Discussion Papers In Economics And Business

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YUKI UCHIDA, TETSUO ONO

Discussion Paper 25-01

January 2025

Graduate School of Economics
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Political Economy of Non-Compliance with the Golden Rule of Public Finance*

Yuki Uchida† Seikei University

Tetsuo Ono‡ The University of Osaka

Abstract

This study examines the limitations of political equilibrium in fiscal policy formation by short-sighted governments that represent only the currently living generations, as compared to an allocation determined by a long-lived planner who values both current and future generations. Using an overlapping-generations model calibrated to Germany, Japan, and the United Kingdom, we evaluate the role of the Golden Rule of Public Finance (GR), which restricts deficit financing to public investment. The findings reveal that (i) reduced GR compliance shifts fiscal burdens from middle-aged voters to future generations and older adults, resulting in spillover effects; (ii) GR compliance is significantly influenced by the elasticity of public capital to investment, preferences for public goods, and GDP growth rates; and (iii) non-compliance with the GR causes political equilibrium to diverge from the planner's optimal allocation.

- Keywords: Fiscal Rule; Golden Rule of Public Finance; Probabilistic Voting; Overlapping Generations; Political Distortions
- JEL Classification: D70, E62, H63

*This paper was previously circulated under the title "Borrowing to Finance Public Investment: A Politico-Economic Analysis of Fiscal Rules." The authors gratefully acknowledge financial support from Grant-in-Aid for Early-Career Scientists from the Japan Society for the Promotion of Science (No. 24K16388, Uchida) and Grant-in-Aid for Scientific Research (C) from the Japan Society for the Promotion of Science (No. 24K04926, Ono).

†Yuki Uchida: Faculty of Economics, Seikei University, 3-3-1 Kichijoji-Kitamachi, Musashino, Tokyo 180-8633, Japan E-mail: yuchida@econ.seikei.ac.jp.

‡Tetsuo Ono: Graduate School of Economics, The University of Osaka, 1-7, Machikaneyama, Toyonaka, Osaka 560-0043, Japan E-mail: tono@econ.osaka-u.ac.jp.

1 Introduction

Many developed countries have incurred large budget deficits, thereby accumulating public debt for the past decades (OECD, 2021). To cope with large budget deficits and their associated problems, they have implemented various types of fiscal rules such as debt, deficit, expenditure, revenue and golden rules (Budina et al., 2012; Schaechter et al., 2012; Wyplosz, 2013; Lledó et al., 2017; Caselli et al., 2018). These fiscal rules are aimed at regulating fiscal policy from a long-term perspective, and in some countries, they are constitutionally mandated, making their revision exceedingly rare. In contrast, fiscal policies, including government debt issuance, are decided annually through parliamentary deliberations, provided they in principle adhere to the fiscal rules. As a result, accounting for the political influence of fiscal rules on fiscal policy decisions is crucial for assessing the welfare implications of fiscal policy, a topic that has attracted considerable attention from prominent researchers (e.g., Bisin et al., 2015; Halac and Yared, 2018; Coate and Milton, 2019; Bouton et al., 2020; Dovis and Kirpalani, 2020, 2021; Piguillem and Riboni, 2020, 2024).

Introducing fiscal rules constrains fiscal policy, particularly the issuance of public debt, which is expected to affect capital accumulation, economic growth, and welfare across generations through market equilibrium. Political economy studies examining these impacts include Bassetto and Sargent (2006), Azzimonti et al. (2016), Barseghyan and Battaglini (2016), Arai et al. (2018), Andersen (2019), and Uchida and Ono (2021). These studies assume that fiscal policies are determined by governments composed of elected politicians or represented by median voters, who adhere to fiscal rules. Specifically, Bassetto and Sargent (2006) focused on the Golden Rule of Public Finance (hereafter abbreviated as GR), which was adopted in Germany and is currently in effect in countries such as Japan and the United Kingdom (Kumar et al., 2009).[1] The GR permits budget deficits to finance public investment but prohibits their use for current expenditures (Buiter et al., 1993; Corsetti and Roubini, 1996; Robinson, 1998). To evaluate the GR's role from a social welfare perspective, Bassetto and Sargent (2006) calibrated an overlapping-generations model to the U.S. economy and demonstrated that the GR may approximate the first-best allocation that maximizes Benthamite-type social welfare under resource and technology constraints. Their findings suggest that, as long as governments comply with the GR, it can contribute to achieving the first-best allocation.

The assumption that governments consistently formulate fiscal policies in compliance with the GR is not always realistic. As depicted in Figure 1, Germany, Japan, and the United Kingdom exhibit a lower public investment-to-GDP ratio than the fiscal deficit-to-GDP ratio for many years, indicating that non-compliance with the GR had become commonplace in these countries.[2] This observation raises the following questions: Why did these countries fail to

[1]Under the 2009 amendment to its Basic Law, Germany abolished the GR and introduced a new fiscal rule (Kumar et al., 2009).

[2]In Japan, a waiver of the GR has been approved by the Diet almost every year since 1975 (Kumar et al., 2009). In the United Kingdom, the GR was adopted in 1997, but it was met only for the first few years (Wyplosz,

comply with the GR despite having adopted it? How does non-compliance with the GR affect fiscal policy formation and, consequently, the intergenerational distribution of fiscal burdens? To what extent does non-compliance with the GR result in allocative inefficiency, measured against the allocation that maximizes the Benthamite-type social welfare, when accounting for individuals' responses to fiscal policy?

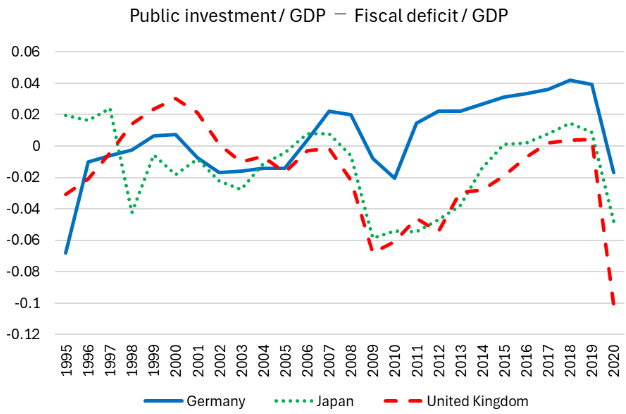

Figure 1: Time series data of the difference between the public investment-to-GDP ratio and the fiscal deficit-to-GDP ratio, 1995-2020, for Germany, Japan, and the United Kingdom.
Note. The indicator in the figure takes a negative value if a fiscal deficit arises and the public investment-to-GDP ratio is less than the fiscal deficit-to-GDP ratio. It takes a positive value if either (i) a fiscal deficit arises but the public investment-to-GDP ratio exceeds the fiscal deficit-to-GDP ratio, or (ii) a fiscal surplus arises.

To address the abovementioned questions, we employ a three-period overlapping-generations model with physical and public capital. Each generation consists of identical individuals who live through three stages: the young, who make no economic decisions; the middle-aged, who work; and the older adults, who are retired. Public investment, along with the inherited public capital from previous periods, serves as an input in the public capital formation process, thereby contributing to labor productivity. Governments are short-sighted in that they act as elected representatives for only living generations. They finance public investment in public capital and the provision of unproductive public goods through taxes on capital and labor, as well as through public debt issuance.

Although taxes and public debt issuance are in principle constrained by the GR, the short-sighted government may choose to issue public debt in excess of the GR compliance without facing immediate penalties. The government first determines the degree of compliance with the GR, which may include the temporary suspension of fiscal rules, activation of escape clauses, or modification of fiscal limits.[3] Based on the chosen degree of compliance, the government then

2013).

[3]Recent examples of procedures for suspending or modifying fiscal rules in parliamentary debates across various countries are detailed by Davoodi et al. (2022). Regarding the three countries discussed in this study—Germany, Japan, and the United Kingdom—Tetlow et al. (2024) provide an in-depth analysis of the United Kingdom, Mause and Groeteke (2012) cover Germany, and Ministry of Finance Japan (2019) offers detailed information on Japan.

sets appropriate levels of capital and labor taxes, public investment, and public goods provision to ensure budgetary balance. To illustrate this two-step process, we assume a probabilistic voting framework, wherein middle-aged and older adults vote on candidates in each period. The government selects both the degree of the GR compliance and the corresponding fiscal policies in turn to maximize a political objective function, which is defined as the weighted sum of the utilities of the middle-aged and older adult populations. The resulting allocation is referred to as a political equilibrium.

Within this framework, we begin our analysis of the political equilibrium at the second stage by examining the politics of fiscal policy formation under a given degree of compliance with the GR. In the political process described above, current policy decisions influence future choices through the accumulation of both physical and public capital. This intertemporal linkage generates three key forces that shape fiscal policy: a general equilibrium effect via the interest rate, a disciplining effect through the next period capital income tax rate, and a disciplining effect via the next period provision of public goods. These three forces, which were not comprehensively captured in previous studies, play a crucial role in illustrating the impact of the degree of rule compliance on fiscal policy formation.

These forces also shape the intergenerational distribution of fiscal burdens. A lower degree of rule compliance enables the current middle-aged population to reduce their labor income tax burden by shifting fiscal responsibilities to future generations through the issuance of additional public debt. At the same time, the current older adult population bears the increased repayment burden of public debt issued by previous generations, who deferred their fiscal responsibilities. To address these heightened repayment costs, the current government raises the capital income tax rate, thereby increasing the tax burden on the current older adults. As a result, lower rule compliance reduces the fiscal burden on the current middle-aged while imposing greater burdens on both the current older adults and future generations. In other words, the tax burden spills over not only from the current working generation to younger generations, who do not participate in voting, but also to the current older adults, who do participate in voting. This outcome represents a novel characteristic of non-compliance with the GR in public debt issuance.

We next examine the first stage to determine how the government selects its degree of compliance with the GR. We show that this degree hinges on two structural parameters: θ, which captures preferences for public goods, and η, which represents the elasticity of public capital with respect to investment. A lower θ weakens the disciplining effect provided by next-period public goods, thereby reducing the government's incentive to limit public debt issuance. Likewise, a lower η diminishes the marginal benefit of public investment, further discouraging the government from curtailing the proportion of debt-financed public investment. These results imply that low θ or low η makes it hard to comply with the GR. We further show that higher GDP growth rates make compliance with the GR more challenging. In political equilibrium, both debt issuance and public investment increase at the same rate as GDP, keeping their ratio constant. However, the GR requires that fiscal deficits, accounting for outstanding public debt, remain below public investment, complicating compliance during periods of economic growth.

We then calibrate the model economy to Germany, Japan, and the United Kingdom over the period 2002-2007, during which the GR was in effect in all three countries. During this period, the ratio of government deficit to GDP is higher than that of public investment to GDP on average in all three countries, which implies non-compliance with the GR. Our calibration results indicate that the key factors for non-compliance with the GR differ among the three countries. In Germany, low θ and low η play a crucial role, whereas, in other countries, high GDP growth ratio does.

This observed non-compliance with the GR reflects the disparity between short-sighted governments' optimization and long-term social welfare maximization. To explore this further, we examine the political equilibrium in contrast to the perspective of a long-lived planner, referred to as a Ramsey planner, who selects fiscal policy to maximize the sum of utilities of both present and future generations, subject to the same constraints faced by short-sighted governments. In the allocation chosen by the Ramsey planner, known as the Ramsey allocation, the ratio of the fiscal deficit to public investment remains below one, indicating adherence to the GR of fiscal policy. In contrast, in the political equilibrium, governments fail to comply with the GR due to their short-sighted nature. This suggests that by setting fiscal policy in non-compliance with the GR, governments achieve a less balanced allocation of resources across generations, ultimately diverging significantly from the Ramsey allocation.

This research contributes to the literature on the political economy of fiscal rules in three significant ways. First, it builds on the work of Bassetto and Sargent (2006), who examined the political economy of the GR within an overlapping generations model. Unlike their approach, which assumes an exogenous interest rate, this study incorporates the general equilibrium effects of fiscal policy through the interest rate and the disciplining effects through future capital income taxation and public expenditure. Moreover, while Bassetto and Sargent (2006) assume full compliance with the GR, this study demonstrates how and why a short-sighted government may not comply with the GR in its fiscal policy formation and identifies the economic conditions under which such non-compliance occurs. Furthermore, this study adopts the Ramsey planner's allocation, rather than the first-best allocation employed by Bassetto and Sargent (2006), as the benchmark for welfare evaluation. This approach allows for a more precise characterization of optimal allocations by accounting for individuals' responses to fiscal policies and better capturing the political distortions arising from short-sighted governments' non-compliance with the GR.

Second, this study contributes to the dynamic political economy literature by examining the effects of fiscal policies constrained by fiscal rules on dynamic resource allocation. Much of the existing literature either abstracts from the general equilibrium effects of fiscal policy through the interest rate (Azzimonti et al., 2016; Barseghyan and Battaglini, 2016; Arai et al., 2018; Andersen and Bhattacharya, 2020) or overlooks the disciplining effect through future public goods provision (Uchida and Ono, 2021). In contrast, this study offers a comprehensive examination of both the general equilibrium and disciplining effects of fiscal policy, evaluating the impact of fiscal rule non-compliance on policy choices and the resulting resource allocation from the Ramsey planner's perspective.

Finally, this research makes a significant contribution to the analysis of the GR in growth models with public capital. Previous studies have demonstrated that the GR can improve growth and welfare across generations (e.g., Greiner and Semmler, 2000; Ghosh and Mourmouras, 2004a,b; Greiner, 2008; Yakita, 2008; Minea and Villieu, 2009; Agénor and Yilmaz, 2017; Ueshina, 2018). However, these studies assume tax rates and/or expenditures as fixed, meaning their conclusions rest on the assumption that fiscal policy instruments are independent of fiscal rules or the degree of rule compliance. In reality, fiscal rules heavily influence governments' fiscal policy choices (e.g., Bisin et al., 2015; Halac and Yared, 2018; Coate and Milton, 2019; Bouton et al., 2020; Dovis and Kirpalani, 2020, 2021; Piguillem and Riboni, 2020, 2024). Thus, this study's consideration of the endogenous response of fiscal policies to fiscal rule compliance provides a new perspective on the implications of fiscal rules for growth and welfare.

The remainder of this paper is organized as follows. Section 2 introduces the model and characterizes an economic equilibrium that describes the behavior of individuals and firms for a given set of fiscal policies. Section 3 presents the politics of fiscal policy formation, characterizes a political equilibrium for a given degree of rule compliance, and investigates the effects of the degree of rule compliance on fiscal policy formation. It also demonstrates the political determination of the degree of rule compliance and investigates its property. Section 4 characterizes the Ramsey allocation and evaluates the political equilibrium from the viewpoint of the Ramsey planner. Section 5 provides concluding remarks. All proofs are presented in the appendix.

2 Model

The discrete time economy starts in period 0 and consists of overlapping generations. Individuals are identical within a generation and live for three periods: youth, middle age, and older adult age. Each middle-aged individual has $1+n$ children. The middle-aged population for period t is N_t and the population grows at a constant rate of $n(>-1) : N_{t+1} = (1+n)N_t$.

2.1 Individuals

Individuals display the following economic behavior over their life cycles. During youth, they make no economic decisions and live under the financial support and protection of their parents. In middle age, individuals work, receive market wages, and pay taxes. They use after-tax income for consumption and savings. Individuals retire in their older adult age and consume returns from savings.[4]

Consider an individual born in period $t-1$. In period t, the individual is middle-aged and endowed with one unit of labor. The individual supplies one unit of labor inelastically in the labor market, and obtains labor income w_t, where w_t is the wage rate per unit of labor in period t. After paying tax $\tau_t w_t$, where $\tau_t \in (0,1)$ is the period-t labor income tax rate, the individual distributes the after-tax income between consumption c_t and savings invested in physical capital s_t. Therefore, the period-t budget constraint for the middle-aged becomes $c_t + s_t \leq (1-\tau_t)w_t$.

The period-$t+1$ budget constraint in older adult age is $d_{t+1} \leq \left(1-\tau_{t+1}^k\right) R_{t+1} s_t$, where d_{t+1} is consumption, τ_{t+1}^k is the period-$t+1$ capital income tax rate, $R_{t+1}(>0)$ is the gross return from investment in physical capital, and $R_{t+1}s_t$ is the return from savings. The results are qualitatively unchanged if the capital income tax is on the net return from savings rather than the gross return from savings.

The preferences of the middle-aged in period t are specified by the following lifetime utility function in the logarithmic form, $U_t^M = \ln c_t + \theta \ln g_t + \beta \left(\ln d_{t+1} + \theta \ln g_{t+1}\right)$, where g is per capita public goods provision, $\beta \in (0,1)$ is the discount factor, and $\theta(>0)$ is the degree of preferences for public goods. The preferences of older adults in period t are given by $U_t^O = \ln d_t + \theta \ln g_t$.

We substitute the budget constraints into the utility function of the middle-aged, U_t^M, to form the following unconstrained maximization problem:

$$\max_{\{s_t\}} \ln\left((1-\tau_t)w_t - s_t\right) + \theta \ln g_t + \beta \left(\ln \left(1-\tau_{t+1}^k\right) R_{t+1} s_t + \theta \ln g_{t+1}\right), \tag{1}$$

where the factor prices, $\{w_t, R_{t+1}\}$, and the fiscal policy variables, $\{\tau_t, \tau_{t+1}^k, g_t, g_{t+1}\}$, are taken as given. By solving the problem in (1), we obtain the following savings and consumption functions:

$$s_t = \frac{\beta}{1+\beta}(1-\tau_t)w_t, \tag{2}$$

$$c_t = \frac{1}{1+\beta}(1-\tau_t)w_t, \tag{3}$$

$$d_{t+1} = \left(1-\tau_{t+1}^k\right) R_{t+1} \frac{\beta}{1+\beta}(1-\tau_t)w_t. \tag{4}$$

2.2 Firms and Public Capital

Each period contains a continuum of identical firms that are perfectly competitive profit maximizers. According to Cobb–Douglas technology, the firms produce a final good Y_t using two inputs: aggregate physical capital K_t and aggregate labor N_t. Aggregate output is given by $Y_t = A_t \left(K_t\right)^{\alpha} \left(N_t\right)^{1-\alpha}$, where A_t represents a period-t technology and $\alpha \in (0,1)$ denotes the physical capital share. The production function in intensive form is $y_t = A_t \left(k_t\right)^{\alpha}$ where

[4]The role of the young is not directly incorporated into the subsequent analysis. Nonetheless, their presence is assumed as we calibrate the model economy to align with the key statistics of the sample countries outlined in Section 3.

$y_t \equiv Y_t/N_t$ and $k_t \equiv K_t/N_t$ denote output and physical capital per middle-aged individual, respectively.

The first-order conditions for profit maximization with respect to N_t and K_t are

$$w_t = (1-\alpha)A_t\,(k_t)^{\alpha}\,, \tag{5}$$

$$R_t = \alpha A_t\,(k_t)^{\alpha-1}\,, \tag{6}$$

where w_t and R_t are labor wages and the gross return on physical capital, respectively. The conditions state that firms hire labor and physical capital until the marginal products are equal to the factor prices. Physical capital is assumed to depreciate fully within each period.

The productivity A_t is assumed to be proportional to the per labor public capital: $A_t \equiv A\,(K_t^p/N_t)^{1-\alpha}$, where K_t^p denotes the aggregate public capital in period t. Thus, public capital involves a technological externality of the type often used in endogenous-growth theories (Barro, 1990). Under this assumption, the first-order conditions in (5) and (6) are rewritten as follows:

$$w_t = w\,(k_t, k_t^p) \equiv (1-\alpha)A\,(k_t)^{\alpha}\,(k_t^p)^{1-\alpha}\,, \tag{7}$$

$$R_t = R\,(k_t, k_t^p) \equiv \alpha A\,(k_t)^{\alpha-1}\,(k_t^p)^{1-\alpha}\,, \tag{8}$$

and the associated production function in intensive form becomes:

$$y_t = y\,(k_t, k_t^p) \equiv A\,(k_t)^{\alpha}\,(k_t^p)^{1-\alpha}\,. \tag{9}$$

The public capital evolves in the following way. The aggregate public capital in period $t+1$, K_{t+1}^p, is a function of public investment, x_t, and the public capital inherited from the past, K_t^p. In particular, K_{t+1}^p is assumed to be formulated using the following equation:

$$K_{t+1}^p = \tilde{D}\cdot(N_{t+1}x_t)^{\eta}\,(K_t^p)^{1-\eta}\,,$$

where $\tilde{D}\,(>0)$ is a scale factor, $N_{t+1}x_t$ is the aggregate public investment evaluated on a generation $t+1$ population basis, and $\eta \in (0,1)$ denotes the elasticity of public capital with respect to public investment. Let $k_t^p \equiv K_t^p/N_t$ denote public capital per middle-aged individual. Then the equation is reformulated as

$$k_{t+1}^p = D\cdot(x_t)^{\eta}\,(k_t^p)^{1-\eta}\,, \tag{10}$$

where $D \equiv \tilde{D}\cdot(1+n)^{\eta-1}$.

Equation (10) suggests that the production function of public capital is homogeneous of degree one in both x and k^p. This function is similar to the functional form postulated by Barseghyan and Battaglini (2016), who assume k^p as the overall productivity of the economy, driven by government investment. In contrast, the present study assumes k^p to represent public capital, but the fundamental properties of the production function remain the same in both cases. An alternative formulation would be to assume a linear public capital accumulation equation, as proposed by Bassetto and Sargent (2006). The present study adopts the formulation of (10) in line with the framework of Barseghyan and Battaglini (2016) for the purpose of analytical tractability.

2.3 Government Budget Constraint and Fiscal Rule

Government expenditure items are an investment in public capital and expenditure on (unproductive) public goods provision. They are financed by capital and labor taxes, as well as public debt issues. Let B_t denote the aggregate inherited debt. The aggregate government budget constraint in period t is

$$B_{t+1} + \tau_t^k R_t s_{t-1} N_{t-1} + \tau_t w_t N_t = N_{t+1} x_t + G_t + R_t B_t, \tag{11}$$

where B_{t+1} is newly issued public debt; $\tau_t^k R_t s_{t-1} N_{t-1}$ is the aggregate capital tax revenue; $\tau_t w_t N_t$ is the aggregate labor tax revenue; $N_{t+1} x_t$ is the aggregate public investment; G_t is aggregate public goods provision, and $R_t B_t$ is debt repayment. We assume a one-period debt structure to derive analytical solutions from the model. We also assume that the government in each period is committed to not repudiating the debt. The associated government budget constraint per middle-aged individual in period t is

$$(1+n) b_{t+1} + \tau_t^k R_t \frac{s_{t-1}}{1+n} + \tau_t w_t = (1+n) x_t + \frac{2+n}{1+n} g_t + R_t b_t, \tag{12}$$

where $b_t \equiv B_t / N_t$ and $g_t \equiv G_t / (N_{t-1} + N_t)$ denote public debt per middle-aged individual and public good per capita, respectively.

Consider a situation in which government expenditures are constrained by fiscal rules. The fiscal rules we assume here are long-term rules that remain effective across periods. We focus specifically on fiscal rules that limit the financing of public investment, known as the Golden Rule of Public Finance (abbreviated as GR). The GR requires that the fiscal deficit remains below public investment expenditures: $B_{t+1} - B_t \leq N_{t+1} x_t$. In other words, under the GR, while a portion of public investment can be financed through the fiscal deficit, the GR does not permit the financing of other current expenditures through the fiscal deficit. When the GR is expressed on a per middle-aged individual basis, it is represented as

$$(1+n) b_{t+1} - b_t \leq (1+n) x_t. \tag{13}$$

We now consider the possibility of government non-compliance with the GR in each period. To explore this possibility, we formulate the relationship between public investment and public debt issuance as $B_{t+1} = \phi_t N_{t+1} x_t$, or:

$$(1+n) b_{t+1} = \phi_t (1+n) x_t, \tag{14}$$

where $\phi_t \, (\geq 0)$, determined through voting (as explained below), represents the period-t debt-investment ratio. Using the expression in (14), we can reformulate the GR in (13) as follows:

$$\phi_t (1+n) x_t - b_t \leq (1+n) x_t \Leftrightarrow (1-\phi_t)(1+n) x_t + b_t \geq 0. \tag{15}$$

The expression in (15) shows that if the ratio ϕ_t is set to satisfy $\phi_t \leq 1$, the GR in (13) is satisfied. In other words, $\phi_t \leq 1$ serves as a sufficient condition for verifying whether the government sets fiscal policies in compliance with the GR. The ratio can be viewed as representing the degree of rule compliance: a lower (higher) ϕ_t implies a higher (lower) degree of rule compliance. Hereafter, we refer to ϕ_t as the debt-investment ratio or the degree of rule compliance interchangeably. In the following, we first proceed with the analysis focusing on ϕ_t, and then numerically explore the conditions under which the fiscal policies align with the GR in (13).[5]

2.4 Economic Equilibrium

Public debt is traded in the domestic asset market. The asset market clearing condition is $B_{t+1}+K_{t+1}=N_t s_t$, which expresses the equality of total savings by the middle-aged population in period t, $N_t s_t$ to the sum of the stocks of aggregate public debt and aggregate physical capital at the beginning of period $t+1$, $B_{t+1}+K_{t+1}$. Using $k_{t+1}\equiv K_{t+1}/N_{t+1}$, $k_t^p=K_t^p/N_t$, the savings function in (2), and the profit-maximization condition in (5), we can rewrite the abovementioned asset market clearing condition as

$$(1+n)\left(k_{t+1}+b_{t+1}\right)=s\left(\tau_t;k_t,k_t^p\right)\equiv\frac{\beta}{1+\beta}(1-\tau_t)w\left(k_t,k_t^p\right). \tag{16}$$

Using the private budget constraints, $c_t+s_t=(1-\tau_t)\,w_t$ and $d_t=\left(1-\tau_t^k\right)R_t s_{t-1}$, the firms' profit maximization conditions in (7) and (8), and the government budget constraint in (12) with (14), we can reformulate the asset market clearing condition in (16) as follows:

$$y_t=c_t+\frac{d_t}{1+n}+(1+n)\,k_{t+1}+(1+n)\,x_t+\frac{2+n}{1+n}g_t.$$

This is the resource constraint (i.e., the final good market clearing condition), describing that the output is allocated to consumption, physical and public capital investment, and public goods expenditure.

Building on the aforementioned framework, the timing in period t can be delineated as follows. Initially, with k_t, k_t^p, and b_t as given, the government representing existing generations determines the debt-investment ratio (or equivalently, the degree of rule compliance), ϕ_t, with the aim of maximizing the political objective function (which will be introduced later). Subsequently, based on the value of ϕ_t determined in the initial stage, as well as the state variables k_t, k_t^p, and b_t, the government sets τ_t, τ_t^k, x_t, and g_t within the aim of maximizing the political objective function. Finally, given the fiscal parameters and factor prices, individuals and firms make their decisions, leading to market clearing.

The ratio, ϕ_t and the policy functions, τ_t, τ_t^k, x_t, and g_t obtained through the two-step procedure are identical to those derived from a simultaneous determination procedure. The aim of employing the two-step procedure is to examine the impact of ϕ_t on other policy variables.

[5] More precisely, the degree of compliance with GR should be expressed as $(1+n)b_{t+1}-b_t=\tilde{\phi}_t(1+n)x_t$, where $\tilde{\phi}_t\leq 1$ implies compliance with GR, while $\tilde{\phi}_t>1$ implies non-compliance. However, for the sake of analytical convenience, we adopt the expression in (14), which shows that a larger ϕ_t corresponds to a lower degree of compliance with the GR. The following analysis demonstrates that even with (14), the degree of compliance with the GR can be effectively identified.

For instance, in Japan, the legislature debates and approves exemptions from the Public Finance Act, which restricts the issuance of public bonds, before proceeding with the deliberation of the budget for the following fiscal year (Ministry of Finance Japan, 2019). Consequently, the degree of rule compliance approved by the legislature influences subsequent budget formulation and the resulting policies. Our two-step procedural approach allows us to illustrate this process and capture its effects.

The following defines the economic equilibrium that characterizes the optimizing behavior of individuals and firms, as well as the resulting market clearing, given the policies and rules in the present model.

Definition 1 Given a sequence of policies and rules, $\{\tau_t^k, \tau_t, x_t, g_t, \phi_t\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations $\{c_t, d_t, s_t, k_{t+1}, b_{t+1}, k_{t+1}^p\}_{t=0}^{\infty}$ and prices $\{w_t, R_t\}_{t=0}^{\infty}$ with initial conditions $k_0(>0), b_0(\geq 0)$ and $k_0^p(>0)$, such that (i) given $\left(w_t, R_{t+1}, \tau_{t+1}^k, \tau_t, x_t, g_t, g_{t+1}\right)$, (c_t, d_{t+1}, s_t) solves the utility-maximization problem to satisfy (2) – (4); (ii) given (w_t, R_t), k_t solves the firm's profit maximization problem to satisfy (7) and (8); (iii) given (w_t, k_t^p, R_t, b_t) and ϕ_t, $\left(\tau_t^k, \tau_t, x_t, b_{t+1}\right)$ satisfies the government budget constraint with the debt-investment relationship in (14); (vi) the public capital evolves according to (10), and (v) the asset market clears as in (16).

Definition 1 allows us to reduce the economic equilibrium conditions to a system of difference equations that characterizes the motion of (k_t, b_t, k_t^p). The system includes the optimality conditions of firms in (5) and (6), the public capital formation function in (10), the government budget constraint in (12) with the debt-investment ratio in (14), and the asset market clearing condition in (16). The government budget constraint in (12) is reformulated as

$$TR^K\left(\tau_t^k; k_t, k_t^p, b_t\right) + TR\left(\tau_t; k_t, k_t^p\right) = (1-\phi_t)(1+n)x_t + \frac{2+n}{1+n}g_t + R\left(k_t, k_t^p\right)b_t, \tag{17}$$

where $TR^K\left(\tau_t^k; k_t, k_t^p, b_t\right)$ and $TR\left(\tau_t; k_t, k_t^p\right)$, representing the tax revenues from capital and labor income, respectively, are defined as follows:

$$\begin{aligned} TR^K\left(\tau_t^k; k_t, k_t^p, b_t\right) &\equiv \tau_t^k R\left(k_t, k_t^p\right)\left(k_t + b_t\right), \\ TR\left(\tau_t; k_t, k_t^p\right) &\equiv \tau_t w\left(k_t, k_t^p\right). \end{aligned}$$

In economic equilibrium, the indirect utility of the middle-aged population in period t, V_t^M, and that of the older adult population in period t, V_t^O, can be expressed as functions of fiscal policy variables, physical and public capital, and public debt as follows:

$$\begin{aligned} V_t^M &= V^M\left(\tau_t, x_t, g_t, \tau_{t+1}^k, g_{t+1}; k_{t+1}, k_{t+1}^p, b_{t+1}, k_t, k_t^p\right) \\ &\equiv \ln c\left(\tau_t; k_t, k_t^p\right) + \theta \ln g_t + \beta\left(\ln d\left(\tau_{t+1}^k; k_{t+1}, k_{t+1}^p, b_{t+1}\right) + \theta \ln g_{t+1}\right), \end{aligned} \tag{18}$$

$$V_t^O = V^O\left(\tau_t^k, g_t; k_t, b_t, k_t^p\right) \equiv \ln d\left(\tau_t^k; k_t, k_t^p, b_t\right) + \theta \ln g_t, \tag{19}$$

where $c\left(\tau_t;k_t,k_t^p\right)$ and $d\left(\tau_t^k;k_t,k_t^p,b_t\right)$, representing consumption in middle and older adult ages, respectively, are defined as follows:

$$c\left(\tau_t;k_t,k_t^p\right) \equiv \frac{1}{1+\beta}\left(1-\tau_t\right)w\left(k_t,k_t^p\right),$$
$$d\left(\tau_t^k;k_t,k_t^p,b_t\right) \equiv \left(1-\tau_t^k\right)R\left(k_t,k_t^p\right)(1+n)\left(k_t+b_t\right).$$

3 Politics

Based on the characterization of the economic equilibrium in Section 2.4, we consider the politics of rule compliance and fiscal policy formation. In particular, we employ probabilistic voting à la Lindbeck and Weibull (1987). In this voting scheme, two office-seeking candidates engage in electoral competition. Each candidate announces a platform specifying the debt-investment ratio and a set of fiscal policies subject to the government budget constraint. As demonstrated by Persson and Tabellini (2000), in equilibrium, the platforms of the two candidates converge to the same one, which maximizes the weighted average utility of voters (i.e., the middle-aged and older adults).

In the present framework, the young, the middle-aged, and older adults have the incentive to vote. While the young may benefit from public investment through public capital accumulation, we assume that their preferences are not considered by politicians. We impose this assumption, which is often used in the literature (e.g., Saint-Paul and Verdier, 1993; Bernasconi and Profeta, 2012; Lancia and Russo, 2016), for tractability reasons. However, this assumption could be supported in part by the fact that a significant proportion of the young are below the voting age.

The political objective is defined as the weighted sum of the utility of the older adults and middle-aged, given by $\tilde{\Omega}_t \equiv \omega V_t^O + (1+n)(1-\omega)V_t^M$, where $\omega \in (0,1)$ and $1-\omega$ are the political weights placed on older adults and middle-aged, respectively. The weight on the middle-aged is adjusted by the gross population growth rate, $(1+n)$, to reflect their share of the population. We divide $\tilde{\Omega}_t$ by $(1+n)(1-\omega)$ and redefine the objective function as follows:

$$\Omega_t = \frac{\omega}{(1+n)(1-\omega)}V_t^O + V_t^M, \tag{20}$$

where the coefficient $\omega/(1+n)(1-\omega)$ of V_t^O represents the relative political weight on older adults.

The details of the timing of rule compliance and fiscal policy decisions described in Section 2.4 is as follows. (i) First, the two office-seeking candidates, denoted as O^A and O^B, simultaneously and non-cooperatively, announce their debt-investment ratios, denoted by ϕ^A and ϕ^B, respectively. (ii) Given ϕ^A and ϕ^B, the candidates then simultaneously and non-cooperatively announce their policy platforms, $\left(\tau^A, \tau^{kA}, x^A, g^A\right)$ and $\left(\tau^B, \tau^{kB}, x^B, g^B\right)$, respectively. The elected candidate will then implement the announced debt-investment ratio and policy platform. This two-stage game is analyzed using backward induction. The result of the second-stage game is presented in Section 3.1, and the implications of this stage are explored in Section 3.2. The result of the first-stage game is analyzed in Section 3.3, with a comprehensive discussion of the findings provided in Section 3.4.

3.1 Political Equilibrium Fiscal Policy

The political objective function in (20) suggests that the current policy choice affects the decision on future policy via physical and public capital accumulation. In particular, the period-t choices of τ_t^k, x_t, g_t, and b_{t+1} affect the formation of physical and public capital in period $t+1$. This, in turn, influences the decision making on the period-$t+1$ fiscal policy. To demonstrate such an intertemporal effect, we employ the concept of a Markov-perfect equilibrium under which fiscal policy in the present period depends on the current payoff-relevant state variables.

In our framework, the payoff-relevant state variables are physical capital k_t, public debt b_t, and public capital k_t^p. Thus, the expected capital income tax rate and public goods provision for the next period, τ_{t+1}^k and g_{t+1}, are given by the functions of the period-$t+1$ state variables, $\tau_{t+1}^k = \hat{T}^k\left(k_{t+1}, b_{t+1}, k_{t+1}^p\right)$ and $g_{t+1} = \hat{G}\left(k_{t+1}, b_{t+1}, k_{t+1}^p\right)$, respectively. We denote the arbitrary lower limits of τ and τ^k by $-\underline{\tau}(<0)$ and $-\underline{\tau}^k(<0)$, respectively. By using recursive notation with z' denoting the next period z, we define a Markov-perfect political equilibrium that determines fiscal policy for a given ϕ as follows.

Definition 2 Given ϕ, a *Markov-perfect political equilibrium* is a set of functions, $\langle \hat{T}, \hat{T}^k, \hat{X}, \hat{G}, \hat{B}\rangle$, where $\hat{T} : \Re_+^3 \to (-\underline{\tau}, 1)$ is the labor income tax rule, $\tau = \hat{T}(k, b, k^p)$, $\hat{T}^k : \Re_+^3 \to \left(-\underline{\tau}^k, 1\right)$ is a capital income tax rule, $\tau^k = \hat{T}^k(k, b, k^p)$, $\hat{X} : \Re_+^3 \to \Re_+$ is a public investment rule, $x = \hat{X}(k, b, k^p)$, and $\hat{G} : \Re_+^3 \to \Re_+$ is a public goods provision rule, $g = \hat{G}(k, b, k^p)$, so that (i) given k, b, and k^p, $\langle \hat{T}(k, b, k^p), \hat{T}^k(k, b, k^p), \hat{X}(k, b, k^p), \hat{G}(k, b, k^p)\rangle$ is a solution to the problem of maximizing Ω in (20), subject to the public capital formation function in (10), the asset market clearing condition in (16), and the government budget constraint in (17); and (ii) $\hat{B} : \Re_+^3 \to \Re_+$ is a public debt rule, $b' = \hat{B}(k, b, k^p)$, that follows the debt-investment relationship in (14).[6]

We derive the political equilibrium policy functions and the associated sequence of physical and public capital for a given ϕ. To obtain the set of policy functions, we conjecture the capital income tax rate and public goods provision in the next period as follows:

$$\tau^{k\prime} = 1 - T^k \cdot \frac{1}{\alpha\left(1 + \frac{b'}{k'}\right)}, \tag{21}$$

$$g' = G \cdot y\left(k', k^{p\prime}\right), \tag{22}$$

where $T^k(>0)$ and $G(>0)$ are constant and represent the ratio of after-tax capital income to

[6] The state variables do not line up in compact sets because they grow across periods. To define the equilibrium more precisely, we need to redefine the equilibrium as a mapping from a compact set to a compact set by introducing the following notations: $\hat{x}_t \equiv x_t / y\left(k_t, k_t^p\right)$, $\hat{g}_t \equiv g_t / y\left(k_t, k_t^p\right)$, and $\hat{b}_{t+1} \equiv b_{t+1} / y\left(k_t, k_t^p\right)$. However, for simplicity, we define the equilibrium as in Definition 2.

GDP and the ratio of public goods provision to GDP, respectively.[7] The conjectures in (21) and (22) suggest that at the aggregate level, the after-tax capital income and the government expenditure on public goods provision are linearly related to GDP.

Given these conjectures, we consider the optimization problem in Definition 2, and obtain the following first-order derivatives:

$$\tau^k : \tfrac{\omega}{(1+n)(1-\omega)} \tfrac{d_{\tau^k}}{d} + \lambda TR^K_{\tau^k} = 0, \tag{23}$$

$$\tau : \frac{c_\tau}{c} + \beta \left(\frac{d'_\tau}{d'} + \theta \frac{g'_\tau}{g'} \right) + \lambda TR_\tau = 0, \tag{24}$$

$$g : \left(\frac{\omega}{(1+n)(1-\omega)} + 1 \right) \frac{\theta}{g} - \lambda \frac{2+n}{1+n} = 0, \tag{25}$$

$$x : \beta \left(\frac{d'_x}{d'} + \theta \frac{g'_x}{g'} \right) - \lambda (1-\phi)(1+n) = 0, \tag{26}$$

where $\lambda\,(\geq 0)$ is the Lagrangian multiplier associated with the government budget constraint in (17). The choice of x determines the level of public debt issuance according to the debt-investment relationship in (14). The impact of debt issuance decisions through the relationship in (14) is contained in the term $\left(\frac{d'_\tau}{d'} + \theta \frac{g'_\tau}{g'} \right)$ of (24) and the terms $\beta \left(\frac{d'_x}{d'} + \theta \frac{g'_x}{g'} \right)$ and $\lambda (1-\phi)(1+n)$ of (26).

The first-order conditions in (23)–(26) are summarized as follows, focusing on the marginal benefit of public goods provision appearing on the left-hand side of (25):

$$\left(\frac{\omega}{(1+n)(1-\omega)} + 1 \right) \frac{\theta}{g} = -\frac{2+n}{1+n} \cdot \frac{\frac{\omega}{(1+n)(1-\omega)} \frac{d_{\tau^k}}{d}}{TR^K_{\tau^k}}, \tag{27}$$

$$\left(\frac{\omega}{(1+n)(1-\omega)} + 1 \right) \frac{\theta}{g} = -\frac{2+n}{1+n} \cdot \frac{\frac{c_\tau}{c} + \beta \left(\frac{d'_\tau}{d'} + \theta \frac{g'_\tau}{g'} \right)}{TR_\tau}, \tag{28}$$

$$\left(\frac{\omega}{(1+n)(1-\omega)} + 1 \right) \frac{\theta}{g} = \frac{2+n}{1+n} \cdot \frac{\beta \left(\frac{d'_x}{d'} + \theta \frac{g'_x}{g'} \right)}{1+n}. \tag{29}$$

According to the expressions in (27)–(29), the government chooses policies to equate the marginal benefits of public goods provision that appear on the left-hand side with the marginal benefits of capital income tax cut, marginal net benefits of the labor income tax cut, and marginal benefits of public investment that appear on the right-hand side, respectively.

A detailed interpretation of these conditions is as follows. The intuition is straightforward as for the marginal benefit of public goods provision appearing on the left-hand side of each equation. An increase in public goods leads to an increase in the marginal utility of public goods for older adults and for the middle-aged. The right-hand side of (27) represents the marginal benefit of capital income tax cut. A decrease in the capital income tax rate raises the consumption of older adults, and thus makes them better off. We evaluate this effect based on

[7] The conjecture in (21) is reformulated as $T^k = \left(1-\tau^{k\prime}\right) R\left(k', k^{p\prime}\right) s\left(\tau; k, k^p\right) \frac{1}{1+n} / y\left(k', k^{p\prime}\right)$. This is further reformulated as $T^k = \left(1-\tau^{k\prime}\right) R\left(k', k^{p\prime}\right) s\left(\tau; k, k^p\right) N/Y'$, showing the ratio of after-tax capital income to GDP.

the change in tax revenue through capital income taxation represented by the term $TR^K_{\tau^k}$ in the denominator. The right-hand sides of (28) and (29) include the following three inter-temporal effects: the general equilibrium effect of physical capital through the interest rate, R' (named as "Effect R_i"); the disciplining effect through the capital income tax rate, $\tau^{k\prime}$ (named as "Effect τ^k_i"); and the disciplining effect through public goods provision, g' (name as "Effect g_i"). As these effects arise across multiple dimensions, they are numbered as $i = 1, 2, \cdots$ for classification in the following description. These effects play crucial roles in shaping fiscal policy, which are explained in turn below.

First, consider the right-hand side of (28), showing the marginal benefit of the labor income tax cut. A cut of the labor income tax rate causes disposable income to rise, thereby increasing the consumption of the middle-aged, as represented by the term c_τ/c. The term $\beta\left(d'_\tau/d' + \theta g'_\tau/g'\right)$ includes the marginal costs and benefits of the labor income tax cut that the current middle-aged population are expected to receive when they are older adults. The cut of the labor income tax rate increases the disposable income of the middle-aged, and thus expands their savings, which in turn raises their consumption when they are older adults. In addition, the increase in savings works to lower the return from savings, R', and thus reduces their consumption when they are older adults (Effect R_1). Simultaneously, the increase in savings promotes physical capital accumulation, lowers the interest rate and thus reduces the burden of debt repayment. This enables the government to lower the capital income tax rate in the next period, $\tau^{k\prime}$, which raises the consumption of the middle-aged when they are older adults (Effect τ^k_1). These effects are represented by the term d'_τ/d'. The term $\theta g'_\tau/g'$ shows that the labor income tax cut stimulates savings and physical capital accumulation, and thus promotes public goods provision in the next period (Effect g_1). The right-hand side evaluates these effects based on the change in the tax revenue through the labor income tax cut, as represented by the term TR_τ in the denominator.

Next, consider the right-hand side of (29), showing the marginal net benefit of public investment. The investment promotes public capital accumulation and thus, raises the return from savings, R' (Effect R_2). This leads to an increase in consumption for the older adults. Simultaneously, the increase in the public capital level leads to an increase in public goods provision in the next period (Effect g_2). In addition, under the debt-investment relationship in (14), an increase in public investment expands public debt issuance, which slows down physical capital accumulation. This raises the interest rate R', and increases consumption in older adult age (Effect R_3). At the same time, the impeded accumulation of physical capital leads to an increase in the interest rate. This rise in the interest rate, coupled with increased debt issuance, exacerbates the burden of debt repayment. This prompts the government to raise the capital income tax rate in the subsequent period, $\tau^{k\prime}$, ultimately leading to a decline in the consumption of older adults (Effect τ^k_2). Furthermore, impeded physical capital accumulation lowers public goods provision in the next period, g' (Effect g_3). The right–hand side of (29) includes these five marginal costs and benefits, which affect the middle-aged when they are older adults.

Among the effects discussed above, some are addressed in the preceding studies by Song et al. (2012) and Uchida and Ono (2021), while others are not. First, Song et al. (2012) assume a small open economy, which leads them to abstract from Effects R_i $(i = 1, 2, 3)$ in their analysis. Moreover, their model excludes capital income taxation, thereby omitting Effects τ^k_i $(i = 1, 2)$. Additionally, by excluding public capital, they neglect the effects of public investment on the interest rate via public capital and public debt issuance, which we refer to as Effect g_i $(i = 2, 3)$. Second, Uchida and Ono (2021) assume a closed economy with capital income taxation and thus demonstrate Effects R_i $(i = 1, 2, 3)$ and τ^k_i $(i = 1, 2)$. However, they abstract from public goods in their model, resulting in the exclusion of Effect g_i $(i = 1, 2, 3)$. The present study departs from these previous studies by explicitly demonstrating Effects g_i $(i = 1, 2, 3)$ through the incorporation of public investment and public goods. Therefore, this study comprehensively captures all the effects that were partially addressed in these two previous studies.

In deriving the policy functions using the first-order conditions in (23)–(26), alongside the debt-investment relationship in (14) and the government budget constraint in (17), we further conjecture that the policy functions of τ and x are given by

$$\tau = T, \tag{30}$$

$$(1 + n)x = X \cdot y(k, k^p), \tag{31}$$

where T and X are constant. We can verify the conjectures in (21), (22), (30), and (31) and obtain the following result:

Proposition 1 *Given the debt-investment ratio, ϕ, in (14), there is a Markov-perfect political equilibrium such that the policy functions of τ^k, g, x, τ, and b' are given by*

$$\begin{aligned}
\tau^k &= 1 - T^k \cdot \frac{1}{\alpha\left(1 + \frac{b}{k}\right)}, \\
\frac{2+n}{1+n} g &= G \cdot y(k, k^p), \\
(1 + n)x &= X \cdot y(k, k^p), \\
\tau &= T, \\
(1 + n)b' &= \phi X \cdot y(k, k^p),
\end{aligned}$$

where T^k, G, X, and T, defined in Appendix A.1, are constant and dependent on ϕ.

Proof. See Appendix A.1.

Proposition 1 implies that given the debt-investment ratio in (14), the policy functions have the following features. First, the capital income tax rate is increasing in public debt but decreasing in physical capital. A higher level of public debt increases the burden of debt repayment. The government responds to the increased burden by raising the capital income tax rate. By contrast, a higher level of physical capital lowers the interest rate and thus reduces the burden of debt repayment. This enables the government to lower the capital income tax rate. Second, the labor income tax rate is independent of the state variables and is constant across periods, whereas the levels of public goods provision and public investment are linear functions of output, y. Given the debt-investment ratio in (14), this property, with the first one, is necessary for the government budget constraint in (17) to be satisfied in each period.

Having established the policy functions, we are now ready to present physical and public capital accumulation. We substitute the policy functions presented in Proposition 1 into the public capital formation function in (10) and the asset market clearing condition in (16), and obtain

$$\frac{k'}{k} = \frac{1}{1+n}\left(\frac{\beta}{1+\beta}(1-T)(1-\alpha)-\phi X\right)A\left(\frac{k^p}{k}\right)^{1-\alpha}, \tag{32}$$

$$\frac{k^{p\prime}}{k^p} = D\left(\frac{1}{1+n}XA\left(\frac{k}{k^p}\right)^{\alpha}\right)^{\eta}. \tag{33}$$

Given the initial condition, $\{k_0, k_0^p\}$, the sequence $\{k_t, k_t^p\}$ is characterized by the above two equations in (32) and (33). A *steady state* is defined as a political equilibrium with $k^{p\prime}/k' = k^p/k$. In other words, the ratio of public to physical capital is constant across periods. Equations (32) and (33) lead to:

$$\frac{k^{p\prime}}{k'} = \Psi\left(\frac{k^p}{k}\right) \equiv \frac{D\left(\frac{1}{1+n}XA\right)^{\eta}}{\frac{1}{1+n}\left(\frac{\beta}{1+\beta}(1-T)(1-\alpha)-\phi X\right)A}\left(\frac{k^p}{k}\right)^{\alpha(1-\eta)},$$

with the property of $\Psi'(\cdot) > 0$ and $\Psi''(\cdot) < 0$. This property shows that there is a unique stable steady state for the path of $\{k^p/k\}$.

3.2 Effects of Debt-Investment Ratio

The result in Subsection 3.1 indicates that the debt-investment ratio, denoted as ϕ, taken as an institutional parameter at the second stage, has a decisive effect on the formation of fiscal policies. In particular, ϕ has direct impacts on τ (the labor income tax rate) via the term $\beta\left(d'_\tau/d'/+\theta g'_\tau/g'\right)$ in (24) and on x (the public investment) via the terms $\beta\left(d'_x/d'+\theta g'_x/g'\right)$ and $\lambda(1-\phi)(1+n)$ in (26). However, ϕ has no such direct impact on τ^k (the capital income tax rate) and g (public goods provision) although they might be affected by ϕ via the choice of τ and x. Thus, we first focus on the first-order conditions with respect to τ in (24) and x in (26) to investigate the direct effects of ϕ and then move on to the analysis of the indirect effects of ϕ on τ^k and g.

To investigate the direct effects of ϕ on τ and x, we reformulate (24) and (26) as follows:

$$\tau : \underbrace{\frac{-1}{1-\tau}}_{\equiv \frac{c_\tau}{c}} + \underbrace{\frac{-\alpha\beta(1+\theta)\frac{\beta}{1+\beta}w(k,k^p)}{\frac{\beta}{1+\beta}(1-\tau)w(k,k^p)-\phi(1+n)x}}_{\equiv \beta\left(\frac{d'_\tau}{d'}+\theta\frac{g'_\tau}{g'}\right)} + \underbrace{\lambda w(k,k^p)}_{\equiv \lambda TR_\tau} = 0, \tag{34}$$

$$x : \underbrace{\frac{-\alpha\beta(1+\theta)\phi(1+n)}{\frac{\beta}{1+\beta}(1-\tau)w(k,k^p)-\phi(1+n)x} + \frac{\beta\eta(1+\theta)(1-\alpha)}{x}}_{\equiv \beta\left(\frac{d'_x}{d'}+\theta\frac{g'_x}{g'}\right)} - \lambda(1-\phi)(1+n) = 0. \tag{35}$$

Equations (34) and (35) show that the debt-investment ratio affects the formation of the labor income tax rate and the ratio of public investment to GDP, respectively. Given the difficulty of showing its qualitative impacts, we clarify the effects quantitatively based on numerical methods.

We calibrate the model economy so that the steady-state political equilibrium allocation and policies align with key statistics for each of the sample countries: Germany, Japan, and the United Kingdom. We use the period 2002–2007 for calibration because the GR was in effect in these countries during this time (Kumar et al., 2009), and the period also precedes the financial crisis, during which the fiscal deficit-to-GDP ratios of all three countries were stable.

We assume that each period of the present model lasts 30 years; this assumption is standard in quantitative analyses of two- or three-period overlapping-generations models (e.g., Gonzalez-Eiras and Niepelt, 2008; Lancia and Russo, 2016). The population growth rate, n, is obtained from the average of each sample country during the 2002–2007 period. We assume the share of physical capital, α, is common to the three countries and fix at $\alpha = 1/3$, in line with Song et al. (2012) and Lancia and Russo (2016). The productivity parameter of human capital is set at $D = 5$.[8]

We calibrate the country-specific parameters β, ω, η, θ, and A to simultaneously match the average statistics of the labor income tax rate (τ), the ratios of general government final consumption expenditure (G/Y), public investment ($N'x/Y$), and government deficit ($(B' - B)/Y$) to GDP and the growth rate of GDP per middle-aged individual (y'/y) for each country during the period 2002–2007. Table 1 reports the average statistics and the estimated parameter values for each sample country. Appendix A.2 provides details on calibration.

Figure 2 illustrates the numerical result of the effects of an increased ϕ on the labor income tax rate, T (Panel (a)), and the ratio of public investment to GDP, X (Panel (b)), for the three countries. On the basis of the result in Figure 2, we can interpret (34) and (35) as follows. First, (34) shows that ϕ affects the formation of the labor income tax rate. The greater ϕ is, the greater the share of public investment financed by public debt issuance. Debt issuance lowers physical capital accumulation through crowding-out effects, reduces the next-period public goods provision, and generates the following two opposing effects on older adult

[8] A lower D is associated with a higher A. To keep the value of A within a moderate range, D is set at 5. While the value of D influences the determination of A, it does not affect the values of other parameters. Therefore, the setting of D is independent of the estimation of other parameters conducted below.

Country	POP	TAX	GOV	INV	DEF	GRO
Germany	0.9995	0.3945	0.1872	0.0203	0.0260	1.0207
Japan	1.0009	0.2292	0.1772	0.0422	0.0504	1.0231
United Kingdom	1.0065	0.2495	0.1930	0.0215	0.0276	1.0176
Country	n	β	ω	η	θ	A
Germany	-0.0160	0.32574	0.59076	0.24613	0.24925	3.1626
Japan	0.0265	0.49221	0.52152	0.29854	0.24953	3.2751
United Kingdom	0.2142	0.22769	0.50252	0.27595	0.25851	4.0192

Table 1: Data and calibrated parameters for Germany, Japan, and the United Kingdom. The first six columns are the annual gross population growth rate (POP), the labor income tax rate (TAX), the ratio of general government consumption expenditure to GDP (GOV), the ratio of public investment to GDP (INV), the ratio of deficit to GDP (DEF), and the annual growth rate of GDP per middle-aged individual (GRO) during 2002–2007. The last six columns are the calibrated parameter values of n, β, ω, η, θ, and A. Sources of the data are provided in Appendix A.2.

consumption: a positive effect via an increase in the interest rate, and a negative effect via an increase in the next-period capital income tax rate. These effects are observed in the second term of the left-hand side in (34). In the present framework, the latter effect dominates the former one; thus, an increase in ϕ leads to an increase in the marginal cost of τ. To offset this increase in the marginal cost, the government has the incentive to lower the labor income tax rate as ϕ increases. Therefore, an increase in ϕ leads to a decrease in the labor income tax rate, as we can see from Figure 2(a).

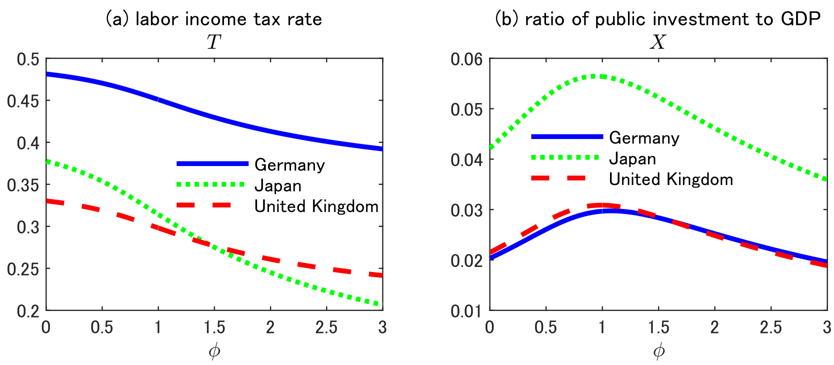

Figure 2: Changes in the labor income tax rate, T and the ratio of public investment to GDP, X against changes in ϕ.

Second, (35) shows that ϕ has two conflicting effects on the choice of public investment, x. The first is a negative effect on x observed in the first term on the left-hand side; the effect is qualitatively similar to the negative effect on τ, as mentioned previously. The second is a positive effect on x, as observed in the third term on the left-hand side. This effect arises because the tax burden associated with public investment, x, decreases as the debt-financed proportion of public investment, ϕ, increases. Given these conflicting effects, there is a threshold value of ϕ at which the two effects are balanced, and an increase in ϕ has an inverse U-shaped effect on the ratio of public investment to GDP around the threshold value, as illustrated in Figure 2(b).

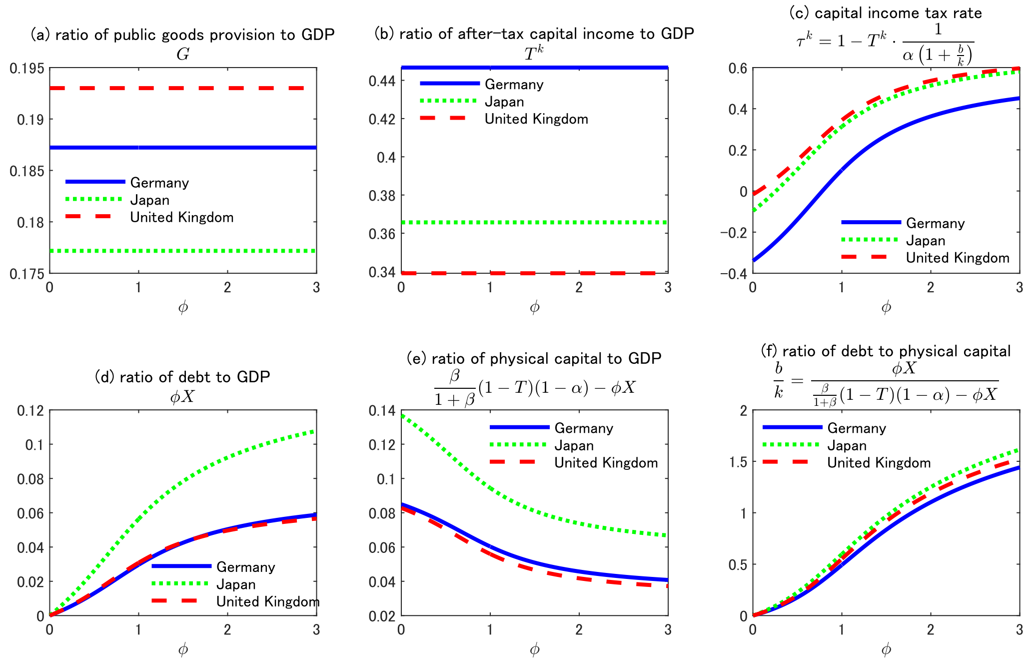

Figure 3: Changes in the ratio of public goods provision to GDP, G, the ratio of after-tax capital income to GDP, T^k, the capital income tax rate, τ^k, the ratio of debt to GDP, ϕX, the ratio of physical capital to GDP, $\frac{\beta}{1+\beta}(1-T)(1-\alpha) - \phi X$, and the ratio of debt to physical capital, $b/k = \phi X / \left[\frac{\beta}{1+\beta}(1-T)(1-\alpha) - \phi X\right]$ against changes in ϕ.
Note. Panel (c) demonstrates that τ^k becomes negative ($\tau^k < 0$) when ϕ is sufficiently small. However, as will be shown in Table 2, the realistic, calibration-based values of ϕ range from 2.35 to 2.85, and within this interval, τ^k remains positive ($\tau^k > 0$).

The first three panels of Figure 3 illustrate the numerical results of the effects of an increase in ϕ on the ratio of public goods provision to GDP, G (Panel (a)), the ratio of after-tax capital income to GDP, T^k (Panel (b)), and the capital income tax rate, τ^k (Panel (c)). The figure indicates that the parameters G and T^k are independent of ϕ, as the associated first-order conditions in (23) and (25) do not depend on ϕ. In other words, the effects of ϕ are mutually offset through adjustments in τ, x, and b'. However, ϕ influences the capital income tax rate, τ^k, indirectly through its impact on debt and physical capital accumulation, as inferred from the policy function for τ^k demonstrated in Proposition 1. Therefore, it is crucial to examine the effects of ϕ on the ratio of debt to physical capital to fully understand its indirect impact on the capital income tax rate.

The last three panels of Figure 3 present the numerical results of the effects of an increase in ϕ on the ratio of debt to GDP, ϕX (Panel (d)), the ratio of physical capital to GDP, $\frac{\beta}{1+\beta}(1-T)(1-\alpha) - \phi X$ (Panel (e)), and the ratio of debt to physical capital, $b/k = \phi X / \left[\frac{\beta}{1+\beta}(1-T)(1-\alpha) - \phi X\right]$ (Panel (f)). The result depicted in Figure 3 helps us to understand the effects of ϕ on the ratio of debt to physical capital in the following way. First, the ratio of debt to GDP, ϕX, increases as ϕ rises (see Panel (d)). This increase raises the cost of public debt repayment, leading to a higher capital income tax rate. Second, an increase in ϕ promotes savings and, consequently, physical capital accumulation through a reduction in the labor income tax rate. However, it also triggers a crowding-out effect due to increased public debt issuance, with the latter impact outweighing the former (see Panel (e)). This results in a rise in the interest rate, which further elevates the capital income tax rate through the increased cost of public debt repayment. As a result of these two effects, an increase in ϕ leads to a higher ratio of debt to physical capital (see Panel (f)), which, in turn, raises the capital income tax rate, τ^k.

The results described thus far have the following implications for the intergenerational distribution of fiscal burdens. An increase in ϕ enables the current middle-aged population to reduce their labor income tax burden by shifting fiscal responsibilities to future generations through the issuance of additional public debt. Simultaneously, the current older adult population bears the increased repayment burden of public debt issued by previous generations, who deferred their fiscal responsibilities. To address these heightened repayment costs, the current government chooses to raise the capital income tax rate, thereby increasing the tax burden on the current older adults. As a result, a higher ϕ reduces the fiscal burden on the current middle-aged while imposing greater burdens on both the current older adults and future generations. In other words, the tax burden spills over not only from the current working generation to younger generations who do not participate in voting but also to the current older adults, who do participate in voting. This characteristic represents a novel outcome arising from non-compliance with the GR in public debt issuance.

3.3 Politically Preferred Debt-Investment Ratio

Thus far, we have considered the debt-investment ratio, ϕ, as institutionally given and investigated the impact of ϕ on fiscal policy decisions. However, as we discussed in Section 2.4, the ratio itself is generally introduced after deliberation and voting in parliaments. Thus, it is natural to assume that they are also determined through the political process of voting. In this section, we introduce such a process into the model and show how the process and the resulting debt-investment ratio are affected by the structural parameters of the model economy.[9]

The debt-investment ratio determined through voting, called a *politically preferred debt-investment ratio*, is defined as ϕ, which maximizes the political objective function subject to the policy functions presented in Proposition 1. In writing down the political objective function at the first stage, recall the indirect utility functions of the middle-aged and older adults derived

[9]As mentioned in Section 2.3, the equilibrium policy functions ultimately obtained are the same whether ϕ and fiscal policy are chosen simultaneously or if ϕ is chosen first and fiscal policy is decided thereafter. The reason for dividing the determination of ϕ and fiscal policy into two stages is to observe the influence of the ratio ϕ on the subsequent determination of fiscal policy as disussed in Section 3.2.

in Section 2. Using the policy function obtained in Proposition 1 and the equilibrium condition of the asset market in (16), the political objective function in (20) at the first stage is written as follows

$$\Omega = \frac{\omega}{(1+n)(1-\omega)}\left(\ln d\left(\tau^k; k, k^p, b\right) + \theta \ln g\right)$$
$$+ \ln c\left(\tau; k, k^p\right) + \theta \ln g + \beta\left(\ln d'\left(\tau, x, b'; k, k^p\right) + \theta \ln g'\left(\tau, x, b'; k, k^p\right)\right), \qquad (36)$$

where the derivation of (36) is given in Appendix A.3. Thus, the problem of the government at the first stage is to choose ϕ to maximize Ω in (36) subject to the debt-investment relationship in (14), the government budget constraint in (17), and the policy functions derived in Proposition 1. The solution to the problem, denoted by $\phi_{political}$, is provided in the following proposition.

Proposition 2 *A politically preferred debt-investment ratio, $\phi_{political}$, along the Markov-perfect political equilibrium characterized in Proposition 1 is*

$$\phi_{political} = \frac{1-\alpha(1+\theta)}{(1+\theta)\,\eta\,(1-\alpha)}. \qquad (37)$$

Proof. See Appendix A.3.

The result demonstrated in Proposition 2 suggests that $\phi_{political}$ is constant across periods, and that θ, representing the preferences for public goods, and η, representing the elasticity of public capital with respect to public investment, are crucial to the determination of $\phi_{political}$. A greater θ implies that voters attach a larger weight to public goods expenditure, and thus provide a stronger incentive for the government to cut public debt issuance through the disciplining effect. A greater η implies a greater marginal benefit of public investment, thus, incentivizing the government to lower the debt-financed proportion of public investment, ϕ, from the viewpoint of balancing the marginal costs and benefits of public investment, as we can see in (35). Thus, $\phi_{political}$ decreases as θ and η increase.

Based on the result in Proposition 2 with the calibration reported in Table 1, we compute the politically preferred debt-investment ratio $\phi_{political}$ for the three countries, as reported in Table 2. Japan exhibits the lowest $\phi_{political}$, followed by the United Kingdom and Germany. This ranking is primarily driven by the values of η, which are highest in Japan, followed by the United Kingdom and Germany. However, in comparing Japan and the United Kingdom, the relative magnitude of θ also plays a crucial role, as θ is higher but η is lower in the United Kingdom compared to Japan. Table 2 reports that $\phi_{political}$ of Japan is lower than that of the United Kingdom, suggesting that the effect of η overcomes the effect of θ in determining $\phi_{political}$ of these two countries.

Apart from the property mentioned above, the result of Proposition 2 highlights three critical observations. First, compared to an economy without public goods ($\theta = 0$), the issuance of public debt is restrained when public goods provision is present. This is due to the disciplining effect, unique to this study, which operates through public goods provision. Second, without the

Country	$\phi_{political}$	$Y/Y_{-1} = (1+n)y/y_{-1}$
Germany	2.847	1.818
Japan	2.346	2.038
United Kingdom	2.507	2.052

Table 2: The politically preferred debt-investment ratio, $\phi_{political}$, and the gross growth rate of GDP, $Y/Y_{-1} = (1+n)y/y_{-1}$.

disciplining effects through public goods provision and the capital income tax rate, the political objective function in (36) is monotonically increasing with respect to ϕ. In other words, public debt issuance would occur up to its maximum limit. The disciplining effects thus play a crucial role in restraining public debt issuance. Third, assuming a full depreciation of public capital ($\eta = 1$) simplifies the analysis and results in the GR compliance ($\phi_{political} < 1$). However, the assumption of $\eta = 1$ contradicts the calibration results reported in Table 1. Therefore, the assumption of $\eta < 1$ is necessary to maintain the empirical validity of the analysis.

3.4 Non-compliance with the GR in Political Equilibrium

In this section, we examine the relationship between the GR in (13) and the politically preferred debt-investment ratio, $\phi_{political}$, determined through voting, as presented in Proposition 2. Recall that the GR is exogenously imposed in the model, and the government, representing existing generations, faces this constraint in each period. However, in the choice of fiscal policy within political equilibrium, we have allowed for the possibility that the government might not comply with the GR. Specifically, in determining fiscal policy through voting, we have required that the public investment and debt issuance chosen by the government satisfy the debt-investment relationship in (14), including the possibility of non-compliance with the GR in (13). Furthermore, we have described the situation where the parameter ϕ, which governs (14), is determined through voting in the pre-policy stage and demonstrated its characteristics.

The central question here is under what conditions the fiscal policy characterized in the political equilibrium (Proposition 1) and the corresponding debt-investment ratio (Proposition 2) ultimately fulfill the GR described by (13). To address the question, we rewrite the GR in (13) as follows:

$$(1+n)\,b' - b \le (1+n)\,x \Leftrightarrow B' - B \le xN' \Leftrightarrow \frac{\frac{Y}{Y_{-1}}\frac{B'}{Y} - \frac{B}{Y_{-1}}}{\frac{Y}{Y_{-1}}\frac{xN'}{Y}} \le 1, \tag{38}$$

where Y_{-1} denotes the GDP in the pervious period. From the above expression, it can be inferred that higher growth rates of GDP, Y/Y_{-1}, make it more difficult to comply with the GR.

To understand the background of this implication, recall that in the political equilibrium, B' and xN' are constant proportions of GDP, and so are linear with respect to Y. Therefore, as the GDP growth rate increases, both debt issuance B' and public investment xN' increase at the same rate. In other words, the debt issuance/public investment ratio remains unchanged. However, the GR requires that the fiscal deficit, which subtracts the outstanding public debt,

B, from debt issuance, B', must be less than public investment, xN', not the debt issuance itself. The existence of B, which appears in the numerator of the left-hand side of (38), makes compliance with the GR more challenging during periods of economic growth. Indeed, as the left-hand side of (38) increases with the GDP growth rate Y/Y_{-1}, higher growth rates make it increasingly difficult to comply with the GR.

To derive the implications for the political equilibrium, we reformulate the GR in (13), using the politically preferred debt-investment ratio from Proposition 2, as follows:

$$(1+n)\,b' - b \leq (1+n)\,x \Leftrightarrow \frac{(1+n)\,b'}{(1+n)\,x} - \frac{b}{(1+n)x} \leq 1 \Leftrightarrow \phi_{political}\left[1 - \left((1+n)\,\frac{y}{y_{-1}}\right)^{-1}\right] \leq 1.$$

From this expression, we derive the following additional implications. First, if the gross growth rate of GDP is less than one and so the GDP growth rate is negative (i.e., $(1+n)\,y/y_{-1} = Y/Y_{-1} < 1$), the GR will be satisfied regardless of the level of ϕ determined in the political equilibrium. Specifically, the lower the population growth rate or the lower the growth rate of GDP per middle-aged individual, the easier it becomes to comply with the GR. Second, when the GDP growth rate is positive ($(1+n)\,y/y_{-1} = Y/Y_{-1} > 1$), whether the GR is satisfied depends on the level of ϕ determined in the political equilibrium. If $\phi_{political} \leq 1$, the GR will be satisfied regardless of the GDP growth rate. Furthermore, even if $\phi_{political} > 1$ and the constraint on debt issuance is relaxed, the GR could still be satisfied as long as the GDP growth rate is low.

The above analysis extends to three sample countries: Germany, Japan, and the United Kingdom. As shown in Figure 1, non-compliance with the GR was prevalent in all three countries during the period under examination (2002–2007), except for the final two years in Germany and Japan. To understand the underlying causes of this outcome, refer to Table 2, which reports the gross GDP growth rates (Y/Y_{-1}) and the politically preferred debt-to-investment ratios $\phi_{political}$ for the three sample countries. In all cases, the GDP growth rate is positive ($Y/Y_{-1} > 1$), and the politically preferred debt-to-investment ratio exceeds 1 ($\phi_{political} > 1$). This suggests that the GR is not satisfied in these countries on average during the period.

Let us now examine each country in further detail. Germany exhibits the lowest GDP growth rate among the three sample countries, while its politically preferred debt-to-investment ratio is the highest. This high ratio contributes to the failure to meet the GR. As indicated by the parameter estimates in Table 1 and the results of Proposition 2, the smaller values of θ and η in Germany, compared to the other two countries, result in this higher ratio. In contrast, Japan and the United Kingdom display higher GDP growth rates than Germany, which leads to the failure to comply with the GR. The primary factors behind these higher growth rates are the high population growth rate in the United Kingdom and the high growth rate of GDP per middle-aged individual in Japan. The analysis so far suggests that whether the GR is satisfied in the political equilibrium depends critically on the economic growth rate and the politically preferred debt-to-investment ratio, both of which are strongly influenced by the structural parameters of each country.

4 Assessing Political Distortions

In Section 3.3, we have demonstrated the politically preferred debt-investment ratio, $\phi_{political}$, set through voting. We have also shown that the fiscal policy is not in accordance with the GR in the three countries, Germany, Japan, and the United Kingdom during the period under examination (2002–2007). The result may be because $\phi_{political}$ and the associated fiscal policy are the choices that consider only the generations existing at the time of voting and does not take into account unborn future generations. Thus, evaluating the political equilibrium allocation presented in Section 3 from a long-term perspective reveals the inherent issues in the short-sighted behavior of elected politicians.

To evaluate the political equilibrium allocation from a long-term perspective, we begin by assuming a Ramsey planner who sets the fiscal policy sequence in the initial period so as to maximize the discounted sum of utilities of current and future generations subject to the same set of constraints faced by the short-lived governments. Assuming such a planner, we characterize the allocation chosen by the Ramsey planner and compare it to the political equilibrium allocation. This comparison should provide us with some insight into the setting of fiscal policy that takes unborn future generations into account.

The Ramsey planner is assumed to value the welfare of all generations. In particular, its objective is to maximize a discounted sum of the lifecycle utility of all current and future generations, $SW = \gamma^{-1}V_0^O + \sum_{t=0}^{\infty}\gamma^t V_t^M$, $0 < \gamma < 1$, subject to the public capital function in (10), the government budget constraint in (12), and the asset market clearing condition in (16), where k_0, k_0^p, and b_0 are given. The individuals' optimal saving and consumption in (2), (3), and (4) and firms' optimal demand for labor and physical capital in (7) and (8) are included in V_0^O and V_t^M. The solution to this problem is called a "Ramsey allocation." The parameter $\gamma \in (0,1)$ in the objective function is the discount factor of the Ramsey planner.

Solving the problem leads to the following characterization of the policy functions in the Ramsey allocation.

Proposition 3 *Given k_0, k_0^p, and b_0, the policy functions in the Ramsey allocation are given by the following:*

$$\tau_t^k = 1 - \bar{T}^k \cdot \frac{1}{\alpha\left(1 + \frac{b_t}{k_t}\right)}, \tag{39}$$

$$\frac{2+n}{1+n} g_t = \bar{G} \cdot y\left(k_t, k_t^p\right), \tag{40}$$

$$(1+n)x_t = \bar{X} \cdot y\left(k_t, k_t^p\right), \tag{41}$$

$$\tau_t = \bar{T}, \tag{42}$$

$$(1+n)\, b_{t+1} = \bar{B} \cdot y\left(k_t, k_t^p\right), \tag{43}$$

where $\bar{T}^k$, $\bar{G}$, $\bar{X}$, $\bar{T}$, and $\bar{B}$, defined in Appendix A.4, are constant and dependent on γ.

Proof. See Appendix A.4.

The result of Proposition 3 highlights two essential points. First, the parameters $\bar{T}^k$ $\bar{G}$, $\bar{X}$, $\bar{T}$, and $\bar{B}$ in the policy functions of Proposition 3 are contingent on the discount factor γ of the Ramsey planner. In simpler terms, the policy maximizing social welfare depends on the importance the Ramsey planner places on future generations relative to the current generation. Second, although the policy functions in Proposition 3 may initially seem akin to those in the political equilibrium from Proposition 1, the parameters in the policy functions chosen by the Ramsey planner differ from those in the political equilibrium.

To explore these points further, we present the parameters of the policy functions in the Ramsey allocations for Germany, Japan, and the United Kingdom, based on the calibrations in Section 3.2. Figure 4 illustrates $\bar{T}$, $\bar{T}^k$, $\bar{B}$, $\bar{X}$, and $\bar{G}$, respectively, against the Ramsey planner's discount factor γ on the horizontal axis for the three countries. The figure reveals the following trends. First, as γ increases, the ratio of public investment to GDP ($\bar{X}$) rises, while the ratio of after-tax capital income to GDP ($\bar{T}^k$) and the ratio of public goods to GDP ($\bar{G}$) decrease. Second, the labor income tax rate ($\bar{T}$) displays a U-shaped pattern, decreasing initially and then rising with an increase in γ. Conversely, the ratio of public debt to GDP ($\bar{B}$) follows an inverse U-shaped pattern, initially rising and then falling as γ increases. When γ reaches a certain level, the ratio of public debt to GDP ($\bar{B}$) becomes zero due to zero issuance of public debt. In what follows, we discuss the mechanisms behind these findings.

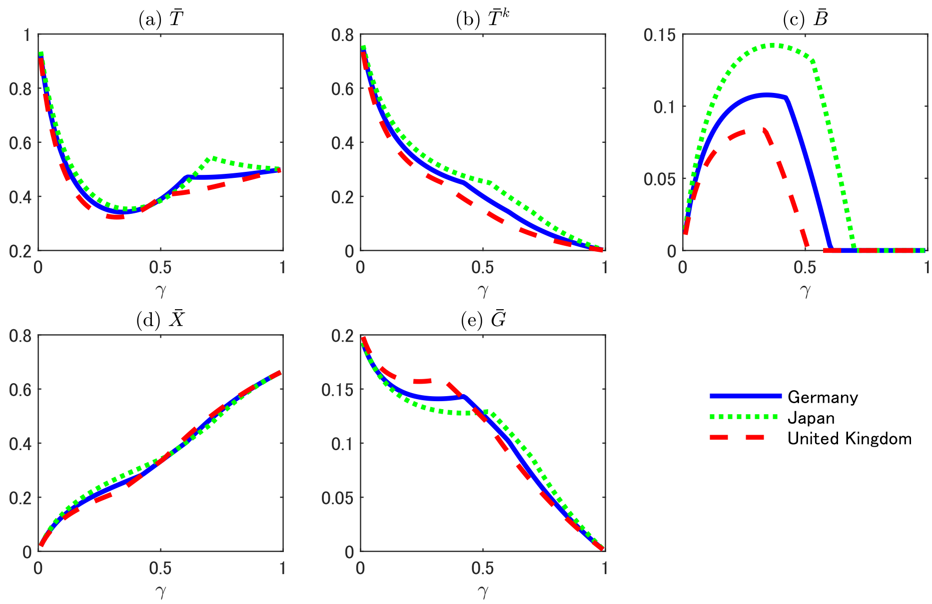

Figure 4: Changes in $\bar{T}$, $\bar{T}^k$, $\bar{B}$, $\bar{X}$, and $\bar{G}$ against changes in γ.

Beginning with the impact of the discount factor γ on $\bar{X}$ and $\bar{G}$, an increase in public investment incurs three costs or benefits. First, the current supply of public goods decreases because of limited government budgets, causing a utility loss for the current older adults. However, this loss is less significant for the Ramsey planner as γ increases, encouraging public investment. Second, increased public investment raises wages and interest rates for both current and future generations, promoting overall welfare. Third, the wage increase effect leads to higher savings and increased physical capital in successive periods. This, in turn, boosts production and the supply of public goods in future periods, benefiting all future generations. Consequently, an increase in γ promotes public investment and thus the ratio of public investment to GDP ($\bar{X}$), while it reduces public goods provision and thus the ratio of public goods to GDP ($\bar{G}$) due to limited budget constraint.

Next, when considering the impact of an increase in γ on $\bar{T}^k$, it is important to acknowledge that private savings remain unaffected by the capital income tax rate (τ_t^k) due to the logarithmic utility function. This independence implies that τ_t^k does not generate spillover effects on future generations through physical and public capital accumulation. Consequently, the effects of τ_t^k are static in the following manner. As τ_t^k rises, the consumption of the current older adults decreases. However, the Ramsey planner values this utility loss less as γ increases. Simultaneously, the increased tax revenues from a higher capital income tax rate enhance the supply of public goods for the current generation. This benefit is relatively less valued with a larger γ. Thus, a larger γ leads to a higher τ_t^k and thus a lower $\bar{T}^k$.

Finally, upon assessing the impact of γ on variables $\bar{B}$ and $\bar{T}$, an increase in debt issuance entails distinct costs and benefits. Regarding the cost of debt issuance, an increased debt issuance yields two adverse effects on future generations. Firstly, it triggers a negative spillover effect on physical capital through crowding out, and secondly, it constricts the government's budget due to increased debt repayment costs. These effects gain greater significance with a higher γ, signifying a stronger consideration by the Ramsey planner toward future generations. Conversely, the benefit of debt issuance lies in the government's ability to alleviate the tax burden on the middle-aged by funding public expenditures through debt issuance. However, this positive effect diminishes in value with a larger γ. Furthermore, the reduced tax burden on the middle-aged mitigates the negative spillover effect on physical capital via increased savings, a consequence that gains prominence as γ increases. Initially low γ results in an augmented debt-GDP ratio ($\bar{B}$), while higher initial γ levels lead to a decrease in $\bar{B}$. Beyond a certain threshold of γ, $\bar{B}$ becomes zero due to a cessation of debt issuance. Conversely, the impact on the labor income tax rate ($\bar{T}$) contrasts that on $\bar{B}$. This discrepancy arises from the increase (or decrease) in debt issuance revenue being offset by a decrease (or increase) in labor income tax revenue.

Figure 5 illustrates the effects of the discount factor (γ) on various economic indicators through the policy variables across the three countries, taking γ on the horizontal axis. These indicators include the growth rate of public capital (panel (a)), the growth rate of physical capital (panel (b)), the growth rate of GDP (panel (c)), the ratio of fiscal deficit to GDP (panel (d)), and the ratio of fiscal deficit to public investment (panel (e)). The first three indicators are measured on a per middle-aged individual basis. When γ is low, $K'/Y = (1+n)k/y = 0$ holds because a low γ implies a high tax rate $\bar{T}$, which reduces private savings. These limited savings are fully allocated to government debt issuance through the asset market, resulting in a physical capital stock of zero. Consequently, the ratio of physical capital to public capital in the steady state becomes $k/k^p = 0$, leading to $k'/k = k^{p\prime}/k^p = y'/y = 0$. In the following analysis, we focus on the range of γ that satisfies $K'/Y > 0$, $k'/k > 0$, $k^{p\prime}/k^p > 0$, and $y'/y > 0$, and examine the impact of γ on the concerned indicators.

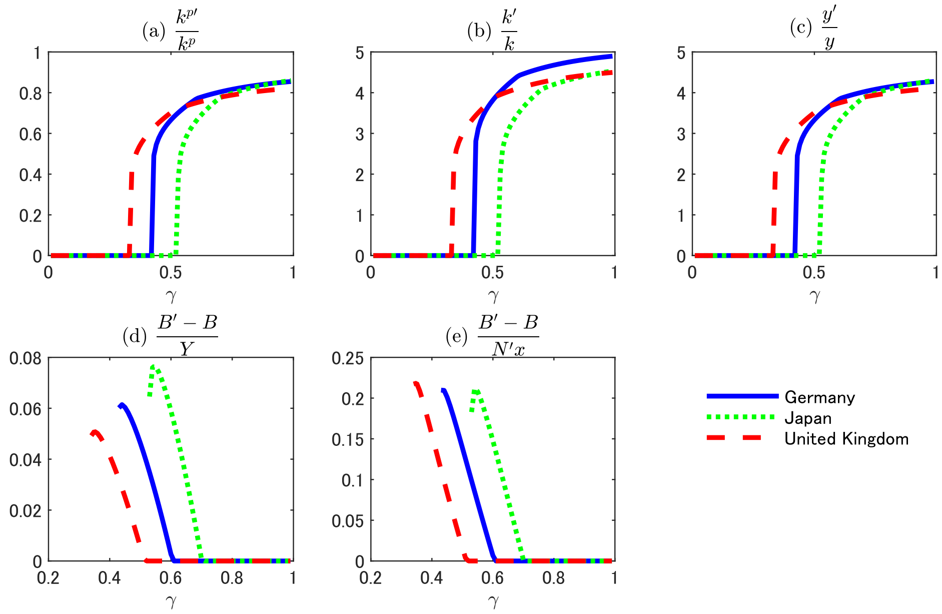

Figure 5: Changes in the growth rate of public capital, the growth rate of physical capital, the growth rate of GDP, the ratio of fiscal deficit to GDP, and the ratio of fiscal deficit to public investment against changes in γ.
Note. Panels (d) and (e) do not present the indicators for low values of γ, because the physical capital stock is zero and consequently no economic activity occurs at those values of γ.

First, as γ increases, the accumulation of physical and public capital accelerates, resulting in a higher growth rate. To understand the mechanism behind this result, recall the accumulation equations for physical and public capital in (32) and (33) under the political equilibrium. By substituting the policy functions in (32) and (33) with those corresponding to the Ramsey allocation under the condition $\phi\bar{X} = \bar{B}$, the accumulation equations in the Ramsey allocation can be expressed as follows:

$$\frac{k'}{k} = \frac{1}{1+n}\left(\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha) - \bar{B}\right) A \left(\frac{k^p}{k}\right)^{1-\alpha},$$
$$\frac{k^{p\prime}}{k^p} = D\left(\frac{1}{1+n}\bar{X}A\left(\frac{k}{k^p}\right)^{\alpha}\right)^{\eta}.$$

The above expressions suggest that as γ increases, the planner places greater emphasis on future generations, leading to an increase in public investment (see Figure 4(d)). This, in turn, stimulates the accumulation of public capital (Figure 5(a)). The rise in public investment increases income, which subsequently enhances savings and promotes the accumulation of physical capital. While an increase in γ also produces additional effects, including non-monotonic impacts on the labor income tax rate (Figure 4(a)) and public debt issuance (Figure 4(c)), these two factors tend to offset one another. Consequently, the impact of increased public investment dominates, resulting in a higher growth rate of physical capital (Figure 5(b)). Through these combined effects on physical and public capital, growth rates increase as γ rises (Figure 5(c)).

Second, as depicted in Figures 4(c) and 5(c), within the range of γ where the growth rate $y'/y > 0$, $\bar{B}$ decreases with respect to γ. This implies that as γ increases, the ratio of government debt issuance to GDP declines. Consequently, the ratio of fiscal deficit to GDP also decreases as γ increases (Figure 5(d)). In particular, each country has a threshold for γ, above which government debt issuance becomes zero, resulting in a zero fiscal deficit to GDP ratio. Third, while fiscal deficits decrease with an increase in γ, public investment increases (see Figure 4(d)). Thus, the ratio of fiscal deficit to public investment declines with rising γ, and beyond the aforementioned threshold for γ, this ratio also becomes zero (Figure 5(e)).

A particularly noteworthy feature is that, in the Ramsey allocation, the ratio of the fiscal deficit to public investment is below 1 in all three countries (see Figure 5(e)). This suggests that the Ramsey planner with a long-term perspective would design fiscal policies in accordance with the GR. However, as demonstrated in Section 3, a government consisting of short-sighted politicians fails to adhere to the GR. This highlights the need for a long-term institutional mechanism to limit such excessive fiscal issuance. The GR can serve as a benchmark for identifying excessive public debt. Specifically, if the current fiscal policy does not adhere to the GR, the government should revise its policies to ensure compliance with the GR, thereby bringing the resulting resource allocation closer to the Ramsey allocation.

5 Conclusion

This study explores the influence of fiscal rules, particularly the Golden Rule of Public Finance (GR), on fiscal policy formation, physical and public capital accumulation, and intergenerational welfare. Using a three-period overlapping-generations model calibrated to Germany, Japan, and the United Kingdom, we demonstrate that political decisions regarding fiscal rule compliance significantly impact resource allocation. Our results suggest that, even in instances of non-compliance, fiscal rules like the GR serve as critical benchmarks for evaluating welfare, as they enable fiscal policies to approximate an allocation chosen by the Ramsey planner with a long-term perspective. However, we find that factors such as GDP growth rates and preferences for debt issuance impact a country's ability to fully comply with the GR, with high growth rates often leading to structural non-compliance.

Future research could benefit from examining fiscal rule compliance in a broader set of countries, especially those with distinct economic structures or emerging market dynamics. Expanding the model to incorporate additional demographic variables, such as aging populations or migration, would also enhance its applicability. Furthermore, exploring the effects of different fiscal rule thresholds or alternative political economy frameworks could offer insights into how various rule designs impact both short-term policy decisions and long-term welfare. Such research could contribute to refining fiscal rule structures to accommodate diverse economic contexts while promoting sustainable public finance.

A Proofs and Supplementary Explanations

A.1 Proof of Proposition 1

Given the conjecture in (30) and (31), we can reformulate the first-order condition with respect to x in (26) as follows:

$$\lambda = \frac{\beta(1+\theta)}{(1-\phi)\,y(k,k^p)}\left[(-1)\frac{\alpha\phi(1+\theta,)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X}+\frac{\eta(1-\alpha)}{X}\right]. \tag{A.1}$$

We can also reformulate the first-order condition with respect to τ in (24) as follows:

$$\lambda = \frac{1}{(1-\alpha)\,y(k,k^p)}\left[\frac{1}{1-T}+\frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X}\right]. \tag{A.2}$$

With (A.1) and (A.2), we obtain

$$\frac{\beta(1+\theta)\eta(1-\alpha)}{(1-\phi)}\cdot\frac{1}{X} = \frac{1}{(1-\alpha)(1-T)}+\frac{\alpha\beta(1+\theta)\left[\frac{\beta}{1+\beta}+\frac{\phi}{1-\phi}\right]}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X}, \tag{A.3}$$

where (A.3) includes two undetermined constants, T and X. Denote the left-hand (right-hand) side of (A.3) by $LHS^{(A.3)}$ ($RHS^{(A.3)}$). They have the following properties: $\partial LHS^{(A.3)}/\partial X<0$, $\lim_{X\to 0}LHS^{(A.3)}=+\infty$, $\lim_{X\to+\infty}LHS^{(A.3)}=0$, $\partial RHS^{(A.3)}/\partial X>0$, $\lim_{X\to 0}RHS^{(A.3)}\in(0,+\infty)$, and $\lim_{X\to\frac{\beta}{1+\beta}(1-\alpha)(1-T)/\phi}RHS^{(A.3)}=+\infty$. These properties imply that given T, there is a unique X that satisfies (A.3): $X=X(T)$.

Given T and X, the first-order condition with respect to τ^k in (23) is reformulated as

$$1-\tau^k = \frac{\omega}{(1+n)(1-\omega)}(1-\alpha)\left[\frac{1}{1-T}+\frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X}\right]^{-1}\frac{1}{\alpha\frac{k+b}{k}}. \tag{A.4}$$

Eq. (A.4) shows that the conjecture of τ^k in (21) is correct as long as T and X are constant.

Next, given T and X, the first-order condition with respect to g in (25) is reformulated as

$$\frac{2+n}{1+n}g = \left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\theta\left[\frac{1}{1-T}+\frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X}\right]^{-1}(1-\alpha)\,y(k,k^p). \tag{A.5}$$

Eq. (A.5) shows that the conjecture of g in (22) is correct as long as T and X are constant.

The remaining task is to show that the conjectures of T in (30) and X in (31) are correct. Consider the government budget constraint in (17). We substitute the policy functions derived thus far into the constraint and rearrange the terms to obtain

$$\begin{aligned}
&\alpha-\frac{\omega}{(1+n)(1-\omega)}\left[\frac{1}{1-T}+\frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X}\right]^{-1}(1-\alpha)+T(1-\alpha)\\
&=\left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\theta\left[\frac{1}{1-T}+\frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X}\right]^{-1}(1-\alpha)+(1-\phi)X.
\end{aligned} \tag{A.6}$$

The expression in (A.6) is independent of the state variables, k, k^p, and b, and times. Thus, we can verify that the two unknown parameters, X and T, are constant and solved for using (A.3) and (A.6). They are given by

$$X \equiv \frac{I(\phi)}{(1-\phi)\phi}\left\{1+\left[\frac{\omega}{(1+n)(1-\omega)}(1+\theta)+\theta\right]\frac{\frac{\beta}{1+\beta}-\frac{I(\phi)}{1-\phi}}{\frac{\beta}{1+\beta}[1+\alpha\beta(1+\theta)]-\frac{I(\phi)}{1-\phi}}+\frac{I(\phi)}{\phi}\right\}^{-1},$$

$$T \equiv 1-\frac{1}{1-\alpha}\left\{1+\left[\frac{\omega}{(1+n)(1-\omega)}(1+\theta)+\theta\right]\frac{\frac{\beta}{1+\beta}-\frac{I(\phi)}{1-\phi}}{\frac{\beta}{1+\beta}[1+\alpha\beta(1+\theta)]-\frac{I(\phi)}{1-\phi}}+\frac{I(\phi)}{\phi}\right\}^{-1},$$

where $I(\phi)$ appeared in the expressions of X and T is

$$I(\phi) \equiv \frac{H_1(\phi)-\sqrt{(H_1(\phi))^2-4H_2(\phi)}}{2},$$

and $H_1(\phi)$ and $H_2(\phi)$ are

$$H_1(\phi) \equiv \beta(1+\theta)(\alpha+\eta(1-\alpha))\phi+\frac{\beta}{1+\beta}[1+\alpha\beta(1+\theta)](1-\phi),$$

$$H_2(\phi) \equiv (1-\phi)\phi\beta(1+\theta)\eta(1-\alpha)\frac{\beta}{1+\beta}.$$

Substitution of X and T into (A.4) and (A.5) lead to the policy functions of $1-\tau^k$ and $\frac{2+n}{1+n}g$ as presented in Proposition 1, where T^k and G are given by

$$T^k \equiv \frac{\omega}{(1+n)(1-\omega)}(1-\alpha)\left[\frac{1}{1-T}+\frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X}\right]^{-1},$$

$$G \equiv \left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\theta\left[\frac{1}{1-T}+\frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X}\right]^{-1}(1-\alpha).$$

■

A.2 Data and Calibration

A.2.1 Sources of the Data for Table 1

The sources of the data are as follows: World Development Indicators (`https://databank.worldbank.org/source/world-development-indicators`, accessed on September 9, 2022) for data on POP and GOV; the data archive of Professor McDaniel (`https://www.caramcdaniel.com/research`, accessed on April 6, 2021) for data on TAX; OECD gross domestic product (GDP) indicator (`doi:10.1787/dc2f7aec-en`, accessed on April 12, 2022), OECD investment ($GFCF$) indicator (`doi:10.1787/b6793677-en`, accessed on April 12, 2022), and OECD investment by sector indicator (`doi:10.1787/abd72f11-en`, accessed on April 12, 2022) for data on INV; OECD library, General government deficit indicator (`https://doi.org/10.1787/cc9669ed-en`, accessed on February 25, 2021) for data on DEF of Germany and the United Kingdom; and FRED economic data (`https://fred.stlouisfed.org/series/GGNLBAJPA188N`, accessed on July 24, 2024) for data on DEF of Japan; and World Development Indicator of GDP per capita (`https://databank.worldbank.org/source/world-development-indicators`, accessed on June 4, 2024) for data on GRO.

A.2.2 Calibration on n, β, ω, η, θ, and A

Country specific parameters, n, β, ω, η, θ, and A are calibrated in the following way. The population growth rate, n, is obtained from the average of each sample country during the 2002—2007 period. Let POP_j denote the annual gross population growth rate of country j. The net population growth rate for 30 years is $(POP_j)^{30} - 1$.

To determine the remaining five parameters, β, ω, η, θ, and A, we focus on the labor income tax rate, τ, the ratio of the public goods provision to GDP, G/Y, the ratio of public investment to GDP, $N'x/Y$, the ratio of deficit to GDP, $(B' - B)/Y$, and the growth rate of GDP per middle-aged individual, y'/y. These five variables are given by

$$\tau_j = TAX_j, \tag{A.7}$$

$$\left.\frac{G}{Y}\right|_j = GOV_j, \tag{A.8}$$

$$\left.\frac{N'x}{Y}\right|_j = INV_j, \tag{A.9}$$

$$\left.\frac{B' - B}{Y}\right|_j = DFF_j, \tag{A.10}$$

$$\left.\frac{y'}{y}\right|_j = GRO_j, \tag{A.11}$$

where the subscript j is the country code. We use the data of TAX_j, GOV_j, INV_j, DFF_j, and GRO_j for each sample country during 2002–2007 and solve the five equations in (A.7) – (A.11) for β_j, ω_j, η_j, θ_j, and A_j. The result is presented in Table 1.

The capital income tax rate is not a target for the calibration. Using the values of the parameters estimated based on the calibration, we need to compute the capital income tax rate and check whether it is consistent with the data of the three sample countries. Table 3 reports the capital income tax rates we compute on the basis of the calibration and those obtained from Professor McDaniel's data archive.[10] The result in Table 3 shows that the computed capital income tax rates are higher than the actual rates for the three countries. This suggests that there is room for improvement in the calibration, but the computed capital income tax rates fall within the range $(0, 1)$. We, therefore, use the calibration result in Table 1 to carry out the analysis in the main text.

[10]https://www.caramcdaniel.com/ (accessed on February 17, 2022).

Country	estimation	data
Germany	0.4420	0.1701
Japan	0.5431	0.1965
United Kingdom	0.5733	0.2808

Table 3: Estimated and actual capital income tax rates of the sample three countries.
Source: Actual capital income tax rates are obtained from Professor McDaniel's data archive: `https://www.caramcdaniel.com/` (accessed on February 17, 2022).

A.2.3 Data on Public Investment

To estimate the ratio of public investment to GDP, we use the following three sorts of the data: the gross domestic product (GDP), gross fixed capital formation ($GFCG$), and the ratio of the investment by sector, general government ($ISGG$). Using this data, we first compute the ratio of public investment to GDP of country i in year t, denoted by $INV_{i,t}$, as follows:

$$INV_{i,t} = \frac{(ISGG_{i,t}/100) \cdot GFCF_{i,t}}{GDP_{i,t}}.$$

Then, INV_i, representing the average ratio of public investment to GDP of country i during 2002–2007, is computed based on the following equation:

$$INV_i = \frac{1}{6} \sum_{t=2002}^{2007} INV_{i,t}.$$

■

A.3 Supplement to Subsection 3.3

A.3.1 Derivation of Ω in (36)

Recall the indirect utility function of the middle in (18), which is reformulated using the recursive notation as follows:

$$V^M\left(\tau, x, g, \tau^{k\prime}, g'; k, k^p, k', k^{p\prime}, b'\right) = \ln c\left(\tau; k, k^p\right) + \theta \ln g + \beta \left[\ln d'\left(\tau^{k\prime}; k', k^{p\prime}, b'\right) + \theta \ln g'\right]. \tag{A.12}$$

The term d' in (A.12) is reformulated as follows:

$$\begin{aligned} d'(\cdot) &= \left(1-\tau^{k\prime}\right) R\left(k', k^{p\prime}\right)(1+n)\left(k'+b'\right) \\ &= T^k \frac{k'}{\alpha\left(k'+b'\right)} \alpha A \left(k'\right)^{\alpha-1}\left(k^{p\prime}\right)^{1-\alpha}(1+n)\left(k'+b'\right) \\ &= (1+n)T^k \cdot y\left(k', k^{p\prime}\right), \end{aligned} \tag{A.13}$$

where the equality in the first line comes from the budget constraint for older adults, $d' = \left(1-\tau^{k\prime}\right) R's$ and the asset market clearing condition, $(1+n)(k'+b') = s$; the equality in the second line comes from the conjecture of $\tau^{k\prime}$ in (21) and the first-order condition with respect to physical capital in (6), and the equality in the third line comes from the production function, $y(k,k^p) = A\,(k)^{\alpha}\,(k^p)^{1-\alpha}$. The term $g'(\cdot)$ is reformulated, using the conjecture of g' in (22), as

$$g'(\cdot) = G \cdot y\left(k', k^{p\prime}\right). \tag{A.14}$$

The term $y\left(k', k^{p\prime}\right)$, which appeared in (A.13) and (A.14), is further reformulated as follows:

$$\begin{aligned} y\left(k', k^{p\prime}\right) &= A\left[\frac{1}{1+n}\frac{\beta}{1+\beta}(1-\tau)\,(1-\alpha)\,y(k,k^p) - b'\right]^{\alpha}\left[D\,(k^p)^{1-\eta}\,(x)^{\eta}\right]^{1-\alpha} \\ &= y\left(\tau, x, b'; k, k^p\right) \end{aligned} \tag{A.15}$$

where we use the asset market clearing condition in (16) and the public capital formation function in (10) in deriving (A.15). Thus, the terms $d'(\cdot)$ in (A.13) and $g'(\cdot)$ in (A.14) are now given by

$$\begin{aligned} d' &= d'\left(\tau, x, b'; k, k^p\right) \equiv (1+n)T^k \cdot y\left(\tau, x, b'; k, k^p\right), \\ g' &= g'\left(\tau, x, b'; k, k^p\right) \equiv G \cdot y\left(\tau, x, b'; k, k^p\right), \end{aligned}$$

respectively, and the indirect utility function of the middle-aged in (A.12) is reformulated as follows:

$$\begin{aligned} V^M\left(\tau, x, g, b'; k, k^p\right) &= \ln c\,(\tau; k, k^p) + \theta \ln g \\ &\quad + \beta\left[\ln d'\left(\tau, x, g, b'; k, k^p\right) + \theta \ln g'\left(\tau, x, g, b'; k, k^p\right)\right]. \end{aligned} \tag{A.16}$$

We substitute the indirect utility function of older adults in (19) and that of the middle-aged in (A.16) into the political objective function in (20) and then obtain (36).

A.3.2 Proof of Proposition 2

The problem of the government is to choose ϕ to maximize Ω in (36) subject to the government budget constraint in (17), the fiscal rule in (14), and the policy functions presented in Proposition 1. Substituting (17) and (14) into (36) and rearranging the terms, we have

$$\begin{aligned} \Omega \approx\ & \frac{\omega}{(1+n)(1-\omega)} \ln d\left(\tau^k; k, k^p, n\right) + \ln c\,(\tau; k, k^p) \\ & + \left(\frac{\omega}{(1+n)(1-\omega)} + 1\right)\theta \ln \underbrace{\frac{1+n}{2+n}\left[TR^K\left(\tau^k; k, k^p\right) + TR\,(\tau; k, k^p) - (1-\phi)(1+n)x - R\,(k, k^p)\,b\right]}_{=g} \\ & + \beta\,(1+\theta)\,\alpha \ln \underbrace{\left[\frac{\beta}{1+\beta}\,(1-\tau)\,(1-\alpha)\,y(k,k^p) - \phi(1+n)x\right]}_{=(1+n)k'} + \beta\,(1+\theta)\,\eta\,(1-\alpha)\ln x, \end{aligned} \tag{A.17}$$

where the politically unrelated terms are omitted from the expression in (A.17).

We write the policy functions of τ^k, τ, and x derived in Proposition 1 in the following implicit form:

$$\begin{aligned} \tau^k &= \tau^k\,(\phi; k, k^p), \\ \tau &= \tau\,(\phi), \\ x &= x\,(\phi; k, k^p). \end{aligned}$$

We substitute these policy functions into (A.17) and obtain, by using the envelope theorem, the first-order condition with respect to ϕ as follows:

$$\frac{\partial \Omega}{\partial \phi} = 0 \Leftrightarrow \left(\frac{\omega}{(1+n)(1-\omega)} + 1 \right) \theta \frac{\frac{1+n}{2+n}}{g} = \frac{\beta (1+\theta) \alpha}{(1+n)k'}. \tag{A.18}$$

To solve (A.18) for ϕ, recall the policy functions of g presented in Proposition 1,

$$\frac{2+n}{1+n} g = \left(\frac{\omega}{(1+n)(1-\omega)} + 1 \right) \theta \left[\frac{1}{1-T} + \frac{\alpha\beta (1+\theta) \frac{\beta}{1+\beta} (1-\alpha)}{\frac{\beta}{1+\beta} (1-\alpha)(1-T) - \phi X} \right]^{-1} (1-\alpha)\, y(k, k^p), \tag{A.19}$$

and the asset market clearing condition in (16) with the fiscal rule in (14),

$$(1+n)k' = \frac{\beta}{1+\beta} (1-\tau)(1-\alpha)\, y(k, k^p) - \phi X y(k, k^p). \tag{A.20}$$

We substitute (A.19) and (A.20) into (A.18) and rearrange the terms to obtain:

$$\frac{1}{1-T} = \frac{\alpha (1+\theta) \frac{\beta}{1+\beta} (1-\alpha)}{\frac{\beta}{1+\beta} (1-\alpha)(1-T) - \phi X},$$

or,

$$\frac{\beta}{1+\beta} (1 - \alpha (1+\theta)) = \frac{I(\phi)}{1-\phi}, \tag{A.21}$$

where we use the definition of T and X in Proposition 1 in deriving (A.21).

Substituting the definition of $I(\phi)$ into (A.21), we obtain

$$\frac{\beta}{1+\beta} (1 - \alpha (1+\theta)) = \frac{H_1(\phi) - \sqrt{(H_1(\phi))^2 - 4H_2(\phi)}}{2(1-\phi)},$$

or,

$$-H_2(\phi) = -\frac{\beta}{1+\beta} (1 - \alpha (1+\theta))(1-\phi) H_1(\phi) + \left[\frac{\beta}{1+\beta} (1 - \alpha (1+\theta))(1-\phi) \right]^2. \tag{A.22}$$

With the use of the definition of $H_1(\phi)$ and $H_2(\phi)$, (A.22) is further reformulated as in (37).

■

A.4 Proof of Proposition 3

Recall the indirect utility of the middle-aged population in period t in (18) and that of older adult population in period 0 in (19), both of which are restated as follows:

$$\begin{aligned} V_t^M &= \ln \frac{1}{1+\beta} (1-\tau_t)\, w(k_t, k_t^p) + \theta \ln g_t \\ &\quad + \beta \ln \left(1 - \tau_{t+1}^k\right) R\left(k_{t+1}, k_{t+1}^p\right) \frac{\beta}{1+\beta} (1-\tau_t)\, w(k_t, k_t^p) + \beta\theta \ln g_{t+1}, \end{aligned} \tag{A.23}$$

$$V_0^O = \left(1 - \tau_0^k\right) R(k_0, k_0^p)(1+n)(k_0 + b_0) + \theta \ln g_0. \tag{A.24}$$

With (A.23) and (A.24), we can reformulate $SW = \gamma^{-1} V_0^O + \sum_{t=0}^{\infty} \gamma^t V_t^M$ as follows:

$$
\begin{aligned}
SW \approx & \frac{1}{\gamma} \ln\left(1-\tau_0^k\right) + (1+\beta)\ln(1-\tau_0) + \alpha(1+\beta)\ln k_0 + (1-\alpha)(1+\beta)\ln k_0^p + \theta\left(\frac{1}{\gamma}+1\right)\ln g_0 \\
& + \sum_{t=1}^{\infty} \gamma^t \left[\frac{\beta}{\gamma}\ln\left(1-\tau_t^k\right) + (1+\beta)\ln(1-\tau_t) + \left(\frac{\beta(\alpha-1)}{\gamma} + \alpha(1+\beta)\right)\ln k_t \right. \\
& \left. + (1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right)\ln k_t^p + \theta\left(\frac{\beta}{\gamma}+1\right)\ln g_t\right].
\end{aligned} \tag{A.25}
$$

The government budget constraint in period t in (12) is rewritten as:

$$
g_t = \frac{1+n}{2+n}\left[R\left(k_t, k_t^p\right)(k_t+b_t)\tau_t^k + w\left(k_t, k_t^p\right)\tau_t + (1+n)b_{t+1} - (1+n)x_t - R\left(k_t, k_t^p\right)b_t\right]. \tag{A.26}
$$

Substitution of (A.26) into (A.25) leads to:

$$
\begin{aligned}
SW \approx & \frac{1}{\gamma}\ln\left(1-\tau_0^k\right) + (1+\beta)\ln(1-\tau_0) + \alpha(1+\beta)\ln k_0 + (1-\alpha)(1+\beta)\ln k_0^p \\
& + \theta\left(\frac{1}{\gamma}+1\right)\ln\left[R\left(k_0, k_0^p\right)(k_0+b_0)\tau_0^k + w\left(k_0, k_0^p\right)\tau_0 + (1+n)b_1 - (1+n)x_0 - R\left(k_0, k_0^p\right)b_0\right] \\
& + \sum_{t=1}^{\infty}\gamma^t\left\{\frac{\beta}{\gamma}\ln\left(1-\tau_t^k\right) + (1+\beta)\ln(1-\tau_t)\right. \\
& + \left(\frac{\beta(\alpha-1)}{\gamma} + \alpha(1+\beta)\right)\ln k_t + (1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right)\ln k_t^p \\
& \left. +\theta\left(\frac{\beta}{\gamma}+1\right)\ln\left[R\left(k_t, k_t^p\right)(k_t+b_t)\tau_t^k + w\left(k_t, k_t^p\right)\tau_t + (1+n)b_{t+1} - (1+n)x_t - R\left(k_t, k_t^p\right)b_t\right]\right\}.
\end{aligned} \tag{A.27}
$$

The problem of the Ramsey planner is to choose a sequence $\left\{\tau_t, \tau_t^k, b_{t+1}, x_t\right\}_{t=0}^{\infty}$ that maximizes SW in (A.27). The associated sequence $\{g_t\}_{t=0}^{\infty}$ is determined by substituting the solution of $\left\{\tau_t, \tau_t^k, b_{t+1}, x_t\right\}$ into the government budget constraint in (A.26). Hereafter, for simplicity of notation, $w\left(k_t, k_t^p\right)$ and $R\left(k_t, k_t^p\right)$ are denoted as w_t and R_t, respectively. The first order conditions with respect to τ_t, τ_t^k, b_{t+1}, and x_t are as follows:

$$\begin{aligned}\tau_t : 0 =& -\frac{1+\beta}{1-\tau_t} + \theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_t}{R_t\left(k_t+b_t\right)\tau_t^k + w_t\tau_t + (1+n)\,b_{t+1} - (1+n)\,x_t - R_t b_t} \\ &+ \sum_{j=1}^{\infty}\gamma^j\left\{\left(\frac{\beta\left(\alpha-1\right)}{\gamma} + \alpha\left(1+\beta\right)\right)\frac{1}{k_{t+j}} + \theta\left(\frac{\beta}{\gamma}+1\right)\right. \\ &\left.\times\frac{\frac{\partial R_{t+j}}{\partial k_{t+j}}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + R_{t+j}\tau_{t+j}^k + \frac{\partial w_{t+j}}{\partial k_{t+j}}\tau_{t+j} - \frac{\partial R_{t+j}}{\partial k_{t+j}}b_{t+j}}{R_{t+j}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + w_{t+j}\tau_{t+j} + (1+n)\,b_{t+j+1} - (1+n)\,x_{t+j} - R_{t+j}b_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial \tau_t},\end{aligned} \tag{A.28}$$

$$\tau_t^k : 0 = -\frac{\beta}{\gamma}\frac{1}{1-\tau_t^k} + \theta\left(\frac{\beta}{\gamma}+1\right)\frac{R_t\left(k_t+b_t\right)}{R_t\left(k_t+b_t\right)\tau_t^k + w_t\tau_t + (1+n)\,b_{t+1} - (1+n)\,x_t - R_t b_t}, \tag{A.29}$$

$$\begin{aligned}b_{t+1} : 0 =& \;\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{R_t\left(k_t+b_t\right)\tau_t^k + w_t\tau_t + (1+n)\,b_{t+1} - (1+n)\,x_t - R_t b_t} \\ &+ \gamma\theta\left(\frac{\beta}{\gamma}+1\right)\frac{R_{t+1}\tau_{t+1}^k - R_{t+1}}{R_{t+1}\left(k_{t+1}+b_{t+1}\right)\tau_{t+1}^k + w_{t+1}\tau_{t+1} + (1+n)\,b_{t+2} - (1+n)\,x_{t+1} - R_{t+1}b_{t+1}} \\ &+ \sum_{j=1}^{\infty}\gamma^j\left\{\left(\frac{\beta\left(\alpha-1\right)}{\gamma} + \alpha\left(1+\beta\right)\right)\frac{1}{k_{t+j}} + \theta\left(\frac{\beta}{\gamma}+1\right)\right. \\ &\left.\times\frac{\frac{\partial R_{t+j}}{\partial k_{t+j}}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + R_{t+j}\tau_{t+j}^k + \frac{\partial w_{t+j}}{\partial k_{t+j}}\tau_{t+j} - \frac{\partial R_{t+j}}{\partial k_{t+j}}b_{t+j}}{R_{t+j}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + w_{t+j}\tau_{t+j} + (1+n)\,b_{t+j+1} - (1+n)\,x_{t+j} - R_{t+j}b_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial b_{t+1}},\end{aligned} \tag{A.30}$$

$$\begin{aligned}x_t : 0 =& -\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{R_t\left(k_t+b_t\right)\tau_t^k + w_t\tau_t + (1+n)\,b_{t+1} - (1+n)\,x_t - R_t b_t} \\ &+ \sum_{j=1}^{\infty}\gamma^j\left\{(1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right)\frac{1}{k_{t+j}^p} + \theta\left(\frac{\beta}{\gamma}+1\right)\right. \\ &\left.\times\frac{\frac{\partial R_{t+j}}{\partial k_{t+j}^p}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + \frac{\partial w_{t+j}}{\partial k_{t+j}^p}\tau_{t+j} - \frac{\partial R_{t+j}}{\partial k_{t+j}^p}b_{t+j}}{R_{t+j}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + w_{t+j}\tau_{t+j} + (1+n)\,b_{t+j+1} - (1+n)\,x_{t+j} - R_{t+j}b_{t+j}}\right\}\frac{\partial k_{t+j}^p}{\partial x_t} \\ &+ \sum_{j=2}^{\infty}\gamma^j\left\{\left(\frac{\beta\left(\alpha-1\right)}{\gamma} + \alpha\left(1+\beta\right)\right)\frac{1}{k_{t+j}} + \theta\left(\frac{\beta}{\gamma}+1\right)\right. \\ &\left.\times\frac{\frac{\partial R_{t+j}}{\partial k_{t+j}}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + R_{t+j}\tau_{t+j}^k + \frac{\partial w_{t+j}}{\partial k_{t+j}}\tau_{t+j} - \frac{\partial R_{t+j}}{\partial k_{t+j}}b_{t+j}}{R_{t+j}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + w_{t+j}\tau_{t+j} + (1+n)\,b_{t+j+1} - (1+n)\,x_{t+j} - R_{t+j}b_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial x_t}.\end{aligned} \tag{A.31}$$

To simplify notation, we present the following definition:

$$I_{t+j} \equiv R_{t+j}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + w_{t+j}\tau_{t+j} + (1+n)\,b_{t+j+1} - (1+n)\,x_{t+j} - R_{t+j}b_{t+j}, \tag{A.32}$$

$$J_{t+j} \equiv \frac{\partial R_{t+j}}{\partial k_{t+j}^p}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + \frac{\partial w_{t+j}}{\partial k_{t+j}^p}\tau_{t+j} - \frac{\partial R_{t+j}}{\partial k_{t+j}^p}b_{t+j}, \tag{A.33}$$

$$L_{t+j} \equiv \frac{\partial R_{t+j}}{\partial k_{t+j}}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + R_{t+j}\tau_{t+j}^k + \frac{\partial w_{t+j}}{\partial k_{t+j}}\tau_{t+j} - \frac{\partial R_{t+j}}{\partial k_{t+j}}b_{t+j}. \tag{A.34}$$

By employing (A.32) – (A.34), we can streamline the expressions of the first-order conditions in (A.35) – (A.38) as follows:

$$
\begin{aligned}
\tau_t : 0 = & -\frac{1+\beta}{1-\tau_t} + \theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_t}{I_t} \\
& + \sum_{j=1}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}} + \theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial \tau_t}, \qquad (A.35) \\
\tau_t^k : 0 = & -\frac{\beta}{\gamma}\frac{1}{1-\tau_t^k} + \theta\left(\frac{\beta}{\gamma}+1\right)\frac{R_t(k_t+b_t)}{I_t}, \qquad (A.36) \\
b_{t+1} : 0 = & \ \theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{I_t} + \gamma\theta\left(\frac{\beta}{\gamma}+1\right)\frac{R_{t+1}\tau_{t+1}^k - R_{t+1}}{I_{t+1}} \\
& + \sum_{j=1}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}} + \theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial b_{t+1}}, \qquad (A.37) \\
x_t : 0 = & -\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{I_t} \\
& + \sum_{j=1}^{\infty}\gamma^j\left\{(1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right)\frac{1}{k_{t+j}^p} + \theta\left(\frac{\beta}{\gamma}+1\right)\frac{J_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}^p}{\partial x_t} \\
& + \sum_{j=2}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}} + \theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial x_t}. \qquad (A.38)
\end{aligned}
$$

The process for identifying policy functions that satisfy the conditions in (A.35), (A.36), (A.37) and (A.38) involves the following steps.

- Step 1. Simplify the expression of the first-order conditions by substituting the summation notation with alternative expressions.
- Step 2. Conjecture a set of policy functions that fulfill the first-order conditions, guided by Proposition 3, substitute the conjectured solution into first-order conditions, and rigorously verify its correctness.

A.4.1 Step 1

The derivatives of k_{t+j} with respect to τ_t, b_{t+1}, and x_t are, respectively,

$$\frac{\partial k_{t+j}}{\partial \tau_t} = \begin{cases} \frac{\partial k_{t+1}}{\partial \tau_t} & \text{for } j=1, \\ \prod_{i=1}^{j-1} \frac{\partial k_{t+i+1}}{\partial k_{t+i}} \frac{\partial k_{t+1}}{\partial \tau_t} & \text{for } j \geq 2, \end{cases} \tag{A.39}$$

$$\frac{\partial k_{t+j}}{\partial b_{t+1}} = \begin{cases} \frac{\partial k_{t+1}}{\partial b_{t+1}} & \text{for } j=1, \\ \prod_{i=1}^{j-1} \frac{\partial k_{t+i+1}}{\partial k_{t+i}} \frac{\partial k_{t+1}}{\partial b_{t+1}} & \text{for } j \geq 2, \end{cases} \tag{A.40}$$

$$\frac{\partial k_{t+j}}{\partial x_t} = \begin{cases} 0 & \text{for } j=1, \\ \frac{\partial k_{t+2}}{\partial k^p_{t+1}} \frac{\partial k^p_{t+1}}{\partial x_t} & \text{for } j=2, \\ \frac{\partial k_{t+j}}{\partial k^p_{t+j+1}} \left(\prod_{l=1}^{j-2} \frac{\partial k^p_{t+l+1}}{\partial k^p_{t+l}} \right) \frac{\partial k^p_{t+1}}{\partial x_t} + \sum_{i=2}^{j-2} \left[\left(\prod_{l=j-i+1}^{j-1} \frac{\partial k_{t+l+1}}{\partial k_{t+l}} \right) \frac{\partial k_{t+j-i+1}}{\partial k_{t+j-i}} \right. & \\ \quad \times \left. \left(\prod_{l=1}^{j-i-1} \frac{\partial k^p_{t+l+1}}{\partial k^p_{t+l}} \right) \frac{\partial k^p_{t+1}}{\partial x_t} \right] + \left(\prod_{l=2}^{j-1} \frac{\partial k_{t+l+1}}{\partial k_{t+l}} \right) \frac{\partial k_{t+2}}{\partial k^p_{t+1}} \frac{\partial k^p_{t+1}}{\partial x_t} & \text{for } j \geq 3, \end{cases} \tag{A.41}$$

$$\frac{\partial k^p_{t+j}}{\partial x_t} = \begin{cases} \frac{\partial k^p_{t+1}}{\partial x_t} & \text{for } j=1, \\ \prod_{i=1}^{j-1} \frac{\partial k^p_{t+i+1}}{\partial k^p_{t+i}} \frac{\partial k^p_{t+1}}{\partial x_t} & \text{for } j \geq 2. \end{cases} \tag{A.42}$$

Using (A.39), (A.40), and (A.41), we reformulate the first-order conditions in (A.35) – (A.38).

First, we substitute (A.39) into (A.35) and (A.40) into (A.37), and obtain

$$0 = -\frac{1+\beta}{1-\tau_t} + \theta \left(\frac{\beta}{\gamma} + 1 \right) \frac{w_t}{I_t} + \frac{\partial k_{t+1}}{\partial \tau_t} \Lambda_t, \tag{A.43}$$

$$0 = \theta \left(\frac{\beta}{\gamma} + 1 \right) \frac{1+n}{I_t} + \gamma\theta \left(\frac{\beta}{\gamma} + 1 \right) \frac{R_{t+1}\tau^k_{t+1} - R_{t+1}}{I_{t+1}} + \frac{\partial k_{t+1}}{\partial b_{t+1}} \Lambda_t, \tag{A.44}$$

where Λ_t is defined as

$$\begin{aligned} \Lambda_t \equiv\ & \gamma \left\{ \left(\frac{\beta(\alpha-1)}{\gamma} + \alpha(1+\beta) \right) \frac{1}{k_{t+1}} + \theta \left(\frac{\beta}{\gamma} + 1 \right) \times \frac{L_{t+1}}{I_{t+1}} \right\} \\ & + \sum_{j=2}^{\infty} \gamma^j \left\{ \left(\frac{\beta(\alpha-1)}{\gamma} + \alpha(1+\beta) \right) \frac{1}{k_{t+j}} + \theta \left(\frac{\beta}{\gamma} + 1 \right) \times \frac{L_{t+j}}{I_{t+j}} \right\} \prod_{i=1}^{j-1} \frac{\partial k_{t+i+1}}{\partial k_{t+i}}. \end{aligned} \tag{A.45}$$

Removing the term Λ_t from (A.43) and (A.44) by putting them together, and using $\partial k_{t+1}/\partial \tau_t = -\frac{1}{1+n}\frac{\beta}{1+\beta}(1-\alpha)y_t$ and $\partial k_{t+1}/\partial b_{t+1} = -1$, we obtain the following expression:

$$0 = \theta \left(\frac{\beta}{\gamma} + 1 \right) \frac{1+n}{I_t} - \gamma\theta \left(\frac{\beta}{\gamma} + 1 \right) \frac{R_{t+1}\left(1-\tau^k_{t+1}\right)}{I_{t+1}} - \frac{-\frac{1+\beta}{1-\tau_t} + \theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_t}{I_t}}{\frac{1}{1+n}\frac{\beta}{1+\beta}(1-\alpha)y_t}. \tag{A.46}$$

Second, recall the first-order condition with respect to x_t in (A.38). Taking one-period ahead and rearranging the terms, we obtain

$$\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{I_{t+1}}=\sum_{j=2}^{\infty}\gamma^j\left\{(1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right)\frac{1}{k^p_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{J_{t+j}}{I_{t+j}}\right\}\frac{\partial k^p_{t+j}}{\partial x_{t+i}}$$
$$+\sum_{j=3}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial x_{t+i}}. \quad \text{(A.47)}$$

Using the definition of I_{t+j} in (A.32), J_{t+j} in (A.33), and L_{t+j} in (A.34), we can reformulate the first-order condition with respect to x_t in (A.38) as follows:

$$0=-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{I_t}$$
$$+\underbrace{\sum_{j=2}^{\infty}\gamma^j\left\{(1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right)\frac{1}{k^p_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{J_{t+j}}{I_{t+j}}\right\}\frac{\partial k^p_{t+j}}{\partial x_t}}_{(\#1\_A.48)}$$
$$+\gamma\left\{(1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right)\frac{1}{k^p_{t+1}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{J_{t+1}}{I_{t+1}}\right\}\frac{\partial k^p_{t+1}}{\partial x_t}$$
$$+\underbrace{\sum_{j=3}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial x_t}}_{(\#2\_A.48)}$$
$$+\gamma^2\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+2}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+2}}{I_{t+2}}\right\}\frac{\partial k_{t+2}}{\partial x_t}. \quad \text{(A.48)}$$

By using (A.42), we can reformulate the term $\partial k^p_{t+j}/\partial x_t$ (#1_A.48) in (A.48) as

$$\frac{\partial k^p_{t+j}}{\partial x_t}=\frac{\frac{\partial k^p_{t+2}}{\partial k^p_{t+1}}\frac{\partial k^p_{t+1}}{\partial x_t}}{\frac{\partial k^p_{t+2}}{\partial x_{t+1}}}\frac{\partial k^p_{t+j}}{\partial x_{t+1}}. \quad \text{(A.49)}$$

By using (A.41), we can also reformulate the term $\partial k_{t+j}/\partial x_t$ (#2_A.48) in (A.48) as:

$$\frac{\partial k_{t+j}}{\partial x_t}=\left(\frac{\frac{\partial k_{t+j}}{\partial x_{t+1}}}{\frac{\partial k^p_{t+2}}{\partial x_{t+1}}}\frac{\partial k^p_{t+2}}{\partial k^p_{t+1}}+\prod_{i=2}^{j-1}\frac{\partial k_{t+i+1}}{\partial k_{t+i}}\frac{\partial k_{t+2}}{\partial k^p_{t+1}}\right)\frac{\partial k^p_{t+j}}{\partial x_t}. \quad \text{(A.50)}$$

Plugging (A.47), (A.49), and (A.50) into (A.48) and rearranging the terms, we have

$$0=-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{I_t}+\gamma\left\{(1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right)\frac{1}{k^p_{t+1}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{J_{t+1}}{I_{t+1}}\right\}\frac{\partial k^p_{t+1}}{\partial x_t}$$
$$+\gamma^2\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+2}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+2}}{I_{t+2}}\right\}\frac{\partial k_{t+2}}{\partial x_t}$$
$$+\frac{\frac{\partial k^p_{t+2}}{\partial k^p_{t+1}}\frac{\partial k^p_{t+1}}{\partial x_t}}{\frac{\partial k^p_{t+2}}{\partial x_{t+1}}}\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{I_{t+1}}$$
$$+\frac{\partial k_{t+2}}{\partial k^p_{t+1}}\frac{\partial k^p_{t+1}}{\partial x_t}\cdot\underbrace{\sum_{j=3}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\prod_{i=2}^{j-1}\frac{\partial k_{t+i+1}}{\partial k_{t+i}}}_{(\#1\_A.51)}. \quad \text{(A.51)}$$

Using the first-order condition with respect to τ_t in (A.35) and the derivative of k_{t+j} with respect to τ_t in (A.39), we can simplify the term (#1_A.51) in (A.51) as follows:

$$\sum_{j=3}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\prod_{i=2}^{j-1}\frac{\partial k_{t+i+1}}{\partial k_{t+i}}$$
$$=\frac{1}{\frac{\partial k_{t+2}}{\partial k_{t+1}}\frac{\partial k_{t+1}}{\partial \tau_t}}\left\{\frac{1+\beta}{1-\tau_t}-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_t}{I_t}-\sum_{j=1,2}\gamma^j\left[\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right]\frac{\partial k_{t+j}}{\partial \tau_t}\right\}. \tag{A.52}$$

Plugging (A.52) into (A.51), we obtain

$$0=-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{I_t}+\gamma\left\{(1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right)\frac{1}{k^p_{t+1}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{J_{t+1}}{I_{t+1}}\right\}\frac{\partial k^p_{t+1}}{\partial x_t}$$
$$+\gamma^2\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+2}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+2}}{I_{t+2}}\right\}\frac{\partial k_{t+2}}{\partial x_t}$$
$$+\frac{\frac{\partial k^p_{t+2}}{\partial k^p_{t+1}}\frac{\partial k^p_{t+1}}{\partial x_t}}{\frac{\partial k^p_{t+2}}{\partial x_{t+1}}}\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{I_{t+1}}+\frac{\frac{\partial k_{t+2}}{\partial k^p_{t+1}}\frac{\partial k^p_{t+1}}{\partial x_t}}{\frac{\partial k_{t+2}}{\partial k_{t+1}}\frac{\partial k_{t+1}}{\partial \tau_t}}$$
$$\times\left\{\frac{1+\beta}{1-\tau_t}-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_t}{I_t}-\sum_{j=1,2}\gamma^j\left[\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right]\frac{\partial k_{t+j}}{\partial \tau_t}\right\}. \tag{A.53}$$

Finally, recall again the first-order condition with respect to τ_t in (A.35). Taking one-period ahead and rearranging the terms, we have

$$\sum_{j=2}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial \tau_{t+1}}$$
$$=\frac{1+\beta}{1-\tau_{t+1}}-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_{t+1}}{I_{t+1}}. \tag{A.54}$$

In addition, using (A.39), we can reformulate the first-order condition with respect to τ_t in (A.35) as

$$0=-\frac{1+\beta}{1-\tau_t}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_t}{I_t}$$
$$+\sum_{j=2}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial \tau_t}$$
$$+\gamma\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+1}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+1}}{I_{t+1}}\right\}\frac{\partial k_{t+1}}{\partial \tau_t}$$
$$=-\frac{1+\beta}{1-\tau_t}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_t}{I_t}$$
$$+\frac{\frac{\partial k_{t+2}}{\partial k_{t+1}}\frac{\partial k_{t+1}}{\partial \tau_t}}{\frac{\partial k_{t+2}}{\partial \tau_{t+1}}}\sum_{j=2}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial \tau_{t+1}}$$
$$+\gamma\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+1}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+1}}{I_{t+1}}\right\}\frac{\partial k_{t+1}}{\partial \tau_t}. \tag{A.55}$$

We substitute (A.54) into (A.55) and obtain

$$0=-\frac{1+\beta}{1-\tau_t}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_t}{I_t}+\frac{\frac{\partial k_{t+2}}{\partial k_{t+1}}\frac{\partial k_{t+1}}{\partial \tau_t}}{\frac{\partial k_{t+2}}{\partial \tau_{t+1}}}\left[\frac{1+\beta}{1-\tau_{t+1}}-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_{t+1}}{I_{t+1}}\right]$$
$$+\gamma\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+1}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+1}}{I_{t+1}}\right\}\frac{\partial k_{t+1}}{\partial \tau_t}. \tag{A.56}$$

We have so far derived three conditions, namely (A.46), (A.53), and (A.56), from the first-order conditions (A.35), (A.37), and (A.38) corresponding to the fiscal variables τ, b', and x, respectively. In conjunction with the first-order condition with respect to τ_t^k in (A.36), these four conditions collectively define the optimal fiscal variables, (τ, τ^k, b', x), that maximize social welfare in (A.27).

A.4.2 Step 2

To find the policy functions, τ, τ^k, b', and x that satisfy the four first-order conditions, (A.36), (A.46), (A.53), and (A.56), we conjecture the policy functions as follows:

$$\begin{cases}\tau_t=\bar{T},\\ 1-\tau_t^k=\bar{T}^k\frac{1}{\alpha\left(1+\frac{b_t}{k_t}\right)},\\ (1+n)\,b_{t+1}=\bar{B}\cdot y\left(k_t,k_t^p\right),\\ (1+n)x_t=\bar{X}\cdot y\left(k_t,k_t^p\right),\end{cases} \tag{A.57}$$

where $\bar{T}$, $\bar{T}^k$, $\bar{B}$, and $\bar{X}$ are constant. We substitute these conjectured solutions into the four first-order conditions, (A.36), (A.46), (A.53), and (A.56) and verify their correctness.

For presentation of the analysis, recall w_t in (7) and R_t in (8), which have the following properties:

$$\begin{cases}\frac{\partial w_t}{\partial k_t}=\alpha(1-\alpha)\frac{y_t}{k_t};\ \frac{\partial w_t}{\partial k_t^p}=(1-\alpha)^2\frac{y_t}{k_t^p};\\ \frac{\partial R_t}{\partial k_t}=\alpha(\alpha-1)\frac{y_t}{(k_t)^2};\ \frac{\partial R_t}{\partial k_t^p}=\alpha(1-\alpha)\frac{y_t}{k_tk_t^p}\end{cases} \tag{A.58}$$

Given the properties in (A.58) and the conjectured policy functions in (A.57), I_t, J_t, and L_t defined in (A.32), (A.33), and (A.34), respectively, are reformulated as follows;

$$I_t=\left(\alpha-\bar{T}^k+(1-\alpha)\bar{T}+\bar{B}-\bar{X}\right)y_t, \tag{A.59}$$

$$J_t=(1-\alpha)\left(\alpha-\bar{T}^k+(1-\alpha)\bar{T}\right)\frac{y_t}{k_t^p}, \tag{A.60}$$

$$L_t=\left(-\alpha(1-\alpha)-(1-\alpha)\bar{T}^k+\alpha+(1-\alpha)\bar{T}\right)\frac{y_t}{k_t}-\bar{T}^k\frac{y_t}{k_t+b_t}. \tag{A.61}$$

Using the asset market clearing condition in (16) with (A.57) and (A.58), we have

$$\begin{cases}\frac{\partial k_{t+1}}{\partial \tau_t}=-\frac{1}{1+n}\frac{\beta}{1+\beta}(1-\alpha)\,y_t;\\ \frac{\partial k_{t+1}}{\partial k_t}=\frac{1}{1+n}\frac{\beta}{1+\beta}\left(1-\bar{T}\right)\alpha(1-\alpha)\frac{y_t}{k_t};\\ \frac{\partial k_{t+1}}{\partial b_{t+1}}=-1;\\ \frac{\partial k_{t+1}}{\partial k_t^p}=\frac{1}{1+n}\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)^2\frac{y_t}{k_t^p}.\end{cases} \tag{A.62}$$

From (10), we also have the following properties of the public capital formation function:

$$\frac{\partial k_{t+1}^p}{\partial k_t}=\eta\frac{k_{t+1}^p}{x_t};\ \frac{\partial k_{t+1}^p}{\partial k_t^p}=(1-\eta)\frac{k_{t+1}^p}{k_t^p}. \tag{A.63}$$

With (A.57) – (A.63), we reformulate the four conditions, (A.36), (A.46), (A.53), and (A.56) in the following way. First, with (A.57), (A.58), and (A.59), the condition in (A.36) is reduced to:

$$0=-\frac{\beta}{\gamma}\cdot\frac{1}{\bar{T}^{k}}+\frac{\theta\left(\frac{\beta}{\gamma}+1\right)}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}. \tag{A.64}$$

Second, with (A.57), (A.58), (A.59) and (A.61), the condition in (A.46) is reduced to:

$$\begin{aligned}&-\frac{1+\beta}{1-\bar{T}}\left(\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}\right)+\theta\left(\frac{\beta}{\gamma}+1\right)(1-\alpha)\\&=\frac{\beta}{1+\beta}(1-\alpha)\left[\theta\left(\frac{\beta}{\gamma}+1\right)-\gamma\theta\left(\frac{\beta}{\gamma}+1\right)\frac{\bar{T}^{k}}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)}\right].\end{aligned} \tag{A.65}$$

Third, with (A.57), (A.61) and (A.62), the condition in (A.56) is reduced to:

$$\begin{aligned}0=&-\frac{1+\beta}{1-\bar{T}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1-\alpha}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}\\&+\frac{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)\alpha(1-\alpha)}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)-\bar{B}}\left[\frac{1+\beta}{1-\bar{T}}-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1-\alpha}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}\right]\\&-\gamma\frac{\beta}{1+\beta}(1-\alpha)\left\{\frac{\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)-\bar{B}}\right.\\&\left.+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{\frac{-\alpha(1-\alpha)-(1-\alpha)\bar{T}^{k}+\alpha+\alpha(1-\alpha)\bar{T}}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)-\bar{B}}-\frac{\bar{T}^{k}}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)}}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}\right\}.\end{aligned} \tag{A.66}$$

Finally, with (A.57), (A.59), (A.61) and (A.63), the condition in (A.53) is reduced to:

$$\begin{aligned}0=&-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{\bar{X}}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}\\&+\gamma\left\{(1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right)+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{(1-\alpha)\left(\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}\right)}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}\right\}\\&+\gamma^{2}\left\{\frac{\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)-\bar{B}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{\frac{-\alpha(1-\alpha)-(1-\alpha)\bar{T}^{k}+\alpha+\alpha(1-\alpha)\bar{T}}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)-\bar{B}}-\frac{\bar{T}^{k}}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)}}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}\right\}\\&\times\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)^{2}\\&-\frac{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)-\bar{B}}{\alpha\frac{\beta}{1+\beta}}\left\{\frac{1+\beta}{1-\bar{T}}-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1-\alpha}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}\right.\\&+\gamma\frac{\beta}{1+\beta}(1-\alpha)\left[1+\frac{\gamma\frac{\beta}{1+\beta}\left(1-\bar{T}\right)\alpha(1-\alpha)}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)-\bar{B}}\right]\\&\left.\left[\frac{\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)-\bar{B}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{\frac{-\alpha(1-\alpha)-(1-\alpha)\bar{T}^{k}+\alpha+\alpha(1-\alpha)\bar{T}}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)-\bar{B}}-\frac{\bar{T}^{k}}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)}}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}\right]\right\}.\end{aligned} \tag{A.67}$$

Conditions (A.64), (A.65), (A.66) and (A.67) all include only constant terms, $\bar{T}$, $\bar{T}^k$, $\bar{B}$, and $\bar{X}$, and they are independent of the state variables. Therefore, (A.64), (A.65), (A.66) and (A.67) are solved for $\bar{T}$, $\bar{T}^k$, $\bar{B}$, and $\bar{X}$ when these four variables are constant. This verifies the conjecture in (A.57).

■

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