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Abstract

This study examines how corporate social responsibility (CSR) practices by oligopolistic firms impact pollution levels in a steady state. I develop a dynamic game model for polluting firms that adopt CSR. The analysis reveals that a firm's CSR awareness drives its production strategy to align with the socially optimal level in both open-loop Nash equilibrium and Markov perfect Nash equilibrium. Achieving this social optimum is possible if firms are fully committed to CSR. The study explores two scenarios: excess pollution or underproduction, which depend on the pollutant's impact on utility. Notably, when the pollutant's damage to utility is significant, even a modest commitment to CSR can effectively reduce excessive pollution. These findings offer valuable insights for government policy, suggesting that stringent environmental regulations might be less necessary if firms are attentive to CSR.

Keywords: Corporate social responsibility · Pollution · Oligopoly · Differential games

JEL Classification Numbers: H41 · L13 · Q20

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1 Introduction

1.1 Introduction

Corporate Social Responsibility (CSR) is a widely recognized concept where companies consider not only their profits but also the societal impact of their production activities. This global trend has seen many major firms engage in various CSR initiatives, such as environmental protection and human capital investment. The topic has garnered significant attention from economists and has been explored through both theoretical and empirical analysis.

In theoretical models of CSR, a firm maximizes social welfare including firm profits and consumer surplus. However, the focus here is specifically on Environmental CSR (ECSR), which examines the environmental impact of a firm's production practices when adopting ECSR. Most studies focus on one-shot or short-term analysis, with few addressing long-term effects. Environmental issues demand more than short-term solutions because individual economic agents often struggle to collaborate effectively. Long-term analysis is crucial for ensuring a sustainable environment for future generations. This study explores how firms implementing ECSR influence their long-term pollution levels. It introduces a payoff model for firms concerned with ECSR in a dynamic pollution game within an oligopolistic market. Assuming firms are homogeneous and have equal levels of CSR, each firm observes its pollution stock and adjusts its output to maximize its payoff. This study compares the social optimum with non-cooperative outcome to assess the relationship between CSR levels and long-term pollution stock.

I analyze two types of equilibria: the open-loop Nash equilibrium and the Markov perfect Nash equilibrium, which differ based on how firms use information about pollution level to make decisions. In the open-loop Nash equilibrium, firms set their strategies based on other firms' strategies time path, and these strategies remain consistent throughout. In contrast, in the Markov perfect Nash equilibrium, firms adjust their strategies based on the current pollution stock and rivals' strategies. Both equilibria are time-consistent, meaning firms have no incentive to deviate from their chosen strategies. Additionally, the Markov perfect Nash equilibrium meets the subgame perfect equilibrium. Given that the outcomes in both equilibria are similar in this study, I perform a qualitative analysis of the open-loop Nash equilibrium and a quantitative analysis of the Markov perfect Nash equilibrium to better understand firms' actual actions.

I find that increased awareness of CSR leads firms to adopt production strategies closer to the socially optimal level in both types of equilibria. The social optimum can be achieved if firms are fully committed to CSR. I consider two scenarios: excess pollution or underproduction, which depend on the pollutant's impact on utility. Notably, a slight increase in CSR is more effective in reducing pollution when the pollutant's impact on utility is high. Additionally, even in cases of underproduction and when pollution levels are below the socially optimal level, a high CSR level enhances social welfare. These findings suggest that CSR positively impacts the environment, benefiting firms that are attentive to CSR under various conditions.

1.2 Related literature

This study is closely related to two key areas of research. First, it connects with literature examining the impact of CSR on firm strategy in oligopolistic and other imperfect markets. Matsumura and Ogawa (2014) incorporated CSR into an endogenous timing game, demonstrating that CSR does not influence timing decisions when firm payoffs are symmetric. Wirl (2014) analyzed the strategic dynamics of CSR, finding that CSR activities are significantly higher in the Markov perfect Nash equilibrium compared to the cooperative solution and the open-loop Nash equilibrium. Kim et al. (2019) explored CSR and privatization in a mixed oligopoly, assuming that only private firm adopt CSR and analyzing the optimal privatization policy.

Second, this study contributes to the literature on differential game models for pollution problems. Differential games have been used as a valuable tool for dynamic pollution control. Van der Ploeg and De Zeeuw (1992) and Long (1992) were pioneers in applying differential game models to pollution issues. Dockner and Long (1993) provided first-best solutions and non-cooperative strategies, while Rubio and

Casino (2002) assessed the conditions under which Tsutsui and Mino's (1990) methods were not applicable. Subsequent studies, such as those by Kossioris et al. (2008), Li (2014), and Chen and Li (2023), have explored various settings. These studies typically address transboundary pollution caused by countries or regions. In contrast, Benchekroun and Long (1998) focused on optimal tax and subsidy policies for polluting oligopolists to achieve socially optimal production level, and Benchekroun and Chaudhuri (2011) examined how taxes can induce stable cartelization in polluting industries. Additionally, the analysis of CSR and environmental externalities has been addressed in static models by Hirose et al. (2017, 2020) and Lambertini and Tampieri (2023), as well as in dynamic game models by Lambertini, Palestini, and Tampieri (2016) and Feichtinger et al. (2016).

The dynamic pollution game in this study builds on the literature concerning polluting oligopolies and follows the approach of Benchekroun and Chaudhuri (2011). I develop a differential game involving n firms that are aware of CSR, incorporating the damage caused by pollution. Following Matsumura and Ogawa (2014), each firm's payoff is represented as a weighted sum of profit and social welfare, with the weight reflecting the level of CSR. The study most closely related to this work is Lambertini, Palestini, and Tampieri (2016). However, this study differs in its approach to incorporating CSR into the model. Lambertini, Palestini, and Tampieri (2016) use distinct differential parameters for social welfare and pollution damage and include both a CSR firm and a profit-maximizing firm in their model introducing firm heterogeneity.

The remainder of this article is organized as follows: Section 2 presents the model and derives the social optimum solution. Section 3 qualitatively derives and analyzes the open-loop Nash equilibrium. Section 4 examines the Markov perfect Nash equilibrium through numerical examples. Finally, Section 5 concludes the paper and discusses future research directions.

2 The model

2.1 Game model

Consider a simple model of polluting oligopolists based on Benchekroun and Long (1998) and Benchekroun and Chaudhuri (2011). n firms compete in quantity over continuous time $t \in [0, \infty)$. These firms produce homogeneous products and implement CSR practices. The inverse demand function is given by $P(Q) = \alpha - \beta Q$, where $Q = \sum_{i=1}^{n} q_i$ represents the total output and q_i is the output of firm i. The marginal cost is constant at c, with $\alpha > c$.

Let e_i denote each firm's emissions and $S(t)$ the cumulative amount of pollution. The dynamics of the pollution stock are as follows:

$$\frac{dS(t)}{dt} = \sum_{i=1}^{n} e_i - \delta S, \tag{1}$$

where $\delta \in [0, 1)$ is the pollution decay rate. Since emissions are linked to production, I set $e_i = q_i$. The assumption is that the damage caused by pollution follows a quadratic form, expressed as $\frac{\gamma}{2} S^2$, where γ is a constant.

Firms aim to maximize the weighted sum of their profits and social welfare. Following Matsumura and Ogawa (2014), the payoff for firm i is:

$$V_i = (1 - \theta)\pi_i + \theta SW, \tag{2}$$

where $\theta \in [0, 1)$, SW represents social welfare, and π_i denotes firm i's profit. The parameter θ reflects the weight of CSR in each firm's payoff. Since all firms are homogeneous in this study, the level of CSR is assumed to be constant across all firms. The profit π_i and social welfare SW are defined as follows:

$$\pi_i = P(Q)q_i - cq_i = (P(Q) - c)q_i = (\alpha - \beta Q - c)q_i, \tag{3}$$

$$SW = \sum_{i=1}^{n} \pi_i + CS - \frac{\gamma}{2}S^2 = (\alpha - c)Q - \frac{\beta}{2}Q^2 - \frac{\gamma}{2}S^2, \tag{4}$$

where $CS = \int P(Q)dQ - P(Q)Q = \frac{\beta}{2}Q^2$.

Under these settings, I analyze three types of equilibria: the social optimum, the open-loop Nash equilibrium, and the Markov perfect Nash equilibrium. In the socially optimal solution, a social planner determines the total production level that maximizes social surplus, taking into account the damage caused by pollution. This outcome serves as a benchmark and is compared with the other two equilibria.

2.2 The social optimum

In this subsection, I determine the optimal solution when firms can be directly controlled by a social planner. The planner selects the time path of total output, $Q(t)$, to maximize social surplus, given by (4), subject to (1)[1] and the initial condition $S(0) = S_0$. To summarize, the social planner's optimization problem is:

$$\max_{Q \geq 0} \int_0^\infty e^{-\rho t} \left[(\alpha - c)Q - \frac{\beta}{2}Q^2 - \frac{\gamma}{2}S^2 \right] dt,$$
$$\text{s.t. } \dot{S} = Q - \delta S, \; S(0) = S_0 > 0.$$

The solution to this maximization problem in the steady state can be expressed as

$$Q^S = \frac{\delta(\alpha - c)(\rho + \delta)}{\gamma + \beta\delta(\rho + \delta)}, \tag{5}$$

$$S^S = \frac{(\alpha - c)(\rho + \delta)}{\gamma + \beta\delta(\rho + \delta)}. \tag{6}$$

[1] A dot represents the time derivative in this study.

where Q^S represents the socially optimal total output and S^S denotes the socially optimal pollution stock in the long run. I derive the equilibrium outcome and compare it with the socially optimal solution as follows.

3 Open-loop

In this section, I determine the open-loop Nash equilibrium of the model. Following Dockner et al. (2000, p.85), each firm i chooses its time path $q_i(t)$ as its strategy to maximize its payoff, considering the time paths chosen by other firms. An open-loop Nash equilibrium consists of the set of time paths $\{q_1(t), \cdots, q_n(t)\}_{t=0}^{\infty}$ where firms commit to their chosen strategies for all future periods.

Firm i adopts the strategies of its rivals Q_{-i} and the initial condition S_0 as given, and maximizes (2) subject to (1) as follows:

$$\max_{q_i(t)\geq 0} \int_0^{\infty} e^{-\rho t}\{(1-\theta)\pi_i + \theta SW\}dt,$$
$$\text{s.t. } \dot{S} = (q_i + Q_{-i}) - \delta S, \ S(0) = S_0 > 0.$$

Let λ_i denote the shadow price of the pollution stock in firm i's optimal control problem. The Hamiltonian for this problem is given by:

$$\mathcal{H}_i = (1-\theta)(\alpha - c - \beta(q_i + Q_{-i}))q_i + \theta\left[(\alpha - c)(q_i + Q_{-i}) - \frac{\beta}{2}(q_i + Q_{-i})^2 - \frac{\gamma}{2}S^2\right] + \lambda_i(q_i + Q_{-i} - \delta S). \tag{7}$$

The first-order conditions are as follows:

$$\frac{\partial \mathcal{H}_i}{\partial q_i} = 0 : \alpha - c - \beta((2-\theta)q_i + Q_{-i}) + \lambda_i = 0$$
$$\dot{\lambda_i} = \rho\lambda_i - \frac{\partial \mathcal{H}_i}{\partial \lambda_i} : \dot{\lambda_i} = (\rho+\delta)\lambda_i + \theta\gamma S$$
$$\text{TVC} : \lim_{t\to\infty} e^{-\rho t}\lambda_i S = 0.$$

I consider the symmetric equilibrium where $q_i = q_j = q^o$ in the steady state. Using these first-order conditions and (1), each firm's strategy, total output, and the pollution stock in the open-loop equilibrium (q^0, Q^O, S^o) is:

$$q^o = \frac{\delta(\alpha-c)(\rho+\delta)}{\beta\delta(\rho+\delta)(n+1) + \theta(\gamma n - \beta\delta(\rho+\delta))}, \tag{8}$$
$$Q^o = nq^o, \tag{9}$$
$$S^o = \frac{nq^o}{\delta}. \tag{10}$$

Before examining the effect of θ, I first consider the case where firms focus solely on profit maximization $(\theta = 0)$.

Proposition 1. Suppose the firms are profit-seeking, $(\theta = 0)$. In this case, it follows that:

$$Q^o \gtreqless Q^S \text{and } S^o \gtreqless S^S$$

if and only if

$$\gamma n \gtreqless \beta\delta(\rho+\delta). \tag{11}$$

■

Proof. By taking the difference between Q^o and Q^S and using (5), (6), (8)-(10), I obtain:

$$Q^o - Q^S = \delta(S^o - S^S) = \frac{(1-\theta)\delta(\alpha - c)(\rho+\delta)}{[\beta\delta(\rho+\delta)(n+1) + \theta(\gamma n - \beta\delta(\rho+\delta))][\gamma + \beta\delta(\rho+\delta)]}(n\gamma - \beta\delta(\rho+\delta)) \gtreqless 0$$
$$\Leftrightarrow n\gamma \gtreqless \beta\delta(\rho+\delta).$$

■

This proposition outlines the conditions under which overproduction or underproduction occurs in the open-loop Nash equilibrium when all firms seek profit, relative to the social optimum. Two key factors determine whether firms overproduce or underproduce. The first factor is the "free rider effect," which is commonly used to explain issues related to pollution and resource extraction. In this model, it corresponds to the case when the number of firms is sufficiently large or the pollutant's impact on utility is significant. Conversely, when the number of firms is small, oligopolistic firms intentionally reduce their production to increase profits, leading to what is known as the "oligopoly effect." As a result, the total output in equilibrium is lower than the social optimum. Next, I examine how a firm's level of CSR affects overproduction or underproduction. Differentiating q^o with respect to θ yields:

$$\frac{\partial q^o}{\partial \theta} = -(\gamma n - \beta\delta(\rho+\delta))\frac{\delta(\alpha-c)(\rho+\delta)}{H(\theta)^2}, \ \frac{\partial^2 q^o}{\partial \theta^2} = \frac{2\delta(\alpha-c)(\rho+\delta)(\gamma n - \beta\delta(\rho+\delta))^2}{H(\theta)^3} \geq 0$$

where $H(\theta) \equiv \beta\delta(\rho+\delta)(n+1) + \theta(\gamma n - \beta\delta(\rho+\delta)) > 0$. As with Proposition 1, I can consider two cases under condition (11). When the "free rider effect" dominates the "oligopoly effect," both total output and pollution stock, which are proportional to each other, decrease with higher θ. These levels approach the socially optimal levels at $\theta = 1$. Conversely, when the "oligopoly effect" dominates the "free rider effect," total output and pollution stock increase towards the socially optimal levels. Additionally, Q^o and S^o are convex downward with respect to θ, as illustrated in Figures 1 and 2. Based on these results, the following proposition can be stated:

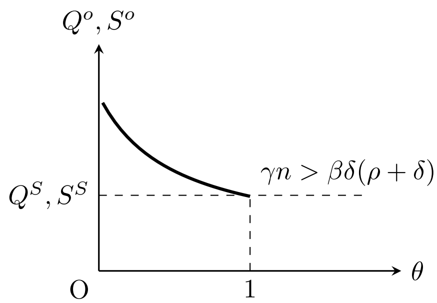

Figure 1: Q^o and S^o as a function of θ for $\gamma n > \beta\delta(\rho+\delta)$.

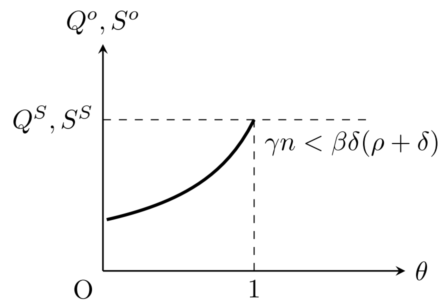

Figure 2: Q^o and S^o as a function of θ for $\gamma n < \beta\delta(\rho+\delta)$.

Proposition 2. In an open-loop Nash equilibrium,

1. if $\gamma n = \beta\delta(\rho+\delta)$ holds, the total output (Q^o) and the pollution stock, (S^o), in the steady state, correspond to the socially optimal levels (Q^S, S^S) without depending on θ.
2. If $\gamma n > \beta\delta(\rho+\delta)$ holds, $Q^o \geq Q^S$ and $S^o \geq S^S$ for any $\theta \in [0,1]$. Q^o and S^o decline as θ increases.
3. If $\gamma n < \beta\delta(\rho+\delta)$ holds, $Q^o \leq Q^S$ and $S^o \leq S^S$ for any $\theta \in [0,1]$. Q^o and S^o also increase as θ increases.

Moreover, a socially optimal level can be achieved in the open-loop Nash equilibrium if the firms are fully committed to CSR ($\theta = 1$). ■

For $\theta = 1$, the results are straightforward. In this scenario, the optimal strategy for each firm aligns with the social planner's objectives. Socially optimal outputs can be achieved if all firms in the oligopoly are fully aware of CSR. However, this assumption is often too stringent for the real economy. Therefore, examining the case with limited concern for CSR ($\theta \in [0,1)$). The first case of Proposition 2 shows that unique, noncooperative firms can always achieve a social optimum, regardless of θ. In the second case, where ($\gamma n > \beta\delta(\rho+\delta)$), firms overproduce, leading to an increase in pollution stock in the steady state, as described in Proposition 1. However, as θ increases, both total output and pollution stock in the long run decrease. This suggests that a stronger concern for CSR improves the pollution problem. In the third case, where ($\gamma n < \beta\delta(\rho+\delta)$), firms opt to reduce their output to increase their profits through oligopoly effects. This reduction in output, coupled with higher market prices, results in an economic loss. Here, we observe that total output and pollution stock increase with θ, but concern for CSR helps to mitigate the economic loss.

Finding a situation where firms priorities social welfare over profits may be challenging (i.e., $\theta > 0.5$). I explore how increasing θ from zero affects a firm's strategy and pollution stock within an open-loop Nash equilibrium.

Proposition 3. Increasing firm awareness of CSR can more effectively reduce excess pollution emissions than it can address economic losses resulting from an oligopoly, provided that the two damage parameters are sufficiently high.

Proof. See Appendix A. ■

Proposition 3 is a significant result, demonstrating that increased CSR awareness can effectively reduce excessive pollution. I assumed two cases: γ_1, γ_2 such that $n\gamma_1 > \beta\delta(\rho+\delta)$ and $n\gamma_2 < \beta\delta(\rho+\delta)$. By comparing the absolute values of the differential coefficients for each γ_i around $\theta = 0$, I find that the absolute value for γ_1 is larger than that for γ_2 when the sum of the two damage parameters is sufficiently high (i.e., when the free-rider effect is more dominant). This suggests that if CSR activities are taken into account, the government may not need to impose stringent pollution emission controls on oligopolistic markets.

4 Markov perfect Nash equilibrium

I now consider the scenario where firms use Markovian strategies. According to Dockner et al. (2000, p.86), each firm i observes the current pollution stock and selects a strategy, $q_i(t) = \phi(S(t))$ to maximize its payoff, taking into account the Markovian strategies of the other firms, $(\phi_1(.), \phi_2(.), \cdots, \phi_{i-1}, \phi_{i+1}, \cdots, \phi_n(.))$. A Markov perfect Nash equilibrium consists of a set of Markovian strategies $(\phi_1(.), \cdots, \phi_n(.))$ that satisfy subgame perfect equilibrium. This type of equilibrium is more realistic than the open-loop Nash equilibrium because each firm bases its optimal strategy on the observed pollution stock at time t. Although the Markov perfect Nash equilibrium produces outcomes similar to those of the open-loop Nash equilibrium, it more accurately reflects a firm's actual actions. However, analyzing the outcome of the open-loop Nash equilibrium is more straightforward. Therefore, while the previous section provides a qualitative analysis, I will now conduct a quantitative analysis of the Markov perfect Nash equilibrium.

4.1 Markov perfect Nash equilibrium

Given the rivals' strategies $Q_{-i} = \sum_{j \neq i} q_j(t) = \sum_{j \neq i} \phi_j(S(t))$, firm i's optimization problem is:

$$\max_{\phi_i(S(t))} \int_0^\infty e^{-\rho t} \left[(1-\theta)(\alpha - c - \beta(\phi_i + Q_{-i}))\phi_i + \theta \left\{ (\alpha - c)(\phi_i + Q_{-i}) - \frac{\beta}{2}(\phi_i + Q_{-i})^2 - \frac{\gamma}{2} S^2 \right\} \right] dt,$$
$$\text{s.t. } \dot{S} = \phi_i + Q_{-i} - \delta S.$$

I assume that firm i's value function is $W_i(S)$. The Hamilton-Jacobi-Bellman (HJB) equation for i's maximization problem at time t is:

$$\begin{aligned} \rho W_i(S) = \max_{q_i(t) \geq 0} & [(1-\theta)(\alpha - c - \beta(q_i + Q_{-i}))q_i \\ & + \theta \left\{ (\alpha - c)(q_i + Q_{-i}) - \frac{\beta}{2}(q_i + Q_{-i})^2 - \frac{\gamma}{2} S^2 \right\} + W_i'(S)(q_i + Q_{-i} - \delta S) \Big] \end{aligned} \tag{12}$$

The first-order condition of this problem is

$$\alpha - c - \beta[(2-\theta)q_i + Q_{-i}] + W_i'(S) = 0. \tag{13}$$

Assuming symmetric equilibrium $q_i = \phi_j(S) = q^M(S)$, the firms adopt the following production strategy:

$$q^M(S) = \frac{\alpha - c + W_i'(S)}{\beta(1 - \theta + n)}. \tag{14}$$

Let $W(S) = \frac{1}{2}ES^2 + FS + G$ for $0 \leq S \leq \bar{S} \equiv -\frac{F+\alpha-c}{E}$ where E, F, and G are constants to be determined[2]. See Appendix B for the derivation of these constants. By comparing the coefficients of S and S^2, we can determine E, F, and G because I have specified the value function as $W(S)$. In the Markov perfect Nash equilibrium, the pollution stock path is given by:

$$S(t) = e^{\eta t}(S_0 - S^M) + S^M,$$

where $S^M = -\dfrac{n(F + \alpha - c)}{\eta\beta(1 - \theta + n)}$, and $\eta = \dfrac{nE}{\beta(1 - \theta + n)} - \delta < 0$. Here, S^M represents the long-run pollution stock in the Markov perfect Nash equilibrium.

I can easily verify that the outcomes in the Markov perfect Nash equilibrium when $\theta = 0, 1$ correspond to those in the open-loop Nash equilibrium. When $\theta = 0$, the dynamic system of pollution stock does not impact the firm's optimization problem. Thus, the firm's optimal control problem simplifies to a static profit maximization problem. This result aligns with Proposition 1: If the firm is fully concerned about CSR ($\theta = 1$), then its strategy, total output, and steady-state pollution stock correspond to the socially optimal levels.

Example 1. For simplicity in analyzing the Markov perfect Nash equilibrium, I specify the following parameters: $\alpha - c = 1,\ \beta = 1,\ \delta = 1$. With these values, the following results hold:

1. The firm's equilibrium production strategy decreases ($q^M \downarrow$) as the harm caused by pollution damage increases ($\gamma \uparrow$).

2. Suppose pollution damage is sufficiently small($\gamma \downarrow$). When the pollution stock(S) is relatively low, the firm's Markov perfect Nash equilibrium increases ($q^M \uparrow$) as the firm's CSR level rises ($\theta \uparrow$). Conversely, when the pollution stock($S(t)$) is high, the firm's Markov perfect Nash equilibrium decreases ($q^M \downarrow$) as the CSR level increases ($\theta \uparrow$).

Proof. See Appendix C. ■

[2]The time script, t, in the value function is omitted due to the assumption of symmetric strategies.

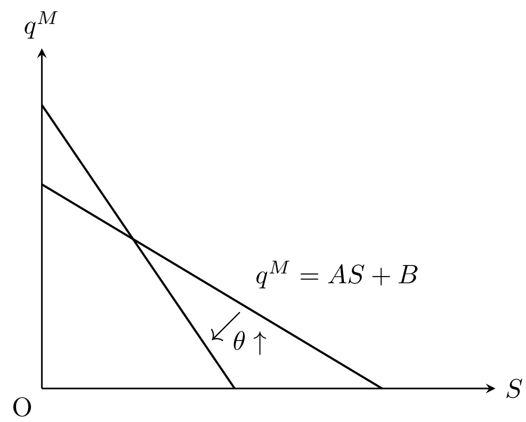

Figure 3: q^M with increasing θ if γ is sufficiently small.

The second item in Example 1 is illustrated in Figure 3.

4.2 Numerical examples

Analyzing the case where θ is between 0 and 1 is challenging due to the complexity of the analytical solution. Therefore, I use numerical examples to illustrate the effects. One case involves harmful pollution damage ($\gamma = 3$), and the other involves less harmful pollution damage ($\gamma = 0.1$). For both examples, the parameters are set as follows: $\alpha = 2$, $c = 1$, $\beta = 1$, $\delta = 1$, $\rho = 0.1$. These parameter values are consistent with the general literature on polluting oligopolies[3]. Figure 4 depicts the scenario with harmful damage, $\gamma = 3$, $n = 2$ and $n = 4$. In this case, the socially optimal pollution stock level is $S^S|_{\gamma=3} = 0.2683$. Excess pollution occurs in both situations, but it decreases as θ increases. Figure 5 illustrates the scenario with less harmful damage $\gamma = 0.1$, $n = 2$ and $n = 4$. The socially optimal pollution stock level is $S^S|_{\gamma=0.1} = 0.9167$. This scenario resembles a general oligopoly market problem, leading to lower production by firms. Consequently, the equilibrium pollution stock is lower than the socially optimal level, and the difference between these values decreases as θ increases.

To assess how the CSR level impacts pollution and economic losses, I define the deviation rate between

[3] Benchekroun and Chaudhuri (2011) adopt these parameter values.

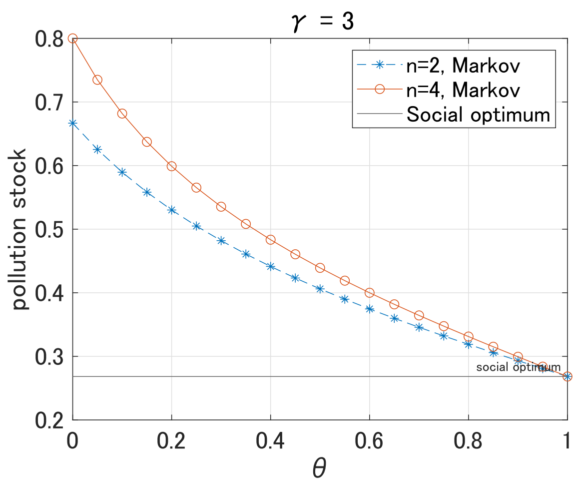

Figure 4: Pollution stock level for $\gamma = 3$

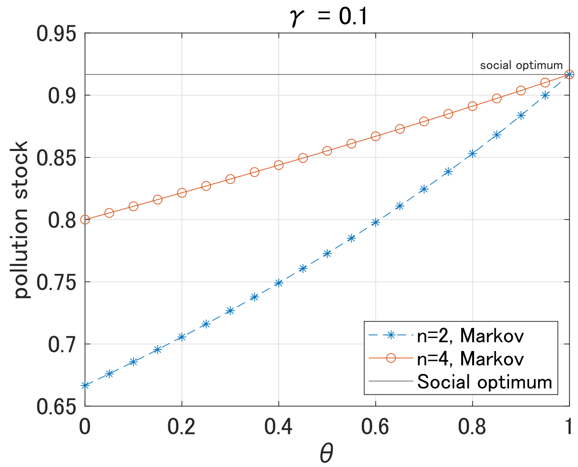

Figure 5: Pollution stock level for $\gamma = 0.1$

the pollution stock in the Markov perfect Nash equilibrium and the social optimum as: $\Delta S(\theta, \gamma, n) \equiv |\frac{S^M(\theta,\gamma,n) - S^S(\gamma)}{S^S(\gamma)}|$. The results are shown in Table 1. For $n = 2$ firms, when the damage is harmful ($\gamma = 3$), the equilibrium pollution stock is approximately 148% higher than the optimal pollution level when firms are profit-seeking ($\theta = 0$). Increasing the CSR value by 10% results in a reduction of approximately 28% in the deviation. This reduction continues at rates of approximately 22%, 18%, 15%, $\cdots$ for each additional 10% increase in θ. In the case of minimal damage ($\gamma = 0.1$), the deviation rate at $\theta = 0$ is about 27%. When θ increases from 0 to 0.1, this rate decreases by approximately 2.1%. However, as θ continues to increase, the deviation rate rises by approximately 2.2%, 2.3%, $\cdots$, 3.6%. These numerical examples indicate that improving CSR levels is effective in reducing excess pollution, even when the damage caused by pollution is relatively severe. Conversely, when pollution damage is minimal, achieving a sufficiently high CSR level is necessary to address economic losses due to oligopolistic behavior.

θ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$\Delta S(\theta, 3, 2)$	1.4848	1.1974	0.9759	0.7964	0.6450	0.5134	0.3959	0.2886	0.1884	0.0929	0
$\Delta S(\theta, 3, 4)$	1.9818	1.5411	1.2323	0.9952	0.8018	0.6367	0.4908	0.3580	0.2341	0.1157	0
$\Delta S(\theta, 0.1, 2)$	0.2727	0.2520	0.2302	0.2072	0.1829	0.1571	0.1297	0.1005	0.0693	0.0359	0
$\Delta S(\theta, 0.1, 4)$	0.1273	0.1156	0.1038	0.0918	0.0795	0.0670	0.0542	0.0412	0.0728	0.0141	0

Table 1: Deviation rate between the pollution stock in the MPNE and that of the social optimum.

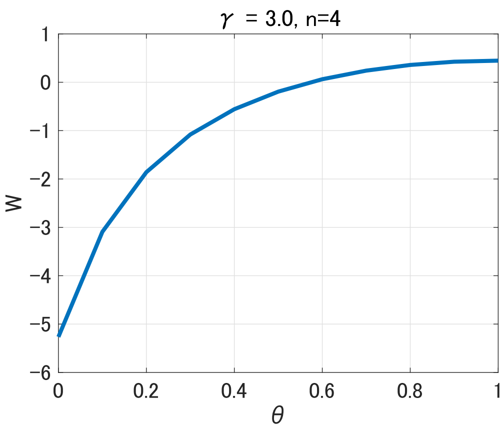

Figure 6: Aggregate social welfare for $\gamma = 3.0$, $n = 4$

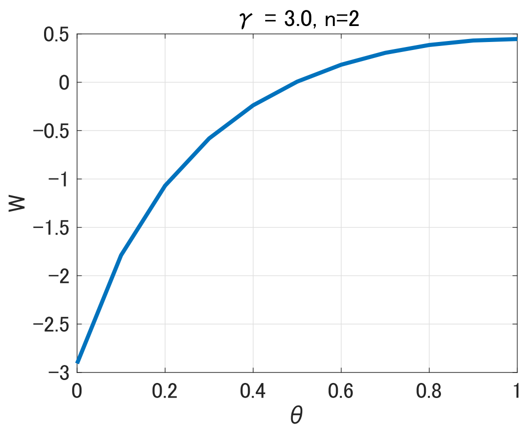

Figure 7: Aggregate social welfare for $\gamma = 3.0$, $n = 2$

4.3 Welfare analysis

Given that the damage caused by pollution is sufficiently small, the pollution stock remains below the socially optimal level for $\theta \in [0, 1)$ and increases as θ rises. This raises the question: do higher CSR levels negatively impact social welfare? To address this, I consider a numerical example using the parameter values outlined in the previous subsection. Using (4) and the parameter values specified in Section 5.2, the aggregate social welfare is defined as follows:

$$W \equiv \int_0^{\infty} e^{-\rho t} SW(t) dt = \int_0^{\infty} e^{-\rho t} \left\{ Q(S,t) - \frac{1}{2} Q(S,t)^2 - \frac{\gamma}{2} S(t)^2 \right\} dt.$$

Figures 6 and 7 depict the scenario where the damage is harmful $\gamma = 3.0$. Figures 8 and 9 illustrate cases where the damage is minimal $\gamma = 0.1$. In both scenarios, a higher θ positively impacts both reduced pollution and increased aggregate social welfare, regardless of whether the damage is harmful or minimal.

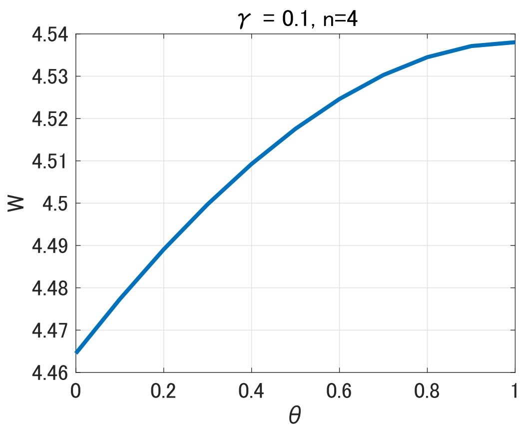

Figure 8: Aggregate social welfare for $\gamma = 0.1$, $n = 4$

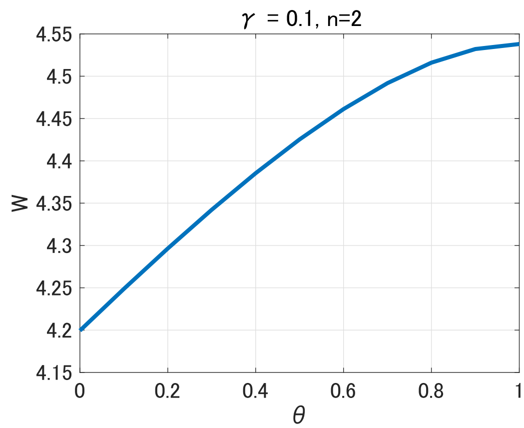

Figure 9: Aggregate social welfare for $\gamma = 0.1$, $n = 2$

5 Concluding remarks

I investigated how CSR impacts a firm's production strategy and pollution levels in an oligopolistic market. A firm's increased awareness of CSR leads its production strategy to align with the socially optimal level in both the open-loop Nash equilibrium and the Markov perfect Nash equilibrium. Socially optimal outcomes can be achieved if all firms are fully committed to CSR. I identified two scenarios where overproduction or underproduction can occur, depending on the number of firms and the impact of pollution on utility. In both cases, raising CSR levels helps mitigate excessive pollution or economic loss. Specifically, greater CSR awareness by firms reduces both production and pollution, particularly when pollution significantly harms utility. This finding has implications for government policy, suggesting that a high CSR level might eliminate the need for stringent environmental regulations. Moreover, even in the absence of excessive pollution, a higher CSR level enhances social welfare. Future research should explore how firms balance CSR awareness with their production strategies. In practice, firms would adjust their CSR levels as part of their strategic decisions, so modeling a dynamic two-stage game that includes both CSR level and production decisions would be a valuable direction for future studies.

Appendix

A Proof of Proposition 3

Assume values γ_1 and γ_2 exist such that $\gamma_1 n > \beta\delta(\rho+\delta)$ and $\gamma_2 n < \beta\delta(\rho+\delta)$. Then, the difference between the two differential coefficients for $\gamma = \gamma_1, \gamma_2$ is given by:

$$\left|\frac{\partial}{\partial\theta}q_o(\gamma_1)\right| - \left|\frac{\partial}{\partial\theta}q_o(\gamma_2)\right| = \frac{\delta(\alpha-c)(\rho+\delta)(\gamma_1 n - \beta\delta(\rho+\delta))}{H^2(\theta;\gamma_1)} - \frac{\delta(\alpha-c)(\rho+\delta)(\beta\delta(\rho+\delta)-\gamma_2 n)}{H^2(\theta;\gamma_2)}$$
$$= \underbrace{\frac{\delta(\alpha-c)(\rho+\delta)}{H^2(\theta;\gamma_1)H^2(\theta;\gamma_2)}}_{+} f(\theta) \tag{A1}$$

where $f(\theta) \equiv [(n(\gamma_1+\gamma_2)-2\psi)(n+1)^2\psi^2 + \theta^2(\gamma_1 n-\psi)(\gamma_2 n-\psi)(n(\gamma_1+\gamma_2)-2\psi) + 4\theta(n+1)\psi(\gamma_1 n - \psi)(\gamma_2 n-\psi)]$ and $\psi \equiv \beta\delta(\rho+\delta)$. From (A1), the sign of the difference depends on the sign of $f(\theta)$, and $f(0)$ becomes:

$$f(0) = (n(\gamma_1+\gamma_2)-2\psi)(n+1)^2\psi^2.$$

When γ_1, γ_2, and n are sufficiently large, the sign of $f(\theta)$ around $\theta = 0$ becomes positive, indicating that overproduction occurs in this scenario. Consequently, Proposition 3 is upheld.

B Derivation of coefficients

In line with the general literature on differential games of pollution (Dockner et al. (2000); Benchekroun and Chaudhuri (2011)), each firm's value function is expressed in a linear quadratic form. The value function is given by:

$$W(S) = \frac{1}{2}ES^2 + FS + G, \tag{A2}$$

where E, F and G are constants to be determined. Substituting (14) and (A2) into (12) yields the following equation:

$$
\begin{aligned}
\rho\left(\frac{1}{2}ES^2 + FS + G\right) &= (1-\theta)(\alpha - c - \beta n q^M)q^M + \theta\left[(\alpha - c)nq^M - \frac{\beta}{2}(nq^M)^2 - \frac{\gamma}{2}S^2\right] + (ES+F)(nq^M - \delta S) \\
&= \left[\frac{nE^2}{\beta(1-\theta+n)} - \delta E - \frac{\gamma\theta}{2} - \frac{n(1-\theta+n\theta/2)E^2}{\beta(1-\theta+n)^2}\right]S^2 \\
&+ \left[\frac{(\alpha-c)(1-\theta+\theta n)+Fn}{\beta(1-\theta+n)}E - \frac{2nE(\alpha-c+F)(1-\theta+n\theta/2)}{\beta(1-\theta+n)^2} + \frac{nE(\alpha-c+F)}{\beta(1-\theta+n)} - \delta F\right]S \\
&+ \frac{(\alpha-c+F)\{(\alpha-c)(1-\theta+\theta n)+Fn\}}{\beta(1-\theta+n)} - \frac{n(1-\theta+n\theta/2)(\alpha-c+F)^2}{\beta(1-\theta+n)^2}.
\end{aligned}
$$

Comparing the coefficients of S and S^2 on both sides of the above equations, the constants E, F and G are given by:

$$
\begin{aligned}
E &= \frac{1-\theta+n}{2n^2(2-\theta)}[\beta(2\delta+\rho)(1-\theta+n) - \sqrt{\beta^2(2\delta+\rho)^2(1-\theta+n)^2 + 4n^2\beta\gamma\theta(2-\theta)}] \leq 0, \\
F &= \frac{(\alpha-c)[(1-\theta+n)^2 - (1-\theta)(2-\theta)n]E}{\beta(\rho+\delta)(1-\theta+n)^2 - n^2(2-\theta)E} \leq 0, \\
G &= \frac{\alpha-c+F}{\beta\rho(1-\theta+n)^2}\left[(1-\theta)^2 + n\theta(1-\theta) + \frac{n^2\theta}{2}\right].
\end{aligned}
$$

Using E and F, (14) can be rewritten as follows:

$$
q^M(S) = \frac{E}{\beta(1-\theta+n)}S + \frac{\alpha-c+F}{\beta(1-\theta+n)} = AS + B, \tag{A3}
$$

where

$$
\begin{aligned}
A &= \frac{1}{2\beta n^2(2-\theta)}[\beta(2\delta+\rho)(1-\theta+n) - \sqrt{\beta^2(2\delta+\rho)^2(1-\theta+n)^2 + 4n^2\beta\gamma\theta(2-\theta)}] \leq 0, \\
B &= \frac{(\alpha-c)(\rho+\delta) - (\alpha-c)(n-1)(1-\theta)A}{\beta(\rho+\delta)(1-\theta+n) - \beta n^2(2-\theta)A} > 0.
\end{aligned}
$$

C Proof of Lemma 1

For simplicity, some parameters are specified as $\alpha - c = 1$, $\beta = 1$ and $\delta = 1$. Differentiating A and B with respect to γ yields:

$$\begin{aligned}
\frac{\partial A}{\partial \gamma} &= -\frac{\theta}{\sqrt{h(\theta,\gamma)}} \leq 0, \\
\frac{\partial B}{\partial \gamma} &= \frac{1+\rho}{(\beta(\rho+\delta)(1-\theta+n) - \beta n^2(2-\theta)A)^2}\frac{\partial A}{\partial \gamma}[n^2(2-\theta) - (n-1)(1-\theta)(1-\theta+n)] \leq 0.
\end{aligned}$$

where $h(\theta,\gamma) = (2+\rho)^2(1-\theta+n)^2 + 4n^2\gamma\theta(2-\theta)$. Therefore, $\dfrac{\partial q^M}{\partial \gamma} \leq 0$ holds. Next, differentiating A with respect to θ yields:

$$\frac{\partial A}{\partial \theta} = \frac{1}{2n^2(2-\theta)^2}\left[-(2-\theta)\left\{2+\rho+\frac{h'(\theta,\gamma)}{2\sqrt{h(\theta,\gamma)}}\right\} + \underbrace{(2+\rho)(1-\theta+n) - \sqrt{h(\theta,\gamma)}}_{\leq 0}\right].$$

where

$$\begin{aligned}
2+\rho+\frac{h'(\theta,\gamma)}{2\sqrt{h(\theta,\gamma)}} &= \frac{1}{\sqrt{h(\theta)}}\left((2+\rho)\sqrt{h(\theta,\gamma)} + \frac{1}{2}h'(\theta,\gamma)\right) \\
&= \frac{1}{\sqrt{h(\theta,\gamma)}}[(2+\rho)(\underbrace{\sqrt{h(\theta,\gamma)} - (2+\rho)(1-\theta+n)}_{\geq 0}) + 4n^2\gamma(1-\theta)] \geq 0.
\end{aligned}$$

Therefore, $\dfrac{\partial A}{\partial \theta} \leq 0$ holds. In addition, differentiating B with respect to θ yields:

$$\frac{\partial B}{\partial \theta} = \frac{-n(1+\rho)A - n^2(n-1)A^2 - A'(1+\rho)(n^2+(1-\theta)(\theta n+1-\theta))}{((1+\rho)(1-\theta+n) - n^2(2-\theta)A)^2}\begin{cases}\leq 0 \text{ if } \theta \text{ and } \gamma \text{ are sufficiently large,} \\ \geq 0 \text{ otherwise.}\end{cases}$$

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