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Optimal Fiscal Spending and Deviation Rules under Political Uncertainty*

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Abstract

This paper characterizes optimal fiscal rules within a model integrating fiscal rule deviations in a two-period political turnover framework. The incumbent party aims to secure favored spending through increased debt issuance due to potential power loss. The study introduces spending and deviation rules, requiring legislative approval for deviations from the spending rule. Analysis shows the optimal deviation rule, favoring flexible responses to stringent spending rules. Furthermore, larger initial debt balances warrant tighter spending rules, while the optimal deviation rule remains unaffected. Additionally, political conflict influences deviation rule permissiveness, aligning more with the incumbent party's preferences as conflicts escalate.

Key words: Fiscal rules, Government debt, Political turnover.

JEL Classification: D72, D78, H62, H63

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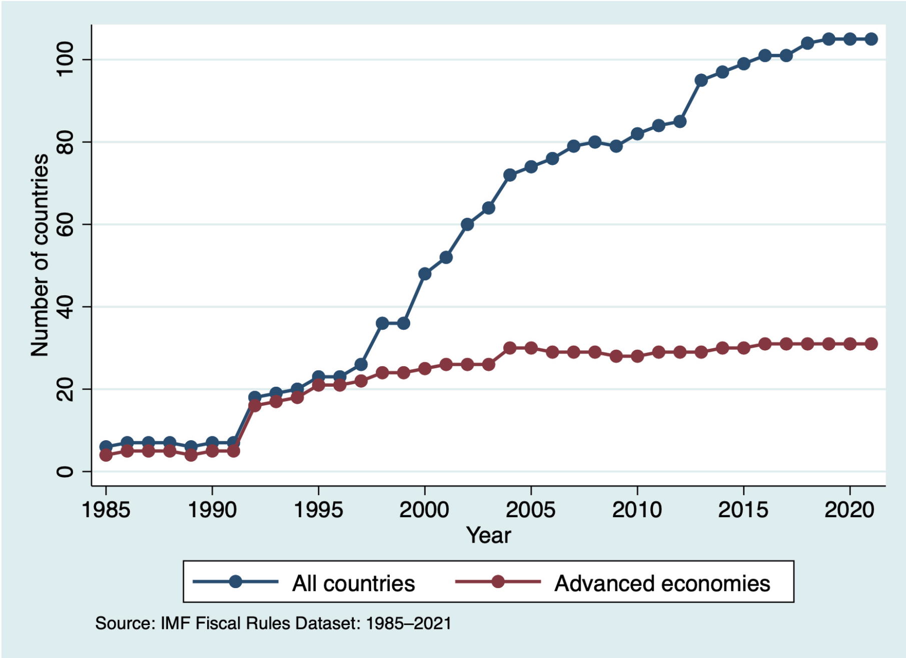

Figure 1: Trends in the number of countries adopting fiscal rules.

1 Introduction

Since the onset of the 21st century, addressing excessive budget deficits and the continuous rise in government debt has emerged as a common policy challenge among advanced countries. To tackle this challenge, an increasing number of countries have implemented fiscal rules (see Figure 1). Nevertheless, empirical evidence indicates that the adoption of fiscal rules has not effectively restrained excessive budget deficits and the accumulation of government debt. One contributing factor to this phenomenon is the lax enforcement of fiscal rules. Legal frameworks and constitutional provisions establishing fiscal rules often incorporate 'deviation clauses,' permitting departures from these rules under specific conditions (Eyraud, et al., 2018).

Figure 2 illustrates the prevalence of deviation clauses in fiscal rules across countries. The data indicates that such clauses have primarily been incorporated into the fiscal frameworks of advanced countries since the mid-2000s, with subsequent adoption observed in developing countries. Utilizing international panel data, Davoodi et al. (2022) estimate that the likelihood of compliance with fiscal rules among countries adopting them between 2004 and 2021 is approximately 50 percent.[1] This finding underscores the notable impact of deviation clauses on the implementation of fiscal rules in recent years.

The deviation clause presents both advantages and disadvantages. It facilitates flexible fiscal adjustments during significant external shocks, such as the 2008 financial crisis or the 2020 COVID-19 crisis, enabling timely responses to economic fluctuations. Conversely, poorly crafted deviation clauses can render fiscal rules ineffective, undermining the ability to adequately

[1] Deviations from BBRs and DRs have been common across countries, even before the pandemic. On average, countries exceeded the deficit and debt limits about 50 and 42 percent of the time during 2004-21, respectively (Daboodi et al., 2022).

(a) (b)

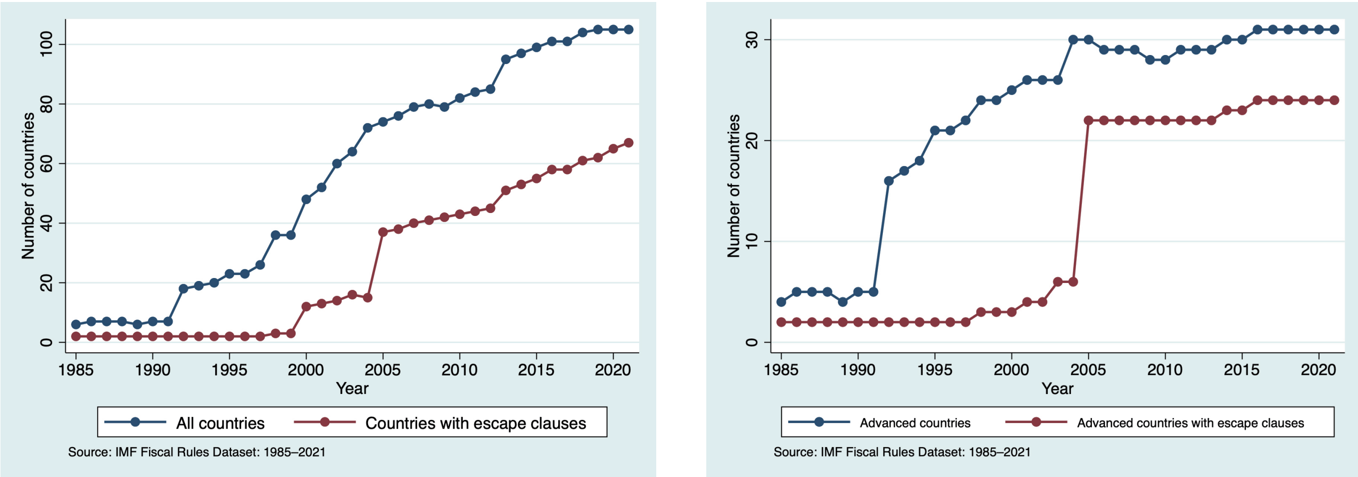

Figure 2: Trends in the number of countries adopting the deviation clauses. Panel (a) shows the trends for all countries and panel (b) shows the trends for advanced countries.

manage excessive budget deficits. Hence, the design of deviation clauses plays a crucial role in shaping the effectiveness of fiscal rules.

The objective of this paper is to theoretically characterize the design of optimal deviation clauses, or optimal deviation rules. To achieve this aim, the paper constructs a model that incorporates a deviation from the fiscal rule (Piguillem and Riboni, 2021) within a two-period framework of political turnover, drawing upon the frameworks proposed by Persson and Svensson (1989) and Alesina and Tabellini (1990). The model considers two distinct categories of public expenditures and accounts for two types of voters with differing preferences for public spending, each represented by a political party. It is assumed that voters' preferences for public spending evolve in response to changes in the social and economic context. Consequently, each party faces the risk of alternation in power due to stochastic shifts in the voter composition

The incumbent party, confronted with the prospect of political turnover, is motivated to augment its current preferred public spending through increased issuance of public debt. This strategy stems from the concern that its favored public spending might face reductions should it lose power in the future. Consequently, political disputes over public spending and the potential for changes in power lead to excessive budget deficits. To address these deficits, this study supplements the standard model of political turnover with two types of rules: a 'spending rule' (Piguillem and Riboni, 2021) that imposes a cap on public spending, and a 'deviation rule' that outlines conditions for deviating from the spending rule. Specifically, the study examines an environment where deviations from the fiscal rule are permissible if more than δ of the share in legislature members approve the deviation. This feature distinguishes the model employed in this study.

Within this framework, this study examines the optimal deviation rule from three perspectives. Firstly, we explore the magnitude of the optimal deviation rule δ under a given spending rule. The analysis reveals that the optimal δ for any spending rule always remains below 1. This indicates that a rigid spending rule requiring strict adherence is undesirable. Furthermore, we find a greater inclination towards a flexible deviation rule as the spending rule becomes more stringent. This suggests that in cases where fiscal spending is tightly restricted by spending rules, it is preferable to design deviation rules loosely to allow for flexible responses to changes in economic environments.

Secondly, we clarify the relationship between the fiscal situation and the optimal deviation rule. Specifically, we analyze the characteristics of the optimal deviation rule given the initial debt balance. Our findings indicate that as the initial debt balance increases, a stricter spending rule becomes optimal, while the optimal deviation rule remains unaffected by the initial debt balance. The mechanism driving this result is as follows: Debt levels influence social welfare solely through public goods provision. However, the deviation rule does not impact public goods provision because the incumbent party determines it after knowing whether the deviation rule can be applied. Hence, the deviation rule cannot control public goods provision based on the initial debt balance. Consequently, it is suggested that the deviation rule should maintain consistency over the long term following the economic structure, rather than being altered in response to changes in the fiscal situation.

Thirdly, we clarify the relationship between the magnitude of political conflict among voters (between parties) and the optimal deviation rule. Specifically, we investigate how differences in preferences for public goods spending among voters (between parties) impact the optimal deviation rule. The analysis demonstrates that as political conflict among voters (between parties) increases, the optimal deviation rule becomes more permissive. Given the greater number of supporters of the incumbent party compared to the opposition party, the optimal deviation rule aligns more with the preferences of the incumbent party as political conflict evolves. Consequently, a larger disparity in preferences leads to a preference for more permissive deviation and spending rules.

The remainder of this article is as follows. Section 2 reviews the relevant literature. Section 3 presents the model. Section 4 examines the optimal structure of the fiscal rule, comprising a spending rule and a deviation rule. Section 5 conducts a comparative statics analysis to explore the effects of initial public debt levels and variations in preferences between parties on the optimal fiscal rule. Section 6 provides concluding remarks. The appendix provides the proofs of lemmas and propositions.

2 Relation to the Literature

The present study is closely related to Piguillem and Riboni (2021) who developed a strategic debt model where preference misalignment between current and future governments creates incentives for the incumbent to over-issue public debt (Persson and Sevenson, 1989; Alesina and Tabellini, 1990). They introduce the possibility for politicians to override fiscal rules if there is consensus between the incumbent and the opposition. Within this framework, they examine which fiscal rule is most effective in promoting inter-party compromise and reducing debt. Our study adopts their framework but formulates regulations for deviations from fiscal rules, rather than allowing their negotiation between parties. In this alternative framework, we characterize the optimal deviation rule from the perspective of maximizing social welfare. Our findings provide a basis for evaluating deviation rules, currently being implemented in numerous countries, from a social welfare standpoint.

Apart from the study mentioned above, the present study is also related to the literature of the political economy of fiscal rule violation (Coate and Milton, 2019; Dovis and Kirpalani, 2020; Halac and Yared, 2022; Arawatari and Ono, 2021, 2022). In Coate and Milton's (2019) analysis, two pivotal roles emerge: the constitutional designer, representing citizens, and the politician, inclined towards higher taxes. The designer establishes a fiscal limit to curb the politician's bias, yet the politician can override it with citizen approval. The study focuses on determining the optimal fiscal limit and examining its sensitivity to potential overrides. The static analytical framework, however, leads to the exclusion of expenditure financing through bond issues and the resulting budget deficits. Consequently, fiscal rules controlling deficits are beyond the study's scope.

Dovis and Kirpalani (2020) introduce instances when fiscal rules were established but lacked ex-post enforcement, as evidenced in the Stability and Growth Pact within the European Union. They present a model populated by local and central governments to elucidate these occurrences. Within their framework, local governments may violate fiscal rules and engage in excessive borrowing, but their study does not address the determination of optimal fiscal rules for such cases.

Halac and Yared (2022) examine a government displaying present bias toward public spending while possessing private information about shocks impacting the value of such spending. In this context, society selects a fiscal rule, aiming to balance the advantages of committing the government to avoid overspending against the advantages of allowing flexibility to respond to shocks. Within this framework, the authors demonstrate that violation occurs under sufficiently high shocks only when the penalties in place are weak, and such shocks are relatively unlikely. However, the study does not delve into the design of optimal fiscal rules in the presence of the possibility of fiscal rule violation.

Arawatari and Ono (2021, 2022) also examine the behavior of a present-biased government as in Halac and Yared (2022). They utilize the model developed by Bisin et al. (2015), which incorporates time-inconsistent voters. In the framework of Bisin et al. (2015), voters with present bias are inclined to increase present consumption by raising public debt issuance. However, the impact of such present-biased behavior can be reduced through the imposition of a debt ceiling. Arawatari and Ono (2021, 2022) extend the analysis by allowing for the override of the debt ceiling. They illustrate instances of such overrides (Arawatari and Ono, 2021) and present the optimal debt rule when faced with the possibility of override (Arawatari and Ono, 2022). However, they do not address the design of the deviation rule. The present study instead aims to demonstrate the optimal deviation rule in the political economy framework, a facet not fully covered in previous studies.

3 The Model

The model in this paper is based on a standard two-period framework that incorporates political turnover (Persson and Svensson, 1989; Alesina and Tabellini, 1990). The economic setting involves two distinct public spending variables, denoted as g^A and g^B. The associated two categories of voters, defined by A and B, are characterized by disparate preferences towards public spending. In particular, the preferences of each type of voter in each period are specified as follows:

$$u_A(g_A, g_B) = \frac{(g_A)^{1-\sigma}}{1-\sigma} + \theta \cdot \frac{(g_B)^{1-\sigma}}{1-\sigma}, \tag{1}$$

$$u_B(g_A, g_B) = \frac{(g_B)^{1-\sigma}}{1-\sigma} + \theta \cdot \frac{(g_A)^{1-\sigma}}{1-\sigma}. \tag{2}$$

where $\sigma \in (0,1)$ and $\theta \in [0,1)$ are parameters shaping preferences of each type of voters. The functions $u_A(g^A, g^B)$ and $u_B(g^A, g^B)$ represent the preferences of voters favoring public spending g^A and g^B, respectively. A smaller value of θ intensifies the political conflict surrounding public spending.

The preferences of each type of voter are stochastically determined in each period. This stochastic determination can be interpreted as a form of preference shock, reflecting changes in voters' preferences for public spending in response to shifts in the social and economic environment. Considering the existence of two political parties, A and B, representing voters of type A and B respectively, and given their symmetric preferences, we denote the current preferences of the incumbent and opposition as follows:

$$u_I(g_I, g_O) = \frac{(g_I)^{1-\sigma}}{1-\sigma} + \theta \cdot \frac{(g_O)^{1-\sigma}}{1-\sigma}, \tag{3}$$

$$u_O(g_I, g_O) = \frac{(g_O)^{1-\sigma}}{1-\sigma} + \theta \cdot \frac{(g_I)^{1-\sigma}}{1-\sigma}, \tag{4}$$

where the indices I and O signify incumbent and opposition, respectively.

Assuming that public spending denoted as $\{g_I, g_O\}$ is funded through tax revenues and public debt issuances, the government's budget constraint for each period is expressed as follows:

$$\tau + b' \geq (1+r)b + g_I + g_O. \tag{5}$$

Here, $\tau > 0$ represents exogenous tax revenue in each period, b, represents the outstanding public debt at the beginning of a period, b' denotes the amount of public debt issued during this period, and r is the interest rate on public debt. It is assumed that the nation functions as a small open economy, engaging in borrowing within foreign asset markets at the given interest rate r.

The uncertainty of political turnover arises due to the stochastic variation in the number of voters for each type. With $\theta < 1$, the incumbent party is incentivized to augment its current preferred public spending g_I by increasing public debt issuance b', considering the risk that its preferred public spending may diminish upon transitioning into the opposition in the future. Essentially, political conflict over public spending and the prospect of political turnover contribute to the emergence of excessive budget deficits.

Let p denote the number of seats obtained by the incumbent from the previous period through elections in the current period. The variable p is assumed to follow a uniform distribution within the interval $[0, 1]$. The party securing a majority of seats holds power. Under these assumptions, there exists a 1/2 probability of political turnover in each period.

3.1 Spending Rule and Deviation Rule

We consider two types of fiscal rules: the spending rule and the deviation rule. We follow Piguillem and Riboni (2021) and specify the spending rule as follows:

$$g_I + g_O \leq \alpha \cdot (\tau - rb), \quad \alpha \geq 0. \tag{6}$$

where $\alpha \geq 0$ serves as a parameter showing the stringency of the spending rule. The spending rule in (6) dictates that the party in office is restricted to allocate only a certain percentage of tax revenue, net of interest payments on debt. A smaller value of α corresponds to a more rigorous spending rule. Specifically, when $\alpha = 0$, the scenario adheres to a shutdown rule, entailing no discretionary spending ($g_I + g_O = 0$). Conversely, for $\alpha = 1$, adherence to a balanced budget rule is observed, resulting in no alteration to the public debt balance ($g_I + g_O + rb = \tau$). When $\alpha = \tau/(\tau - rb)$, a primary balance equilibrium rule is established, ensuring that public spending is entirely covered by tax revenues ($g_I + g_O = \tau$).

The pivotal distinction inherent in our model, setting it apart from Piguillem and Riboni (2021), lies in the explicit consideration of the following deviation rule. We assume that if a political party wins an election and obtains the proportion of seats above $\delta \in [1/2, 1]$, it is entitled to implement fiscal policies without being constrained by the spending rule. The parameter δ is an exogenously assigned institutional parameter defining the deviation rule. The deviation rule like this one is currently employed in various countries including Japan, Germany, and Switzerland.

Building upon the work of Piguillem and Riboni (2021), we designate the scenario wherein the incumbent secures more than δ seats as the "dictator state," while the scenario with fewer than δ seats is termed the "rule state." Figure 3 illustrates the relationship between the number of seats p held by party A that was in office in the previous period and the applicable spending rule for the party in office in the current period. If party A continues to retain the office in the current period, adherence to the spending rule is required when the probabilistically determined number of seats p falls within $1/2 \leq p < \delta$ (rule state). Conversely, if $\delta \leq p$, party A may invoke the deviation rule, bypassing the spending rule (dictator state). The equilibrium fiscal policy in the dictator state is denoted as $\{g_I^{d*}, g_O^{d*}, b'^{d^*}_d\}$, while the equilibrium fiscal policy in the rule state is represented as $\{g_I^{r*}, g_O^{r*}, b'^{r^*}\}$.

For the scenario where $\delta = 1/2$, the party in office wins the seats $p \geq 1/2$, so it consistently operates without fiscal rules, akin to the standard regime change model devoid of fiscal

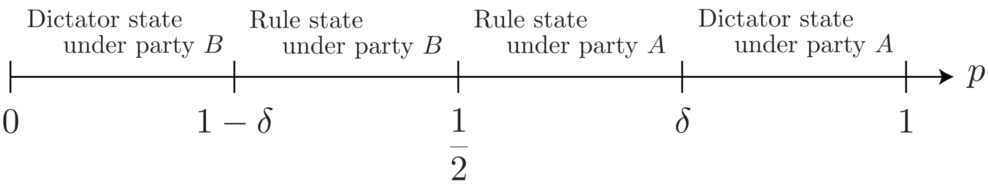

Figure 3: Relationship between the random variable p and the incumbent party in the current period if the incumbent party in the previous period was party A.

constraints. Conversely, when $\delta = 1$, it is obligated to consistently adhere to the fiscal rule, aligning with the regime change model commonly employed in prior studies on fiscal rules. By specifically examining cases where $1/2 < \delta < 1$, we scrutinize the impact of the deviation rule on equilibrium fiscal policy and explore the optimal interplay between the spending rule and the deviation rule.

3.2 Timing of Events

The policy decisions within a two-period economy start as follows. In the first period, the randomly assigned variable $p \in [0, 1]$ determines how seats are divided between the political parties A and B. The party that gets more than half of the seats runs for office, with the authority to shape fiscal policies, including public goods provision and public debt issuance. In the rule state where p falls within the range $[1/2, \delta)$, the party in office creates and enacts fiscal policies following the spending rule. However, in the dictator state where p falls within the range $[\delta, 1]$, the party in the office can decide on fiscal policies without adhering to the spending rule.

Moving to the second period, a new random assignment of $p \in [0, 1]$ determines if the incumbent party from the previous period still stays in office. Recall that we have assumed a uniform distribution for the number of seats p. The incumbent party from the first period faces the uncertainty of possibly becoming the opposition in the second period. As a result, the incumbent party in the first period decides fiscal policies, taking this possibility into account. We address the second-period problem before delving into the intricacies of the first-period problem.

3.3 Period-2 Incumbent Party's Problem

In period 2, the incumbent party has no opportunity to issue additional debt since the economy ends at the end of period 2. The incumbent party allocates tax revenues to spending $\{g_{I,2}, g_{O,2}\}$ and the debt repayment, $(1+r)b_1$. This renders the spending rule irrelevant, eliminating the need to categorize the problem based on whether it is the dictator or rule state. Consequently, the problem faced by the period-2 incumbent party is as follows:

$$V_{I,2}(b_1) = \max_{\{g_{I,2}, g_{O,2}\}} \{u_I(g_{I,2}, g_{O,2})\}, \tag{7}$$

$$\text{s.t. } \tau \geq (1+r)b_1 + g_{I,2} + g_{O,2}, \tag{8}$$

where $V_{I,2}(b_1)$ is the value function of the incumbent party in period 2.

Let $\{g^*_{I,2}, g^*_{O,2}\}$ denote the supply of public goods chosen by the incumbent party in period 2. The supply of $\{g^*_{I,2}, g^*_{O,2}\}$ is given by

$$g^*_{I,2}(b_1) = \frac{\tau - (1+r)b_1}{1+\theta^{\frac{1}{\sigma}}}, \quad g^*_{O,2}(b_1) = \theta^{\frac{1}{\sigma}} \cdot \frac{\tau - (1+r)b_1}{1+\theta^{\frac{1}{\sigma}}}. \tag{9}$$

From (9), it is established that $g^*_{O,2}(b_1)/g^*_{I,2}(b_1) = \theta^{\frac{1}{\sigma}} < 1$, indicating that as θ decreases, reflecting heightened political conflict, the opposition party tends to prefer a relatively smaller supply of their preferred public goods $g_{O,2}$. The result also suggests that a larger public debt outstanding in period 2, b_1, leads to a smaller supply of public goods. This finding implies that the incumbent party in period 1 can influence the fiscal policy of the party in office in period 2 through the public debt balance b_1 left to the party in office in period 2.

Substituting (9) into (7), we obtain the value function of the incumbent party in period 2 as follows:

$$V_{I,2}(b_1) = u_I\left(g^*_{I,2}(b_1), g^*_{O,2}(b_1)\right) = \frac{1+\theta^{\frac{1}{\sigma}}}{1-\sigma} \cdot \left(\frac{\tau - (1+r)b_1}{1+\theta^{\frac{1}{\sigma}}}\right)^{1-\sigma}. \tag{10}$$

The value function of the opposition party in period 2 is:

$$V_{O,2}(b_1) = u_O\left(g^*_{I,2}(b_1), g^*_{O,2}(b_1)\right) = \frac{\theta+\theta^{\frac{1-\sigma}{\sigma}}}{1-\sigma} \cdot \left(\frac{\tau - (1+r)b_1}{1+\theta^{\frac{1}{\sigma}}}\right)^{1-\sigma}. \tag{11}$$

By comparing the terms $1+\theta^{1/\sigma}$ in (10) and $\theta+\theta^{(1-\sigma)/\sigma}$ in (11), we find that

$$1+\theta^{\frac{1}{\sigma}} - \left\{\theta+\theta^{\frac{1-\sigma}{\sigma}}\right\} = (1-\theta)\left(1-\theta^{\frac{1-\sigma}{\sigma}}\right) > 0. \tag{12}$$

Derived from the condition in (12), $V_{I,2}(b_1) > V_{O,2}(b_1)$ holds. This implies that the value of the incumbent party is higher than that of the opposition party, due to political conflict. This is a risk of political turnover for the incumbent party in period 1.

3.4 Period-1 Incumbent Party's Problem

Consider the period-1 incumbent party's problem. The public debt from the outset of period 1, denoted as b_0, is a predetermined variable and thus an initial condition for the incumbent party. For this value of b_0, the following assumption is made.

Assumption 1 $b_0 < \dfrac{2+r}{(1+r)^2} \cdot \tau \equiv b^{NDL}$.

Notice that b^{NDL} denotes the natural debt ceiling. The problem confronting the incumbent party in period 1 hinges on whether its number of seats exceeds δ (the dictator state) or falls below δ (the rule state). We investigate the decision-making process of the period-1 incumbent party for both the dictator and rule state scenarios.

3.4.1 Dictator State Case

Consider the situation where the incumbent party's number of seats in period 1 lies within the range $[\delta, 1]$ and so the dictator state is realized. Under this circumstance, the incumbent party possesses the flexibility to implement fiscal policies without being constrained by the spending rule. Consequently, the problem faced by the period-1 incumbent party can be defined as follows.

$$V_{I,1}^{d}(b_0) = \max_{\{g_{I,1}, g_{O,1}, b_1\}} \left\{ u_I(g_{I,1}, g_{O,1}) + \beta W_2(b_1) \right\}, \tag{13}$$

$$\text{s.t. } \tau + b_1 \geq (1+r)b_0 + g_{I,1} + g_{O,1}, \tag{14}$$

where $\beta \in (0, 1)$ is the discount factor and $W_2(b_1)$ is the expected value function of the next period.

The allocation of seats for each party in the subsequent period is determined irrespective of its current seat count. Under the assumption of the uniform distribution of p, the probabilities of being in and out of office in period 2 are equal, standing at $1/2$. Consequently, the expected value function for the next period, denoted as $W_2(b_1)$, remains identical for both the incumbent and opposition parties and can be articulated as follows.

$$\begin{aligned} W_2(b_1) &= \frac{1}{2} \cdot V_{I,2}(b_1) + \frac{1}{2} \cdot V_{O,2}(b_1) \\ &= \phi \cdot \frac{1}{1-\sigma} \cdot \left(\frac{\tau - (1+r)b_1}{1 + \theta^{\frac{1}{\sigma}}} \right)^{1-\sigma}, \end{aligned} \tag{15}$$

where ϕ is defined as follows:

$$\phi \equiv \frac{1 + \theta^{\frac{1}{\sigma}} + \theta + \theta^{\frac{1-\sigma}{\sigma}}}{2}. \tag{16}$$

To clarify the relationship between spending and deviation rules and fiscal policy, we introduce the following assumption.

Assumption 2 $\beta(1 + r) = 1$.

This assumption abstracts from the incumbent party's incentive to save and borrow, considering intertemporal optimization. Put differently, the motivation to issue public debt arises solely from the risk of power alternation. This assumption is also a prerequisite for steady-state stability in the infinite-horizon small open economy model.

In the dictator state, the set of fiscal policies selected by the incumbent party is represented as $\{b_1^{d*}(b_0), g_{I,1}^{d*}(b_0), g_{O,1}^{d*}(b_0)\}$. The policies are expressed as

$$b_1^{d*}(b_0) = \frac{\tau - \left(\frac{\phi}{1+\theta^{\frac{1}{\sigma}}} \right)^{\frac{1}{\sigma}} \cdot [\tau - (1+r)b_0]}{(1+r) + \left(\phi / (1 + \theta^{\frac{1}{\sigma}}) \right)^{\frac{1}{\sigma}}}, \tag{17}$$

$$g_{I,1}^{d*}(b_0) = \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right]}{(1+\theta^{\frac{1}{\sigma}})\left[(1+r) + \left(\phi/(1+\theta^{\frac{1}{\sigma}})\right)^{\frac{1}{\sigma}}\right]}, \tag{18}$$

$$g_{O,1}^{d*}(b_0) = \theta^{\frac{1}{\sigma}} \cdot \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right]}{(1+\theta^{\frac{1}{\sigma}})\left[(1+r) + \left(\phi/(1+\theta^{\frac{1}{\sigma}})\right)^{\frac{1}{\sigma}}\right]}, \tag{19}$$

where $\tau - rb_0 > 0$ holds under Assumption 1.

From (18) and (19), the following lemma holds.

Lemma 1 *The greater the conflict between the parties (with a smaller θ), the larger the government expenditure $\left(g_{I,1}^{d*}(b_0) + g_{O,1}^{d*}(b_0)\right)$ in the dictator state.*

(Proof) See Appendix A.

The result presented in Lemma 1 implies that the risk of political turnover and conflict between political parties result in excessive fiscal spending, a finding qualitatively akin to Alesina and Tabellini (1990: Proposition 5). The mechanism behind this result is as follows: in period 1, the incumbent party faces the risk of political turnover, potentially losing its position in period 2. Since $g_{O,2}^{*}(b_1)/g_{I,2}^{*}(b_1) = \theta^{\frac{1}{\sigma}} < 1$ as indicated by (9), if the party becomes the opposition in period 2, it opts for a lower level of preferred public goods provision. Consequently, the incumbent party in period 1 is incentivized to allocate more resources to its preferred public good while in power today. The lower the value of θ and the higher the conflict between parties, the more pronounced these incentives become, leading to heightened public spending and public debt issuance.

When no spending rule is imposed, the provision of public goods is determined by (18) and (19). With the introduction of the spending rule α, the condition under which the incumbent party is motivated to deviate from the spending rule can be expressed as follows.

$$g_{I,1}^{d*}(b_0) + g_{O,1}^{d*}(b_0) > \alpha \cdot (\tau - rb_0) \;\Leftrightarrow\; \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right]}{(\tau - rb_0)\left[(1+r) + \left(\phi/(1+\theta^{\frac{1}{\sigma}})\right)^{\frac{1}{\sigma}}\right]} > \alpha, \tag{20}$$

When (20) holds, the spending rule (6) is binding. Under this circumstance, the incumbent party in period 1 finds an incentive to deviate from the spending rule. Put differently, in the rule state, the incumbent party must determine its fiscal policy within the confines of both the spending rule and the deviation rule. We introduce the following assumption for this case.

Assumption 3 $\alpha < \dfrac{\tau + (1+r)\left[\tau - (1+r)b_0\right]}{(\tau - rb_0)\left[(1+r) + \left(\phi/(1+\theta^{\frac{1}{\sigma}})\right)^{\frac{1}{\sigma}}\right]} \equiv \bar{\alpha}(b_0),$

$\bar{\alpha}(b_0)$ is a decreasing function of both b_0 and θ. The greater the initial public debt balance (b_0) or the lesser the conflict between parties (resulting in a larger θ), the weaker the incentive for the period-1 incumbent party to provide excessive public goods. Consequently, the spending rule does not bind unless it is stronger. In addition, as will be demonstrated later, the optimal spending rule that maximizes social welfare consistently adheres to the condition $\alpha^* < \bar{\alpha}(b_0)$

Derived from the definitions of (13), (15), (17), and $\bar{\alpha}(b_0)$, the value functions for both the incumbent and opposition parties in the dictator state are as follows.

$$V_{I,1}^d(b_0) = \frac{1}{1-\sigma} \cdot \left[1 + \theta^{\frac{1}{\sigma}} + \beta(1+\theta^{\frac{1}{\sigma}}) \left(\frac{\phi}{1+\theta^{\frac{1}{\sigma}}}\right)^{\frac{1}{\sigma}}\right] \cdot \left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{1-\sigma}, \tag{21}$$

$$V_{O,1}^d(b_0) = \frac{1}{1-\sigma} \cdot \left[\theta + \theta^{\frac{1-\sigma}{\sigma}} + \beta(1+\theta^{\frac{1}{\sigma}}) \left(\frac{\phi}{1+\theta^{\frac{1}{\sigma}}}\right)^{\frac{1}{\sigma}}\right] \cdot \left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{1-\sigma}, \tag{22}$$

where $V_{I,1}^d(b_0) > V_{O,1}^d(b_0)$ holds from (12).

3.4.2 Rule State Case

In the scenario where the incumbent party possesses fewer seats than δ in the initial period, the rule state is established. The incumbent party is obligated to formulate fiscal policy in adherence to the spending rule. This results in framing the incumbent party's fiscal problem as follows:

$$V_{I,1}^b(b_0) = \max_{\{g_{I,1}, g_{O,1}, b_1\}} \left\{u_I(g_{I,1}, g_{O,1}) + \beta W_2(b_1)\right\}, \tag{23}$$

$$\text{s.t. } \tau + b_1 \geq (1+r)b_0 + g_{I,1} + g_{O,1}, \tag{24}$$

$$g_{I,1} + g_{O,1} \leq \alpha(\tau - rb_0). \tag{25}$$

The problem faced by the incumbent party in the rule state differs from that in the dictator state in that it involves adhering to the spending rule of the form (25). Furthermore, under Assumption 3, the spending rule holds with equality. This requires that (25) holds with an equality sign in the analysis of the ruling state.

Let $\{b_1^{r*}(b_0,\alpha), g_{I,1}^{r*}(b_0,\alpha), g_{O,1}^{r*}(b_0,\alpha)\}$ denote the fiscal policy chosen by the incumbent party in the rule state. Through the utilization of $g_{O,1}^{r*} = \theta^{\frac{1}{\sigma}} g_{I,1}^{r*}$ derived from the first-order condition, we ascertain that the pair of public spending $\{g_{I,1}^{r*}(b_0,\alpha), g_{O,1}^{r*}(b_0\alpha)\}$ satisfies the following equation:

$$g_{I,1}^{r*} + g_{O,1}^{r*} = (1+\theta^{\frac{1}{\sigma}}) g_{I,1}^{r*} = \alpha(\tau - rb_0). \tag{26}$$

Utilizing the condition in (26) met by the pair of public spending and the budget constraint formula (24), we derive the fiscal policy in the rule state:

$$b_1^{r*}(b_0,\alpha) = b_0 + (\alpha - 1)(\tau - rb_0), \tag{27}$$

$$g_{I,1}^{r*}(b_0,\alpha) = \frac{\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}, \tag{28}$$

$$g_{O,1}^{r*}(b_0,\alpha) = \theta^{\frac{1}{\sigma}} \cdot \frac{\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}. \tag{29}$$

Expressions in (27), (28), and (29) indicate that the spending rule influences fiscal policy in period-1 of the rule state. Specifically, a stronger spending rule (smaller α) leads to a more effective control over excessive budget deficits. In essence, the spending rule serves to restrain unwarranted fiscal expenditures arising from the risk of political turnover and conflict, as highlighted in Lemma 1.

Substitution of the required fiscal policy, as indicated by (27), (28), and (29), to the incumbent party's objective function (13) and the expected value of the next period (15), leads to the following value functions of the incumbent and opposition parties under the rule state:

$$V^r_{I,1}(b_0,\alpha) = \frac{1}{1-\sigma}\cdot\Bigg\{\left(1+\theta^{\frac{1}{\sigma}}\right)\cdot\left[\frac{\alpha(\tau-rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{1-\sigma} +\beta\phi\cdot\left\{\frac{\tau+(1+r)\left[\tau-(1+r)b_0\right]-(1+r)\alpha(\tau-rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right\}^{1-\sigma}\Bigg\}, \tag{30}$$

$$V^r_{O,1}(b_0,\alpha) = \frac{1}{1-\sigma}\cdot\Bigg\{\left(\theta+\theta^{\frac{1-\sigma}{\sigma}}\right)\cdot\left[\frac{\alpha(\tau-rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{1-\sigma} +\beta\phi\cdot\left\{\frac{\tau+(1+r)\left[\tau-(1+r)b_0\right]-(1+r)\alpha(\tau-rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right\}^{1-\sigma}\Bigg\}. \tag{31}$$

Equations (30) and (31) demonstrate that the spending rule, denoted as α, exerts dual effects on the value functions $V^r_{I,1}(b_0,\alpha)$ and $V^r_{O,1}(b_0,\alpha)$. The initial term in (30) and (31) functions as an increasing factor with respect to α; a more permissive spending rule, indicated by a larger α, results in an augmented provision of public goods $\{g^{r*}_{I,1}(b_0,\alpha), g^{r*}_{O,1}(b_0,\alpha)\}$ during the first period. Conversely, the subsequent term in (30) and (31) behaves as a decreasing function of α. A looser spending rule corresponds to an increased propensity of the first-period incumbent party to excessively provide public goods and issue public debt. This, in turn, diminishes the availability of public goods in the second period. Therefore, an increase in α, accompanied by a relaxation of the spending rule, yields both positive and negative impacts on the value associated with both the incumbent and opposition parties. The optimal spending rule, which will be investigated in the next section, represents the point at which these opposing effects are balanced.

In concluding this section, we pinpoint the spending rule that maximizes the value functions for both the incumbent and opposition parties. By examining (30) and (31), we can derive the conditions that determine the optimal rule for the incumbent and opposition parties, respectively, as outlined below.

$$\frac{\partial V^r_{I,1}(b_0,\alpha)}{\partial\alpha} \gtreqless 0 \;\Leftrightarrow\; \alpha \lesseqgtr \frac{\tau+(1+r)\left[\tau-(1+r)b_0\right]}{(\tau-rb_0)\left[(1+r)+\left(\phi/(1+\theta^{\frac{1}{\sigma}})\right)^{\frac{1}{\sigma}}\right]} \equiv \bar{\alpha}(b_0), \tag{32}$$

$$\frac{\partial V^r_{O,1}(b_0,\alpha)}{\partial\alpha} \gtreqless 0 \;\Leftrightarrow\; \alpha \lesseqgtr \frac{\tau+(1+r)\left[\tau-(1+r)b_0\right]}{(\tau-rb_0)\left[(1+r)+\left(\phi/\left(\theta+\theta^{\frac{1-\sigma}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]} < \bar{\alpha}(b_0). \tag{33}$$

Expression in (32) suggests that under Assumption 3, the incumbent party favors a relaxed spending rule denoted by ($\alpha = \bar{\alpha}(b_0)$). Expression in (33) indicates that the spending rule maximizing the value for the opposition party is determined as an interior solution. In the next section, we explore the derivation of the optimal spending rule within the context of social welfare, in accordance with the identified preferences of the opposition party.

4 Optimal Fiscal Rules

Up to this point, we have examined the spending and deviation rules, α and δ, as predetermined. In this section, we derive optimal spending and deviation rules aimed at maximizing social welfare, formed by aggregating voters' utilities, and scrutinize their characteristics.

4.1 Social Welfare Function

Without loss of generality, let p denote the number of seats for party A in period 1. As illustrated in Figure 3, party A attains the position of the incumbent party when $p \geq \frac{1}{2}$, while party B takes on this role when $p < \frac{1}{2}$. If $p > \delta$ or $p < 1 - \delta$, the state is categorized as the dictator state; otherwise, it is classified as the rule state.

The variable p serves a dual purpose, representing both the number of seats for party A and the number of voters of type A. The magnitude of p directly correlates with the number of voters for type A and inversely correlates with the number for type B. In the context of assuming a Benthamite social welfare function, it becomes pertinent to weigh the value of each political party by the number of seats. Consequently, we introduce the social welfare function $W(b_0; \delta, \alpha)$, defined as follows.

$$
\begin{aligned}
W(b_0, \delta, \alpha) =& \mathbb{E}\left[pV_{A,1} + (1-p)V_{B,1}\right] \\
=& \underbrace{\int_{\delta}^{1} \cdot \left[pV_{I,1}^{d}(b_0) + (1-p)V_{O,1}^{d}(b_0)\right] \mathrm{d}p}_{\text{dictator state under party } A} + \underbrace{\int_{1/2}^{\delta} \cdot \left[pV_{I,1}^{r}(b_0,\alpha) + (1-p)V_{O,1}^{r}(b_0,\alpha)\right] \mathrm{d}p}_{\text{rule state under party } A} \\
&+ \underbrace{\int_{1-\delta}^{1/2} \cdot \left[pV_{O,1}^{r}(b_0;\alpha) + (1-p)V_{I,1}^{r}(b_0;\alpha)\right] \mathrm{d}p}_{\text{rule state under party } B} + \underbrace{\int_{0}^{1-\delta} \cdot \left[pV_{O,1}^{d}(b_0) + (1-p)V_{I,1}^{d}(b_0)\right] \mathrm{d}p}_{\text{dictator state under party } B} \\
=& (1-\delta^2)V_{I,1}^{d}(b_0) + (1-\delta)^2 V_{O,1}^{d}(b_0) + \frac{4\delta^2-1}{4} \cdot V_{I,1}^{r}(b_0,\alpha) - \frac{4(1-\delta)^2-1}{4} \cdot V_{O,1}^{r}(b_0,\alpha),
\end{aligned} \tag{34}
$$

where p is uniformly distributed in the $[0, 1]$ interval.

In the following analysis, we initially determine the optimal expenditure rule α corresponding to a given deviation rule δ (in Subsection 4.2). Next, we identify the optimal δ for a given α (in Subsection 4.3). Finally, we investigate the optimal combination of α and δ (in Subsection 4.4).

4.2 Optimal Spending Rule

Consider the optimal spending rule $\alpha^*(b_0, \delta)$ with a given deviation rule δ. If $\delta = 1/2$, the incumbent party acts like a dictator regardless of seat count, rendering the spending rule entirely meaningless. Consequently, our analysis concentrates on cases where $\delta > 1/2$ and derives the following lemma.

Lemma 2 *Suppose a deviation rule $\delta \in (1/2, 1]$ is provided. The optimal spending rule $\alpha^*(b_0, \delta)$ is then determined by*

$$\alpha^*(b_0, \delta) = \frac{\tau + (1+r) \cdot [\tau - (1+r)b_0]}{\tau - rb_0} \cdot \frac{\Gamma(\delta)}{1 + (1+r)\Gamma(\delta)} \in (0, \bar{\alpha}(b_0)) \ \ \forall \delta \in (1/2, 1], \tag{35}$$

where $\Gamma(\delta)$ *is defined by*

$$\Gamma(\delta) \equiv \left[1 + \frac{(2\delta - 1)\left(1 + \theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right)}{2\left(1 + \theta^{\frac{1}{\sigma}} + \theta + \theta^{\frac{1-\sigma}{\sigma}}\right)} \right]^{\frac{1}{\sigma}} > 1. \tag{36}$$

(Proof) See Appendix B.

Lemma 2 indicates that for any deviation rule $\delta \in (1/2, 1]$, the optimal interior solution for the spending rule is determined as follows. As observed from (34), if the rule state is realized, the spending rule does not affect social welfare. The optimal spending rule is determined to maximize expected welfare when the rule state occurs. In such cases, the incumbent party prefers the loosest possible spending rule ($\alpha = \bar{\alpha}(b_0)$) (see (32) and (33)). Conversely, we observe that the opposition party prefers a specific interior solution for the spending rule. Through the analysis of both scenarios, it becomes evident that the spending rule $\alpha^*(b_0, \delta)$ that maximizes social welfare is obtained as an interior solution.

The implication of this result is as follows: the result of Lemma 1 suggests that the incumbent party has an incentive to overspend when there is a conflict between the parties on the provision of public goods. Stricter spending rules can mitigate such excessive spending and improve the welfare of the opposition's supporters. However, tighter spending rules worsen the welfare of supporters of the incumbent party. Therefore, the optimal spending rule is the one with an interior point that effectively balances these trade-offs.

The result presented in Lemma 2 reveals that the maximum allowable spending, denoted as $\alpha^*(b_0, \delta) \cdot (\tau - rb_0)$ under the optimal spending rule, is a decreasing function of b_0. This observation implies that it is optimal to impose stronger spending restrictions for larger initial public debt balances. Moreover, $\alpha^*(b_0, \delta)$ is an increasing function of δ, indicating that the tighter the deviation rule (demanding greater compliance with the spending rule), the optimal spending rule becomes more lenient. In essence, there exists a substitutability between the strictness of the deviation rule and the spending rule.

The rationale behind this substitutability is as follows. When the deviation rule is stringent, the incumbent party, with its substantial seats and supporters, bound by the spending rule, finds it easier to achieve its objectives. In such cases, many voters supporting the incumbent party are constrained by the spending rule. This implies that the welfare loss associated with enforcing a stringent spending rule becomes significant, making a more lenient spending rule optimal.

The implications of these findings for real spending rules warrant consideration. Taking Japan as an example, where deviation from the spending rule is facilitated by the requirement for a special debt law to be passed by both houses of the Diet. Nevertheless, the spending rule remains stringent, allowing no deficit government debt except for construction debt aimed at funding public investment, consistent with the relationship outlined in Lemma 2. As a result, the spending rule enforced in Japan may be viewed as suitable in terms of maximizing social welfare.

4.3 Optimal Deviation Rule

Next, given the spending rule α, we find the optimal deviation rule $\delta^*(b_0, \alpha)$.

Lemma 3 *For a given spending rule $\alpha \in [0, \bar{\alpha}(b_0))$, the optimal deviation rule $\delta^*(b_0, \alpha)$ is:*

$$\delta^*(b_0, \alpha) = \frac{V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0)}{[V^d_{I,1}(b_0) - V^r_{I,1}(b_0, \alpha)] + [V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0)]} < 1. \tag{37}$$

In addition, $\delta^*(b_0, \alpha) > 1/2$ holds if the following condition is satisfied:

$$V^d_{I,1}(b_0) - V^r_{I,1}(b_0, \alpha) < V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0). \tag{38}$$

(Proof) See Appendix C.

Lemma 3 shows that, for any spending rule $\alpha < \bar{\alpha}(b_0)$, under specific conditions, the optimal deviation rule is determined by the interior point. If δ is small and so the deviation rule is too loose, the dictator state is likely, rendering the spending rule meaningless. This results in overspending by the incumbent party and welfare losses for the opposition. Strengthening the enforceability of the spending rule involves implementing more stringent deviation rules, curbing excessive fiscal spending, and improving the welfare of the opposition's supporters. However, this comes at the cost of worsening the welfare of the incumbent party's supporters. The optimal deviation rule, therefore, delicately balances these tradeoffs, offering a nuanced solution that mitigates excessive spending while considering the welfare of both parties.

Next, consider the condition in (38) for $\delta^*(b_0, \alpha) > 1/2$ to be satisfied. If $\delta = 1/2$, the incumbent party, holding more than half of the seats, can consistently deviate from the spending rule. When the condition (38) holds, and therefore $\delta^*(b_0, \alpha) > 1/2$, the utility gain experienced by the supporters of the opposition party during the transition from the dictator state to the rule state $(V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0))$ exceeds the utility loss of the incumbent party's supporters $(V^d_{I,1}(b_0) - V^r_{I,1}(b_0, \alpha))$. Given that there are more supporters of the incumbent party than supporters of the opposition party, disregarding the spending rule at all times becomes optimal from the viewpoint of social welfare maximization if the condition (38) is violated.

4.4 Optimal Pair of Fiscal Rules

In Subsection 4.2, we derived the optimal spending rule for a specified deviation rule. In Subsection 4.3, we established the optimal deviation rule corresponding to a given spending rule. Building upon these findings, our objective is to identify the optimal combination of spending and deviation rules that maximize social welfare. Formally, we define the optimal combination of the fiscal rules as follows.

Definition 1 *An optimal pair of fiscal rules is represented by a pair* $\{\alpha^{opt}(b_0), \delta^{opt}(b_0)\}$ *that simultaneously satisfies the following two equations:*

$$\alpha^{opt}(b_0) = \alpha^*(b_0, \delta^{opt}), \quad \delta^{opt}(b_0) = \delta^*(b_0, \alpha^{opt}). \tag{39}$$

From Lemma 2 and Lemma 3 we obtain the following proposition.

Proposition 1 *Suppose that the following inequality condition holds:*

$$Z(x) \equiv \left[(1+r) + x^{\frac{1}{\sigma}}\right]^{\sigma} \cdot \left[1 + (1+r)\left(\frac{1}{2} + \frac{1}{2x}\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} - (1+r)\left(\frac{1}{2} + \frac{1}{2x}\right)^{\frac{1-\sigma}{\sigma}} - x > 0, \tag{40}$$

$$x \equiv \frac{\phi}{1+\theta^{\frac{1}{\sigma}}} \in \left[\frac{1}{2}, 1\right). \tag{41}$$

There is an interior optimal pair of fiscal rules that satisfy $\alpha^{opt}(b_0) \in (0, \bar{\alpha}(b_0))$ *and* $\delta^{opt}(b_0) \in (1/2, 1)$.

(Proof) See Appendix D.

It is in general difficult to demonstrate analytically that the condition specified by the expression in (40) is universally satisfied. To resolve this difficulty, we conduct a numerical check, confirming that the condition in (40) holds across a broad range of parameters. Figure 4 illustrates the function $Z(x)$ with $r = \{0.01, 0.1, 1\}$ and $\sigma = \{0.1, 0.5, 0.99\}$. It is evident that in all cases, $Z(x) > 0$ holds for any $x \in [1/2, 1)$. In what follows, we proceed with the analysis under the assumption that the condition in (40) is satisfied.

Figure 5 illustrates the reaction functions of $\delta^*(b_0, \alpha)$ and $\alpha^*(b_0, \delta)$ for a given initial public debt b_0. The intersection of the two reaction functions is the combination of the optimal fiscal rules $\{\alpha^{opt}(b_0), \delta^{opt}\}$ that satisfies Definition 1. In the numerical example depicted in Figure 5, $\alpha^*(b_0, \delta)$ is an increasing function of δ, while $\delta^*(b_0, \alpha)$ is an increasing function of α. This highlights that a looser spending rule is optimal under a tighter deviation rule, while a tighter deviation rule is optimal under a looser spending rule. In other words, it represents a substitutable relationship between the spending rule and the deviance rule.

Figure 5 demonstrates that the optimal pair of fiscal rules at the interior point is attained, as indicated by Proposition 1. Notably, the optimal deviation rule satisfies $\delta^{opt}(b_0) < 1$. The

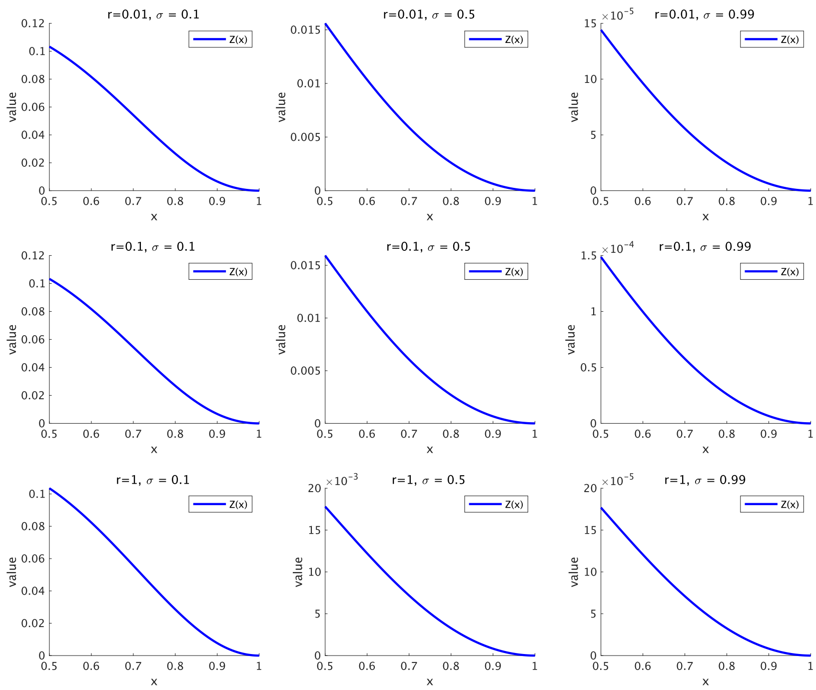

Figure 4: The function $Z(x)$ with $r = \{0.01, 0.1, 1\}$ and $\sigma = \{0.1, 0.5, 0.99\}$.

implication of this result is that the rule that prohibits any deviation is not optimal. If no deviations are permitted at all, even the incumbent party, which has a large number of supporters, is obliged to adhere to the spending rule from the perspective of maximizing social welfare. In other words, a rule that causes welfare loss to the majority of members of society cannot be deemed optimal.

5 Comparative Statics

So far, we have analyzed the optimal fiscal rule, considering the initial public debt balance b_0 and the extent of political opposition to public spending among voters denoted as θ as predetermined. These parameters signify variations in the fiscal environment of each country and wield a crucial influence in shaping the optimal fiscal rule for each country. In this section, we investigate the impact of the initial public debt balance b_0 and political conflict θ on the formulation of the optimal fiscal rules through comparative statics analysis.

5.1 Effects of Initial Public Debt Balance

We examine the impact of the initial public debt balance b_0 on the optimal combination of fiscal rules, $\alpha^{opt}(b_0)$ and $\delta^{opt}(b_0)$. The results of the comparative statics analysis can be summarized through the following proposition.

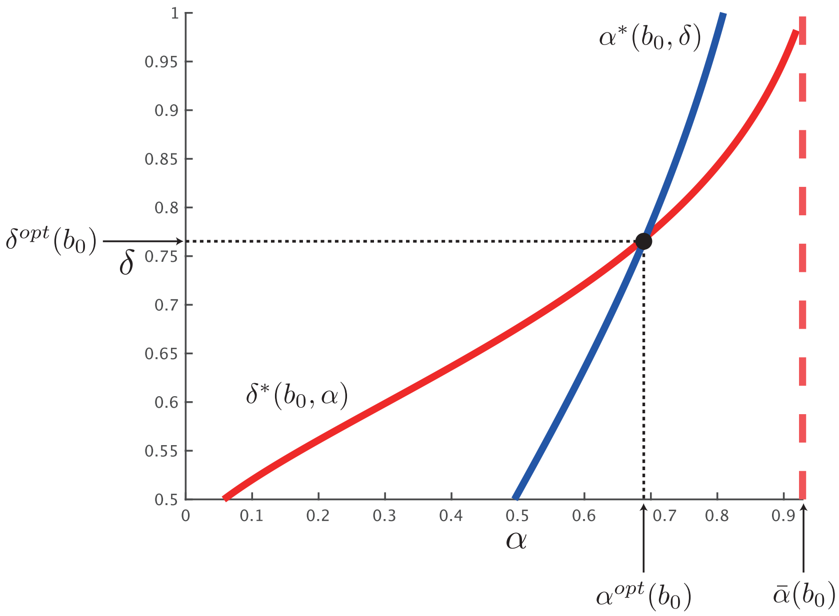

Figure 5: Numerical calculations of the optimal spending rule $\alpha^*(b_0, \delta)$ given the deviation rule δ, and the optimal deviation rule $\delta^*(b_0, \alpha)$ given the spending rule α. Note: $b_0 = 1$, $\sigma = 0.1$, $r = 0.02$ ($\beta = 1/1.02$), $\theta = 0.5$, $\tau = 1$.

Proposition 2 *The optimal deviation rule* δ^{opt} *remains independent of* b_0*, whereas the optimal spending rule* $\alpha^{opt}(b_0)$ *is decreasing in* b_0*.*

(Proof) See Appendix E.

Figure 6 illustrates the relationship between b_0 and $\{\alpha^{opt}(b_0), \delta^{opt}\}$ as presented in Proposition 2, supported by numerical examples. The mechanism underlying the result of Proposition 2 unfolds as follows. As evident from the equations (17)-(19) and (27)-(29), the impact of the initial public debt balance b_0 on social welfare is confined solely to the provision of public goods. The incumbent party determines the supply of public goods based on whether the dictator state or the rule state materializes. Consequently, the deviation rule δ does not influence the supply of public goods. In other words, the deviation rule cannot regulate public good provision based on the initial public debt balance, rendering it independent of the latter. Regarding the spending rule, $\alpha^{opt}(b_0)$ demonstrates a decreasing trend for b_0, signifying that a more restrictive spending rule is optimal for higher initial public debt levels. This trend arises because as b_0 increases, stronger constraints on excessive public spending are necessary to meet the terminal condition $b_2 = 0$. Therefore, a larger initial public debt balance necessitates an optimal spending rule with tighter restrictions.[2]

By Proposition 2, the optimal spending rule $\alpha^{opt}(b_0)$ is contingent upon the initial public debt balance (b_0), whereas the optimal deviation rule δ^{opt} establishes its optimal level independently of the initial public debt balance. This implies that policymakers should adjust the optimal

[2]This implication might not change even if an infinite-period model is considered, because even in an infinite-period model, the terminal condition that the outstanding public debt must not exceed the natural debt limit must be satisfied.

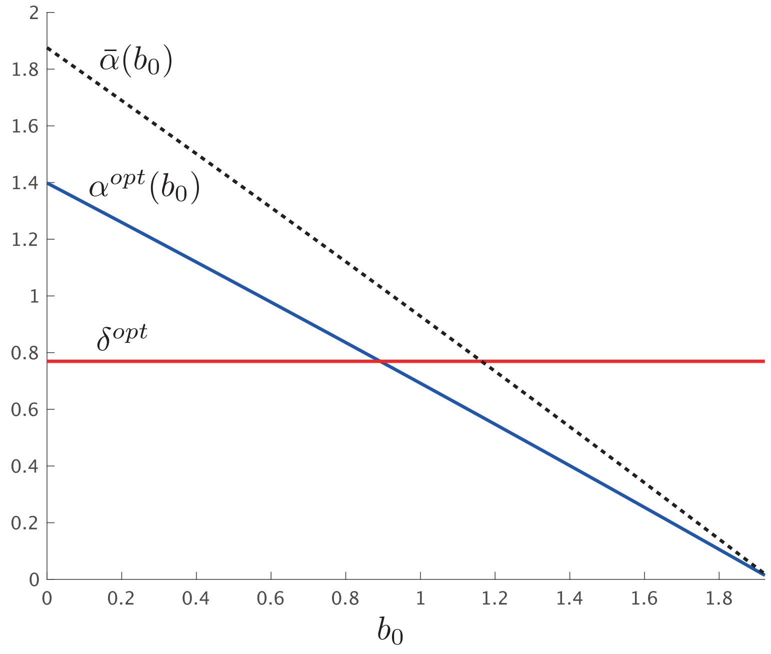

Figure 6: Illustration of $\alpha^{opt}(b_0)$ and δ^{opt}, taking b_0 on the horizontal axis. For other parameters, $\sigma = 0.1$, $r = 0.02$ ($\beta = 1/1.02$), $\theta = 0.5$ and $\tau = 1$.

spending rule in response to current fiscal conditions. Once the optimal level of the deviation rule is determined, it ought to be maintained irrespective of the fiscal situation. This result implies that, as observed in Germany, for instance, it is socially beneficial to establish a mechanism where the spending rule is flexibly adjusted by law, while the deviation rule is constitutionally administered over the long term. This implication, derived from the present analysis, represents one of the significant conclusions of this section, addressing a dimension not explored in previous studies.

5.2 Effects of Differences in Preferences

The parameter θ represents the disparity in preferences between the incumbent party and the opposition party. A smaller value of θ indicates a more significant difference in preferences, highlighting increased conflict between the incumbent and opposition parties. Since θ functions as a parameter characterizing the shape of the voters' (parties') utility function, its modification not only affects the equilibrium supply of public goods and the issuance of public debt but also shapes social welfare through utility. This impact complicates the derivation of a closed-form solution. To address this complexity, we turn to a numerical example to analyze how a change in θ influences the optimal fiscal rules.

Figure 7 illustrates a numerical example depicting the relationship between θ and $\{\alpha^{opt}(b_0), \delta^{opt}\}$. The figure demonstrates that as θ decreases, $\alpha^{opt}(b_0)$ increases, and δ^{opt} decreases. In simpler terms, stronger conflict between parties corresponds to more relaxed optimal spending and deviation rules. The underlying mechanism for this result is as follows: when θ decreases and party conflict intensifies, the change in preferences among supporters of the incumbent party exerts a

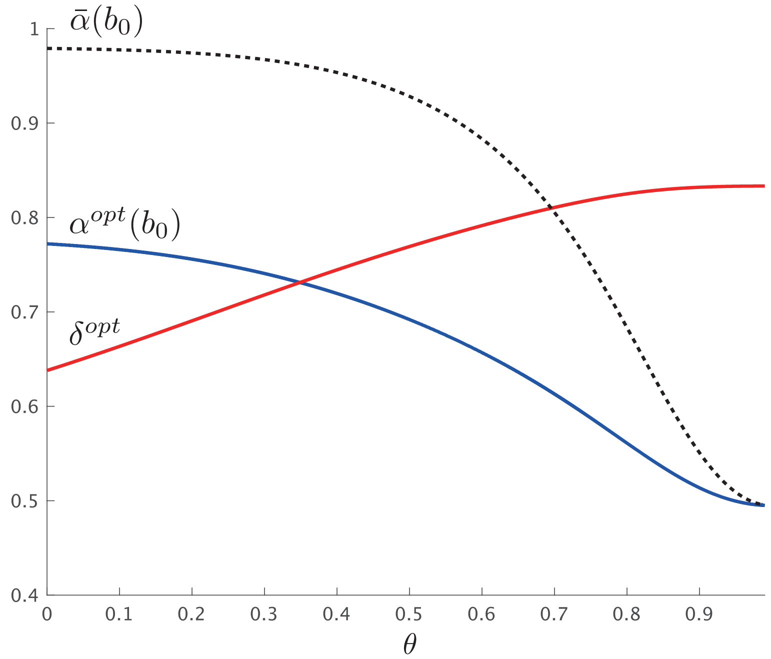

Figure 7: Illustration of $\alpha^{opt}(b_0)$ and δ^{opt}, taking θ on the horizontal axis. For other parameters, $\sigma = 0.1$, $r = 0.02$ ($\beta = 1/1.02$), $b_0 = 1$, and $\tau = 1$.

stronger influence on social welfare than the change among opposition party supporters. As θ decreases, the optimal fiscal rule that maximizes social welfare becomes more oriented towards the incumbent party. Drawing on the insights from Lemma 1, the heightened conflict between political parties results in increased public spending in the dictator state, where the spending rule is irrelevant. Consequently, a smaller θ leads to a more relaxed optimal spending rule. Moreover, an optimal looser deviation rule is preferred, as a stricter deviation rule adversely affects the welfare of incumbent party supporters.

6 Conclusion

This paper theoretically examines the characteristics of the optimal deviation rule from the spending rule, assuming the latter imposes a ceiling on public spending. The analysis focuses on scenarios where deviations from the spending rule are permissible if the incumbent party holds more than δ seats in parliament, investigating the determination of the optimal δ level that maximizes social welfare. The primary findings are as follows. Firstly, a fiscal rule disallowing any deviations is deemed socially undesirable. In instances of skewed voter selection favoring the incumbent party with a significant seat majority, temporarily relaxing the spending rule may prove beneficial.

Secondly, the optimal deviation rule remains independent of outstanding public debt. This is attributed to the fact that public debt affects social welfare solely through public spending levels. Consequently, while the deviation rule dictates adherence to the spending rule, public expenditure is regulated by the latter. Thus, public debt influences the optimal spending rule but not the optimal deviation rule. This underscores the necessity of adjusting the spending rule in line with current fiscal conditions, while the optimal deviation rule remains constant regardless of fiscal circumstances once established. Thirdly, greater disparities in preferences for public goods among voters (or between parties) favor a looser deviation rule. As supporters of the incumbent party outnumber those of the opposition, political conflicts tilt the optimal deviation rule towards the incumbent party's favor, warranting a looser deviance rule.

Several issues warrant further investigation. Firstly, while this study examines the optimal deviation rule concerning the spending rule, similar analyses could explore other fiscal rule types. Notably, studying the optimal deviation rule characteristics for rules governing the ceiling on outstanding public debt in EU countries would be insightful. Secondly, while this study bases deviations on the number of seats held by the incumbent party, in practice, even with a significant seat majority, deviation often requires opposition party agreement. Addressing this by incorporating negotiations between ruling and opposition parties into the model would be beneficial. Lastly, although this study employs a two-period model for analytical tractability, extending it to an infinite-horizon version would facilitate analyzing the relationship between the deviation rule and the dynamics of the public debt balance. These issues will be the focus of future research.

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Appendix

A Proof of Lemma 1

It is sufficient to show that $\left(g_{I,1}^{d*}(b_0)+g_{O,1}^{d*}(b_0)\right)$ is a decreasing function of θ. From (18) and (19), government spending in the dictator state is:

$$g_{I,1}^{d*}(b_0)+g_{0,1}^{d*}(b_0)=\frac{\tau+(1+r)\left[\tau-(1+r)b_0\right]}{(1+r)+\left(\phi/(1+\theta^{\frac{1}{\sigma}})\right)^{\frac{1}{\sigma}}}. \tag{A.1}$$

Here, noting (16) and $\sigma\in(0,1),\theta\in[0,1)$, we obtain the following equation.

$$\begin{aligned}\frac{\partial}{\partial\theta}\left(\frac{\phi}{(1+\theta^{\frac{1}{\sigma}}}\right)&=\frac{1-\frac{1-\sigma}{\sigma}\cdot\theta^{\frac{1}{\sigma}}+\frac{1-\sigma}{\sigma}\cdot\theta^{\frac{1}{\sigma}-2}-\theta^{\frac{2}{\sigma}-2}}{2\left(1+\theta^{\frac{1}{\sigma}}\right)^2}\\&=\frac{1+\frac{1-\sigma}{\sigma}\cdot\left(\theta^{\frac{1}{\sigma}-2}-\theta^{\frac{1}{\sigma}}\right)-\theta^{\frac{2}{\sigma}-2}}{2\left(1+\theta^{\frac{1}{\sigma}}\right)^2}\\&=\frac{1-\theta^{\frac{2(1-\sigma)}{\sigma}}+\frac{1-\sigma}{\sigma}\cdot\theta^{\frac{1}{\sigma}}\left(\frac{1}{\theta^2}-1\right)}{2\left(1+\theta^{\frac{1}{\sigma}}\right)^2}\\&>0.\end{aligned}$$

Therefore, $\left(g_{I,1}^{d*}(b_0)+g_{O,1}^{d*}(b_0)\right)$ is a decreasing function of θ.

Q.E.D.

B Proof of Lemma 2

Suppose that $\delta>1/2$ is given. From Assumption 2 and (34), we have

$$\begin{aligned}\frac{\partial W(b_0,\delta,\alpha)}{\partial\alpha}&=\left[\frac{4\delta^2-1}{4}\cdot\left(1+\theta^{\frac{1}{\sigma}}\right)+\frac{-4\delta^2+8\delta-3}{4}\cdot\left(\theta+\theta^{\frac{1-\sigma}{\sigma}}\right)\right]\cdot\alpha^{-\sigma}\cdot\left(\frac{\tau-rb_0}{1+\theta^{\frac{1}{\sigma}}}\right)^{1-\sigma}\\&\quad+\left(\frac{4\delta^2-1}{4}+\frac{-4\delta^2+8\delta-3}{4}\right)\cdot\beta\phi\cdot\left\{\frac{\tau+(1+r)\left[\tau-(1+r)b_0\right]-(1+r)\alpha(\tau-rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right\}^{-\sigma}\\&\qquad\times\frac{-(1+r)(\tau-rb_0)}{1+\theta^{\frac{1}{\sigma}}}\\&=(2\delta-1)\cdot\frac{(2\delta+1)\left(1+\theta^{\frac{1}{\sigma}}\right)+(-2\delta+3)\left(\theta+\theta^{\frac{1-\sigma}{\sigma}}\right)}{4}\cdot\alpha^{-\sigma}\cdot\left(\frac{\tau-rb_0}{1+\theta^{\frac{1}{\sigma}}}\right)^{1-\sigma}\\&\quad+(2\delta-1)(-1)\cdot\phi\cdot\left\{\frac{\tau+(1+r)\left[\tau-(1+r)b_0\right]-(1+r)\alpha(\tau-rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right\}^{-\sigma}\cdot\frac{\tau-rb_0}{1+\theta^{\frac{1}{\sigma}}}\\&=(2\delta-1)\cdot\left\{\frac{(2\delta-1)\left(1+\theta^{\frac{1}{\sigma}}-\theta-\theta^{\frac{1-\sigma}{\sigma}}\right)+2\left(1+\theta^{\frac{1}{\sigma}}+\theta+\theta^{\frac{1-\sigma}{\sigma}}\right)}{4}\cdot\alpha^{-\sigma}\cdot\left(\frac{\tau-rb_0}{1+\theta^{\frac{1}{\sigma}}}\right)^{1-\sigma}\right.\end{aligned}$$

$$
-\phi \cdot \left\{ \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}} \right\}^{-\sigma} \cdot \frac{\tau - rb_0}{1+\theta^{\frac{1}{\sigma}}} \Bigg\}
$$

$$
= \frac{(2\delta-1)(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}} \cdot \left\{ \frac{(2\delta-1)\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) + 4\phi}{4} \cdot \left[\frac{\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{-\sigma} \right.
$$

$$
\left. -\phi \cdot \left\{ \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}} \right\}^{-\sigma} \right\}. \tag{B.1}
$$

We verify that the second-order condition is satisfied.

$$
\frac{\partial^2 W(b_0,\delta,\alpha)}{\partial \alpha^2} = \frac{(2\delta-1)(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}} \cdot (-\sigma) \cdot \left\{ \frac{(2\delta-1)\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) + 4\phi}{4} \cdot \alpha^{-\sigma-1} \cdot \left(\frac{\tau - rb_0}{1+\theta^{\frac{1}{\sigma}}}\right)^{-\sigma} \right.
$$

$$
\left. +\phi \cdot \left\{ \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}} \right\}^{-\sigma-1} \cdot \frac{(1+r)(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}} \right\}.
$$

From (12), $\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) > 0$ is satisfied; and according to Assumption 1, $(\tau - rb_0 > 0)$ is also true. Furthermore, Assumption 1 implies $b_0 < b^{NDL}$, and Assumption 3 ensures $\alpha < \bar{\alpha}(b_0)$. Therefore, the following expression holds.

$$
\begin{aligned}
\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)(\tau - rb_0)\alpha &> \tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)(\tau - rb_0)\bar{\alpha}(b_0) \\
&= \left\{\tau + (1+r)\left[\tau - (1+r)b_0\right]\right\} \cdot \frac{\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}}{(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}} \\
&> \left\{\tau + (1+r)\left[\tau - (1+r)b^{NDL}\right]\right\} \cdot \frac{\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}}{(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}} \\
&= 0.
\end{aligned} \tag{B.2}
$$

Given that we are currently examining the case where $\delta > 1/2$, $(\partial^2 W(b_0;\delta,\alpha))/(\partial\alpha^2) < 0$ is affirmed based on the aforementioned formula. Consequently, the second-order condition is fulfilled.

Next, we explore the first-order condition of optimization. In the ensuing discussion, we represent the optimal level of the spending rule given δ as α^*. It is important to note that we are addressing the scenario where $\delta > 1/2$, and utilizing the formula (B.1), we derive the following:

$$
\frac{\partial W(b_0,\delta,\alpha^*)}{\partial \alpha} = 0
$$

$$
\Leftrightarrow \frac{(2\delta-1)\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) + 4\phi}{4} \cdot \left[\frac{\alpha^*(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{-\sigma}
$$

$$= \phi \cdot \left\{ \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)\alpha^*(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}} \right\}^{-\sigma}$$

$$\Leftrightarrow \left[\frac{(2\delta - 1)\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) + 4\phi}{4\phi} \right]^{\frac{1}{\sigma}} \cdot \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)\alpha^*(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}$$

$$= \alpha^* \cdot \left[\frac{\tau - rb_0}{1+\theta^{\frac{1}{\sigma}}} \right].$$

Let define $\Gamma(\delta)$ as follows:

$$\Gamma(\delta) \equiv \left[\frac{(2\delta - 1)\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) + 4\phi}{4\phi} \right]^{\frac{1}{\sigma}} \geq 1. \tag{B.3}$$

We then obtain the following expression.[3]

$$\alpha^*(b_0, \delta) = \frac{\tau + (1+r)\cdot\left[\tau - (1+r)b_0\right]}{\tau - rb_0} \cdot \frac{\Gamma(\delta)}{1 + (1+r)\Gamma(\delta)}. \tag{B.4}$$

From Assumption 1, the following two conditions hold:

$$\tau + (1+r)\cdot\left[\tau - (1+r)b_0\right] > \tau + (1+r)\cdot\left[\tau - (1+r)b^{NDL}\right] = 0, \tag{B.5}$$

$$\tau - rb_0 > \tau - rb^{NDL} = \frac{1}{(1+r)^2} > 0, \tag{B.6}$$

These two conditions ensure that $\alpha^*(b_0, \delta) > 0$. Additionally, based on Assumption 3 and condition in (B.5) and (B.6), the following conditions are fulfilled.

$$\begin{aligned}
\alpha^*(b_0,\delta) < \bar{\alpha}(b_0) \ &\Leftrightarrow \ \Gamma(\delta) \cdot \left(\phi / \left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}} < 1 \\
&\Leftrightarrow \ (2\delta - 1)\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) + 4\phi < 4 \cdot \left(1+\theta^{\frac{1}{\sigma}}\right) \\
&\Leftrightarrow \ \delta < \frac{3}{2}.
\end{aligned}$$

From the above, for any $\delta \in (1/2, 1]$, $\alpha^*(b_0, \delta) < \bar{\alpha}(b_0)$ must hold.

Q.E.D.

C Proof of Lemma 3

Suppose $\alpha < \bar{\alpha}(b_0)$ holds. From (34), the first-order condition is given as follows.

$$\frac{\partial W(b_0, \delta^*, \alpha)}{\partial \delta} = -2 \cdot \left[\delta^* V^d_{I,1}(b_0) + (1-\delta^*)V^d_{O,1}(b_0) - \delta^* V^r_{I,1}(b_0, \alpha) - (1-\delta^*)V^r_{O,1}(b_0,\alpha)\right]. \tag{C.1}$$

[3]Note that we are considering the range $\delta > 1/2$ and that $\Gamma(\delta) \geq 1$ always holds from (12).

The second-order condition is

$$\frac{\partial^2 W(b_0,\delta,\alpha)}{\partial\delta^2} = -2\cdot\left[V^d_{I,1}(b_0) - V^r_{I,1}(b_0,\alpha) + V^r_{O,1}(b_0,\alpha) - V^d_{O,1}(b_0)\right]. \tag{C.2}$$

From Assumption 3, $\alpha < \bar{\alpha}(b_0)$ holds; and from (12), $1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}} > 0$ holds. Given these two conditions with (21), (22), (30), and (31), we obtain

$$\begin{aligned}
&V^d_{I,1}(b_0) - V^r_{I,1}(b_0,\alpha) + V^r_{O,1}(b_0,\alpha) - V^d_{O,1}(b_0)\\
&= \frac{1}{1-\sigma}\cdot\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right)\cdot\left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{1-\sigma}\\
&\quad - \frac{1}{1-\sigma}\cdot\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right)\cdot\left[\frac{\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{1-\sigma}\\
&= \frac{\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right)(\tau - rb_0)^{1-\sigma}}{(1-\sigma)\left(1+\theta^{\frac{1}{\sigma}}\right)^{1-\sigma}}\cdot\left[\bar{\alpha}(b_0)^{1-\sigma} - \alpha^{1-\sigma}\right]\\
&> 0.
\end{aligned} \tag{C.3}$$

(C.2) and (C.3) ensures that the seond-order condition in (C.2) holds. Thus, from (C.1), the optimal δ is

$$\delta^*(b_0,\alpha) = \frac{V^r_{O,1}(b_0,\alpha) - V^d_{O,1}(b_0)}{[V^d_{I,1}(b_0) - V^r_{I,1}(b_0,\alpha)] + [V^r_{O,1}(b_0,\alpha) - V^d_{O,1}(b_0)]}. \tag{C.4}$$

Next, we examine the condition for $\delta^*(b_0,\alpha) < 1$ to be met. Bearing in mind the formula in (C.3), we derive the following equation.

$$\delta^*(b_0,\alpha) < 1 \Leftrightarrow V^d_{I,1}(b_0) - V^r_{I,1}(b_0,\alpha) > 0. \tag{C.5}$$

Considering the expression in (32), $V^r_{I,1}(b_0;\alpha)$ is monotonically increasing in α within the parameter range where Assumption 3 holds. It is important to acknowledge the definition of $\bar{\alpha}(b_0)$ as stipulated by Assumption 3, leading us to the following expression.

$$\begin{aligned}
V^r_{I,1}(b_0,\bar{\alpha}(b_0)) &= \frac{1}{1-\sigma}\cdot\left\{\left(1+\theta^{\frac{1}{\sigma}}\right)\cdot\left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{1-\sigma}\right.\\
&\quad\left. +\beta\phi\cdot\left\{\frac{\bar{\alpha}(b_0)(\tau - rb_0)\left[(1+r) + \left(\phi/(1+\theta^{\frac{1}{\sigma}})\right)^{\frac{1}{\sigma}}\right] - (1+r)\bar{\alpha}(b_0)(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right\}^{1-\sigma}\right\}\\
&= \frac{1}{1-\sigma}\cdot\left[1+\theta^{\frac{1}{\sigma}} + \beta(1+\theta^{\frac{1}{\sigma}})\left(\frac{\phi}{1+\theta^{\frac{1}{\sigma}}}\right)^{\frac{1}{\sigma}}\right]\cdot\left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{1-\sigma}\\
&= V^d_{I,1}(b_0).
\end{aligned} \tag{C.6}$$

Hence, for any $\alpha < \bar{\alpha}(b_0)$, it holds that $0 < V^d_{I,1}(b_0) - V^r_{I,1}(b_0, \alpha)$. Put differently, as long as Assumption 3 is met, the condition $\delta^*(b_0, \alpha) < 1$ remains consistently satisfied.

Finally, (C.3) leads to the following condition.

$$\delta^*(b_0, \alpha) > \frac{1}{2} \Leftrightarrow V^d_{I,1}(b_0) - V^r_{I,1}(b_0, \alpha) < V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0).$$

Q.E.D.

D Proof of Proposition 1

From (35) and (37), incorporating $\bar{\alpha}(b_0)$ from Assumption 3, the optimal pair of fiscal rules, denoted as $\alpha^{opt}, \delta^{opt}$, is determined by solving the following simultaneous equations.

$$\delta = \frac{V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0)}{[V^d_{I,1}(b_0) - V^r_{I,1}(b_0, \alpha)] + [V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0)]}, \tag{D.1}$$

$$\alpha = \bar{\alpha}(b_0) \cdot \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right] \cdot \frac{\Gamma(\delta)}{1+(1+r)\Gamma(\delta)}. \tag{D.2}$$

We reformulate both the numerator and denominator of (D.1). Initially, considering (22), (31), and (D.2), the numerator is reformulated as follows.

$$\begin{aligned}
&V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0) \\
&= \frac{1}{1-\sigma} \cdot \left\{ \left(\theta + \theta^{\frac{1-\sigma}{\sigma}}\right) \cdot \left[\frac{\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{1-\sigma} \right. \\
&\quad \left. + \beta\phi \cdot \left\{\frac{\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right\}^{1-\sigma} \right\} \\
&\quad - \frac{1}{1-\sigma} \cdot \left[\theta + \theta^{\frac{1-\sigma}{\sigma}} + \beta(1+\theta^{\frac{1}{\sigma}})\left(\frac{\phi}{1+\theta^{\frac{1}{\sigma}}}\right)^{\frac{1}{\sigma}}\right] \cdot \left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{1-\sigma} \\
&= \frac{1}{1-\sigma} \cdot \left\{ \left(\theta + \theta^{\frac{1-\sigma}{\sigma}}\right) \cdot \left[\bar{\alpha}(b_0) \cdot \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right] \cdot \frac{\Gamma(\delta)}{1+(1+r)\Gamma(\delta)} \cdot \frac{\tau - rb_0}{1+\theta^{\frac{1}{\sigma}}}\right]^{1-\sigma} \right. \\
&\quad \left. + \beta\phi \cdot \left\{\frac{\bar{\alpha}(b_0) \cdot (\tau - rb_0) \cdot \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]}{(1+\theta^{\frac{1}{\sigma}})\left[1+(1+r)\Gamma(\delta)\right]}\right\}^{1-\sigma} \right\} \\
&\quad - \frac{1}{1-\sigma} \cdot \left[\theta + \theta^{\frac{1-\sigma}{\sigma}} + \beta(1+\theta^{\frac{1}{\sigma}})\left(\frac{\phi}{1+\theta^{\frac{1}{\sigma}}}\right)^{\frac{1}{\sigma}}\right] \cdot \left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{1-\sigma} \\
&= \frac{1}{1-\sigma} \cdot \left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{(1+\theta^{\frac{1}{\sigma}})\left[1+(1+r)\Gamma(\delta)\right]}\right]^{1-\sigma} \\
&\quad \times \left\{ \left(\theta + \theta^{\frac{1-\sigma}{\sigma}}\right) \cdot \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \cdot \Gamma(\delta)^{1-\sigma} \right.
\end{aligned}$$

$$+ \beta\phi \cdot \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}$$

$$- [1+(1+r)\Gamma(\delta)]^{1-\sigma} \cdot \left[\theta + \theta^{\frac{1-\sigma}{\sigma}} + \beta(1+\theta^{\frac{1}{\sigma}})\left(\frac{\phi}{1+\theta^{\frac{1}{\sigma}}}\right)^{\frac{1}{\sigma}}\right]\Bigg\}. \quad \text{(D.3)}$$

Next, from (C.3) and (D.2), the denominator can be rewritten as follows:

$$\begin{aligned}
&[V_{I,1}^d(b_0) - V_{I,1}^r(b_0,\alpha)] + [V_{O,1}^r(b_0,\alpha) - V_{O,1}^d(b_0)] \\
&= \frac{\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right)(\tau - rb_0)^{1-\sigma}}{(1-\sigma)\left(1+\theta^{\frac{1}{\sigma}}\right)^{1-\sigma}} \cdot \left[\bar{\alpha}(b_0)^{1-\sigma} - (\alpha)^{1-\sigma}\right] \\
&= \frac{\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right)(\tau - rb_0)^{1-\sigma}}{(1-\sigma)\left(1+\theta^{\frac{1}{\sigma}}\right)^{1-\sigma}} \\
&\quad \times \left[\bar{\alpha}(b_0)^{1-\sigma} - \left\{\bar{\alpha}(b_0) \cdot \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right] \cdot \frac{\Gamma(\delta)}{1+(1+r)\Gamma(\delta)}\right\}^{1-\sigma}\right] \\
&= \frac{1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}}{1-\sigma} \cdot \left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{(1+\theta^{\frac{1}{\sigma}})\left[1+(1+r)\Gamma(\delta)\right]}\right]^{1-\sigma} \\
&\quad \times \left\{[1+(1+r)\Gamma(\delta)]^{1-\sigma} - \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \cdot \Gamma(\delta)^{1-\sigma}\right\}. \quad \text{(D.4)}
\end{aligned}$$

Finally, upon substituting (D.3) and (D.4) into (D.1), we derive the condition necessary for determining the optimal value of δ.

$$\delta = \frac{\left[\begin{aligned}&\left(\theta + \theta^{\frac{1-\sigma}{\sigma}}\right) \cdot \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \cdot \Gamma(\delta)^{1-\sigma} \\ &\quad + \beta\phi \cdot \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \\ &\quad - [1+(1+r)\Gamma(\delta)]^{1-\sigma} \cdot \left[\theta + \theta^{\frac{1-\sigma}{\sigma}} + \beta(1+\theta^{\frac{1}{\sigma}})\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]\end{aligned}\right]}{\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) \cdot \left\{[1+(1+r)\Gamma(\delta)]^{1-\sigma} - \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \cdot \Gamma(\delta)^{1-\sigma}\right\}}$$

$$\equiv RHS(\delta). \quad \text{(D.5)}$$

From the above expression, the optimal deviation rule δ^{opt} is defined as the value of δ that satisfies $\delta = \text{RHS}(\delta)$.

Next, we establish the condition for the existence of a $\delta \in (1/2, 1)$ that fulfills (D.5). Initially, we demonstrate the positivity of the denominator in (D.5). Utilizing (12), the requisite condition for the denominator to be positive is as follows.

$$[1+(1+r)\Gamma(\delta)]^{1-\sigma} - \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \cdot \Gamma(\delta)^{1-\sigma} > 0$$

$$
\begin{aligned}
&\Leftrightarrow\ 1+(1+r)\Gamma(\delta) > \left[(1+r)+\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]\cdot\Gamma(\delta)\\
&\Leftrightarrow\ 1 > \left[\frac{\phi}{1+\theta^{\frac{1}{\sigma}}}\cdot\frac{2(1+\theta^{\frac{1}{\sigma}}+\theta+\theta^{\frac{1-\sigma}{\sigma}})+(2\delta-1)(1+\theta^{\frac{1}{\sigma}}-\theta-\theta^{\frac{1-\sigma}{\sigma}})}{2(1+\theta^{\frac{1}{\sigma}}+\theta+\theta^{\frac{1-\sigma}{\sigma}})}\right]^{\frac{1}{\sigma}}\\
&\Leftrightarrow\ 1 > \frac{2(1+\theta^{\frac{1}{\sigma}}+\theta+\theta^{\frac{1-\sigma}{\sigma}})+(2\delta-1)(1+\theta^{\frac{1}{\sigma}}-\theta-\theta^{\frac{1-\sigma}{\sigma}})}{4(1+\theta^{\frac{1}{\sigma}})}\\
&\Leftrightarrow\ (3-2\delta)(1+\theta^{\frac{1}{\sigma}})+(2\delta-1)(\theta+\theta^{\frac{1-\sigma}{\sigma}})>0.
\end{aligned}
$$

As $\delta\in[1/2,1]$, the inequality in the above equation must be satisfied. Consequently, the denominator of the formula in (D.5) is always positive.

If the following two conditions are satisfied, then there exists at least one $\delta\in(1/2,1)$ such that (D.5) holds.

$$
RHS\left(\frac{1}{2}\right)>\frac{1}{2},\quad RHS(1)<1.
$$

Initially, we demonstrate the consistent satisfaction of $RHS\left(\frac{1}{2}\right)>\frac{1}{2}$. To establish this, it is crucial to note that $\Gamma\left(\frac{1}{2}\right)=1$ (as indicated in (36)) and that $\beta(1+r)=1$ (according to Assumption 2). Moreover, considering the definition of ϕ provided in (16), we derive the following conditions from (D.5).

$$
\begin{aligned}
&RHS\left(\frac{1}{2}\right)>\frac{1}{2}\\
&\Leftrightarrow\ \left[2\left(\theta+\theta^{\frac{1-\sigma}{\sigma}}\right)+2\beta\phi+\left(1+\theta^{\frac{1}{\sigma}}-\theta-\theta^{\frac{1-\sigma}{\sigma}}\right)\right]\cdot\left[(1+r)+\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}\\
&\qquad > (2+r)^{1-\sigma}\cdot\left[\left(1+\theta^{\frac{1}{\sigma}}-\theta-\theta^{\frac{1-\sigma}{\sigma}}\right)+2\left(\theta+\theta^{\frac{1-\sigma}{\sigma}}\right)+2\beta\left(1+\theta^{\frac{1}{\sigma}}\right)\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]\\
&\Leftrightarrow\ \left[\left(1+\theta^{\frac{1}{\sigma}}+\theta+\theta^{\frac{1-\sigma}{\sigma}}\right)+2\beta\phi\right]\cdot\left[(1+r)+\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}\\
&\qquad > (2+r)^{1-\sigma}\cdot\left[\left(1+\theta^{\frac{1}{\sigma}}+\theta+\theta^{\frac{1-\sigma}{\sigma}}\right)+2\beta\left(1+\theta^{\frac{1}{\sigma}}\right)\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]\\
&\Leftrightarrow\ 2\phi(1+\beta)\cdot\left[(1+r)+\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}\\
&\qquad > 2\phi\beta(2+r)^{1-\sigma}\cdot\left[(1+r)+\frac{1+\theta^{\frac{1}{\sigma}}}{\phi}\cdot\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)^{\frac{1}{\sigma}}\right)\right]\\
&\Leftrightarrow\ (2+r)^{\sigma}\cdot\left[(1+r)+\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}-\left[(1+r)+\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}-1}\right]>0. \qquad \text{(D.6)}
\end{aligned}
$$

Define x as $x\equiv\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)$ and denote the left-hand side of the above equation as $W(x)$. The expression in (D.6) can be reformulated as follows.

$$
RHS\left(\frac{1}{2}\right)>\frac{1}{2}\ \Leftrightarrow\ W(x)=(2+r)^{\sigma}\cdot\left[(1+r)+x^{\frac{1}{\sigma}}\right]^{1-\sigma}-\left[(1+r)+x^{\frac{1}{\sigma}-1}\right]>0. \qquad \text{(D.7)}
$$

The differentiation of $W(x)$ with respect to x leads to:

$$
\begin{aligned}
\frac{\partial W(x)}{\partial x} &= (1-\sigma)(2+r)^{\sigma}\cdot\left[(1+r)+x^{\frac{1}{\sigma}}\right]^{-\sigma}\cdot\frac{1}{\sigma}\cdot x^{\frac{1}{\sigma}-1}-\frac{1-\sigma}{\sigma}\cdot x^{\frac{1}{\sigma}-2}\\
&= \frac{1-\sigma}{\sigma}\cdot x^{\frac{1}{\sigma}-2}\cdot\left\{\left[\frac{2+r}{(1+r)+x^{\frac{1}{\sigma}}}\right]^{\sigma}\cdot x-1\right\}.
\end{aligned}
$$

The condition for $\partial W(x)/\partial x$ to be negative is as follows.

$$
\begin{aligned}
\frac{\partial W(x)}{\partial x}<0 \;&\Leftrightarrow\; \left[\frac{2+r}{(1+r)+x^{\frac{1}{\sigma}}}\right]^{\sigma}\cdot x<1\\
&\Leftrightarrow\; \frac{2+r}{(1+r)+x^{\frac{1}{\sigma}}}\cdot x^{\frac{1}{\sigma}}<1\\
&\Leftrightarrow\; (1+r)x^{\frac{1}{\sigma}}<(1+r)\\
&\Leftrightarrow\; x<1. \qquad\qquad \text{(D.8)}
\end{aligned}
$$

From (12) and (16), we have

$$
x\equiv\frac{\phi}{1+\theta^{\frac{1}{\sigma}}}=\frac{1+\theta^{\frac{1}{\sigma}}+\theta+\theta^{\frac{1-\sigma}{\sigma}}}{2\left(1+\theta^{\frac{1}{\sigma}}\right)}=\frac{1}{2}+\frac{\theta+\theta^{\frac{1-\sigma}{\sigma}}}{2\left(1+\theta^{\frac{1}{\sigma}}\right)}<\frac{1}{2}+\frac{1}{2}=1, \qquad \text{(D.9)}
$$

By demonstrating that (D.8) consistently holds, we establish that $W(x)$ is a monotonically decreasing function of x. Given that $x<1$, we derive the following result.

$$
\begin{aligned}
W(x)>W(1) &= (2+r)^{\sigma}\cdot[(1+r)+1]^{1-\sigma}-[(1+r)+1]\\
&= (2+r)-(2+r)\\
&= 0. \qquad\qquad \text{(D.10)}
\end{aligned}
$$

As $W(x)>0$ consistently holds, (D.7) guarantees that $RHS\left(\frac{1}{2}\right)>\frac{1}{2}$ is always satisfied.

Next, we derive the condition for $RHS\,(1)<1$ to be satisfied.

$$
\begin{aligned}
&RHS\,(1)<1\\
&\Leftrightarrow\; \left[\left(1+\theta^{\frac{1}{\sigma}}\right)\Gamma(1)^{1-\sigma}+\beta\phi\right]\cdot\left[(1+r)+\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}\\
&\qquad<\left[\left(1+\theta^{\frac{1}{\sigma}}\right)+\beta\left(1+\theta^{\frac{1}{\sigma}}\right)\cdot\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]\cdot[1+(1+r)\Gamma(1)]^{1-\sigma}\\
&\Leftrightarrow\; \beta\left(1+\theta^{\frac{1}{\sigma}}\right)\left[(1+r)\Gamma(1)^{1-\sigma}+\frac{\phi}{1+\theta^{\frac{1}{\sigma}}}\right]\cdot\left[(1+r)+\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}\\
&\qquad<\beta\left(1+\theta^{\frac{1}{\sigma}}\right)\left[(1+r)+\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]\cdot[1+(1+r)\Gamma(1)]^{1-\sigma}
\end{aligned}
$$

$$\Leftrightarrow \; 0 < \left[(1+r) + \left(\phi / \left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{\sigma} \cdot [1 + (1+r)\Gamma(1)]^{1-\sigma} - \left[(1+r)\Gamma(1)^{1-\sigma} + \frac{\phi}{1+\theta^{\frac{1}{\sigma}}}\right]. \tag{D.11}$$

We denote the right-hand side of (D.11) as $Z(x)$. Utilizing the definition of $\Gamma(\delta)$ provided in Lemma 2, we derive the following expression.

$$\Gamma(1) = \left[1 + \frac{1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}}{2\left(1+\theta^{\frac{1}{\sigma}} + \theta + \theta^{\frac{1-\sigma}{\sigma}}\right)}\right]^{\frac{1}{\sigma}} = \left(\frac{1}{2} + \frac{1}{2x}\right)^{\frac{1}{\sigma}}, \tag{D.12}$$

Using this expression, (D.11) is rewritten as follows

$$\begin{aligned} RHS\,(1) < 1 \;\Leftrightarrow\; Z(x) = \left[(1+r) + x^{\frac{1}{\sigma}}\right]^{\sigma} \cdot \left[1 + (1+r)\left(\frac{1}{2} + \frac{1}{2x}\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \\ - (1+r)\left(\frac{1}{2} + \frac{1}{2x}\right)^{\frac{1-\sigma}{\sigma}} - x > 0. \end{aligned} \tag{D.13}$$

If the inequality in (D.13) is met, then the condition $RHS(1) < 1$ is also fulfilled, indicating the existence of at least one $\delta \in (1/2, 1)$ that satisfies the equality in (D.5).

Lastly, as per Lemma 2, if the condition $\delta^{opt} \in (1/2, 1)$ is met, then it follows that $\alpha^{opt}(b_0) = \alpha^*(b_0, \delta^{opt}) \in (0, \bar{\alpha}(b_0))$ is also satisfied.

Q.E.D.

E Proof of Prpopsition 2

We initially establish that δ^{opt} is independent of b_0. The optimal deviation rule, denoted as δ^{opt}, is the value of δ that satisfies the expression in (D.5). Importantly, the expression in (D.5) does not include b_0. Consequently, we conclude that δ^{opt} is independent of b_0.

We proceed to demonstrate that α^{opt} is a decreasing function of b_0. It is noteworthy that δ^{opt} is independent of b_0. Moreover, considering the expression in (35), we observe that

$$\begin{aligned} \frac{\partial \alpha^{opt}}{\partial b_0} &= \frac{\Gamma(\delta^{opt})}{1+(1+r)\Gamma(\delta^{opt})} \cdot \frac{\partial}{\partial b_0}\left(\frac{\tau + (1+r)\left[\tau - (1+r)b_0\right]}{\tau - rb_0}\right) &\text{(E.1)} \\ &= \frac{\Gamma(\delta^{opt})}{1+(1+r)\Gamma(\delta^{opt})} \cdot \frac{-\tau}{(\tau - rb_0)^2} &\text{(E.2)} \\ &< 0, &\text{(E.3)} \end{aligned}$$

showing that α^{opt} is a decreasing function of b_0.

Q.E.D.