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Optimal Government Expenditures on Production and Extraction Technologies in a Small Open Economy with a Non-renewable Natural Resource [*]

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Abstract

This paper sets a small open economy model in which the government can raise the efficiencies of production and extraction of a non-renewable natural resource by spending the budget raised through income tax. Using this model, we examine the optimal government expenditure on the production technology and that on the extraction technology. The results are as follows. If revenue of natural resource is high, the optimal expenditure on the extraction technology is high, while the optimal expenditure on production technology is independent of the natural resource revenue. Moreover, if the ratio of the optimal expenditure for improvement of extraction technology to the resource revenue is higher (lower) than the optimal tax rate under the depletion of the natural resource, the optimal tax rate before the depletion is higher (lower) than that after the depletion.

Keywords: non-renewable natural resource, labor productivity, optimal tax, small open economy

JEL classification: H21, H54, O41, Q32

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1. Introduction

The obvious characteristics of non-renewable natural resource extraction is limited capability to support income over extended periods; once the resource is exhausted the economy loses one of the growth factors. Over the years, a lot of models theorized to reflect the mechanisms of incorporation of natural resources to achieve long-term development goals. Groth (2007) summarizing the role of non-renewable natural resource on endogenous growth theory emphasizes declining nature of growth with exogenous or endogenous technical change and suggests with hesitation that, endogenous technical change might bring about the technological basis for a rising per capita consumption in the long run or at least non-decreasing per capita consumption. Developing a model similar to Stiglitz (1974) and Groth and Schou (2002), Sasaki (2021) considers positive or negative population growth and increasing or decreasing returns to both capital and labor and shows 4 cases where positive per capita output growth rate is achievable by positive or negative set of parameters. Such mixed outcome of theories is also accompanied by empirical findings. The literature survey by Mousavi and Clark (2021) concludes that majority of literature studying the effects of natural resource abundance or dependence on human capital accumulation, a key factor in development and economic growth, find adverse effects of natural resource abundance or dependence on education and health, but that a small to sizeable minority find mixed or beneficial effects. Even if we consider the literature covering past few years on the role of natural resource extraction on labor productivity, still there is no consensus and results are mixed. S. Rahim, M. Murshed, S. Umarbeyli et al. (2021) conduct an empirical study on so called Next Eleven countries, consisted of Bangladesh, Egypt, Indonesia, Iran, Korea, Mexico, Nigeria, Pakistan, Philippines, Turkey, and Vietnam over the period of 1990 – 2019. The results reveal a significant negative effect of natural resource rents on the economic growth and concluded that the resource curse hypothesis exists for these nations, however, the human capital is found to positively influence the growth through a joint impact with natural resources. Ramli et.al (2022) investigate the relationship between public investment in human capital (education and health) and economic growth in Algeria and by using an ARDL approach, conclude that public investment in human capital and growth are not cointegrated in the long run suggesting policy failure. The empirical study by Le Clech et.al. (2023) investigate the effect of non-renewable natural resources on the accumulation of human capital for the period of 1995 – 2018 in a sample of eighteen Latin American and Caribbean countries and conclude that, non-renewable natural resources have a positive impact on human capital accumulation in the long term, although, estimates demonstrate relatively low impact. However, they identify physical capital, institutional framework and the economic openness as the determinants of the human capital and estimate that increasing the physical capital by 1% would increase the human capital by approximately another 1%. Institutions and economic openness also promote human capital accumulation. As pointed out in Pritchett (2000), it is difficult to confirm that all of the public investments creates economically valuable capital. Thus, this argument along with omitted peculiar factors generally represented as characteristics of a given group of economies or regions can explain inconclusive outcome of empirical literature on the efficiency of the investment into labor productivity on economic growth.

The research question of this paper is to determine whether the government can facilitate welfare maximization through its policies optimizing natural resource extraction and non-resource output. In a simplified approach where the factors of production in economy reduced to physical assets, labor and natural resources, the only option to improve welfare after the exhaustion of natural resource stock is to enhance the output of either both of physical assets and labor or at least any of them. In this paper we are going to study welfare maximization perspectives of resource exporting country through improvement of labor productivity. Thus we introduce a theoretical framework of a small open economy where the government plays a central role in organizing the extraction of natural resource and supporting the productivity improvement in non-resource output, since concentrating the efforts on narrow group of economies with more common features might provide better insights. We incorporate the basic model introduced in Aliyev (2023) and modify the non-resource output to reflect the government support of productivity improvement. Our model is distinct from Sun et al. (2018) in that, the natural resource extraction inputs neither physical nor human capital.

This paper is organized as the following. The Section 2 describes the model and variables introduced. Section 3 describes the optimizing behavior of firms and households in this economy. Section 4 focuses on welfare maximizing behavior of the government deriving optimal values of extraction expenditure, investment into productivity improvement, optimal rate of income tax before and after the depletion of non-renewable natural resource. Section 5 summarizes the analysis with concluding remarks.

2. The Model

In this model the firms maximize their profit producing output inputting labor and capital, the natural resource is extracted from non-renewable stock and exchanged to foreign assets and the households derive utility from consumption. For this analysis we decided to use the same approach as we did in Aliyev (2023) for some of the components of the model. It is a small open economy with non-renewable natural resource stock. The natural resource is extracted using similar technology and exchanged to foreign assets $b_t$at a price p relative to non-resource output. The labor, world interest rate and subjective discount rate are assumed to be constant as well. Particularly, the preference of the households remains same, to derive utility form consumption only given as,

$$u[0] = \int_0^\infty e^{-\rho t} u(c_t) dt \,, \tag{1}$$

where ρ denotes the subjective discount rate.

We describe the non-resource output in a popular Solow-Swan style, well covered in economic literature, where the output is labor augmented. Firms produce non-resource output Y_t inputting capital K_t, labor L and the labor is subject to improvements through another variable. That is, generally referred as knowledge or productivity parameter[1], the labor augmenting variable in turn is a function of investment. The government invests some g_t

[1] See Barro, Robert J., Sala-i-Martin, Xavier (2004) *Economic Growth, second edition*, The MIT Press, chapter 1.1, pp 23-26 or Aghion, Philippe, Howitt, Peter (2009) The Economics of Growth, The MIT Press, chapter 1.2.2, pp 27-29.

into labor productivity, however, this investment is exogenous to firms' profit maximization problem and yields diminishing returns to scale on productivity. The production function in a general form can be given as

$$Y_t = F\left(K_t, g_t^{\eta} L\right), \eta \leq 1,$$

and the profit maximization conditions would be $F_K = r, F_L = w$ where r and w denote the interest rate and the wage rate respectively.

The stock of natural resource S_t diminishes at higher rate than the extraction X_t obtained. We can show the extraction technology as

$$\dot{S_t} = -\Gamma(h_t) X_t, \tag{2}$$

where $S_t \in [S_0, 0]$, $\Gamma(h_t)$>1 denotes the (in)efficiency of extraction which is a function of expenditures h_t incurred to extract. The expenditure does not accumulate over time. $\Gamma'(h_t) < 0$ and $\Gamma''(h_t) > 0$ show that the efficiency of extraction increases with diminishing return as expenditures increase.

The government runs an intertemporal budget collecting taxes to finance the natural resource extraction and improvement of labor productivity in non-resource output technology. The whole output of the economy will be taxed. Hence, the budget constraint of the government is given as

$$\int_0^T e^{-rt} \tau [f(k_t, g_t) + pX_t] dt = \int_0^T e^{-rt} (g_t + h_t) dt\,. \tag{3}$$

3. Optimal Paths

3.1. Firms

We specify the following Cobb-Douglas production function for non-resource output:

$$F\left(K_t, g_t^{\eta} L\right) = AK_t^{\alpha} \left(g_t^{\eta} L\right)^{1-\alpha},$$

where A is constant parameter of the technology. Then, using the profit maximization condition of firms with respect to capital we obtain

$$F_K = r: \qquad \alpha A \left(\frac{k_t}{g_t^{\eta}}\right)^{\alpha - 1} = r,$$

where $k \equiv \frac{K}{L}$ denotes capital per capita. After rearranging this condition, we can show that

$$k = \left(\frac{\alpha A}{r}\right)^{\frac{1}{1-\alpha}} g^{\eta}. \tag{4}$$

Since the amount of investment into labor productivity is exogenous into households' problem, the level of capital becomes constant and in order (2) to hold, implying that the investment rate becomes constant too.

Then we determine the per capita output of non-resource sector as

$$y \equiv \frac{Y}{L} = \frac{F(K, g^{\eta} L)}{L} = F(k, g^{\eta}) = Ak^{\alpha} g^{\eta(1-\alpha)},$$

and substituting the value of capital stock per capita from (4) and rearranging the above, express the non-resource output in terms of technology, government investment and interest rate:

$$y = \bar{A} g^{\eta}, \qquad \bar{A} \equiv A^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}. \tag{5}$$

3.2. Households

Households maximize their discounted utility subject to natural resource constraint and foreign assets constraint. The problem can be separated into two steps, where in the first step the natural resource earnings maximized with respect to natural resource constraint and in the second step the discounted utility is maximized with respect to foreign assets constraint. Since the extraction X_t is a choice variable and its power is one, the Hamiltonian function of this problem in the first step is linear in extraction rate. This means that the rate of extraction is constant over time and has the highest possible finite value. Moreover, the stock of natural resource does not appear in Hamiltonian function, thus making the shadow value of natural resource constant, which in turn together with constant extraction rate mean that the economy deliberately will exhaust the stock of non-renewable natural resource. Consequently, the extraction expenditure becomes constant over time. This part of our model is derived exactly the same way as in Aliyev (2023), hence we skip it here[2]. After determining the optimal paths of resource stock, its shadow value, extraction rate and its expenditure, we can show the next step of households' problem as the following:

$$\max \int_0^{\infty} e^{-\rho t} u(c_t) dt$$

subject to:

$$\begin{aligned} \dot{b}_t &= r b_t + (1-\tau)[f(k,g) + p\bar{X}] - c_t \quad \text{for } t \in [0, T], \\ \dot{b}_t &= r b_t + (1-\bar{\tau}) f(k,\bar{g}) - c_t \quad \text{for } t \in (T, \infty) \end{aligned} \tag{6}$$

The intertemporal budget constraint of household is obtained from foreign assets constraint as the following:

$$\int_0^{\infty} e^{-rt} c_t dt = b_0 + \frac{1-e^{-rT}}{r} Z + \frac{e^{-rT}}{r} \bar{Z}, \quad Z \equiv (1-\tau)[f(k,g) + p\bar{X}], \quad \bar{Z} \equiv (1-\bar{\tau}) f(k,\bar{g}). \tag{7}$$

The solution and the result to this step of households' maximization problem is also similar to Aliyev (2023)[3], hence by skipping it we only show the level of consumption for households, which we obtain from (7):

$$\bar{c} = r b_0 + (1-e^{-rT})(1-\tau)[f(k,g) + p\bar{X})] + e^{-rT}(1-\bar{\tau})[f(k,\bar{g})]. \tag{8}$$

Hence, the consumption becomes constant and equals the sum of interest earnings from initial foreign assets, and discounted after-tax earnings from natural extraction and non-resource output over the duration of resource extraction.

4. Government

The process of natural resource extraction is financed through the government and performed at highest possible rate of extraction. As turns out, the expenditures are constant over time too. Thus the level of expenditure

[2] See Aliyev (2023), pages 151-152 to derive these results.
[3] See Aliyev (2023), pages 153-154.

affects the duration of extraction until exhaustion of the natural resource stock affecting the (in)efficiency of extraction technology. Denoting the duration by T we can express it in terms of initial natural resource stock, extraction rate and technology[4]:

$$T = \frac{S_0}{\Gamma(h)\bar{X}}. \tag{9}$$

Hence, tax rate affects the value of T.

Using the optimal values from households' and firms' maximization problems the intertemporal budget of the government can be given as

$$\int_0^T e^{-rt}\{\tau[f(k,g) + p\bar{X}] - (g+h)\}dt = 0,$$

$$\Rightarrow \frac{1-e^{-rT}}{r}\{\tau[f(k,g) + p\bar{X}] - (g+h)\} = 0$$

$$\therefore \tau[f(k,g) + p\bar{X}] = g + h \text{ for } t \in [0,T]. \tag{10}$$

After the exhaustion of the natural resource the government supports the labor productivity with investment by imposing tax on the output of non-resource industry and runs the following balanced budget:

$$\bar{\tau}f(k,\bar{g}) = \bar{g} \;\; for\; t \in (T,\infty).$$

The government maximizes the welfare of households by determining the optimal levels of tax rate, extraction expenditures and investment into labor productivity improvement. Deriving the optimality condition for $\bar{Z}$ is easier, so we give it first.

Since the extraction is not financed anymore after the depletion of the natural resource, substituting (5) for output in above we can show that

$$\bar{g} = (\bar{\tau}\bar{A})^{\frac{1}{1-\eta}}, \tag{11}$$

and substituting (11) into $\bar{Z}$ in (7) gives us:

$$\bar{Z} = (1-\bar{\tau})\bar{\tau}^{\frac{\eta}{1-\eta}}\bar{A}^{\frac{1}{1-\eta}}. \tag{12}$$

$\bar{\tau}$ is determined so as maximize $\bar{Z}$. Hence, the optimal tax rate can be given as

$$\frac{\partial\bar{Z}}{\partial\bar{\tau}} = 0\!: \; -\bar{A}^{\frac{1}{1-\eta}}\frac{\bar{\tau}^{-\frac{1-2\eta}{1-\eta}}(\bar{\tau}-\eta)}{1-\eta} = 0,$$

$$\Rightarrow \bar{\tau}^* = \eta. \tag{13}$$

We applied the following approach to obtain the solution to maximization of Z by the government.

Substituting (5) into (10) turns the budget constraint into the following:

$$(g+h) = \tau[\bar{A}g^{\eta} + p\bar{X}], \tag{14}$$

which determines g. We let $\phi(\tau,h)$ denote the implicit function of g. Then the maximization problem of the government can be given from Z at (7) as

[4] See Aliyev (2023) pages 152-153 to derive this result.

$$(1-\tau)[\bar{A}g^{\eta}+p\bar{X}]=(1-\tau)[\bar{A}\phi(\tau,h)^{\eta}+p\bar{X}],$$

where the government chooses τ to maximize. The first-order condition with respect to τ is:

$$(1-\tau)\bar{A}\eta\phi(\tau,h)^{\eta-1}\phi_{\tau}=\bar{A}\phi(\tau,h)^{\eta}+p\bar{X}. \tag{15}$$

Since $\phi(\tau,h)$ satisfies (14), substituting for g to get:

$$\phi+h=\tau[\bar{A}\phi^{\eta}+p\bar{X}].$$

Further, differentiating both sides with respect to τ yields:

$$\phi_{\tau}=[\bar{A}\phi^{\eta}+p\bar{X}]+\tau\bar{A}\eta\phi^{\eta-1}\phi_{\tau},$$

which after rearranging gives us:

$$\phi_{\tau}=\frac{\bar{A}\phi^{\eta}+p\bar{X}}{1-\tau\eta\bar{A}\phi^{\eta-1}}. \tag{16}$$

Substituting this into (15) and simplifying gives us an expression which represent the optimal level of investment, g^{*} as the following:

$$\phi=(\eta\bar{A})^{\frac{1}{1-\eta}}\equiv g^{*}. \tag{17}$$

From (11) and (13), this is the same as the optimal expenditure after the depletion of the natural resource. From the definition of $\bar{A}$ in (15), we obtain the following proposition.

Proposition 1. If the productivity of the non-resource good production A is higher, the optimal expenditure for production technology g^{*} is higher. If the world interest rate r is higher, the optimal expenditure for production technology g^{*} is lower.

Using the optimal values obtained in (14) we can show that:

$$h=\tau[\bar{A}g^{*\eta}+p\bar{X}]-g^{*}. \tag{18}$$

Next, the government's maximization problem can be reduced to:

$$\max_{\tau}\frac{1-e^{-rT}}{r}(1-\tau)\,[\bar{A}g^{*\eta}+p\bar{X}]+\frac{e^{-rT}}{r}\bar{Z}$$

subject to

$$T=\frac{S_0}{\Gamma(h)\bar{X}},$$

$$h=\tau[\bar{A}g^{*\eta}+p\bar{X}]-g^{*}.$$

The first-order condition with respect to τ is obtained as the following:

$$e^{-rT}\{(1-\tau)[\bar{A}g^{*\eta}+p\bar{X}]-\bar{Z}\}\frac{\partial T}{\partial h}\frac{\partial h}{\partial \tau}=\frac{1-e^{-rT}}{r}[\bar{A}g^{*\eta}+p\bar{X}], \tag{19}$$

$$\frac{\partial T}{\partial h}=-\frac{S_0}{\Gamma(h)^2\bar{X}}\Gamma'(h),$$

$$\frac{\partial h}{\partial \tau}=[\bar{A}g^{*\eta}+p\bar{X}]>0. \tag{20}$$

Then we simplify (19) by defining $-\frac{\Gamma'(h)h}{\Gamma(h)} \equiv \varepsilon$ and using (9), its derivative with respect to h and (20) to obtain:

$$\frac{(1-\tau)[\bar{A}g^{*\eta} + p\bar{X}] - \bar{Z}}{h}\varepsilon = \frac{e^{rT}-1}{rT} \ . \tag{21}$$

Proposition 2. There is a single value of extraction expenditures which maximizes income earnings of the households.

Proof: Refer to (21) and substitute (18) into it. At the left-hand side h increases as τ increases, however, the left-hand side is a decreasing function of τ overall.

$$\frac{(\bar{A}g^{*\eta} + p\bar{X}) - \tau(\bar{A}g^{*\eta} + p\bar{X}) - \bar{Z}}{h},$$

$$\therefore \frac{(\bar{A}g^{*\eta} + p\bar{X}) - g^{*} - \bar{Z} - h}{h}\varepsilon = \frac{e^{rT}-1}{rT}.$$

From $g^* = \overline{g^*}$ and the definition of $\bar{Z}$, $\bar{A}g^{*\eta} - g^* = \bar{Z}$. Therefore, we obtain

$$\frac{p\bar{X} - h}{h}\varepsilon = \frac{e^{rT}-1}{rT}. \tag{22}$$

The right-hand side is an increasing function of T, whereas T is an increasing function of h. Hence, h is an increasing function of τ. As shown in Figure 2, there is a single intersection of the left-hand side and the right-hand side of (22), which constitute the optimal value. QED

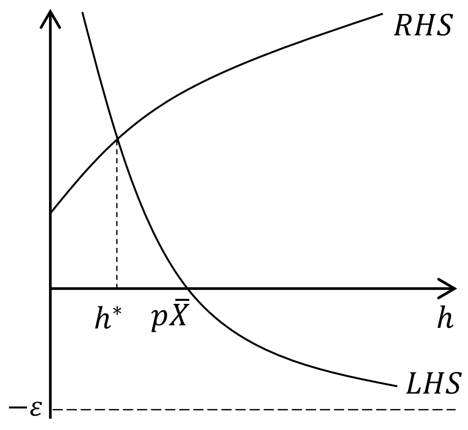

Figure 1. Determination of optimal level of extraction expenditures

Proposition 3. If $p\bar{X}$ and ε are higher, the optimal expenditure for extraction technology h is higher, and thus the duration of natural resource extraction is longer.

Increasing the values of $p\bar{X}$ and ε would shift the left-hand side of (22) upward shifting the intersection point h^* shown in Figure 1 rightward. Since T is an increasing function of h the duration of natural resource extraction would be longer as a result of better extraction efficiency. QED

Finally, we compare the optimal tax rate before the depletion of the natural resource and that after the depletion. Letting h^* the optimal expenditure on productivity of extraction, from (18), the optimal income tax rate is given by $\tau^* = \frac{g^*+h^*}{\bar{A}g^{*\eta}+p\bar{X}}$.

$$\tau^* - \bar{\tau}^* = \frac{g^* + h^*}{\bar{A}g^{*\eta} + p\bar{X}} - \bar{\tau}^* = \frac{g^* + h^* - \bar{\tau}^*(\bar{A}g^{*\eta} + p\bar{X})}{\bar{A}g^{*\eta} + p\bar{X}}.$$

From (13) and (17), $g^* = \bar{\tau}^*\bar{A}g^{*\eta}$. Substituting this into the equation, we obtain

$$\tau^* - \bar{\tau}^* = \frac{h^* - \bar{\tau}^* p\bar{X}}{\bar{A}g^{*\eta} + p\bar{X}}. \tag{23}$$

Thus, if $\frac{h^*}{p\bar{X}} > (<)\bar{\tau}^*$,then $\tau^* > (<)\bar{\tau}^*$.

5. Conclusion

In this research we considered a situation where government can support the labor productivity. In our model, government imposes income tax to invest into labor productivity and support the natural resource extraction. While the economy gains from the extraction of non-renewable natural resource along with the output produced inputting labor and capital, the optimal amount of expenditure for natural resource extraction depends on the relative share of resource earnings, the (in)efficiency of extraction technology as well as the parameters of non-resource output technology. The higher the world interest rate, the lower the value of optimal investment is.

Our key finding is that, when the extracted natural resource is not used as an input for another output in the economy, the optimization of extraction and productivity improvement in remaining aggregate output are separate problems for the government. That is, if revenue of natural resource is high, the optimal expenditure on the extraction technology is high, while the optimal expenditure on production technology is independent of the natural resource revenue.

The optimal tax rate is determined to provide the optimal level of expenditures while the natural resource is being extracted. However, after depletion of natural resource, the tax rate to support labor productivity equals to output elasticity of investment into labor productivity when the whole output is produced labor and capital. Comparison of tax rates before and after the depletion shows that if the ratio of the optimal extraction expenditure to resource revenue is higher (lower) than the optimal tax rate after the depletion of the natural resource, the optimal tax rate before the depletion is higher (lower) than that after the depletion.

An empirical study might test our key theoretical finding as the possible cause of the “resource curse” from the context of growth theory.

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