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Market Commodity Structure in General Equilibrium Model: The Commodification Technology

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Abstract

In this paper, we formalize the concept of *commodification technology* determining a commodity structure of the market as an extension of the standard Arrow-Debreu general equilibrium framework. Like the *commodity differentiation* approach in Mas-Colell (1975) and the *hedonic* approach in Rosen (1974), we treat a commodity as a bandle of *characteristics*. Such *quality* of each commodity as well as the *market viability* problem under asymmetric-information arguments like Dubey et al. (2000) and Bisin et al. (2011) to determine the *market structure* will be treated uniformly as an endogenous equilibrium state in a strict sense of the general equilibrium theory. In our model, a market participant (a *consumer* or a *producer*) is supposed to have a *commodification technology* that enables each agent to prepare for market commodities based on their *endowments* and the transformation *technology* on basic characteristics, representing the costs for *standardization*, *advertisement*, *signaling*, etc.

Keywords: General Equilibrium Model, Commodity Differentiation, Market Structure, Asymmetoric Information, Commodification Technology

JEL classification: C62; D51; D82; L11; L15

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1 Introduction

The purpose of this paper is to provide a general equilibrium framework to treat endogenously the formation of *market commodity structure*. Here, the market commodity structure means both the *quality* of commodities (as in the commodity differentiation under the hedonic approach) and the *kinds* of commodities emerging in an economy (as in the case with market viability under the asymmetric information approach). Hence, we provide a unified general equilibrium viewpoint on both approaches to endogenize the market structure problem.

For the quality of commodities, there are classical (including general equilibrium) frameworks like Rosen (1974), Lancaster (1966), and Mas-Colell (1975), called the *hedonic* or *product-differentiation* approach. In such models, commodities embody some properties called *characteristics*, which are sources of utilities and implicitly determine the value of commodities. Since discussions in their papers focus on describing the equilibrium states with commodity differentiation, they simply set each commodity as an *exogenously* given bundle of characteristics. So the framework can be extended to investigate the relationship between the determination of the quality of commodities and market mechanism. We have similar motivations as classical monopolystic and oligopolistic product differentiation frameworks like Chamberlin (1937). In addition to their implications, our general-equilibrium framework can also make the determination of the quality of commodities tie to social optimality.

On the other hand, problems about what kind of commodities are actually traded have been treated as the *market viability problem*. This is one viewpoint of the adverse selection problem discussed by Akerlof (1970) and/or Rothschild and Stiglitz (1976). For the general-equilibrium treatment, the approaches of Dubey et al. (2000) and Bisin and Gottardi (1999) are groundbreaking. Problems such as adverse selection assume that informational asymmetry is a structure that depends not only on individuals but also on their selling and/or buying standpoints. Dubey et al. (2000) and Bisin and Gottardi (1999) realizes such selling-buying informational asymmetry by considering a system of *pooled insurance* in an economy. Subsequently to these groundbreaking works, Bisin et al. (2011), Correia-da-Silva (2012), and Meier et al. (2014) treat selling-buying informational asymmetry in a static pure exchange economy with different market frameworks (as contracts for delivery or generalized goods with the description of agents' names). One of the authors also treat this problem (Urai et al. 2017) in a standard static Arrow-Debreu production economy by considering the concept of *commodification technology* that can treat upper bounds for individual market trade amounts endogenously through a certain kind of cost structure, which is an important starting point for this paper's theme of *endogenization of market structure*.

What was treated in Bisin et al. (2011) as a general equilibrium problem of *future market contracts* under information asymmetry is recast here as a *market structure endogenization* problem. The key is *commodification technology* introduced in Urai et al. (2017), where the problem is restricted to the endogenization of delivery upperbounds.

It goes without saying that asymmetric information problems such as signaling and adverse selection play an important role in determining market structure as a "market viability problem." However, as mentioned above, market structure is not *only* a problem related to them. The "market viablity problem" caused by information asymmetry and the "market quality problem" for *product differentiation* (combining characteristics such as the hedonic approach) can be treated in an integrated manner by attributing them to the "(production) technology" problem of *commodification*. This is what this paper attempts to do. In addition, a general and integrated view can be given toward such "technology" in a way that also encompasses technology in the normal competitive market. With these various perspectives, the "contract" model of Bisin et al. (2011) and Urai et al. (2017) are reformulated as our "market commodity" model. The objective of our model is to address the key issue of "endogenous determination of commodity structure" in a general equilibrium market.

Here, we give a brief overview of the general equilibrium model of this paper. As in the hedonic approach, we assume that each commodity is produced from a basic variety of materials, referred to below as *characteristics*. The number of all (potentially) possible commodities will be denoted by an integer $\lambda \geqq 1$ and the number of characteristics by $\ell \geqq 1$. We assume that exchange is possible only in the market, through *commodities*. On the other hand, what gives people utility and serves as an input for production are the characteristics, i.e., the basic material levels mentioned above. In other words, consumption and production are considered here as *private* activities, and the question is how the *social* activity (structure) of the market is built upon them. Toward the potential commodities, $\kappa = 1, \ldots, \lambda$, for each seller i (a consumer or a firm), we suppose that there is a technology to transform characteristics (a vector in R^ℓ) into commodities, *commodification technology* C_i. For more detail, technology C_i enables for each seller i, to transform a list of certain amounts of characteristics, $(v_i^\kappa \in R^\ell)_{\kappa=1}^\lambda$, into certain amounts of commodities, $(\phi_i^\kappa \in R)_{\kappa=1}^\lambda$, together with certain real deliveries of characteristics, $(d_i^\kappa \in R^\ell)_{\kappa=1}^\ell$ as the "quality" of commodities, $\kappa = 1, \ldots, \lambda$. Hence, the commodification technology of i is defined as a set, C_i, including the triple, $((v_i^\kappa)_{\kappa=1}^\lambda, (\phi_i^\kappa)_{\kappa=1}^\lambda, (d_i^\kappa)_{\kappa=1}^\lambda)$ as its element. (See Fig.1 for an image of these.) Notation ϕ_i^κ and d_i^κ for each commodity κ come from the usage of variables for future market contracts in Bisin et al (2011). Together with the commodification technologies, our model also allows the selling-buying asymmetry by considering a system of pooled insurance. That is, for each commodity κ, each agent i, as a buyer, buys z_i^κ units of that commodity while expecting the actual delivery of characteristics per unit from the market to be $s^\kappa \in R^\ell$, which is equal to its average value derived from what all sellers delivered to the market (see Fig.2). We will treat $s^\kappa \in R^\ell$ (i.e., the quality of commodity κ) for each $\kappa = 1, \ldots, \lambda$, as a parameter like the price. Then, the determination of qualities and quantities (the market structure) will be identified with a rational expectation equilibrium state including parameters $s = (s^1, \ldots, s^\lambda) \in (R^\ell)^\lambda$ and $p \in R^\lambda$ (see Fig.3).

In summary, this paper provides a general equilibrium model which determines market structure in the context of the hedonic approach and of the market viability problem. Even though there are many approaches that are based on imperfect competition frameworks, our model gives a perfect competition general equilibrium forms on the market structure determination problem. We believe, specifically, that this framework can also be a ground for describing an economy with platformization by incorporating a firm-formation argument like Urai et al. (2023).

The rest of the paper is organized as follows. In Section 2, we provide a two-characteristics and one-commodity example to establish a concrete image of our model. In Section 3, The Model, we focus on agent i's ($i = 1, \cdots, m+n$) interaction with markets as a seller and a buyer, referring to some assumptions on the commodification technology. We mention producer's and consumer's maximization problems and equilibrium in Section 4. In Section 5, the theorem and optimality argument are referred. We provide a specific calculation of the example and proof of theorems in Appendix.

We denote by R the set of real numbers and R^n the n-dimensional Euclidean space. On R^n, there are three types of order relations, $\leqq$, $<$, and $\ll$, such that for each $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ in R^n, we have $(x \leqq y) \iff (\forall i = 1, 2, \ldots, n,\ x_i \leqq y_i)$, $(x < y) \iff (x \leqq y \text{ and } x \neq y)$, and $(x \ll y) \iff (\forall i = 1, 2, \ldots, n,\ x_i < y_i)$. By R^n_+ and R^n_{++}, we denote $\{x \in R^n | \, 0 \leqq x\}$ and $\{x \in R^n | \, 0 \ll x\}$, respectively. For n-dimensional Euclidean space R^n and subset $A \subset \{1, \ldots, n\}$, let R^A be the subspace of R^n containing the elements whose k-th coordinate is 0 if $k \notin A$. For each finite set A, we denote by $\sharp A$ the number of elements of A. R^A_+ denote the non-negative orthant, and R^A_{++} denote the strictly positive orthant of R^A, where we identify R^A with the $\sharp A$-dimensional Euclidean space.

2 An Example (Two Characteristics and One Commodity)

Consider an economy containing two consumers, $i = 1, 2$, and two characteristics (Grain and Vegetable) $k = G, V$. We denote the $(grain, vegetable)$ characteristics space by R^2_+. Characteristics are sources of utility so that the consumption set X_i is a compact subset of R^2_+ and initially characteristics are endowed as $\omega_1 = (1, 0)$, one unit of Grain for agent 1, and $\omega_2 = (0, 1)$, one unit of vegetable for agent 2. In this example we specifically define a utility function for each $i = 1, 2$ as

$$u_i(x_i^G, x_i^V) = x_i^G x_i^V, \tag{1}$$

where $x_i = (x_i^G, x_i^V) \in X_i$.

Suppose that the characteristics cannot be exchanged directly. Instead, a commodity named LUNCH formed by these two characteristics will be traded in the market. For simplification, in this example, we assume that there is at most one kind of commodity embodied with characteristics Grain and Vegetable traded in the market (so $\lambda = 1$ and the general equilibrium market-structure discussion is now reduced to the viability-quality problem for the single commodity market of Lunch).

[**As Sellers**]**:** Each consumer $i = 1, 2$ as a *seller* has a commodification technology, C_i, to produce commodity, LUNCH, from characteristics, Grain and Vegetable. In this example, consumer i's commodification technology is defined as follows:

$$(\phi_i, d_i, v_i) \in C_i \Leftrightarrow \phi_i = v_i^G + v_i^V \text{ and } (v_i^G, v_i^V) \geqq d_i \geqq \frac{1}{2}(v_i^G, v_i^V), \tag{2}$$

where element $(\phi_i, d_i, v_i) \in C_i$ means that for the market of LUNCH, consumer i sells ϕ_i units of LUNCH and delivers $d_i \in R^2_+$ amount of characteristics while preparing $v_i \in R^2_+$ amount of characteristics. (See Fig. 1. Note that in this example, all agents as sellers and/or buyers are Consumers. The value of 1/2, which indicates the lower limit, has no particular significance.)

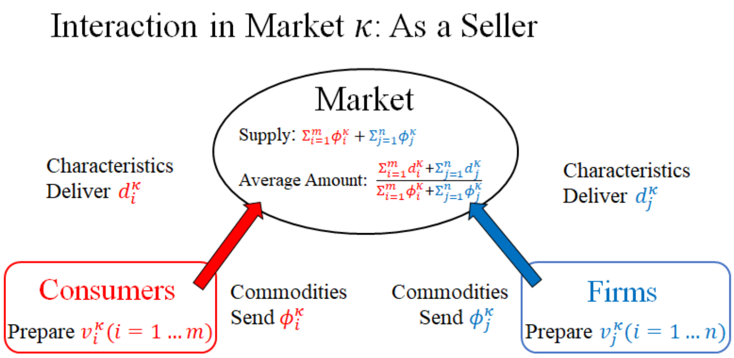

Fig. 1: Interaction with Markets as A Seller

[As Buyers]: On the other hand, for the commodity, consumers as *buyers* have a common expectation $s = (s_G, s_V) \in R^2_+$ of receiving a certain quantity of each characteristic per unit purchase of commodity LUNCH through the market. For instance, if $z_i \in R_+$ is the units of the commodity that consumer i purchases, consumer i expects to receive $z_i s_G$ quantities of characteristic Grain and $z_i s_V$ quantities of characteristic Vegetable. (See Figure 2.)

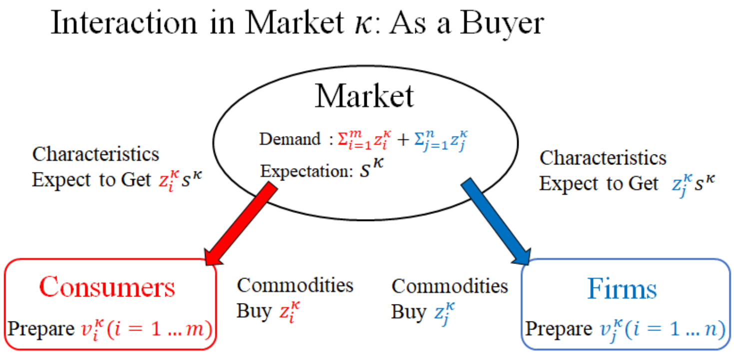

Fig. 2: Interaction with Markets as A Buyer

[Maximization Problems and Equilibria]: Given commodity price $p \in R_{++}$ and common expectation s, consumer i's *utility maximization problem* can be described as follows:

$$\text{Max} \qquad u_i(x_i^G, x_i^V) = x_i^G x_i^V, \tag{3}$$

sub. to

$$x_i^G = s_G z_i + \omega_i^G - v_i^G, \tag{4}$$

$$x_i^V = s_V z_i + \omega_i^V - v_i^V, \tag{5}$$

$$p\phi_i = p z_i, \tag{6}$$

$$(\phi_i^\kappa, d_i^\kappa, v_i^\kappa) \in C_i^\kappa. \tag{7}$$

Two equations (4) and (5) implies that consumer i decide (i) what to consume x_i^G, x_i^V, (ii) what to buy z_i (as a buyer), and (iii) what to prepare v_i^G, v_i^V (as a seller). The equation (6) is a budget constraint. Since there is only one market in this economy, budget constraint is $\phi_i = z_i$ as long as $p > 0$.

In accordance with the above settings, we now discuss whether equilibrium exists in this economy. A list $((x_i^*, \phi_i^*, d_i^*, v_i^*, z_i^*)_{i=1,2}, p^*, s^*)$ is an *equilibrium* if the following conditions are satisfied:

(i) (**utility maximization**) given p^* and s^*, $(x_i^*, \phi_i^*, d_i^*, v_i^*, z_i^*)$ is a solution of consumer i's utility maximization problem for each $i = 1, 2$;

(ii) (**market clear**) the market is clear, $\sum_{i=1}^2 z_i^* = \sum_{i=1}^2 \phi_i^*$;

(iii) (**expectation specification**) as long as $\sum_{i=1}^2 \phi_i^* > 0$, the expectation s^* in an equilibrium expectation corresponds with the average amount of characteristics

$$\frac{\sum_{i=1}^2 d_i^*}{\sum_{i=1}^2 \phi_i^*} = \left(\frac{\sum_{i=1}^2 d_i^*}{\sum_{i=1}^2 z_i^*} \right) = s^*. \tag{8}$$

For the interpretation of the expectation specification condition, see Figure 3.

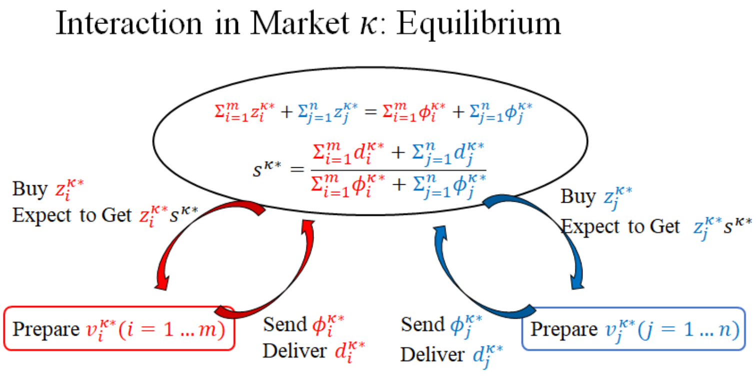

Fig. 3: Market Equilibrium

Next, we derive equilibria under $\hat{s}^* = s_G^* = s_V^*$ and $\frac{1}{2} \geq \hat{s}^* \geq 0$. (Here, we focus on symmetric solutions.)

(i) When $\frac{1}{2} \geq \hat{s}^* \geq \frac{1}{3}$, equilibria are given as

$$x_1^* = \left(\frac{1}{2}, \frac{\hat{s}^*}{2(1-\hat{s}^*)}\right), \phi_1^* = \frac{1}{2(1-\hat{s}^*)}, d_1^* = \left(\frac{\hat{s}^*}{1-\hat{s}^*}, 0\right), v_1^* = \left(\frac{1}{2(1-\hat{s}^*)}, 0\right), z_1^* = \frac{1}{2(1-\hat{s}^*)},$$

$$x_2^* = \left(\frac{\hat{s}^*}{2(1-\hat{s}^*)}, \frac{1}{2}\right), \phi_2^* = \frac{1}{2(1-\hat{s}^*)}, d_2^* = \left(0, \frac{\hat{s}^*}{1-\hat{s}^*}\right), v_2^* = \left(0, \frac{1}{2(1-\hat{s}^*)}\right), z_2^* = \frac{1}{2(1-\hat{s}^*)},$$

$$p^* = 1, \frac{1}{2} \geq \hat{s}^* \geq \frac{1}{3}.$$

In section 6, we check that the profile is an equilibrium. If $\frac{1}{4} > \hat{s} > 0$, there is no equilibrium due to commodification technology.

(ii) When $\hat{s} = 0$, no characteristics is traded in a market ($v_i^G = v_i^V = 0$). In this case, although $x_1^* = (1, 0)$, $x_2^* = (0, 1)$, $\phi_1^* = \phi_2^* = z_1^* = z_2^* = 0$, $d_1^* = v_1^* = d_2^* = v_2^* = (0, 0)$, $p^* = 1$, $\hat{s}^* = 0$ is also an equilibrium (the market is not viable). Buyer's expectation might affect the viability of market.

For the discussion of efficiency, we discuss the attainability of a consumption state, (x_1, x_2). A pair (x_1, x_2) is said to be attainable if $x_i \in X_i$ for $i = 1, 2$ and satisfies the following (9).

$$x_1 + x_2 \leq \omega_1 + \omega_2. \tag{9}$$

Clearly, the consumption state, (x_1^*, x_2^*), in the equilibrium profile above is attainable.

A list $(x_i, \phi_i, d_i, v_i, z_i)_{i=1,2}$ is Pareto optimal if there is no other attainable state (x_1', x_2') such that for all $i = 1, 2$, $u_i(x_i') \geq u_i(x_i)$ and at least one i, $u_j(x_i') > u_i(x_i)$. If $\frac{1}{2} > \hat{s} > \frac{1}{4}$, there is a loss $(v_1^1 - d_1^{1*}) > 0$ and $(v_2^2 - d_2^{2*}) > 0$. This causes a Pareto inefficiency. However, when $\hat{s} = \frac{1}{2}$ and $p^* = 1$, the profile $x_1^* = x_2^* = (\frac{1}{2}, \frac{1}{2}), \phi_1^* = \phi_2^* = z_1^* = z_2^* = 1, d_1^* = (1, 0), d_2^* = (0, 1)$ establishes the Pareto optimality.

As we have seen above, some equilibria can achieve the Pareto efficiency but others are not. Urai et al. (2017) shows that the first welfare theorem can be established under specific condition, and this is an extension of the research. In contrast to Urai et al. (2017)'s model, d_i is not uniquely determined by v_i. Buyer's expectation has more significant effect on the Pareto efficiency. We argue that taking care of buyer's expectation (about s) is a key to achieve the Pareto efficiency.

3 The Model

3.1 Characteristics and Market Commodity

In this model, we regard *commodity* as a bundle of characteristics or basic raw materials from which consumers benefit and producers produce products. [1] In other words, there is a distinction between the *private* use of goods and services and the exchange of "commodities" through *social* "market" institutions, both for the consumer and for the producer = firm.[2]

The number of *characteristics* are $\ell \geqq 1$ indexed by $k = 1, \ldots \ell$. We denote the set of indices by $L = \{1, 2, \ldots, \ell\}$. From the standpoint of treating a commodity as a bundle of characteristics, it would be reasonable to start with a subset of L as an index of a commodity. Let us consider a family of non-empty subsets of L, $\{L_\kappa\}_{\kappa=1}^{\lambda}$, $\lambda \geqq 1$, as the list of all possible *commodities*. Each commodity κ $(= 1, \ldots, \lambda)$ may include more than one characteristic in L_κ, so is potentially a mixture of $\sharp L_\kappa$ kinds of characteristics. Prices are identically given to each market commodity, not characteristics. Hence, the price space is λ-dimensional, and in this sense, we deal with a commodity differentiation model.[3]

3.2 Agents as Sellers and Commodification Technologies

There are $m > 0$ individual consumers indexed by $i = 1, \ldots, m$, and $n > 0$ firms indexed by $j = m+1, \ldots m+n$. We often use a common index i=1,2,...,m+n for consumers and firms to emphasize that they face the market as sellers. Each firm j has a *production technology* on characteristics, $Y_j \subset R^\ell$. While only firms have production technologies, every agents have a *commodification technology*, a set denoted by $(\phi_i, d_i, v_i) \in C_i \subset R_+^\lambda \times \prod_{\kappa=1}^{\lambda} R_+^{L_\kappa} \times (R_+^\ell)^\lambda$ $(i = 1, \ldots, m+n)$. For an element, (ϕ_i, d_i, v_i), of C_i, where $\phi_i = (\phi_i^1, \ldots, \phi_i^\lambda)$, $d_i = (d_i^1, \ldots, d_i^\lambda)$, and $v_i = (v_i^1, \ldots, v_i^\lambda)$, we can provide an interpretation as follows. For each commodity κ, agent i can sell $\phi_i^\kappa \in R_+$ units of the market commodity κ and deliver $d_i^\kappa \in R_+^{L_\kappa}$ units of characteristics while preparing $v_i^\kappa \in R_+^\ell$ as inputs. Note that we assign a price to each market commodity but consumers have their preference not over market commodities but over characteristics.[4]

We make some assumptions on commodification technology C_i as follows. Note that condition (C3) does not necessarily apply to the case $\sharp L_\kappa = 1$ involving a normal Arrow-Debreu economy, i.e., where one characteristic can be regarded as one commodity.

(C1) C_i is a closed and convex set containing $(0, 0, 0) \in R_+^\lambda \times \prod_{\kappa=1}^{\lambda} R_+^{L_\kappa} \times (R_+^\ell)^\lambda$ for all $i = 1, \ldots, m+n$.

(C2) For any $(\phi_i, d_i, v_i) \in C_i$, $v_i^\kappa \geqq d_i^\kappa$ for all $i = 1, \ldots, m+n$ and $\kappa = 1, \ldots, \lambda$. Moreover, for any $(\phi_i, d_i, v_i) \in C_i$, $\phi_i^\kappa = 0$ implies $v_i^\kappa = 0$.

[1] In this sense, it may be possible to treat all sellers as a kind of producer, but here we will refer specifically to the agents who have the transformation technology Y_j among the raw materials (characteristics) as producers. Such transformation technology Y_j between raw materials will be distinguished from commodification technology C_j in the following.

[2] For example, commodification in this sense means removing crooked cucumbers, or making them uniform in size. As such, characteristics and their "market product com-mod-it-ization" are dealt with below.

[3] Note that we do not exclude the case where $L_\kappa = L_{\kappa'}$ although $\kappa \neq \kappa'$.

[4] Among these notation, the pair, $(d_j^\kappa, \phi_j^\kappa)$, for each commodity, κ, can be identified with a "contract" treated in Bisin et al. (2011) as an element of a compact domain, $\Phi^\kappa \times D^\kappa$. So our model can be identified with an extension of their contract-delivery model.

(C3) For each $i = 1, \ldots, m+n$, for each $\kappa = 1, \ldots, \lambda$ and each sequence $(\phi_i^\nu, d_i^\nu, v_i^\nu)_{\nu=1}^\infty$ in C_i, $\phi_i^{\kappa\nu} \to \infty$ $(\nu \to \infty)$ implies $\sum_{k=1}^{\ell}(v_{ik}^{\kappa\nu} - d_{ik}^{\kappa\nu}) \to \infty$ $(\nu \to \infty)$, where the subscript k means the k-th coordinate, except for the case that $\sharp L_\kappa = \{k\}$ for a certain k and $\phi_j^{\kappa\nu} = v_{jk}^{\kappa\nu} = d_{jk}^\kappa$ for all i and ν.

(C4) If $(\phi_i, d_i, v_i) \in C_i$, then for every $\kappa = 1, \ldots, \lambda$ there exists a compact convex set $\Delta^\kappa \subset R_+^{L_\kappa}$ such that for all $i = 1, \ldots, m+n$, $\frac{d_i^\kappa}{\phi_i^\kappa} \in \Delta^\kappa$ where $\phi_i^\kappa \neq 0$.

Figures 4 and 5 display a typical form of commodification technology.

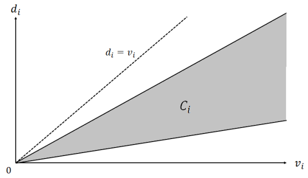

Fig. 4: *d*-*v*

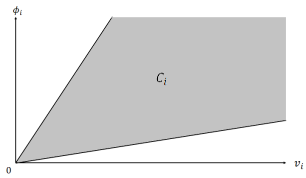

Fig. 5: *ϕ*-*v*

The condition (C1) is similar to the standard condition of production technologies. The assumption (C2) is for making the equilibrium fit the reality. Condition (C3) means that except for the market such that $\sharp L_\kappa = 1$ and there are no standardization, selection and/or dressing costs, each agent cannot supply a market commodity without a positive cost per unit at least for all sufficiently large amounts. The exception for market $\sharp L_\kappa = 1$ is important since by this, our model include the standard market delivery contract situation for the Arrow-Debreu type model in which unbounded short position supply contracts without costs are allowed for.[5] Condition (C4) is weak, it only says that for one unit of delivery of characteristics in market κ, the variety of real deliveries can be confined in a bounded set.

3.3 Agents as Buyers and Expectations of Real Receipts

Commodity κ in the market is really a mixture of $\sharp L_\kappa$ kinds of characteristics and agents as buyers are assumed to have a common *expectation of their receipts* for each of their trade (demand) contracts before choosing their actions. For each market commodity κ, let $\phi^\kappa \in R_{++}$ be the expected aggregate amount which is contracted to be supplied to the market, and let $d^\kappa \in R_+^{L_\kappa}$ be the vector of expected aggregate amounts of real goods and/or services that are actually delivered to the market κ. Then, for each unit of market commodity κ that is demanded, agents as buyers expect that the average amount $d^\kappa/\phi^\kappa \in R_+^{L_\kappa}$ will be delivered. We use valuable $s^\kappa \in R^{L_\kappa}$ to represent for such an average, d^κ/ϕ^κ, for each $\kappa = 1, 2, \ldots, \lambda$. By (C4), we can take s^κ in Δ^κ for an equilibrium.

[5] To include the standard unbounded short sales contracts in our model, we have used one of the simplest method that directly assigns the cost-exception condition in (C3) of commodification technologies. It would also be possible to obtain such a condition based on the shapes of commodification technologies.

4 Maximization Problems and Equilibrium

4.1 Producers' Problems

Producer $j = m+1, \ldots, m+n$ has a production technology $Y_j \subset R^\ell$ and a commodification technology $C_j \subset R_+^\lambda \times \prod_{\kappa=1}^\lambda R_+^{L_\kappa} \times R_+^\ell$ Given a price $p \in \Delta = \{(p_1, \ldots, p_\lambda) | \, p_1 \geqq 0, \ldots, p_\lambda \geqq 0, \sum_{\kappa=1}^\lambda p_\kappa = 1\}$ and the expectation of receipts for each market commodity $s = (s^1, \ldots, s^\lambda) = ((s_k^1)_{k \in L_1}, \ldots, (s_k^\lambda)_{k \in L_\lambda}) \in \prod_{\kappa=1}^\lambda \Delta^\kappa \subset \prod_{\kappa=1}^\lambda R^{L_\kappa}$, producer j chooses a production plan and market transaction plans, $(y_j,\ \phi_j,\ d_j,\ v_j,\ z_j)$, where $z_j = (z_j^1, \ldots, z_j^\lambda)$, to solve the following profit maximization problem:

$$\max \qquad p \cdot \phi_j - p \cdot z_j \tag{10}$$

subject to

$$v_j = y_j + z_j^1 s^1 + z_j^2 s^2 + \cdots + z_j^\lambda s^\lambda, \tag{11}$$

$$y_j \in Y_j, \tag{12}$$

$$(\phi_j, d_j, v_j) \in C_j, \tag{13}$$

$$z_j \in R_+^\lambda. \tag{14}$$

Equation (9) implies that agent j should prepare $v_j = (v_j(1), \ldots, v_j(\ell))$ through his production y_j and purchase $z_j^1 s^1 + \cdots + z_j^\lambda s^\lambda$. Note that $z_j^\kappa s^\kappa$ is a point in R^ℓ and (9) is a condition in R^ℓ. Restrictions (10) and (11) mention that y_j is producible and firm j is able to interact with markets through commodification technology C_j. (12) implies the buying amount should be positive.

4.2 Consumers' Problems

Consumer $i = 1, \ldots, m$ has an initial endowment $\omega_i \in R^\ell$ of characteristics and a consumption set $X_i \subset R^\ell$ with a commodification technology $(\phi_i, d_i, v_i) \in C_i \subset R_+^\lambda \times \prod_{\kappa=1}^\lambda R^{L_\kappa^+} \times R_+^\ell$. Given a price system p and the expectation of their receipts for each commodity in the market, $s = (s^1, \ldots, s^\lambda)$, consumer i chooses a plan with market transaction plans $(x_i, \phi_i, d_i, v_i, z_i) \in X_i \times C_i \times R_+^\lambda$ $(z_i = (z_i^1, \ldots, z_i^\lambda))$ to solve the following maximization problem:

$$\max \qquad u_i(x_i) \tag{15}$$

subject to

$$v_i + x_i = \omega_i + z_i^1 s^1 + z_i^2 s^2 + \cdots + z_i^\lambda s^\lambda, \tag{16}$$

$$p \cdot z_i = p \cdot \phi_i + \sum_{j=m+1}^{m+n} \theta_{ij} \pi_j(p, s), \tag{17}$$

$$x_i \in X_i, \tag{18}$$

$$(\phi_i, d_i, v_i) \in C_i, \tag{19}$$

$$z_i \in R_+^\lambda. \tag{20}$$

where u_i is a utility function of i, $\pi_j(p,s)$ is the profit of j under price p and expectation $s = (s^\kappa)_{\kappa=1}^\lambda$ (under the maximization problem (10) – (14) and θ_{ij} denotes consumer i's share of the profit of producer j (non-negative real numbers satisfying $\sum_{i=1}^m \theta_{ij} = 1$ for each j). Interpretations of restrictions are almost same with producers problems. Restriction (15) is the budget constraint in the usual general equilibrium settings.

4.3 Equilibrium

Denote by $\mathcal{E} := ((X_i, C_i, \omega_i, u_i, (\theta_{ij})_{j=m+1}^{m+n})_{i=1}^m, (Y_j, C_j)_{j=m+1}^{m+n})$ the economy described before.

Definition 1. (Equilibrium) For an economy $\mathcal{E}$, a state $((x_i, \phi_i, d_i, v_i, z_i)_{i=1}^m, (y_j, \phi_j, d_j, v_j, z_j)_{j=m+1}^{m+n}) \in \prod_{i=1}^m (X_i \times C_i \times R_+^\lambda) \times \prod_{j=m+1}^{m+n} (Y_j \times C_j \times R_+^\lambda)$ and $(p, s) \in \Delta \times \prod_{\kappa=1}^\lambda \Delta^\kappa$ is an *equilibrium* if it is a solution of the profit maximization problem (8)–(12), and utility maximization problem (13)–(18), and satisfies the *market clearing condition* (21) with *expectation specification* (22) for each $\kappa \in \{1, \dots, \lambda\}$ and $k \in L_\kappa$.

The market clearing condition and expectation specification are defined as follows.

$$\sum_{i=1}^{m+n} z_i^\kappa = \sum_{i=1}^{m+n} \phi_i^\kappa, \tag{21}$$

$$\frac{\sum_{i=1}^{m+n} d_i^\kappa}{\sum_{i=1}^{m+n} \phi_i^\kappa} \left(= \frac{\sum_{i=1}^{m+n} d_i^\kappa}{\sum_{i=1}^{m+n} z_i^\kappa} \right) = s^\kappa \quad \text{as long as} \quad \sum_{i=1}^{m+n} \phi_i^\kappa \left(= \sum_{i=1}^{m+n} z_i^\kappa \right) > 0. \tag{22}$$

Condition (21) is the standard market clearing condition. Condition (22) says that actual mixture ratio of market κ (the left hand side) is rationally expected by all the agents (the right hand side). Note that we only consider Eq. (22) when $\sum_{i=1}^{m+n} \phi_i^\kappa$ (that equals $\sum_{i=1}^{m+n} z_i^\kappa$) is positive. Hence, if $\sum_{i=1}^{m+n} \phi_i^\kappa \left(= \sum_{i=1}^{m+n} z_i^\kappa \right) = 0$, we have no restriction on the expectation specifications.

Even though our equilibrium is defined by means of trade quantities $(z_i)_{i=1}^{n+m}$ and the trading process includes costs in real (characteristics) goods and/or services that are represented by agents' commodification technologies, it is easy to check (by condition (C3)) that any equilibrium *characteristics state* $((x_i)_{i=1}^m, (y_j)_{j=m+1}^{m+n})$ satisfies the following condition:

$$\sum_{i=1}^m x_i \leq \sum_{j=m+1}^{m+n} y_j + \sum_{i=1}^m \omega_i. \tag{23}$$

We define an *attainable* characteristics state. The pair $((x_i)_{i=1}^m, (y_j)_{j=m+1}^{m+n})$ is attainable if conditions $x_i \in X_i$, $y_j \in Y_j$, and (23) are satisfied. We consider an allocation $((x_i)_{i=1}^m, (y_j)_{j=m+1}^{m+n})$ without any restriction and cost of market interaction. Let us define the *attainable characteristics set* for each agent $i = 1, \dots, n+m$ as follows:

$$\tilde{X}_i \overset{\text{def.}}{=} \{x_i \in X_i \,|\, x_i \text{ is a component of a pair } ((x_i)_{i=1}^m, (y_j)_{j=m+1}^{m+n}) \text{ which satisfies (23).}\}$$
$$\tilde{Y}_j \overset{\text{def.}}{=} \{y_j \in Y_j \,|\, y_j \text{ is a component of a pair } ((x_i)_{i=1}^m, (y_j)_{j=m+1}^{m+n}) \text{ which satisfies (23).}\}$$

If a state $((x_i, \phi_i, d_i, v_i, z_i)_{i=1}^m, (y_j, \phi_j, d_j, v_j, z_j,)_{j=m+1}^{m+n}, p, s)$ satisfies (9), (14), (19), and (20), then each agents' real state (x_i or y_j) is attainable; $x_i \in \tilde{X}_i$ $(i = 1, \dots, n)$ and $y_j \in \tilde{Y}_j$ $(j = m+1, \dots, m+n)$. We define the pareto optimality based on the attainable set.

Definition 2. (Pareto Optimality) A state $((x_i, \phi_i, d_i, v_i, z_i)_{i=1}^m, (y_j, \phi_j, d_j, v_j, z_j,)_{j=m+1}^{m+n}) \in \prod_{i=1}^m (X_i \times C_i \times R_+^\lambda) \times \prod_{j=m+1}^{m+n} (Y_j \times C_j \times R_+^\lambda)$ is *pareto optimal* if there is no other attainable pair $((\hat{x}_i)_{i=1}^m, (\hat{y})_{j=m+1}^{m+n})$ such that for all $i = 1, \cdots, m$, $u_i(\hat{x}_i) \geq u_i(x)$ and at least one i, $u_i(\hat{x}_i) > u_i(x_i)$.

We argue that under specific conditions, an equilibrium can be pareto optimal next section.

5 Equilibrium Existence and Optimality Arguments

We now state a general-equilibrium existence theorem for the economy:

Theorem 1. *Economy* $\mathcal{E} := ((X_i, C_i, \omega_i, u_i, (\theta_{ij})_{j=m+1}^{m+n})_{i=1}^{m}, (Y_j, C_j)_{j=m+1}^{m+n})$ *has an equilibrium,* $((x_i^*, \phi_i^*, d_i^*, v_i^*, z_i^*, v_i^*)_{i=1}^{m},\ (y_j^*, \phi_j^*, d_j^*, v_j^*, z_j^*)_{j=m+1}^{m+n}, p^*, s^*)$, *if the following conditions are satisfied:*

(Consumers) *Each consumer* $i = 1, \ldots, m$ *has a non-empty closed convex consumption set* $X_i \subset R_+^{\ell}$ *that is bounded from below, a convex preference induced by a strictly monotonic* [6] *and continuous utility function* $u_i : X_i \to R_+$, *and a strictly positive initial endowment* $\omega_i \in R_{++}^{\ell}$.

(Producers) *For each* $j = m+1, \ldots, m+n$, Y_j *is a closed convex set containing* $-R_+^{\ell}$.[7]

(Attainable Set) *Each agents' attainable set* ($\tilde{X}_i\,(i = 1, \ldots, m)$ *or* $\tilde{Y}_j\,(j = m+1, \ldots, m+n)$) *is bounded.*

(Commodification Technologies) *For each agent* $i = 1, \ldots, m+n$, *commodification technology* C_i *satisfies conditions* $(C1)$ – $(C4)$.

In our setting, we must treat demand and supply (z_i and v_i) as being distinguished from consumption and production (x_i and y_j), as in the case with transactions in asset markets, and treat producers or consumers whose actions are restricted not only by their technologies or standard budgets but also by their buying and selling constraints (9) and (14). Since transaction plans z_i and v_i are not bounded, and the expectation $s = (s^{\kappa})_{\kappa=1}^{\lambda}$ decides the estimation of real receipts, continuity of excess demands with respect to prices and expectations may not be warranted in some boundary cases such as $s_k^{\kappa} \to 0$ for some κ and $k \in L_{\kappa}$ together with an agent's demand like $z_i^{\kappa} \to \infty$. In this paper, we overcome this problem by condition (C3) for commodification technologies, which is a natural condition for them as a cost structure due to their technological limits. Note that such a discontinuity problem is likely to occur in a market in which the adverse selection problem ($s_k^{\kappa} \to 0$) exists, but our method do not exclude the existence of the adverse selection behaviors at all. We only exclude the possibility of discontinuity by considering the cost structure in supplying market commodities.

Condition (C3) is crucial for the existence result. To see this, let us confirm the attainability condition for the real goods and services. Consider each k-th coordinate of $\sum_{i=1}^{m}(\omega_i - x_i) + \sum_{j=m+1}^{m+n} y_j$:

$$\begin{aligned}\sum_{i=1}^{m}(\omega_{ik} - x_{ik}) + \sum_{j=m+1}^{m+n} y_{jk} &\overset{(9),(14)}{=} (\sum_{i=1}^{m+n} v_{ik}^{1}) + \cdots + (\sum_{i=1}^{m+n} v_{ik}^{\lambda}) - (\sum_{i=1}^{m+n} z_i^{1})s_k^{1} - \cdots - (\sum_{i=1}^{m+n} z_i^{\lambda})s_k^{\lambda} \\ &= \sum_{\kappa=1}^{\lambda}\left[\sum_{i=1}^{m+n} v_{ik}^{\kappa} - (\sum_{i=1}^{m+n} z_i^{\kappa})s_k^{\kappa}\right]. \end{aligned} \tag{24}$$

If $\sum_{i=1}^{m+n} \phi_i^{\kappa} = 0$ for some κ, we have $v_{ik}^{\kappa} = 0$ for all i by condition (C2). If $\sum_{i=1}^{m+n} \phi_i^{\kappa} > 0$ for some κ, the κ-th term of the right hand side of equation (24) is equal by equations (12) and (13) to:

$$\sum_{i=1}^{m+n} (v_{ik}^{\kappa} - d_{ik}^{\kappa}), \tag{25}$$

[6] A function $u : R^{\ell} \to R$ is *strictly monotonic* if $x' \geqq x$, $x' \neq x$ implies $u_i(x') > u_i(x)$.

[7] Here, a production technology may have a non empty intersection with the positive part of the first quadrant. Otherwise, note that there may exist a trivial equilibrium, $((x_i^*, \phi_i^*, d_i^*, v_i^*, z_i^*, v_i^*)_{i=1}^{m},\ (y_j^*, \phi_j^*, d_j^*, v_j^*, z_j^*)_{j=m+1}^{m+n}, p^*, s^*) = ((0,0,0,0,0,0)_{i=1}^{m},\ (0,0,0,0,0)_{j=m+1}^{m+n}, 0, 0)$.

which is non-negative by (C2). Hence, the equilibrium state is feasible and the amount $v^\kappa_{ik} - d^\kappa_{ik}$ for each i and κ (if it is not equal to zero) necessarily cause a welfare loss as long as the preferences are strictly monotonic (*suboptimality property*). [8]

Note here that conditions (C2) and (C3) do not exclude the case that $v^\kappa_i - d^\kappa_i = 0$ at least for all v^κ_i in a certain bounded domain, $K \subset R^l$. Furthermore, let us turn our attention to the simple case, where each commodification technology is represented by a function. That is, on some bounded domain (including K) of v^κ_i, let d^κ_i and ϕ^κ_i be represented by functions, $d^\kappa_i(v^\kappa_i)$ and $\phi^\kappa_i(v^\kappa_i)$, respectively. To be more specific, suppose d^κ_i be identity function ($d^\kappa_i(v^\kappa_i) = v^\kappa_i$) and $\phi^\kappa_i(v^\kappa_i) = \Sigma_{k\in L^\kappa} v^\kappa_{ik}$ in that domain. This still leaves our model as a generalization of Bisin et al. (2011). This is because the unit of "contract" in their model can be interpreted as some v^κ_i such that $\phi^\kappa_i(v^\kappa_i) = 1$, and their model has a delivery upper bound. We then obtain the following proposition about the optimality of the equilibrium.

Theorem 2. (Optimality): *Suppose that equilibrium v^κ_i is in the bounded domain mentioned above for each i, (i.e. the above simplification $d^\kappa_i(v^\kappa_i) = v^\kappa_i$ and condition of $\phi^\kappa_i = \Sigma_{k\in L^\kappa} v^\kappa_{ik}$ holds for each i in equilibrium). Suppose additionally that at equilibrium, for each characteristic k, there exists a market κ such that $s^\kappa_k > 0$, then the equilibrium state is Pareto-optimal.*[9]

The above suboptimality argument clearly suggested that the avoidance of the condition (C3) (non-essentiality of the adverse selection problem) is a necessary condition for the optimality of equilibrium (non-essentiality of government intervention) in the endogenization problem of market structure. If we consider this theorem in conjunction with this, it can be said to suggest that the avoidance of the condition (C3) (non-essentiality of the adverse selection problem) is also a sufficient condition for the optimality of equilibrium. More precisely, see the following remark.

Remark (Optimality and Suboptimality) When the presence of adverse selection is essential to ensure the existence of equilibrium, the condition (C3) is indispensable. Hence as we argued in (25), an equilibrium is sub-optimal. However, consider that (ϕ, d) are functions of v as in theorem 2, the equilibrium belongs to a bounded set, and $d_i = v_i$ is valid. The condition (C3) is not necessary. If (C3) is not essentially the key to the existence, then the equilibrium is pareto optimal. It implies that (C3), as a condition of a cost structure between d and v, describes problems of adverse selection. In general, ϕ^κ_i and d^κ_i may not be functions of v^κ_i. That is, consider an element $(\phi_i, d_i, v_i) \in C_i$, where $\phi_i = (\phi^1_i, \cdots, \phi^\lambda_i)$ and $d_i = (d^1_i, \cdots, d^\lambda_i)$. There may be a market κ and a pair $(\hat{\phi}^\kappa_i, \hat{d}^\kappa_i) \neq (\phi^\kappa_i, d^\kappa_i)$ such that $((\phi^1_i, \cdots, \phi^{\kappa-1}_i, \hat{\phi}^\kappa_i, \phi^{\kappa+1}_i \cdots, \phi^\lambda_i), (d^1_i, \cdots, d^{\kappa-1}_i, \hat{d}^\kappa_i, d^{\kappa+1}_i, \cdots, d^\lambda_i), v_i)$ is also an element of C_i. In this case, as we mentioned in section 2, a pareto inefficient equilibrium and pareto efficient equilibrium may exist at the same time. Thus, the above property may cause the existence of pareto inefficient equilibrium, because the expectation s is a given paremeter to solve utility maximization and profit maximization problems,

6 Concluding Remarks

1. (An Alternative Notion of Optimality) As we have noted in the end of section 4, our equilibrium situation with the commodification-cost like (C3) is necessarily related to the suboptimality property. It

[8] The discussion of optimality is only on the attainable set, not market-attainable. That is, $((x^*_i)^m_{i=1}, (y^*_j)^{m+n}_{j=m+1})$ is pareto optimal if there is no attainable pair $(x'_i)^m_{i=1}, (y'_j)^{m+n}_{j=m+1}$ such that $\forall i = 1, \cdots, m, u_i(x'_i) \geq u_i(x^*_i)$ and $\exists i, u_i(x'_i) > u_i(x^*_i)$.

[9] This is because our model do not use the trading upper bounds. For proof, see Theorem 1 of Urai, Yoshimachi, and Shiozawa (2017).

seems, however, that we may consider two kinds of optimality concepts in this model, (i) the optimality that individuals can establish by exchanging their characteristics *directly* in the economy (without using the commodification technology), as defined in this paper, and (ii) the optimality that individuals can achieve by interacting with markets. While an economy do not generally achieve the Pareto optimality in the meaning of (i), it may happen that with some conditions, the economy can establish optimality in the sense of (ii). Also, discussion (ii) suggests that an inefficiency may happen merely through the expectation among buyers. The treatment of expectation, therefore, is economically important even if an economy does not establish the Pareto optimality in the meaning of (i).

2. (Asymmetric Information and Market Viability Problem) The models of Bisin et al.(2011) and Urai et al. (2017) are generalized applications of the DGS bankruptcy model (Dubey et al. 2005) to the asymmetric information problem. In these models, asymmetric information is resolved by self-fulfilling (average) expectations in the market, and in our model the cost of building such a market structure corresponds to the (C3)-condition represented by the signaling cost. The problem of endogenization of market structure under various product differentiation possibilities is here solved as a market viability problem, i.e., the problem of resolving asymmetric information through self-fulfilling expectations.

3. (Incomplete Market Dynamics) It would be worthwhile to extend our result to the dynamic general equilibrium models with incomplete markets. As is well known that under the incomplete market (rational expectation asset pricing) framework, it is difficult to treat production because of the existence of new assets based on a future production action (see, e.g., Mas-Colell et al. 1995, Ch. 19). On the other hand, the need for a model that can include entities creating new markets as an equilibrium action is even greater today. A, so called, platform firm like GAFAM can be identified (from the general equilibrium viewpoint) with *a technology creating new commodities and a new market structure*, i.e., based on a commodification technology treated here.

4. (Firm Formation Dynamics) The importance of dynamics pointed out in the previous remark, should also be argued with the problem how we could incorporate the creation of new firms or the endogenized firm structure. It would be interesting to relate our result with the general equilibrium (together with the cooperative game) argument including firm formation like Boehm (1974), Greenburg (1979), Ichiishi (1997) and some resent researchs of the authors, Urai-Murakami-Chen (2023), Urai et al. (2023) including Shiozawa, etc. In preparation for such dynamicization, the authors have begun to apply the endogenization problem of market structure treated in this paper to a monetary dynamic economy with an overlapping generation structure (Murakami et al. 2023).

5. (Numbers of Characteristics and Commodities) In this paper, the number of basic characteristics, l, and the possible number of commodities, λ, are taken to be finite. From the mathematical viewpoint, it would be natural to consider how our result can be extended to the infinite number of the basic characteristics (as in Mas-Colell 1975) and/or possible commodities. Finally, to emphasize, our model does not exclude that the same bundle of (the names of) characteristics correspond to two different commodities, so our model includes the treatment of commodity differentiation like Mas-Colell (1975), in this respect.

Appendix

Appendix A : Example Calculation Detals

This appendix provides the details of the example calculation given in Section 2. The notation for characteristics, G and V, used in Section 2 are replaced here by 1 and 2, respectively, to simplify the notation.

Suppose a pure exchange economy with two consumers indexed by $i = 1, 2$, and there are two types of characteristics $k = 1, 2\,(\,\ell = 2\,)$ and is only one market $(\lambda = 1)$. Consumers have preference over characteristics and the quality of goods will correspond with a common expectation of consumers $s \in R^{\ell}$ in equilibrium. We let $\omega_i^k \geq 0$ be player i's initial endowment of characteristics. Initial endowment is given as $\omega_1 = (1, 0), \omega_2 = (0, 1)$ so that two consumers have incentives to trade through the market. Each player's utility function is given as $u_i(x_i^1, x_i^2) = (x_i^1)(x_i^2)$, where x_i^1 and x_i^2 are player i's consumption of character 1 and 2.

$$(\phi_i^1, d_i^1, v_i^1) \in C_i \;\Leftrightarrow\; v_i^1 \geq 0,\; v_i^2 \geq 0 \quad \phi_i = (v_i^1 + v_i^2) \quad (v_i^1, v_i^2) \;\geq\; d_i \;\geq\; \frac{1}{2}(v_i^1, v_i^2). \tag{26}$$

In this setting, we consider a situation where consumers have discretion over the delivery of characteristics, while agents do not have in Urai et al (2017)'s model. In Urai et al (2017), it is verified that any equilibrium can achieve pareto efficiency. In contrast, our framework can provide the case of inefficiency of equilibrium, because, to a certain extent, people have a control over their delivery. Following the setting, the utility maximization problem is

$$\max \quad u_i(x_i^1, x_i^2) = (x_i^1)(x_i^2), \tag{27}$$

sub.to

$$x_i^1 = s_1 z_i + \omega_i^1 - v_i^1, \tag{28}$$

$$x_i^2 = s_2 z_i + \omega_i^2 - v_i^2, \tag{29}$$

$$p\phi_i = p z_i, \tag{30}$$

$$(\phi_i, d_i, v_i) \in C_i. \tag{31}$$

Hereafter, we consider a case of $\frac{1}{2} \geq \hat{s} := s_1 = s_2$.
Define a Languladian function as follows:

$$\mathcal{L}(v_i^1, v_i^2, \nu_1, \nu_2) := (\omega_i^1 + (\hat{s} - 1)v_i^1 + \hat{s}v_i^2)(\omega_i^2 + \hat{s}v_i^1 + (\hat{s} - 1)v_i^2) - \nu_1 v_i^1 - \nu_2 v_i^2.$$

By Karush-Kuhn-Tucker condition, we have

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial v_i^1} &= (\hat{s} - 1)(\omega_i^2 + \hat{s}v_i^1 + (\hat{s} - 1)v_i^2) + \hat{s}(\omega_i^1 + (\hat{s} - 1)v_i^1 + \hat{s}v_i^2) - \nu_1 = 0, \\
\frac{\partial \mathcal{L}}{\partial v_i^2} &= \hat{s}(\omega_i^2 + \hat{s}v_i^1 + (\hat{s} - 1)v_i^2) + (\hat{s} - 1)(\omega_i^1 + (\hat{s} - 1)v_i^1 + \hat{s}v_i^2) - \nu_2 = 0, \\
v_i^1 &= 0 \text{ or } \nu_1 = 0, \\
v_i^2 &= 0 \text{ or } \nu_2 = 0.
\end{aligned}$$

(i) $v_i^1 = v_i^2 = 0$. Then $x_i^1 = \omega_i^1$, and $x_i^2 = \omega_i^2$. Then the utility is $\omega_i^1 \omega_i^2$.
(ii) $\nu_1 = 0, \nu_2 = 0$.

$$\begin{pmatrix} -2\hat{s}(1-\hat{s}) & \hat{s}^2 + (1-\hat{s})^2 \\ \hat{s}^2 + (1-\hat{s})^2 & -2\hat{s}(1-\hat{s}) \end{pmatrix} \begin{pmatrix} v_i^1 \\ v_i^2 \end{pmatrix} = \begin{pmatrix} -\hat{s} & 1-\hat{s} \\ 1-\hat{s} & -\hat{s} \end{pmatrix} \begin{pmatrix} \omega_i^1 \\ \omega_i^2 \end{pmatrix} \tag{32}$$

Then the determinant of the matrix $\begin{pmatrix} -2\hat{s}(1-\hat{s}) & \hat{s}^2 + (1-\hat{s})^2 \\ \hat{s}^2 + (1-\hat{s})^2 & -2\hat{s}(1-\hat{s}) \end{pmatrix}$ is not zero if $\hat{s} \neq \frac{1}{2}$.
When $\hat{s} \neq \frac{1}{2}$,

$$\begin{pmatrix} v_i^1 \\ v_i^2 \end{pmatrix} = \frac{1}{(\hat{s}^2 - (1-\hat{s})^2)^2} \begin{pmatrix} 2\hat{s}(1-\hat{s}) & \hat{s}^2 + (1-\hat{s})^2 \\ \hat{s}^2 + (1-\hat{s})^2 & 2\hat{s}(1-\hat{s}) \end{pmatrix} \begin{pmatrix} -\hat{s} & 1-\hat{s} \\ 1-\hat{s} & -\hat{s} \end{pmatrix} \begin{pmatrix} \omega_i^1 \\ \omega_i^2 \end{pmatrix}. \tag{33}$$

Since $\begin{pmatrix} x_i^1 - \omega_i^1 \\ x_i^2 - \omega_i^2 \end{pmatrix} = -\begin{pmatrix} 1-\hat{s} & -\hat{s} \\ -\hat{s} & 1-\hat{s} \end{pmatrix} \begin{pmatrix} v_i^1 \\ v_i^2 \end{pmatrix}$, and

$$\frac{-1}{(\hat{s}^2 - (1-\hat{s})^2)^2} \begin{pmatrix} 1-\hat{s} & -\hat{s} \\ -\hat{s} & 1-\hat{s} \end{pmatrix} \begin{pmatrix} 2\hat{s}(1-\hat{s}) & \hat{s}^2 + (1-\hat{s})^2 \\ \hat{s}^2 + (1-\hat{s})^2 & 2\hat{s}(1-\hat{s}) \end{pmatrix} \begin{pmatrix} -\hat{s} & 1-\hat{s} \\ 1-\hat{s} & -\hat{s} \end{pmatrix}$$

$$= \frac{1}{\hat{s}^2 - (1-\hat{s})^2} \begin{pmatrix} \hat{s} & 1-\hat{s} \\ 1-\hat{s} & \hat{s} \end{pmatrix} \begin{pmatrix} -\hat{s} & 1-\hat{s} \\ 1-\hat{s} & -\hat{s} \end{pmatrix} = -\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \text{ we have } x_i^1 = x_i^2 = 0.$$

Then, the utility $u_i = 0$.
If $\hat{s} = \dfrac{1}{2}$, then (32) yields

$$\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} v_i^1 \\ v_i^2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \omega_i^1 \\ \omega_i^2 \end{pmatrix} \tag{34}$$

There are infinitely many solutions as long as both v_i^1 and v_i^2 are non-negative and satisfy $v_i^1 - v_i^2 = \omega_i^1 - \omega_i^2$. In this case, $x_i^1 = x_i^2 = \dfrac{\omega_i^1 + \omega_i^2}{2}$. Then $u_i = \left(\dfrac{\omega_i^1 + \omega_i^2}{2}\right)^2$.[10]
(iii) $\nu_1 = 0$, $v_i^2 = 0$.
The Karush-Kuhn-Tucker condition becomes

$$\frac{\partial \mathcal{L}}{\partial v_i^1} = (\hat{s}-1)(\omega_i^2 + \hat{s} v_i^1) + \hat{s}(\omega_i^1 + (\hat{s}-1)v_i^1) = 0, \tag{35}$$

$$\frac{\partial \mathcal{L}}{\partial v_i^2} = \hat{s}(\omega_i^2 + \hat{s} v_i^1) + (\hat{s}-1)(\omega_i^1 + (\hat{s}-1)v_i^1) - \nu_2 = 0. \tag{36}$$

By (35), $2\hat{s}(1-\hat{s})v_i^1 = \hat{s}\,\omega_i^1 - (1-\hat{s})\,\omega_i^2$. If $\dfrac{1-\hat{s}}{\hat{s}}\,\omega_i^2 > \omega_i^1$, $0 > v_i^1$. This violates a condition that v_i^1 is non-negative. We consider the case where the initial endowment satisfies the condition $\omega_i^1 > \dfrac{1-\hat{s}}{\hat{s}}\,\omega_i^2$. We have $v_i^1 = \dfrac{\hat{s}\,\omega_i^1 + (\hat{s}-1)\,\omega_i^2}{2\hat{s}\,(1-\hat{s})}$. By (36), $\nu_2 = -(1-\hat{s})\,\omega_i^1 + \hat{s}\,\omega_i^2 + (\hat{s}^2 + (1-\hat{s})^2)\,v_i^1$. Finally, the utility $u_i = \left(\dfrac{1}{2}\,\omega_i^1 + \dfrac{1-\hat{s}}{2\hat{s}}\,\omega_i^2\right)\left(\dfrac{\hat{s}}{2(1-\hat{s})}\omega_i^1 + \dfrac{1}{2}\,\omega_i^2\right)$.

[10] It will be stated that, in this case, both consumer one and consumer two can achieve Pareto Optimality.

(iv) $v_i^1 = 0$, $\nu_2 = 0$.
The Karush-Kuhn-Tucker condition becomes

$$\frac{\partial \mathcal{L}}{\partial v_i^1} = (\hat{s}-1)(\omega_i^2 + (\hat{s}-1)v_i^2) + \hat{s}(\omega_i^1 + \hat{s}v_i^2) - \nu_1 = 0, \tag{37}$$

$$\frac{\partial \mathcal{L}}{\partial v_i^2} = \hat{s}(\omega_i^2 + (\hat{s}-1)v_i^2) + (\hat{s}-1)(\omega_i^1 + \hat{s}v_i^2) = 0. \tag{38}$$

If $\omega_i^1 > \dfrac{\hat{s}}{1-\hat{s}}\,\omega_i^2$, $0 > v_i^2$, which violates that v_i^2 is non-negative. We suppose that $\dfrac{\hat{s}}{1-\hat{s}}\,\omega_i^2 \geq \omega_i^1$. Then $v_i^2 = \dfrac{(\hat{s}-1)\,\omega_i^1 + \hat{s}\,\omega_i^2}{2\hat{s}\,(1-\hat{s})}$ and $\nu_1 = (\hat{s}^2 + (1-\hat{s})^2)\,\dfrac{(\hat{s}-1)\omega_i^1 + \hat{s}\omega_i^2}{2\hat{s}\,(1-\hat{s})} + \hat{s}\omega_i^1 + (1-\hat{s})\omega_i^2$. Then the utility $u_i = \left(\dfrac{1}{2}\,\omega_i^1 + \dfrac{\hat{s}}{2(1-\hat{s})}\,\omega_i^2\right)\left(\dfrac{1-\hat{s}}{2\hat{s}}\,\omega_i^1 + \dfrac{1}{2}\,\omega_i^2\right)$.
We apply the analysis above for the case where initial endowments are given as $\omega_1 = (1,0), \omega_2 = (0,1)$. If $\frac{1}{2} \geq \hat{s}$, comsumer 1 and 2's utility $u_1 = u_2 = \dfrac{\hat{s}}{4(1-\hat{s})}$. Next, we discuss equilibria. When $v_1^2 = v_2^1 = 0$, then $v_1^1 = v_2^2 = \dfrac{1}{2(1-\hat{s})}$. In addition, $d_1^2 = d_2^1 = 0$ by the condition (31). Since the expectation corresponds the average amount of characteristics,

$$\begin{pmatrix} \hat{s} \\ \hat{s} \end{pmatrix} = \begin{pmatrix} \dfrac{d_1^1 + d_1^2}{\phi_1 + \phi_2} \\ \\ \dfrac{d_2^1 + d_2^2}{\phi_1 + \phi_2} \end{pmatrix} = \begin{pmatrix} d_1^1(1-\hat{s}) \\ \\ d_2^2(1-\hat{s}) \end{pmatrix} \tag{39}$$

Then, we have $d_1^1 = d_2^2 = \dfrac{\hat{s}}{1-\hat{s}} = 2\,\hat{s}\,v_1^1 = 2\,\hat{s}\,v_2^2$. By (31), $\frac{1}{2} \geq \hat{s} \geq \frac{1}{4}$. If $\hat{s}$ is less than $\frac{1}{2}$, Pareto efficiency cannot be established. The example is an extensive case of Urai et al(2017). According to a proposition in Urai et al (2017), Pareto efficiency can be achieved under a particular condition. However, we have a different conclusion in our model. One reason is that people have a certain discretion over the delivery of characteristics.

Appendix B: Proof of Theorem 1.

The proof arguments on the existence theorem would be reduced to the standard general equilibrium ones except for the next two points: (i) we have to appropriately select valuables for constructing expectation parameter s^κ for each κ in (22) to define a fixed-point mapping, and check that for the fixed-point to be called an equilibrium, such selections are harmless, and (ii) with respect to expectation parameter s^κ, the convergence, $s_k^\kappa \to 0$ for some k, causes the non-continuity (non-closedness) problem on agents' constraint correspondences.

For the first problem, we will only use ϕ_i^κ in constructing s^κ through the equation in (22). For the second problem, we can appeal to the ordinary truncation argument, i.e., setting a large cube and taking its limit. The price space is $\Delta = \{(p_1, \ldots, p_\lambda) | \, p_1 \geqq 0, \ldots, p_\lambda \geqq 0, \sum_{\kappa=1}^{\lambda} p_\kappa = 1\}$. Moreover, by (C4), we have Δ^κ, the set of all real-receipt expectations for market κ, for $\kappa = 1, \ldots, \lambda$.

Producers: Each real technology $Y_j \subset R^\ell$, $j = m+1, \ldots, m+n$ as well as commodificaton technology $C_j \subset R_+^\lambda \times \Pi_{\kappa=1}^{\lambda} R^{L_\kappa} \times R_+^l$, $j = m+1, \ldots, m+n$, is assumed to be closed and convex, and to contain 0. Hence, the set of all solutions to the maximization problem (10)–(14) under price $p \in \Delta$ and expectation $s =$

$(s^\kappa)_{\kappa=1}^{\lambda} \in \prod_{\kappa=1}^{\lambda} \Delta^\kappa$, $\eta_j(p,s) \subset R^\ell \times R_+^{\lambda+\lambda\ell+\ell+\lambda}$ is closed and convex. Now, take an arbitrarily large number $t > 0$ and consider maximization problem (10) subject to (9)–(12) with $(y_j, \phi_j, d_j, v_j, z_j) \in [-t,t]^\ell \times [0,t]^{\lambda+\lambda\ell+\ell+\lambda}$. In words, producers problems are restricted to $[-t,t]^\ell \times [0,t]^{\lambda+\lambda\ell+\ell+\lambda}$. We denote by $\eta_j^t(p,s)$ the set of solutions to the restricted maximization problem. The non-emptiness, closedness, and convexity of $\eta_j^t(p,s)$ are clear. We can also prove that the correspondence $\eta_j^t : \Delta \times \prod_{\kappa=1}^{\lambda} \Delta^\kappa \to [-t,t]^\ell \times [0,t]^{\lambda+\lambda\ell+\ell+\lambda}$ has a closed graph. Indeed, the constraint correspondence $(p,s) \mapsto \{\, (y_j, \phi_j, d_j, v_j, z_j) \,|\, (y_j, \phi_j, d_j, v_j, z_j) \in [-t,t]^\ell \times [0,t]^{\lambda+\lambda\ell+\ell+\lambda}$ satisfies (9)–(12) under $(p,s)\,\}$ has a closed graph and is lower semi-continuous, and thus also continuous. Hence, Berge's maximum theorem (cf.Debreu (1959) , p. 19, Theorem (4)) is applicable. Here, the continuity of the profit function of this truncated problem, $\pi_j^t(p,s)$, is simultaneously assured by Berge's theorem.

Consumers: As in the producer case, the set of all solutions to the maximization problem (15), subject to (14)–(18) under $p \in \Delta$ and $s \in \prod_{\kappa=1}^{\lambda} \Delta^\kappa$, $\xi_i(p,s)$, is closed and convex. Let $\xi_i^t(p,s)$ denote the set of solutions to maximization problem (15) subject to (14)–(18) with $(x_i, \phi_i, d_i, v_i, z_i) \in [-t,t]^\ell \times [0,t]^{\lambda+\lambda\ell+\ell+\lambda}$, and with each profit $\pi_j(p,s)$ in Eq. (16) replaced by $\pi_j^t(p,s)$, which is the maximized profit of producers in the truncated maximization problem. We assume that each consumer has a strictly positive initial endowment, $\omega_i \in R_{++}^\ell$. Then it is also possible to verify that the correspondence $\xi_i^t : \Delta \times \prod_{\kappa=1}^{\lambda} \Delta^\kappa \to [-t,t]^\ell \times [0,t]^{\lambda+\lambda\ell+\ell+\lambda}$ is non-empty closed convex valued and has a closed graph. In particular, for the closed graph of ξ_i^t, check that the constraint correspondence ,$(p,s) \mapsto \{(x_i, \phi_i, d_i, v_i, z_i) | \, (x_i, \phi_i, d_i, v_i, z_i) \in [-t,t]^\ell \times [0,t]^{\lambda+\lambda\ell+\ell+\lambda}$ satisfies (14)–(18) under (p,s) where π_j in Eq. (9) is replaced by $\pi_j^t\}$, has a closed graph and is lower semi-continuous. Then apply Berge's maximum theorem again.

Fixed Points and Limit Arguments: Take a number $t > 0$ sufficiently large for all the bounded attainable sets, $\tilde{Y}_j (j = m+1, \ldots, m+n)$ and $\tilde{X}_i (i = 1, \ldots, n)$, to be a subset of the interior of $[-t,t]^\ell$. Restrict the individual maximization problems (10)-(14) and (15)-(20) to the set $[-t,t]^\ell \times [0,t]^{\lambda+\lambda\ell+\ell+\lambda}$. Consider η_j^t, ξ_i^t, and the product map Φ of these correspondences:

$$\Phi : \Delta \times \prod_{\kappa=1}^{\lambda} \Delta^\kappa \ni (p,s) \mapsto \prod_{i=1}^{m} \xi_i^t(p,s) \ \times \prod_{j=m+1}^{m+n} \eta_j^t(p,s) \subset \left([-t,t]^\ell \times [0,t]^{\lambda+\lambda\ell+\ell+\lambda}\right)^{m+n}. \tag{40}$$

The mapping Φ has a closed graph since each of η_j^t and ξ_i^t is so.

Define a price-expectation manipulation correspondence as follows:

$$\Psi : ([-t,t]^\ell \times [0,t]^{\lambda+\lambda\ell+\ell+\lambda})^{m+n} \ni (\phi_i, z_i, v_i)_{i=1}^{m+n} \mapsto \Theta((\phi_i, z_i)_{i=1}^{m+n}) \times \Xi((v_i)_{i=1}^{m+n}) \subset \Delta \times \prod_{\kappa=1}^{\lambda} \Delta^\kappa, \tag{41}$$

where Θ denotes the price manipulation mapping such that, for each $(z_i)_{i=1}^m$, $\Theta((z_i)_{i=1}^m)$ assigns a set of prices which evaluate the excess-demand with the highest value $\{p \in \Delta | \, \forall q \in \Delta, q \cdot \sum_{i=1}^{m+n}(z_i - \phi_i) \leqq p \cdot \sum_{i=1}^{m+n}(z_i - \phi_i)\}$, and Ξ is the correspondence that assigns the real mixture ratio of the goods for each market. More precisely, we define the κ-th coordinate of Ξ by

$$\Xi_\kappa((v_i)_{i=1}^{m+n}) = \frac{\sum_{i=1}^{m+n} d_i^\kappa}{\sum_{i=1}^{m+n} \phi_i^\kappa} \in R^{L_\kappa}, \tag{42}$$

as long as $\sum_{i=1}^{m+n} \phi_i^\kappa \neq 0$, and otherwise by $\Xi_\kappa((v_i)_{i=1}^{m+n}) = \Delta^\kappa$. Note that the right hand side of Eq. (42) is always an element of Δ^κ when $\sum_{i=1}^{m+n} \phi_i^\kappa \neq 0$ by condition (C4) for $(\phi_i^\kappa, d_i^\kappa)_{\kappa=1}^{\lambda}$. It is easy to check that Θ and Ξ are non-empty closed convex valued correspondence with a closed graph. In particular, Ξ has a closed graph since the right hand side of Eq. (42) is continuous when $\sum_{i=1}^{m+n}\left(\sum_{k \in L_\kappa} \phi_{ik}\right) \neq 0$.

Now, the product of the mappings Φ and Ψ,

$$\Phi\times\Psi : \big([-t,t]^\ell\times[0,t]^{\lambda+\lambda\ell+\ell+\lambda}\big)^{m+n}\times\Delta\times\Big(\prod_{\kappa=1}^{\lambda}\Delta^\kappa\Big) \to \big([-t,t]^\ell\times[0,t]^{\lambda+\lambda\ell+\ell+\lambda}\big)^{m+n}\times\Delta\times\Big(\prod_{\kappa=1}^{\lambda}\Delta^\kappa\Big), \quad (43)$$

is a non-empty closed convex valued correspondence with a closed graph. Then all of the conditions for Kakutani's fixed point theorem are satisfied. $\Phi\times\Psi$ has a fixed point $\big((x_i^t,\phi^t,d^t,v_i^t,z_i^t)_{i=1}^m,\ (y_j^t,\ \phi_j^t,\ d_j^t,\ v_j^t,\ z_j^t)_{j=1}^n,\ p^t,\ s^t\big)\in\big([-t,t]^\ell\times[0,t]^{\lambda+\lambda\ell+\ell+\lambda}\big)^{m+n}\times\Delta\times\big(\prod_{\kappa=1}^{\lambda}\Delta^\kappa\big)$. Equation (17) with (10) gives Walras' Law:

$$(p,s)\in\Delta\times\prod_{\kappa=1}^{\lambda}\Delta^\kappa \quad\text{and}\quad z-\phi\in-\sum_{i=1}^{m}\xi_i^t(p,s)-\sum_{j=m+1}^{m+n}\eta_j^t(p,s) \quad\Longrightarrow\quad p\cdot(z-\phi)=0. \qquad (44)$$

Under the standard argument, this means that, by the definition of Θ, the summation of $(\phi_i^t,z_i^t)_{i=1}^{m+n}$ must satisfy $q\cdot(\sum_{i=1}^{m+n}z_i^t-\phi_i^t)\leqq p^t\cdot(\sum_{i=1}^{m+n}z_i^t-\phi_i^t)=0$ for all $q\in\Delta$, and so for each $\kappa=1,\dots,\lambda$, the κ-th coordinates of $(\phi_i^t,z_i^t)_{i=1}^{m+n}$, $(\phi_i^{\kappa t},z_i^{\kappa t})_{i=1}^{m+n}$, must be such that $\sum_{i=1}^{m+n}(z_i^{\kappa t}-\phi_i^{\kappa t})\leqq 0$, where $\sum_{i=1}^{m+n}(z_i^{\kappa t}-\phi_i^{\kappa t})<0$ if and only if the price of κ-th commodity, p_κ^t, equals 0. Moreover each of the price p_κ in market κ is strictly positive since each consumers utility function is strictly monotone. Therefore, it follows that the state $\big((x_i^t,\phi_i^t,d_i^t,v_i^t,z_i^t)_{i=1}^m,(y_j^t,\phi_j^t,d_j^t,v_j^t,z_j^t)_{j=m+1}^{m+n},p^t,s^t\big)$ satisfies (21) and (22). We have that this state satisfies (9), (14), (21) and (22), so the real states x_i^t and y_j^t are in the bounded attainable sets $\tilde{X}_i$ and $\tilde{Y}_j$ (see the end of section 2).

Since we take $t>0$ sufficiently large for the bounded attainable set to be a subset of the interior of $[-t,t]^\ell$, all x_i^t or y_j^t are interior points of $[-t,t]^\ell$. Therefore, the *t-equilibrium state* $\big((x_i^t,\phi_i^t,d_i^t,v_i^t,z_i^t)_{i=1}^m,(y_j^t,\phi_j^t,d_j^t,v_j^t,z_j^t)_{j=m+1}^{m+n},p^t,s^t\big)$ is not an equilibrium of the original economy $\mathcal{E}$ only when (ϕ_i^t,d_i^t,v_i^t) is a boundary point of $[0,t]^{\lambda+\lambda\ell+\ell}$ for some $i=1,\dots,m+n$. Hence, if (ϕ_i^t,d_i^t,v_i^t) is bounded for all $i=1,\dots,n+m$, then an equilibrium of the original economy $\mathcal{E}$ exists.

Suppose, on the contrary, that (ϕ_i^t,d_i^t,v_i^t) can not be rearranged (without changing each agent's maximization condition) to be an interior point for some $i\in\{1,\dots,n+m\}$ with respect to valuables of a certain market, κ, for for all t. This implies that $\phi_i^{\kappa'}\to\infty$ as $t\to\infty$ for some κ' since the boundedness of $\{\|\phi_i^t\|\,|\,t>0\}$ means the boundedness of $\{\|d_i^t\|\,|\,t>0\}$ and makes $\{\|v_i^t\|\,|\,t>0\}$ to be bounded. If this is an exceptional case of (C3), i.e., $L_{\kappa'}=\{k\}$ and $\phi_i^\kappa=v_{ik}^t$ for all t, then $s^{\kappa'}$ is a real number in $[0,1]$ by (C3) and (22).

Hence, by condition (C2) with equations (8) and (9) (resp., equations (13) and (14)) together with the free disposal condition for producers (resp., the monotonic preference condition for consumers), they can realize at least as much as the same profit (resp., the same utility) by decreasing the value of $\phi_i^{\kappa'}$ as long as $d_i^{\kappa' t}>0$. Since this is contradictory to the impossibility of such rearrangements, we have to think that $d_i^{\kappa' t}=0$ for all t, and agent i must obtain good k from other market $\kappa''\neq\kappa'$ for all t. For such market κ'', $\sharp L_{\kappa''}\geqq 2$, non-exceptional situation of (C3) would be applicable. It follows that now we can only consider the non-exceptional case of (C3). Note that we have the following inequality:

$$\sum_{k=1}^{\ell}\Big(\sum_{i=1}^{m}(\omega_{ik}-x_{ik}^t)+\sum_{j=m+1}^{m+n}y_j^t\Big)\geq\sum_{k=1}^{\ell}(v_{ik}^{\kappa' t}-d_{ik}^{\kappa'}) \qquad (45)$$

for all κ'.[11] However, if $\phi_i^{\kappa' t}\to\infty$ as $t\to\infty$ for some i and κ', then condition (C3) requires that

[11] See equations (22) and (23). In particular, it is sufficient to consider the case $\sum_{i=1}^{m+n}\phi_i^{\kappa' t}>0$ since $\phi_i^{t\kappa'}\to\infty$. Through the above argument, we can get $\sum_{i=1}^{m}(\omega_{ik}-x_{ik}^t)+\sum_{j=m+1}^{m+n}y_{jk}^t\geqq v_{ik}^{\kappa' t}-d_{ik}^{\kappa' t}$ for all $i=1,\dots,n+m$ and all $k=1,\dots,\ell$. Hence, (29) follows by taking summation side by side.

$\sum_{k=1}^{\ell}(v_{ik}^{\kappa' t} - d_{ik}^{\kappa' t}) \to \infty$ as $t \to \infty$. This contradicts to the fact that x_i^t and y_j^t are bounded.

Appendix C: Proof of Theorem 2

We deal with the pareto optimality when $d_i^\kappa(v_i^\kappa) = v_i^\kappa$ and homogeneity of ϕ_i^κ holds for each i in equilibrium and $\forall\kappa = 1, \cdots, \lambda,\ \forall k = 1, \cdots, \ell,\ s_k^\kappa > 0$. These conditions are similar to the following theorem 1. in Urai et al. (2017).[12]

Theorem 2. (Optimality): *Suppose that there exists an equilibrium such that v_i^κ is in the bounded domain K mentioned above for each i, (i.e. the above simplification $d_i^\kappa(v_i^\kappa) = v_i^\kappa$ and $\phi_i^\kappa(v_i^\kappa) = \Sigma_{k\in L^\kappa} v_{ik}^\kappa$ holds for each i in equilibrium). Suppose additionally that at equilibrium, for each characteristic k, there exists a market κ such that $s_k^\kappa > 0$, then the equilibrium state is Pareto-optimal.*[13]

(**Proof**:) Suppose on the contrary that there is an attainable allocation $((\hat{x}_i)_{i=1}^m, (\hat{y})_{j=m+1}^{m+n})$ such that for all $i = 1, \cdots, m$, $u_i(\hat{x}_i) \geq u_i(x^*)$ and at least one i', $u_{i'}(\hat{x}_{i'}) > u_{i'}(x_{i'}^*)$. Note that we are assuming here that: (i) $d_i^\kappa(v_i^\kappa) = v_i^\kappa$, (ii) $\phi_i^\kappa(v_i^\kappa) = \Sigma_{k\in L^\kappa} v_{ik}^\kappa$ holds for each i, and (iii) for each characteristic k, there exists a market κ such that $s_k^\kappa > 0$. Then, it can be shown that for any attainable allocations $((\hat{x}_i)_{i=1}^m, (\hat{y})_{j=m+1}^{m+n})$, we can take corresponding quadiples $(\hat{x}_i, \hat{\phi}_i, \hat{d}_i, \hat{v}_i, \hat{z}_i)_{i=1}^{m+n} \in (C_i \times R_+^\lambda)$ such that for all $i = 1, \cdots, m$, $(\hat{x}_i, \hat{\phi}_i, \hat{d}_i, \hat{v}_i, \hat{z}_i)_{i=1}^m \in (X_i \times C_i \times R_+^\lambda)$ satisfies (16)-(20), and for all $j = m+1, \cdots, m+n$, $(\hat{y}_j, \hat{\phi}_j, \hat{d}_j, \hat{v}_j, \hat{z}_j) \in \Pi_{j=m+1}^{m+n}(Y_j \times C_j \times R_+^\lambda)$ satisfies (11)-(14). In fact, the implications of assumptions (i), (ii), and (iii) are as follows. All unnecessary characteristics can be sold back in the market where they were acquired, in the quantities guaranteed in (ii). Through this, any characteristic k can be obtained independently at price $p_k = p^\kappa$ in the market κ where it can be obtained, guaranteed by condition $s_k^\kappa > 0$ in (iii). This resale process does not involve any resource loss as guaranteed in (i). Moreover, the price p_k of such characteristic k is unique as long as it is defined from the equilibrium price. This is because if there can be different values, one can arbitrage through the process described above to increase one's budget constraint as much as possible. It follows that under the equilibrium price, the budget constraint for each agent under (16)-(20) or (11)-(14) can be identified with the budget under price system $(p_k)_{k=1}^\ell$ for characteristics space R^ℓ. Since $(x_i^*, \phi_i^*, d_i^*, v_i^*, z_i^*)_{i=1}^m \in (X_i \times C_i \times R_+^\lambda)$ maximizes the utility for all $i = 1, \cdots, m$, we have the following statements.

$$\hat{x}_i \succ_i x_i^* \Longrightarrow p^*(\hat{z}_i - \hat{\phi}_i) > \Sigma_{j=1}^n \theta_{ij}\pi_j(p^*, s^*) \tag{46}$$

$$\hat{x}_i \succsim_i x_i^* \Longrightarrow p^*(\hat{z}_i - \hat{\phi}_i) \geq \Sigma_{j=m+1}^{m+n} \theta_{ij}\pi_j(p^*, s^*) \tag{47}$$

The profit maximization assumption of the equilibrium state implies $\Sigma_{j=m+1}^{m+n} p^*(\hat{z}_j - \hat{\phi}_j) \geq \Sigma_{j=m+1}^{m+n} p^*(z_i^* - \phi_i^*)$. Note that since $d_i^\kappa(v_i^\kappa) = v_i^\kappa \in R{+}{+}^{L^\kappa}$ and $\phi_i^\kappa(v_i^\kappa) = \Sigma_{k\in L^\kappa} v_{ik}^\kappa$, the sum of the coordinates of $s^\kappa \in R^{L^\kappa}$ equals to 1 ($\kappa = 1, \cdots, \lambda$, $\Sigma_{k\in L^\kappa} s_k^\kappa = 1$). Then $\Sigma_{i=1}^m p^*(\hat{\phi}_i - \hat{z}_i) = \Sigma_{i=1}^m \Sigma_{k\in L^\kappa} p^*(\hat{v}_{ik} - \hat{z}_k s_k^\kappa) \overset{(16)}{=} \Sigma_{k\in L^\kappa} p^*(\omega_{ik} - \hat{x}_{ik}) \overset{(46),(47)}{<} \Sigma_{j=m+1}^{m+n} p^*(\phi_j^* - z_j^*) = \Sigma_{j=m+1}^{m+n} \Sigma_{k\in L^\kappa} p^*(\hat{v}_{ik} - \hat{z}_i s_k^\kappa) \overset{(11)}{=} \Sigma_{j=m+1}^{m+n} \Sigma_{k\in L^\kappa} p^*(-\hat{y}_{jk})$. This is a contradiction to the attainability of the allocation $((\hat{x}_i)_{i=1}^m, (\hat{y}_j)_{j=m+1}^{m+n})$.

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[12] We adopt notations in this paper, not same as Urai et al (2017), but it does not essentially matter.

[13] This is because our model do not use the trading upperbounds. For proof, see Theorem 1 of Urai et al. (2017).

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