent firm's profits increases not only for a two-firm oligopoly, but also for an oligopoly with three or more firms. Thus, sharing technology for free could improve the profits of incumbent firms.

Keywords: Monopolistic competition; Endogenous variety of intermediate goods; Technology sharing; Intermediate goods; Technology transfer

JEL classification: D43; L13; L16

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† Graduate School of Economics, Osaka University, 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, Japan

E-mail address: u979318f@ecs.osaka-u.ac.jp

1. Introduction

There are several real-life examples of firms sharing their skills and knowledge with other firms. In 2019, Toyota announced that it would make around 24,000 licenses available free of charge until 2030, for parts, such as motors, power control units, and system controls, required in electronic car manufacturing (Nikkei Business Daily, May 21, 2019). Moreover, The Victor Company of Japan (JVC) has been proactively sharing technology with both domestic and foreign manufacturers and engaging in original equipment manufacturing since the firm began to sell video home system (VHS)-style video tape recorders (VTRs) in 1976. According to the late Shizuo Takano, who led the development of VTRs as a JVC executive, "the best way to expand the adoption of VHS-style VTRs was to expand the VHS family." Powerful firms such as Matsushita Electronic Industrial Co., Hitachi, and Mitsubishi Electronics joined the VHS family successively, expanding their shares in both domestic and foreign markets at an accelerated pace (Nihon Keizai Shimbun, November 12, 1983). Furthermore, the Ford Motor Company, which achieved an unprecedented level of efficiency at the start of the 20th century, publicly disseminated its know-how (Nevins, 1954; Ishida, Matsumura and Matsushima, 2011).

Prima facie, it may appear that technology sharing allows other firms to emerge, spurs competition, and weakens the profits of incumbent firms. However, the possibility of incumbent firms experiencing a rise in profits by sharing their technology free of charge cannot be ruled out. The present study thus theoretically analyzes the effects that free sharing of technology may have on an incumbent firm's profits.

This study leverages the following model: Both final and intermediate goods-producing firms exist. Through free technology sharing by the incumbent monopolistic final-good producer, the final-good market transforms into a two-firm oligopoly. Subsequently, under this two-firm oligopoly, the final-good producers compete in terms of quantity. Intermediate-goods firms produce differentiated intermediate goods, and the number of emergent firms is determined endogenously through free-entry conditions. We analyze a case in which a final-good firm shares technology with another firm and allows it to enter the final-good market, thereby potentially increasing the incumbent firms' profits. The results show that it is possible for the price of the final good to decline, leading to higher demand based on its price elasticity and a rise in the profits of the final-good firms. This is explained by the following two effects: First, when the final-good market transforms from a single-firm monopoly to a two-firm oligopoly, the demand for intermediate goods by final-good firms expands, resulting in an increase in the number of intermediate-goods firms and thus, a reduction in the unit cost of the final good. Second, the entry of the other final-good firm lowers the markup.

Some related studies show the potential for firms to increase profits through licensing and the provision of technology to other firms. Mukherjee, Broll and Mukherjee (2008) analyze monopolistic final-good producers that provided skills to other firms for a fee to show that for an imperfectly competitive input market, a monopolistic firm can potentially benefit by licensing its technology and creating competition. In other words, if a unionized labor market exists, monopolistic firms' profits may rise when licenses are paid for, because a well-designed licensing contract may help ease competition and lower the labor wage rate owing to the threat of a decline in labor demand. In principle, the potential for monopolistic firms to increase returns arises due to the reduction in wage costs as a result of intensifying competition, surpassing the effects of any drop in firm profits.

These results indicate that there are no incentives for a monopolistic firm to provide technology to competitors, even for a fee, unless specific conditions are met. However, the present study analyzes the effect of a monopolistic firm providing technology free of charge to show that the firm potentially increases its profits, and that there are incentives for the firm to complete technology sharing without a fee.

Moreover, Mukherjee and Balasubramanian (2001) and Mukherjee, Broll and Mukherjee (2008) establish that firms that have monopolized technology can potentially increase their profits through technology licensing and provision. However, the present study differs from the existing literature regarding the mechanism by which owner firms' technology provision translates into improved profits.

The present paper takes into account that technology is shared free of charge in an economy where the number of intermediate-goods firms is endogenously determined. It shows that when technology is shared free of charge and the number of final-good firms changes, demand for intermediate goods rises among final-good firms. This promotes the entry of intermediate-goods firms and reduces the unit cost of the final good. This effect surpasses any negative effect of increased competition on profits. Therefore, this study demonstrates the possibility that monopolistic firms benefit from increased competition if they share their technology free of charge with other firms. [1]

In addition, Lewis and Winkler (2015) and Goh and Oliver (2002) share similarities with the present paper in that they use a general equilibrium model in which the emergence of intermediate goods is endogenously determined. However, the focus of these studies is different to the present paper, with Lewis and Winkler (2015) analyzing optimal taxation, and Goh and Oliver (2002) analyzing optimal patent protections in economies with upstreams and downstreams. Moreover, Devereux and Lee (2001) conduct a dynamic analysis using a general equilibrium model, in which the number of firms in the consumer goods industry is endogenously determined by a free-entry condition. The use of a general equilibrium model is one area in which that study resembles the present one. However, in the present paper the analysis is carried out with a model in which the entry of intermediate goods is endogenously determined. Moreover, Ju (2003) features a nested

[1] Wang (1998) and Wang and Yang (1999) also analyze the effects of technology provision and licensing on firm profits. However, these studies do not consider the situation where the number of intermediate-goods firm entrants is endogenously determined.

structure and analyzes firms that produce many varieties of products using an oligopolistic competition model. The present paper is similar to Ju (2003) in that both feature a nested structure. However, while Ju (2003) posits that the emergence of final goods is endogenously determined, the present paper posits that the emergence of intermediate goods is endogenously determined.

The remainder of this paper is organized as follows. Section 2 presents the model setting and the results. Section 3 presents the discussion of the results. Section 4 explains a case in which there are at least two firms in the final-goods market, and section 5 provides the conclusions of the present study.

2. Model

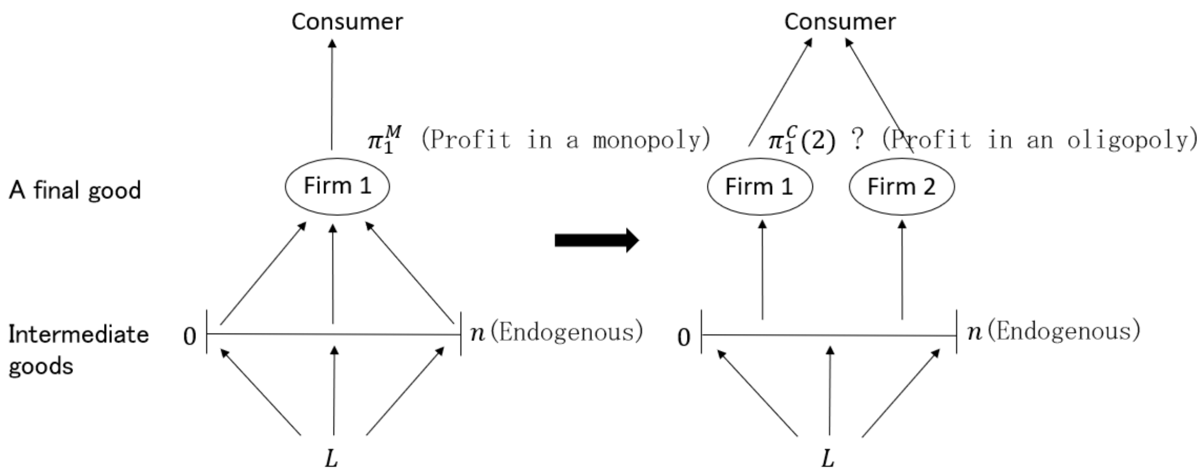

Figure 1: Structure of the model

Model overview

Intermediate goods are produced using labor inputs and are subsequently used to produce the final good for the consumer. The variety of intermediate goods is endogenously determined. Here, we consider the following two cases: First, there exists one final-good firm in the final-good market, and second, there exist two final-good firms in the final-good market. Fig. 1 summarizes the structure of the model.

This section is structured as follows. Section 2.1 analyzes consumers' decision-making. Section 2.2 features an analysis of the first case, wherein a monopolistic firm exists in the final-good market, and section 2.3 features an analysis of the second case, wherein two oligopolistic firms exist in the final-good market. In section 2.4, we compare the incumbent final-good firm's profits in the monopolistic and oligopolistic final-good markets. In section 2.5. we analyze social welfare for the monopolistic as well as the oligopolistic final-good market.

2.1. Consumer's decision-making

A consumer maximizes his or her utility subject to the budget constraint. The labor supply is endogenously determined. We use labor as the numeraire. The consumer problem is represented as follows.

$$max \ \ C^{\gamma} - \phi L \quad , 0 < \gamma < 1,$$

$$s.t. \ PC = L + \Pi,$$

where C is the consumption of the final good, L is the labor supply, P is the price of the final good, Π is the profit received by consumers, which is generated at the final-good firms, and γ is a parameter of the utility function. From the consumer's first-order utility maximization condition, we obtain:

$$C = \left(\frac{\phi}{\gamma} P\right)^{-\frac{1}{1-\gamma}}. \tag{1}$$

2.2. Monopolistic final-good market analysis

2.2.1. Final-good firm

The final good is produced using intermediate goods. Initially, only firm 1 exists in the final-good sector, that is, firm 1 is a monopolistic firm. The demand for each intermediate good, which is a factor of production, is determined by the final-good firm's cost minimization. When the final-good firm tries to minimize cost, it takes the output of the final good as given. We assume that x_{i1} is firm 1's demand for intermediate goods i; x_i represents the entire final-good sector's demand for intermediate goods i, which, for a monopoly, is equal to x_{i1}. Therefore, the cost minimization considered by the final-good firm, which is a monopolistic firm, is as follows:

$$\min \int_0^n p_i x_i \, di,$$

$$s.t. \ \left[\int_0^n (x_i)^{\alpha} \, di\right]^{\frac{1}{\alpha}} = C_1 \quad , 0 < \alpha < 1,$$

where C_1 is the production volume of firm 1, p_i represents the price of intermediate good i, n is the variety of intermediate goods, that is, the number of firms entering the intermediate-goods sector. Here, we assume that P_I denotes the unit cost of the final good and is expressed as follows:

$$P_I \equiv \left[\int_0^n (p_i)^{-\frac{\alpha}{1-\alpha}} di\right]^{-\frac{1-\alpha}{\alpha}} \tag{2}$$

The demand for intermediate good i by the final-good firm is defined as

$$x_i = (p_i)^{-\frac{1}{1-\alpha}} (P_I)^{\frac{1}{1-\alpha}} C_1. \tag{3}$$

Thus, the profit maximization behavior of the final-good firm is expressed as follows (see Appendix 1):

$$\max PC_1 - \int_0^n p_i x_i di, \tag{4}$$

where P denotes the price of the final good. By calculating (4), we obtain

$$P^M = \frac{1}{\gamma} P_I. \tag{5}$$

where P^M denotes the price of the final good when the final-good market is a monopoly. From (5), we find that the final-good firm sets the markup as $\frac{1}{\gamma}$ when the final-good market is a monopoly. Thus, from (1), (2), (3), and (5), the demand for intermediate good i by the final-good firm is

$$x_i = (p_i)^{-\frac{1}{1-\alpha}} \left[\int_0^n (p_i)^{-\frac{\alpha}{1-\alpha}} di\right]^{-\frac{1}{\alpha}} \left(\frac{\phi}{\gamma}\frac{P_I}{\gamma}\right)^{-\frac{1}{1-\gamma}}.$$

In addition, the profit of the final-good firm is expressed as follows:

$$\pi_1 = \left(\frac{1}{\gamma} - 1\right) \phi^{-\frac{1}{1-\gamma}} \gamma^{\frac{2}{1-\gamma}} (P_I)^{-\frac{\gamma}{1-\gamma}}.$$

2.2.2. Intermediate-goods firms

Each intermediate-good firm needs one unit of labor to produce one unit of an intermediate good. The intermediate-goods sector is assumed to be a monopolistic competition, wherein a fixed cost f expressed as labor input is required to launch an intermediate-good firm. Each intermediate-good firm produces one differentiated intermediate good and sets its price, p_i, which maximizes the firm's profit by taking into account the final-good firm's demand for that intermediate good. However, in the intermediate-goods sector, the equilibrium profit of each firm is zero because of free entry. As the intermediate-goods sector is a monopolistic competition, the variety of intermediate goods is large and continuous. The number of firms entering the intermediate-goods sector is equivalent to the variety of intermediate goods and is represented by n. Here, we first consider intermediate-goods firms' profit maximization behavior. The intermediate-good firm i's problem is set up as follows:

$$\max_{p_i} p_i x_i - x_i - f,$$

where p_i denotes the price of the intermediate good i, x_i is the final-good sector's demand for the intermediate good i, and f is the fixed cost to launch an intermediate-good firm. From $p_i = \frac{1}{\alpha}$, the unit cost of the final good is calculated as follows:

$$P_I = \frac{1}{\alpha} n^{-\frac{1-\alpha}{\alpha}}. \tag{6}$$

Here, the intermediate-goods sector is in a state of monopolistic competition; under the assumption of free entry, as long as there are profits, entry into the intermediate-goods sector continues, stopping only when all positive profits are exhausted. Therefore, the equilibrium profit for every intermediate-good firm is zero. By rearranging the condition that requires the intermediate-goods firms' profits to be zero for a monopoly in the final-good market, the number of intermediate-goods firms, or the variety of intermediate goods, at equilibrium can be calculated. From the free-entry condition, we obtain the following equation:

$$\left(\frac{1}{\alpha}\right)^{-\frac{1}{1-\alpha}} \left(\frac{1}{\alpha} n^{-\frac{1-\alpha}{\alpha}}\right)^{\frac{1}{1-\alpha}} \left(\frac{\phi}{\gamma} \frac{\frac{1}{\alpha} n^{-\frac{1-\alpha}{\alpha}}}{\gamma}\right)^{-\frac{1}{1-\gamma}} = \frac{\alpha}{1-\alpha} f. \tag{7}$$

Here, we assume that n^M denotes the number of intermediate-goods firms at equilibrium when the final-good market is a monopoly. We then calculate the number of intermediate-goods sector entrants $n = n^M$ that satisfies Equation (7) as follows:

$$n^M = (1-\alpha)^{\frac{\alpha(1-\gamma)}{\alpha-\gamma}} \alpha^{\frac{\alpha\gamma}{\alpha-\gamma}} f^{-\frac{\alpha(1-\gamma)}{\alpha-\gamma}} \phi^{-\frac{\alpha}{\alpha-\gamma}} \gamma^{\frac{2\alpha}{\alpha-\gamma}}.$$

Thus, the unit cost of the final good, P_I^M, when the final-good market is a monopoly, is as follows:

$$P_I^M = \alpha^{-\frac{\alpha(1-\gamma)}{\alpha-\gamma}} \phi^{\frac{1-\alpha}{\alpha-\gamma}} \gamma^{-\frac{2(1-\alpha)}{\alpha-\gamma}} (1-\alpha)^{-\frac{(1-\gamma)(1-\alpha)}{\alpha-\gamma}} f^{\frac{(1-\gamma)(1-\alpha)}{\alpha-\gamma}}.$$

By using the value of the unit cost of the final good, the profit of the incumbent final-good firm, firm 1, is

$$\pi_1^M = (1-\gamma)\phi^{-\frac{\alpha}{\alpha-\gamma}} \gamma^{\frac{\alpha+\gamma}{\alpha-\gamma}} \alpha^{\frac{\alpha\gamma}{\alpha-\gamma}} (1-\alpha)^{\frac{(1-\alpha)\gamma}{\alpha-\gamma}} f^{-\frac{(1-\alpha)\gamma}{\alpha-\gamma}}, \quad (8)$$

where π_1^M denotes firm 1's profit.

2.3. Analysis of a two-firm oligopolistic final-good market with free technology sharing

We assume that the final-good market transforms from a monopoly with firm 1, to an oligopoly with firms 1 and 2, due to free technology sharing by firm 1. We analyze the case in which the final-good market is a two-firm oligopoly by defining the production volumes of firms 1 and 2 as C_1 and C_2, respectively.

2.3.1. Final-good firms

The unit cost of the final good when the final-good market is a two-firm oligopoly is expressed in (2). In a two-firm oligopoly, the final-good firms 1 and 2 engage in quantity competition. Firm 1 maximizes its profit as follows (see Appendix 2):

$$\max_{C_1} P(c_1 + C_2)C_1 - \int_0^n p_i x_{i1}\, di.$$

By solving the above, we obtain the following equation for firm 1's first-order condition:

$$(C_1 + C_2)^{-(2-\gamma)}(\gamma C_1 + C_2) = \frac{\phi}{\gamma} P_I. \tag{9}$$

Firm 2's first-order condition for profit maximization is as follows:

$$(C_1 + C_2)^{-(2-\gamma)}(\gamma C_2 + C_1) = \frac{\phi}{\gamma} P_I.$$

Therefore, combining firm 1's and firm 2's first-order conditions for profit maximization, we obtain the production volumes for firms 1 and 2 as follows:

$$(C_1^C, C_2^C) = \left(\left(\frac{\phi}{\gamma}\frac{1}{1+\gamma}2^{(2-\gamma)}P_I\right)^{-\frac{1}{1-\gamma}}, \left(\frac{\phi}{\gamma}\frac{1}{1+\gamma}2^{(2-\gamma)}P_I\right)^{-\frac{1}{1-\gamma}}\right).$$

The market price in the final-good market under the Cournot-Nash equilibrium, P^C, is as follows:

$$P^C = \frac{2}{1+\gamma} P_I,$$

where x_i^C represents the entire final-good sector's demand for the intermediate good i in a two-firm oligopolistic final-good market. We calculate the final-good sector's demand for the intermediate good i below. For firms 1 and 2 in the final-good sector, we consider cost minimization at the production volume under the Cournot-Nash equilibrium. Here, the entire final-good sector's demand for intermediate good i is the sum of x_{i1} and x_{i2}, which minimizes firm 1 and firm 2's costs, respectively. Thus, the entire final-good sector's demand for intermediate good i is given by

$$x_i^C = 2(p_i)^{-\frac{1}{1-\alpha}}(P_I)^{\frac{1}{1-\alpha}}\left(\frac{\phi}{\gamma}\frac{1}{1+\gamma}2^{(2-\gamma)}P_I\right)^{-\frac{1}{1-\gamma}}.$$

2.3.2. Intermediate-goods firms

The behavior of intermediate-goods firms when the final-goods market is a two-firm oligopoly at the Cournot-Nash equilibrium is identical to the behavior of intermediate-goods firms when the final-good market is a monopoly. However, the intermediate-goods firm i considers the two-firm oligopolistic final-good market's demand in the Cournot-Nash equilibrium while determining its choices. Here, we calculate the number of intermediate-goods sector entrants when the final-good market is a two-firm oligopoly. From the free-entry condition, we obtain the following equation:

$$2\left(\frac{1}{\alpha}\right)^{-\frac{1}{1-\alpha}}\left(\frac{1}{\alpha}n^{-\frac{1-\alpha}{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{\phi}{\gamma}\frac{1}{1+\gamma}2^{(2-\gamma)}\frac{1}{\alpha}n^{-\frac{1-\alpha}{\alpha}}\right)^{-\frac{1}{1-\gamma}}=\frac{\alpha}{1-\alpha}f. \tag{10}$$

In this situation, the number of firms entering the intermediate-goods sector, $n=n^C(2)$, is as follows:

$$n^C(2)=2^{-\frac{\alpha}{\alpha-\gamma}}(1-\alpha)^{\frac{\alpha(1-\gamma)}{\alpha-\gamma}}\alpha^{\frac{\alpha\gamma}{(\alpha-\gamma)}}f^{-\frac{\alpha(1-\gamma)}{\alpha-\gamma}}\phi^{-\frac{\alpha}{\alpha-\gamma}}\gamma^{\frac{\alpha}{\alpha-\gamma}}(1+\gamma)^{\frac{\alpha}{\alpha-\gamma}}. \tag{11}$$

The unit cost of the final good when the final-good market is a two-firm oligopoly is expressed in (6). By substituting (11) into (6), we obtain

$$P_I^C(2)=2^{\frac{1-\alpha}{\alpha-\gamma}}(1-\alpha)^{-\frac{(1-\gamma)(1-\alpha)}{\alpha-\gamma}}\alpha^{-\frac{\alpha(1-\gamma)}{\alpha-\gamma}}f^{\frac{(1-\gamma)(1-\alpha)}{\alpha-\gamma}}\phi^{\frac{1-\alpha}{\alpha-\gamma}}\gamma^{-\frac{1-\alpha}{\alpha-\gamma}}(1+\gamma)^{-\frac{1-\alpha}{\alpha-\gamma}},$$

where $P_I^C(2)$ denotes the unit cost of the final good when the final-good market is a two-firm oligopoly.

2.3.3. The profit of the final-good firm in a two-firm oligopolistic final-goods market

Using the unit cost of the final good, the incumbent final-goods firm 1's profit, when the final-good market is a two-firm oligopoly, $\pi_1^C(2)$, is as follows:

$$\pi_1^C(2)=(1-\gamma)\phi^{-\frac{\alpha}{\alpha-\gamma}}\gamma^{\frac{\alpha}{\alpha-\gamma}}(1+\gamma)^{\frac{\gamma}{\alpha-\gamma}}2^{\frac{-2\alpha+\gamma}{\alpha-\gamma}}(1-\alpha)^{\frac{(1-\alpha)\gamma}{\alpha-\gamma}}\alpha^{\frac{\alpha\gamma}{\alpha-\gamma}}f^{-\frac{(1-\alpha)\gamma}{\alpha-\gamma}}. \tag{12}$$

By comparing $\pi_1^C(2)$ with π_1^M, the incumbent firm's profit when the final-good market is a monopoly, we consider the condition that $\pi_1^C(2)$ is greater than π_1^M.

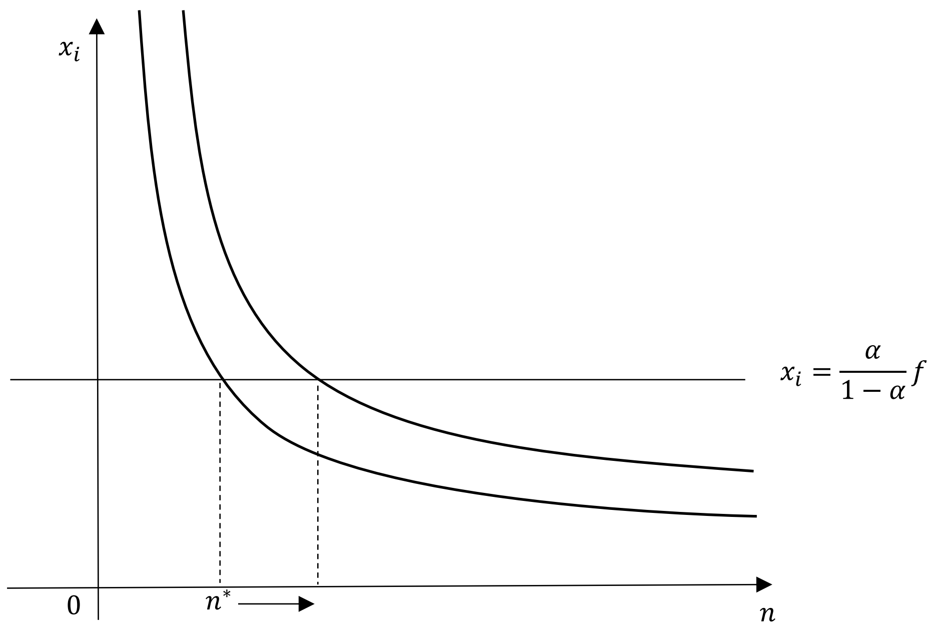

Figure 2: The relationship between the number of firms entering the intermediate-goods sector and the demand for intermediate good i when $\alpha > \gamma$.

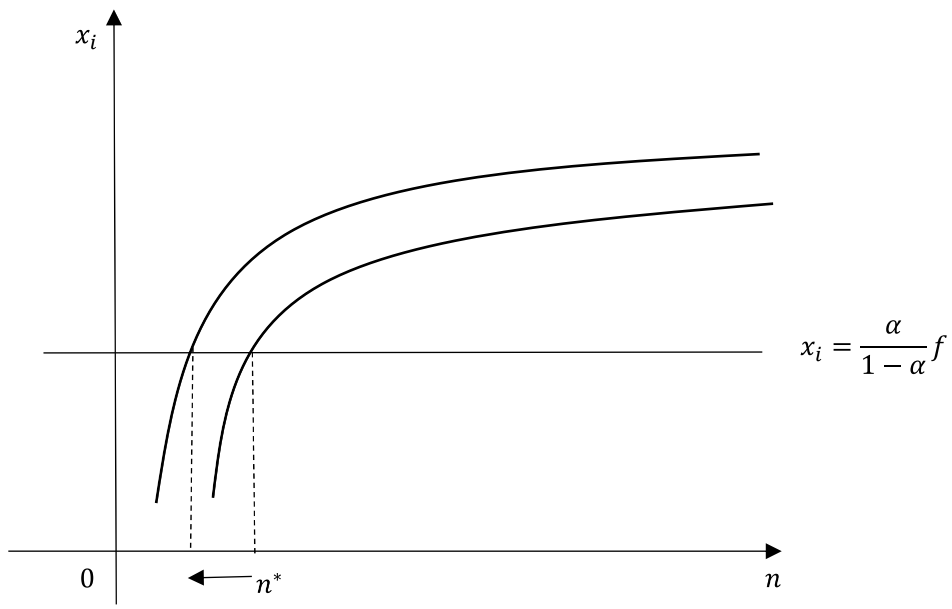

Figure 3: The relationship between the number of firms entering the intermediate-goods sector and the demand for intermediate good i when $\alpha < \gamma$.

In Figs. 2 and 3, the lower and upper curves show the relationship between the number of intermediate-goods sector entrants and the demand for intermediate good i when the final-good market is a monopoly and a two-firm oligopoly, respectively. In addition, n^* represents the equilibrium number of firms entering the intermediate-goods sector. When the number of firms entering the intermediate-goods sector increases, the unit cost of the final good decreases, which leads to a decrease in the price of the final good and subsequently, an increase in their demand. The effect of the price decrease on the increase in demand for the final good becomes larger as γ increases. Therefore, if γ is too large, the increasing demand effect for intermediate goods becomes larger; as the number of firms entering the intermediate-goods sector increases, an individual intermediate-good firm's profit increases, leading to an unstable free-entry equilibrium. For a stable equilibrium, there should be a limit on the magnitude of γ. Specifically, using Equations (7) and (10), we obtain the following inequality:

$$-\frac{1}{\alpha}+\frac{\frac{1-\alpha}{\alpha}}{1-\gamma}<0.$$

By rearranging the inequality above,

$$\gamma<\alpha.$$

Therefore, in the analysis that follows, we assume that $\alpha>\gamma$.

2.4. Comparing the incumbent's profit in monopolistic and two-firm oligopolistic final-good markets

The condition that the incumbent final-good firm's profit is greater in the two-firm oligopoly case than in the monopoly case, is as follows:

$$\pi_1^C(2)>\pi_1^M. \tag{13}$$

By substituting (8) and (12) into (13) and rearranging the inequality, we obtain the following inequality:

$$2^{-2\alpha+\gamma}>\left(\frac{\gamma}{1+\gamma}\right)^{\gamma}. \tag{14}$$

By rearranging Inequality (14), we obtain the following condition:

$$\alpha < \frac{\gamma}{2}\left[1 - log_2\left(\frac{\gamma}{1+\gamma}\right)\right]. \tag{15}$$

Therefore, we obtain Proposition 1.

Proposition 1. When the condition $\alpha < \frac{\gamma}{2}\left[1 - log_2\left(\frac{\gamma}{1+\gamma}\right)\right]$ holds, the incumbent final-good firm's profit is greater when the final-good market is a two-firm oligopoly due to free technology sharing by the incumbent than when the final-good market is a monopoly. Therefore, when $\alpha < \frac{\gamma}{2}\left[1 - log_2\left(\frac{\gamma}{1+\gamma}\right)\right]$ holds, the incumbent has an incentive to share its technology free of charge with the other final-good firm.

2.5. Welfare

U^M denotes welfare when the final-good market is a monopoly, while $U^C(2)$ denotes the welfare when the final-good market is a two-firm oligopoly. We can calculate U^M as follows:

$$U^M = \phi^{-\frac{\gamma}{\alpha-\gamma}} \alpha^{\frac{\alpha\gamma}{\alpha-\gamma}} (1-\alpha)^{\frac{\gamma(1-\alpha)}{\alpha-\gamma}} f^{-\frac{\gamma(1-\alpha)}{\alpha-\gamma}} \left(\gamma^{\frac{2\gamma}{\alpha-\gamma}} - \gamma^{\frac{2\alpha}{\alpha-\gamma}}\right).$$

We can calculate $U^C(2)$ as follows:

$$U^C(2) = \phi^{-\frac{\gamma}{\alpha-\gamma}} \alpha^{\frac{\alpha\gamma}{\alpha-\gamma}} (1-\alpha)^{\frac{\gamma(1-\alpha)}{\alpha-\gamma}} f^{-\frac{\gamma(1-\alpha)}{\alpha-\gamma}} \left\{\left[\frac{\gamma(1+\gamma)}{2}\right]^{\frac{\gamma}{\alpha-\gamma}} - \left[\frac{\gamma(1+\gamma)}{2}\right]^{\frac{\alpha}{\alpha-\gamma}}\right\}.$$

The above calculation shows that $U^C(2) > U^M$, leading to Proposition 2 (see Appendix 3).

Proposition 2. Welfare is greater when the final-good market is a two-firm oligopoly than when the final-good market is a monopoly.

We define the markup of the final good for a monopoly as k^M and for a two-firm oligopoly as k^C. Then, $[\pi_1^C(2)/\pi_1^M]$ is equal to the product of $[(k^C-1)/(k^M-1)]$, $\{[P_I^C(2)]/P_I^M\}$, and (C_1^C/C_1^M). If we assume that $[\pi_1^C(2)/\pi_1^M] > 1$, we obtain the following inequality:

$$\left(\frac{\gamma}{1+\gamma}\right)^{-\frac{\gamma}{\alpha-\gamma}} 2^{\frac{-2\alpha+\gamma}{\alpha-\gamma}} > 1. \tag{16}$$

By calculating both sides of Inequality (16) to the $(\alpha-\gamma)^{\text{th}}$ power under the assumption that γ is less than α and rearranging the inequality, we obtain the following inequality:

$$2^{-2\alpha+\gamma} > \left(\frac{\gamma}{1+\gamma}\right)^{\gamma}. \tag{17}$$

Inequality (17) is identical to Inequality (14) (see Appendix 4).

Here, we consider the condition that the unit cost of the final good is lower when the final-good market is a two-firm oligopoly at the Cournot-Nash equilibrium than when the final-good market is a monopoly (see Appendix 5). From the condition, $P_I^C < P_I^M$, we obtain the following inequality:

$$2^{\frac{1-\alpha}{\alpha-\gamma}} \gamma^{\frac{1-\alpha}{\alpha-\gamma}} (1+\gamma)^{-\frac{1-\alpha}{\alpha-\gamma}} < 1. \tag{18}$$

By rearranging (18), we obtain the following condition:

$$\gamma < 1. \tag{19}$$

Thus, under the assumption that $0 < \gamma < 1$, the unit cost of the final good is lower when the final-good market is a two-firm oligopoly than when the final-good market is a monopoly.

3. Discussion of the results

In this study, we analyze a situation in which the final-good market changes from a monopoly to a two-firm oligopoly due to free sharing of technology by the incumbent final-good firm, firm 1, resulting in the final-good market attaining its Cournot-Nash equilibrium. The results show that the incumbent's profit grows when the final-good market changes from a monopolistic market to a two-firm oligopoly if certain parameter conditions are met, namely, when α is less than $\frac{\gamma}{2}\left[1 - log_2\left(\frac{\gamma}{1+\gamma}\right)\right]$. The results of the analysis regarding the number of firms entering the intermediate-goods sector also indicate that $n^C > n^M$ is necessary for the incumbent firm's profit in a monopoly to be greater than that in an oligopoly.

For (P_I^C/P_I^M), by dividing the unit cost of the final good in an oligopoly by that in a monopoly, we obtain the following expression:

$$\frac{P_I^C}{P_I^M} = \frac{\frac{1}{\alpha}(n^C)^{-\frac{1-\alpha}{\alpha}}}{\frac{1}{\alpha}(n^M)^{-\frac{1-\alpha}{\alpha}}} = \left(\frac{n^C}{n^M}\right)^{-\left(\frac{1}{\alpha}-1\right)}.$$

The term $((1/\alpha) - 1)$ in the above equation represents the strength of the effect of an increase in the number of firms entering the intermediate-goods sector on the unit cost of the final good. Therefore, the larger the magnitude of $(1/\alpha)$, the greater the reduction in the unit cost of the final good due to the increase in the number of firms entering the intermediate-goods sector. In other words, the larger the magnitude of $(1/\alpha)$, the greater the increase in productivity. Therefore, the condition $\alpha < \frac{\gamma}{2}\left[1 - log_2\left(\frac{\gamma}{1+\gamma}\right)\right]$ in Proposition 1 requires α to be less than the constant value, that is, $\frac{\gamma}{2}\left[1 - log_2\left(\frac{\gamma}{1+\gamma}\right)\right]$, and the increase in the number of firms entering the intermediate-goods sector largely contributes to the unit cost reduction of the final good.

When the final-good market changes from a monopoly to an oligopoly, certain negative and positive effects change firm 1's profits. This change directly weakens the profit by lowering the markup. Furthermore, the profit per firm is reduced as consumer demand is split between the two firms. A positive effect may also be seen in terms of a decline in the final good's price, a consequential increase in its demand based on price elasticity, and a rise in the profits of the final-good firms. This results from two effects: First, through the transformation in the final-good market structure, the final-good firms' demand for intermediate goods expands, leading to an increase in the number of firms entering the intermediate-goods sector, which ultimately translates into a reduced unit cost of the final good. Second, the entry of the other final-good firm lowers the profit markup. The profit of firm 1 rises when the final-good market changes from a monopoly to a two-firm oligopoly as the positive effect outweighs the negative effects, as represented by the condition, $\alpha < \frac{\gamma}{2}\left[1 - log_2\left(\frac{\gamma}{1+\gamma}\right)\right]$, in Proposition 1.

In addition, the profit of the incumbent final-good firm π_1 is expressed as follows:

$$\pi_1 = (k-1)P_I C_1,$$

where k and P_I are the markup and the unit cost of the final good, respectively, and C_1 is the production volume of firm 1.

Firm 1's profit is represented by the product of $(k-1)$ and $P_I C_1$. When $(k-1)$ decreases as the final-good market changes from a monopoly to a two-firm oligopoly, there is a possibility that the incumbent final-good firm's profit rises.

4. The analysis when the final good market is an oligopoly with two or more firms

We also analyze how the incumbent firm's profit changes when one or more final-good firms enter the market due to free technology sharing by the incumbent final-good firm. For the situation in which the final-good market changes from a monopolistic market to an oligopoly with m firms due to technology sharing (see Appendix 6), the market price under the Cournot-Nash equilibrium is as follows:

$$P^C = \frac{m}{m-1+\gamma} P_I.$$

Considering the free-entry condition of intermediate-goods firms, we obtain the following equation:

$$m\left(\frac{1}{\alpha}\right)^{-\frac{1}{1-\alpha}}\left(\frac{1}{\alpha}n^{-\frac{1-\alpha}{\alpha}}\right)^{\frac{1}{1-\alpha}} m^{-\frac{2-\gamma}{1-\gamma}}(m-1+\gamma)^{\frac{1}{1-\gamma}}\phi^{-\frac{1}{1-\gamma}}\gamma^{\frac{1}{1-\gamma}}\left(\frac{1}{\alpha}n^{-\frac{1-\alpha}{\alpha}}\right)^{-\frac{1}{1-\gamma}} = \frac{\alpha}{1-\alpha}f.$$

Here, similar to the case where the final-good market changes from a monopolistic market to a two-firm oligopoly, we assume $\gamma < \alpha$, such that the profit of each intermediate-good firm is a decreasing function of the number of firms entering the intermediate-goods sector, n, which is calculated as follows:

$$n^C(m) = \alpha^{\frac{\alpha\gamma}{\alpha-\gamma}} m^{-\frac{\alpha}{\alpha-\gamma}}(m-1+\gamma)^{\frac{\alpha}{\alpha-\gamma}}\phi^{-\frac{\alpha}{\alpha-\gamma}}\gamma^{\frac{\alpha}{\alpha-\gamma}}(1-\alpha)^{\frac{\alpha(1-\gamma)}{\alpha-\gamma}} f^{-\frac{\alpha(1-\gamma)}{\alpha-\gamma}}.$$

The unit cost of the final good is calculated as follows:

$$P_I^C(m) = \alpha^{-\frac{\alpha(1-\gamma)}{\alpha-\gamma}} m^{\frac{1-\alpha}{\alpha-\gamma}}(m-1+\gamma)^{-\frac{1-\alpha}{\alpha-\gamma}}\phi^{\frac{1-\alpha}{\alpha-\gamma}}\gamma^{-\frac{1-\alpha}{\alpha-\gamma}}(1-\alpha)^{-\frac{(1-\alpha)(1-\gamma)}{\alpha-\gamma}} f^{\frac{(1-\alpha)(1-\gamma)}{\alpha-\gamma}}.$$

The profit of the incumbent final-good firm is as follows:

$$\pi_1^C(m) = (1-\gamma)\phi^{-\frac{\alpha}{\alpha-\gamma}}\gamma^{\frac{\alpha}{\alpha-\gamma}}\alpha^{\frac{\alpha\gamma}{\alpha-\gamma}}(1-\alpha)^{\frac{(1-\alpha)\gamma}{\alpha-\gamma}} f^{-\frac{(1-\alpha)\gamma}{\alpha-\gamma}}(m-1+\gamma)^{\frac{\gamma}{\alpha-\gamma}} m^{\frac{-2\alpha+\gamma}{\alpha-\gamma}}.$$

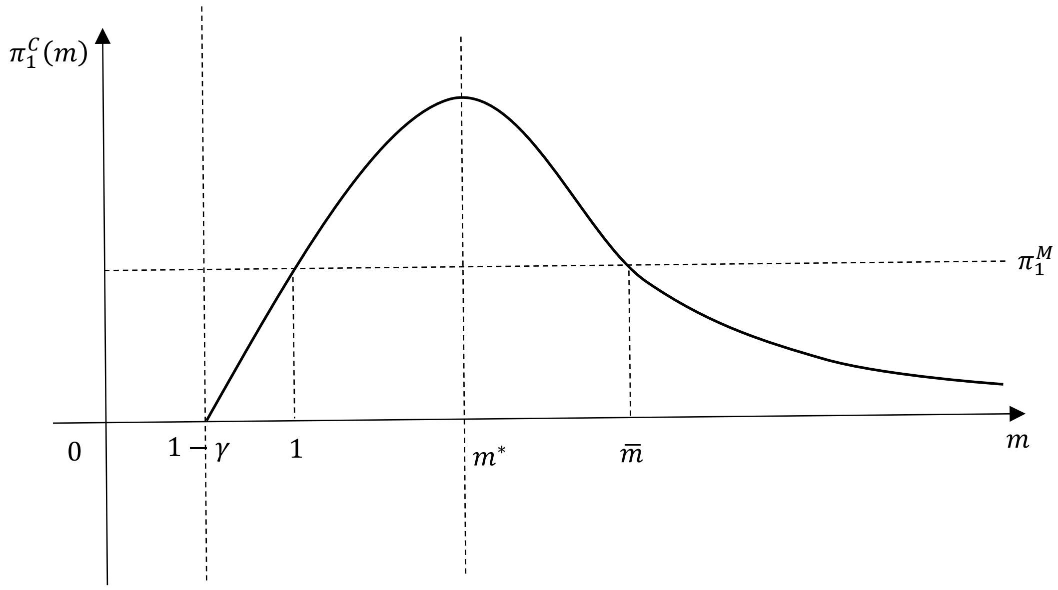

Figure 4: The relationship between the number of final-good firms, m, and the incumbent firm's profit, $\pi_1^C(m)$.

The function of $\pi_1^C(m)$ has an inverted-U shape, as shown in Fig. 4, where m^* is the optimal number of firms for the incumbent firm in the final-good market. As α approaches 1, m^* decreases (see Appendix 7).

We now analyze the maximum number of final-good firms. Let $\bar{m}$ be the maximum number of final-good firms such that the profit of the incumbent final-good firm when there exist $\bar{m}$ final-good firms is equal to the profit in the case of a monopolistic final-good market. In the equation that determines $\bar{m}$, as α increases, $\bar{m}$ decreases (see Appendix 7).

Proposition 3. Suppose that $\pi_1^C(2) > \pi_1^M$ and there exists $\bar{m} > 2$. If $m < \bar{m}$, free technology sharing with $(m-1)$ firms raises the incumbent's profit, thereby incentivizing it to share its technology free of charge. The larger the α, the lower the maximum number of final-good firms, $\bar{m}$.

The larger the α, that is, the lower the elasticity of final-good output to the variety of intermediate goods or the productivity of the number of intermediate goods in the production of the final good, the lower the $\bar{m}$. Therefore, when α is large, that is, when the productivity is low, it is better for the incumbent final-good firm if the other final-good firms do not enter the market.

5. Conclusions

In the present paper, we consider a model in which another firm enters the final-good market due to free technology sharing, changing the market into a two-firm oligopoly. We analyze the effect of free technology sharing by the monopolistic final-good firm with the other final-good firm using the Cournot-Nash equilibrium. The findings indicate that there is a possibility that free technology sharing is beneficial for the incumbent final-good firm in the way of increasing profits. Additionally, an analysis of the case wherein the final-good market changes from a monopoly to an oligopoly with m firms due to free technology sharing by the incumbent highlights the possibility that the incumbent's profit increases not only when the final-good market changes from a monopolistic market to a two-firm oligopoly but also when it transforms into an oligopoly with three or more firms.

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Appendix

Appendix 1

When the final-good market is a monopoly, the final-good firm maximizes its profit as follows:

$$max\, PC_1 - \int_0^n p_i x_i di\,,$$

As $\int_0^n p_i x_i \, di = P_I C_1$, we obtain the following expression:

$$\pi_1 = PC_1 - P_I C_1$$

As $C_1 = \left(\frac{\phi}{\gamma}P\right)^{-\frac{1}{1-\gamma}}$, rearranging π_1 yields

$$\pi_1 = \left(\frac{\phi}{\gamma}\right)^{-\frac{1}{1-\gamma}} P^{-\frac{\gamma}{1-\gamma}} - \left(\frac{\phi}{\gamma}\right)^{-\frac{1}{1-\gamma}} P_I P^{-\frac{1}{1-\gamma}}.$$

From the first-order condition for P, we obtain the following expression:

$$\frac{\partial \pi_1}{\partial P} = 0.$$

By calculating this, we obtain

$$-\frac{\gamma}{1-\gamma}\left(\frac{\phi}{\gamma}\right)^{-\frac{1}{1-\gamma}} P^{-\frac{1}{1-\gamma}} - \left(-\frac{1}{1-\gamma}\right)\left(\frac{\phi}{\gamma}\right)^{-\frac{1}{1-\gamma}} P_I P^{-\frac{1}{1-\gamma}-1} = 0.$$

Rearranging this equation provides the following:

$$-\frac{1}{1-\gamma}\left(\frac{\phi}{\gamma}\right)^{-\frac{1}{1-\gamma}} P^{-\frac{1}{1-\gamma}}\left(\gamma - \frac{P_I}{P}\right) = 0.$$

Thus, we obtain expression (5):

$$P^M = \frac{1}{\gamma} P_I.$$

Appendix 2

Firm 1's profit maximization problem is

$$\max_{C_1} P(C_1 + C_2)C_1 - \int_0^n p_i x_i \, di.$$

As $\int_0^n p_i x_i \, di = P_I C_1$, we obtain the following expression:

$$\max_{C_1}(P(C_1 + C_2) - P_I)C_1.$$

Suppose $C = C_1 + C_2$. The consumer demand function yields the following expression:

$$C = \left(\frac{\phi}{\gamma}P\right)^{-\frac{1}{1-\gamma}}.$$

By rearranging this expression, we obtain

$$P = \frac{\gamma}{\phi}C^{-(1-\gamma)}.$$

Therefore, firm 1's profit is

$$\pi_1 = \left[\frac{\gamma}{\phi}(C_1 + C_2)^{-(1-\gamma)} - P_I\right]C_1.$$

Thus, firm 1's profit maximization problem can be expressed as follows:

$$\max_{C_1}\left[\frac{\gamma}{\phi}(C_1 + C_2)^{-(1-\gamma)} - P_I\right]C_1.$$

From $\frac{\partial \pi_1^C}{\partial C_1^C} = 0$, we obtain the following expression:

$$\frac{\partial}{\partial C_1}\left[\frac{\gamma}{\phi}(C_1 + C_2)^{-(1-\gamma)}C_1 - P_I C_1\right] = 0.$$

By calculating this expression, we obtain the following equation:

$$\frac{\gamma}{\phi}[-(1-\gamma)](C_1+C_2)^{-(1-\gamma)-1}C_1+\frac{\gamma}{\phi}(C_1+C_2)^{-(1-\gamma)}-P_I=0.$$

Rearranging the above equation yields

$$(C_1+C_2)^{-(1-\gamma)}[1-(1-\gamma)(C_1+C_2)^{-1}C_1]-\frac{\phi}{\gamma}P_I=0.$$

Thus, we obtain Equation (9), which expresses the first-order condition for firm 1's maximization problem, as follows:

$$(C_1+C_2)^{-(2-\gamma)}(\gamma C_1+C_2)=\frac{\phi}{\gamma}P_I.$$

Appendix 3

To prove that $U^C(2)>U^M$, we set $\Omega(s)$ as follows:

$$\Omega(s)=s^\gamma-s^\alpha.$$

Therefore, we obtain the following expression:

$$\Omega'(s)=\gamma s^{\gamma-1}-\alpha s^{\alpha-1}.$$

When $\Omega'(s)=0$, substituting the above expression into $\Omega'(s)=0$ yields

$$\gamma s^{\gamma-1}=\alpha s^{\alpha-1}.$$

By rearranging this equation, we obtain

$$s=\left(\frac{\gamma}{\alpha}\right)^{\frac{1}{\alpha-\gamma}}<1.$$

Therefore, $\Omega(s)$ increases as s increases when s is less than $\left(\frac{\gamma}{\alpha}\right)^{\frac{1}{\alpha-\gamma}}$ and decreases as s increases when s is greater than $\left(\frac{\gamma}{\alpha}\right)^{\frac{1}{\alpha-\gamma}}$. We assume $\gamma<\alpha$ and $\alpha-\gamma<1$. As $0<\gamma<1$, we obtain the following inequality:

$$\gamma > \frac{\gamma(1+\gamma)}{2} > \frac{\gamma(2\gamma)}{2}.$$

That is,

$$\gamma > \frac{\gamma(1+\gamma)}{2} > \gamma^2.$$

Here, as $\gamma < \frac{\gamma}{\alpha}$, we obtain

$$\gamma^2 < \frac{\gamma(1+\gamma)}{2} < \gamma < \frac{\gamma}{\alpha}. \tag{A3.1}$$

As $\alpha - \gamma < 1$, we obtain

$$\frac{1}{\alpha - \gamma} > 1. \tag{A3.2}$$

From (A3.1) and (A3.2), we obtain

$$(\gamma^2)^{\frac{1}{\alpha-\gamma}} < \left[\frac{\gamma(1+\gamma)}{2}\right]^{\frac{1}{\alpha-\gamma}} < \left(\frac{\gamma}{\alpha}\right)^{\frac{1}{\alpha-\gamma}}.$$

Therefore, considering that $(\gamma^2)^{\frac{1}{\alpha-\gamma}}$ and $\left[\frac{\gamma(1+\gamma)}{2}\right]^{\frac{1}{\alpha-\gamma}}$ are both less than $\left(\frac{\gamma}{\alpha}\right)^{\frac{1}{\alpha-\gamma}}$, and $\left[\frac{\gamma(1+\gamma)}{2}\right]^{\frac{1}{\alpha-\gamma}}$ is greater than $(\gamma^2)^{\frac{1}{\alpha-\gamma}}$ because $\Omega(s)$ increases as s increases when s is less than $\left(\frac{\gamma}{\alpha}\right)^{\frac{1}{\alpha-\gamma}}$, we obtain

$$\Omega\left[(\gamma^2)^{\frac{1}{\alpha-\gamma}}\right] < \Omega\left\{\left[\frac{\gamma(1+\gamma)}{2}\right]^{\frac{1}{\alpha-\gamma}}\right\}.$$

Therefore, we obtain the following:

$$\Omega\left(\gamma^{\frac{2}{\alpha-\gamma}}\right) < \Omega\left\{\left[\frac{\gamma(1+\gamma)}{2}\right]^{\frac{1}{\alpha-\gamma}}\right\}. \tag{A3.3}$$

U^M and $U^C(2)$ are as follows:

$$U^M = \phi^{-\frac{\gamma}{\alpha-\gamma}} \alpha^{\frac{\alpha\gamma}{\alpha-\gamma}} (1-\alpha)^{\frac{\gamma(1-\alpha)}{\alpha-\gamma}} f^{-\frac{\gamma(1-\alpha)}{\alpha-\gamma}} \left(\gamma^{\frac{2\gamma}{\alpha-\gamma}} - \gamma^{\frac{2\alpha}{\alpha-\gamma}}\right),$$

$$U^C(2) = \phi^{-\frac{\gamma}{\alpha-\gamma}} \alpha^{\frac{\alpha\gamma}{\alpha-\gamma}} (1-\alpha)^{\frac{\gamma(1-\alpha)}{\alpha-\gamma}} f^{-\frac{\gamma(1-\alpha)}{\alpha-\gamma}} \left\{\left[\frac{\gamma(1+\gamma)}{2}\right]^{\frac{\gamma}{\alpha-\gamma}} - \left[\frac{\gamma(1+\gamma)}{2}\right]^{\frac{\alpha}{\alpha-\gamma}}\right\}.$$

That is,

$$U^M = \phi^{-\frac{\gamma}{\alpha-\gamma}} \alpha^{\frac{\alpha\gamma}{\alpha-\gamma}} (1-\alpha)^{\frac{\gamma(1-\alpha)}{\alpha-\gamma}} f^{-\frac{\gamma(1-\alpha)}{\alpha-\gamma}} \left[\left(\gamma^{\frac{2}{\alpha-\gamma}}\right)^{\gamma} - \left(\gamma^{\frac{2}{\alpha-\gamma}}\right)^{\alpha}\right], \tag{A3.4}$$

$$U^C(2) = \phi^{-\frac{\gamma}{\alpha-\gamma}} \alpha^{\frac{\alpha\gamma}{\alpha-\gamma}} (1-\alpha)^{\frac{\gamma(1-\alpha)}{\alpha-\gamma}} f^{-\frac{\gamma(1-\alpha)}{\alpha-\gamma}} \left\{\left\{\left[\frac{\gamma(1+\gamma)}{2}\right]^{\frac{1}{\alpha-\gamma}}\right\}^{\gamma} - \left\{\left[\frac{\gamma(1+\gamma)}{2}\right]^{\frac{1}{\alpha-\gamma}}\right\}^{\alpha}\right\}. \tag{A3.5}$$

Therefore,

$$U^M = \phi^{-\frac{\gamma}{\alpha-\gamma}} \alpha^{\frac{\alpha\gamma}{\alpha-\gamma}} (1-\alpha)^{\frac{\gamma(1-\alpha)}{\alpha-\gamma}} f^{-\frac{\gamma(1-\alpha)}{\alpha-\gamma}} \Omega\left(\gamma^{\frac{2}{\alpha-\gamma}}\right),$$

$$U^C(2) = \phi^{-\frac{\gamma}{\alpha-\gamma}} \alpha^{\frac{\alpha\gamma}{\alpha-\gamma}} (1-\alpha)^{\frac{\gamma(1-\alpha)}{\alpha-\gamma}} f^{-\frac{\gamma(1-\alpha)}{\alpha-\gamma}} \Omega\left\{\left[\frac{\gamma(1+\gamma)}{2}\right]^{\frac{1}{\alpha-\gamma}}\right\}.$$

Comparing (A3.4) and (A3.5), U^M and $U^C(2)$ both have the term $\phi^{-\frac{\gamma}{\alpha-\gamma}} \alpha^{\frac{\alpha\gamma}{\alpha-\gamma}} (1-\alpha)^{\frac{\gamma(1-\alpha)}{\alpha-\gamma}} f^{-\frac{\gamma(1-\alpha)}{\alpha-\gamma}}$ in common. Further, from (A3.3), $\Omega\left\{\left[\frac{\gamma(1+\gamma)}{2}\right]^{\frac{1}{\alpha-\gamma}}\right\} > \Omega\left(\gamma^{\frac{2}{\alpha-\gamma}}\right)$, that is, $\left\{\left[\frac{\gamma(1+\gamma)}{2}\right]^{\frac{1}{\alpha-\gamma}}\right\}^{\gamma} - \left\{\left[\frac{\gamma(1+\gamma)}{2}\right]^{\frac{1}{\alpha-\gamma}}\right\}^{\alpha} > \left(\gamma^{\frac{2}{\alpha-\gamma}}\right)^{\gamma} - \left(\gamma^{\frac{2}{\alpha-\gamma}}\right)^{\alpha}$. Thus, $U^C(2) > U^M$.

Appendix 4

By calculating $[(k^C - 1)/(k^M - 1)]$, we obtain

$$\frac{k^C - 1}{k^M - 1} = \frac{\gamma}{1+\gamma}. \qquad \text{(A4.1)}$$

From (A4.1), $[(k^C - 1)/(k^M - 1)] < 1$. We calculate the ratio of the unit cost in the case where the final-good market is a two-firm oligopoly to the unit cost in the case where the final-good market is a monopoly as follows:

$$\frac{P_I^C(2)}{P_I^M} = 2^{\frac{1-\alpha}{\alpha-\gamma}}\gamma^{\frac{1-\alpha}{\alpha-\gamma}}(1+\gamma)^{-\frac{1-\alpha}{\alpha-\gamma}}.$$

By rearranging this expression, we obtain

$$\frac{P_I^C(2)}{P_I^M} = \left(\frac{2\gamma}{1+\gamma}\right)^{\frac{1-\alpha}{\alpha-\gamma}}.$$

The production volume of the final good produced by firm 1 when the market of the final good is a monopoly is

$$C_1^M = \left(\frac{\phi}{\gamma}\frac{1}{\gamma}P_I^M\right)^{-\frac{1}{1-\gamma}}.$$

By rearranging this expression, we obtain

$$C_1^M = \phi^{-\frac{1}{\alpha-\gamma}}\gamma^{\frac{2}{\alpha-\gamma}}\alpha^{\frac{\alpha}{\alpha-\gamma}}(1-\alpha)^{\frac{1-\alpha}{\alpha-\gamma}}f^{-\frac{1-\alpha}{\alpha-\gamma}}.$$

The production volume of the final good produced by firm 1 when the final-good market is a two-firm oligopoly is

$$C_1^C = \left(\frac{\phi}{\gamma}\frac{1}{1+\gamma}2^{(2-\gamma)}P_I^C\right)^{-\frac{1}{1-\gamma}}.$$

By rearranging this, we obtain the following expression:

$$C_1^C = \phi^{-\frac{1}{\alpha-\gamma}} \gamma^{\frac{1}{\alpha-\gamma}} (1+\gamma)^{\frac{1}{\alpha-\gamma}} 2^{-\frac{1+\alpha-\gamma}{\alpha-\gamma}} \alpha^{\frac{\alpha}{\alpha-\gamma}} (1-\alpha)^{\frac{1-\alpha}{\alpha-\gamma}} f^{-\frac{1-\alpha}{\alpha-\gamma}}.$$

Therefore, we can calculate $\frac{C_1^C}{C_1^M}$ as follows

$$\frac{C_1^C}{C_1^M} = \frac{1}{2}\left(\frac{2\gamma}{1+\gamma}\right)^{-\frac{1}{\alpha-\gamma}}.$$

The ratio of $\pi_1^C(2)$ to π_1^M is given by

$$\frac{\pi_1^C(2)}{\pi_1^M} = \frac{(k^C-1)P_I^C(2)C_1^C}{(k^M-1)P_I^M C_1^M} = \frac{(k^C-1)}{(k^M-1)} \frac{P_I^C(2)}{P_I^M} \frac{C_1^C}{C_1^M}.$$

We can thus calculate $[\pi_1^C(2)/\pi_1^M]$ as follows:

$$\frac{\pi_1^C(2)}{\pi_1^M} = \frac{\gamma}{1+\gamma}\left(\frac{2\gamma}{1+\gamma}\right)^{\frac{1-\alpha}{\alpha-\gamma}} \frac{1}{2}\left(\frac{2\gamma}{1+\gamma}\right)^{-\frac{1}{\alpha-\gamma}}.$$

By rearranging this expression, we obtain

$$\frac{\pi_1^C(2)}{\pi_1^M} = \frac{1}{2}\frac{\gamma}{1+\gamma}\left(\frac{2\gamma}{1+\gamma}\right)^{-\frac{\alpha}{\alpha-\gamma}}.$$

Assuming that $[\pi_1^C(2)/\pi_1^M] > 1$, we obtain

$$\left(\frac{\gamma}{1+\gamma}\right)^{-\frac{\gamma}{\alpha-\gamma}} 2^{\frac{-2\alpha+\gamma}{\alpha-\gamma}} > 1.$$

Under the assumption that γ is less than α, we can calculate both sides of the $(\alpha-\gamma)^{\text{th}}$ power as follows:

$$\left(\frac{\gamma}{1+\gamma}\right)^{-\gamma} 2^{-2\alpha+\gamma} > 1.$$

By rearranging this, we obtain Inequality (17), which is equivalent to Inequality (14):

$$2^{-2\alpha+\gamma} > \left(\frac{\gamma}{1+\gamma}\right)^{\gamma}.$$

Thus, we obtain Condition (15), that is, the profit of the incumbent final-good firm under a two-firm oligopoly is greater than that under a monopoly.

Appendix 5

This appendix outlines the calculations for the condition: $P_I^C(2) < P_I^M$. The condition that the unit cost of the final good when the final-good market is a two-firm oligopoly at the Cournot-Nash equilibrium is lower than the unit cost of the final good when the final-good market is a monopoly is as follows:

$$2^{\frac{1-\alpha}{\alpha-\gamma}}\gamma^{\frac{1-\alpha}{\alpha-\gamma}}(1+\gamma)^{-\frac{1-\alpha}{\alpha-\gamma}} < 1. \qquad \text{(A5.1)}$$

By rearranging this inequality, we obtain the following expression:

$$\left(\frac{2\gamma}{1+\gamma}\right)^{\frac{1-\alpha}{\alpha-\gamma}} < 1.$$

Here, under the assumption $\alpha > \gamma$, calculating both sides of (A5.1) to the $(\alpha-\gamma)^{\text{th}}$ power, we obtain

$$\left(\frac{2\gamma}{1+\gamma}\right)^{1-\alpha} < 1.$$

Calculating both sides of the $[1/(1-\alpha)]^{\text{th}}$ power, we obtain the following inequality:

$$\frac{2\gamma}{1+\gamma} < 1.$$

By rearranging this inequality, we obtain Inequality (19):

$$\gamma < 1.$$

Therefore, the unit cost of the final good is lower when the final-good market is a two-firm oligopoly than when it is a monopoly, assuming that $0 < \gamma < 1$.

Appendix 6

The final-good firm k's profit maximization problem is expressed as follows:

$$\max_{C_k} P(C_1 + C_2 + \cdots + C_k + \cdots + C_m)C_k - \int_0^n p_i x_{ik}\, di.$$

As $\int_0^n p_i x_{ik}\, di = P_I C_k$, the final-good firm k's profit maximization problem is

$$\max_{C_k} [P(C_1 + C_2 + \cdots + C_k + \cdots + C_m) - P_I]C_k.$$

We assume that $C = C_1 + C_2 + \cdots + C_k + \cdots + C_m$. Considering the demand function, we obtain

$$P = \frac{\gamma}{\phi}(C_1 + C_2 + \cdots + C_k + \cdots + C_m)^{-(1-\gamma)}.$$

Therefore, the final-good firm k's profit maximization problem is given by

$$\max_{C_k} \left[\frac{\gamma}{\phi}(C_1 + C_2 + \cdots + C_k + \cdots + C_m)^{-(1-\gamma)} - P_I\right] C_k.$$

In the Cournot-Nash equilibrium, we obtain the following expression:

$$C_1 = C_2 = \cdots = C_k = \cdots = C_m = y,$$

where y is defined as the production volume of each firm under the Cournot-Nash equilibrium. Since the expression above holds at the Cournot-Nash equilibrium, the first-order condition for firm k is as follows:

$$(my)^{-(1-\gamma)}[1 - (1-\gamma)(my)^{-1}y] = \frac{\phi}{\gamma} P_I.$$

By rearranging this equation, we obtain the following expression:

$$y = \left[m^{(2-\gamma)} \frac{1}{(m-1+\gamma)} \frac{\phi}{\gamma} P_I\right]^{-\frac{1}{1-\gamma}}.$$

Thus,

$(C_1^C, C_2^C, \dots, C_m^C)$

$$= \left\{ \left[m^{(2-\gamma)} \frac{1}{(m-1+\gamma)} \frac{\phi}{\gamma} P_I \right]^{-\frac{1}{1-\gamma}}, \left[m^{(2-\gamma)} \frac{1}{(m-1+\gamma)} \frac{\phi}{\gamma} P_I \right]^{-\frac{1}{1-\gamma}}, \dots, \left[m^{(2-\gamma)} \frac{1}{(m-1+\gamma)} \frac{\phi}{\gamma} P_I \right]^{-\frac{1}{1-\gamma}} \right\}.$$

Therefore, the production volume of the market is

$$C^C = my = m \left[m^{(2-\gamma)} \frac{1}{(m-1+\gamma)} \frac{\phi}{\gamma} P_I \right]^{-\frac{1}{1-\gamma}}.$$

The market price under the Cournot-Nash equilibrium is as follows:

$$P = \frac{\gamma}{\phi} C^{-(1-\gamma)}.$$

Therefore, we obtain

$$P^C = \frac{m}{m-1+\gamma} P_I.$$

The profit of the incumbent final-good firm, firm 1, is expressed as follows:

$$\pi_1 = P^C C_1 - P_I C_1.$$

By rearranging this expression, we obtain the following expression:

$$\pi_1 = \left(\frac{m}{m-1+\gamma} P_I - P_I \right) \left[m^{(2-\gamma)} \frac{1}{(m-1+\gamma)} \frac{\phi}{\gamma} P_I \right]^{-\frac{1}{1-\gamma}}.$$

Therefore,

$$\pi_1 = \frac{1-\gamma}{m-1+\gamma} \left(\frac{\phi}{\gamma} \frac{1}{m-1+\gamma} m^{(2-\gamma)} \right)^{-\frac{1}{1-\gamma}} (P_I)^{-\frac{\gamma}{1-\gamma}}.$$

Here, the incumbent final-good firm's (firm 1) demand for intermediate good i is as follows:

$$x_{i1} = (p_i)^{-\frac{1}{1-\alpha}} (P_I)^{\frac{1}{1-\alpha}} C_1.$$

By rearranging this expression, we obtain

$$x_{i1}^{C} = (p_i)^{-\frac{1}{1-\alpha}} (P_I)^{\frac{1}{1-\alpha}} \left[m^{(2-\gamma)} \frac{1}{(m-1+\gamma)} \frac{\phi}{\gamma} P_I \right]^{-\frac{1}{1-\gamma}}.$$

Therefore, the entire final-good sector's demand for intermediate good i is as follows:

$$x_i^C = m x_{i1}^C = m^{-\frac{1}{1-\gamma}} (p_i)^{-\frac{1}{1-\alpha}} (P_I)^{\frac{\alpha-\gamma}{(1-\alpha)(1-\gamma)}} (m-1+\gamma)^{\frac{1}{1-\gamma}} \phi^{-\frac{1}{1-\gamma}} \gamma^{\frac{1}{1-\gamma}}.$$

The intermediate-good firm i's profit maximization problem is

$$\max_{p_i} p_i x_i - x_i - f.$$

Therefore, the intermediate-good firm i's profit maximization problem is expressed as follows:

$$\max_{p_i} m^{-\frac{1}{1-\gamma}} (P_I)^{\frac{\alpha-\gamma}{(1-\alpha)(1-\gamma)}} (m-1+\gamma)^{\frac{1}{1-\gamma}} \phi^{-\frac{1}{1-\gamma}} \gamma^{\frac{1}{1-\gamma}} \left[(p_i)^{-\frac{\alpha}{1-\alpha}} - (p_i)^{-\frac{1}{1-\alpha}} \right] - f.$$

The first-order condition for p_i yields

$$p_i = \frac{1}{\alpha}.$$

Considering the free-entry condition of intermediate-goods firms, we obtain

$$p_i x_i - x_i - f = 0.$$

By rearranging this equation, we obtain the following expression:

$$x_i = \frac{\alpha}{1-\alpha} f. \qquad \text{(A6.1)}$$

As $x_i^C = m x_{i1}^C$, $x_i^C = m (p_i)^{-\frac{1}{1-\alpha}} (P_I)^{\frac{1}{1-\alpha}} \left[m^{(2-\gamma)} \frac{1}{(m-1+\gamma)} \frac{\phi}{\gamma} P_I \right]^{-\frac{1}{1-\gamma}}$. Therefore, the expression (A6.1) is arranged as follows

$$m(p_i)^{-\frac{1}{1-\alpha}}(P_I)^{\frac{1}{1-\alpha}}\left[m^{(2-\gamma)}\frac{1}{(m-1+\gamma)}\frac{\phi}{\gamma}P_I\right]^{-\frac{1}{1-\gamma}}=\frac{\alpha}{1-\alpha}f.$$

By rearranging this expression, we obtain

$$m\left(\frac{1}{\alpha}\right)^{-\frac{1}{1-\alpha}}\left(\frac{1}{\alpha}n^{-\frac{1-\alpha}{\alpha}}\right)^{\frac{1}{1-\alpha}}m^{-\frac{2-\gamma}{1-\gamma}}(m-1+\gamma)^{\frac{1}{1-\gamma}}\phi^{-\frac{1}{1-\gamma}}\gamma^{\frac{1}{1-\gamma}}\left(\frac{1}{\alpha}n^{-\frac{1-\alpha}{\alpha}}\right)^{-\frac{1}{1-\gamma}}=\frac{\alpha}{1-\alpha}f.$$

By calculating this equation, we obtain the following expression:

$$n=\left[\alpha^{-\frac{\gamma}{1-\gamma}}m^{\frac{1}{1-\gamma}}(m-1+\gamma)^{-\frac{1}{1-\gamma}}\phi^{\frac{1}{1-\gamma}}\gamma^{-\frac{1}{1-\gamma}}(1-\alpha)^{-1}f\right]^{-\frac{\alpha(1-\gamma)}{\alpha-\gamma}}.$$

Therefore, when there are m firms in the final-good market, the number of firms entering the intermediate-goods sector is calculated as follows:

$$n^C(m)=\alpha^{\frac{\alpha\gamma}{\alpha-\gamma}}m^{-\frac{\alpha}{\alpha-\gamma}}(m-1+\gamma)^{\frac{\alpha}{\alpha-\gamma}}\phi^{-\frac{\alpha}{\alpha-\gamma}}\gamma^{\frac{\alpha}{\alpha-\gamma}}(1-\alpha)^{\frac{\alpha(1-\gamma)}{\alpha-\gamma}}f^{-\frac{\alpha(1-\gamma)}{\alpha-\gamma}}. \tag{A6.2}$$

The unit cost of the final good when there exist m firms in the final-good market is calculated as follows:

$$P_I=\frac{1}{\alpha}(n^C)^{-\frac{1-\alpha}{\alpha}}.$$

By substituting (A6.2) into the expression above, we obtain

$$P_I=\frac{1}{\alpha}\left[\alpha^{\frac{\alpha\gamma}{\alpha-\gamma}}m^{-\frac{\alpha}{\alpha-\gamma}}(m-1+\gamma)^{\frac{\alpha}{\alpha-\gamma}}\phi^{-\frac{\alpha}{\alpha-\gamma}}\gamma^{\frac{\alpha}{\alpha-\gamma}}(1-\alpha)^{\frac{\alpha(1-\gamma)}{\alpha-\gamma}}f^{-\frac{\alpha(1-\gamma)}{\alpha-\gamma}}\right]^{-\frac{1-\alpha}{\alpha}}.$$

By rearranging this expression, we obtain

$$P_I^C(m)=\alpha^{-\frac{\alpha(1-\gamma)}{\alpha-\gamma}}m^{\frac{1-\alpha}{\alpha-\gamma}}(m-1+\gamma)^{-\frac{1-\alpha}{\alpha-\gamma}}\phi^{\frac{1-\alpha}{\alpha-\gamma}}\gamma^{-\frac{1-\alpha}{\alpha-\gamma}}(1-\alpha)^{-\frac{(1-\alpha)(1-\gamma)}{\alpha-\gamma}}f^{\frac{(1-\alpha)(1-\gamma)}{\alpha-\gamma}},$$

where $P_I^C(m)$ denotes the unit cost of the final good when there are m firms in the final-good market.

Therefore, the incumbent final-good firm's profit, $\pi_1^C(m)$, when there are m final-good firms in the market, is as follows:

$$\pi_1^C(m) = (1-\gamma)\phi^{-\frac{\alpha}{\alpha-\gamma}}\gamma^{\frac{\alpha}{\alpha-\gamma}}\alpha^{\frac{\alpha\gamma}{\alpha-\gamma}}(1-\alpha)^{\frac{(1-\alpha)\gamma}{\alpha-\gamma}}f^{-\frac{(1-\alpha)\gamma}{\alpha-\gamma}}(m-1+\gamma)^{\frac{\gamma}{\alpha-\gamma}}m^{\frac{-2\alpha+\gamma}{\alpha-\gamma}}$$

Appendix 7

The incumbent final-good firm's profit is as follows

$$\pi_1^C(m) = (1-\gamma)\phi^{-\frac{\alpha}{\alpha-\gamma}}\gamma^{\frac{\alpha}{\alpha-\gamma}}\alpha^{\frac{\alpha\gamma}{\alpha-\gamma}}(1-\alpha)^{\frac{(1-\alpha)\gamma}{\alpha-\gamma}}f^{-\frac{(1-\alpha)\gamma}{\alpha-\gamma}}(m-1+\gamma)^{\frac{\gamma}{\alpha-\gamma}}m^{\frac{-2\alpha+\gamma}{\alpha-\gamma}}.$$

Here, the condition that the incumbent final-good firm's profit is greater when there exist m final-good firms than when the final-good market is a monopoly is

$$\pi_1^C(m) > \pi_1^M.$$

By calculating this inequality, we obtain the following expression:

$$(m-1+\gamma)^{\frac{\gamma}{\alpha-\gamma}}m^{\frac{-2\alpha+\gamma}{\alpha-\gamma}} > \gamma^{\frac{\gamma}{\alpha-\gamma}}.$$

The first-order derivative of $\pi_1^C(m)$ is

$$\frac{d}{dm}\pi_1^C(m) = (1-\gamma)\phi^{-\frac{\alpha}{\alpha-\gamma}}\gamma^{\frac{\alpha}{\alpha-\gamma}}\alpha^{\frac{\alpha\gamma}{\alpha-\gamma}}(1-\alpha)^{\frac{(1-\alpha)\gamma}{\alpha-\gamma}}f^{-\frac{(1-\alpha)\gamma}{\alpha-\gamma}}(\alpha-\gamma)^{-1}(m-1+\gamma)^{\frac{-\alpha+2\gamma}{\alpha-\gamma}}m^{\frac{-3\alpha+2\gamma}{\alpha-\gamma}}[(2\alpha-\gamma)(1-\gamma)-2(\alpha-\gamma)m].$$

When $\frac{d}{dm}\pi_1^C(m) = 0$, we obtain

$$m = m^* = \frac{(2\alpha-\gamma)(1-\gamma)}{2(\alpha-\gamma)} = \left[1+\frac{\gamma}{2(\alpha-\gamma)}\right](1-\gamma) > 1-\gamma > 0.$$

The $\pi_1^C(m)$ function takes an inverted-U shape, as shown in Fig. 4, where m^* is the optimal number of firms for the incumbent firm in the final-good market. When $m > m^*$, the incumbent's profit decreases with m.

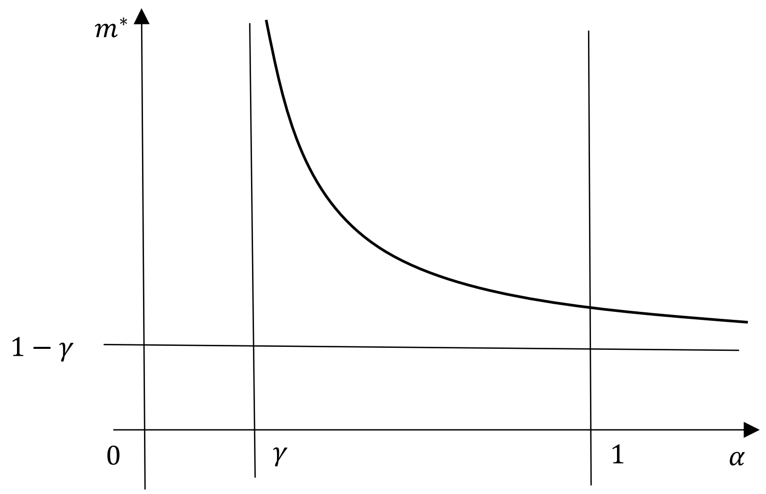

Figure 5: The relationship between α and the optimal number of firms in the final-good market for the incumbent firm, m^*.

Thus, $\frac{\partial m^*}{\partial \alpha}$ can be calculated as follows:

$$\frac{\partial m^*}{\partial \alpha} = -\frac{\gamma(1-\gamma)}{2(\alpha-\gamma)^2}.$$

As α increases toward 1, m^* decreases, as shown in Fig. 5.

Hereafter, we analyze the maximum number of final-good firms. Let $\overline{m}$ be the maximum number of final-good firms such that the incumbent final-good firm's profit when there exist $\overline{m}$ final-good firms is equal to its profit when the final-good market is a monopoly. In other words, if the final-good market changes to an oligopoly with more than $\overline{m}$ firms because of free technology sharing, the incumbent's profit is less for an oligopolistic final-goods market than when the final-good market is a monopoly. The equation that determines $\overline{m}$ is as follows:

$$\overline{m} - (1-\gamma) = \gamma(\overline{m})^{\frac{2\alpha}{\gamma}-1}.$$

Here, we define $h(m) \equiv m - (1-\gamma)$, $\psi(m) \equiv \gamma m^{\frac{2\alpha}{\gamma}-1}$. The equation $h(m) = \psi(m)$ holds when

$m = 1$. We consider the three cases separately. In the first case, $h(m)$ is tangent to $\psi(m)$. In the second case, $h(m)$ and $\psi(m)$ intersect at two points whose m-coordinates are 1 and $\bar{m} > 1$, respectively. In the third case, $h(m)$ and $\psi(m)$ intersect at two points whose m-coordinates are 1 and $\bar{m} < 1$, respectively. We analyze the second case with respect to $\psi(m) \equiv \gamma m^{\frac{2\alpha}{\gamma}-1}$, as $\gamma < \alpha$, $\frac{2\alpha}{\gamma} - 1 > 1$. Therefore, as shown in Fig. 6, when $\psi'(1) < 1$, $\bar{m} > 1$. From the condition $\psi'(1) < 1$, considering that $\gamma < \alpha$, we obtain the following condition:

$$\gamma < \alpha < \frac{1+\gamma}{2}.$$

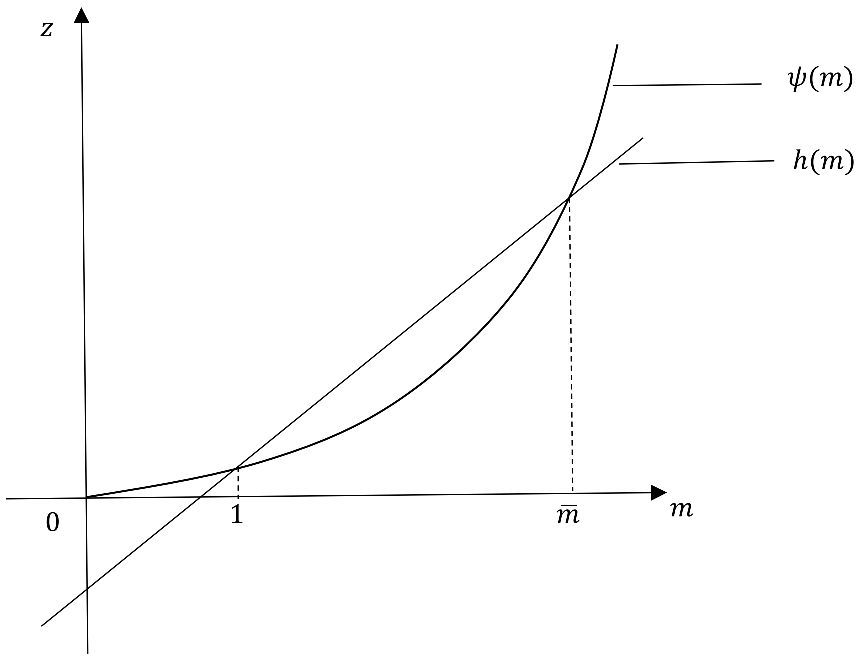

Figure 6: Maximum number of final-good firms, $\bar{m}$.

We now consider the relationship between α and $\bar{m}$. When α increases, $\frac{2\alpha}{\gamma} - 1$, which is the exponent of m in $\psi(m) = \gamma m^{\frac{2\alpha}{\gamma}-1}$, increases, and $z = \psi(m)$ shifts upward in $m > 1$, such that the point of intersection of $\psi(m)$ and $h(m)$ that is not 1 shifts leftward, as shown in Fig. 6. Therefore, the m-coordinate of the intersection of $\psi(m)$ and $h(m)$, $\bar{m}$, decreases as α increases.