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Stochastic shocks and consumption externalities

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Abstract

This study introduces uncertainty into a simple growth model of common capital accumulation by considering consumption externalities. We consider two equilibrium concepts, including the Markov-perfect Nash equilibrium and cooperative solution, and examine how uncertainty affects the difference in the growth of common capital. Our results show that individuals' attitudes toward uncertainty change depending on the consumption externality type. Consumption externality types exist wherein the expected growth rate of common capital increases as uncertainty increases. We conclude that the problem of the "tragedy of the commons" is improved by greater uncertainty if individuals demonstrate jealousy and "keeping up with the Joneses," or admiration and "running away from the Joneses."

Keywords: Stochastic differential games; Resource extraction; Consumption externalities; Markov-perfect Nash equilibrium

JEL Classification Numbers: C73; E21; Q20

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1 Introduction

The "tragedy of the commons" is an extensively discussed significant environmental concern. This study extends this problem by considering the effects of consumption externalities and stochastic shocks on a dynamic capital accumulation game. Recent literature on the commons shows that consumption externalities worsen the overconsumption of commons. However, stochastic shocks can capture the uncertain impacts of climate change or disasters caused by global warming on common resources. Therefore, we develop a simple stochastic capital accumulation game model with consumption externalities and investigate how consumption externalities affect an individual's strategy under uncertainty.

Hardin (1968) clarified why common resources are over-consumed. He argues that each individual uses common land for private benefit; however, no one covers the maintenance cost, eventually making the land desolate and unusable. Levhari and Mirman (1980) first considered this problem comprehensively by analyzing a game-like situation of repeated fishing between two countries, where fishery resources increase at a constant rate each year and are consumed by the two countries. Lane and Tornell (1996) considered the voracity effect and found that positive productivity shocks negatively affect the growth rate of common capital in equilibrium when the number of players is sufficiently high. Additionally, similar studies exist on capital accumulation games, such as Fershtman and Nitzan (1991), Benhabib and Radner (1992), Tornell and Velasco (1992), Dockner and Sorger (1996), Dockner and Nishimura (2005), Dockner and Wagener (2014), and Mitra and Sorger (2014).

Resource extinction concern has recently expanded from a behavioral economics perspective. For example, one case considers that an individual's utility derives from their consumption and that of others. This property, called "consumption externality," has been examined by theoretical and empirical studies. Several studies, such as Long and Wang (2009) and Katayama and Long (2010), show that consumption externalities worsen the excessive extraction of common property resources. Hori and Shibata (2010) compared two cases that each agent committed to their action or did not. They show that the growth rate in the latter case can be higher than that in the former if agents exhibit a substantial admiration for others' consumption. Futagami and Nakabo (2021) consider a capital accumulation game incorporating present bias and consumption externalities and two equilibrium concepts, including a non-cooperative Nash equilibrium and a cooperative equilibrium. They show that the welfare level in the former equilibrium can be higher than that in the latter in the initial period; however, this relationship can be reversed later. Additionally, other studies exist on consumption externalities in growth models, such as Ljungqvist and Uhlig (2000), Liu and Turnovsky (2005), Turnovsky and Monteiro (2007), Mino (2008), Mino and Nakamoto (2016), Pham (2019), Yanase and Karasawa-Ohtashiro (2019), and Dioikitopoulos, Turnovsky, and Wendner (2020). Empirical evidence on consumption externalities was examined by Dynan and Ravina (2007), Stiglitz (2012), and Petach and Tavani (2021).

However, global climate change has necessitated growth models with common resources to account for stochastic shocks. For example, increasing seawater temperature causes fish previously caught to evacuate the area, resulting in catch fluctuations. Haurie, Krawczyk, and Roche (1994) consider a stochastic differential game based on monitoring an implicit cooperative solution with two fisheries. Jorgensen and Yeung (1996) present a stochastic differential game model of an n-firm fishery, where the value functions are nonlinear in the state and stochastic shocks influence the feedback strategies. Wirl (2008) included uncertainty in the pollution stock evolution and characterized equilibrium strategies for reversible and irreversible pollution. Other applications of the stochastic resource extraction model include those of Wang and Ewald (2010) and Sylenko (2021).[1] Apart from the aforementioned consumption externality, a renewable resource model is required to manage stochastic shocks.

This study incorporates stochastic shocks and consumption externalities into a simple growth model of common capital accumulation to explore the effect of consumption externalities and a stochastic shock on the "tragedy of the commons." According to Dupor and Liu (2003), consumption externalities can be classified into four types, and we consider two equilibrium concepts: a Markov-perfect Nash equilibrium and a cooperative solution. We hypothesize that uncertainty strengthens the effect that the property of

[1]For examples of other situations of stochastic differential games, see Yeung and Petrosyam (2008), Koethenburger and Lockwood (2010), and Leong and Huang (2010).

consumption externalities causes the overconsumption of a common property resource. Moreover, if the type of consumption externality is "jealousy and keeping up with the Joneses (KUJ)" or "admiration and running away from the Joneses (RAJ)," then each individual decreases consumption as uncertainty increases. Therefore, the expected growth rate increases with greater uncertainty. Conversely, if individuals exhibit "jealousy and RAJ" or "admiration and KUJ," they increase consumption, and the expected growth rate decreases as uncertainty increases. This is because the range of the degree of relative risk aversion of the utility function is determined by the type of consumption externality, which changes the behavior toward uncertainty. Furthermore, we compare how the expected growth rate changes as uncertainty increases in the two equilibrium solutions. If individuals exhibit jealousy toward others' consumption, the expected growth rate in the Markov-perfect Nash equilibrium changes more significantly than that in the cooperative solution as uncertainty increases. This study is novel because it combines consumption externalities with stochastically determined capital accumulation, which helps illustrate a realistic economy. Our study extends the work of Hori and Shibata (2010), who did not consider uncertain situations such as natural disasters and climate change.

The remainder of this paper is organized as follows. Section 2 sets up the stochastic dynamic game model with consumption externalities. Section 3 characterizes the Markov-perfect Nash equilibrium and examines the effect of uncertainty on the equilibrium outcomes. Section 4 examines the cooperative solutions. Section 5 compares the two expected growth rates. Finally, Section 6 summarizes the conclusions of this study.

2 Model

This study constructed a stochastic dynamic game model for common capital accumulation with consumption externalities. Individuals are concerned about their and their peer groups' consumption levels. Common capital fluctuates due to stochastic shocks, such as global climate change or disasters. Time is continuous and is denoted by $t \in [0, \infty)$.

2.1 Individuals

This economy has N homogeneous individuals who extract common capital stocks. The lifetime utility function of each individual is supposed to be additively separable over time. We denote ρ as the individual subjective discount rate and η as the intertemporal elasticity of substitution. The problem for individual i is to maximize the following discounted sum of the stream of instantaneous utilities:

$$\int_0^\infty u_{it} \exp(-\rho t) dt, \ i = 1, \cdots, N, \tag{1}$$

where u_{it} is the instantaneous utility function of individual i at period $t \geq 0$; $\rho > 0$. Instantaneous utility u_{it} is specified as follows:

$$u_{it} = \frac{\eta}{\eta - 1}(c_{it} \cdot (\bar{c}_{-it})^{-\alpha})^{1-\frac{1}{\eta}}, \ i = 1, \cdots, N,$$

where $\bar{c}_{-it} = \sum_{j \neq i} c_{jt}/(N-1)$, $\alpha < 1$, $\eta > 0$, and $\eta \neq 1$. Let c_{it} and $\bar{c}_{-it}$ denote the consumption of individual i in period t and the level of the average consumption of other individuals' consumption in period t, respectively; α represents the attitude toward the consumption of other individuals and the magnitude of this external effect. Each individual's utility is affected by others' average consumption level. According to Dupor and Liu (2003), consumption externalities are defined as follows:

Definition 1. *We define the consumption externality attitude as*

1. *jealousy if* $\partial u_i/\partial \bar{c}_{-i} < 0$ $(\alpha > 0)$ *and admiration if* $\partial u_i/\partial \bar{c}_{-i} > 0$ $(\alpha < 0)$, *and*
2. *"keeping up with the Joneses (KUJ)" if* $\partial^2 u_i/\partial c_i \partial \bar{c}_{-i} > 0$ $(\alpha(1-\eta) > 0)$ *and "running away from the Joneses (RAJ)" if* $\partial^2 u_i/\partial c_i \partial \bar{c}_{-i} < 0$ $(\alpha(1-\eta) < 0)$.

■

We used the following classification cases as the types: (a) jealousy and KUJ; (b) jealousy and RAJ; (c) admiration and KUJ; (d) admiration and RAJ. The KUJ (RAJ) indicates the case that the marginal utility of an individual's consumption increases (decreases) as the average level of others' consumption increases. KUJ (RAJ) indicates that the individual derives greater utility from their own increased (decreased) consumption when others increase their consumption.[2] Figure 1 shows the type of consumption externalities depending on α and η.

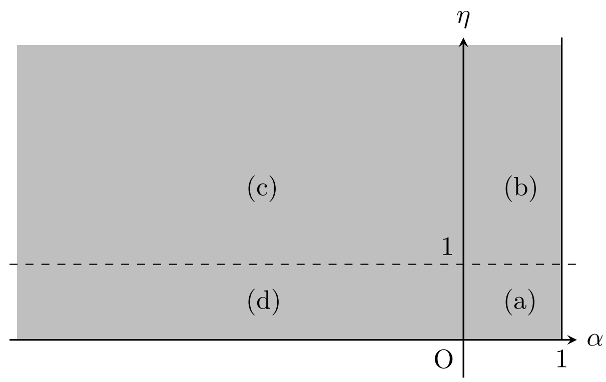


Figure 1: Four types of consumption externalities on an α - η plane

2.2 Capital accumulation

In this economy, each individual has access to physical capital, including fish stocks and grassland, and produces final goods using this common capital; the production technology has an Ak form. Previous studies have adopted this form of production technology, including Tornell and Velasco (1992), Long and Wang (2009), and Fujiwara (2011). Individuals divide common capital into two types: consumption and accumulation. The accumulation of natural resources is stochastically influenced by changes in the natural environment, such as disasters and climate change. Therefore, we incorporated a stochastic factor into capital accumulation.[3] The common capital dynamics is given by:

$$dk = \left(Ak - \sum_{j=1}^{N} c_j \right) dt + \sigma k dz, \text{ given } k_0, \tag{2}$$

[2]Futagami and Nakabo (2021) interpret KUJ (RAJ) as the belief that an individual wants (does not want) to consume in the same way that others do.

[3]We refer to Steger (2005) and Wälde (2011) for expressing the stochastic endogenous growth model.

where $k(>0)$ is the capital stock at t, $A(>0)$ is a constant productivity parameter, $k_0(>0)$ is the initial stock, and $dz(t)$ is a simple Brownian motion; that is, over period $(t, t+dt)$, change $dz_i \equiv z(t+dt)-z(t)$ is normally distributed with variance $\sigma^2 dt$ and zero mean, and non-overlapping increments are stochastically independent.

This study assumes the following:

Assumption 1. A, N, α, ρ, ψ satisfy

$$\psi < \frac{\rho}{A} < \min\left(1, \frac{1-\alpha}{N}\right), \tag{3}$$

where $\psi = (1-\alpha)\left(1-\frac{1}{\eta}\right)$.

The first segment of (3) of Assumption 1 ensures that the individual's lifetime utility is bounded. The second segment implies that the consumption growth rate becomes positive in the absence of uncertainty. This problem can be well defined by Assumption 1.

3 Non-cooperative resource extraction

Herein, we derive a Markov-perfect Nash equilibrium. In this case, each individual acts non-cooperatively, and their consumption depends on the capital stock at t. We assume that the strategy of individual i is a function, $\phi_i : \mathbb{R}_+ \mapsto \mathbb{R}_+$, that maps the current stock k_t to the individual's consumption rate c_{it}, that is, $c_{it} = \phi_i(k_t)$. The strategy space of individual i is the set of all the functions. A strategy profile is an N-tuple $(\phi_1, \phi_2, \cdots, \phi_N)$ comprising one strategy for each of the N individuals. We define a Markov-perfect Nash equilibrium as follows:

Definition 2. A Markov-perfect Nash equilibrium of the game is an N-tuple of strategies $(\phi_1^*(k), \phi_2^*(k), \cdots, \phi_N^*(k))$ such that, for each player i $(i = 1, 2, \cdots, N)$, function $\phi_i^*(k)$ is player i's best response that maximizes (1) to the $N-1$ tuple of Markovian strategies of others, $(\phi_1^*(k),\ \phi_2^*(k),\ \cdots, \phi_{i-1}^*(k),\ \phi_{i+1}^*(k),\ \cdots,\ \phi_N^*(k))$.

We denote that $c_{it}^* = \phi_{it}^*(k)$ is i's Markov-perfect Nash equilibrium strategy, and the time notation, t, is omitted for simplicity. The value function of i can be expressed as follows:

$$U_i^n(k) = \mathbb{E}\int_t^\infty \frac{\eta}{\eta-1}(\phi_i^*(k)\cdot[\bar{\phi}_{-i}^*(k)]^{-\alpha})^{1-\frac{1}{\eta}}\exp(-\rho(s-t))ds,$$

where $\bar{\phi}_{-i}^*(k) = \sum_{j\neq i}\phi_j^*(k)/(N-1)$. If the value function is continuously differentiable, the Hamilton-Jacobi-Bellman equation of this problem is given as follows:[4]

$$\rho U_i^n(k) = \max_{c_i}\left[\frac{\eta}{\eta-1}(c_i\cdot[\bar{\phi}_{-i}^*(k)]^{-\alpha})^{1-\frac{1}{\eta}} + \frac{dU_i^n}{dk}\left(Ak - c_i - \sum_{j\neq i}\bar{\phi}_j^*\right) + \frac{1}{2}\sigma^2k^2\frac{d^2U_i^n}{dk^2}\right], \tag{4}$$

subject to

$$c_i\begin{cases} \geq 0 & \text{if } k \geq 0, \\ = 0 & \text{if } k = 0. \end{cases}$$

We assume a symmetric equilibrium and linear strategies, that is, $c_i = \bar{\phi}_{-i}^*(k) = \phi(k) = \beta k + \gamma$, where β and γ are constants to be determined. Since we specified the objective function and the dynamics of capital accumulation, we can obtain the values of β and γ by comparing coefficients of k and k^2. Appendix A presents the necessary conditions for this maximization problem. We summarize the preceding arguments in the following lemma.

Lemma 1. *Assume that $\psi < \rho/A < \min\left(1, \frac{1-\alpha}{N}\right)$, and the variance is sufficiently small. The ratio of consumption to capital in the Markov-perfect Nash equilibrium, $\omega^n(= c/k)$, is, for any $k \in \mathbb{R}_+$,*

$$\omega^n = \frac{\rho - \psi A + \frac{\sigma^2}{2}\psi(1-\psi)}{1-\alpha-\psi N}, \tag{5}$$

[4]See Section 22 of Part II of the textbook of Kamien and Schwartz (2012) for an application to economics.

and the value function is

$$U^n(k) = \frac{1}{\psi}(\omega^n)^{\psi-1}k^{\psi}.$$

If $\sigma = 0$, ω^n coincides with ω_{BGP} in Hori and Shibata (2010), where $\omega_{BGP} = \dfrac{\rho - \psi A}{1-\alpha-\psi N}$. Depending on the degree of uncertainty, the consumption-capital ratio may hit zero if individuals exhibit either (a) jealousy and KUJ or (d) admiration and RAJ. Therefore, the conditions under which the consumption-capital ratio has a positive value must be considered. For $\psi < 0$,

$$-\frac{\rho - \psi A}{\psi(1-\psi)} > \frac{\sigma^2}{2}. \tag{6}$$

In contrast, $\psi > 0$ holds when the type of consumption externalities is either (b) jealousy and RAJ or (c) admiration and KUJ. In this case, consumption is positive even when the variance holds a significant value. The results are summarized as follows.

Lemma 2. *If each individual exhibits either (a) jealousy and KUJ or (d) admiration and RAJ, sufficiently small uncertainty guarantees positive consumption.*

We can state that the consumption-capital ratio is always positive when individuals exhibit the consumption externalities of (b) jealousy and RAJ or (c) admiration and KUJ. However, if they exhibit consumption externalities of (a) jealousy and KUJ or (d) admiration and RAJ, the ratio may be non-positive. The following proposition clarifies the effects of consumption externalities on the consumption-capital ratio in the Markov-perfect Nash equilibrium.

Proposition 1. *Suppose that $\psi < \rho/A < \min\left(1, \frac{1-\alpha}{N}\right)$ is satisfied. In the Markov-perfect Nash equilibrium, it holds that*

$$\frac{\partial \omega^n}{\partial \alpha} > 0 \text{ and } \frac{\partial}{\partial \sigma^2}\left(\frac{\partial \omega^n}{\partial \alpha}\right) > 0.$$

Proof. See Appendix B. ■

This proposition shows that both the consumption-capital ratio and the impact of uncertainty on the ratio increase as individuals become increasingly jealous of others' consumption levels. Moreover, since we can verify that $\partial\omega^n/\partial\alpha > \partial\omega_{BGP}/\partial\alpha$, uncertainty strengthens the effect that the property of consumption externalities causes overconsumption.

When all individuals choose symmetric strategies, we have $c_i = c$ for all $i = 1, \cdots, N$. Using (2), we can define the expected growth rate g as follows:

$$g \equiv \frac{\mathbb{E}[dk]}{kdt} = N(\omega_{SS} - \omega), \tag{7}$$

where $\omega_{SS} = A/N$, $\omega = c/k$. Using (5) and (7), we obtain the expected growth rate in the Markov-perfect Nash equilibrium[5] as follows:

$$g^n = N(\omega_{SS} - \omega^n) = \frac{A(1-\alpha) - [\rho + \frac{\sigma^2}{2}\psi(1-\psi)]N}{1-\alpha-\psi N}, \tag{8}$$

where g^n denotes the expected growth rate of the economy in the Markov-perfect Nash equilibrium, and the terminal condition holds (See Appendix A).

Lemma 3. *When the type of individuals' consumption externality is (a) jealousy and KUJ or (d) admiration and RAJ, always $g^n > 0$. When they exhibit the other type, $g^n > 0$ if*

$$\frac{A(1-\alpha) - \rho N}{\psi(1-\psi)N} > \frac{\sigma^2}{2}.$$

This indicates that a sufficiently significant uncertainty causes a negative expected growth rate when

[5] In this paper, when all individuals take linear strategy $c = \omega k$, (2) becomes

$$dk = (A - N\omega)kdt + \sigma k dz.$$

Now, the common capital dynamics is modeled as a geometric Brownian motion. As long as the initial value is positive, the capital stock is positive with probability 1 at all times, even though the Wiener process $z(t)$ is supposed to have unbounded negative values with positive probability. See Example 5.1.1 in Oksendal (2003).

individuals exhibit either (b) jealousy and RAJ or (c) admiration and RAJ.

Finally, we consider the effect of uncertainty on the consumption-capital ratio and expected growth rate.

Proposition 2. *Suppose that $\psi < \rho/A < \min\left(1, \frac{1-\alpha}{N}\right)$ is satisfied. In the Markov-perfect Nash equilibrium,*

1. $\dfrac{\partial \omega^n}{\partial \sigma^2} < 0$ *and* $\dfrac{\partial g^n}{\partial \sigma^2} > 0$ *in the case of (a) jealousy and KUJ and (d) admiration and RAJ;*
2. $\dfrac{\partial \omega^n}{\partial \sigma^2} > 0$ *and* $\dfrac{\partial g^n}{\partial \sigma^2} < 0$ *in the case of (b) jealousy and RAJ and (c) admiration and KUJ.*

Proof. See Appendix C. ■

This proposition shows that individuals who show consumption externalities of (a) jealousy and KUJ or (d) admiration and RAJ reduce their present consumption for future consumption when uncertainty increases, increasing the expected growth rate. In this case, the problem of the "tragedy of the commons" is improved by greater uncertainty. Instead, individuals who exhibit the consumption externalities of (b) jealousy and RAJ or (c) admiration and KUJ consume more common capital as uncertainty increases. In this case, the "tragedy of the commons" becomes worse by greater uncertainty. This proposition can be interpreted considering the relative risk-averse (RRA) parameters. Suppose that the equilibrium is symmetric. The objective function of the Markov-perfect Nash equilibrium is $\frac{\eta}{\eta-1}(c)^{\psi}$; then, $-c \cdot u''/u' = 1-\psi$ holds. Thus, $1-\psi$ is interpreted as an RRA parameter. If individuals exhibit the type of either (a) or (d) [(b) or (c)], $1-\psi > 1$ $[0 < 1-\psi < 1]$ holds. Steger (2005) noted that under risk aversion, the income effect reduces contemporaneous consumption, leading to increased savings and faster capital stock growth. Applying this in the current study shows that individuals conduct precautionary savings if they are type (a) or (d). However, if they are type (b) or (c), the substitution effect dominates the income effect; therefore, growth slows as uncertainty increases. Table 1 presents the key results.

Table 1: Key results of the Markov-perfect Nash equilibrium

Type of consumption externality			
(a) jealousy and KUJ	$\psi < 0$	$\partial\omega^n/\partial\sigma^2 < 0$	$\partial g^n/\partial\sigma^2 > 0$
(b) jealousy and RAJ	$\psi > 0$	$\partial\omega^n/\partial\sigma^2 > 0$	$\partial g^n/\partial\sigma^2 < 0$
(c) admiration and KUJ	$\psi > 0$	$\partial\omega^n/\partial\sigma^2 > 0$	$\partial g^n/\partial\sigma^2 < 0$
(d) admiration and RAJ	$\psi < 0$	$\partial\omega^n/\partial\sigma^2 < 0$	$\partial g^n/\partial\sigma^2 > 0$

Note: KUJ: keeping up with the Joneses; RAJ: running away from Joneses.

4 Cooperative solution

Herein, we consider the cooperative cases.[6] In this situation, each individual maximizes the sum of their utilities, where all individuals are aware of their influence on their opponents' utility. We examine how individuals with consumption externalities and stochastic shocks cooperate. Suppose that individuals are symmetric; they choose the exact consumption level in the solution, that is, $c_i = \bar{c}_{-i} = c$. Let c^{**} be the solution to this problem, and the value function $U^c(k)$ is given as

$$U^c(k) = \mathbb{E}\int_t^\infty N\frac{\eta}{\eta-1}(c^{**})^{(1-\alpha)(1-\frac{1}{\eta})}\exp(-\rho(s-t))ds,$$

We choose c^{**} to maximize this value function subject to (2). If the value function is continuously differentiable, the Hamilton-Jacobi-Bellman equation for this problem is given as

$$\rho U^c(k) = \max_c \left[N\frac{\eta}{\eta-1}(c)^{\psi} + \frac{dU^c}{dk}(Ak - Nc) + \frac{\sigma^2}{2}k^2\frac{d^2U^c}{dk^2}\right]. \tag{9}$$

We solve this maximization problem similarly to the Markov-perfect Nash equilibrium. The solution is a linear form, that is, $c(k) = \lambda k + \mu$, where λ and μ are constants to be determined. For the derivation of the solution, see Appendix D. Subsequently, we obtain the ratio of consumption to common capital in the cooperative solution as follows:

Lemma 4. *Assume that $\psi < \rho/A < 1$, and the variance is sufficiently small. The ratio of consumption*

[6] We refer to Subsection 3.2 in Futagami and Nakabo (2021) for an explanation of the cooperative solution.

to capital in the cooperative solution, $\omega^c (= c/k)$*, is, for any* $k \in \mathbb{R}_+$*,*

$$\omega^c = \frac{\rho - \psi A + \frac{1}{2}\sigma^2 \psi(1-\psi)}{N(1-\psi)}, \tag{10}$$

and the value function is

$$U^c(k) = \frac{1-\alpha}{\psi}(\omega^c)^{\psi-1} k^{\psi}. \tag{11}$$

In this Lemma, we assume that the variance is sufficiently small such that consumption does not hit zero. The condition that the variance satisfies is the same as in (6) of Section 3. Using (7) and (10), we obtain the expected growth rate in the cooperative solution as follows:

$$g^c = N\left(\omega_{SS} - \omega^c\right) = \frac{A-\rho}{1-\psi} - \frac{\sigma^2}{2}\psi. \tag{12}$$

Moreover, we obtain the conditions under which the expected growth rate becomes positive as follows:

Lemma 5. *In the cooperative solution, if individuals exhibit consumption externalities of (a) jealousy and KUJ or (d) admiration and RAJ, then it is always* $g^c > 0$*. When they exhibit the other type,* $g^c > 0$ *if*

$$\frac{A-\rho}{\psi(1-\psi)} > \frac{\sigma^2}{2}.$$

■

Analogous to the case of the Markov-perfect Nash equilibrium, we consider the effect of uncertainty on the ratio of consumption to capital and the expected growth rate.

Proposition 3. *Suppose that* $\psi < \rho/A < 1$ *is satisfied. In the cooperative solution, it holds that*

1. $\frac{\partial \omega^c}{\partial \sigma^2} < 0$ *and* $\frac{\partial g^c}{\partial \sigma^2} > 0$ *in the case of (a) jealousy and KUJ and (d) admiration and RAJ, and*

2. $\dfrac{\partial \omega^c}{\partial \sigma^2} > 0$ *and* $\dfrac{\partial g^c}{\partial \sigma^2} < 0$ *in the case of (b) jealousy and RAJ and (c) admiration and KUJ.*

Proof. See Appendix E. ∎

This proposition is similar to that in Section 3. We conclude that, in the cooperative case, the impact of uncertainty on consumption and the expected growth rate is similar to the Markov-perfect Nash equilibrium. The key results are summarized in Table 2.

Table 2: Key results of the cooperative solution

Type of consumption externality			
(a) jealousy and KUJ	$\psi < 0$	$\partial\omega^c/\partial\sigma^2 < 0$	$\partial g^c/\partial\sigma^2 > 0$
(b) jealousy and RAJ	$\psi > 0$	$\partial\omega^c/\partial\sigma^2 > 0$	$\partial g^c/\partial\sigma^2 < 0$
(c) admiration and KUJ	$\psi > 0$	$\partial\omega^c/\partial\sigma^2 > 0$	$\partial g^c/\partial\sigma^2 < 0$
(d) admiration and RAJ	$\psi < 0$	$\partial\omega^c/\partial\sigma^2 < 0$	$\partial g^c/\partial\sigma^2 > 0$

Note: KUJ: keeping up with the Joneses; RAJ: running away from Joneses.

5 Markov-perfect Nash equilibrium and cooperative solution Comparison

We compare the expected growth rates in the Markov-perfect Nash equilibrium and cooperative solution. The impact of the degree of consumption externalities and the increased uncertainty on the difference between the two growth rates are discussed.

5.1 Comparison of two expected growth rates

We compared the differences between the two expected growth rates. Taking the difference in the expected growth rates, (8) and (12), we obtain

$$\Delta g \equiv g^c - g^n = \frac{N-(1-\alpha)}{1-\alpha-\psi N}\left(\frac{\rho-\psi A}{1-\psi} + \frac{\sigma^2}{2}\psi\right). \tag{13}$$

Hence, we can state the following:

Proposition 4. Under Assumption 1, $g^n > g^c$ if and only if

$$1 - \alpha > N. \tag{14}$$

Proof. See Appendix F. ■

This proposition states that the expected growth rate of the Markov-perfect Nash equilibrium is higher than that of the cooperative solution if individuals exhibit substantial admiration for others' consumption levels, regardless of the degree of uncertainty. This aligns with the findings of Futagami and Nakabo (2021). Additionally, we considered the effect of uncertainty on the gap between the two expected growth rates. Thus, we propose the following proposition.

Proposition 5. *If individuals exhibit either (a) jealousy and KUJ or (d) admiration and RAJ, it holds that* $\dfrac{\partial(\Delta g)}{\partial \sigma^2} < 0$. *In contrast, it holds that* $\dfrac{\partial(\Delta g)}{\partial \sigma^2} > 0$ *in the case of either (b) jealousy and RAJ or (c) admiration and KUJ.*

This proposition states that the gap between g^n and g^c closes [expands] as uncertainty increases during (a) or (d) [(b) or (c)]. As noted previously, the value of the RRA parameter affects the relationship between uncertainty and the expected growth rate of common capital. Later, we consider the relationship between the expected growth rate and uncertainty for each type of consumption externality. These results were obtained by incorporating stochastic shocks into the model, which is crucial to this study. Notably, uncertainty shocks affect the Markov-perfect Nash equilibrium more than the cooperative solutions when $1-\alpha < N$ holds. For example, consider cases of (a) and (b). The absolute rate of change in the expected growth rate of the Markov-perfect Nash equilibrium is greater than that of the cooperative solution. In other words, when individuals exhibit jealousy toward others' consumption, the impact of uncertainty on their consumption strategies is more significant than that of the cooperative solution. The main results are summarized in Table 3.

Table 3: Correlation of some variables with the variance

Type of consumption externality			
(a) jealousy and KUJ	$0 < 1-\alpha < 1$	$g^c > g^n$	$\partial\|\Delta g\|/\partial\sigma^2 < 0$
(b) jealousy and RAJ	$0 < 1-\alpha < 1$	$g^c > g^n$	$\partial\|\Delta g\|/\partial\sigma^2 > 0$
(c) weak admiration and KUJ	$1 < 1-\alpha < N$	$g^c > g^n$	$\partial\|\Delta g\|/\partial\sigma^2 > 0$
strong admiration and KUJ	$1-\alpha > N$	$g^c < g^n$	$\partial\|\Delta g\|/\partial\sigma^2 > 0$
(d) weak admiration and RAJ	$1 < 1-\alpha < N$	$g^c > g^n$	$\partial\|\Delta g\|/\partial\sigma^2 < 0$
strong admiration and RAJ	$1-\alpha > N$	$g^c < g^n$	$\partial\|\Delta g\|/\partial\sigma^2 < 0$

Note: KUJ: keeping up with the Joneses; RAJ: running away from Joneses.

5.1.1 (a) Jealousy and KUJ

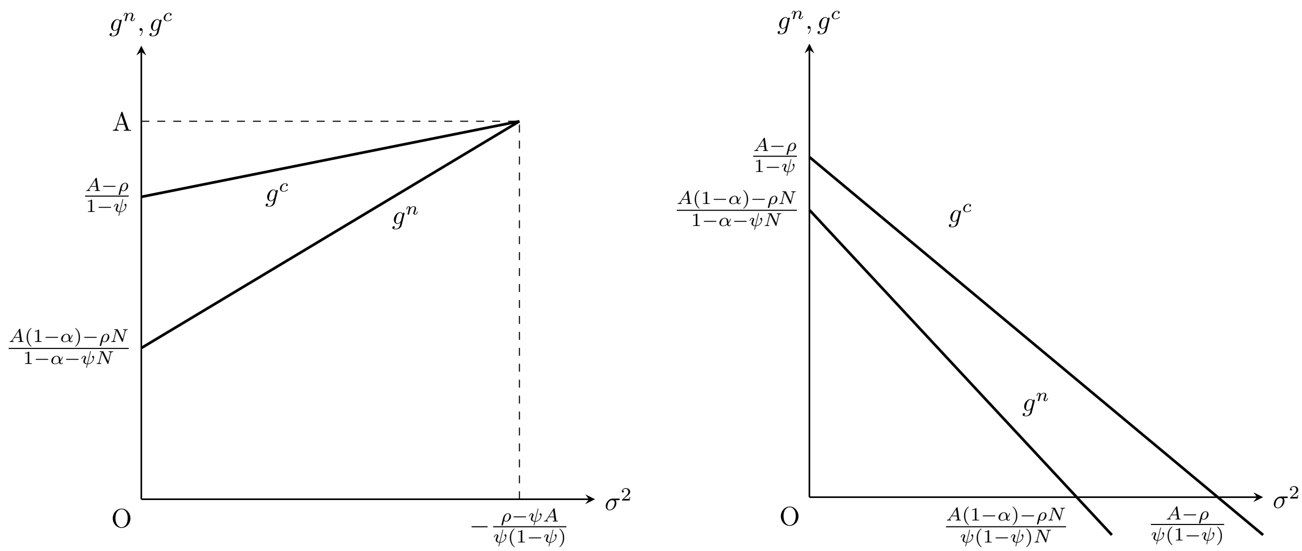
When individuals exhibit (a) jealousy and KUJ, the expected growth rate of the cooperative solution g^c is higher than that of the Markov-perfect Nash equilibrium g^n, and their gap shrinks as uncertainty increases. Figure 2-(a) shows the relationship between the expected growth rate and the uncertainty of this type. In this case, as uncertainty increases, the "tragedy of the commons," that is, the over-consumption of the common capital, is suppressed.

5.1.2 (b) Jealousy and RAJ

When individuals exhibit (b) jealousy and RAJ, the expected growth rate of the cooperative solution g^c is higher than that of the Markov-perfect Nash equilibrium g^n and the gap expands as uncertainty increases. Figure 2-(b) shows the relationship between the two expected growth rates for type (b). In this case, greater uncertainty worsens the "tragedy of the commons."

5.1.3 (c) Admiration and KUJ

When individuals show type (c), we need to distinguish between weak admiration, $1 < 1-\alpha < N$, and strong admiration, $N < 1-\alpha$. The former case is similar to (b) jealousy and RAJ. In the latter case, the expected growth rate of the Markov-perfect Nash equilibrium g^n is higher than that of the cooperative solution g^c and the gap expands as uncertainty increases. Figure 3 shows the relationship between the two expected growth rates and uncertainty for this type.



(a) expected growth rates of type (a)

(b) expected growth rates of type (b)

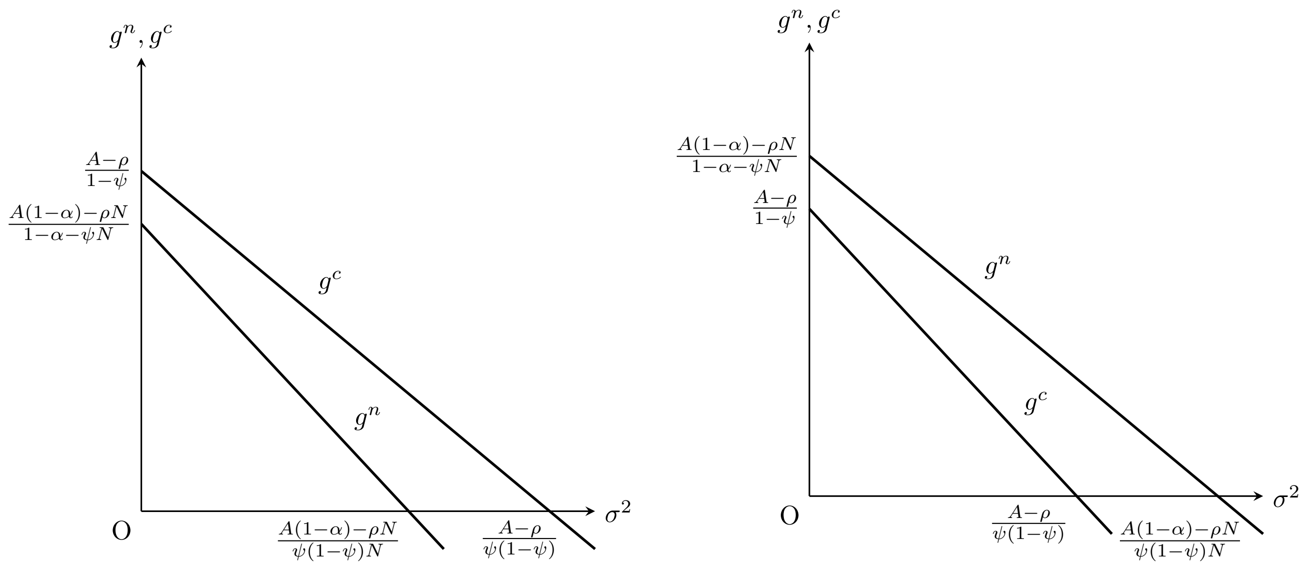
Figure 2: Relationship between the expected growth rate and uncertainty when individuals exhibit (a) or (b)

5.1.4 (d) Admiration and RAJ

When individuals show (d) similar to (c), we distinguish between weak admiration ($1 < 1 - \alpha < N$) and strong admiration ($N < 1 - \alpha$). The former case is similar to (a) jealousy and KUJ. In the latter case, the expected growth rate of the Markov-perfect Nash equilibrium g^n is higher than that of the cooperative solution g^c and the gap closes as uncertainty increases. Figure 4 shows the relationship between the two expected growth rates and uncertainty for this type.

6 Conclusion

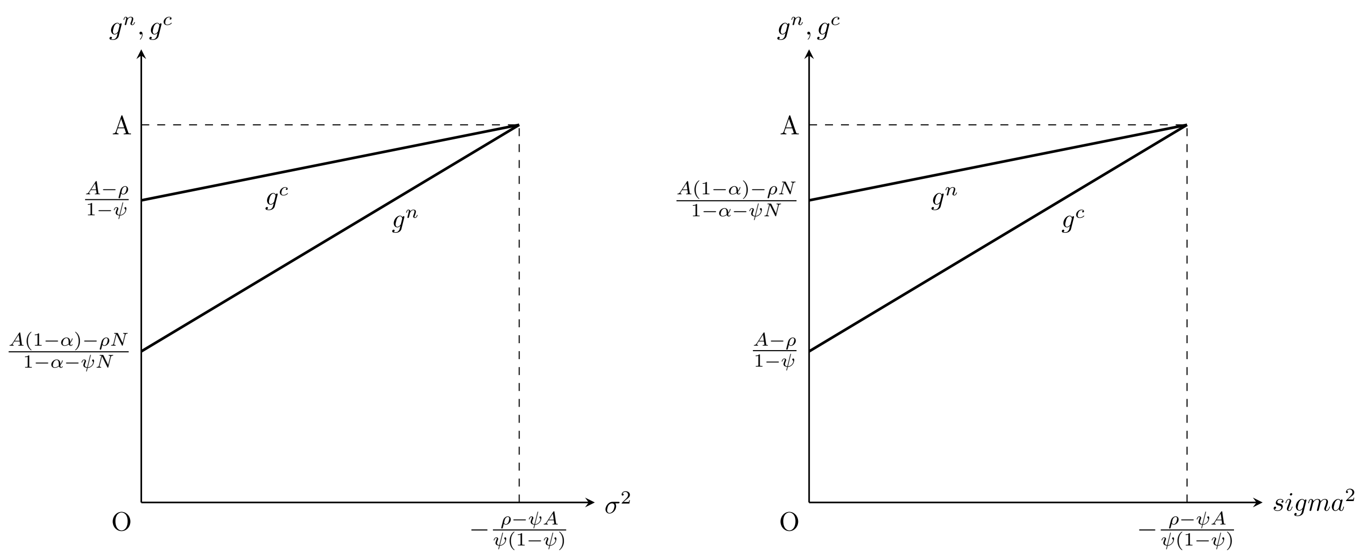
This study examines a stochastic capital accumulation game by considering different types of consumption externalities. Consumption externalities have the effect of overconsumption of a common property resource; however, we show that this effect is further strengthened by an increase in uncertainty. Moreover, the type of consumption externality changes individuals' attitudes toward uncertainty. If in-



(a) expected growth rates of type (c) with weak admiration

(b) expected growth rates of type (c) with strong admiration

Figure 3: Relationship between the expected growth rate and uncertainty when individuals exhibit (c)



(a) expected growth rates of type (d) with weak admiration

(b) expected growth rates of type (d) with strong admiration

Figure 4: Relationship between the expected growth rate and uncertainty when individuals exhibit (d)

dividuals exhibit either (a) jealousy and KUJ or (d) admiration and RAJ, they prepare for the stochastic shock and reduce consumption, causing a significant accumulation of common capital. In such cases, common capital can be sustainable without becoming extinct. Conversely, if individuals exhibit either (b) jealousy and RAJ or (c) admiration and KUJ, consumption increases and the expected growth rate decreases as uncertainty increases, worsening the problem of the "tragedy of the commons." Government interventions may be required to preserve common resources.

Appendix

A Derivation of (5)

The first-order condition of the maximization problem of (4) is

$$c_i^{-\frac{1}{\eta}} \cdot (\bar{\phi}^*_{-i}(k))^{-\alpha(1-\frac{1}{\eta})} = \frac{dU_i^n}{dk}. \tag{A1}$$

We assume a symmetric equilibrium and linear strategies, that is, $c_i = \bar{\phi}^*{}_{-i}(k) = \phi(k) = \beta k + \gamma$, where β and γ are constants to be determined. Hence, from (A1), we obtain

$$(\beta k + \gamma)^{\psi - 1} = \frac{dU^n}{dk}. \tag{A2}$$

Note that $\psi = (1-\alpha)(1-\frac{1}{\eta})$. Differentiating (A2) with respect to k yields

$$\frac{d^2 U^n}{dk^2} = \beta(\psi - 1)(\beta k + \gamma)^{\psi - 2}. \tag{A3}$$

At the equilibrium, from integrating (A1), it holds that,

$$U^n(k) = \frac{1}{\beta\psi}\phi(k)^\psi + \delta,$$

where δ denotes an integral constant. Using (A1)-(A3), we can rewrite (4) as follows:

$$\frac{\rho}{\beta\psi}\phi(k)^\psi + \rho\delta = \left[\frac{\eta}{\eta - 1} - N + A\frac{k}{\beta k + \gamma} + \frac{\sigma^2}{2}(\psi - 1)\frac{\beta k^2}{(\beta k + \gamma)^2}\right]\phi(k)^\psi. \tag{A4}$$

For (A4) to hold for any $k \in \mathbb{R}_+$, it must hold that

$$\gamma = \delta = 0 \text{ and } \beta = \frac{\rho - \psi A + \frac{\sigma^2}{2}\psi(1-\psi)}{1 - \alpha - \psi N},$$
$$\psi < \frac{\rho}{A} < \min\left(1, \frac{1-\alpha}{N}\right).$$

Subsequently, if individuals exhibit either (a) jealousy and KUJ or (d) admiration and RAJ, $\psi < 0$ holds. Therefore, variance σ^2 must be sufficiently small for the numerator of β to be positive. Thus, if individuals adopt the linear strategy and symmetric strategy, the ratio of consumption to common capital, $\omega^n = c/k$, is

$$\omega^n = \frac{\rho - \psi A + \frac{\sigma^2}{2}\psi(1-\psi)}{1 - \alpha - \psi N}.$$

Moreover, the value function becomes

$$U^n(k) = \frac{1}{\psi}(\omega^n)^{\psi - 1}k^\psi.$$

This equilibrium satisfies the terminal condition. From (8), we can rewrite k as $k_0 e^{g_n t}$. Introducing this expression into the above equation, we obtain

$$U^n(k) = \frac{1}{\psi}(\omega^n)^{\psi-1}k_0^\psi \exp\left(\psi\frac{A(1-\alpha)-[\rho+\frac{\sigma^2}{2}\psi(1-\psi)]N}{1-\alpha-\psi N}t\right).$$

Then, we have

$$\lim_{t\to\infty} U^n(k)\exp(-\rho t) = \frac{1}{\psi}(\omega^n)^{\psi-1}k_0^\psi \lim_{t\to\infty} \exp\left(\frac{(1-\alpha)(\psi A-\rho)-\frac{\sigma^2}{2}\psi^2(1-\psi)N}{1-\alpha-\psi N}t\right) = 0,$$

since $(1-\alpha) > 0$ and $\psi A - \rho < 0$. Therefore, the terminal conditions hold true.

B Proof of Proposition 1

To verify the effect of consumption externalities on the consumption-capital ratio in the Markov-perfect Nash equilibrium, we differentiate (5) with respect to α as follows:

$$\frac{\partial \omega^n}{\partial \alpha} = \frac{\rho + \frac{\sigma^2}{2}\psi^2}{(1-\alpha)(1-\alpha-\psi N)} > 0.$$

Additionally, we confirm the impact of uncertainty on the consumption-capital ratio. Differentiate (5) with respect to σ^2 and α yields

$$\frac{\partial^2 \omega^n}{\partial \alpha(\partial \sigma^2)} = \frac{\psi^2/2}{(1-\alpha)(1-\alpha-\psi N)} > 0.$$

Therefore, we have Proposition 2.

C Proof of Proposition 2

Differentiating the ratio of consumption to capital, (5), and the expected growth rate, (8), with respect to σ^2, we obtain

$$\frac{\partial \omega^n}{\partial \sigma^2} = \frac{1}{2} \cdot \frac{\psi(1-\psi)}{1-\alpha-\psi N} \text{ and } \frac{\partial g^n}{\partial \sigma^2} = -\frac{N}{2}\frac{\psi(1-\psi)}{1-\alpha-\psi N}.$$

In the case of (a) jealousy and KUJ [$\alpha > 0$ and $\alpha(1-\eta) > 0$] and (d) admiration and RAJ [$\alpha < 0$ and $\alpha(1-\eta) < 0$], $\psi < 0$. Therefore, $\frac{\partial \omega^n}{\partial \sigma^2} < 0$ and $\frac{\partial g^n}{\partial \sigma^2} > 0$ hold. Conversely, in the case of (b) jealousy and RAJ [$\alpha > 0$ and $\alpha(1-\eta) < 0$] and (c) admiration and KUJ, [$\alpha < 0$ and $\alpha(1-\eta) > 0$], $\psi > 0$. Therefore, we obtain $\frac{\partial \omega^n}{\partial \sigma^2} > 0$ and $\frac{\partial g^n}{\partial \sigma^2} < 0$.

D Derivation of (10)

The first-order condition of (9) is

$$(1-\alpha)c^{\psi-1} = \frac{dU^c}{dk}. \tag{A5}$$

Hence, we consider the linear solution, $c(k) = \lambda k + \mu$, where λ and μ are constants to be determined. From (A5), we obtain

$$U^c(k) = \frac{1-\alpha}{\lambda\psi}(\lambda k + \mu)^{\psi} + \nu, \tag{A6}$$

where ν is an integral constant, and second-order differentiating (A6) with respect to k yields

$$\frac{d^2 U^c}{dk^2} = (1-\alpha)(\psi-1)\lambda(\lambda k + \mu)^{\psi-2}. \tag{A7}$$

Using (A5)-(A7), we can rewrite (9) as follows:

$$\frac{\rho(1-\alpha)}{\lambda\psi}(c)^{\psi}+\rho\nu=\left[N\frac{\eta}{\eta-1}+(1-\alpha)\frac{(Ak-Nc)}{\lambda k+\mu}+\frac{\sigma^2}{2}(1-\alpha)(\psi-1)\frac{\lambda k^2}{(\lambda k+\mu)^2}\right](c)^{\psi}. \tag{A8}$$

For (A8) to hold for any $k \in \mathbb{R}_+$, it must hold that

$$\mu=\nu=0 \text{ and } \lambda=\frac{\rho-\psi A+\frac{\sigma^2}{2}\psi(1-\psi)}{(1-\psi)N},$$

$$\psi<\frac{\rho}{A}<1.$$

Then, if individuals exhibit either (a) jealousy and KUJ or (d) admiration and RAJ, $\psi<0$ holds. Therefore, variance σ^2 must be sufficiently small for the numerator of λ to be positive. Thus, the consumption-capital ratio is

$$\omega^c=\frac{\rho-\psi A+\frac{1}{2}\sigma^2\psi(1-\psi)}{N(1-\psi)}.$$

The value function becomes

$$U^c(k)=\frac{1-\alpha}{\psi}(\omega^c)^{\psi-1}k^{\psi}.$$

Subsequently, we determine whether the cooperative solution satisfied the terminal conditions. From (12), we can rearrange k as $k_0e^{g_ct}$ and substitute this form into $U^c(k)$ as follows:

$$U^c(k)=\frac{1-\alpha}{\psi}(\omega^c)^{\psi-1}k_0^{\psi}\exp\left(\left(\frac{A-\rho}{1-\psi}-\frac{\sigma^2}{2}\psi\right)\psi t\right).$$

Then, we obtain

$$\lim_{t\to\infty}U^c(k)\exp(-\rho t)=\frac{1-\alpha}{\psi}(\omega^c)^{\psi-1}k_0^{\psi}\lim_{t\to\infty}\exp\left(-\left(\frac{\rho-\psi A}{1-\psi}+\frac{\sigma^2}{2}\psi^2\right)t\right)=0,$$

since $\rho - \psi A > 0$ and $1 - \psi > 0$ from Assumption 1. Thus, the terminal condition is satisfied.

E Proof of Proposition 3

Differentiating (10) and (12) with respect to σ^2, we obtain

$$\frac{\partial \omega^c}{\partial \sigma^2} = \frac{\psi}{2N} \text{ and } \frac{\partial g^c}{\partial \sigma^2} = -\frac{\psi}{2}.$$

When individuals show the consumption externalities of (a) and (d), $\psi < 0$. In contrast, if they show (b) or (c), $\psi > 0$. Therefore, we obtain the following proposition.

F Proof of Proposition 4

First, we show that the sign of the last parenthesis in (13) is positive. If individuals exhibit (a) or (d) ($\psi < 0$), this can be confirmed using (6). If individuals exhibit (b) or (c) ($\psi > 0$), it is confirmed. Subsequently, under the assumption, $1 - \alpha - \psi N > 0$ holds. Therefore, the expected growth rate of the Markov-perfect Nash equilibrium is higher than that of the cooperative solution if and only if (14) holds.

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Declarations

Conflict of interest The author has no relevant financial or non-financial interests to disclose.

Ethical Approval Not applicable.

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