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Optimal capital structure with earnings above a floor *

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Abstract

This paper derives the optimal capital structure of a firm whose earnings follow a geometric Brownian motion with a lower reflecting barrier. The barrier can be interpreted as a market intervention threshold (e.g., a price floor) by the government or an exit threshold of weak competitors in the market. Unlike in the standard model with no barrier, the firm is able to issue riskless debt to a certain capacity determined by the barrier. The higher the barrier, the larger the riskless debt capacity, and the firm prefers riskless capital structure rather than risky capital structure. Notably, with intermediate barrier levels, the firm can choose riskless capital structure with lower leverage than the level with no barrier. This mechanism can help explain debt conservatism observed in practice. The paper also entails several implications of public intervention by examining the lowest barrier (i.e., the weakest intervention) to achieve riskless capital structure.

JEL Classification Codes: G13; G28; G32.

Keywords: Capital structure; Real options; Regulated market; Price floor; Competitive advantage.

1 Introduction

This paper analyzes an optimal capital structure model of a firm that receives stochastic flows of earnings above a floor. The floor can be interpreted as the government's intervention to protect particular companies or industries (e.g., transportation, utility, agricultural, or financial industries) against downside risks. Apart from the regulated markets, the floor can be interpreted as an exit threshold of relatively weak competitors. Then, the model can approximate a firm with a certain competitive advantage or protection against downside risks. In the model, we reveal that a firm

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can determine capital structure by a mechanism different from standard trade-off theory; indeed, a firm can optimally choose capital structure with no risky debt.

The baseline model builds on the standard real options models of optimal capital structure (e.g., Leland (1994), Goldstein, Ju, and Leland (2001), and Shibata and Nishihara (2012)). As in the standard literature, we assume that a firm has an option to issue consol debt at an initial time and that shareholders of the firm have an option to default debt in place. The firm's earnings are modeled by a geometric Brownian motion (GBM) with a lower reflecting barrier (i.e., a floor), which is a difference from the standard models. We also extend the baseline model to a model with a debt financing constraint. In the models with a barrier, we analytically derive the equity, debt, firm values, leverage, and credit spreads, as well as their sensitivities to barrier levels. The results are explained below.

A most notable difference from the standard results with no barrier (e.g., Leland (1994), Goldstein, Ju, and Leland (2001), and Shibata and Nishihara (2012)) is that a barrier generates a capacity of riskless debt financing. Naturally, the higher the barrier, the larger the riskless debt capacity. Compared to risky debt, riskless debt has an advantage of no bankruptcy cost but a disadvantage of the debt level being limited by the capacity. If the barrier is lower than a critical level (i.e., the riskless debt capacity is insufficient), then the firm prefers risky capital structure. In this case, the presence of a barrier hardly affects the equity, debt, firm values, leverage, and credit spreads because the firm chooses leverage by the standard trade-off between the tax benefits and bankruptcy costs of debt.

If the barrier is higher than the critical level (i.e., the riskless debt capacity is sufficient), then the firm prefers riskless capital structure. In the no-default case, the barrier level greatly affects all the values because the riskless debt capacity (depending on the barrier level) rather than the standard trade-off is a key determinant of capital structure. Notably, the barrier close to the critical level leads to lower debt and leverage levels than the optimal levels with no barrier. That is, in contrast to the straightforward intuition that the firm increases debt with a floor, the firm voluntarily reduces debt to take advantage of having no bankruptcy risk. This result can help explain empirical observations of debt conservatism (e.g., Graham (2000), Strebulaev and Yang (2013), and El Ghoul, Guedhami, Kwok, and Zheng (2018)); some firms have quite low leverage and bankruptcy risk compared to the optimal levels predicted by standard trade-off theory. Indeed, our model suggests that firms do not choose risky capital structure based on the standard trade-off but optimally choose riskless capital structure with low leverage if they have certain degrees of competitive advantage or protection against downside risks.

We also examine the comparative statics with respect to key parameters. With a given barrier level, a higher volatility, bankruptcy cost, lower growth rate, corporate tax rate, and stronger debt issuance constraint tend to lead to the no-default case. The switch to the no-default case can cause the comparative static results to differ from those of the standard trade-off models. For instance, in the no-default case, a higher growth rate increases equity value more than riskless debt capacity, decreasing leverage contrary to the standard result. This mechanism can explain empirical evidence of a negative relation between leverage and profitability, which is well known as an inconsistency between trade-off theory and practice (e.g., Frank and Goyal (2015) and Demarzo (2019)).

This paper focuses on the lowest barrier to achieve the no-default case, because it can be interpreted as the weakest intervention by the government that prevents the firm from bankruptcy. We show that the critical level is lower than the level necessary to save the firm from bankruptcy ex post. This result emphasizes the importance of the ex ante information disclosure of the intervention policy. The appropriate commitment by the government leads a firm to adopt riskless capital structure with low leverage rather than leading the firm to take moral hazard behavior of increasing debt. A higher volatility and lower growth rate decrease the critical barrier level but increase the frequency of hitting the barrier. A higher bankruptcy cost, lower corporate tax rate, and stronger debt issuance constraint decrease advantages of risky debt, decreasing the critical barrier level and the frequency of hitting the barrier. These results suggest that the government can weaken and reduce market interventions with a more stringent bankruptcy law (i.e., a higher bankruptcy cost), lower corporate tax rate, and stronger leverage regulation.

Last, we will briefly explain technically related literature. Dixit and Pindyck (1994) solve the investment and exit timing models with a price ceiling and floor and entail many implications of market competition and regulation. By extending the models, Dobbs (2004) shows that the optimal price ceiling delays investment in a monopoly, whereas Roques and Savva (2009) show that it accelerates investment in an oligopoly. Evans and Guthrie (2012) study a firm's production capacity adjustments under a price ceiling and quantity floor and show that with economies of scale, the firm invests in smaller, more frequent, increments than the social planner. Adkins, Paxson, Pereira, and Rodrigues (2019) examine the optimal duration of regulation in the investment timing model with a finite/retractable price ceiling and floor. Unlike this paper, the above papers assume all-equity firms and do not examine any capital structure problem.

Sarkar (2016) develops a Leland-type capital structure model with a price ceiling and shows that the price ceiling significantly increases leverage. He also shows that the price ceiling can counterintuitively decrease consumer welfare. Rodrigues (2022) is closest to this paper. He investigates the investment timing and capital structure model with a revenue ceiling and floor. His model, which includes the investment timing and price ceiling, is more generalized than our baseline model, but due to the model complexity, most of the results are shown numerically. Unlike Rodrigues (2022), this paper shows the explicit mechanism of how the price floor generates riskless debt capacity and leads to riskless capital structure in the simple model. In contrast to Sarkar (2016), this paper shows that leverage can either increase or decrease (i.e., it can be nonmonotonic) with floor levels.

The paper is organized as follows. Section 2 explains the model setup. In Section 3.1, we explain the solutions in the benchmark model with no barrier, and in Section 3.2, we derive the explicit solutions in the baseline model with a barrier. We also analytically derive the sensitivities to barrier levels. In Section 3.3, we derive the explicit solutions in the extended model with a debt issuance constraint. Section 4 numerically examines the sensitivities to the key parameters, and Section 5 concludes.

2 Model setup

The baseline model builds on the standard capital structure model based on trade-off theory (e.g., Leland (1994) and Goldstein, Ju, and Leland (2001)). Consider a firm that receives continuous streams of earnings before interest and taxes (EBIT) $X(t)$ until bankruptcy. EBIT $X(t)$ follows a GBM

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}B(t) \ (t > 0), \quad X(0) = x$$

with lower reflecting barrier $x_B(> 0)$, where $B(t)$ denotes the standard Brownian motion defined in a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\})$ and growth rate μ, volatility $\sigma(> 0)$, and initial value $X(0) = x(\geq x_B)$ are constants. For convergence, $r > \mu$ is assumed, where a positive constant r denotes the risk-free interest rate, and $X(0) = x$ is assumed to be sufficiently high level so that the firm is not bankrupt at time 0.

At time 0, the firm issues consol debt to maximize the firm value, where the tax benefits and bankruptcy costs of debt will be clarified in the next section. For debt in place, shareholders can stop coupon payments (i.e., declare default) to maximize the equity value. In the default case, shareholders receive nothing, and debt holders receive the post-bankruptcy firm value, which is equal to the unlevered firm value multiplied by $(1-\alpha)$. This means that a fraction $\alpha \in (0, 1)$ of the unlevered firm value is lost to the deadweight costs of bankruptcy. Equity, debt, and firm values are fairly priced based on the rational expectation of ex post shareholders' default behavior.

The presence of lower reflecting barrier x_B is a difference from the standard capital structure model.[1] Intuitively, lower reflecting barrier x_B means that $X(t)$ is pulled back to x_B and moves again from x_B immediately after $X(t)$ falls below x_B. It is different from the assumption that $X(t)$ equals a GBM $S(t)$ (i.e., the shadow process) for $S(t) \geq x_B$ but remains at x_B for $S(t) < x_B$. Although the shadow process model requires the value function in regions $S(t) < x_B$ and $S(t) \geq x_B$, the reflecting barrier model requires only one region, $X(t) \geq x_B$.[2] By this technical simplicity, we can solve the model explicitly in the next section.

The model with barrier x_B can approximate the following two situations. First, governments might try to intervene in markets to protect specific firms or industries (e.g., transportation, utility, agricultural, or financial industries) against downside risks. For instance, European Union countries' governments purchase particular agricultural products to prevent their prices from dropping to unsustainably low levels. Although such market interventions require direct and indirect costs,

[1] Reflecting barriers frequently appear in dynamic financing and payout models. For instance, in Bolton, Chen, and Wang (2011), the state variable moves between a lower reflecting barrier (i.e., the financing threshold) and an upper reflecting barrier (i.e., the payout threshold), although unlike in our model, the barrier levels are determined endogenously in their model.

[2] Most of the literature studies the shadow process models (e.g., Sarkar (2016), Adkins, Paxson, Pereira, and Rodrigues (2019), and Rodrigues (2022)). An exception is Chapter 8 of Dixit and Pindyck (1994), where the reflecting barrier models are examined. We do not think that the technical difference matters in terms of economic implications. Indeed, the same results on floors as in our results are observed in Rodrigues (2022), who studies a more complicated model based on the shadow process. For instance, a higher floor also leads to riskless capital structure in Rodrigues (2022).

governments can adopt the market measures if the bankruptcy costs of these firms, including indirect costs, such as threats to national security, are higher than the intervention costs. In this case, x_B is interpreted as the intervention threshold.[3] Chapter 9 of Dixit and Pindyck (1994), Adkins, Paxson, Pereira, and Rodrigues (2019), and Rodrigues (2022) also examine real options models with floors (and ceilings) in terms of public intervention. Sections 3.3 and 4.6 study the effects of leverage regulation in addition to the public intervention by incorporating a debt issuance constraint into the baseline model.

Second, the baseline model may capture firms with strong competitive advantage against downside risks. For instance, consider oil prices. Relatively weak shale oil producers tend to exit the markets when oil prices fall to unsustainably low levels for them. After the exit of shale oil producers, oil prices are likely to rebound. Thus, the biggest oil companies, which have sufficient competitive advantage to survive downturns, could receive cash flows above certain levels. More generally, the cash flow dynamics of resilient firms may have such a trend. In this case, x_B is regarded as the exit threshold of weak competitors. Chapter 8 of Dixit and Pindyck (1994) also examine entry and exit timing models with lower and upper reflecting barriers in terms of the competitive market.

3 Model solutions

3.1 EBIT with no barrier

This subsection explains the benchmark model with no barrier (i.e., $x_B = 0$). The following results are well known in previous literature (e.g., Goldstein, Ju, and Leland (2001), Shibata and Nishihara (2012), and Sundaresan, Wang, and Yang (2015)), and hence, the details of derivation are omitted. First, suppose that the firm issues consol debt with coupon C. For given coupon C, the equity, debt, and firm values are expressed as

$$E_0(x;C) = \pi x - \frac{(1-\tau)C}{r} + \left(\frac{x}{x_0(C)}\right)^{\gamma}\left(\frac{(1-\tau)C}{r} - \pi x_0(C)\right) \tag{1}$$

$$D_0(x;C) = \frac{C}{r} - \left(\frac{x}{x_0(C)}\right)^{\gamma}\left(\frac{C}{r} - (1-\alpha)\pi x_0(C)\right) \tag{2}$$

$$F_0(x;C) = \pi x + \frac{\tau C}{r} - \left(\frac{x}{x_0(C)}\right)^{\gamma}\left(\alpha\pi x_0(C) + \frac{\tau C}{r}\right) \tag{3}$$

for $x \geq x_0(C)$, where τ denotes a corporate tax rate, $\pi = (1-\tau)/(r-\mu)$ denotes the unlevered firm's coefficient, $\gamma = 0.5 - \mu/\sigma^2 - \sqrt{(\mu/\sigma^2 - 0.5)^2 + 2r/\sigma^2}$ denotes a negative characteristic root, and $x_0(C)$ denotes the default threshold. Throughout the paper, subscript 0 stands for the benchmark model with no barrier. The first, second, and last terms in equity value (1) correspond to the unlevered firm value, perpetual coupon payments, and the value of the default option, respectively. The first and second terms in debt value (2) are the perpetual coupon receipts (i.e., the riskless

[3]When the product price follows a GBM with a floor, EBIT also follows a GBM with a floor in the standard setups (e.g., Dixit and Pindyck (1994)). Then, for simplicity, this paper directly assumes EBIT with a floor.

debt value) and loss due to default risk, respectively. The first, second, and last terms in firm value (3) are the unlevered firm value, perpetual tax benefits of debt, and bankruptcy costs, respectively.

Note that shareholders determine $x_0(C)$ to maximize its own value $E_0(x;C)$ for debt in place. By solving $\arg\max_{x_0(C)\geq 0} E_0(x;C)$, we obtain default threshold

$$x_0(C) = C/\delta, \tag{4}$$

where δ is a constant given by

$$\delta = \frac{(\gamma-1)r}{\gamma(r-\mu)} \ (>1).$$

Now, consider the optimal capital structure. The firm chooses coupon C to maximize firm value $F_0(x;C)$ based on the trade-off between the tax benefits and bankruptcy costs of debt. By solving $\arg\max_{C\geq 0} F_0(x;C)$, we obtain optimal coupon

$$C_0(x) = \delta x/h, \tag{5}$$

where h is a constant given by

$$h = \left[1-\gamma\left(1-\alpha+\frac{\alpha}{\tau}\right)\right]^{-\frac{1}{\gamma}} \ (>1).$$

The optimally levered firm value is

$$F_0(x;C_0(x)) = \psi\pi x, \tag{6}$$

where ψ is a constant given by

$$\psi = 1+\frac{\tau}{(1-\tau)h} \ (>1)$$

and is interpreted as the leverage effect. Indeed, the levered firm value $F_0(x;C_0(x))$ is the unleverd firm value πx multiplied by $\psi(>1)$. The firm's leverage is

$$LV_0(x) = \frac{D_0(x;C_0(x))}{F_0(x;C_0(x))} = \frac{\gamma-1}{\gamma}\frac{(1-\tau)(1-\xi)}{h\psi}, \tag{7}$$

and credit spreads are

$$CS_0(x) = \frac{C_0(x)}{D_0(x;C_0(x))} - r = \frac{r\xi}{1-\xi}, \tag{8}$$

where ξ is a constant defined by

$$\xi = \left(1-(1-\alpha)(1-\tau)\frac{\gamma}{\gamma-1}\right)h^{\gamma} \ (\in(0,1)).$$

3.2 EBIT with a lower reflecting barrier

This subsection solves the baseline model with lower reflecting barrier $x_B > 0$. First, suppose that the firm issues debt with coupon C. For given C, shareholders choose whether they default. Then, equity value $E(x;C)$ is expressed as

$$E(x;C) = \max\{E_d(x;C), E_n(x;C)\}, \tag{9}$$

where $E_d(x;C)$ and $E_n(x;C)$ represent the equity values in the default-possible and no-default cases, which will be defined later. The next proposition shows the equity, debt, and firm values, denoted by $E(x;C), D(x;C)$, and $F(x;C)$, respectively, for given coupon C. For the proof, see Appendix A.

Proposition 1 *For $C > \delta x_B$, the firm goes bankrupt at default threshold $x_0(C) = C/\delta$ (i.e., the default-possible case). The equity, debt, and firm values are given by*

$$E(x;C) = E_d(x;C) = E_0(x;C), \tag{10}$$

$$D(x;C) = D_d(x;C) = \underbrace{\frac{C}{r} - \left(\frac{x}{x_0(C)}\right)^{\gamma}\left(\frac{C}{r} - (1-\alpha)\pi x_0(C)\right)}_{=D_0(x;C)} - \left(\frac{x}{x_B}\right)^{\gamma}\frac{(1-\alpha)\pi x_B}{\gamma}, \tag{11}$$

$$F(x;C) = F_d(x;C) = \underbrace{\pi x + \frac{\tau C}{r} - \left(\frac{x}{x_0(C)}\right)^{\gamma}\left(\alpha\pi x_0(C) + \frac{\tau C}{r}\right)}_{=F_0(x;C)} - \left(\frac{x}{x_B}\right)^{\gamma}\frac{(1-\alpha)\pi x_B}{\gamma}. \tag{12}$$

Otherwise, the firm never goes bankrupt (i.e., the no-default case). The equity, debt, and firm values are given by

$$E(x;C) = E_n(x;C) = \pi x - \frac{(1-\tau)C}{r} - \left(\frac{x}{x_B}\right)^{\gamma}\frac{\pi x_B}{\gamma}, \tag{13}$$

$$D(x;C) = D_n(x;C) = \frac{C}{r}, \tag{14}$$

$$F(x;C) = F_n(x;C) = \pi x + \frac{\tau C}{r} - \left(\frac{x}{x_B}\right)^{\gamma}\frac{\pi x_B}{\gamma}. \tag{15}$$

Note that $C > \delta x_B$ is equivalent to $x_0(C) > x_B$. First, we explain the default-possible case, in which $x_0(C)$ is higher than x_B. Equity value $E_d(x;C)$ does not depend on x_B because $X(t)$ does not hit x_B before bankruptcy. Then, $E_d(x;C)$ is the same as the benchmark value $E_0(x;C)$ (see (10)). However, debt and firm values, $D_d(x;C)$ and $F_d(x;C)$, respectively, benefit by x_B. The first and second terms in (11) coincide with $D_0(x;C)$, and the last term is the additional value by x_B (note that $\gamma < 0$). The additional value arises from the fact that $X(t)$ can hit x_B after default. That is, the post-default value, which debt holders obtain, increases with higher x_B.[4] Firm value $F_d(x;C)$ has the same benefit from x_B (see the last terms in (11) and (12)) because of $F_d(x;C) = E_0(x;C) + D_d(x;C)$.

Now, we explain the no-default case. By $C \leq \delta x_B$, $E_n(x_B;C) \geq 0$ holds in (13). Then, for any $X(t) \geq x_B$, shareholders are better off continuing operation with coupon payments rather than declaring default. Shareholders benefit by x_B because $X(t)$ can hit x_B. The first, second, and third terms in (13) represent the unlevered firm value, perpetual coupon payments, and additional value by x_B (note that $\gamma < 0$). Debt holders also benefit by x_B because x_B removes the default risk.

[4]As in Leland (1994) and Goldstein, Ju, and Leland (2001), our model does not specify either liquidation or reorganization bankruptcy but assumes that the post-default firm value is the unlevered value discounted by bankruptcy costs. The unlevered value increases in x_B.

Then, $D_n(x;C)$ agrees with the riskless debt value in (14). In (15), firm value $F_n(x;C)$ consists of the unlevered firm value, perpetual tax benefits, and additional value by x_B. Unlike $F_0(x;C)$, $F_n(x;C)$ does not include any term representing bankruptcy costs.

Proposition 1 implies that $D_n(x;\delta x_B) = \delta x_B/r$ is the capacity of riskless debt. Of course, for $C \leq x_B$, the firm always receives nonnegative cash flows $X(t) - C$, and hence, shareholders continue operation perpetually. Note that $\delta > 1$. Considering the possibility that $X(t)$ goes beyond x_B due to volatility σ, shareholders prefer to operate perpetually for $C \leq \delta x_B$. Indeed, the expected cash flows of perpetual operation are nonnegative (i.e., $E_n(x;C) \geq 0$) for $C \leq \delta x_B$. This is how the presence of barrier x_B creates the riskless debt capacity $\delta x_B/r$. Proposition 1 nests the benchmark case with no barrier as the limiting case of $x_B \to 0$. Indeed, $\lim_{x_B \to 0} E(x;C) = E_0(x;C), \lim_{x_B \to 0} D(x;C) = D_0(x;C)$, and $\lim_{x_B \to 0} F(x;C) = F_0(x;C)$ hold.

Next, consider the optimal capital structure. We need to solve $\max_{C \geq 0} F(x;C)$. By (5) and (12), we have $\arg\max_{C \geq 0} F_d(x;C) = \arg\max_{C \geq 0} F_0(x;C) = C_0(x)$, which reflects the standard trade-off between the tax benefits and bankruptcy costs of debt. By (15), $F_n(x;C)$ increases linearly in C, implying $\arg\max_{C \in [0,\delta x_B]} F_n(x;C) = \delta x_B$. This reflects the fact that a higher debt level increases firm value via greater tax benefits in the no-default case. Comparing (12) and (15), we have $F_d(x;C) < F_n(x;C)$ for any (x,C) because $F_d(x;C)$, unlike $F_n(x;C)$, includes the term of bankruptcy costs. Therefore, $\max_{C \geq 0} F(x;C) = \max\{F_d(x;C_0(x)), F_n(x;\delta x_B)\}$ holds. For graphical images of $F(x;C)$, see Figure 1. Define the difference as

$$\begin{aligned} G(x;x_B) &= F_n(x;\delta x_B) - F_d(x;C_0(x)) \\ &= \frac{(\gamma-1)\tau x_B}{\gamma(r-\mu)} - \frac{\tau x}{(r-\mu)h} - \left(\frac{x}{x_B}\right)^{\gamma} \frac{\alpha\pi x_B}{\gamma}, \end{aligned} \tag{16}$$

where we used (6), (12), and (15) to obtain (16). The next proposition shows the equity, debt, firm values, coupon, leverage, and credit spreads, denoted by $E(x), D(x), F(x), C(x), LV(x)$, and $CS(x)$ respectively, under optimal capital structure. For the proof, see Appendix B.

Proposition 2 *There exists a unique solution* $x_B^* \in (0, \gamma x/(\gamma-1)h)$ *to* $G(x;x_B^*) = 0$, *and* $x_B < x_B^*$ *is equivalent to* $G(x;x_B) < 0$.

For $x_B < x_B^*$, *the firm issues risky debt with coupon* $C_0(x)$ *and goes bankrupt at default threshold* $x_0(C_0(x)) = x/h$ *(i.e., the default-possible case). The equity, debt, firm values, coupon, leverage, and credit spreads are given by* $E(x) = E_0(x;C_0(x)), D(x) = D_d(x;C_0(x)), F(x) = F_d(x;C_0(x)), C(x) = C_0(x), LV(x) = D_d(x;C_0(x))/F_d(x;C_0(x))$, *and* $CS(x) = C_0(x)/D_d(x;C_0(x)) - r$, *respectively.*

Otherwise, the firm issues riskless debt with coupon δx_B *and never goes bankrupt (i.e., the no-default case). The equity, debt, firm values, coupon, leverage, and credit spreads are given by* $E(x) = E_n(x;\delta x_B), D(x) = \delta x_B/r, F(x) = F_n(x;\delta x_B), C(x) = \delta x_B, LV(x) = \delta x_B/rF_n(x;\delta x_B)$, *and* $CS(x) = 0$, *respectively.*

First, we explain the default-possible case (i.e., $x_B < x_B^*$). The firm prefers to issue risky debt due to the insufficient riskless debt capacity. In this case, the optimal coupon and default timing are the same as those of the benchmark model with no barrier. The other values are obtained by substituting coupon $C_0(x)$ into the default-possible case of Proposition 1. Note that $C_0(x) > \delta x_B$ holds by $F_n(x;\delta x_B) < F_d(x;C_0(x)) < F_n(x;C_0(x))$.

Next, we focus on the no-default case (i.e., $x_B \geq x_B^*$). The firm is better off issuing riskless debt $\delta x_B/r$ because of the sufficient riskless debt capacity. Notably, δx_B is not necessarily higher than $C_0(x)$. Both $\delta x_B < C_0(x)$ and $F_n(x;\delta x_B) \geq F_d(x;C_0(x))$ can be satisfied because $F_n(x;\delta x_B)$, unlike $F_d(x;C_0(x))$, includes no term of bankruptcy costs (e.g., see the top panel of Figure 1). In other words, the firm chooses riskless capital structure if the gain from having no bankruptcy risk dominates the inefficiency from the upper limit of riskless debt.

In Proposition 2, x_B^* stands for the lowest barrier to achieve the no-default case.[5] The critical level x_B^* is lower than the benchmark default threshold $x_0(C_0(x)) = x/h$ by $x_B^* < \gamma x/(\gamma-1)h < x/h$. This result has the following implication for public intervention. Suppose that the government attempts to prevent the firm from bankruptcy. Without the government's ex ante commitment of an intervention threshold, the firm issues debt with coupon $C_0(x)$ (as in the benchmark case with no barrier). In this case, by Proposition 1 with $C = C_0(x)$, the government needs intervention threshold $x_B = C_0(x)/\delta = x/h$ to prevent the firm from bankruptcy. However, the ex ante commitment of intervention threshold $x_B^* (< x/h)$ prevents the firm from bankruptcy. This is because by considering intervention threshold x_B^*, the firm strategically reduces debt (i.e., $\delta x_B^* < C_0(x)$) and chooses riskless capital structure. That is, the government's credible commitment does not cause the firm's moral hazard (i.e., increasing debt) but instead improves the efficiency of the public intervention policy. As we will check numerically in Section 4, the ex ante required level x_B^* is much lower than the ex post required level x/h, which highlights the importance of the credible commitment of the public intervention policy.

We can analytically prove the comparative statics with respect to barrier x_B because Proposition 2 derives all the values explicitly. For the proof, see Appendix C.

Proposition 3 *For $x_B < x_B^*$ (i.e., the default-possible case), $D(x), F(x)$, and $LV(x)$ increase in x_B, $CS(x)$ decreases in x_B, and $C(x) = C_0(x)$, $x_0(C_0(x)) = x/h$, and $E(x) = E_0(x;C_0(x))$ are constant.*

At $x_B = x_B^*$ (i.e., the switching point), $E(x)$ jumps upward, $D(x), C(x), LV(x)$, and $CS(x)$ jump downward, and $F(x)$ is continuous.

For $x_B \geq x_B^*$ (i.e., the no-default case), $D(x), F(x), C(x) = \delta x_B$, and $LV(x)$ increase in x_B, $E(x)$ decreases in x_B, and $CS(x)$ is 0.

Note that the values approach the benchmark values with no barrier for $x_B \to 0$. At $x_B = x_B^*$, the values, except firm value $F(x)$, jump because the firm switches coupon $C(x)$ from $C_0(x)$ (i.e., the default-possible case) to δx_B^* (i.e., the no-default case). The switch results from maximization

[5] By (16), x_B^* depends on initial EBIT $X(0) = x$. Define $y_B = x_B/x$ and $G(x;x_B) = xg(y_B)$. Then, there is a unique solution $y_B^* \in (0, \gamma/(\gamma-1)h)$ to $g(y_B^*) = 0$, and y_B^* is the threshold between the default-possible and no-default cases. In other words, the standardized barrier level x_B/x matters to the firm.

of $F(x)$, and hence, the switch does not cause a jump in $F(x)$. Interestingly, $E(x), D(x), C(x)$, and $LV(x)$ are nonmonotonic with respect to x_B because of the switch. Section 4.1 will show the quantitative effects of x_B on the results in numerical examples.

The no-default case with x_B close to x_B^* is most notable. In this region, $D(x)$ and $LV(x)$ are lower than $D_0(x)$ and $LV_0(x)$ due to $\delta x_B^* < C_0(x)$. As explained previously, this result implies that by the credible commitment of the market intervention threshold, the government can decrease the firm's debt and remove its bankruptcy risk. Furthermore, this result can help explain debt conservatism. It is well known as debt conservatism that some firms have quite low leverage and bankruptcy risk compared to the optimal level predicted by trade-off theory (e.g., Graham (2000), Strebulaev and Yang (2013), and El Ghoul, Guedhami, Kwok, and Zheng (2018)). Debt conservatism is often explained by theories of dynamic (and infrequent) leverage adjustment and financial slack for future investments and downside risks, but our model adds an alternative mechanism. Indeed, a firm can optimally choose riskless capital structure with low leverage if it has a certain degree of competitive advantage or protection against downside risks.

Following the standard models (e.g., Leland (1994) and Goldstein, Ju, and Leland (2001)), our model assumes the post-bankruptcy value as the discounted value of the unlevered value. However, some papers, including Mella-Barral and Perraudin (1997), Lambrecht and Myers (2008), and Shibata and Nishihara (2018), assume a constant component of the post-bankruptcy value (e.g., constant scrap value). The presence of constant liquidation value generates the possibility of riskless debt financing, but its mechanism is different from that of this paper. In these models, shareholders can retire the principal of debt by a part of the constant liquidation value and obtain the residual value. In such a situation, the firm exits the market, but debt becomes riskless. In contrast, in our model, protection against downside risks can lead the firm to operate perpetually in the market, which makes debt riskless.

3.3 Debt issuance constraint

This subsection interprets x_B as the public intervention threshold. In such regulated markets (e.g., transportation, utility, agricultural, and financial industries), the government might not only save firms from financial distress but also regulate excessive uses of debt to remove bankruptcy risk. To explore the effects of such a leverage regulation on the outcome, this subsection extends the baseline model to a model with an upper limit of debt issuance. The extended model assumes that coupon C must satisfy $C \leq \bar{C}$ for a given upper limit $\bar{C}(> 0)$. This is interpreted as a constraint on the book value of debt (i.e., $C/r \leq \bar{C}/r$). However, as we will see in Section 4.5, $D(x)$ and $LV(x)$ monotonically increases in $\bar{C}$. Hence, the results will remain unchanged even if we assume a constraint on the market value of debt or leverage. Assume that $\bar{C} < C_0(x)$ because the firm is unconstrained for $\bar{C} \geq C_0(x)$.

For $\bar{C} \leq \delta x_B$, the firm optimally chooses the maximum coupon $\bar{C}$ and obtain firm value $F_n(x; \bar{C})$ because there is no possibility of bankruptcy (see Proposition 1). For $\bar{C} \in (\delta x_B, C_0(x))$, the firm solves $\max\{F_d(x; \bar{C}), F_n(x; \delta x_B)\}$ because $F_d(x; C)$ increases in $C \leq C_0(x)$. As in the previous subsection, define the difference as

$$\bar{G}(x;x_B) = F_n(x;\delta x_B) - F_d(x;\bar{C})$$
$$= \frac{\tau(\delta x_B - \bar{C})}{r} - \left(\frac{x}{x_B}\right)^\gamma \frac{\alpha\pi x_B}{\gamma} + \left(\frac{x}{x_0(\bar{C})}\right)^\gamma \left(\alpha\pi x_0(\bar{C}) + \frac{\tau\bar{C}}{r}\right), \tag{17}$$

where we used (12) and (15) to obtain (17). The next proposition shows the equity, debt, firm values, coupon, leverage, and credit spreads, denoted by $\bar{E}(x), \bar{D}(x), \bar{F}(x), \bar{C}(x), \bar{LV}(x)$, and $\bar{CS}(x)$, respectively, under the debt issuance constraint. For the proof, see Appendix D.

Proposition 4 *There exists a unique solution $\bar{x_B} \in (0, \min\{x_B^*, \bar{C}/\delta\})$ to $\bar{G}(x;\bar{x_B}) = 0$. The solution $\bar{x_B}$ increases in $\bar{C}$.*

For $x_B < \bar{x_B}$, the firm issues risky debt with coupon $\bar{C}$ and goes bankrupt at default threshold $x_0(\bar{C})$ (i.e., the default-possible case). The equity, debt, firm values, coupon, leverage, and credit spreads are given by $\bar{E}(x) = E_0(x;\bar{C}), \bar{D}(x) = D_d(x;\bar{C}), \bar{F}(x) = F_d(x;\bar{C}), \bar{C}(x) = \bar{C}, \bar{LV}(x) = D_d(x;\bar{C})/F_d(x;\bar{C})$, and $\bar{CS}(x) = \bar{C}/D_d(x;\bar{C}) - r$, respectively.

For $x_B \in [\bar{x_B}, \bar{C}/\delta]$, the firm riskless debt with coupon δx_B and never goes bankrupt (i.e., the no-default case). The equity, debt, firm values, coupon, leverage, and credit spreads are given by $\bar{E}(x) = E_n(x;\delta x_B), \bar{D}(x) = \delta x_B/r, \bar{F}(x) = F_n(x;\delta x_B), \bar{C}(x) = \delta x_B, \bar{LV}(x) = \delta x_B/rF_n(x;\delta x_B)$, and $\bar{CS}(x) = 0$, respectively.

For $x_B > \bar{C}/\delta$, the firm riskless debt with coupon $\bar{C}$ and never goes bankrupt (i.e., the no-default case). The equity, debt, firm values, coupon, leverage, and credit spreads are given by $\bar{E}(x) = E_n(x;\bar{C}), \bar{D}(x) = \bar{C}/r, \bar{F}(x) = F_n(x;\bar{C}), \bar{C}(x) = \bar{C}, \bar{LV}(x) = \bar{C}/rF_n(x;\bar{C})$, and $\bar{CS}(x) = 0$, respectively.

Proposition 4 can be interpreted in the same way as Proposition 2. Indeed, $\bar{x_B}$ represents the lowest barrier to achieve the no-default case. For $x_B < \bar{x_B}$ (i.e., the default-possible case), riskless debt capacity $\delta x_B/r$ is not sufficient, and hence, the firm issues risky debt with the maximum coupon $\bar{C}$. For $x_B \in [\bar{x_B}, \bar{C}/\delta]$ (i.e., the no-default case), riskless debt capacity $\delta x_B/r$ is large enough to lead the firm to choose riskless debt. For $x_B > \bar{C}/\delta$ (i.e., the no-default case), debt with any $C \le \bar{C}$ becomes riskless, and hence, the firm issues riskless debt with the maximum coupon $\bar{C}$.[6] Proposition 4 nests Proposition 2 as the limiting case of $\bar{C} \to C_0(x)$ because $\bar{G}(x;x_B)$ and $\bar{x_B}$ agree with $G(x;x_B)$ and x_B^* in the limiting case.

Proposition 4 shows that lower $\bar{C}$ decreases $\bar{x_B}$ and firm value $F_n(x;\delta x_B)$. That is, with a stronger leverage regulation, the government can weaken market intervention, but the firm value lowers. In reality, the government's regulation and intervention require direct and indirect costs, causing spillover effects on other firms and industries. The combination of strong regulation (i.e., low $\bar{C}$) and weak intervention (i.e, low $\bar{x_B}$) will decrease the intervention cost but increase the regulation cost. The government chooses one from the set $\{(\bar{C}, \bar{x_B}) \mid 0 \le \bar{C} \le C_0(x)\}$ so that it

[6] Although the debt issuance constraint is imposed even for riskless debt in this paper, it may be imposed only for risky debt to reduce default risk (see Nishihara, Shibata, and Zhang (2023)). In that setup, the firm issues riskless debt $\delta x_B/r$ for any $x_B \ge \bar{x_B}$, and it does not matter whether $\bar{x_B}$ is higher than $\bar{C}/r$.

can minimize the total costs based on the trade-off. It is beyond the scope of this paper to model the total social costs and derive the optimal choice. Note that $\bar{x_B}$ is lower than $x_0(\bar{C}) = \bar{C}/\delta$. As in Proposition Proposition 2, this implies that by the ex ante commitment, the government can improve the efficiency of the market intervention policy.

Although this paper interprets $\bar{C}$ as a leverage regulation, it can be be interpreted as a financing constraint imposed by debt holders. In this context, some papers investigate the effects of a borrowing constraint on the investment and financing timing problems with no barrier (e.g., Shibata and Nishihara (2012), Shibata and Nishihara (2015), and Shibata and Nishihara (2018)). In particular, Shibata and Nishihara (2018) and Nishihara, Shibata, and Zhang (2023) show that under very hard borrowing constraints, the firm tends to issue riskless debt in the models with constant liquidation value.[7] The previous results align with our result that lower $\bar{C}$ decreases $\bar{x_B}$.

As in Proposition 3, we can analytically prove the comparative statics with respect to barrier x_B. For the proof, see Appendix E.

Proposition 5 *For $x_B < \bar{x_B}$ (i.e., the default-possible case), $\bar{D}(x), \bar{F}(x)$, and $\bar{LV}(x)$ increase in x_B, $\bar{CS}(x)$ decreases in x_B, and $\bar{C}(x) = \bar{C}$, $x_0(\bar{C}) = \bar{C}/\delta$, and $\bar{E}(x) = E_0(x; \bar{C})$ are constant.*

At $x_B = \bar{x_B}$ (i.e., the switching point), $\bar{E}(x)$ jumps upward, $\bar{D}(x), \bar{C}(x), \bar{LV}(x)$, and $\bar{CS}(x)$ jump downward, and $\bar{F}(x)$ is continuous.

For $x_B \in [\bar{x_B}, \bar{C}/\delta]$ (i.e., the no-default case), $\bar{D}(x), \bar{F}(x), \bar{C}(x)$, and $\bar{LV}(x)$ increase in x_B, $\bar{E}(x)$ decreases in x_B, and $\bar{CS}(x)$ is 0.

For $x_B > \bar{C}/\delta$ (i.e., the no-default case), $\bar{E}(x)$ and $\bar{F}(x)$ increase in x_B, $\bar{LV}(x)$ decrease in x_B, and $\bar{D}(x) = \bar{C}/r, \bar{C}(x) = \bar{C}$, $\bar{CS}(x) = 0$ are constant.

Proposition 5 shows that the comparative statics with respect to barrier x_B are mostly unchanged from Proposition 3, even if the model includes the debt issuance constraint. At $x_B = \bar{x_B}$, the values, except firm value $F(x)$, jump because coupon $\bar{C}(x)$ jumps from $\bar{C}$ (i.e., the default-possible case) to δx_B^* (i.e., the no-default case). All the values are continuous at $x_B = \bar{C}/\delta$ because $\bar{C}(x)$ is continuous. Due to the switching point $x_B = \bar{x_B}$, $\bar{E}(x), \bar{D}(x), \bar{C}(x)$, and $\bar{LV}(x)$ become nonmonotonic with respect to x_B.

4 Numerical analysis and implications

4.1 Baseline results

This section conducts numerical analyses, including comparative statics with respect to barrier x_B, volatility σ, growth rate μ, bankruptcy cost α, and upper limit $\bar{C}$. The baseline parameter values are set as in Table 1, where the values of r, μ, σ, τ, and α are standard in dynamic corporate finance

[7] In these previous models, a firm optimally chooses risky debt financing in the first-best case with no financing constraint. This differs from this paper's result (cf. Proposition 2). Regarding this issue, Shibata and Nishihara (2023) show that with a high degree of information asymmetry between managers and shareholders. the firm can use riskless debt financing.

literature and reflect a typical S&P firm (e.g., Morellec (2001), Arnold (2014), and Nishihara, Shibata, and Zhang (2023)). The initial EBIT value is normalized as $x = X(0) = 1$. For these parameter values, the lowest barrier to achieve the no-default case becomes $x_B^* = 0.153$, which amounts to 15.3% of the initial EBIT. In the baseline case, we set $x_B = 0.2$ (i.e., the no-default case), which is close to $x_B^* = 0.153$, so that the outcome will switch between the no-default and default-possible cases with varying levels of other parameters (cf. Sections 4.3, 4.4, and 4.5). Sections 4.1–4.5 focus on the baseline model, and Section 4.6 examines the model with a debt issuance constraint.

Figure 1 depicts firm value $F(x;C)$ for varying levels of C and x_B. As shown by (15) in Proposition 1, $F(x;C)$ increases linearly in C up to $C = \delta x_B = 0.435, 0.333$, and 0.217 in the top, center, and bottom panels, respectively. In each panel, $F(x;C)$ jumps downward after point $x = \delta x_B$ because $F_n(x;\delta x_B) > F_d(x;\delta x_B)$ holds in (12) and (15). All the results are shown by line graphs in Section 4, and lines that look vertical stand for jumps. For the baseline parameter values, we have $C_0(x) = 0.623$. For $C > \delta x_B$, $F(x;C) = F_d(x;C)$ takes its maximum value at $C = C_0(x) = 0.623$, whereas for $C \leq \delta x_B$, $F(x;C) = F_n(x;C)$ takes its maximum value at $C = \delta x_B = 0.435, 0.333$, and 0.217 in the panels. In the center panel (i.e., $x_B = x_B^* = 0.153$), $F_n(x;\delta x_B)$ agrees with $F_d(x;C_0(x))$, and hence, the firm is indifferent to the choice between coupon $\delta x_B = 0.333$ or $C_0(x) = 0.623$. In the baseline case (i.e., $x_B = 0.2$; see the top panel), $F_n(x;\delta x_B)$ is higher than $F_d(x;C_0(x))$, and the firm chooses $C(x) = \delta x_B = 0.435$ (i.e., riskless capital structure). In the bottom panel (i.e., $x_B = 0.1$), $F_n(x;\delta x_B)$ is lower than $F_d(x;C_0(x))$, and the firm chooses $C(x) = C_0(x) = 0.623$ (i.e., risky capital structure).

Tables 2 and 3 show the baseline results (i.e., $x_B = 0.2$) and benchmark results with no barrier, respectively. As mentioned above, in the baseline model, the firm prefers riskless capital structure (i.e., the no-default case). The firm issues debt up to the riskless debt capacity (i.e., $D(x) = \delta x_B/r = 8.7$) to obtain the maximum tax benefits. Leverage becomes $LV(x) = 0.38$, but credit spreads are $CS(x) = 0$ because of riskless debt. In the benchmark model with no barrier, the firm chooses coupon $C_0(x) = 0.623$ based on the trade-off between the tax benefits and bankruptcy costs of debt. The firm will go bankrupt when $X(t)$ falls to $x_0(C_0(x)) = 0.287$. Hence, $D_0(x) = 10.84$ is discounted from $C_0(x)/r = 12.46$ due to default risk, and credit spreads are positive (i.e., $CS_0(x) = 0.00751$).

In Tables 2 and 3, it holds that $C(x) < C_0(x), E(x) > E_0(x), D(x) < D_0(x), F(x) > F_0(x), LV(x) < LV_0(x)$, and $CS(x) < CS_0(x)$. It is straightforward that $F(x) > F_0(x)$ and $CS(x) < CS_0(x)$, and $E(x) > E_0(x)$ readily follows from $C(x) < C_0(x)$. Inequalities $C(x) < C_0(x), D(x) < D_0(x)$, and $LV(x) < LV_0(x)$ are notable. As discussed after Proposition 3, these inequalities imply that barrier $x_B = 0.2$ leads the firm to strategically reduce debt to take advantage of riskless capital structure rather than to increase debt. In particular, $LV(x) = 0.3798$ is much lower than $LV_0(x) = 0.4854$. As explained after Proposition 3, the model can help explain firms with low leverage and no bankruptcy risk observed in the real world.

4.2 Effects of barrier x_B

Figure 2 depicts $C(x), x_0(C(x)), E(x), D(x), F(x), LV(x)$, and $CS(x)$ for varying levels of barrier x_B.[8] The other parameter values are set as in Table 1. Region $x_B < x_B^* = 0.153$ is the default-possible case, whereas region $x_B \geq x_B^* = 0.153$ is the no-default case. Default threshold $x_0(C(x))$ is depicted only in the default-possible case. For comparison, Figure 2 also depicts the benchmark results with no barrier by dashed lines. The benchmark results do not depend on x_B.

Although Proposition 3 has already shown the comparative static results analytically, Figure 2 shows them more closely and quantitatively. For instance, we find that the effects of x_B on $D(x), F(x), LV(x)$, and $CS(x)$ are very weak in the default-possible region (i.e., $x_B < x_B^* = 0.153$). This is because $C(x) = C_0(x)$ and $E(x) = E_0(x)$ do not depend on x_B and the third term in (11) is very small. Thus, barrier x_B does not largely change $D(x), F(x), LV(x)$, and $CS(x)$ from the benchmark values $D_0(x), F_0(x), LV_0(x)$, and $CS_0(x)$. However, in the no-default region (i.e., $x_B \geq x_B^* = 0.153$), the effects of x_B on $E(x), D(x), F(x)$, and $LV(x)$ are strong because $C(x) = \delta x_B$ increases linearly in x_B. As discussed after Proposition 3, $C(x), D(x)$, and $LV(x)$ are lower than $C_0(x), D_0(x)$, and $LV_0(x)$ for x_B close to $x_B^* = 0.153$, whereas $C(x), D(x)$, and $LV(x)$ are higher than $C_0(x), D_0(x)$, and $LV_0(x)$ for $x_B > 0.3$.

These results entail several implications. First, we interpret x_B as the degree of competitive advantage. Then, the model shows that leverage can be nonmonotonic with respect to the degree of competitive advantage. Indeed, firms with intermediate levels of competitive advantage can take low leverage with only riskless debt.

Next, we interpret x_B as the strength of public intervention. The critical intervention threshold $x_B^* = 0.153$ (i.e., 15.3% of the initial EBIT) is not very high. In absence of the ex ante commitment of the intervention threshold, as explained after Proposition 2, the government would need intervention threshold $x_B = x_0(C_0(x)) = 0.287$ to prevent the firm from bankruptcy ex post. With a credible commitment, the government can prevent the firm from bankruptcy by almost a half intervention threshold (i.e., $x_B^* = 0.153$). Of course, in the real world including uncertainty and diversity of firm parameter values, it may be difficult for the government to match $x_B = x_B^*$ perfectly. A low market intervention threshold (i.e., $x_B < x_B^* = 0.153$) hardly influences capital structure, bankruptcy probability, and firm value, whereas a high market intervention threshold (say, $x_B > 0.3$) prevents bankruptcy but leads to the firm's moral hazard (i.e., increasing leverage to gain tax benefits). It is important to set an appropriate intervention threshold (i.e., $x_B \approx x_B^* = 0.153$) to prevent bankruptcy and reduce leverage effectively.

[8] Rodrigues (2022) also studies the comparative statics with respect to floor levels in numerical examples. Although his model is more complicated than our model, the effects of floor levels on capital structure are qualitatively unchanged from our results. Indeed, Rodrigues (2022) also shows that a higher floor leads to riskless capital structure.

4.3 Effects of volatility σ

Figure 3 depicts $C(x), x_0(C(x)), E(x), D(x), F(x), LV(x), CS(x), x_B^*$, and $(x/x_B^*)^\gamma$ for varying levels of volatility σ.[9] The other parameter values are set as in Table 1. Region $\sigma < 0.17$ is the default-possible case, whereas region $\sigma \geq 0.17$ is the no-default case. Default threshold $x_0(C(x))$ is depicted only in the default-possible case. For comparison, Figure 3 also depicts the benchmark results with no barrier by dashed lines.

In the default-possible region (i.e., $\sigma < 0.17$), each value moves in the same way as in the benchmark case with no barrier. In fact, higher σ decreases $C(x), x_0(C(x)), D(x), F(x)$, and $LV(x)$ and increases $E(x)$. These results can be intuitively interpreted as follows. Higher σ increases bankruptcy risk, and the firm reduces leverage to mitigate bankruptcy risk. However, decreased leverage does not fully offset increased bankruptcy risk with higher σ, and hence, credit spreads increase in σ. Firm value decreases in σ due to the lower leverage effect, although equity value increases due to decreased coupon payments. These results align with the standard results in previous literature (e.g., Leland (1994)). As in Figure 2, Figure 3 also shows that the presence of x_B hardly affects the values in the default-possible region.

At $\sigma = 0.17$, the result switches from the default-possible case to the no-default case. Then, $C(x), E(x), D(x), LV(x)$, and $CS(x)$ jump at this point. In the no-default region (i.e., $\sigma \geq 0.17$), $C(x), E(x), D(x), F(x)$, and $LV(x)$ move contrary to the benchmark case with no barrier. The comparative statics are explained by the sensitivity of riskless debt capacity $\delta x_B/r$ to σ. Note that $\delta x_B/r$ increases in σ by $\partial\delta/\partial\sigma > 0$. Then, $C(x) = \delta x_B, D(x) = \delta x_B/r, F(x)$, and $LV(x)$ increase in σ, whereas $E(x)$ decreases in σ due to increased $C(x)$. The results imply that the effects of volatility on firm value and capital structure for firms with sufficient competitive advantage or protection against downside risks can greatly differ from those for ordinary firms.

Note that the above results are based on the assumption of constant barrier level $x_B = 0.2$. Barrier $x_B = 0.2$ is more effective with higher σ because the probability of $X(t)$ hitting $x_B = 0.2$ increases with higher σ. The bottom-right panel of Figure 3 shows that the critical level x_B^* and the state price $(x/x_B^*)^\gamma$[10] decrease and increase, respectively, in σ. The comparative statics of x_B^* are explained by the decrease in $F_d(x)$ and increase in $F_n(x)$ with higher σ (see $F(x)$ of Figure 3). By these two effects, x_B^*, which is the unique solution to (16), decreases in σ. Despite the decrease in σ, $(x/x_B^*)^\gamma$ increases in σ due to $\partial\gamma/\partial\sigma > 0$. In other words, higher σ makes $X(t)$ more volatile and increases the probability of $X(t)$ hitting x_B^*. In terms of public intervention, these results suggest that the government needs a lower market intervention threshold but more frequent interventions to prevent a more volatile firm from bankruptcy.

[9] Rodrigues (2022) also studies the comparative statics with respect to volatility in numerical examples. The effects of volatility on capital structure in his shadow process model are qualitatively the same as in our reflecting barrier model.

[10] The state price denotes the present values of \$1 contingent on $X(t)$ hitting x_B^*.

4.4 Effects of growth rate μ

Figure 4 depicts $C(x), x_0(C(x)), E(x), D(x), F(x), LV(x), CS(x), x_B^*$, and $(x/x_B^*)^\gamma$ for varying levels of growth rate μ.[11] The other parameter values are set as in Table 1. Region $\mu \leq 0.0252$ is the no-default case, whereas region $\mu > 0.0252$ is the default-possible case. Default threshold $x_0(C(x))$ is depicted only in the default-possible case. For comparison, Figure 4 also depicts the benchmark results with no barrier by dashed lines.

As in Figures 2 and 3, Figure 4 shows that all the values in the baseline case are almost the same as those in the benchmark case in the default-possible region (i.e., $\mu > 0.0252$). One reason is that the firm choose the same coupon $C(x) = C_0(x)$, and the other reason is that the state price contingent on $X(t)$ hitting x_B (i.e., $(x/x_B)^\gamma$) is very low. We omit explaining the details of the comparative statics in the default-possible case because they are the same as those in the standard model with no barrier (e.g., Leland (1994)).

At the switching point $\mu = 0.0252$, $C(x), E(x), D(x), LV(x)$, and $CS(x)$ jump. Even in the no-default region (i.e., $\mu \leq 0.0252$), $C(x), E(x), D(x)$, and $F(x)$ change with μ in the same way as in the benchmark values. More notably, $LV(x)$ decreases in μ, contrary to $LV_0(x)$. The reason is as follows. Riskless debt capacity $D(x) = \delta x_B/r$ increases in μ by $\partial\delta/\partial\mu > 0$, and equity value $E(x) = E(x;\delta x_B)$ also increases in μ by $\partial\pi/\partial\mu > 0$ in (13). The latter effect dominates the former effect, and hence $LV(x)$ decreases in μ. This sensitivity is novel and contrasted with the standard result. In fact, the standard trade-off models (e.g., Leland (1994)) predict a positive relation between leverage and growth rate (see $LV_0(x)$ in Figure 4), but empirical studies (e.g., Titman and Wessels (1988) and Frank and Goyal (2015)) show a negative relation. This is well known as a deficit of the standard trade-off models (e.g., Demarzo (2019)). Our model may help resolve the problem. Indeed, the model predicts a negative relation between leverage and cash flows for firms with sufficient competitive advantage or protection against downside risks.[12] This is because such a firm can set debt level by riskless debt capacity rather than the trade-off between the tax benefits and bankruptcy costs.

The bottom-right panel of Figure 4 shows that x_B^* and $(x/x_B^*)^\gamma$ increase and decrease, respectively, in μ. The former result is caused by $F_d(x)$ increasing in μ more than $F_n(x)$ does. Despite the increase in σ, $(x/x_B^*)^\gamma$ decreases in μ due to $\partial\gamma/\partial\mu < 0$ (i.e., higher μ decreases the probability of $X(t)$ hitting x_B^*). These results entail a policy implication that the government needs a higher market intervention threshold but less frequent market interventions to prevent a high-growth firm from bankruptcy.

[11] Rodrigues (2022) also studies the comparative statics with respect to growth rate in numerical examples. The effects of growth rate on capital structure in his shadow process model are qualitatively the same as in our reflecting barrier model.

[12] The relation between $X(0) = x$ and $LV(x)$ is also negative in the no-default region because as explained in footnote 5, the sensitivities to x is inverse to the sensitivities to x_B (see Figure 2).

4.5 Effects of bankruptcy cost α

Figure 5 depicts $C(x), x_0(C(x)), E(x), D(x), F(x), LV(x), CS(x), x_B^*$, and $(x/x_B^*)^\gamma$ for varying levels of bankruptcy cost α. The other parameter values are set as in Table 1. Region $\alpha < 0.199$ is the no-default case, whereas region $\alpha \geq 0.199$ is the default-possible case. Default threshold $x_0(C(x))$ is depicted only in the default-possible case. For comparison, Figure 5 also depicts the benchmark results with no barrier by dashed lines.

In the no-default region (i.e., $\alpha \geq 0.199$), neither value depends on α because the firm will never go bankrupt. In the default-possible region (i.e., $\alpha < 0.199$), all the values change with α in the same way as in the benchmark values with no barrier. In this region, higher α increases the disadvantages of debt and hence decreases $C(x), D(x)$, and $LV(x)$. Firm value $F(x)$ and $CS(x)$ also decrease in α due to the decreased leverage effect, whereas $E(x)$ increases in α due to decreased coupon payments.

By (16) and $\partial h/\partial \alpha > 0$, we can easily prove that x_B^* decreases in α. The bottom-right panel of Figure 3 numerically verifies the sensitivity of x_B^* to α. Note that $(x/x_B^*)^\gamma$ changes in the same way as x_B^* because γ does not depend on α. This result is intuitively explained as follows. Higher α increases the disadvantages of risky debt and decreases the leverage effect. Then, the firm is more likely to be better off using riskless debt rather than relying on risky debt. For the same reason, lower corporate tax rate τ decreases x_B^* and $(x/x_B^*)^\gamma$. although we omit depicting a figure with varying levels of τ. Indeed, lower τ decreases the tax advantages of debt, which decreases the firm's motive to use risky debt. The comparative static results have the following implications of public intervention. The government can prevent the firm from bankruptcy by weaker and fewer market interventions if it imposes a lower corporate tax rate and a more stringent bankruptcy law with higher bankruptcy penalty. This is because with such public policies, the firm has fewer advantages from issuing risky debt and is more likely to choose riskless capital structure.

4.6 Effects of upper limit $\bar{C}$

So far, we have examined the effects of the key parameters on the results in the baseline model. This subsection studies the effects of upper limit $\bar{C}$ in the extended model of Section 3.3. For the baseline parameter values (i.e., Table 1), the no-default case holds by $x_B^* = 0.1529 < x_B = 0.2$ in absence of $\bar{C}$. By Proposition 4, we have $\bar{x_B} < x_B^* = 0.153 < x_B = 0.2$ for any $\bar{C}$, and hence, the no-default case holds for any $\bar{C}$. We reset $x_B = 0.1$ to depict both the no-default and default-possible cases. The other parameter values are set as in Table 1. Figure 6 depicts $\bar{C}(x), x_0(\bar{C}(x)), \bar{E}(x), \bar{D}(x), \bar{F}(x), \bar{LV}(x), \bar{CS}(x), \bar{x_B}$, and $(x/\bar{x_B})^\gamma$ for varying levels of $\bar{C}(\leq C_0(x) = 0.623)$. Note that $\bar{C}$ does not bind the firm for $\bar{C} \geq C_0(x) = 0.623$. Region $\bar{C} \leq 0.261$ is the no-default region, whereas region $\bar{C} > 0.261$ is the default-possible region. Default threshold $x_0(\bar{C}(x))$ is depicted only in the default-possible case. The no-default region is classified into region $\bar{C} \in [0.218, 0.261]$, where $\bar{C}(x) = \delta x_B = 0.218$ does not depend on $\bar{C}$, and region $\bar{C} < 0.218$, where $\bar{C}(x) = \bar{C}$ (see Proposition 4). For comparison, Figure 5 also depicts the benchmark results with no barrier under upper limit $\bar{C}$ by dashed lines. In this benchmark case, the firm chooses coupon $\bar{C}$ because firm value $F_0(x;C)$ (see (3)) monotonically increases in C up to $C = C_0(x) = 0.623$.

In the default-possible region (i.e., $\bar{C} > 0.261$) of Figure 6, the presence of x_B hardly affects each value. The main reason is that the firm chooses the maximum coupon $\bar{C}$ regardless of x_B. All the comparative static results are straightforward and the same as the benchmark results with no barrier. Higher $\bar{C}$ increases $\bar{D}(x)$ and $\bar{LV}(x)$. The increased leverage effects increase $\bar{F}(x)$, although the increased coupon payments decrease $\bar{E}(x)$ and increase $\bar{CS}(x)$. Note that each value agrees with that of the unconstrained baseline model for $\bar{C} = C_0(x) = 0.623$ (i.e., the right end of each panel of Figure 6).

The no-default region $\bar{C} \in [0.218, 0.261]$ is most notable. In this region, the firm chooses riskless capital structure because $\bar{x_B} \leq x_B = 0.1$ (see the bottom-right panel of Figure 6). Riskless debt capacity $\delta x_B = 0.218$ rather than debt issuance limit $\bar{C}$ binds the firm due to $\delta x_B = 0.218 \leq \bar{C}$. Then, coupon $\bar{C}(x) = \delta x_B = 0.218$ is constant in this region. This also implies that $\bar{E}(x), \bar{D}(x), \bar{F}(x)$, and $\bar{LV}(x)$ are constant in this region. These results are contrasted with the benchmark results with no barrier.

Last, we turn to the no-default region $\bar{C} < 0.218$. In this region, debt issuance limit $\bar{C}$ rather than riskless debt capacity $\delta x_B = 0.218$ binds the firm due to $\bar{C} < \delta x_B = 0.218$. Then, the firm chooses the maximum coupon $\bar{C}$ as in the benchmark case with no barrier. The comparative static results other than $\bar{CS}(x) = 0$ are the same with the standard results with no barrier. Note that each value converges to that of the all-equity firm for $\bar{C} \to 0$ (i.e., the left end of each panel of Figure 6).

As shown by Proposition 4, the bottom-right panel of Figure 6 shows that the critical level $\bar{x_B}$ increases in $\bar{C}$. State price $(x/\bar{x_B})^\gamma$ similarly increases in $\bar{C}$ because γ does not depend on $\bar{C}$. These results show that by regulating leverage, the government can reduce the market intervention threshold and frequency to prevent the firm from bankruptcy. As discussed after Proposition 4, the optimal policy would lie in $\{(\bar{C}, \bar{x_B}) \mid 0 \leq \bar{C} \leq C_0(x)\}$, but it may be difficult for the government to find a perfectly optimal pair $(\bar{C}, \bar{x_B})$. In fact, the government tends to impose a uniform regulation and protection policy over firms within the same industry, although cash flows are affected by firm-specific factors and risks. That is, $(\bar{C}, \bar{x_B})$ differs over firms in the industry, but the government must choose one policy for all the firms. Regulation that is too weak cannot prevent bankruptcy (cf. the region $\bar{C} > 0.261$), whereas regulation that is too strong decreases firm value inefficiently (cf. the region $\bar{C} < 0.218$). Even if the government cannot find a perfect solution for all the firms, it can choose a policy within the plausible region (cf. the region $\bar{C} \in [0.218, 0.261]$).

5 Conclusion

This paper investigates the capital structure model with earnings above a reflecting barrier. The model can approximate a firm with competitive advantage or public protection against downside risks. In the former, the barrier represents an exit threshold of competitors, whereas in the latter, it represents a public intervention threshold. This paper explicitly derives the equity, debt, firm values, leverage, and credit spreads and shows their comparative statics with respect to barrier level. The main results are summarized below.

First, and most notably, the barrier generates the riskless debt capacity, and the firm chooses either riskless or risky capital structure by comparing the values with the maximum riskless debt and with risky debt. The higher the barrier, the larger the riskless debt capacity, and the firm tends to prefer riskless capital structure. With intermediate barrier levels, the firm chooses lower leverage than the level with no barrier to take advantage of riskless debt.

This result can help explain debt conservatism observed in the real world. Indeed, the model predicts that firms with certain degrees of competitive advantage or public protection can issue lower levels of riskless debt rather than adjusting risky debt levels based on the trade-off between the tax benefits and bankruptcy costs of debt. In the no-default case, contrary to the results in standard trade-off theory, leverage increases with higher volatility and lower growth rate. The latter result can account for empirical findings of a negative relation between leverage and profitability.

The model also entails several implications of public intervention to protect specific firms or industries from financial distress. Using the ex ante commitment of an appropriate intervention threshold, the government can efficiently lead firms to adopt riskless capital structure with low leverage. With a more stringent bankruptcy law (i.e., higher bankruptcy cost), lower corporate tax rate, and stronger leverage regulation, the government needs weaker and fewer market interventions to prevent the firms from bankruptcy.

A Proof of Proposition 1

First, derive the equity value of the firm that operates perpetually, i.e., $E_n(x;C)$. The derivation process is the same as in the reflecting barrier models in Chapter 8 of Dixit and Pindyck (1994). Equity value $E_n(x;C)$ satisfies the differential equation

$$\mu\frac{\partial E_n(x;C)}{\partial x} + 0.5\sigma^2\frac{\partial^2 E_n(x;C)}{\partial^2 x} + (1-\tau)(x-C) = rE_n(x;C) \tag{18}$$

for $x > x_B$ with the boundary conditions

$$\frac{\partial E_n(x_B;C)}{\partial x} = 0, \tag{19}$$

$$\lim_{x\to\infty}\frac{E_n(x;C)}{\pi(x)} < \infty. \tag{20}$$

Note that (19) means that the derivative of $E_n(x;C)$ must be 0 at reflecting barrier x_B because $X(t)$ surely increases from $X(0) = x_B$, while (20) stems from the fact the probability of $X(t)$ hitting x_B approaches 0 for $X(0) \to \infty$. By (18) and (20), $E_n(x;C)$ is expressed as

$$E_n(x;C) = \pi x - \frac{(1-\tau)C}{r} + Ax^\gamma,$$

where A is a constant. By (19), we can derive A as

$$A = -\frac{(1-\alpha)\pi x_B^{1-\gamma}}{\gamma}.$$

Then, $E_n(x;C)$ is expressed as (13). Note that $E_n(x_B;C) \geq 0$ holds if and only if $C \leq \delta x_B$. Accordingly, for $C \leq \delta x_B$, $E_n(x;C) \geq 0$ holds for all $x \geq x_B$, which implies that shareholders do not prefer to receive default value 0 by declaring default. Then, equity value $E(x;C)$ becomes $E_n(x;C)$ in this case. Debt is riskless, and hence $D_d(x;C) = C/r$ holds. By summing this and $E_n(x;C)$, we have $F_n(x;C)$ as (15).

On the other hand, for $C > \delta x_B$, $E_n(x_B;C) < 0$ holds, which implies that shareholders prefer to declare default at a sufficiently low threshold $x_d (\geq x_B)$. Note that $C > \delta x_B$ is equivalent to $x_0(C) \geq x_B$. As in the standard literature (e.g., Goldstein, Ju, and Leland (2001), Shibata and Nishihara (2012), and Sundaresan, Wang, and Yang (2015)), the equity value of the firm that defaults at the optimal timing, i.e., $E_d(x;C)$, is expressed as

$$
\begin{aligned}
E_d(x;C) &= \sup_{x_d \geq x_B} \left(\pi x - \frac{(1-\tau)C}{r} + \left(\frac{x}{x_d}\right)^{\gamma} \left(\frac{(1-\tau)C}{r} - \pi x_d \right) \right) \\
&= \pi x - \frac{(1-\tau)C}{r} + \left(\frac{x}{x_0(C)}\right)^{\gamma} \left(\frac{(1-\tau)C}{r} - \pi x_0(C) \right) \\
&= E_0(x;C).
\end{aligned}
$$

Hence, equity value $E(x;C)$ becomes $E_d(x;C) = E_0(x;C)$ in this case. It should be noted that $E_0(x;C) > E_n(x;C)$ holds for $x \geq \max\{x_B, x_0(C)\}$ if and only if $C > \delta x_B$.

Debt value is derived as

$$
\begin{aligned}
D_d(x;C) &= \frac{C}{r} - \left(\frac{x}{x_0(C)}\right)^{\gamma} \left(\frac{C}{r} - (1-\alpha) F_n(x_0(C);0) \right) \\
&= \frac{C}{r} - \left(\frac{x}{x_0(C)}\right)^{\gamma} \left(\frac{C}{r} - (1-\alpha)\pi x_0(C) \right) - \left(\frac{x}{x_B}\right)^{\gamma} \frac{(1-\alpha)\pi x_B}{\gamma}.
\end{aligned}
$$

By summing this and $E_0(x;C)$, we also obtain $F_d(x;C)$ as (12).

B Proof of Proposition 2

By (16) and $\gamma < 0$, $G(x;x_B)$ is continuously increases in $x_B \in [0,x]$. By (16), we also have

$$
G(x;0) = -\frac{\tau x}{(r-\mu)h} < 0 \tag{21}
$$

$$
G(x;\tilde{x_B}) = -\left(\frac{x}{\tilde{x_B}}\right)^{\gamma} \frac{\alpha \pi \tilde{x_B}}{\gamma} > 0, \tag{22}
$$

where $\tilde{x_B} = \gamma x/(\gamma-1)h$. Therefore, a unique solution $x_B^* \in (0, \gamma x/(\gamma-1)h)$ exists to $G(x;x_B^*) = 0$.

For $x_B < x_B^*$, $G(x;x_B) < 0$ holds, which leads to $\max_{C \geq 0} F(x;C) = \max\{F_d(x;C_0(x)), F_n(x;\delta x_B)\} = F_d(x;C_0(x))$. Hence, the firm chooses coupon $C(x) = C_0(x)$ at time 0. Note that $C_0(x) = \delta x/h > \delta x_B$ follows from $x_B < x_B^* < \gamma x/(\gamma-1)h$. Then, the equity, debt, firm values, coupon, default threshold, leverage, and credit spreads are equal to those of the default-possible case with $C = C_0(x)$ in Proposition 1.

For $x_B \geq x_B^*$, $G(x;x_B) \geq 0$ holds, which leads to $\max_{C \geq 0} F(x;C) = \max\{F_d(x;C_0(x)), F_n(x;\delta x_B)\} = F_n(x;\delta x_B)$. Hence, the firm chooses coupon $C(x) = \delta x_B$ at time 0. Then, the equity, debt, firm values, coupon, default threshold, leverage, and credit spreads are equal to those of the no-default case with $C = \delta x_B$ in Proposition 1.

C Proof of Proposition 3

By Propositions 1 and 2, for $x_B < x_B^*$, $D(x) = D_d(x; C_0(x))$ increases in x_B, while $C(x) = C_0(x)$, $x_0(C_0(x)) = x/h$, and $E(x) = E_0(x; C_0(x))$ are constant. Then, $F(x)$ and $LV(x)$ increase in x_B, $CS(x)$ decreases in x_B.

At $x_B = x_B^*$, coupon $C(x)$ changes from $C_0(x) = \delta x/h$ to δx_B^*. It follows from $x_B^* < \gamma x/(\gamma-1)h$ that $C_0(x) = \delta x/h > \delta x_B^*$ (i.e., a downward jump). At $x_B = x_B^*$, $E(x)$ changes from $E_0(x; C_0(x))$ to

$$\begin{aligned}E_n(x; \delta x_B^*) &= \pi x - \frac{(1-\tau)\delta x_B^*}{r} - \left(\frac{x}{x_B^*}\right)^\gamma \frac{\pi x_B^*}{\gamma} \\ &= E_0(x; \delta x_B^*) \\ &> E_0(x; C_0(x))\end{aligned}$$

(i.e., an upward jump), where we obtained the last inequality by $C_0(x) > \delta x_B^*$. By definition of x_B^* (i.e., $G(x; x_B^*) = 0$), $F_d(x; C_0(x))$ continuously changes to $F_n(x; \delta x_B^*))$ at $x_B = x_B^*$. By the continuity of $F(x)$ and the upward jump of $E(x)$, $D(x)$ must jump downward at $x_B = x_B^*$. Then, $LV(x) = D(x)/F(x)$ jumps downward at $x_B = x_B^*$, and $CS(x)$ also jumps downward to 0 (i.e., riskless debt).

By Propositions 1 and 2, for $x_B \geq x_B^*$, $D(x) = \delta x_B/r$ and $F(x) = F_n(x; \delta x_B), C(x) = \delta x_B$ increase in x_B, while $CS(x)$ is 0. Define

$$H(x_B) = E_n(x; \delta x_B) = \pi x - \frac{(1-\tau)\delta x_B}{r} - \left(\frac{x}{x_B}\right)^\gamma \frac{\pi x_B}{\gamma}$$

and compute the derivative

$$\frac{\mathrm{d}H(x_B)}{\mathrm{d}x_B} = -\frac{(\gamma-1)\pi}{\gamma}\left(1 - \left(\frac{x}{x_B}\right)^\gamma\right) < 0,$$

where the last inequality follows from $x > x_B$ and $\gamma < 0$. Hence, $E(x) = E_n(x; \delta x_B)$ decreases in x_B. By the decrease of $E(x)$ and increase of $D(x)$, $LV(x)$ increases in x_B.

D Proof of Proposition 4

By (17), and $\gamma < 0$, $\bar{G}(x; x_B)$ is continuously increases in $x_B \in [0, x]$. By (17), we can show that

$$\begin{aligned}\bar{G}(x; 0) &= -\frac{\bar{C}}{r} + \left(\frac{x}{x_0(\bar{C})}\right)^\gamma \left(\alpha\pi x_0(\bar{C}) + \frac{\tau\bar{C}}{r}\right) \\ &= \pi x - F_0(x; \bar{C}) < 0, \qquad (23) \\ \bar{G}(x; x_B^*) &= F_n(x; \delta x_B^*) - F_d(x; \bar{C}) \\ &= F_d(x; C_0(x)) - F_d(x; \bar{C}) > 0, \qquad (24) \\ \bar{G}(x; \bar{C}/\delta) &= -\left(\frac{\delta x}{\bar{C}}\right)^\gamma \frac{\alpha\pi\bar{C}}{\gamma\delta} + \left(\frac{x}{x_0(\bar{C})}\right)^\gamma \left(\alpha\pi x_0(\bar{C}) + \frac{\tau\bar{C}}{r}\right) > 0,\end{aligned}$$

where we used $\pi x = F_0(x; 0) < F_0(x; \bar{C})$ in (23), and we used $F_n(x; \delta x_B^*) = F_d(x; C_0(x))$ and the optimality of $C_0(x)$ in (24). Hence, a unique solution $\bar{x_B} \in (0, \min\{x_B^*, \bar{C}/\delta\})$ exists to $\bar{G}(x; \bar{x_B}) = 0$.

For $x_B < \bar{x_B}$, $\bar{G}(x; x_B) < 0$ holds, which leads to $\max_{C\in[0,\bar{C}]} F(x;C) = \max\{F_d(x;\bar{C}), F_n(x;\delta x_B)\} = F_d(x;\bar{C})$. Then, the firm chooses coupon $C(x) = \bar{C}$ at time 0. The results follow from the default-possible case with $C = \bar{C}$ in Proposition 1.

For $x_B \in [\bar{x_B}, \bar{C}/\delta]$, $\bar{G}(x; x_B) \geq 0$ holds, which leads to $\max_{C\in[0,\bar{C}]} F(x;C) = \max\{F_d(x;\bar{C}), F_n(x;\delta x_B)\} = F_n(x;\delta x_B)$. Then, the firm chooses coupon $C(x) = \delta x_B$ at time 0. The results follow from the no-default case with $C = \delta x_B$ in Proposition 1.

For $x_B > \bar{C}/\delta$, debt with any coupon $C(\leq \bar{C})$ becomes riskless, which leads to $\max_{C\in[0,\bar{C}]} F(x;C) = \max_{C\in[0,\bar{C}]} F_n(x;\bar{C}) = F_n(x;\bar{C})$. Then, the firm chooses coupon $C(x) = \bar{C}$ at time 0. The results follow from the no-default case with $C = \bar{C}$ in Proposition 1.

E Proof of Proposition 5

By Propositions 1 and 4, for $x_B < \bar{x_B}$, $\bar{D}(x) = D_d(x;\bar{C})$ increases in x_B, while $\bar{C}(x) = \bar{C}$, $x_0(\bar{C}) = \bar{C}/\delta$, and $\bar{E}(x) = E_0(x;\bar{C})$ are constant. Then, $\bar{F}(x)$ and $\bar{LV}(x)$ increase in x_B, while $\bar{CS}(x)$ decreases in x_B.

At $x_B = \bar{x_B}$, coupon $\bar{C}(x)$ changes from $\bar{C}$ to $\delta\bar{x_B}$ By Proposition 4, $\bar{C} > \delta\bar{x_B}$ holds. At $x_B = \bar{x_B}$, $\bar{E}(x)$ changes from $E_0(x;\bar{C})$ to

$$\begin{aligned} E_n(x;\delta\bar{x_B}) &= \pi x - \frac{(1-\tau)\delta\bar{x_B}}{r} - \left(\frac{x}{\bar{x_B}}\right)^{\gamma} \frac{\pi\bar{x_B}}{\gamma} \\ &= E_0(x;\delta\bar{x_B}) \\ &> E_0(x;\bar{C}) \end{aligned}$$

(i.e., an upward jump), where we obtained the last inequality by $\bar{C} > \delta\bar{x_B}$. By definition of $\bar{x_B}$ (i.e., $\bar{G}(x;\bar{x_B}) = 0$), $F_d(x;\bar{C})$ continuously changes to $F_n(x;\delta\bar{x_B})$ at $x_B = \bar{x_B}$. By the continuity of $\bar{F}(x)$ and the upward jump of $\bar{E}(x)$, $\bar{D}(x)$ must jump downward at $x_B = \bar{x_B}$. Then, $\bar{LV}(x)$ jumps downward at $x_B = \bar{x_B}$, and $\bar{CS}(x)$ also jumps downward to 0.

For $x_B \in [\bar{x_B}, \bar{C}/\delta]$, the results follow from the proof of Proposition 3 (see the third paragraph of Appendix C).

By Propositions 1 and 4, for $x_B > \bar{C}/\delta$, $\bar{E}(x) = E_n(x;\bar{C})$ increases in x_B, while $\bar{D}(x) = \bar{C}/r, \bar{C}(x) = \bar{C}$, and $\bar{CS}(x) = 0$ are constant. Then, $\bar{F}(x)$ increases in x_B, and $\bar{LV}(x)$ decreases in x_B.

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Table 1: Baseline parameter values.

r	μ	σ	τ	α	x_B	$x = X(0)$
0.05	0.01	0.2	0.15	0.4	0.2	1

Table 2: Baseline results.

$C(x)$	$E(x)$	$D(x)$	$F(x)$	$LV(x)$	$CS(x)$
0.435	14.21	8.7	22.91	0.38	0

Table 3: Benchmark results with no barrier.

$C_0(x)$	$x_0(C_0(x))$	$E_0(x)$	$D_0(x)$	$F_0(x)$	$LV_0(x)$	$CS_0(x)$
0.623	0.287	11.49	10.84	22.32	0.485	0.00751

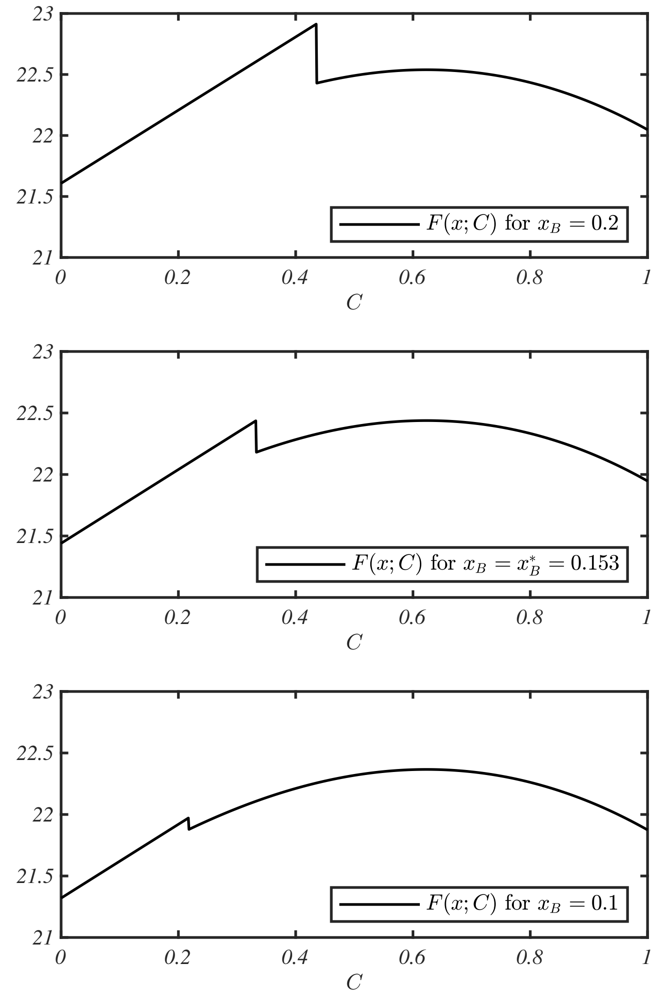

Figure 1: Firm value $F(x;C)$ for varying levels of C. The top, center, and bottom panels show $F(x;C)$ for $x_B = 0.2$ (Baseline), $x_B = x_B^* = 0.153$, and $x_B = 0.1$, respectively, where $F(x;C) = F_n(x;C)$ for $C \leq \delta x_B = 0.435, 0.333$, and 0.217, respectively, and $F(x;C) = F_d(x;C)$ for $C > \delta x_B = 0.435, 0.333$, and 0.217, respectively. In all the panels, $F_d(x;C)$ takes the maximum at $C = C_0(x) = 0.623$.

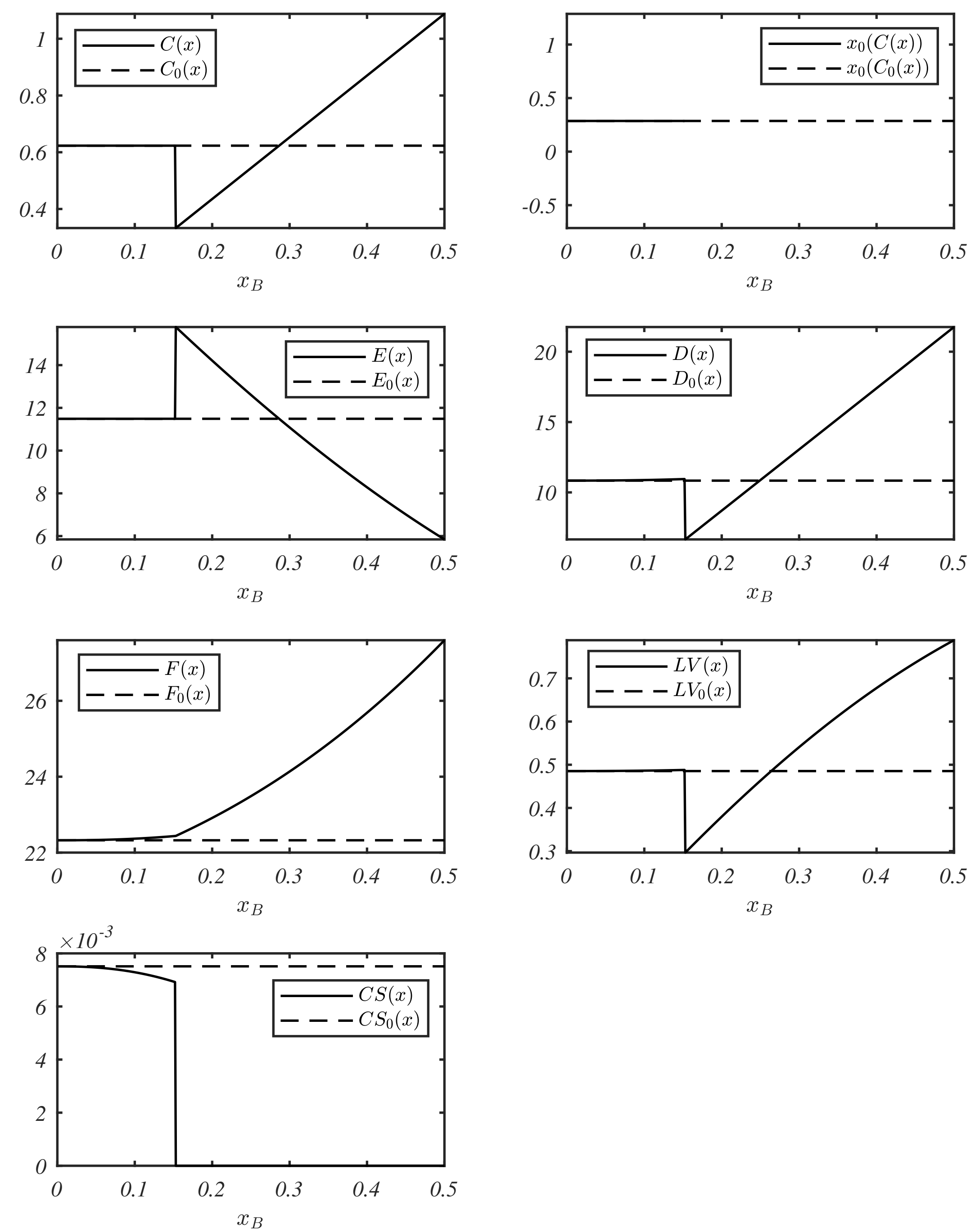

Figure 2: Comparative statics with respect to reflecting barrier x_B. The figure depicts coupon $C(x)$, default threshold $x_0(C(x))$, equity value $E(x)$, debt value $D(x)$, firm value $F(x)$, leverage $LV(x)$, and credit spread $CS(x)$ in the baseline model by solid lines. Region $x_B < x_B^* = 0.153$ is the default-possible case, whereas region $x_B \geq x_B^* = 0.153$ is the no-default case. The dashed lines represent the benchmark results with no barrier.

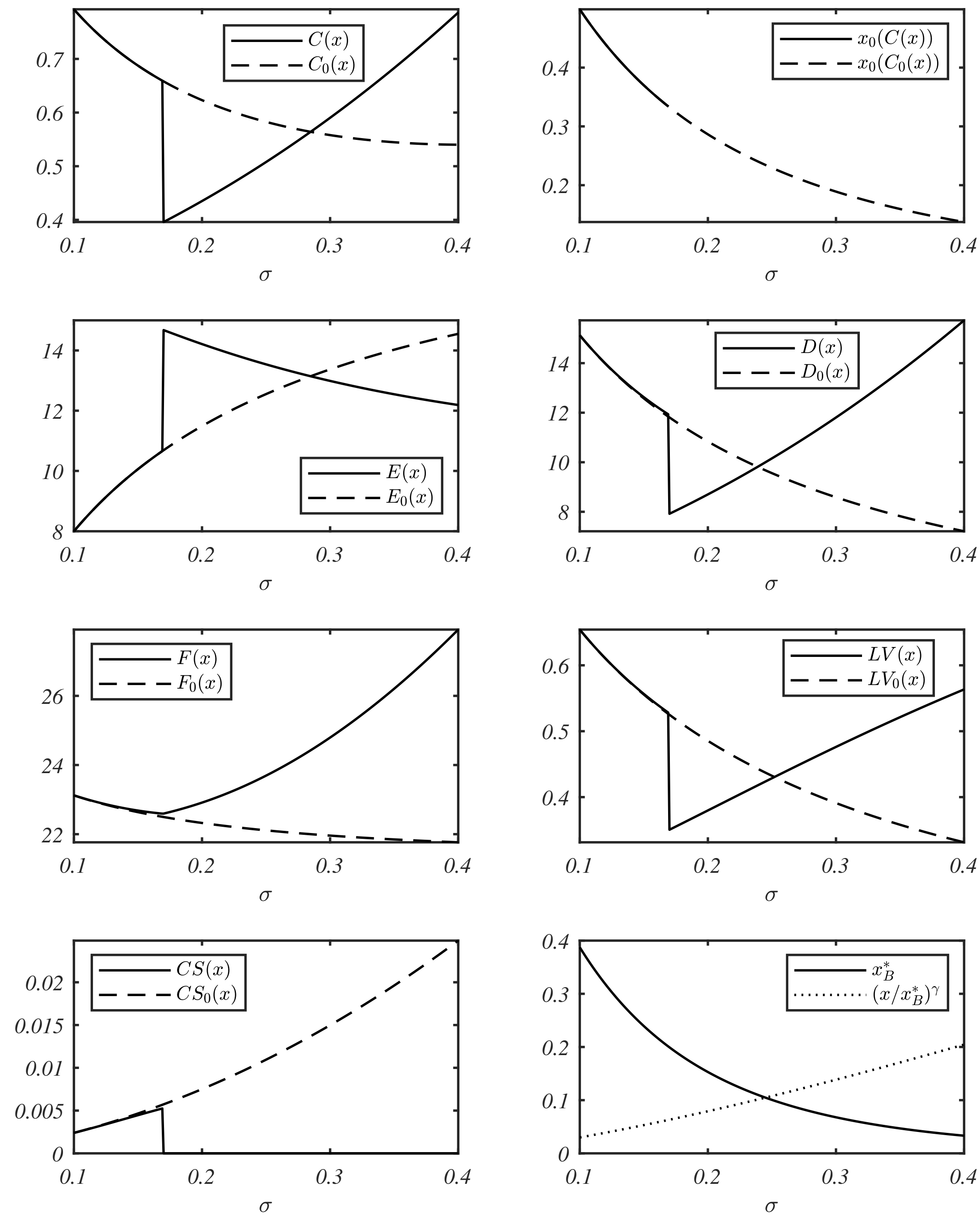

Figure 3: Comparative statics with respect to volatility σ. The figure depicts coupon $C(x)$, default threshold $x_0(C(x))$, equity value $E(x)$, debt value $D(x)$, firm value $F(x)$, leverage $LV(x)$, credit spread $CS(x)$, critical barrier x_B^*, and state price $(x/x_B^*)^\gamma$ in the baseline model by solid lines. Region $\sigma < 0.17$ is the default-possible case, while region $\sigma \geq 0.17$ is the no-default case. The dashed lines represent the benchmark results with no barrier.

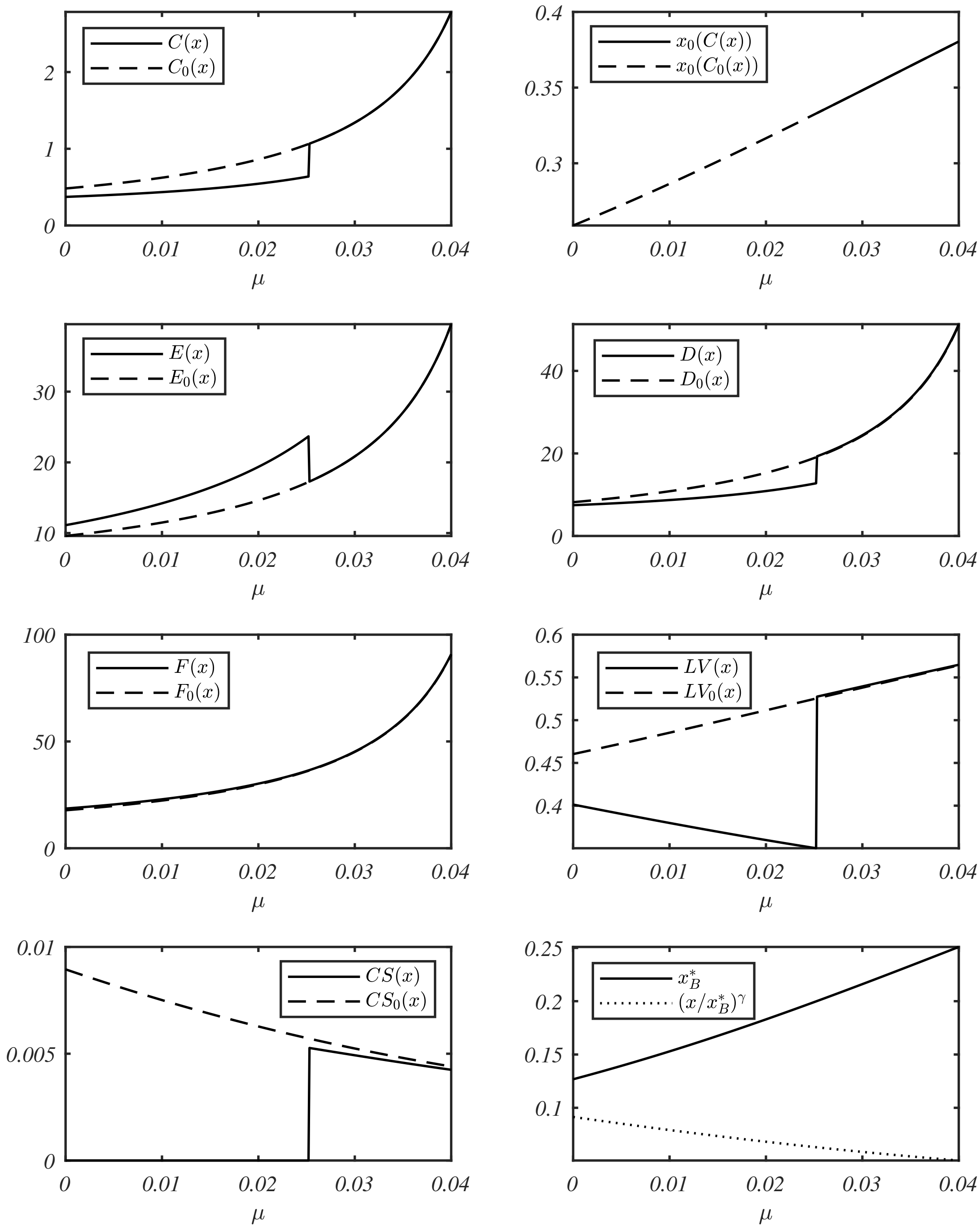

Figure 4: Comparative statics with respect to growth rate μ. The figure depicts coupon $C(x)$, default threshold $x_0(C(x))$, equity value $E(x)$, debt value $D(x)$, firm value $F(x)$, leverage $LV(x)$, credit spread $CS(x)$, critical barrier x_B^*, and state price $(x/x_B^*)^\gamma$ in the baseline model by solid lines. Region $\mu \leq 0.0252$ is the default-possible case, whereas region $\mu > 0.0252$ is the no-default case. The dashed lines represent the benchmark results with no barrier.

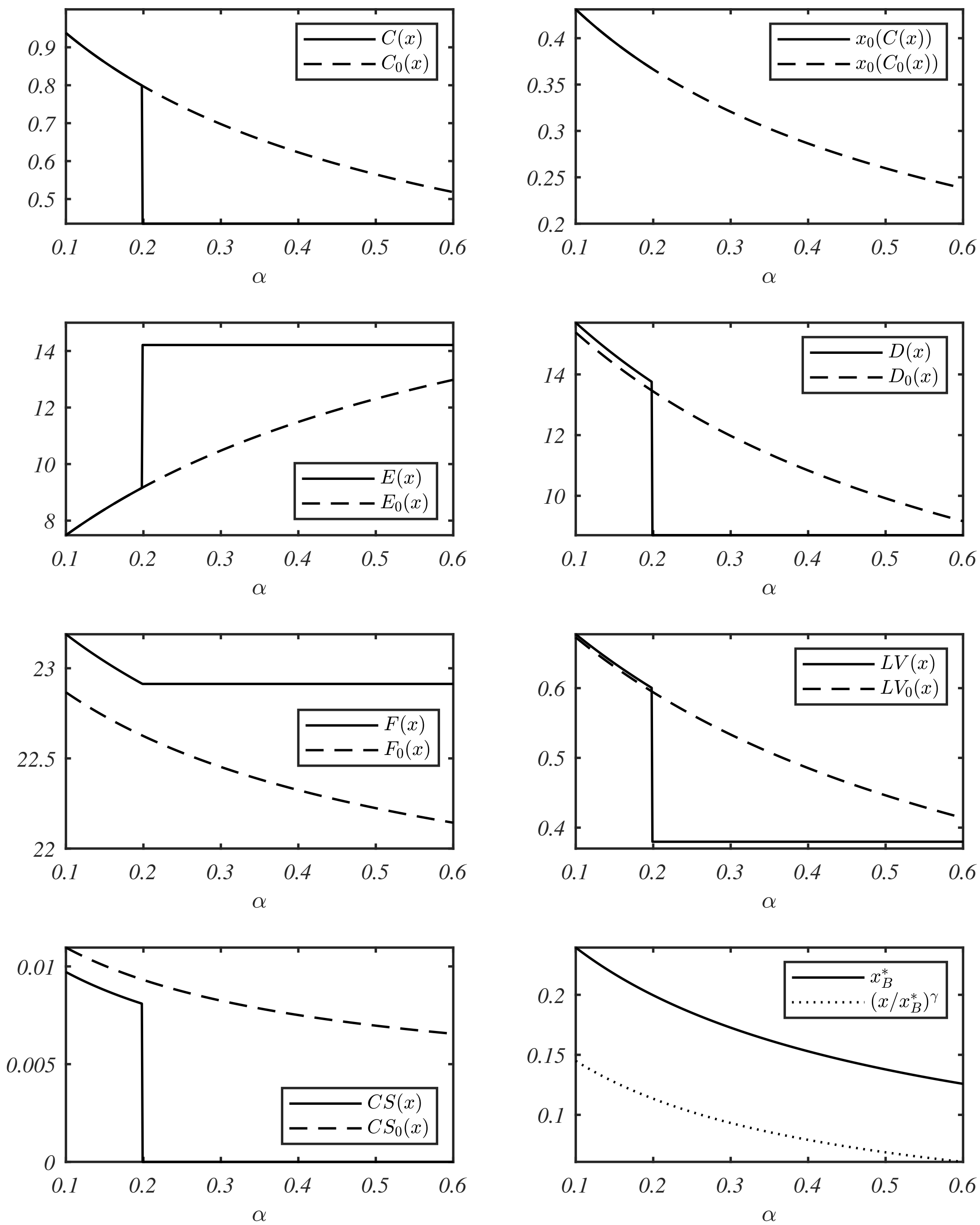

Figure 5: Comparative statics with respect to bankruptcy cost α. The figure depicts coupon $C(x)$, default threshold $x_0(C(x))$, equity value $E(x)$, debt value $D(x)$, firm value $F(x)$, leverage $LV(x)$, credit spread $CS(x)$, critical barrier x_B^*, and state price $(x/x_B^*)^\gamma$ in the baseline model by solid lines. Region $\alpha < 0.199$ is the default-possible case, whereas region $\alpha \geq 0.199$ is the no-default case. The dashed lines represent the benchmark results with no barrier.

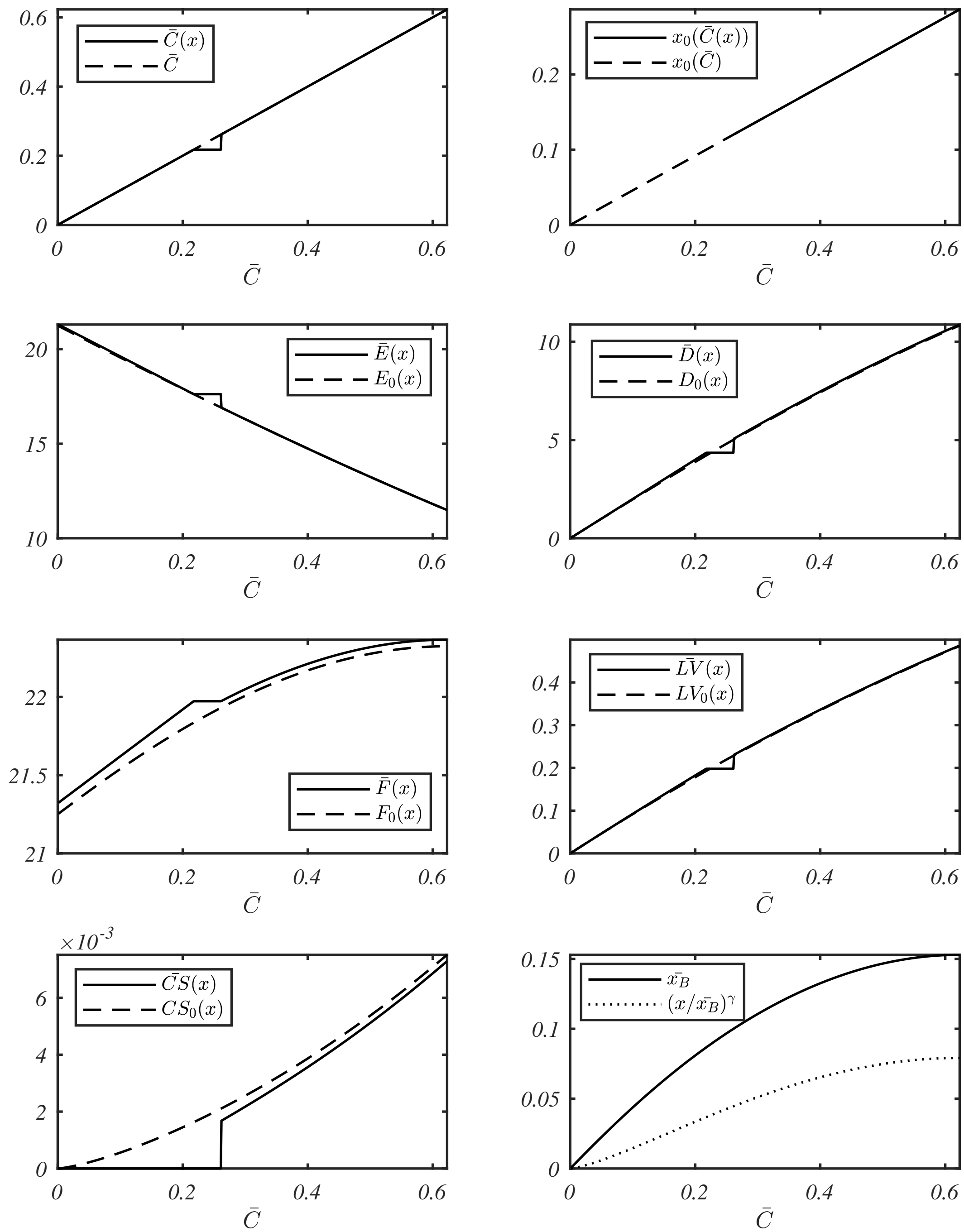

Figure 6: Comparative statics with respect to upper limit $\bar{C}$. The figure depicts coupon $\bar{C}(x)$, default threshold $x_0(\bar{C}(x))$, equity value $\bar{E}(x)$, debt value $\bar{D}(x)$, firm value $\bar{F}(x)$, leverage $\bar{LV}(x)$, credit spread $\bar{CS}(x)$, critical barrier $\bar{x_B}$, and state price $(x/\bar{x_B})^{\gamma}$ in the extended model with upper limit $\bar{C}$ by solid lines. Region $\bar{C} \leq 0.261$ is the no-default case, whereas region $\bar{C} > 0.261$ is the default-possible case. The dashed lines represent the benchmark results with no barrier under upper limit $\bar{C}$.