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Politics of Public Education and Pension with Endogenous Fertility*

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Abstract

Implications of increased life expectancy on parental fertility decisions and subsequent shifts in political influence between younger and older generations carry significant consequences for government policies concerning education and pension. This study introduces an overlapping generations growth model incorporating these effects, qualitatively indicating that increased life expectancy correlates with lower fertility rates, decreased education expenditure-GDP ratio, and increased pension benefit-GDP ratio. A model simulation evaluates the impact of the projected increase in life expectancy until 2100 on four country groups: synthetic rich OECD, synthetic rich OECD Europe, Japan, and the United States. The findings demonstrate similar trends as in the qualitative analysis, yet growth rates are projected to vary significantly across regions and countries due to differing life expectancy increases.

- Keywords: Fertility; Public Pension; Public Education; Probabilistic Voting; Overlapping Generations
- JEL Classification: D70, E62, H52, H55

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1 Introduction

The declining birth rates and increasing life expectancy observed in most Organization for Economic Cooperation and Development (OECD) member countries in recent decades have increased the share of older adults in the voting population (OECD, 2016). This trend is projected to continue over the next few decades (Rouzet et al., 2019). Accordingly, pension benefits for older adults are expected to increase (Gonzalez-Eiras and Niepelt, 2012), while government spending on schemes that may not directly benefit older adults, such as public education (Poterba, 1997; Cattaneo and Wolter, 2009) could decrease. Simultaneously, an aging population reduces the willingness of the working middle-aged population to pay higher taxes to meet the government's growing pension burden (Razin et al., 2002). However, older adults may not object to education spending because of altruistic concerns for the younger generations or because such spending may enhance productivity and ensure a higher level of tax revenues (Gradstein and Kaganovich, 2004). Therefore, these opposing effects lead to the following question: how does the government allocate its limited budget to provide pension for older adults and education for the younger generation in response to population aging?

Recent studies on this topic include Gonzalez-Eiras and Niepelt (2012), Lancia and Russo (2016), Ono and Uchida (2016), and Bishnu and Wang (2017). These studies have used probabilistic voting (Lindbeck and Weibull, 1987; Persson and Tabellini, 2000) to describe intergenerational conflicts in the allocation of government revenue among the public for pension and education. In this voting environment, each office-seeking candidate proposes a policy platform to maximize the probability of winning elections, resulting in the selection of policies that maximize an objective function that weights the utility of each generation by its share of the population.

The aforementioned studies have assumed exogenous fertility and examined the impact of its exogenous decline on equilibrium policy and resulting growth effects across periods. However, fertility is endogenously determined from the optimizing behavior of households (Becker, 1991). Particularly, an increase in life expectancy affects fertility decisions of households (Ehrlich and Lui, 1991; Zhang et al., 2001; Zhang and Zhang, 2005). This consequently affects the population share or political weight of older adults during the subsequent period. Thus, as emphasized by Gonzalez-Eiras and Niepelt (2012) and Bishnu and Wang (2017), the interaction of pension and education policy choices with fertility decisions is an important issue in analyzing the determination of fiscal policies and assessing its impact on economic growth.[1]

To demonstrate the interaction between the determination of education and pension policies

[1] Gonzalez-Eiras and Niepelt (2012) say: "With endogenous fertility, the demographic structure would turn into an endogenous state variable, rendering an analytical solution of the policy game considered in the present paper infeasible. ... We leave an analysis of these feedback effects for future research." Bishnu and Wang (2017) add: "As the intergenerational distribution of political power is tied to the demographic change, which in turn is determined by the changing pattern of fertility and longevity, a natural extension of this study is to accommodate individual choice of fertility and longevity. We leave this for future study."

and parents' decisions on fertility, we utilize the overlapping-generation model with physical and human capital accumulation developed by Gonzalez-Eiras and Niepelt (2012) and Ono and Uchida (2016). We follow de la Croix and Doepke (2004) and extend the model by introducing the decisions regarding children, developed by Becker and Lewis (1973). Specifically, parents care about consumption, the number of children, and the human capital of their children. Parents vote for public education that affects the formation of their children's human capital and pension provisions that benefit retired older adults. Parents spend a part of their lives raising their children. Given the education and pension policies, parents choose consumption, savings, and the number of children, to maximize their lifetime utility.[2]

In this framework, our study uncovers the dynamic interplay between current fertility choices and the determination of future pension benefits. Firstly, current fertility choices influence the political determination of future pension benefits by altering the demographic composition of future older adults. Additionally, these choices shape the labor supply of parents, impacting equilibrium market wages. Consequently, these wage fluctuations propagate through the economy, affecting savings and physical capital accumulation, thus influencing future pension benefits. Conversely, expected future pension benefits reciprocally influence the fertility choices of current parents.

By emphasizing the interrelationship between fertility choices and pension benefits, which has been underrepresented in the literature, we show that increased life expectancy directly affects parents' fertility decisions for a given set of policy variables, as shown by Ehrlich and Lui (1991) and the literature that follows them. We also show that increased life expectancy has an indirect effect through the political decisions regarding pension benefits for older adults, which is new to the literature. An indirect effect occurs through the following four routes: First, increased life expectancy raises the political weight of older adults, increasing pension benefits. Second, an increase in the number of older adults reduces pension benefits per old-aged individuals. Third, increased life expectancy incentivizes middle-aged individuals to save, leading to a lower labor income tax rate and, in turn, lower pension benefits. Fourth, increased savings lead to higher future pension benefits through physical capital accumulation. Thus, increased life expectancy positively influences fertility through the first and fourth routes but negatively impacts through the second and third. Overall, the net impact on fertility through public pension is negative in the present framework.

We also demonstrate that increased life expectancy leads to an increase in the pension benefit-GDP ratio and a decrease in the education expenditure-GDP ratio. The political dimension is pivotal in this process: An increase in life expectancy and the resulting decline in fertility work to raise the political weight of older adults. Consequently, the government prioritizes enhancing pension benefits for older adults over expenditure on education for the young. Furthermore, declining fertility boosts the capital equipment available for working middle-aged individuals,

[2]Life expectancy could be controllable through health investment (Grossman, 1972). However, in this study, we assume it to be exogenous, and focus on the interaction between fertility and policy choices.

stimulating a higher growth rate.

To further investigate the effects of the projected increase in life expectancy, we follow Gonzalez-Eiras and Niepelt (2012) and employ a model simulation that incorporates life expectancy data up to 2100 for four distinct groups of countries: synthetic rich OECD (encompassing all included countries), synthetic rich OECD Europe (comprising exclusively European countries), Japan, and the United States.[3] Our simulation predicts a decline in the fertility rate and the education expenditure-GDP ratio, accompanied by an increase in the pension benefit-GDP ratio. These projections are consistent with qualitative predictions derived from the model. However, the growth rate is expected to decrease for the rich OECD and experience a slight decrease for the rich OECD Europe and Japan, with a temporary decline observed in the United States. This finding contrasts with the comparative statics analysis mentioned earlier, suggesting that varying degrees of increase in life expectancy between periods lead to divergent forecasts of the growth rate.

To evaluate the baseline case mentioned above, we develop and compare two alternative scenarios. The first involves a policy that maintains the pension-GDP ratio at the 2020 level for the future, effectively implying a reduction in pension expenditure. Similar to the baseline case, this scenario forecasts a decline in the fertility rate, albeit at a more rapid pace. Moreover, unlike the baseline case, the fixed pension benefit-GDP ratio helps evade an increase in the tax burden, thus indicating a tendency for the economic growth rate to increase. Therefore, implementing a policy of fixing the pension benefit-GDP ratio entails a trade-off between fertility and growth.

The second scenario omits individual fertility choices from the model and incorporates projection data from the United Nations World Population Prospects. Like the baseline case, the present scenario indicates a trend toward lower fertility rates, higher pension benefit-GDP ratios, and lower education expenditure-GDP ratios. However, these changes are rapid compared to the baseline case. In addition, the present scenario reveals a significant increase in the growth rate. The result implies that endogenous fertility plays an essential role in the pace of fiscal policy changes and economic growth.

The remainder of this study is organized as follows. The next section reviews the related literature. Section 3 describes the proposed model. Section 4 characterizes political equilibrium. Section 5 presents a model-based simulation to predict changes in fiscal policies, fertility rates, and growth rates over time in response to projected improvements in life expectancy. Section 6 concludes with brief remarks. All the proofs are provided in the Appendix.

2 Related Literature

The literature on public education and pensions began with Pogue and Sgontz (1977), who show that pay-as-you-go (PAYG) social security incentivizes public investment in education.

[3]We include the following countries into the set of the rich OECD countries: Australia, Canada, Denmark, France, Germany, Italy, Japan, New Zealand, Sweden, the United Kingdom, and the United States.

Such an incentive has also been indicated by Becker and Murphy (1988), who have demonstrated the role of PAYG social security in garnering political support from the current working population for public investment in education. Subsequent studies by Cremer et al. (1992), Kaganovich and Zilcha (1999), Pecchenino and Utendorf (1999), Boldrin and Montes (2005), Poutvaara (2006), Cremer et al. (2011), and Andersen and Bhattacharya (2017) have focused on how households behave when public education and pensions are provided by the government. Therefore, decisions on these policies through voting are abstracted away from their analyses.

Early studies on the political economy of public education and pensions include those by Bearse et al. (2001), Soares (2006), Iturbe-Ormaetxe and Valera (2012), Kaganovich and Meier (2012), Kaganovich and Zilcha (2012), and Naito (2012). A common feature of these studies is that the two-dimensional voting aspect is reduced to one dimension to simplify the analysis. In other words, they consider a vote on public education for a given pension benefit, or a vote on the allocation of tax revenue for a given tax rate. Therefore, these studies do not indicate how the size of the government (i.e., the tax rate) and the allocation of government spending between education and pensions are jointly determined through voting in the presence of generational conflict.

This problem is resolved by introducing two-dimensional voting based on altruism (Tabellini, 1991), party competition (Levy, 2005), issue-by-issue voting (Poutvaara, 2006), and reputation (Bellettini and Ceroni, 1999; Boldrin and Rustichini, 2000; Rangel, 2003). However, these studies have abstracted physical and/or human capital formation, and thus, have not examined the interaction between policy and capital formation. The capital formation was introduced by Kemnitz (2000), Gradstein and Kaganovich (2004), Holtz-Eakin et al. (2004), Tosun (2008), and Bernasconi and Profeta (2012). These studies have assumed myopic voting, in which current voters consider future policy as a given. In other words, the forward-looking decisions of voters are absent in the analysis of these studies. Therefore, they abstract from the feedback mechanism between current and future redistribution policies through physical and/or human capital accumulation, which plays a crucial role in shaping fiscal policies.

The feedback mechanism is demonstrated by Beauchemin (1998), Forni (2005), Bassetto (2008), Mateos-Planas (2008), Gonzalez-Eiras and Niepelt (2012), Song (2011), Chen and Song (2014), and Arcalean (2018).[4] In particular, the present study is closely related to Gonzalez-Eiras and Niepelt (2012), Lancia and Russo (2016), Ono and Uchida (2016), and Bishnu and Wang (2017), who have analyzed the politics of public education and pensions in the presence of a feedback mechanism in the overlapping generations model. Among these studies, Gonzalez-Eiras and Niepelt (2012) and Ono and Uchida (2016) have explored the effects of exogenously declining population growth rates on policy choices. Nonetheless, they have overlooked the dynamic interplay between policies and fertility decisions triggered by shifts in life expectancy.

[4] The studies of multiple policy instruments other than education spending in the presence of a feedback mechanism include Hassler et al. (2003, 2005, 2007); Arawatari and Ono (2009, 2013); Song et al. (2012); Müller et al. (2016); Röhrs (2016); Arai et al. (2018); and Uchida and Ono (2021).

This study makes a valuable contribution to the existing literature by emphasizing the pivotal role of this interaction in evaluating the consequences of aging on policy determinations and economic growth.

This study contributes to the literature on aging and intergenerational conflict over policy-making through probabilistic voting, from a methodological perspective (Grossman and Helpman, 1998, Hassler et al., 2005, Gonzalez-Eiras and Niepelt, 2008, Song, 2011, Song et al., 2012, Arai et al., 2018 and Uchida and Ono, 2021). The study is, to the best of our knowledge, the first to obtain a closed-form solution of policy functions in a dynamic setting with endogenous fertility. de la Croix and Doepke (2009) and Kimura and Yasui (2009) have analyzed the politics of education when fertility is endogenous. However, their models are static in nature and thus, assume away an intertemporal interaction between fertility and policy choices via physical/human capital accumulation. The present study overcomes this limitation and demonstrates the dynamic impact of fertility on policy decisions and the resulting resource allocation across generations.

This study also contributes to the literature on optimal pension and education policy. Andersen and Bhattacharya (2017), Bishnu et al. (2021, 2023) and Amol et al. (2022) have shown a pension-education package that attains Pareto improving or optimal allocation in a dynamically efficient economy. In particular, Amol et al. (2022) and Bishnu et al. (2023) have presented such a policy package in an environment where fertility is endogenous. Our study shares a concern with theirs in exploring pension and education policies in an endogenous fertility setting, but differs from theirs with respect to the time horizon of the government's implementation of the policies. Their analyses and results have relied on the implicit assumption that an infinitely-lived government can calculate and implement a Pareto-improving or optimal redistribution of resources across generations. This is a common assumption in analyses of decentralization in competitive equilibrium. In contrast, our study, which belongs to the political economy literature, assumes that while the current population can compute an intergenerational redistribution of resources, there is no infinitely lived government that can commit to such a redistribution. In other words, an intergenerational resource reallocation can be performed only by a short-lived government that represents successive generations living in a current period, and this government can only reallocate resources in that period.

3 Model

The discrete-time economy starts in period 0 and consists of overlapping generations. Individuals are identical within a generation and live at most for the following three periods: young, middle, and old age. They face uncertainties in the third period of life. Let $\pi \in [0, 1]$ denote the life expectancy (i.e., the probability of living in old age). This is considered idiosyncratic for all individuals and is constant across periods. Each middle-aged individual gives birth to a number of children denoted by n. The number of middle-aged individuals in period t is represented by

N_t, and the population grows at a rate of n_{t+1} following the equation $N_{t+1} = n_{t+1}N_t$. The gross population growth rate, n_{t+1}, is determined by the fertility decisions of the period-t middle-aged individuals.

3.1 Individuals

Individuals exhibit the following economic behavior during their life cycle. In their youth, individuals do not make any economic decisions and depend on their parents for their livelihood. In middle age, individuals work, receive market wages, pay taxes, and make fertility and saving decisions. In old age, they retire and consume returns from savings.

We consider middle-aged individuals in period t. Each of them is endowed with one unit of time. Raising one child takes fraction $\phi \in (0, 1)$ of time. Each individual devotes ϕn_{t+1} units of time to raising children and supplies the remaining time, $1 - \phi n_{t+1}$, to the labor market. Each middle-aged individual obtains labor-income $(1 - \phi n_{t+1})\, w_t h_t$, where w_t is the wage rate per unit of labor and h_t is the human capital endowment. After paying tax, $\tau_t w_t h_t (1 - \phi n_{t+1})$, where τ_t is the period t labor-income tax rate, the individual distributes the after-tax income between consumption c_t and savings held as an annuity and invested in physical capital, s_t. Therefore, the period-t budget constraint for each middle-aged individual becomes

$$c_t + s_t \leq (1 - \tau_t) w_t h_t (1 - \phi n_{t+1}) \,. \tag{1}$$

The period $t + 1$ budget constraint in old age is

$$d_{t+1} \leq \frac{R_{t+1}}{\pi} s_t + b_{t+1}, \tag{2}$$

where d_{t+1} is consumption, R_{t+1} is the gross return from savings, and b_{t+1} is the pay-as-you-go public pension benefit. If an individual dies at the end of the middle-age period, their annuitized wealth is transferred via the annuity markets, to individuals who live throughout their old age. Therefore, the return on savings becomes R_{t+1}/π under the assumption of perfect annuity markets.

Children's human capital over period $t + 1$, h_{t+1}, is a function of both h_t and x_t, where h_t represents parents' human capital and x_t denotes government expenditure on public education per young individual. Particularly, h_{t+1} is formulated using the following equation:

$$h_{t+1} = \bar{h}(h_t, x_t) \equiv D (h_t)^{1-\eta} (x_t)^{\eta} \,, \tag{3}$$

where $D(> 0)$ is a scale parameter and $\eta \in (0, 1)$ denotes the elasticity of education technology with respect to education spending.

The following two remarks are in order. First, as in Gonzalez-Eiras and Niepelt (2012) and Lancia and Russo (2016), we abstract private education and private old-age support away from the analysis; moreover, heterogeneity within a generation is abstracted from the analysis. This simplification enables us to demonstrate precisely how the results would change when fertility choice of households is introduced in their framework. Second, we do not distinguish between spending on K-12 and higher education. Accordingly, we consider that x, an investment in public education, includes investments in both K-12 and higher education. In a real economy, the benefits of public education expenditure vary from person to person because some people receive higher education while others do not. This model does not explicitly depict such intra-generational heterogeneity. Instead, we focus on a representative agent to demonstrate the extent to which each individual within a generation benefits from public education investment in K-12 and higher education levels on average.

Middle-aged individuals care about consumption, c_t and d_{t+1}, their number of children, n_{t+1}, and the human capital of children, h_{t+1}. The preferences of the middle-aged in period t are specified by the following expected utility function à la de la Croix and Doepke (2003, 2004, 2009):

$$\ln c_t + \delta \ln n_{t+1} h_{t+1} + \beta\pi \ln d_{t+1}, \tag{4}$$

where $\beta \in (0,1)$ is a discount factor, and $\delta(>0)$ is the degree of preference for the children's quantity and quality.

We substitute the budget constraints (1) and (2) into the expected utility function in (4) to form the unconstrained maximization problem:

$$\max_{\{s_t, n_{t+1}\}} \ln\left((1-\tau_t)w_t h_t\left(1-\phi n_{t+1}\right) - s_t\right) + \delta \ln n_{t+1}h_{t+1} + \beta\pi \ln\left(\frac{R_{t+1}}{\pi}s_t + b_{t+1}\right).$$

By solving this problem, we obtain the following fertility, savings, and consumption functions:

$$n_{t+1} = \bar{n}'\left(\tau_t, b_{t+1}, w_t h_t\right) \equiv \frac{1}{\phi}\cdot\frac{\delta}{1+\delta+\beta\pi}\cdot\frac{(1-\tau_t)w_t h_t + \frac{b_{t+1}}{R_{t+1}/\pi}}{(1-\tau_t)w_t h_t}, \tag{5}$$

$$s_t = \bar{s}\left(\tau_t, b_{t+1}, w_t h_t\right) \equiv \frac{\beta\pi}{1+\delta+\beta\pi}\left[(1-\tau_t)w_t h_t - \frac{1+\delta}{\beta\pi}\cdot\frac{b_{t+1}}{R_{t+1}/\pi}\right], \tag{6}$$

$$c_t = \bar{c}\left(\tau_t, b_{t+1}, w_t h_t\right) \equiv \frac{1}{1+\delta+\beta\pi}\left[(1-\tau_t)w_t h_t + \frac{b_{t+1}}{R_{t+1}/\pi}\right], \tag{7}$$

$$d_{t+1} = \bar{d}'\left(\tau_t, b_{t+1}, w_t h_t\right) \equiv \frac{\beta R_{t+1}}{1+\delta+\beta\pi}\left[(1-\tau_t)w_t h_t + \frac{b_{t+1}}{R_{t+1}/\pi}\right], \tag{8}$$

where we drop the argument R_{t+1} from the expressions of $\bar{n}'(\cdot)$, $\bar{s}(\cdot)$, $\bar{c}(\cdot)$, and $\bar{d}'(\cdot)$ because R_{t+1} becomes constant, as demonstrated below. Superscript "$'$" in the expressions for $\bar{n}'(\cdot)$ and $\bar{d}'(\cdot)$ denotes the next period.[5]

3.2 Firms

There exists a continuum of identical firms that are perfectly competitive profit maximizers. Each individual firm is indexed by i. The technology available to firm i is $Y_{it} = A_t\left(K_{it}\right)^{\alpha}\left(L_{it}\right)^{1-\alpha}$

[5]Equation (5) implies a negative relation between income and fertility, a standard finding in the existing literature. However, Doepke et al. (2023) highlight the possibility that this relation may not hold recently in developed countries. This underscores the debatability of the assumption underlying the utility function in (4) and the household's budget constraint in (1). Despite recognizing this aspect, by adhering to the standard utility function and household's budget constraint as expressed in (4) and (1), we can emphasize distinctions from prior studies that presumed the fertility rate to be exogenous.

where Y_{it}, K_{it}, and L_{it} stand for output, physical capital input, and labor input of firm i, respectively. Here, $A_t(> 0)$ represents the general level of factor productivity, given by the individual firm, and $\alpha \in (0, 1)$ is a constant parameter representing the physical capital share in production.

In each period, firm i chooses physical capital and labor to maximize its profit, $A_t (K_{it})^{\alpha} (L_{it})^{1-\alpha} - R_t K_{it} - w_t L_{it}$, where R_t is the gross return on physical capital and w_t is the wage rate. Firms' profit maximization leads to

$$K_{it} : R_t = \alpha A_t (K_{it})^{\alpha-1} (L_{it})^{1-\alpha}, \tag{9}$$

$$L_{it} : w_t = (1-\alpha) A_t (K_{it})^{\alpha} (L_{it})^{-\alpha}. \tag{10}$$

Capital fully depreciates within a single period.

The productivity parameter A_t is assumed to be proportional to the per labor capital $A_t = Q \cdot (K_t/L_t)^{1-\alpha}$, where $K_t = \sum_i K_{it}$ and $L_t = \sum_i L_{it}$ represent the aggregate physical capital stock and labor, respectively, and $Q(> 0)$ is constant. Thus, physical capital investment involves a technological externality of the type often used in endogenous-growth theories. This assumption, called the AK technology, results in a constant interest rate across periods, which is demonstrated below. This approach helps us obtain a closed-form solution of the model.[6] Under this assumption, the first-order conditions in (9) and (10) are rewritten in aggregate terms as follows:

$$R_t = R \equiv \alpha Q, \tag{11}$$

$$w_t = (1-\alpha) Q \frac{K_t}{L_t}. \tag{12}$$

3.3 Government Budget Constraint

Government expenditures include public investment in education and public pension payments. They are financed by taxes on labor income. The government budget constraint in period t is $\tau_t w_t L_t = \pi N_{t-1} b_t + x_t N_{t+1}$, where $\tau_t w_t L_t$ is the aggregate labor-income tax revenue, $\pi N_{t-1} b_t$ is the public pension payment, and $x_t N_{t+1}$ is the aggregate public expenditure on education.

Let $k_t \equiv K_t/N_t$ denote physical capital per middle-aged individual. By using (12) and dividing both sides of the constraint by N_t, we obtain the following expression of the government budget constraint:

$$\tau_t (1-\alpha) Q k_t = \frac{\pi b_t}{n_t} + n_{t+1} x_t. \tag{13}$$

[6] The AK assumption, originally introduced by Romer (1986), has been subsequently employed in models addressing endogenous fertility growth (Yip and Zhang, 1997; Blackburn and Cipriani, 1998; Chang et al., 2013). Given the resemblance of results derived from the AK assumption to those in models of small open economies where the interest rate is exogenously given, these findings may possess limited relevance for large, developed countries. Nevertheless, certain investigations have undertaken the calibration of the model incorporating small open economy assumptions to developed countries (Song et al., 2012; Lancia and Russo, 2016; Müller et al., 2016). This study offers novel insights into these preceding analyses.

3.4 Market Clearing

The market-clearing condition for physical capital is $K_{t+1} = N_t s_t$, which expresses the equality of total savings by the middle-aged individuals in period t, $N_t s_t$, to the stock of aggregate physical capital at the beginning of the period $t+1$. We rewrite the physical capital market clearing condition as

$$n_{t+1}k_{t+1} = \bar{s}\left(\tau_t, b_{t+1}, w_t h_t\right), \tag{14}$$

where $\bar{s}(\cdot)$ is defined in (6).

The market-clearing condition for labor is

$$L_t = \left(1 - \phi n_{t+1}\right) N_t h_t, \tag{15}$$

which expresses the equality of the aggregate labor demand, L_t, to the aggregate supply, $\left(1-\phi n_{t+1}\right) N_t h_t$. Using (12) and (15), we define labor income as follows:

$$w_t h_t = \bar{w}\left(n_{t+1}, k_t\right) \equiv \frac{(1-\alpha)Q}{1-\phi n_{t+1}} k_t. \tag{16}$$

Thus, the labor income, $w_t h_t = \bar{w}\left(n_{t+1}, k_t\right)$, depends on n_{t+1} and k_t, but is independent of h_t.

Using (16), we can reformulate the fertility function in (5) as $n_{t+1} = \bar{n}'\left(\tau_t, b_{t+1}, \bar{w}\left(n_{t+1}, k_t\right)\right)$, which implies that n_{t+1} affects itself through the general equilibrium effect in $\bar{w}\left(n_{t+1}, k_t\right)$. Solving this expression for n_{t+1} leads to:

$$n_{t+1} = n'\left(\tau_t, b_{t+1}, k_t\right) \equiv \frac{1}{\phi} \cdot \frac{\delta}{1+\delta+\beta\pi} \cdot \frac{(1-\tau_t)(1-\alpha)Qk_t + \frac{b_{t+1}}{R/\pi}}{(1-\tau_t)(1-\alpha)Qk_t + \frac{\delta}{1+\delta+\beta\pi} \cdot \frac{b_{t+1}}{R/\pi}}. \tag{17}$$

Equation (17) suggests that an increase in the tax rate τ_t raises the fertility rate n_{t+1} by lowering the relative price of birth. Additionally, (17) indicates that an increase in pension benefits b_{t+1} leads to a rise in n_{t+1} through the income effect.

Using (11), (16) and (17), we can also redefine the saving function in (6) as

$$s_t = s\left(\tau_t, b_{t+1}, k_t\right) \equiv \frac{\beta\pi}{1+\beta\pi}\left[(1-\tau_t)(1-\alpha)Qk_t - \frac{1}{\beta\pi} \cdot \frac{b_{t+1}}{R/\pi}\right]. \tag{18}$$

Equation (18) suggests that an increase in the tax rate τ_t or pension benefits b_{t+1} discourages middle-aged individuals from saving. However, this effect is offset to some extent by the general equilibrium effect in w_t resulting from a decrease in the fertility rate n_{t+1}. Nonetheless, the eventual decrease in savings occur due to the increase in τ_t or b_{t+1}.

4 Political Equilibrium

This section considers voting on fiscal policies. We employ probabilistic voting à la Lindbeck and Weibull (1987), in which there is electoral competition between the two office-seeking candidates. Each candidate announces a set of fiscal policies subject to the government's budget constraints. As Persson and Tabellini (2000) have demonstrated, the two candidates' platforms converge in the equilibrium to the same fiscal policy that maximizes the weighted average utility of voters.

In the present framework, the young, middle-aged, and older adults have an incentive to vote. While the young may benefit from current public education expenditure through human capital accumulation, we assume that their preferences are not considered by politicians. We impose this assumption, which is often used in the literature (Saint-Paul and Verdier, 1993; Bernasconi and Profeta, 2012; Lancia and Russo, 2016; Bishnu and Wang, 2017), for tractability reasons. However, the assumption could be supported in part by the fact that a large number of the young are not allowed to participate in the voting process due to age restriction.

In period t, the office-seeking candidates consider the preferences of the middle-aged and older adults in that period. Their preferences are represented by the indirect utility functions V_t^M and V_t^O, respectively, and can be formulated as follows:

$$V_t^M(\tau_t, x_t, b_{t+1}, k_t, h_t) = \ln \bar{c}\left(\tau_t, b_{t+1}, \bar{w}\left(n'(\tau_t, b_{t+1}, k_t), k_t\right)\right) + \delta \ln n'(\tau_t, b_{t+1}, k_t) \cdot \bar{h}(h_t, x_t) + \beta\pi \ln \bar{d}'\left(\tau_t, b_{t+1}, \bar{w}\left(n'(\tau_t, b_{t+1}, k_t), k_t\right)\right), \tag{19}$$

$$V_t^O(b_t, k_t, n_t) = \ln d(b_t, n_t, k_t), \tag{20}$$

where $\bar{h}(\cdot)$, $\bar{c}(\cdot)$, $\bar{d}'(\cdot)$, $\bar{w}(\cdot)$, and $n'(\cdot)$ are defined in (3), (7), (8), (16), and (17), respectively, and $d(\cdot)$, representing consumption by older adults, is defined as follows:

$$d(b_t, n_t, k_t) \equiv \frac{R}{\pi} n_t k_t + b_t. \tag{21}$$

In the probabilistic voting, the political objective in period t is the weighted sum of the utilities of older and middle-aged adults; this is given by $\pi\omega V_t^O + n_t(1-\omega)V_t^M$, where $\omega \in (0,1)$ and $1-\omega$ are the political weights placed on older and middle-aged adults, respectively. A larger value of ω implies greater political power of older adults. We use the gross population growth rate n_t to adjust the weight of the middle-aged, and life expectancy (i.e., the probability of living in old age) π to adjust the weight of older adults, to reflect their population share. To obtain the intuition behind this result, we divide the objective function by $n_t(1-\omega)$ and redefine it, denoted by Ω_t, as follows:

$$\Omega_t(\tau_t, b_t, x_t, b_{t+1}, k_t, h_t, n_t) = \frac{\pi\omega}{n_t(1-\omega)} V_t^O(b_t, k_t, n_t) + V_t^M(\tau_t, x_t, b_{t+1}, k_t, h_t). \tag{22}$$

The coefficient $\pi\omega/n_t(1-\omega)$ of V_t^O represents the relative political weight of older adults.

The political objective function in (22) suggests that the current policy choice of (τ_t, b_t, x_t) affects the future policy decisions through fertility choices and physical capital accumulation. Specifically, the current choice of τ_t, b_t, and x_t affects fertility decisions and the formation of physical capital in the next period, respectively. This influences political decision-making on pension payments, b_{t+1}, in the next period. Conversely, as seen in (6) and (17), the level of pension benefits in the next period also affects the economic decisions on savings and fertility in the current period.

To demonstrate this mutual interaction between economic and political decisions, we employ the Markov-perfect equilibrium concept, in which today's fiscal policy depends on the current payoff-relevant state variables. In the current framework, the payoff-relevant state variables in period t are the fertility rate, n_t, and physical capital, k_t; the human capital, h_t, is a payoff-irrelevant state variable because of the specification of the human capital formation function in (3) and the assumption of the logarithmic utility function in (4).[7] Thus, we can drop h_t from the arguments of $V_t^M(\cdot)$ and $\Omega_t(\cdot)$. Consequently, the expected provision of public pension in period $t+1$, b_{t+1}, could be given by the function of the period $t+1$ payoff-relevant state variables, k_{t+1} and n_{t+1}: $b_{t+1} = \bar{B}(k_{t+1}, n_{t+1})$.

Using the notation, with z' denoting the next period z, we can define a Markov-perfect political equilibrium in the current framework as follows.

Definition 1 A *Markov-perfect political equilibrium* is a five-tuple , $\left(\hat{T}, \hat{B}, \hat{X}, \hat{S}, \hat{N}\right)$, where $\hat{T}: \Re_+ \times \Re_+ \to [0,1]$ is the tax rule, $\tau = \hat{T}(k,n)$; $\hat{B}: \Re_+ \times \Re_+ \to \Re_+$ is the pension rule, $b = \hat{B}(k,n)$; $\hat{X}: \Re_+ \times \Re_+ \to \Re_+$ is the education expenditure rule, $x = \hat{X}(k,n)$; $\hat{S}: [0,1] \times \Re_+ \to \Re_+$ is the optimal private saving rule, $s = \hat{S}\left(\tau, k \mid \hat{B}\right)$; and $\hat{N}: [0,1] \times \Re_+ \to \Re_+$ is the optimal private fertility rule, $n' = \hat{N}\left(\tau, k \mid \hat{B}\right)$, such that (i) for a given τ, k, and $\hat{B}$, the optimal private saving and fertility rules are the maps, $\hat{S}$ and $\hat{N}$, respectively, that solve

$$\hat{S}\left(\tau, k \mid \hat{B}\right) = s\left(\tau, \hat{B}\left(\hat{S}\left(\tau, k \mid \hat{B}\right), \hat{N}\left(\tau, k \mid \hat{B}\right)\right), k\right),$$
$$\hat{N}\left(\tau, k \mid \hat{B}\right) = n'\left(\tau, \hat{B}\left(\hat{S}\left(\tau, k \mid \hat{B}\right), \hat{N}\left(\tau, k \mid \hat{B}\right)\right), k\right),$$

where $b' = \hat{B}(k', n')$ with $n' = \hat{N}\left(\tau, k \mid \hat{B}\right)$ and $n'k' = \hat{S}\left(\tau, k \mid \hat{B}\right)$; (ii) given the set of initial conditions, (k, n), and the political objective function

$$\Omega\left(\tau, b, x, k, n \mid \hat{B}\right) \equiv \frac{\pi\omega}{n(1-\omega)} V^O(b, k, n) + V^M(\tau, x, b', k),$$

where $b' = \hat{B}(k', n')$ with $n' = \hat{N}\left(\tau, k \mid \hat{B}\right)$ and $n'k' = \hat{S}\left(\tau, k \mid \hat{B}\right)$, the equilibrium fiscal policies solve

$$\left(\hat{T}(k,n), \hat{B}(k,n), \hat{X}(k,n)\right) = \arg\max_{\tau,b,x} \Omega\left(\tau, b, x, k, n \mid \hat{B}\right)$$

subject to the government budget constraint, $\tau(1-\alpha)Qk = \pi b/n + n'x$, and the government budget constraint satisfies:

$$\hat{T}(k,n)(1-\alpha)Qk = \frac{\pi \hat{B}(k,n)}{n} + \hat{N}\left(\hat{T}(k,n), k \mid \hat{B}\right)\hat{X}(k,n).$$

Part (i) defines the functional equations that map the current tax and physical capital stock to achieve optimal private savings and fertility, $s = \hat{S}\left(\tau, k \mid \hat{B}\right)$ and $n' = \hat{N}\left(\tau, k \mid \hat{B}\right)$. This set of rules describes the private sector's response to changes in τ under the expectation

[7] Notice that h_t solely appears within the $\bar{h}(h_t, x_t)$ term of (19). Given the Cobb-Douglas type specification of the function $\bar{h}$ and the assumption of logarithmic utility function, h_t does not influence the optimality conditions with respect to τ_t, b_t, and x_t.

that future pensions will be set according to the equilibrium rule $\hat{B}(k', n')$. Part (ii) describes the government's problem. In each period, the government sets fiscal policies subject to its budget constraints and the private sector's response, consistent with the expectation that future governments will follow the Markov-perfect political equilibrium rule.

4.1 Characterization of Political Equilibrium

To obtain the set of policy functions in Definition 1, we conjecture the following policy function of pension benefits in the next period:

$$\begin{aligned} b' &= \frac{\frac{\pi\omega}{n'(\tau,b',k)(1-\omega)}B + C}{\frac{\pi\omega}{n'(\tau,b',k)(1-\omega)}E + F} \cdot \frac{R}{\pi} n'\left(\tau, b', k\right) k' \\ &= \frac{\frac{\pi\omega}{n'(\tau,b',k)(1-\omega)}B + C}{\frac{\pi\omega}{n'(\tau,b',k)(1-\omega)}E + F} \cdot \frac{R}{\pi} s\left(\tau, b', k\right), \end{aligned} \tag{23}$$

where B, C, E, and F are constant parameters, and the equality in the the second line originates from the capital-market-clearing condition in (14). Equation (23) implies that the amount of the pension benefit, b', is set to match a certain proportion of the savings, $n'k' = s$. The proportion depends on the relative weight given to older adults, $\pi\omega/n'(1-\omega)$, in the the political objective function.

Using the first-order condition with respect to physical capital in (11), the conjecture in (23) is rewritten as follows:

$$\frac{\pi b' N}{Y'} = \frac{\frac{\pi\omega}{n'(\tau,b',k)(1-\omega)}B + C}{\frac{\pi\omega}{n'(\tau,b',k)(1-\omega)}E + F}\alpha.$$

This shows that public pension payments are linearly related to GDP. The conjecture that satisfies this property is based on the results of Gonzalez-Eiras and Niepelt (2008) who show the linear relation of the policy functions on GDP under an exogenous fertility rate. Our conjecture here indicates that the same property holds for the endogenous fertility rate.

The conjecture in (23) suggests that b' affects itself through savings. Specifically, pension benefits b' discourage middle-aged individuals from saving, which leads to a decrease in pension benefits. Solving (23) for b', we obtain

$$b' = \frac{\frac{\frac{\pi\omega}{n'(\tau,b',k)(1-\omega)}B+C}{\frac{\pi\omega}{n'(\tau,b',k)(1-\omega)}E+F}}{1 + \frac{\frac{\pi\omega}{n'(\tau,b',k)(1-\omega)}B+C}{\frac{\pi\omega}{n'(\tau,b',k)(1-\omega)}E+F} \cdot \frac{\frac{\pi}{R}}{1+\beta\pi}} \cdot \frac{\beta\pi}{1+\beta\pi}\left(1-\tau\right)\left(1-\alpha\right)Qk. \tag{24}$$

Expressions in (17) and (24) suggest that a mutual interaction exists between pension benefits, b', and the fertility rate, n'. As shown in (17), an increase in pension benefits b' raises the lifetime income of an individual, thereby improving fertility through the income effect. This is the economic effect of pension benefits on the fertility rate. Conversely, an increase in the fertility rate leads to a decrease in the relative political weight of older adults, as seen in (24), which affects the determination of pension benefits. This is the political effect of fertility on pension benefits. Thus, pension benefits and fertility interact with each other through economic and political effects.

Considering these two interactions, we substitute (17) into (24), and solve for b' to obtain the following equation:

$$b' = G \cdot (1-\tau)\, k, \tag{25}$$

where $G(>0)$, a constant, is defined in Appendix A.1. Equation (25) shows that a higher labor-income tax rate is associated with a lower level of future expected pension benefits. An increase in the tax rate lowers the relative price of births. This incentivizes individuals to increase fertility, which in turn affects the level of pension benefits through the political effect of fertility described above. Simultaneously, a rise in the tax rate diminishes savings by reducing disposable income, thus lowering pension benefits denoted as b'. Consequently, the combined impact of these effects is negative.

We substitute the policy function of b' from (25) into the fertility function presented in (17) and obtain

$$n' = \frac{1}{\phi} \cdot \frac{\delta}{1+\delta+\beta\pi} \cdot \frac{(1-\alpha)Q + \frac{G}{R/\pi}}{(1-\alpha)Q + \frac{\delta}{1+\delta+\beta\pi} \cdot \frac{G}{R/\pi}}. \tag{26}$$

Equation (26) reveals that the fertility rate remains constant across periods, independent of variations in the labor-income tax rate and state variables. This constancy implies that the direct influence of taxes on fertility is counterbalanced by the indirect effect of taxes through public pension benefits, denoted as b'. It is important to note that this result is contingent upon the assumption of a logarithmic utility function. If this assumption is relaxed, the two effects may not necessarily cancel each other out. Nonetheless, even under the current assumption, there remains an endogenous determination of fertility through individuals' utility-maximizing behavior and their responses to changes in structural parameters.

Specifically, we focus on the impact of increased life expectancy on fertility rates. Given the persistent increase in life expectancy, a prevalent trend in developed countries, each generation's weight within the political objective function undergoes transformation due to its impact on fertility choices. This transformation, in turn, influences government policy decisions. Notably, the dynamic interplay between life expectancy and policy formulation through fertility choices sets our study apart from prior research (Gonzalez-Eiras and Niepelt, 2012; Lancia and Russo, 2016). We will explore this unique aspect in Section 5.

Using the pension benefits in (25) and the constant fertility rate in (26), we can reformulate the political objective function in (22) as follows:

$$\Omega = \frac{\pi\omega}{n(1-\omega)} \ln d\,(b,n,k) + \ln c\,(\tau,k) + \delta \ln n' \cdot \bar{h}\,(h,x) + \beta\pi \ln d'\,(\tau,k)\,,$$

where $\bar{h}(h,x) \equiv D\,(h)^{1-\eta}\,(x)^{\eta}$ as in (3), $d(\cdot)$ is defined in (21), and $c(\cdot)$ and $d'(\cdot)$ are sourced from (7) and (8), respectively. Using (25) and (26), we can define the functions $c(\cdot)$ and $d'(\cdot)$ as follows:

$$c(\tau,k) \equiv \frac{1}{1+\delta+\beta\pi} \cdot \left[(1-\tau)\,\bar{w}\left(n',k\right) + \frac{G\cdot(1-\tau)\,k}{R/\pi} \right],$$
$$d'(\tau,k) \equiv \frac{\beta R}{1+\delta+\beta\pi} \cdot \left[(1-\tau)\,\bar{w}\left(n',k\right) + \frac{G\cdot(1-\tau)\,k}{R/\pi} \right].$$

Given the government budget constraint in (13), we derive the first-order conditions with respect to τ, x, and b. They are summarized by a system of the following two functional equations:

$$\frac{\frac{c_\tau}{c} + \beta\pi\frac{d'_\tau}{d'}}{(1-\alpha)Qk} = \frac{\frac{\pi\omega}{n(1-\omega)} \cdot \frac{d_b}{d}}{\frac{\pi}{n}}, \tag{27}$$
$$\frac{\frac{c_\tau}{c} + \beta\pi\frac{d'_\tau}{d'}}{(1-\alpha)Qk} = \frac{\delta\frac{h'_x}{h'}}{n'}. \tag{28}$$

Expression in (27) captures the trade-off between the marginal cost of taxation and the marginal benefit of pension. This trade-off reveals a conflict of interest between older and middle-aged adults. Older adults want a higher tax rate to enjoy more pension benefits. However, a higher tax rate decreases middle-aged adults' consumption (c and d'), because it decreases not only their disposable income but also future pension benefits, as shown in (25). Expression in (28) demonstrates the trade-off between the marginal cost of taxation and the marginal benefit of education expenditure. While education benefits middle-aged adults through altruistic motives, they bear the tax burden to finance it. The government determines education expenditure to balance the aforementioned cost and benefit.

Using the conditions in (27) and (28), alongside the government budget constraint in (13), we verify the conjecture in (23), and obtain the following result.

Proposition 1 *Suppose that the following condition holds:*

$$1+\delta\eta+\beta\pi < \begin{cases} \frac{\pi\omega}{n_0(1-\omega)} \cdot \frac{1-\alpha}{\alpha} & \text{for } t=0, \\ \frac{\pi\omega}{n'(1-\omega)} \cdot \frac{1-\alpha}{\alpha} & \text{for } t\geq 1. \end{cases} \tag{29}$$

There is a Markov perfect political equilibrium such that the policy functions, τ, b, and x, are expressed as follows:

$$\tau = \frac{\frac{\pi\omega}{n(1-\omega)} + \left[\delta\eta - (1+\beta\pi)\frac{\alpha}{1-\alpha}\right]}{\frac{\pi\omega}{n(1-\omega)} + (1+\delta\eta+\beta\pi)} \in [0,1], \tag{30}$$
$$b = \frac{\frac{\pi\omega}{n(1-\omega)}\frac{1-\alpha}{\alpha} - (1+\delta\eta+\beta\pi)}{\frac{\pi\omega}{n(1-\omega)} + (1+\delta\eta+\beta\pi)}\frac{R}{\pi}nk > 0, \tag{31}$$
$$x = \frac{1}{n'} \cdot \frac{\delta\eta}{\frac{\pi\omega}{n(1-\omega)} + (1+\delta\eta+\beta\pi)}Qk > 0, \tag{32}$$

where n' is the fertility rate given by

$$n' = \frac{-(1+\delta+\alpha\beta\pi) + \sqrt{(1+\delta+\alpha\beta\pi)^2 + 4\alpha\beta(1+\delta\eta+\beta\pi)\frac{1-\omega}{\omega}\frac{\delta}{\phi}}}{2\alpha\beta(1+\delta\eta+\beta\pi)\frac{1-\omega}{\omega}}. \tag{33}$$

Proof. See Appendix A.1.

The condition (29) ensures $b > 0$. It indicates that life expectancy has three effects on the government's provision of public pensions. First, the higher the life expectancy, the easier it is for the government to provide pension, given the greater political weight of older adults. This effect is captured by the term π on the right-hand side of (29). Second, the higher the life expectancy, the higher the weight of consumption utility in old age. This provides an incentive for the government to lower its labor-income tax rate to maintain the consumption level of the older population. This effect is captured by the term π on the left-hand side of the equation.

In period $t = 0$, if the effect of the former exceeds that of the latter, pension benefits will be paid to older adults. From period $t = 1$ onward, there exists a third effect in addition to the two aforementioned effects. An increase in life expectancy decreases fertility rates, as will be shown in Proposition 2 below. This increases relative political weight of older adults and strengthens the government's incentive to provide pension benefits. This effect is represented by the term n' on the right-hand side of (29). Thus, because of this additional effect, the increase in life expectancy provides a stronger incentive for the government to provide pension benefits from period 1 onward compared to period 0.

Proposition 1 implies that the fertility and policy functions have the following features: First, the fertility rate remains constant and independent of physical capital over time. This stability arises from the previously mentioned factors: the direct influence of taxes on fertility is offset by the indirect effects of taxes on public pension benefits. Second, the levels of public pension benefits, b', and public education expenditure, x, are linear functions of output, Qk. This property is necessary for generating a balanced growth path for the economy. Finally, the labor-income tax rate is independent of the state variables and is constant across periods. This property is necessary for the government's budget to be balanced each period.

4.2 Effects of Life Expectancy

The result in Proposition 1 suggests that increased life expectancy affects the choice of fertility and policy. These effects extend to economic growth. This section focuses on an unexpected and permanent increase in life expectancy and analyzes its effects on fertility and subsequently, on policies and growth.

First, we consider the effect of life expectancy on the fertility rate in (33), which can be summarized as follows:

Proposition 2 *A higher life expectancy is associated with a lower fertility rate:* $\partial n'/\partial \pi < 0$.

Proof. See Appendix A.1.

To understand the mechanism behind the result in Proposition 2, recall the fertility function in (26), showing the following two types of effects of life expectancy on fertility. First is the direct effect on an individual's decision regarding fertility for a given set of policy variables; the other is the indirect effect on fertility through political decisions regarding the level of pension benefits. In what follows, we examine various factors that contribute to these two effects.

Firstly, we consider the direct effects of the following two routes, named as "Effect n_i" ($i = 1, 2$) for later references. First, an increase in life expectancy increases the weight of old-age consumption utility. This strengthens the incentive for individuals to save, thereby increasing the costs of raising children. This has a negative effect on fertility ("Effect n_1"). Second, an increase in life expectancy lowers the return on savings and thus reduces the incentive for individuals to save. This has a positive effect on fertility ("Effect n_2").

Next, we consider the indirect effects of the following four routes, named as "Effect n_i" ($i = 3, 4, 5, 6$). First, an increase in life expectancy increases the political weight of older adults. This works to increase the level of pension benefits through voting, having a positive effect on fertility ("Effect n_3"). Second, an increase in life expectancy increases the weight of old-age consumption utility. This reduces the labor-income tax rate, which subsequently reduces pension benefits. This negatively affects fertility ("Effect n_4"). Third, an increase in the weight of old-age consumption utility increases the incentive for individuals to save. This leads to an increase in the level of pension benefits, as observed in the policy function of pension benefits. This has a positive effect on fertility ("Effect n_5"). Finally, the pension benefits per old-aged individual are reduced to maintain a constant pension benefit-GDP ratio against an increase in life expectancy. This negatively affects fertility ("Effect n_6"). Overall, there are six conflicting effects. However, in the current framework, the net effect is negative.

As described above, life expectancy affects the government's policy choices. This implies that life expectancy affects the pension benefit-GDP ratio, $\pi b N_{-}/QK$, and the education expenditure-GDP ratio, xN'/QK, where N_{-} and N' denote the previous and next period N, respectively. The following proposition shows the effects of life expectancy on these ratios.

Proposition 3 *An increase in life expectancy results in the following effects: (i) an increase in the pension benefit-GDP ratio; and (ii) a decrease in the education expenditure-GDP ratio:* $\partial \left(\pi b N_{-}/QK\right)/\partial \pi > 0$ *and* $\partial \left(xN'/QK\right)/\partial \pi < 0$.

Proof. See Appendix A.2.

First, we consider the effect of life expectancy on the pension benefit-GDP ratio. As described in the paragraphs following Proposition 1, there exist two positive and one negative effects of life expectancy on the pension benefits per old-aged individual. Additionally, given the pension benefits per old-age individual, an increase in life expectancy leads to an increase in total pension benefits. Overall, there exist three positive and one negative effects, and the former exceeds the latter. Hence, an increase in life expectancy leads to an increase in the pension benefit-GDP ratio.

Next, we consider the education expenditure-GDP ratio. As life expectancy increases, the political weight of older adults also increases. The decrease in fertility brought about by the increase in life expectancy further increases the relative political weight of older adults. These effects reduce education spending through voting. Furthermore, an increase in life expectancy implies an increase in the weight of the utility of old-age consumption, which lowers the current labor-income tax rate and decreases education expenditure. Considering these two negative effects, an increase in life expectancy leads to a decreased education expenditure-GDP ratio.

As discussed, life expectancy has three effects on the labor-income tax rate, named "Effect τ_i" ($i = 1, 2, 3$) for later reference. They can be summarized as follows. First, an increase in life expectancy increases the weight of utility from consumption in old age for middle-aged individuals. This lowers the tax rate to maintain consumption or savings ("Effect τ_1"). Second, an increase in life expectancy raises the political weight of older adults, incentivizing the government to increase pension benefits. This has the effect of raising the tax rate ("Effect τ_2"). Finally, because an increase in life expectancy leads to a decrease in fertility rate (Proposition 2), the political weight of older adults further increases, reinforcing the second effect ("Effect τ_3"). In summary, life expectancy has two positive and one negative effect on the labor-income tax rate, and the net effect is positive or negative, depending on the structural parameter values.

Finally, based on the results presented in Proposition 1, we derive the growth rate of the economy and investigate how it is affected by increased life expectancy. The growth rate of the output per middle-aged individual is calculated as follows:

$$\frac{y'}{y} = \frac{Qk'}{Qk} = \frac{s/n'}{k}. \tag{34}$$

Equation (34) indicates that life expectancy affects growth rate via the fertility rate, n', as well as savings, s.

Proposition 4 *An increase in life expectancy increases the growth rate of GDP per middle-aged individual:* $\partial (y'/y) / \partial \pi > 0$.

Proof. See Appendix A.3.

An increase in life expectancy leads to a decrease in the fertility rate (Proposition 2). As observed in (34), this positively affects the growth rate of GDP per middle-aged individual because the capital equipment per middle-aged individual increases as the fertility rate declines. To observe the growth effect through savings, recall the savings function in (18), which is restated as follows:

$$s = \frac{\beta\pi}{1+\beta\pi}\left[(1-\tau)(1-\alpha)Qk - \frac{1}{\beta}\cdot\frac{b'}{R}\right].$$

Life expectancy affects savings through the following three terms: a coefficient $\beta\pi/(1+\beta\pi)$, representing the propensity to save, the labor-income tax rate, τ, and pension benefits, b'.

As life expectancy increases, the weight on the utility of old-age consumption increases. This strengthens the incentive for individuals to save, and positively affects savings. Second an increase in life expectancy affects the labor income tax rate through Effects τ_1, τ_2, and τ_3 mentioned above. Finally, an increase in life expectancy has three positive and one negative effects on pension benefit determination, as discussed above. Ultimately, the sum of the positive effects exceeds the sum of the negative effects in the current framework. Thus, an increase in life expectancy increases the growth rate.

4.3 Discussion

To assess the predictive capability of the model's propositions outlined in Propositions 2-4, we examine their validity using OECD data. Figure 1 illustrates the correlation between life expectancy and various factors for rich OECD countries. Panel (a) displays a negative correlation with fertility, Panel (b) exhibits a positive correlation with the public pension benefit-GDP ratio, and Panel (c) portrays a negative correlation with the education expenditure-GDP ratio. The model predictions of Propositions 2 and 3 align with the observation based on cross-country data in Panels (a), (b), and (c) of Figure 1. However, in Panel (d), a relatively weak negative correlation between life expectancy and the growth rate is observed. This finding contradicts Proposition 4, which suggests a positive correlation. The inconsistency arises because our comparative statics analysis does not differentiate between current and subsequent period life expectancies.

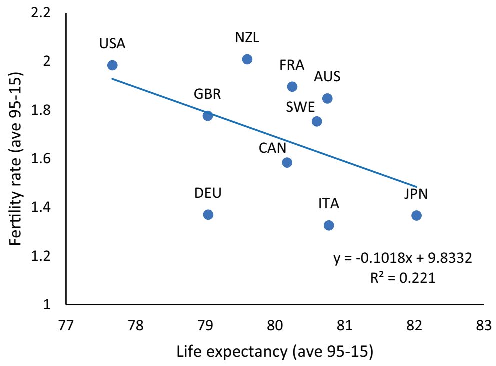
To gain a deeper understanding of this inconsistency, we derive the following expression for the growth rate from the current period to the next period, utilizing the savings function from (18) and the policy functions from Proposition 1:

$$\frac{k'}{k} = \frac{1}{n'\left(\pi', \pi''\right)} \cdot \frac{\beta\pi'}{1+\beta\pi'} \left[(1-\alpha)Q - \frac{1}{\beta} \cdot \frac{G\left(\pi', \pi''\right)}{R}\right] \left(1 - \tau\left(\pi, \pi'\right)\right), \tag{35}$$

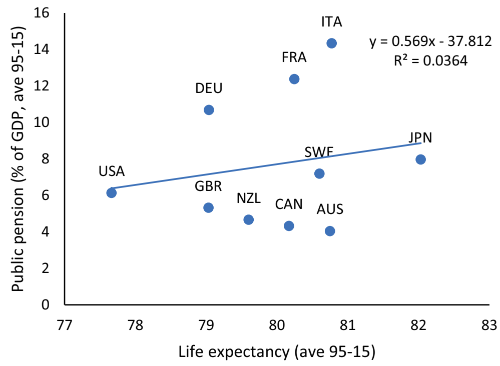
where π, π', and π'' represent the life expectancies for the current period, the next period, and the period after next, respectively. Note that n, G, and τ are now represented as functions of variables π, π', and/or π'' to observe how the timing of changes in life expectancy influences them.

It is noteworthy that the growth rate's dependency on π, π', and π'' is mediated through four distinct terms in (35), denoted as $n'\left(\pi', \pi''\right)$, $\beta\pi'/\left(1+\beta\pi'\right)$, $G\left(\pi', \pi''\right)$, and $\tau\left(\pi, \pi'\right)$. An increase in life expectancy for the current period, π, tends to reduce the growth rate by diminishing the savings of middle-aged individuals in that period through an increased labor income tax rate due to Effects τ_2 and τ_3. In contrast, an increase in life expectancy for the next period, π', tends to stimulate the growth rate by reducing fertility, $n'\left(\pi', \pi''\right)$ through Effects n_1, n_2, n_3, n_5 and n_6, and increasing the savings rate $\beta\pi'/\left(1+\beta\pi'\right)$. In addition, life expectancy for the next period influences pension benefits, represented by $G\left(\pi', \pi''\right)$, and also affects the labor income tax rate through Effects τ_1 and τ_3. Furthermore, life expectancy for the period after next, π'', negatively impacts the fertility rate via Effect n_4 and potentially has an uncertain effect through pension benefits. In summary, the growth effects stemming from these factors remain ambiguous.

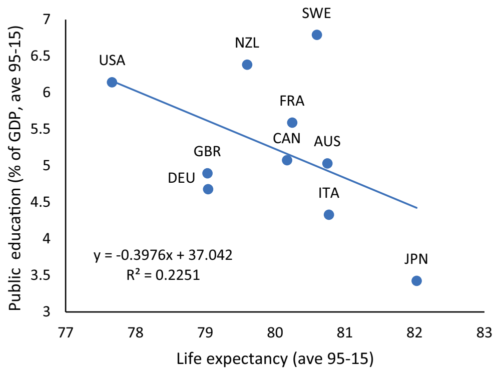
Proposition 4 examined a scenario in which life expectancy in three successive periods increased uniformly from the same level. This analysis revealed a net positive effect of increased



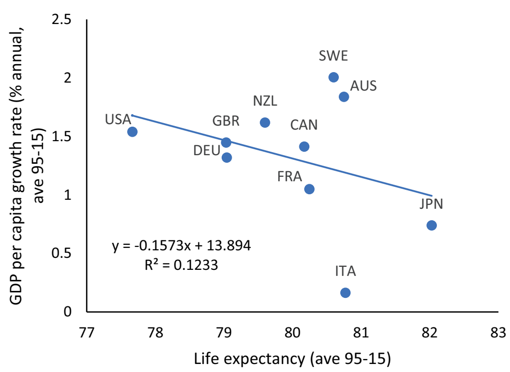
(a)



(b)



(c)



(d)

Figure 1: Association between life expectancy and fertility rates (Panel (a)), the public pension benefit-GDP ratio (Panel (b)), the public education expenditure-GDP ratio (Panel (c)), and the GDP growth rate (Panel (d)) for rich OECD countries during 1995-2015.
Note. The set of rich OECD countries includes Australia, Canada, Denmark, France, Germany, Italy, Japan, New Zealand, Sweden, the United Kingdom, and the United States.
Sources: Data on fertility rates are sourced from the United Nations Department of Economic and Social Affairs Population Division (2022) World Population Prospects 2022, Online Edition (`https://population.un.org/wpp/`) (accessed February 28, 2023). The sources of the other data are described in Appendix A.4.

life expectancy on the growth rate. However, in the real world, the rate of life expectancy increase varies among countries, and the impact of increased life expectancy in one period may not align with that in another. Panel (d) of Figure 1 offers evidence suggesting that the negative effect is more pronounced in Japan and Italy, while the positive effect is stronger in Australia and Sweden. These divergent patterns will be subject to more comprehensive investigation in the model predictions and simulations detailed in Section 5.

5 Numerical Analysis

For numerical analysis, we calibrate the model economy in such a manner that the steady-state equilibrium matches the key statistics of the average synthetic rich OECD countries over the time period 1995–2015. Following Gonzalez-Eiras and Niepelt (2012), we include the following countries into the set of the rich OECD countries: Australia, Canada, Denmark, France, Germany, Italy, Japan, New Zealand, Sweden, the United Kingdom, and the United States. Appendix A.4 describes the sources of the data used in calibration.

5.1 Calibration

We take one period in the model to correspond to 30 years in the data. This assumption is standard in quantitative analyses of two- or three-period overlapping generations models (e.g., Gonzalez-Eiras and Niepelt, 2008; Lancia and Russo, 2016). The probability of living in old age, π, is derived based on the average life expectancy at birth. The average life expectancy in the rich OECD countries is 79.989 years; therefore, individuals will, on average, live 19.989(= 79.989−60) years into old age. In other words, individuals are expected to live 19.989/30 of their 30 years of old age, thus $\pi = 0.666$.

We fix the share of capital at $\alpha = 0.3$, which is considered a standard value in the literature (e.g., Gonzalez-Eiras and Niepelt, 2012). Our selection of R is 1.04 per year (e.g., Song et al., 2012; Lancia and Russo, 2016). The productivity parameter is $Q = 9.730$ because $Q = R/\alpha (= (1.04)^{30}/0.3)$. The opportunity cost of raising a child is approximately 20% of parents' time. (Kimura and Yasui, 2009). By assuming that the duration of parenthood is 18 years, ϕ is $0.2 \times (18/30) = 0.12$.

To determine the remaining four parameters, δ, η, ω, and β, we focus on the fertility rate, the education expenditure-GDP ratio, the pension benefit-GDP ratio, and the growth rate of output per middle-aged individual. The fertility rate, n', is given by (33), and the two ratios, xN'/Y and $\pi b N_{-}/Y$, and the growth rate, y'/y, are as follows:

$$\frac{xN'}{Y} = \frac{\delta\eta}{\frac{\pi\omega}{n'(1-\omega)} + (1 + \delta\eta + \beta\pi)}, \tag{36}$$

$$\frac{\pi b N_{-}}{Y} = \frac{\frac{\pi\omega}{n(1-\omega)}(1-\alpha) - (1 + \delta\eta + \beta\pi)\,\alpha}{\frac{\pi\omega}{n(1-\omega)} + (1 + \delta\eta + \beta\pi)}, \tag{37}$$

$$\frac{y'}{y} = \frac{k'}{k} = \frac{\alpha Q}{\left(1 - \frac{\beta\pi}{1+\beta\pi}(1-\alpha)\right)\frac{\omega}{(1-\omega)\beta} + \left(1 + \frac{\delta\eta}{1+\beta\pi}\right)\alpha n}. \tag{38}$$

We use the data of the synthetic rich OECD average during 1995–2015 to solve the four equations (33), (36), and (37) and (38) for δ, η, ω, and β. Annual population growth rate data during the covered period, which is 1.0067, is taken as the target fertility rate. Therefore, the population growth rate over 30 years is considered $(1.0067)^{30}$.[8] The average education expenditure-GDP ratio is 0.0523, the average pension benefit-GDP ratio is 0.0770, and the average output growth rate for 30 years is $(1.0131)^{30}$. We substitute these data and the values of

[8] In this framework, the gross population growth rate and fertility rates are identical when π and n remain constant. The population size in period t is calculated by adding πN_{t-1}, N_t, and N_{t+1}, and that in period $t+1$ is obtained by adding πN_t, N_{t+1}, and N_{t+2}. Accordingly, the gross population growth rate from period t to $t+1$ is given by $(\pi N_t + N_{t+1} + N_{t+2}) / (\pi N_{t-1} + N_t + N_{t+1}) = (\pi + n_{t+1} + n_{t+1}n_{t+2}) / (\pi/n_t + 1 + n_{t+1})$. This equation reduces to n when $n_{t+1} = n_{t+2} = n$.

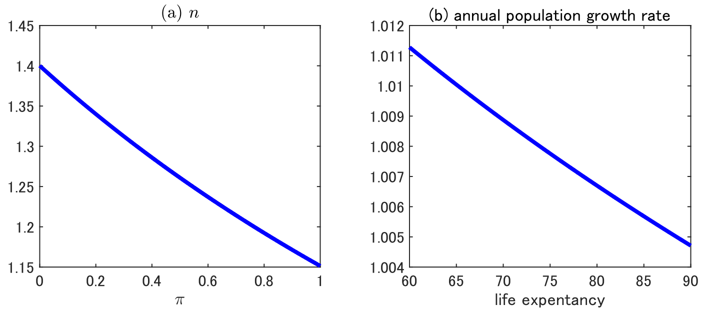


Figure 2: Panel (a) The association between π and n. Panel (b) The association between life expectancy and annual population growth rate.
Note. The life expectancy and annual population growth rate in Panel (b) are computed using $\pi \times 30 + 60$ and $(n)^{1/30}$, respectively.

α, ϕ, and π into (33), (36), (37), and (38), and solve them for δ, η, ω, and β. Accordingly, we obtain $(\delta, \eta, \omega, \beta) = (0.243, 0.573, 0.648, 0.780)$.

Based on this calibration, the impact of changes in life expectancy on fertility and the resulting population growth rate can be estimated, as demonstrated in Figure 2. Panel (a) illustrates the relationship between π, which represents the probability of living to old age, and n, which represents fertility rates, while Panel (b) illustrates the corresponding relationship between life expectancy and the annual population growth rate. The findings in the figure show that a higher life expectancy is associated with a lower fertility rate, as demonstrated in Proposition 2.

Table 1 presents the calibration of life expectancy and its correlation with population growth in the following four country groups: the synthetic rich OECD countries, synthetic rich OECD Europe (consisting exclusively of European countries), Japan, and the United States. Based on the results presented in Table 1, an increase in life expectancy corresponds to a decrease in the annual population growth rate. Particularly in the United States, where life expectancy is comparatively low, a one-year increase in life expectancy results in a reduction of 211 individuals per million. Conversely, in Japan, where life expectancy is higher than that in the United States, a corresponding reduction is 203 individuals per million, which is lower than that in the United States. These findings suggest that the reduction in population growth due to an increase in life expectancy is less significant as life expectancy increases.

	(1)	(2)	(3)
rich OECD	79.989	-0.00020664	-206.644
Japan	82.028	-0.00020297	-202.971
the U.S.	77.664	-0.00021102	-211.022
Europe	79.937	-0.00020674	-206.740

Table 1: Column 1 displays the mean life expectancy between 1995 to 2015. Columns 2 and 3 demonstrate the predicted impact of a one-year increase in life expectancy on the rate of change in the annual population growth (Column 2) and the number of population changes per million (Column 3).

5.2 Projected Changes in Fiscal Policies, Fertility, and Growth Rates

To facilitate our analysis, we utilize a model-based time series approach to predict the changes in fiscal policies, fertility, and growth rates over time in response to projected improvements in life expectancy. In Section 5.1, following Gonzalez-Eiras and Niepelt (2012), we have categorized the OECD countries included in our parameter estimation into four groups: synthetic rich OECD (including all covered countries), synthetic rich OECD Europe (comprising only European countries), Japan, and the United States. In this section we use life expectancy data from the United Nations World Population Prospects,[9] and examine the impact of projected life expectancy on fiscal policies, fertility, and economic growth over time, providing quantitative estimates of the predicted effects.

The analysis proceeds by computing three sequences of model predictions, each spanning a 30-year period. The first sequence includes the years 1990, 2020, 2050, and 2080; the second includes 2000, 2030, 2060, and 2090; and the third includes 2010, 2040, 2070, and 2100. To present the time series predictions, these three sequences are merged into a single time series following the procedure outlined by Gonzalez-Eiras and Niepelt (2012).

Figure 3 displays the predictions of life expectancy in our model for the four groups based on the United Nations World Population Prospects. The other parameter values utilized in the analysis are obtained from the calibration conducted in Section 5.1. It should be noted that instead of using average life expectancy between $t-1$ and t, we utilize the life expectancy at time t, denoted by π_t.[10] Furthermore, π_t for all groups, except the United States, is projected to reach the upper limit of 1 after 2070 in Japan and after 2080 in the synthetic rich OECD and synthetic rich OECD Europe. This is because, in the model, the upper limit of life expectancy is set at 90 years, whereas the estimated life expectancy exceeds 90 years after 2070 or 2080.

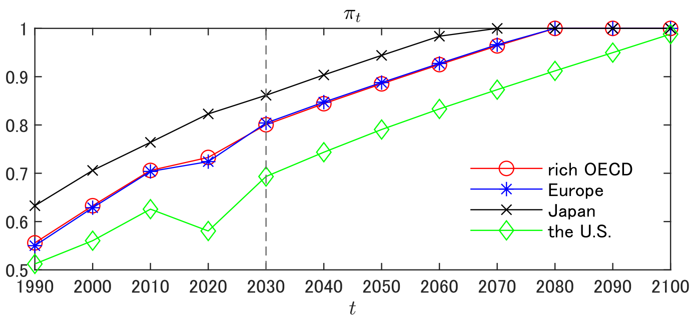


Figure 3: The predictions of π_t for the synthetic rich OECD, the synthetic rich OECD Europe, Japan, and the United States.
Note: The figure plots measured values up to 2020 and projected values after 2030.

It is important to note that the data on π_t of the previous year in each column, that is, 2080, 2090, and 2100, is solely used to calculate n_t at $t = 2050$, 2060, and 2070, respectively. To

[9] See Appendix A.4 for the source of the data.
[10] In the model, π_t represents the probability of survival into old age for middle-aged individuals in period $t-1$.

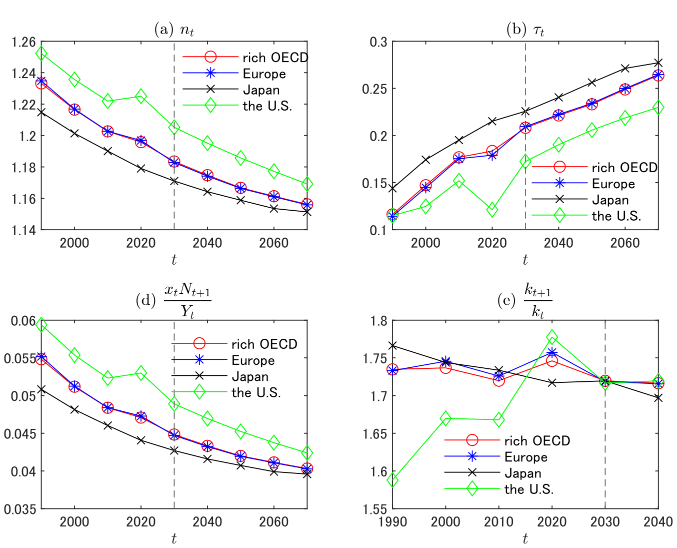


Figure 4: Projected changes in fertility rate (Panel (a)), the labor-income tax rate (Panel (b)), the pension benefit-GDP ratio (Panel (c)), the education expenditure-GDP ratio (Panel (d)), and the growth rate of physical capital per middle-aged individual (Panel (e)).

obtain the fertility rate n_t and policy variables for the last three years of 2080, 2090, and 2100, life expectancy π_t data for 2110, 2120, and 2130, respectively, is required. However, since the available life expectancy estimates are limited to the year 2100, it is not possible to calculate the fertility rates and policy functions for the last three years. Therefore, we forecast fertility rate, policy functions, and the growth rate for each column up to one period before the final year.

Figure 4 illustrates the projections of fertility rates, fiscal policies, and growth rates for the four groups of countries. The figure shows that the fertility rate and education expenditure-GDP ratio will decrease, while the labor-income tax rate and pension benefit-GDP ratio will increase over time. This projection aligns with model-based qualitative predictions in Propositions 2 and 3. However, the growth rate of physical capital per middle-aged individual is projected to decrease for Japan and slightly decrease for the synthetic rich OECD and synthetic rich OECD Europe, with a temporary decline also observed in the United States. This finding contrasts with the result of Proposition 4, which states that the growth rate increases as life expectancy increases, based on comparative statics that assume equal life expectancy increases between the current and future periods. The numerical analysis indicates that the degree of increase in life expectancy differs between periods, leading to different forecasts of economic growth rates.

To better understand the mechanism behind the difference in results, we refer to the equation for the growth rate of physical capital per middle-aged individual in (35). Equation (35) indicates that an increase in life expectancy affects the growth rate through the following four routes: First, a decline in the fertility rate results in an increased physical capital equipment per middle-aged individual. Second, middle-aged individuals are incentivized to save more because of their increased life expectancy. Third, increasing pension benefits discourages individuals from saving. Fourth, an increase in the labor income tax rate reduces savings. Therefore, increased life expectancy positively influences economic growth through the first and second routes but negatively impacts through the third and fourth.

The analysis provided above, based on equation (35), offers insights into the disparity between the model prediction in Proposition 4 and the evidence depicted in Figure 1(d), as well as the divergent projected changes across four country groups. In Japan, where life expectancy is experiencing rapid growth, the adverse effects surpass the positive ones, leading to a decline in economic growth. Conversely, in the United States, where life expectancy temporarily declined between 2010 and 2020, the positive effects outweigh the negative ones, resulting in a temporary upsurge in economic growth from 2030 to 2040. Consequently, the impact of life expectancy on economic growth hinges on the direction of change in life expectancy and the extent to which this trend strengthens in the future.

5.3 Further Quantitative Implications

In this section, we explore two scenarios that differ from the baseline analysis in Section 5.2 to understand their impact on the baseline results. In Section 5.3.1, we analyze the case where the pension benefit-GDP ratio is fixed, while in Section 5.3.2 we examine the scenario where the fertility rates are determined by the United Nations World Population Prospects projections. Both analyses are focused on the rich OECD case. We present only the results and interpretations here, leaving the analysis details to Appendix A.5.

5.3.1 Fixed Pension GDP Ratio

The analysis in Section 5.2 predicts a decline in fertility, an increase in the pension benefit-GDP ratio, and a decline in economic growth against the expected trend of increasing life expectancy in the rich OECD. Specifically, the increase in the pension benefit-GDP ratio is already observed in the rich OECD. To limit the projected further rise in this ratio, many OECD countries are working to curb pension benefits (OECD, 2019). We analyze the impact of this effort on fertility, policy variables, and economic growth. By comparing our analysis results with the baseline results, we can assess the effect of the policy of fixing the pension benefit-GDP ratio.

For the analysis, we assume that the pension benefit-GDP ratio is fixed at the 2020 level after 2030. This policy is assumed to be introduced unexpectedly in 2030. In other words, people do not expect the introduction of this policy before 2020. Figure 5 illustrates the effects of a fixed pension benefit-GDP ratio policy on fertility, tax rates, education expenditure-GDP ratio, and economic growth. The results of the baseline analysis are also shown for comparison.

The scenario with a fixed pension benefit-GDP ratio exhibits the same declining trend in the fertility rate and the education expenditure-GDP ratio as observed in the baseline case. Specifically, while the fertility rate declines rapidly, the education expenditure-GDP ratio de-

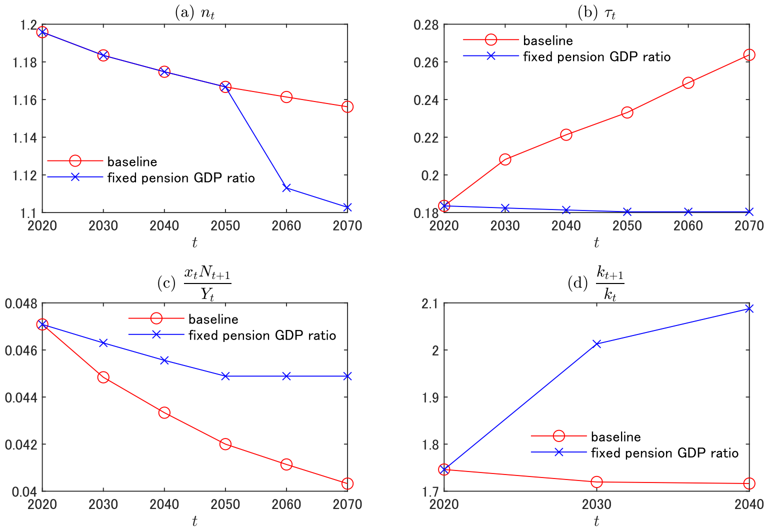


Figure 5: Projected changes in fertility rate (Panel (a)), the labor-income tax rate (Panel (b)), the education expenditure-GDP ratio (Panel (c)), and the growth rate of physical capital per middle-aged individual (Panel (d)).

creases slowly. One significant difference from the baseline is that in the scenario where the pension benefit-GDP ratio is fixed, the labor income tax rate tends to decrease while the growth rate tends to increase. Consequently, reducing future pension benefits by fixing the pension benefit-GDP ratio entails a trade-off between lower fertility and higher economic growth.[11]

The mechanism behind these results is as follows: When the pension benefit-GDP ratio is fixed, decisions regarding x and τ do not impact the utility of older adults. Therefore, the government determines x and τ by considering only the preferences of middle-aged individuals. As life expectancy rises, middle-aged individuals prioritize their old-age consumption, leading to a desire for a smaller tax burden and lower education expenditures. Consequently, the labor income tax rate and the education expenditure-GDP ratio decline.

As noted earlier, a policy of fixing the pension benefit-GDP ratio implies a reduction in future pension benefits. This reduction diminishes the incentive to have and raise children, resulting in a rapid decline in the fertility rate. Simultaneously, the decrease in pension benefits and the lower labor income tax rate resulting from this policy promote physical capital accumulation. Additionally, the decline in fertility increases the per capita capital equipment rate. These effects contribute to an increase in the rate of economic growth.

[11] This trade-off may be lessened by private old-age support. If individuals display altruistic behavior towards their parents - a factor not considered in this study - then future generations might increase private transfers to their parents when confronted with reduced pension benefits. This could offset pension reductions and thus partially alleviate the negative effect of pension cuts on fertility. It should be noted that our model does not incorporate these dynamics associated with private old-age support.

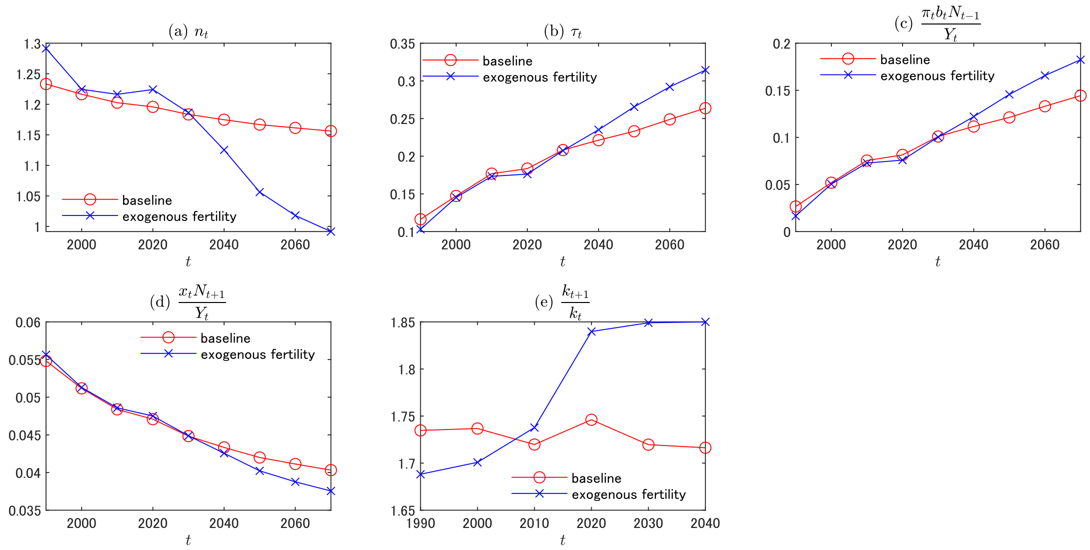


Figure 6: Projected changes in fertility rate (Panel (a)), the labor-income tax rate (Panel (b)), the pension benefit-GDP ratio (Panel (c)), the education expenditure-GDP ratio (Panel (d)), and the growth rate of physical capital per middle-aged individual (Panel (e)).

5.3.2 Exogenous Fertility

The key property of the present study relative to Gonzalez-Eiras and Niepelt (2012) is that fertility is endogenous. This property enables us to demonstrate the fertility decline due to increased life expectancy and its effects on policies and economic growth. In this section, we consider an alternative case where the fertility rate is exogenous, and compare it with the baseline case to see how the assumption of endogenous fertility affects the results.

We utilize data from the United Nations World Population Prospects for the projected fertility rate. For instance, n_{1990} represents the 1990 population/1960 population ratio. Using this estimated fertility data, Figure 6 illustrates the predicted values of the policy variables and the economic growth rate, comparing them with the predicted values in the baseline case. The results in the figure suggest the following: Similar to the baseline case, the exogenous fertility case demonstrates a trend toward lower fertility rates and higher labor income tax rates and pension benefit-GDP ratios, along with lower education expenditure-GDP ratios. However, the pace of these changes is swifter in the exogenous fertility case compared to the baseline case. Another significant difference between the results of the two cases is observed in the economic growth rate. While the baseline case shows minimal change in the growth rate, the exogenous fertility case exhibits a substantial increase in the growth rate.

The mechanism driving these results is as follows: The data from the United Nations World Population Prospects predict a faster decline in fertility compared to the endogenous fertility baseline case. Consequently, the relative political influence of older adults in the exogenous fertility case is greater than in the baseline case, leading to the government's decision to implement higher labor income tax rates and pension benefit-GDP ratios, along with lower education expenditure-GDP ratios. These higher labor income tax rates and pension benefit-GDP ratios diminish the incentive for middle-aged individuals to save, thereby hindering physical capital accumulation and negatively impacting economic growth. However, the decline in the fertility rate increases the physical capital equipment rate per middle-aged individual, which positively influences economic growth. As this positive impact outweighs the negative impact, the exogenous fertility case predicts a higher rate of economic growth than the baseline case.

6 Conclusion

This study investigated the following two key questions: (1) How does the government allocate its limited budget between pension provisions for older adults and education for the younger generation in response to population aging in each period? (2) What are the anticipated shifts in fiscal policies, fertility rates, and economic growth trajectories as a response to projected advancements in life expectancy? To answer these questions, we employed an overlapping-generation model that incorporates physical and human capital accumulation. We further enhanced the model by introducing parental decisions regarding fertility. By endogenizing fertility choices, our approach offers insights into the interplay between political decisions on education and pension expenditures and parental fertility decisions.

Past studies on the political economy of education and pensions have typically treated the population growth rate as an exogenous variable, focusing on the influence of changes in this external factor (specifically, a decline in the population growth rate) on policy formulation. In contrast, our study endogenously models households' fertility decisions and demonstrates their substantial impact on government policy choices and resource allocations. Additionally, our study provides predictions regarding fiscal policies, fertility patterns, and economic growth over time, considering anticipated changes in life expectancy.

While our analysis presents a novel perspective by incorporating fertility as an endogenous variable, several unresolved issues require attention in this study. The first pertains to the generalization of the functional form. The outcomes of this study heavily depend on the assumptions underlying the logarithmic utility function and the AK production technology. Specifically, altering our assumption to a utility function with a constant inter-temporal elasticity of substitution would impact the comparative statics result. Furthermore, relaxing the AK technology assumption would enable us to depict the interplay among capital accumulation, interest rates, and fertility, along with the corresponding endogenous fertility variability. While this exploration holds potential value, our preliminary investigation suggests that this extension would render a closed-form solution impossible, necessitating further quantitative analysis.

The second issue concerns the financial costs associated with child rearing. The current framework concentrates on the opportunity costs of child rearing, sidestepping financial costs due to their lack of intrinsic difference under the logarithmic utility assumption. Introducing financial costs alongside a generalized utility function may yield several novel insights. The third issue involves childcare subsidies, prevalent in many developed countries. This introduces a conflict of interest between older and middle-aged adults and may exert additional influence when combined with the first two extensions. While these extensions hold substantial potential value, they are deferred for future exploration.

A Appendices

A.1 Proofs of Propositions 1 and 2

We substitute the fertility function into (17) to (24) to obtain

$$b' = \frac{\phi^{\frac{1+\delta+\beta\pi}{\delta}\frac{(1-\tau)(1-\alpha)Qk+\frac{\delta}{1+\delta+\beta\pi}\cdot\frac{b'}{R/\pi}}{(1-\tau)(1-\alpha)Qk+\frac{b'}{R/\pi}}}\frac{\pi\omega}{1-\omega}B+C}{\phi^{\frac{1+\delta+\beta\pi}{\delta}\frac{(1-\tau)(1-\alpha)Qk+\frac{\delta}{1+\delta+\beta\pi}\cdot\frac{b'}{R/\pi}}{(1-\tau)(1-\alpha)Qk+\frac{b'}{R/\pi}}}\frac{\pi\omega}{1-\omega}E+F}\cdot\frac{R}{\pi}\cdot\frac{\beta\pi}{1+\beta\pi}\cdot\left[(1-\tau)(1-\alpha)Qk-\frac{1}{\beta\pi}\cdot\frac{b'}{R/\pi}\right], \tag{A.1}$$

or,

$$G_1\cdot\left(b'\right)^2+G_2(1-\tau)k\cdot b'+G_3\cdot((1-\tau)k)^2=0, \tag{A.2}$$

where G_1, G_2, and G_3 are defined as:

$$\begin{aligned}
G_1 &\equiv \frac{\delta}{1+\delta+\beta\pi}\cdot\frac{1}{R/\pi}\cdot\left[\left(\frac{\pi\omega}{1-\omega}E+\frac{F}{\phi}\right)+\left(\frac{\pi\omega}{1-\omega}B+\frac{C}{\phi}\right)\frac{1}{1+\beta\pi}\right],\\
G_2 &\equiv (1-\alpha)\,Q\left\{\left(\frac{\pi\omega}{1-\omega}E+\frac{\delta}{1+\delta+\beta\pi}\cdot\frac{F}{\phi}\right)\right.\\
&\quad\left.+\frac{\beta\pi}{1+\beta\pi}\cdot\left[-\frac{\delta}{1+\delta+\beta\pi}\cdot\left(\frac{\pi\omega}{1-\omega}B+\frac{C}{\phi}\right)+\frac{1}{\beta\pi}\cdot\left(\frac{\pi\omega}{1-\omega}B+\frac{\delta}{1+\delta+\beta\pi}\cdot\frac{C}{\phi}\right)\right]\right\},\\
G_3 &\equiv (-1)\left(\frac{\pi\omega}{1-\omega}B+\frac{\delta}{1+\delta+\beta\pi}\cdot\frac{C}{\phi}\right)\cdot\frac{R}{\pi}\cdot\frac{\beta\pi}{1+\beta\pi}\cdot\left[(1-\alpha)\,Q\right]^2.
\end{aligned}$$

Assuming $G_1 \neq 0$, we solve (A.2) for b' and obtain

$$b' = G(1-\tau)k = \frac{-G_2+\sqrt{(G_2)^2-4G_1G_3}}{2G_1}(1-\tau)k, \tag{A.3}$$

where G is

$$G \equiv \frac{-G_2+\sqrt{(G_2)^2-4G_1G_3}}{2G_1}.$$

We substitute V_t^M into (19) and V_t^O in (20) into Ω_t in (22), and obtain

$$\begin{aligned}
\Omega \simeq{}& \frac{\pi\omega}{n(1-\omega)}\ln\left(\frac{R}{\pi}nk+b\right)+(1+\delta+\beta\pi)\ln\left[(1-\tau)(1-\alpha)Qk+\frac{b'}{R/\pi}\right]\\
&-\delta\ln\left[(1-\tau)(1-\alpha)Qk+\frac{\delta}{1+\delta+\beta\pi}\cdot\frac{b'}{R/\pi}\right]+\delta\eta\ln x.
\end{aligned} \tag{A.4}$$

We use the notation $\simeq$ in (22) because irrelevant terms are omitted from the expressions for Ω_t. We further substitute (A.3) and the government budget constraint in (13) into the political objective function of (A.4), and obtain

$$\Omega \simeq \frac{\pi\omega}{n(1-\omega)}\ln\left(\frac{R}{\pi}nk+b\right)+(1+\beta\pi)\ln(1-\tau)+\delta\eta\ln\left[\tau(1-\alpha)Qk-\frac{\pi b}{n}\right]. \tag{A.5}$$

The first-order conditions with respect to b and τ are:

$$b: \frac{\pi\omega}{n(1-\omega)} \cdot \frac{1}{\frac{R}{\pi}nk+b} - \delta\eta\frac{\frac{\pi}{n}}{\tau(1-\alpha)Qk - \frac{\pi b}{n}} \leq 0, \tag{A.6}$$

$$\tau: \frac{(-1)\,(1+\beta\pi)}{1-\tau} + \frac{\delta\eta(1-\alpha)Qk}{\tau(1-\alpha)Qk - \frac{\pi b}{n}} = 0. \tag{A.7}$$

Equation (A.7) is rewritten as

$$\tau = \frac{\delta\eta(1-\alpha)Qk + (1+\beta\pi)\,\frac{\pi b}{n}}{(1+\delta\eta+\beta\pi)\,(1-\alpha)Qk}. \tag{A.8}$$

Substitution of (A.8) into (A.6) leads to the policy function of pension benefits:

$$b = \frac{\frac{\pi\omega}{n(1-\omega)}\frac{(1-\alpha)}{\alpha} - (1+\delta\eta+\beta\pi)}{\frac{\pi\omega}{n(1-\omega)} + (1+\delta\eta+\beta\pi)}\frac{R}{\pi}nk, \tag{A.9}$$

verifying the conjecture of (23). Equation (A.9) indicates that a public pension is provided, that is, $b>0$ holds if the fertility rate falls below the following critical value:

$$b>0 \Leftrightarrow 1+\delta\eta+\beta\pi < \frac{\pi\omega}{n\,(1-\omega)} \cdot \frac{1-\alpha}{\alpha}. \tag{A.10}$$

We express this condition in (29).

We derive the policy function of τ by substituting (A.9) into (A.7):

$$\tau = \frac{\frac{\pi\omega}{n(1-\omega)} + \left[\delta\eta - (1+\beta\pi)\,\frac{\alpha}{1-\alpha}\right]}{\left[\frac{\pi\omega}{n(1-\omega)} + (1+\delta\eta+\beta\pi)\right]}. \tag{A.11}$$

$\tau<1$ is obtained immediately from the expression in (A.11). $\tau>0$ holds if the sign of the numerator in (A.11) is positive; that is, if

$$\frac{\pi\omega}{n(1-\omega)} + \left[\delta\eta - (1+\beta\pi)\,\frac{\alpha}{1-\alpha}\right] > 0,$$

or,

$$1+\beta\pi - \delta\eta\frac{1-\alpha}{\alpha} < \frac{\pi\omega}{n\,(1-\omega)} \cdot \frac{1-\alpha}{\alpha}. \tag{A.12}$$

(A.12) holds if (A.10) (that is, (29)) holds true. Thus, we obtain $\tau \in [0,1]$ if (29) holds.

Recall that the fertility rate for a given set of policy variables is (17). We must replace τ and b' in (17) with n and k using the policy functions in (A.9) and (A.11). Taking one period lag of (A.9):

$$b' = \frac{\frac{\pi\omega}{n'(1-\omega)}\frac{1-\alpha}{\alpha} - (1+\delta\eta+\beta\pi)}{\frac{\pi\omega}{n'(1-\omega)} + (1+\delta\eta+\beta\pi)}\frac{R}{\pi}n'k' = \frac{z_1(n')}{z_0(n')}\frac{R}{\pi}n'k', \tag{A.13}$$

where $z_0(\cdot)$ and $z_1(\cdot)$ are defined as:

$$z_0(n') \equiv \frac{\pi\omega}{n'(1-\omega)} + (1+\delta\eta+\beta\pi)\,, \tag{A.14}$$

$$z_1(n') \equiv \frac{\pi\omega}{n'(1-\omega)}\frac{1-\alpha}{\alpha} - (1+\delta\eta+\beta\pi)\,. \tag{A.15}$$

The policy function b' in (A.13) is reformulated as follows:

$$\begin{aligned} b' &= \frac{z_1(n')}{z_0(n')}\frac{R}{\pi}s \\ &= \frac{z_1(n')}{z_0(n')}\frac{R}{\pi}\frac{\beta\pi}{1+\beta\pi}\left[(1-\tau)(1-\alpha)Qk - \frac{1}{\beta\pi}\cdot\frac{b'}{R/\pi}\right], \end{aligned}$$

where the equality in the first line comes from (14), and the equality in the second line originates from (6). By rearranging the terms, we have

$$\begin{aligned} \left[1+\frac{z_1(n')}{z_0(n')}\frac{R}{\pi}\frac{\beta\pi}{1+\beta\pi}\frac{1}{\beta\pi}\frac{1}{R/\pi}\right]b' &= \frac{z_1(n')}{z_0(n')}\frac{R}{\pi}\frac{\beta\pi}{1+\beta\pi}(1-\tau)(1-\alpha)Qk \\ &= \frac{z_1(n')}{z_0(n')}\frac{R}{\pi}\frac{\beta\pi}{1+\beta\pi}\frac{1+\beta\pi}{(1-\alpha)z_0(n)}(1-\alpha)Qk, \end{aligned}$$

where the equality in the second line is derived from (A.11). Thus, we obtain

$$b' = \frac{\frac{z_1(n')}{z_0(n')}\frac{R}{\pi}\frac{\beta\pi}{(1-\alpha)z_0(n)}}{1+\frac{z_1(n')}{z_0(n')}\frac{1}{1+\beta\pi}}(1-\alpha)Qk. \tag{A.16}$$

We substitute (A.11) and (A.16) into the fertility function in (17) and obtain the following equation that characterizes the equilibrium fertility rate,

$$n' = \frac{1}{\phi}\cdot\frac{\delta}{1+\delta+\beta\pi}\cdot\frac{1+\frac{z_1(n')}{z_0(n')}}{1+\frac{z_1(n')}{z_0(n')}\cdot\frac{1+\delta}{1+\delta+\beta\pi}}. \tag{A.17}$$

We reformulate the expression in (A.17) by substituting $z_0(\cdot)$ in (A.14) and $z_1(\cdot)$ in (A.15) and rearranging the terms to obtain

$$n' = \frac{\delta}{\phi}\cdot\frac{1}{(1+\delta+\alpha\beta\pi)+\alpha\beta\pi\,(1+\delta\eta+\beta\pi)\,\frac{n'(1-\omega)}{\pi\omega}}. \tag{A.18}$$

Equation (A.18) shows that there exists a unique $n'>0$ satisfying (A.18). Note that (A.18) is rewritten as a quadratic equation for n' :

$$\alpha\beta\,(1+\delta\eta+\beta\pi)\,\frac{1-\omega}{\omega}\,(n')^2 + (1+\delta+\alpha\beta\pi)\,n' - \frac{\delta}{\phi} = 0. \tag{A.19}$$

Thus, we can solve this equation for n as

$$n' = \frac{-(1+\delta+\alpha\beta\pi)+\sqrt{(1+\delta+\alpha\beta\pi)^2+4\alpha\beta\,(1+\delta\eta+\beta\pi)\,\frac{1-\omega}{\omega}\frac{\delta}{\phi}}}{2\alpha\beta\,(1+\delta\eta+\beta\pi)\,\frac{1-\omega}{\omega}}. \tag{A.20}$$

The left-hand side of (A.19) increases in π and n', indicating that a higher π value is associated with a lower n' in (A.19). Thus, we have $\partial n'/\partial\pi<0$.

To obtain the policy function of x, we substitute the policy functions of b in (A.9) and τ in (A.11) into the budget constraints in (13). Then, we have

$$x = \frac{1}{n'}\cdot\frac{\delta\eta}{\frac{\pi\omega}{n(1-\omega)}+(1+\delta\eta+\beta\pi)}Qk > 0. \tag{A.21}$$

■

A.2 Proof of Proposition 3

The aggregate expenditure on public pensions is $\pi b N_-$, and the aggregate GDP is QK. Thus, the pension benefit-GDP ratio is

$$\begin{aligned}\frac{\pi b N_-}{QK} &= \frac{\pi b}{Qkn} \\ &= \frac{\frac{\pi\omega}{n(1-\omega)}(1-\alpha) - (1+\delta\eta+\beta\pi)\alpha}{\frac{\pi\omega}{n(1-\omega)} + (1+\delta\eta+\beta\pi)},\end{aligned}$$

where the second equality comes from the policy function b in (31). After manipulation, we obtain

$$\frac{\pi b N_-}{QK} = \begin{cases} (1-\alpha) - \left[\frac{1}{\frac{1+\delta\eta}{\pi\omega}+\frac{\beta}{\omega}} \cdot \frac{1}{n(1-\omega)} + 1\right]^{-1} & \text{for } t = 0, \\ (1-\alpha) - \left[\frac{1}{\frac{1+\delta\eta}{\pi\omega}+\frac{\beta}{\omega}} \cdot \frac{1}{n'(1-\omega)} + 1\right]^{-1} & \text{for } t \geq 1. \end{cases}$$

As $\partial n'/\partial \pi < 0$, we have $\partial\left[\pi b N_-/QK\right]/\partial \pi > 0$.

The education expenditure-GDP ratio is

$$\begin{aligned}\frac{xN'}{QK} &= \frac{xn'}{Qk} \\ &= \frac{\delta\eta}{\frac{\pi\omega}{n(1-\omega)} + (1+\delta\eta+\beta\pi)},\end{aligned}$$

where the second equality comes from the policy function x in (32). Thus, we have

$$\frac{xN'}{QK} = \begin{cases} \frac{\delta\eta}{\frac{\pi\omega}{n(1-\omega)}+(1+\delta\eta+\beta\pi)} & \text{for } t = 0, \\ \frac{\delta\eta}{\frac{\pi\omega}{n'(1-\omega)}+(1+\delta\eta+\beta\pi)} & \text{for } t \geq 1. \end{cases}$$

As $\partial n'/\partial \pi < 0$, we have $\partial\left(xN'/QK\right)/\partial \pi < 0$.

■

A.3 Proof of Proposition 4

The aggregate output is QK; therefore, the output per middle-aged individual is $QK/N = Qk$. Thus, the gross growth rate of output is $Qk'/Qk = k'/k$. To compute k'/k, we recall the capital market-clearing condition in (14), which we can rewrite as

$$\begin{aligned}n'k' &= \frac{\beta\pi}{1+\beta\pi}\left[(1-\tau)(1-\alpha)Qk - \frac{1}{\beta\pi}\cdot\frac{b'}{R/\pi}\right] \\ &= \frac{\beta\pi}{1+\beta\pi}\left[\frac{1+\beta\pi}{(1-\alpha)z_0(n)}(1-\alpha)Qk - \frac{1}{\beta\pi}\cdot\frac{1}{R/\pi}\cdot\frac{\frac{z_1(n')}{z_0(n')}\frac{Q}{\pi}\frac{\beta\pi}{1+\beta\pi}\frac{1+\beta\pi}{(1-\alpha)z_0(n)}(1-\alpha)Qk}{1+\frac{z_1(n')}{z_0(n')}\frac{Q}{\pi}\frac{\beta\pi}{1+\beta\pi}\frac{1}{\beta\pi}\frac{1}{R/\pi}}\right],\end{aligned}$$

where the first equality is derived using (6) and the second equality is derived by using (A.11) and (A.16). We rearrange the terms in (A.20) and obtain

$$\frac{k'}{k} = \frac{\alpha Q}{\left(1-\frac{\beta\pi}{1+\beta\pi}(1-\alpha)\right)\frac{\omega}{(1-\omega)\beta} + \left(1+\frac{\delta\eta}{1+\beta\pi}\right)\alpha n'}. \tag{A.22}$$

The first term in the denominator on the right-hand side decreases in π, and the second term in the denominator also decreases in π because $\partial n'/\partial\pi < 0$. Thus, we obtain $\partial\left(k'/k\right)/\partial\pi > 0$.

■

A.4 Sources of Data

We source data on the average life expectancy and the average population growth rate in rich OECD countries from the United Nations Department of Economic and Social Affairs Population Division (2022) World Population Prospects 2022, Online Edition (`https://population.un.org/wpp/`) (accessed March 8, 2023). Data on the average education expenditure-GDP ratio are sourced from World Development Indicators (WDI) (`https://datatopics.worldbank.org/world-development-indicators/`) (accessed March 2, 2023). Public education expenditure includes expenditure funding through transfers from international sources to general government. General government refers to local, regional, and central governments.

Data on the pension benefit-GDP ratio are sourced from the OECD (2023), "Pension spending" (indicator), `https://doi.org/10.1787/a041f4ef-en` (accessed on March 2, 2023). Pension spending is defined as all cash expenditure (including lump-sum payments) on old age and survivors' pensions. Data on GDP per capita are from International Comparison Program, World Bank | World Development Indicator Database, World Bank | Eurostat-OECD (`https://datatopics.worldbank.org/world-development-indicators/`) (accessed March 9, 2023). GDP per capita is based on purchasing power parity (PPP).

■

A.5 Supplementary Explanation for Section 5.3

A.5.1 Fixed Pension GDP Ratio

To solve for the government problem under the fixed pension-GDP ratio, we assume that the period-t pension-GDP ratio is fixed at $\varepsilon_t \in (0, 1]$:

$$\frac{\pi_t N_{t-1} b_t}{Y_t} = \varepsilon_t.$$

We reformulate this expression and obtain

$$b_t = \varepsilon_t \frac{n_t}{\pi_t} Q k_t, \tag{A.23}$$

$$b_{t+1} = \varepsilon_{t+1} \frac{n_{t+1}}{\pi_{t+1}} Q k_{t+1}. \tag{A.24}$$

With (A.24), the capital market clearing condition in (18) is reformulated as follows:

$$n_{t+1}k_{t+1} = \frac{\beta\pi_{t+1}}{1+\beta\pi_{t+1}}\left[\left(1-\tau_t\right)\left(1-\alpha\right)Qk_t - \frac{1}{\beta\pi_{t+1}}\frac{\varepsilon_{t+1}}{\alpha}n_{t+1}k_{t+1}\right],$$

or,

$$k_{t+1} = \frac{1}{1+\frac{1}{1+\beta\pi_{t+1}}\frac{\varepsilon_{t+1}}{\alpha}} \cdot \frac{1}{n_{t+1}} \cdot \frac{\beta\pi_{t+1}}{1+\beta\pi_{t+1}}\left(1-\tau_t\right)\left(1-\alpha\right)Qk_t. \tag{A.25}$$

With (A.23), the government budget constraint in (13) is reformulated as follows:

$$\tau_t (1-\alpha) Qk_t = \varepsilon_t Qk_t + n_{t+1} x_t. \tag{A.26}$$

Substitution of (A.25) into the fertility function in (17) leads to:

$$n_{t+1} = \frac{1}{\phi} \cdot \frac{\delta}{1+\delta+\beta\pi_{t+1}} \cdot \frac{n_{t+1} + \frac{\pi_{t+1}}{\alpha Q} \cdot \frac{1}{1+\frac{1}{1+\beta\pi_{t+1}}\frac{\varepsilon_{t+1}}{\alpha}} \cdot \frac{\beta\pi_{t+1}}{1+\beta\pi_{t+1}}}{n_{t+1} + \frac{\delta}{1+\delta+\beta\pi_{t+1}} \cdot \frac{\pi_{t+1}}{\alpha Q} \cdot \frac{1}{1+\frac{1}{1+\beta\pi_{t+1}}\frac{\varepsilon_{t+1}}{\alpha}} \cdot \frac{\beta\pi_{t+1}}{1+\beta\pi_{t+1}}},$$

or,

$$f(n_{t+1}) \equiv (n_{t+1})^2 + \bar{G}_1(\pi_{t+1}, \varepsilon_{t+1}) n_{t+1} - \bar{G}_2(\pi_{t+1}, \varepsilon_{t+1}), \tag{A.27}$$

where $\bar{G}_1(\pi_{t+1}, \varepsilon_{t+1})$ and $\bar{G}_2(\pi_{t+1}, \varepsilon_{t+1})$ are defined by

$$\bar{G}_1(\pi_{t+1}, \varepsilon_{t+1}) \equiv \frac{\delta}{1+\delta+\beta\pi_{t+1}} \cdot \left(\frac{\pi_{t+1}}{\alpha Q} \cdot \frac{1}{1+\frac{1}{1+\beta\pi_{t+1}}\frac{\varepsilon_{t+1}}{\alpha}} \cdot \frac{\beta\pi_{t+1}}{1+\beta\pi_{t+1}} - \frac{1}{\phi} \right),$$

$$\bar{G}_2(\pi_{t+1}, \varepsilon_{t+1}) \equiv \frac{1}{\phi} \cdot \frac{\delta}{1+\delta+\beta\pi_{t+1}} \cdot \frac{\pi_{t+1}}{\alpha Q} \cdot \frac{1}{1+\frac{1}{1+\beta\pi_{t+1}}\frac{\varepsilon_{t+1}}{\alpha}} \cdot \frac{\beta\pi_{t+1}}{1+\beta\pi_{t+1}}.$$

Given the property of $f(0) < 0$, we can solve (A.27) for n_{t+1} to obtain

$$n_{t+1} = \frac{-\bar{G}_1(\pi_{t+1}, \varepsilon_{t+1}) + \sqrt{\left(\bar{G}_1(\pi_{t+1}, \varepsilon_{t+1})\right)^2 + 4\bar{G}_2(\pi_{t+1}, \varepsilon_{t+1})}}{2}. \tag{A.28}$$

Under the assumption in (A.23), the period-t old-age consumption is

$$d_t = \frac{R}{\pi_t} n_t k_t + \varepsilon_t \frac{\pi_t}{n_t} Qk_t. \tag{A.29}$$

Under the assumption in (A.24), the lifetime income becomes

$$\begin{aligned}(1-\tau_t) w_t h_t + \frac{b_{t+1}}{R/\pi_{t+1}} &= (1-\tau_t) \frac{(1-\alpha) Q}{1-\phi n_{t+1}} k_t + \frac{\pi_{t+1}}{\alpha Q} \varepsilon_{t+1} \frac{n_{t+1}}{\pi_{t+1}} Qk_{t+1} \\ &= \left(\frac{1}{1-\phi n_{t+1}} + \frac{\varepsilon_{t+1}}{\alpha} \cdot \frac{1}{1+\frac{1}{1+\beta\pi_{t+1}}\frac{\varepsilon_{t+1}}{\alpha}} \cdot \frac{\beta\pi_{t+1}}{1+\beta\pi_{t+1}} \right) (1-\tau_t)(1-\alpha) Qk_t,\end{aligned} \tag{A.30}$$

where the equality in the first line comes from (16) and the equality in the second line comes from (A.25). We substitute (A.30) into (7) and (8) to obtain

$$c_t = \frac{1}{1+\delta+\beta\pi_{t+1}} \left(\frac{1}{1-\phi n_{t+1}} + \frac{\varepsilon_{t+1}}{\alpha} \cdot \frac{1}{1+\frac{1}{1+\beta\pi_{t+1}}\frac{\varepsilon_{t+1}}{\alpha}} \cdot \frac{\beta\pi_{t+1}}{1+\beta\pi_{t+1}} \right) (1-\tau_t)(1-\alpha) Qk_t,$$

$$d_{t+1} = \frac{\beta R}{1+\delta+\beta\pi_{t+1}} \left(\frac{1}{1-\phi n_{t+1}} + \frac{\varepsilon_{t+1}}{\alpha} \cdot \frac{1}{1+\frac{1}{1+\beta\pi_{t+1}}\frac{\varepsilon_{t+1}}{\alpha}} \cdot \frac{\beta\pi_{t+1}}{1+\beta\pi_{t+1}} \right) (1-\tau_t)(1-\alpha) Qk_t.$$

Consequently, we have

$$\ln c_t \approx \ln(1-\tau_t), \ln d_{t+1} \approx \ln(1-\tau_t). \tag{A.31}$$

The problem of the government in period t is to choose $\{\tau_t, x_t\}$ to maximize

$$\frac{\pi_t \omega}{n_t (1-\omega)} \ln d_t + \ln c_t + \delta \ln n_{t+1} h_{t+1} + \beta \pi_{t+1} \ln d_{t+1}$$

subject to the government budget constraint in (A.26), the fertility function in (A.28), and the consumption functions in terms of utility in (A.31). Substituting the constraints into the objective function, we can reformulate the problem as

$$\max_{\tau_t} (1+\beta\pi_{t+1}) \ln (1-\tau_t) + \delta\eta \ln (\tau_t (1-\alpha) - \varepsilon_t).$$

The first-order condition with respect to τ_t results in the determination of τ_t :

$$\tau_t = \frac{1}{1-\alpha} \cdot \frac{(1+\beta\pi_{t+1})\varepsilon_t + \delta\eta (1-\alpha)}{1+\beta\pi_{t+1}+\delta\eta}. \tag{A.32}$$

Utilizing (A.26) and (A.32), we derive the solution for x_t :

$$x_t = \frac{1}{n_{t+1}} \cdot \frac{\delta\eta (1-\alpha-\varepsilon_t)}{1+\beta\pi_{t+1}+\delta\eta} Q k_t,$$

and by substituting (A.32) into (A.25), we derive the growth rate of physical capital as:

$$\frac{k_{t+1}}{k_t} = \frac{1}{1+\frac{1}{1+\beta\pi_{t+1}}\frac{\varepsilon_{t+1}}{\alpha}} \cdot \frac{1}{n_{t+1}} \cdot \frac{\beta\pi_{t+1}(1-\alpha-\varepsilon_t)}{1+\beta\pi_{t+1}+\delta\eta} Q.$$

A.5.2 Exogenous Fertility

When $\{n_{t+1}\}$ is exogenously given, the individual's utility maximization problem becomes

$$\max_{s_t} \ln ((1-\tau_t) w_t h_t (1-\phi n_{t+1}) - s_t) + \delta \ln n_{t+1} h_{t+1} + \beta\pi_{t+1} \ln \left(\frac{R_{t+1}}{\pi_{t+1}} s_t + b_{t+1} \right)$$

given n_{t+1}. The first-order condition with respect to s_t leads to the following saving function:

$$s_t = \frac{\beta\pi_{t+1}}{1+\beta\pi_{t+1}} \left((1-\tau_t) w_t h_t (1-\phi n_{t+1}) - \frac{1}{\beta\pi_{t+1}} \cdot \frac{b_{t+1}}{R_{t+1}/\pi_{t+1}} \right), \tag{A.33}$$

and the associated consumption functions are as follows:

$$c_t = \frac{1}{1+\beta\pi_{t+1}} \left((1-\tau_t) w_t h_t (1-\phi n_{t+1}) + \frac{b_{t+1}}{R_{t+1}/\pi_{t+1}} \right), \tag{A.34}$$

$$d_{t+1} = \frac{\beta R_{t+1}}{1+\beta\pi_{t+1}} \left((1-\tau_t) w_t h_t (1-\phi n_{t+1}) + \frac{b_{t+1}}{R_{t+1}/\pi_{t+1}} \right). \tag{A.35}$$

Recall the conjecture of the pension benefits in (23):

$$b_{t+1} = \frac{\frac{\pi_{t+1}\omega}{n_{t+1}(1-\omega)}\tilde{B} + \tilde{C}}{\frac{\pi_{t+1}\omega}{n_{t+1}(1-\omega)}\tilde{E} + \tilde{F}} \cdot \frac{R}{\pi_{t+1}} n_{t+1} k_{t+1},$$

where $\tilde{B}$, $\tilde{C}$, $\tilde{E}$, and $\tilde{F}$ are constant. Following the same manner as in the main analysis, the conjecture is reformulated as follows:

$$b_{t+1} = \frac{\frac{\frac{\pi_{t+1}\omega}{n_{t+1}(1-\omega)}\tilde{B}+\tilde{C}}{\frac{\pi_{t+1}\omega}{n_{t+1}(1-\omega)}\tilde{E}+\tilde{F}}}{1+\frac{\frac{\pi_{t+1}\omega}{n_{t+1}(1-\omega)}\tilde{B}+\tilde{C}}{\frac{\pi_{t+1}\omega}{n_{t+1}(1-\omega)}\tilde{E}+\tilde{F}} \cdot \frac{\pi_{t+1}/R_{t+1}}{1+\beta\pi_{t+1}}} \cdot \frac{\beta\pi_{t+1}}{1+\beta\pi_{t+1}} \cdot (1-\tau_t)(1-\alpha) Q k_t. \tag{A.36}$$

Utilizing (16) and (A.36), we can write the lifetime income as follows:

$$
\begin{aligned}
&(1-\tau_t)\,w_t h_t\,(1-\phi n_{t+1}) + \frac{b_{t+1}}{R_{t+1}/\pi_{t+1}} \\
&= \left(1 + \frac{1}{R_{t+1}/\pi_{t+1}} \cdot \frac{\frac{\frac{\pi_{t+1}\omega}{n_{t+1}(1-\omega)}\tilde{B}+\tilde{C}}{\frac{\pi_{t+1}\omega}{n_{t+1}(1-\omega)}\tilde{E}+\tilde{F}} \cdot \frac{\beta\pi_{t+1}}{1+\beta\pi_{t+1}}}{1 + \frac{\frac{\pi_{t+1}\omega}{n_{t+1}(1-\omega)}\tilde{B}+\tilde{C}}{\frac{\pi_{t+1}\omega}{n_{t+1}(1-\omega)}\tilde{E}+\tilde{F}} \cdot \frac{1}{1+\beta\pi_{t+1}}}\right) \cdot (1-\tau_t)\,(1-\alpha)\,Qk_t. \qquad \text{(A.37)}
\end{aligned}
$$

Thus, with (A.34), (A.35), and (A.37), we obtain

$$
\ln c_t \approx \ln(1-\tau_t)\,, \ln d_{t+1} \approx \ln(1-\tau_t)\,. \qquad \text{(A.38)}
$$

The political objective function Ω_t now is given by

$$
\begin{aligned}
\Omega_t &= \frac{\pi_t\omega}{n_t\,(1-\omega)} \ln d_t + \ln c_t + \delta \ln n_{t+1}h_{t+1} + \beta\pi_{t+1}\ln d_{t+1} \\
&\approx \frac{\pi_t\omega}{n_t\,(1-\omega)} \ln\left(\frac{R_t}{\pi_t} n_t k_t + b_t\right) + (1+\beta\pi_{t+1})\ln(1-\tau_t) + \delta\eta \ln x_t.
\end{aligned}
$$

This is identical to that under endogenous fertility (see (A.5)). Thus, policy functions remain consistent regardless of whether fertility is exogenously or endogenously determined. Consequently, the comparative statics outcome outlined in Proposition 3, initially derived with endogenous fertility assumption, holds true even when fertility is assumed exogenous. However, quantitative comparative statics results are contingent upon fertility assumptions, as discussed in Section 5.3.2.

■

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