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Analyzing Collusion Using Set-Identified Marginal Cost Functions*

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Abstract

We propose an approach for analyzing markets in which firms are (suspected of) colluding. Our approach, which is based on set-identified marginal cost functions, enables us to screen for collusion and measure potential welfare losses caused by non-competitive behavior. The key idea is to exploit supernumerary (or excluded) instrumental variables that provide moment restrictions to falsify hypotheses about firm conduct and eliminate cost parameters implied by the falsified hypotheses. The resulting set of cost parameters that remain unfalsified under these restrictions functions as a screening tool for collusion. When competitive behavior is falsified, the corresponding parameter is excluded from the identified set. Additionally, the identified set can be used to measure potential welfare losses when competitive behavior is ruled out. This type of counterfactual welfare analysis is otherwise extremely difficult or nearly impossible to conduct.

Keywords: collusion, set identification, marginal cost functions, interval regression
JEL Classification: C57, L40

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1 Introduction

Identifying markets where competition problems exist and measuring the potential welfare loss caused by non-competitive firm conduct have been central to the field of industrial organization (IO). These objectives are not only of interest to academia but are also of great importance for public policy in general, especially for antitrust policy. For instance, empirical tools to detect evidence of collusion and measure the impact on welfare that can be implemented easily with readily accessible data are useful for cartel investigations.[1]

The empirical IO literature has been developing econometric methods for analyzing markets, some of which are useful for testing for the presence of collusion. For instance, firm conduct tests based on model selection and model assessment tests are often employed for that purpose.[2] In these tests, researchers consider candidate hypotheses or models, such as representative competitive and collusive models, derive firms' marginal costs under these hypotheses, and construct test statistics based on the estimation of marginal cost function. However, in many studies, researchers use (unconstrained) joint-profit maximization or perfect collusion as the collusive benchmark while using oligopolistic competition (Bertrand, Cournot, and etc) as the competitive benchmark.[3]

This paper is motivated by concern regarding this empirical practice used to detect collusion. Specifically, we are concerned that when firms are colluding but, due to various constraints, cannot achieve unconstrained joint-profit maximizing (or perfectly collusive) outcomes, model selection tests incorrectly favor the benchmark model of competition over the collusion model, while model assessment tests reject both models. In the former case, collusion goes undetected and firm conduct is erroneously regarded to be competitive. In the latter case, the rejection of both hypotheses leaves researchers without a clear direction for further analysis. In fact, these results are not artifacts but exactly what we obtain when examining data for the cement market in

[1] Such cost-effective statistical methods are called cartel screens. See Harrington (2008) for a discussion of advances in cartel screen methods and Abrantes-Metz and Bajari (2009) for multiple uses of cartel screens beyond the search for evidence of collusion.

[2] See Duarte, Magnolfi, Sølvsten, and Sullivan (2024) for a detailed explanation of these two testing procedures.

[3] Representative work includes Bresnahan (1987), Gasmi, Laffont, and Vuong (1992), Villas-Boas (2007), Bonnet and Dubois (2010), Doi and Ohashi (2019), Sullivan (2020), and Starc and Wollmann (2025), to name a few.

Japan, where firms were colluding, as presented in Section 3.

These test results arguably stem not from the test procedures themselves, but from the way that they are applied to test firm conduct. The models of joint profit maximization or perfect collusion are too simplistic to accurately describe the actual behaviors of colluding firms. Therefore, a natural direction for improving the testing framework for collusion would be to expand the menu of models to include more realistic models of collusion. However, extending the current framework in this direction would be prohibitively difficult if not impossible, because of the difficulties of modeling, identifying, and estimating models of collusion beyond joint-profit maximization (see the discussion in Section 3).

Against this background, the aim of this paper is to propose an alternative approach for analyzing collusion without explicitly using complicated models of collusion. Following the literature, we base our approach on the estimation of marginal cost functions. However, instead of confining our attention to a limited number of hypotheses about firm conduct – in particular with regard to forms of collusion – we allow a broad spectrum of firm conduct between competitive and collusive benchmarks. In other words, instead of deriving marginal costs under a particular kind of firm behavior, we consider marginal cost intervals bounded by the competitive and collusive benchmarks that we specify. With the marginal cost intervals, we identify a set of marginal cost parameters. This identified set of cost parameters is utilized to test firm conduct and implement the subsequent welfare analysis of possible, but not necessarily perfect, collusion.[4]

The key idea is that, in constructing the identified set, we exploit an extra set of exogenous variables that are not relevant to the marginal cost function but provide moment restrictions that rule out false cost parameters.[5] The resulting identified set has the following desirable properties. First, when the competitive and collusive benchmark behaviors are not compatible with the observed firm behavior, the set

[4]We screen for collusion by testing whether the competitive benchmark is compatible with the data. This is an idea that has been used in many other studies, particularly in the literature on the detection of bid-rigging collusion (Baldwin, Marshall, and Richard (1997), Ishii (2009), Athey, Levin, and Seira (2011), Chassang, Kawai, Nakabayashi, and Ortner (2022), and Kawai and Nakabayashi (2022)).

[5]As explained in Section 4, these additional exogenous variables used for identifying marginal cost parameters are equivalent to excluded instrumental variables in the IO literature (Berry and Haile (2014), Duarte, Magnolfi, Sølvsten, and Sullivan (2024), and Backus, Conlon, and Sinkinson (2021)).

does not contain the cost parameters implied by these benchmarks. This fact suggests that we can test the two benchmark behaviors by examining whether the cost parameters implied by these benchmarks are included in the set. Second, even when these benchmarks are rejected and thus the implied cost parameters are ruled out, the set contains the cost parameters implied by firm conduct other than these benchmarks if these cost parameters satisfy the additional moment restrictions. Therefore, we still have cost parameters and, with these 'unfalsified' cost parameters, we can perform the counterfactual analysis of the effects of (potential) collusion.

Our approach builds on developments in two different sets of literature: the literature on set identification in econometrics and the literature on testing firm conduct in IO. First, because marginal costs are obtained as intervals, our empirical model becomes a regression model with interval-valued outcomes, which has been studied in the econometrics literature (Manski and Tamer (2002), Beresteanu and Molinari (2008), Beresteanu, Molchanov, and Molinari (2011), Kaido and Santos (2014), and Chandrasekhar, Chernozhukov, Molinari, and Schrimpf (2019)). To characterize the identified set of cost parameters, we rely on the work of Bontemps, Magnac, and Maurin (2012). In particular, we leverage their characterization of the identified set in the case of supernumerary instrumental variables (IV), in which the number of moment restrictions is larger than the number of parameters. This case holds particular importance because our approach uses additional exogenous variables to reduce the size of the identified set of cost parameters. Second, the idea of using additional moment restrictions relates to the idea of employing excluded IVs that provide restrictions to falsify oligopolistic models in the IO literature (Bresnahan (1982) and Lau (1982), and, more recently, Berry and Haile (2014), Duarte, Magnolfi, Sølvsten, and Sullivan (2024), and Backus, Conlon, and Sinkinson (2021)). We show that additional exogenous variables reduce the size of the identified set and make it informative because these variables provide falsifying restrictions on hypotheses about firm conduct and rule out false cost parameters.

As far as we know, Rosen (2006) is the only study that uses the concept of set identification in the estimation of marginal cost functions. Without imposing strong assumptions on firm conduct, he derives upper and lower bounds on firms' marginal costs and constructs the identified set of cost parameters.[6] The differences between

[6]Prior to Rosen (2006), Haile and Tamer (2003), analyzing English auction data, consider an incomplete model without imposing any strong assumptions for identification and tractability, and

his study and ours are as follows. First, we highlight the role of supernumerary (or excluded) IVs in reducing the size of the identified set of cost parameters by comparing the identified sets estimated with and without using IVs. On the other hand, Rosen (2006) examines how different assumptions on firm conduct, leading to different sizes of the marginal cost intervals, affect the size of the identified set. Second, from an empirical perspective, we not only estimate marginal cost functions but also conduct inference on the effects of collusion by estimating bounds on price overcharge and consumer welfare losses resulting from collusive behavior.

The novelties of this paper can be summarized as follows. First, we highlight the limitations of current empirical practices for detecting collusion using a cartel in the cement markets in the Chugoku region, Japan. Even though cement firms were colluding, we cannot detect the presence of collusion when following the current empirical practices relying on simple oligopolistic models. Second, we develop an alternative approach for examining markets where firms are (suspected of) colluding. A key advantage of our approach is that it does not require complicated models of collusion. Third, we explain how the addition of supernumerary moment restrictions shrinks the identified set by showing that supernumerary IVs essentially play the same role as excluded IVs in firm conduct testing. Fourth, we apply our approach to analyze the cement cartel in Japan and estimate bounds on the effects of the cartel. Our analysis represents one of only a handful of structural studies using the concept of set identification, outside of the area of empirical analyses of discrete games.[7]

The paper proceeds as follows. Section 2 provides an overview of the cement industry in Japan, with a focus on the criminal cartel that operated in the Chugoku region in the 1980s. In addition, it introduces the data that we use in this study. Section 3, using data for this cartel, critically examines current empirical practices for testing firm conduct. Next, Section 4 presents our novel approach to analyzing

derive informative bounds on the distribution of bidders' valuations. In addition to deriving lower and upper bounds on the valuation distribution, they analyze how auction heterogeneities affect (the mean of) bidders' valuations using the interval regression approach of Manski and Tamer (2002). More recently, Freyberger and Larsen (2025) propose a bounds approach for analyzing e-Bay bargaining data. They derive bounds on private value distributions for buyers and sellers and trade gains using a range of assumptions on behavior and the informational environment.

[7]The concept of set identification has been widely used in empirical analyses of oligopolistic firms' decisions, in particular, firms' discrete choices (Ciliberto and Tamer (2009), Ho (2009), Eizenberg (2015), Grieco (2016), Aradillas-Lopz and Gandhi (2016), Wallmann (2018), Fan and Yang (2020), Ciliberto, Murry, and Tamer (2021), and Magnolfi and Roncoroni (2023)). A comprehensive overview of set identification in applied research is provided by Ho and Rosen (2017).

cartel behavior. Section 5 then applies our approach to the cartel in the Chugoku region to check whether it properly detects the cartel. Finally, we explain the relationship between our approach an traditional approaches such as the conduct parameter methods.

2 The Cement Market and Cartel in the Chugoku Region

This section provides a description of the cement markets and firms in Japan's Chugoku region and the cartel. A description of the data used in this study is also provided.

2.1 The Cement Market and Firms

The Japanese cement industry is broadly divided into 11 regions. Among these regions, the focus of this paper is the Chugoku region, which is the 9th largest market and accounts for around 8% of cement consumption in Japan. The region consists of five prefectures: Hiroshima, Okayama, Shimane, Tottori, and Yamaguchi. In this region, nine cement firms were present during the 1980s. The nine cement firms were Aso Cement, Mitsubishi Cement, Mitsui Mining, Nihon Cement, Nippon Steel Chemical, Onoda Cement, Sumitomo Cement, Tokuyama, and Ube Industries (hereafter, the following short forms are used for these firms: Aso, Mitsubishi, Mitsui, Nippon Steel, Onoda, Sumitomo, and Ube). All of these firms supplied cement to the five prefectures in the region, each of which is treated as a cement market in this study. The number of cement firms in the region remained unchanged until the start of industry consolidation through horizontal mergers in 1994.

Table 1 presents the firms' market shares in this region and other characteristics (as of 1985). There is substantial heterogeneity in firm size. This heterogeneity stems mainly from differences in the size of their distribution networks, which can be gauged by the number of distribution centers or "service stations," and there is a strong link between a firm's number of service stations and its delivery costs. As seen in Table 1, the pattern in firms' number of service stations closely follows that in their market shares.[8] In fact, the correlation between firms' regional supply and their number of

[8]Because it did not have its own supply network (it only started to build its own distribution

service stations is about 0.9, suggesting that the size of the distribution network is an important variable in cement supply.

Table 1: Cement Firms (as of 1985)

	Market Share	No. SS	Plant	Region/Total
Aso	4.9	5	No	12.8
Mitsubishi	11.1	7	No	6.7
Nihon	11.7	7	No	6.6
Onoda	21.6	11	Yes	8.5
Nippon Steel	4.0	3	No	16.1
Sumitomo	10.2	6	No	6.9
Tokuyama	16.4	10	Yes	21.7
Ube	20.1	8	Yes	11.7

"No. SS" is the number of service stations in the Chugoku region. "Plant" indicates whether a cement firm has a plant in the region. "Region/Total" shows the share that the Chugoku region accounts for in a firm's total supply.

Table 1 also shows whether firms have cement producing plants in the region. Only three firms have cement producing plants in the region.[9] Other firms ship cement to the Chugoku region from their plants located in other regions. Shipping cement to (service stations in) a different region is possible and common in Japan because of relatively low transportation costs by sea. The table also shows that the Chugoku region as a whole is a relatively small cement market. The region at most accounted for 22% of a firm's total national supply, while for many it was less than 10%.

Lastly, an important feature that is not taken into account in this study is the vertical relationship between the cement industry and its downstream markets. The vast majority of cement is used by ready-mixed concrete firms, which often have vertical relationships with cement firms. For instance, approximately 20% of ready-mixed concrete firms in downstream markets are, to some degree, vertically integrated with upstream cement firms. In the following sections, when modeling the behavior of cement firms, we abstract away from these vertical relationships. However, we think that this abstraction does not pose a serious issue for the analysis presented in this

centers in the late 1980s), Mitsui relied on Onoda's distribution network and cannot really be regarded as an independent firm. Therefore, in the analysis, Mitsui is treated as having merged with Onoda.

[9]Onoda had two plants. However, the plant located in Yamaguchi prefecture was very small and accounted for only around 1.5% of Onoda's total production. Therefore, the other plant eventually produced almost all the cement supplied to the region.

paper. The reason is that the extent of vertical integration in the cement markets in the Chugoku region is not high. Although approximately 20% of ready-mixed concrete firms are vertically integrated, only a small fraction – less than 10% – are 50% or more owned by a cement firm. While this fraction is not negligible, it is nevertheless quite small. Therefore, we do not think that omitting vertical relationships in our empirical analysis does not cause any serious problems. Moreover, the primary objective of this paper is to propose an empirical approach for analyzing collusion, rather than providing a detailed analysis of vertical integration in the cement and ready-mixed concrete markets.[10]

2.2 The Cartel

The cartel that we focus on in this study is one of the most significant cartel cases in Japanese antitrust history. During the 1970s and 1980s, cement firms were repeatedly found guilty of participating in cartels across various regions. Among these, the cartels in the Chugoku and Hokkaido regions, which were initiated in the summer of 1985 and uncovered in the mid-1990, are among the most notable. These two were the largest cartels in Japan at the time of the conviction in terms of the fines imposed.[11] Moreover, the total amount of fines imposed on the cement firms involved remained the highest for criminal cartels for over 15 years, until 2005.

The cartel, which began in July 1985, persisted for nearly five years before ending in mid-April 1990. All nine aforementioned cement firms were involved in this cartel. The cartel was not detected while the firms were actually colluding. In fact, the cement firms stopped colluding immediately after they realized that the Japan Fair Trade Commission (JFTC) had started cartel investigations into the firms in the Hokkaido region. However, the cartel was eventually uncovered, because the JFTC extended its investigation into all cement markets in Japan.

JFTC documents reveal the workings of the cartel. A group of branch managers of the cement firms in the region organized the cartel. During the cartel, the branch managers controlled the total amount of cement supply in the region to elevate prices.

[10] An analysis of how vertical relationships between cement and concrete firms affect the incentive to collude by focusing on the cement cartel we study in this paper can be found in Nishiwaki (2024).

[11] The main reason this study focuses on the cartel in the Chugoku region is data limitations. Although Hokkaido is a single prefecture, there are local cement markets within Hokkaido because of its geographical size. However, important information, such as firm-level quantities, unfortunately is available at best at the prefecture level and not at a more detailed local level.

They met monthly to decide on the total amount of cement to supply and allocate quotas to the cement firms. The monthly quota of each firm was largely determined by the cartel's share allocation, presented in Table 2, and small adjustments to it.[12] Subject to the quota, each firm determined the quantities it would supply to prefectures in the region.

An important point for the subsequent analysis is that the resulting market shares are essentially the same as those realized in the pre-cartel period (and post-cartel period). As previously explained, a firm's market share is primarily determined by its number of service stations. This means that a firm's cement supply is affected by the number of its own service stations and by the number of service stations operated by other firms. As a result, this variable plays an important role in examining firm conduct in Sections 3 and 5.

Table 2: Share Allocation

	Onoda	Ube	Tokuyama	Nihon	Mitsubishi	Sumitomo	Aso	Nippon Steel
Q1	21.98	20.16	17.00	11.77	11.63	9.17	4.87	2.61
Q2	21.82	21.34	16.26	11.13	11.29	9.12	4.99	3.24
Q3	21.97	21.02	16.50	12.18	11.27	9.09	4.82	2.34
Q4	21.82	20.48	16.66	12.10	11.55	9.42	5.19	1.98

The market shares presented are the firms' market shares in the region. Mitsui's market shares are 0.77 for all quarters but not presented.

While we have detailed information on the workings of the cartel, our objective is not to analyze the cartel using the information. Instead, given that we know that firms were colluding, we use the collusive behavior in the cement markets in the Chugoku region as an empirical laboratory to examining the performance of existing methods and our newly proposed empirical approach for detecting and analyzing collusion.

2.3 Data

The focus of this study is the cartel period from July 1985 to March 1990. The data used for the analysis are collected from multiple sources. The primary data source is the Cement Industry Yearbook. The yearbook provides data on the total cement

[12]For instance, Nippon Steel was allocated 4,300 tons (monthly) in addition to the share-based allocation, whereas 1,700, 1,500, and 1,000 tons were subtracted from the share-based amounts of Onoda, Ube, and Mitsubishi, respectively.

consumption in each prefecture. In addition, it provides data on cement firms' (annual) supply quantity in each prefecture. Further, the yearbook also provides other information on cement distribution centers and producing plants. An important limitation of the yearbook is that it lacks prefecture-level price information. Therefore, cement prices at the prefecture level are obtained from a monthly magazine published by the Construction Research Institute that provides the prices of almost all (raw) materials used in construction.

In addition to these industry data, information on demand and cost-side variables was also collected from various other sources. We obtained monthly prefecture-level construction investment figures from the Ministry of Land, Infrastructure, and Tourism (MLIT). The amount of construction investment is used as a variable that shifts and rotates the cement demand function. Prefecture-level prices of gasoline, which cement firms use in the delivery stages of their product, are obtained from the Retail Price Survey released by the Statistics Bureau, Ministry of Internal Affairs and Communications.

Table 3: Summary Statistics

Variable	Mean	S.D.	Min	Max
Price (1,000 JPY/ton)	15.121	1.191	12.121	17.026
Quantity (1,000 tons)	92.850	45.295	21.212	209.367
Construction Inv. (million JPY)	61.115	33.204	15.204	180.372
Gasoline Price (100 JPY/liter)	1.316	0.108	1.112	1.638
Firm-Prefecture Market Share (%)	12.500	8.571	0.544	32.742
No. SS	1.395	1.093	0.000	5.000

"Construction Inv." is the amount of construction investment, while "No. SS" stands for firms' number of service stations. Price, Quantity, Construction Inv., and Gasoline Price are monthly prefecture-level variables, while Firm-Prefecture Market Share and No. SS are annual prefecture-level variables.

Summary statistics of the variables used in the analysis are provided in Table 3. A few words regarding the construction of the data for firms' supply quantities are in order. Firm-prefecture-level supply quantities are available only on an annual basis, while prefecture-level total supply is available on a monthly basis. Therefore, to estimate firm-prefecture monthly supply quantities, we use prefecture level annual market shares and prefecture-level total supply data. We multiply the monthly prefecture total supply quantity by each firm's market share to estimate monthly firm supply quantities. We believe these estimates of firm-prefecture monthly supply quantities are accurate. The reasons are as follows. First, annual prefecture market shares are

quite stable. Second, it is highly likely that monthly prefecture market shares did not vary substantially within a year because the cartel controlled at least region-level market shares (as in Table 2, the allocated regional market shares did not fluctuate greatly across quarters).

3 Examination of Empirical Practices for Detecting Collusion

Using data from the cartel in the Japanese cement market, we examine whether, by following standard practices for detecting collusion, we can correctly detect the presence of the actual cartel. In particular, our focus is not on the testing procedures themselves but on how these procedures are applied to distinguish between competitive and collusive behaviors. Researchers typically compare competitive models with collusive models. However, these analyses often rely on simplistic representations of collusive behavior, that is, (unconstrained) joint-profit maximization or perfect collusion (Bresnahan (1987), Gasmi, Laffont, and Vuong (1992), Villas-Boas (2007), Bonnet and Dubois (2010), Doi and Ohashi (2019), Sullivan (2020), and Starc and Wollmann (2025)).

While these simplifications facilitate the implementation of firm conduct tests, they raise concerns about the ability of such approaches to capture the complexities of real-world cartel behavior. Specifically, collusion is often imperfect due to a range of constraints, such as firm-level incentive discrepancies stemming from cost or size asymmetries, fluctuations in demand and costs, the risk of detection and antitrust penalties, and the presence of potential competitors or imports. When these factors are present, simple models of collusion may fail to capture aspects of collusive behavior. As a result, current empirical practices may struggle to detect collusion effectively.

The testing results here substantiate these concerns regarding current practices. As explained below, we employ two distinct testing procedures: the Rivers and Vuong (2002) test (RV test) and the Anderson and Rubin (1949) test (AR test). Both the RV and AR tests indicate that joint profit maximization does not fit the data. However, the implications of their results differ. For the RV test, accepting the testing results at face value leads to the incorrect conclusion that competition, rather than collusion,

best explains the observed data. Meanwhile, the results of the AR test fail to provide any meaningful conclusion. This series of unsatisfactory outcomes motivates us to propose a new approach, which we outline in the next section.

3.1 Cost and Demand Functions

The framework for testing the form of competition between firms (or firm conduct) in the IO literature is based on the estimation of firms' marginal cost functions. Under a specific hypothesis about firm conduct, the implied marginal costs are derived and residuals in the marginal cost function estimation are obtained. Different hypotheses yield distinct marginal costs and thus different residuals. Coupled with excluded IVs, these residuals are exploited to evaluate competing hypotheses about firm conduct.[13] We apply this framework for testing firm conduct to data from the cartel in the Japanese cement market.

Marginal Cost Function We consider geographically distinct cement markets with a finite number of heterogeneous firms.[14] We observe these markets T times (months). The (true) marginal cost function of cement firm i in market g in month t is assumed to be as follows:

$$mc^0_{igt} = mc(x_{igt}, \epsilon_{igt}; \beta^0) = x_{igt}\beta^0 + \epsilon_{igt} \quad (i = 1, \ldots, N^F), \tag{1}$$

where x_{igt} denotes the set of K explanatory variables. It is (very) often assumed that firms' marginal costs are constant with respect to supply quantity, q_{igt}. In this specification, x_{igt} includes only exogenous variables, w_{igt}, which consist of firm-specific and common cost shifters. On the other hand, x_{igt} includes endogenous variable q_{igt} when the marginal cost function is not constant but exhibits scale (dis-)economies. Then, $x_{igt} = (w_{igt}, q_{igt})$. Unobserved cost shifter ϵ_{igt} is assumed to satisfy the following condition:

$$E(\epsilon_{igt} \mid w_{gt}, v_{gt}) = 0, \tag{2}$$

[13] An excellent summary of this approach to testing firm conduct is provided by Duarte, Magnolfi, Sølvsten, and Sullivan (2024).

[14] We focus on a homogeneous product industry in this paper. However, this testing framework can be applied to industries characterized by differentiated products as exemplified in Duarte, Magnolfi, Sølvsten, and Sullivan (2024) and Backus, Conlon, and Sinkinson (2021).

where w_{gt} denotes a vector of exogenous cost variables, $w_{gt} = (w_{1gt}, w_{2gt}, \ldots, w_{N^F gt})$, and v_{gt} denotes a set of exogenous variables in the demand function that we will define below.

Demand Function To derive firms' marginal costs under a particular hypothesis, we need to specify the cement demand function. We use the following linear demand specification:

$$Q_{gt} = a_0 - (\alpha_1 + \alpha_2 v_{gt})P_{gt} + \alpha_3 v_{gt} + \xi_{gt}, \tag{3}$$

where Q_{gt} denotes the cement quantity demanded and P_{gt} denotes the price in market g at time t. v_{gt} plays the role of shifting and rotating the demand curve.[15]

3.2 Testing Procedures

Our testing environment is standard. We cannot observe firms' marginal costs and therefore cannot conduct a direct test of firm conduct. Instead, to test certain hypotheses about firm conduct, we use exogenous variables that are not included in the marginal cost function but are relevant to firms' markups, which are denoted by z^e_{igt}, and examine the correlation between residuals obtained from the marginal cost function estimation under these hypotheses and the excluded IVs as proposed by Duarte, Magnolfi, Sølvsten, and Sullivan (2024) and Backus, Conlon, and Sinkinson (2021). That is, the following moment conditions are used:

$$E(z^{e\top}_{igt}\epsilon_{igt}) = 0, \quad \epsilon_{igt} = mc_{igt} - x_{igt}\beta. \tag{4}$$

The rationale for this test is that, if a hypothesis is true, the residuals obtained from the marginal cost estimation under the hypothesis are not correlated with the excluded IVs. Otherwise, the moment condition is not satisfied and the correlation departs from zero. Consequently, this condition serves as a criterion – a lack-of-fit measure – to evaluate the hypothesis. The following two testing procedures based

[15]The introduction of the demand rotator is especially important for distinguishing between different oligopolistic models when the marginal cost function is not constant with respect to the quantity supplied (Bresnahan (1982), Lau (1982), and Duarte, Magnolfi, Quint, Sølvsten, and Sullivan (2025)). On the other hand, when marginal costs are constant, a demand shifter alone is sufficient for testing firm conduct as long as the shifter is relevant to the markups implied by hypotheses about firm conduct.

on such a lack-of-fit measure are used to test firm conduct, with the former being notably more widely used in the literature.

In the remainder of the paper, we drop subscripts g and t for expositional simplicity.

RV Test Let h_l denote a hypothesis about the true firm conduct. Following Duarte, Magnolfi, Sølvsten, and Sullivan (2024),we use the following measure for lack of fit:

$$\hat{Q}_{h_l} = \hat{g}_{h_l}^\top \hat{W} \hat{g}_{h_l}, \tag{5}$$

where $\hat{g}_{h_l} = \frac{1}{N}\sum_{i=1}^{N}\left(\hat{\tilde{z}}_i^{e\top}(mc_i^{h_l} - x_i\hat{\beta}^{h_l})\right)$, which is the sample analogue to (4), and $\hat{W} = \frac{1}{N}\sum_{i=1}^{N}(\hat{\tilde{z}}_i^{e\top}\hat{\tilde{z}}_i^{e})^{-1}$, which is the sample analogue to the inverse of $E(\tilde{z}_i^{e\top}\tilde{z}_i^{e})$.[16] For a pair of two competing hypotheses about firm conduct, h_1 and h_2, the test statistic becomes as follows:

$$\frac{\sqrt{N}(\hat{Q}_{h_1} - \hat{Q}_{h_2})}{\hat{\sigma}_{h_1 h_2}}, \tag{6}$$

where $\hat{\sigma}_{h_1 h_2}$ is the variance of the scaled difference between $\hat{Q}_{h_1}$ and $\hat{Q}_{h_2}$. The test statistic follows the standard normal distribution under the null that the two hypotheses equally fit the data, $H_0 : Q_{h_1} = Q_{h_2}$.

AR Test The AR test evaluates the absolute lack-of-fit of each hypothesis within the set of competing hypotheses. To formulate the test statistic, we again follow Duarte, Magnolfi, Sølvsten, and Sullivan (2024). We consider the following regression equation for the AR test:

$$\hat{\epsilon}_i^{h_l} = \hat{\tilde{z}}_i^{e}\pi^{h_l} + e_i, \tag{7}$$

where $\hat{\epsilon}_i^{h_l}$ denotes the residual obtained from the marginal cost estimation under h_l. In this testing model assessment framework, we test whether π^{h_l} is significantly different from zero for each candidate h_l. When π^{h_l} is significantly different from zero, we reject

[16] In the case of constant marginal costs, $\tilde{z}_i^e = z_i^e - w_i E(w_i^\top w_i)^{-1}E(w_i^\top z_i^e)$. In the case of non-constant marginal costs, $\tilde{z}_i^e = z_i^e - x(z_i)E(x(z_i)^\top x(z_i))^{-1}E(x(z_i)^\top z_i^e)$ where $x(z_i)$ is the linear projection of x_i onto z_i. z_i is the full set of exogenous variables that are used to estimate the marginal cost function and to test for firm conduct, $z_i = (w_i, z_i^e)$.

the hypothesis.

Hypotheses Using these testing procedures, we examine whether the presence of a cartel in the cement markets is accurately detected. In this examination, following current practices, we use (unconstrained) joint-profit maximization as the collusive benchmark hypothesis, while we consider Cournot competition as the competitive benchmark hypothesis.[17] The marginal costs implied by these hypotheses are obtained as follow:

$$mc_i^{h_{JPM}} = P + \frac{\partial P(Q)}{\partial Q} N^F q_i, \quad mc_i^{h_C} = P + \frac{\partial P(Q)}{\partial Q} q_i, \tag{8}$$

where $P(Q)$ denotes the inverse demand function. h_{JPM} denotes the hypothesis of joint-profit maximization, while h_C denotes that of the Cournot competition hypothesis. We use these to estimate the marginal cost function and derive residuals, which are then used to construct the test statistics.

3.3 Results of Firm Conduct Testing

The results of examining the performances of the testing procedures are presented in Table 6, while the demand and cost function estimation results are presented in Tables 4 and 5.[18] In the RV test, a negative sign indicates that the lack-of-fit measure under the joint-profit maximization hypothesis is greater than that under the Cournot competition hypothesis, suggesting that the Cournot hypothesis fits the observed behavior better. When we use the amount of construction investment as the

[17] In their studies of the U.S. cement industry, Ryan (2012) uses a Cournot model with capacity-constrained firms to approximate competition between cement firms, while Miller and Osborne (2014) use a model of spatial differentiation. In this study, we consider the Cournot model as a reasonable approximation of competition within prefectures in Japan's Chugoku region. The reasons are that, first, the Chugoku region is very small compared with the U.S. Southwest, the region that Miller and Osborne (2014) focus on. The land area is less than one tenth of that of California. And second, cement firms can efficiently transport cement from their production plants using ships, as the vast majority of these plants are located in coastal areas. In addition, cement firms reduce the costs of delivery by trucks by setting up several service stations to keep delivery distances short. As a result, there is little room for spatial differentiation, unlike the cement market in the U.S. Southwest. Regarding capacity constraints, the capacity-utilization rates of cement plants have remained at around 85% (on average) even after the period of the cartel. Therefore, capacity constraints of production plants are not a significant concern in this context.

[18] To estimate the variances, we need to take the effect of demand uncertainty into account. To do so, we use bootstrapping and estimate these as the variances of 250 bootstrap samples.

excluded IV, the Cournot hypothesis fits better but the difference between these two hypotheses is not statistically significant. On the other hand, when we use the sum of rival firms' numbers of service stations as the excluded IVs, the Cournot hypothesis is selected (at the 10% significance level).[19] In both cases, the RV test does not choose the hypothesis of joint-profit maximization. In contrast, the AR test rejects both hypotheses, regardless of which variable is used as the excluded IV.

Table 4: Demand Function

	(1) OLS	(2) IV
Quantity	−0.0101	−0.0719
	(0.0022)	(0.0236)
Quantity×Construction Inv.	0.0002	0.0002
	(0.0000)	(0.0000)
Construction Inv.	−0.0437	0.0439
	(0.0050)	(0.0326)
Adj. R^2	0.8944	N.A.
No. Obs.	285	

Note: Month and year fixed effects are included but not reported. The price of gasoline, along with its squared and cubed terms, is used as IVs to estimate the demand function.

Table 5: Cost Function

	(1) Cournot	(2) JPM
Constant	7.760	−8.041
	(1.207)	(7.125)
Gasoline Price	5.562	14.607
	(0.794)	(4.735)
No. SS	−0.293	−2.179
	(0.128)	(0.995)
Adj. R^2	0.997	0.894
No. Obs.	2280	

Note: Year fixed effects are included but not reported. The column labeled "Cournot" shows the estimation results under the assumption of Cournot competition while the column labeled "JPM" presents the results under the assumption of (unconstrained) joint-profit maximization.

[19]The estimation error in the demand function estimation substantially affects the statistical significance. For instance, when this estimation error is ignored, the Cournot competition hypothesis is selected at (at least) the 5% significance level.

Table 6: RV and AR Tests

	RV Test	
	(1)	(2)
Hypothesis	Cournot vs. JPM	Cournot vs. JPM
Test Statistics	−0.671	−1.810
Construction Inv.	Yes	No
Rival No. SS	No	Yes

	AR Test			
	(1)	(2)	(3)	(4)
Hypothesis	Cournot	JPM	Cournot	JPM
Construction Inv.	−0.015	−0.034		
	(0.002)	(0.015)		
Rival No. SS			0.093	1.042
			(0.046)	(0.268)

Note: The panel labeled "RV Test" presents the RV test statistic. The panel labeled "AR Test" presents the estimated values of π ($h \in \{h_C, h_{JPM}\}$), with the standard errors in parenthesis. Construction Inv. is the amount of construction investment, while Rival No. SS is the sum of other firms' No. SS.

In summary, the RV test results may misleadingly suggest that firms were competing with each other during the study period, when in reality they were colluding. The AR test rejects both the Cournot competition hypothesis and the joint-profit maximization hypothesis. Therefore, we have no model and cannot proceed with further analyses.[20]

3.4 Discussion

The above results suggest that relying on simplistic models of collusion to detect firms' collusive behavior and assess its consequences is inadequate.[21] A logical approach to resolve these limitations would be to broaden the menu of candidate models by incorporating more realistic models of collusion.

However, modeling collusion is inherently challenging. A standard approach to

[20] We relegate the results of firm conduct tests under the assumption of non-constant marginal costs to the Supplemental Appendix. The results are qualitatively similar to those in Table 6.

[21] Another implication is that, even if collusion is detected using the RV test, the estimated cost parameters may deviate substantially from their true values when the actual collusive behavior diverges from (unconstrained) joint-profit maximization. This can happen because the RV test compares the relative fits of the hypotheses. If this is the case, subsequent analyses using the model of joint-profit maximization will be unreliable because the marginal costs themselves are underestimated, resulting in the overestimation of the effects of collusion on price and welfare.

representing collusive behavior involves the use of (repeated) game-theoretic models. Developing such models requires addressing several critical aspects of firm behavior under collusion. First, colluding firms need to determine how to implement price increases and quantity reductions while navigating various obstacles and constraints. Factors such as incentive to deviate, antitrust risks, penalties, and the threat of entry influence colluding firms' decisions. Second, collusive profits need to be distributed among member firms. While size-based allocation mechanisms, such as those based on pre-cartel market shares, are commonly used, numerous alternative profit-sharing rules are possible. Third, enforcement mechanisms are needed. To sustain collusion, firms must establish mechanisms to punish deviations. A variety of punishment strategies can be considered.[22] Each of these considerations results in distinct versions of collusion models, depending on the assumptions made. The sheer number of potential combinations and variations can lead to an overwhelmingly large set of possible models.[23]

Even if we were able to develop candidate models, we would need to resolve the identification and estimation problems of these models in order to implement firm conduct testing. The key primitives to identify – whether under tacit or explicit collusion – are firms' cost functions (or cost parameters) and discount factors. Jointly identifying these primitives represents significant challenges. Moreover, models of collusion exhibit multiple equilibria, which introduces an additional layer of complexity to the identification problem. Further, when modeling explicit (illegal) collusion, it is often necessary to incorporate functions related to antitrust risks, such as detection probabilities, fines, and others factors (Harrington (2004)). The inclusion of these functions further exacerbates the identification and estimation challenges.

In summary, while it may not be impossible to address these modeling and identification issues directly, doing so would be prohibitively difficult. Therefore, in this study, we do not pursue this approach. Instead, we propose an alternative approach that avoids the need to explicitly model collusive behavior beyond (unconstrained)

[22]Additionally, it is crucial to specify how firms detect defections and initiate retaliatory actions. This involves determining what information is observable by firms, the time required to detect defections, and the speed at which retaliation occurs.

[23]In analyses of convicted or known cartels, researchers can utilize information on the workings of the cartels. Many studies utilize such information to construct models of collusive behavior by firms (Röller and Steen (2006), Asker (2010), Salvo (2010), Clark and Houde (2013), Igami and Sugaya (2021), and Starc and Wollmann (2025)). However, in the context of finding collusion, such information aiding in the building of models of collusion is not available.

joint-profit maximization.

4 Analyzing Collusion Using Set Identified Marginal Cost Functions

The findings and discussion in the previous section motivate us to propose an alternative approach to analyzing collusion. In our approach, instead of restricting our attention to a limited number of simple models of (or hypotheses about) firm conduct, we allow as many forms of firm conduct as possible, with the aim of incorporating collusive conduct other than (unconstrained) joint-profit maximization in the empirical framework. To this end, we define the most and least competitive forms of competition and assume that the true firm behavior lies between these two competitive and collusive benchmarks. Under this assumption, we derive the lower and upper bounds on firms' marginal costs. With these bounds, we identify a set of marginal cost parameters. In doing so, we exploit an additional set of exogenous variables that provide restrictions to falsify hypotheses about firm conduct and rule out false cost parameters. The resulting identified set is a collection of cost parameters not falsified by these restrictions.

We use this identified set to test for collusion. Our procedure consists of the following steps. First, we evaluate the competitive benchmark behavior by determining whether the cost parameter vector implied by this benchmark falls within the identified set. The rationale for this test is as follows: if the parameter vector implied by the competitive benchmark fails to satisfy the moment restrictions provided by supernumerary IVs, it does not belong to the identified set, indicating that the competitive benchmark behavior is not compatible with the data. Second, we measure the welfare effects of (potential) collusion. We derive lower and upper bounds for the cartel-induced price overcharge and resulting loss in consumer surplus.

4.1 Identification of the Marginal Cost Function

We consider the following markup equation:

$$\Delta_i = P - mc_i^0, \tag{9}$$

where Δ_i denotes the markup of firm i and mc_i^0 is the firm's marginal cost.[24] Δ_i can vary across firms, time, and markets, influenced by numerous factors, including firm characteristics, the market environment, and other things. Among these, the form of competition between firms is particularly significant for the purpose of this paper.[25] We go beyond traditional, simple oligopolistic hypotheses by incorporating the concept developed by Rosen (2006), allowing for a broader range of possible forms of firm conduct. We consider a range of markup values within which we believe the true markup lies. Specifically, for a firm's markup, we define the minimum and maximum values and assume that the true markup lies between these two values. This idea is expressed using the following inequality restrictions:

$$\underline{\Delta}_i \leq P - mc_i^0 \leq \overline{\Delta}_i, \tag{10}$$

where $\underline{\Delta}_i$ denotes the lowest level of firm i's markup and $\overline{\Delta}_i$ denotes the highest level. To derive the lower and upper bounds on the true markup, we need behavioral assumptions regarding the least and most competitive firm conduct. The least competitive firm conduct (collusive benchmark) leads to $\overline{\Delta}_i$, while the most competitive firm conduct (competitive benchmark) leads to $\underline{\Delta}_i$. We will define these shortly.

From the above markup bounds, we derive the following restrictions on the true marginal cost:

$$P - \overline{\Delta}_i \leq mc_i^0 \leq P - \underline{\Delta}_i. \tag{11}$$

Moreover, from these restrictions, we derive the marginal cost interval as follows:

$$H[mc_i^0] = [\underline{mc}_i, \overline{mc}_i] = \left[P - \overline{\Delta}_i, P - \underline{\Delta}_i\right]. \tag{12}$$

Based on the assumptions about the two benchmark firm behaviors, the firm's marginal cost is derived as an interval.

To derive bounds on the true markups, we need to specify the most and least competitive behaviors. In our analysis of the cement cartel, we consider Cournot

[24] We will proceed by focusing on the case of a homogeneous product industry, although the concept can be readily extended to markets with differentiated products.

[25] The markup terms depend on the demand function parameter α. We assume that the demand function is either known to us or has been estimated beforehand. For expositional simplicity, we ignore in this section the dependence of the markups on the demand parameters.

competition as a reasonable competitive benchmark equilibrium, as explained earlier. This competitive equilibrium provides the lower bound on the true markups and in turn the upper bound on the true marginal costs. Conversely, we use (unconstrained) joint-profit maximization as the opposite benchmark. Under these assumptions regarding the benchmarks, the lower and upper bounds on firm i's marginal cost are defined as follows:

$$\underline{mc}_i = P + \frac{\partial P(Q)}{\partial Q} N^F q_i, \quad \overline{mc}_i = P + \frac{\partial P(Q)}{\partial Q} q_i. \tag{13}$$

The interval defined by $\underline{mc}_i$ and $\overline{mc}_i$ includes the marginal costs implied by all possible firm behaviors ranging from Cournot competition to joint-profit maximization.

One issue is that interval-valued marginal costs make regression model (1) incomplete, as the outcome is not a random variable but a random interval. Econometric models with interval-valued outcomes (and/or explanatory variables), which belong to a class of incomplete econometric models, have been studied extensively such as by Manski and Tamer (2002), Stoye (2007), Beresteanu and Molinari (2008), Beresteanu, Molchanov, and Molinari (2011), Bontemps, Magnac, and Maurin (2012), Kaido and Santos (2014), and Chandrasekhar, Chernozhukov, Molinari, and Schrimpf (2019).

Among these studies, the work by Bontemps, Magnac, and Maurin (2012) is most relevant to our study. They examine the identification and estimation of a class of models defined by incomplete linear moment conditions, focusing on a model with interval-valued outcomes as a primary example. They characterize the identified set of parameters of interest not only when the number of moment conditions, M, is equal to the number of parameters, K, but also when M is larger than K. The case where $M > K$ is particularly important for our study. The core idea of our approach involves leveraging exogenous variables that are not included in the (true) marginal cost function but are relevant to firms' markups in order to obtain an informative set of cost parameters.

In what follows, building on the work of Bontemps, Magnac, and Maurin (2012), we first explain the identified set of marginal cost parameters when $M = K$ and then move on to the case when $M > K$, where we demonstrate the role of $M - K$ exogenous variables (IVs) in reducing the size of the identified set.

4.1.1 The Identified Set of Cost Parameters When $M = K$

As just explained, empirical model (1) becomes incomplete because marginal costs are only interval-measured as $H[mc_i^0] = [\underline{mc}_i, \overline{mc}_i]$. Consequently, the cost parameters are only set-identified. Let us first consider the identification of cost parameters when the number of moment restrictions is equal to the number of the parameters.

In this case, the set of marginal cost parameters is defined as the collection of cost parameters satisfying the incomplete linear moment restrictions, as follows:

$$\mathcal{B} = \left\{\beta : E\left[z_i^\top (y - x_i\beta)\right] = 0, \quad y \in [\underline{mc}_i, \overline{mc}_i]\right\}, \tag{14}$$

where z_i is a vector of M exogenous variables used to provide the moment conditions for identifying K marginal cost parameters.[26] In the case of constant marginal costs, there are no endogenous variables in the marginal cost function. Consequently, x_i consists only of exogenous variables w_i, so that $z_i = w_i (= x_i)$. On the other hand, in the case of non-constant marginal costs, x_i includes q_i as well as w_i, that is, $x_i = (w_i, q_i)$. In this case, (at least) one exogenous variable excluded from the marginal cost function is needed as an IV for q_i. Let z_i^{K+0} denote such an IV, which is typically a demand shifter or (the sum of) rival firms' cost shifters. Then, $z_i = (w_i, z_i^{K+0})$.

Because the identified set $\mathcal{B}$ is non-empty, convex, and bounded, we can characterize it using its support function.[27] The support function of $\mathcal{B}$ is defined as

$$\delta^*(q \mid \mathcal{B}) = \sup_{\beta \in \mathcal{B}} q^\top \beta \tag{17}$$

[26] While we employ a simple cost function specification, the functional form can be relaxed. For example, as mentioned by Haile and Tamer (2003), a polynomial can be used to approximate the true functional form.

[27] As explained by Molchanov and Molinari (2014), the support function of a convex closed set $\mathcal{K}$ in direction $u \in \mathbb{R}^K$ is the signed distance of the support plane to $\mathcal{K}$ with exterior normal vector u from the origin:

$$\delta^*(u \mid \mathcal{K}) = \sup\left\{u^\top \kappa : \kappa \in \mathcal{K}\right\}. \tag{15}$$

The support function is homogeneous of degree one. Consequently, its domain is often restricted to the unit sphere $\mathbb{S}^{K-1} = \left\{q \in \mathbb{R}^K \mid \|q\| = 1\right\}$. A closed convex set $\mathcal{K}$ can be expressed as the set of solutions to a system of inequalities given its support function (Rockafellar (1970), p.113):

$$\mathcal{K} = \left\{\kappa \mid q^\top \kappa \leq \delta^*(q \mid \mathcal{K}), \forall q \in \mathbb{S}^{K-1}\right\}. \tag{16}$$

for all directions $q \in \mathbb{S}^{K-1}$. This function traces out the boundary of $\mathcal{B}$, so that knowing the support function of $\mathcal{B}$ is equivalent to knowing $\mathcal{B}$ itself. Following Proposition 2 in Bontemps, Magnac, and Maurin (2012), the support function of $\mathcal{B}$ in direction q is expressed as follows:

$$\delta^*(q \mid \mathcal{B}) = E\left(z_{i,q} y_{i,q}\right) = q^\top \beta_q, \tag{18}$$

where $z_{i,q} = q^\top E(z_i^\top x_i)^{-1} z_i^\top$ and $y_{i,q} = \underline{mc}_i + \mathbf{1}\{z_{i,q} > 0\}(\overline{mc}_i - \underline{mc}_i)$. β_q is the frontier point of $\mathcal{B}$ in direction q:

$$\beta_q = E\left(z_i^\top x_i\right)^{-1} E\left(z_i^\top y_{i,q}\right). \tag{19}$$

Using the support function, $\mathcal{B}$ can be characterized as follows:

$$\mathcal{B} = \left\{\beta \mid q^\top \beta \leq \delta^*(q \mid \mathcal{B}),\ \forall q \in \mathbb{S}^{K-1}\right\}. \tag{20}$$

Because $\mathcal{B}$ is convex, its inner points can be represented by combining two boundary points.

An obvious but important fact about $\mathcal{B}$ is that every possible value in the interval $[\underline{mc}_i, \overline{mc}_i]$ is used to construct $\mathcal{B}$. In other words, it contains any parameter value that is associated with at least one $y \in [\underline{mc}_i, \overline{mc}_i]$. This fact holds important implications for analyzing collusion. To see this, consider the following two hypotheses: one posits that the observed firm behavior over the entire study period aligns with the competitive benchmark, while the other suggests that it aligns with the collusive benchmark. Let β_{q^C} and $\beta_{q^{JPM}}$ denote the vector of cost parameters implied by these two hypotheses, respectively. Using (19), the two vectors can be expressed as follows:

$$\beta_{q^C} = E(z_i^\top x_i)^{-1} E(z_i^\top y_{i,q^C}) = E(z_i^\top x_i)^{-1} E(z_i^\top \overline{mc}_i), \tag{21}$$

$$\beta_{q^{JPM}} = E(z_i^\top x_i)^{-1} E(z_i^\top y_{i,q^{JPM}}) = E(z_i^\top x_i)^{-1} E(z_i^\top \underline{mc}_i), \tag{22}$$

where direction q^C is q such that $z_{i,q} > 0$ for all i and thus $y_{i,q^C} = \overline{mc}_i$. Direction q^{JPM} is q such that $z_{i,q} < 0$ for all i and thus $y_{i,q^{JPM}} = \underline{mc}_i$. These parameter vectors are on the frontier of $\mathcal{B}$, that is, $\beta_{q^C}, \beta_{q^{JPM}} \in \partial\mathcal{B}$. Therefore, $\mathcal{B}$ is not informative because, by construction, it includes the cost parameter vectors implied by the competitive and collusive benchmarks. Consequently, from the identified set, we cannot learn

anything about the form of competition between firms.

4.1.2 The Identified Set of Cost Parameters When $M > K$

However, when the number of moment conditions is greater than the number of parameters, the picture changes. In this case, there are $M - K$ surplus moment conditions that impose further restrictions on cost parameters. Exogenous variables providing these $M-K$ additional moment conditions are referred to as supernumerary IVs by Bontemps, Magnac, and Maurin (2012). Because parameters must also satisfy these $M-K$ moment conditions additionally, the size of the resulting identified set is likely to be smaller. As will become clear, these IVs effectively provide restrictions on $y \in [\underline{mc}_i, \overline{mc}_i]$. That is, only implied marginal costs in the interval $[\underline{mc}_i, \overline{mc}_i]$ that are not correlated with these IVs are admissible, and the resulting identified set consists of the cost parameters associated with these admissible marginal costs. Consequently, when $M > K$, the identified set becomes smaller than $\mathcal{B}$.

Suppose that there are $M - K$ surplus exogenous variables that we can utilize, denoted as $z_i^s = (z_i^{K+1}, \ldots, z_i^M)$. In total, there are M exogenous variables. The vector of M exogenous variables is defined as $z_i = (w_i, z_i^{K+1}, \ldots, z_i^M)$ in the case of constant marginal costs while $z_i = (w_i, z_i^{K+0}, z_i^{K+1}, \ldots, z_i^M)$ in the case of non-constant marginal costs. Let $\mathcal{B}^s$ denote the set of cost parameters that satisfy the incomplete moment conditions provided by z_i. That is, β belongs to $\mathcal{B}^s$ if and only if there exists a $y \in [\underline{mc}_i, \overline{mc}_i]$ that satisfies the following moment conditions:

$$E\left[z_i^\top (y - x_i\beta)\right] = 0. \tag{23}$$

The definition of the identified set is the same as before, except that there are $M-K$ surplus moment conditions here.

To illustrate the role of supernumerary IVs in constructing the identified set of parameters, we decompose the above moment conditions into the following two parts, following Bontemps, Magnac, and Maurin (2012):

$$E\left[z_{F,i}^\top (y - x_i\beta)\right] = 0, \tag{24}$$

$$E\left[z_{H,i}^\top (y - x_i\beta)\right] = E\left[z_{H,i}^\top y\right] = 0. \tag{25}$$

The first set of moment conditions is provided by $z_{F,i}$, whose dimension is K. $z_{F,i}$ is

the orthonormalization of $x(z_i)$, which is the linear projection of x_i onto z_i, $x(z_i) = z_i E(z_i^\top z_i)^{-1} E(z_i^\top x_i)$:

$$z_{F,i} = x(z_i) E\left(x(z_i)^\top x(z_i)\right)^{-\frac{1}{2}}. \tag{26}$$

On the other hand, the second set of moment conditions is provided by $z_{H,i}$. $z_{H,i}$, with a dimension of $M - K$, is the orthonormalization of ζ_i^s, which is the residual from the linear projection of $x(z_i)$ onto z_i^s:

$$z_{H,i} = \zeta_i^s E(\zeta_i^{s\top} \zeta_i^s)^{-\frac{1}{2}}, \quad \zeta_i^s = z_i^s - x(z_i) E\left(x(z_i)^\top x(z_i)\right)^{-1} E\left(x(z_i)^\top z_i^s\right). \tag{27}$$

Because $E\left[\zeta_i^{s\top} x_i\right] = 0$ and thus $E\left[z_{H,i}^\top x_i\right] = 0$, the first equality in (25) holds.[28]

Let us look at the roles of these two sets of moment conditions. First, the role of (24) is to relate $y \in [\underline{mc}_i, \overline{mc}_i]$ to β. In fact, the moment conditions alone provide the following set of cost parameters:

$$\left\{\beta : E\left(z_{F,i}^\top (y - x_i \beta)\right) = 0, \;\; y \in [\underline{mc}_i, \overline{mc}_i]\right\}. \tag{29}$$

This means that the role of the moment restrictions provided by $z_{F,i}$ is identical to that in (14). Specifically, in the case of constant marginal costs, this set of cost parameters is equivalent to $\mathcal{B}$.

Therefore, it is the second set of moment conditions that plays the role of reducing the size of the identified set. As evident from (25), the moment conditions provided by $z_{H,i}$ impose restrictions on y. If $y \in [\underline{mc}_i, \overline{mc}_i]$ is not correlated with $z_{H,i}$, it is admissible for constructing $\mathcal{B}^s$; otherwise, it is ruled out. Therefore, the role of the supernumerary IVs is to determine whether y in the interval $[\underline{mc}_i, \overline{mc}_i]$ is admissible. Only admissible values in the interval are related with admissible β that are elements of $\mathcal{B}^s$. Consequently, supernumerary IVs reduce the size of the identified set.

The Economic Meaning of the Moment Restrictions Provided by the Supernumerary IVs Interestingly, the moment conditions in (25) are essentially

[28] In the case of constant marginal costs, $x_i = w_i$ and $x(z_i) = w_i$. Therefore, the expression simplifies to

$$z_{H,i} = \zeta_i^s E\left(\zeta_i^{s\top} \zeta_i^s\right)^{-\frac{1}{2}}, \;\; \zeta_i^s = z_i^s - w_i E(w_i^\top w_i)^{-1} w_i^\top z_i^s. \tag{28}$$

equivalent to the moment restrictions used to falsify oligopolistic models that are exploited in firm conduct tests in the IO literature (see, e.g., Berry and Haile (2014), Duarte, Magnolfi, Sølvsten, and Sullivan (2024), and Backus, Conlon, and Sinkinson (2021)). Here, we show this connection and explain the economic significance of the supernumerary IVs.

The set of incomplete moment conditions in (25) can be rewritten as follows:

$$E\left(z_{H,i}^{\top}(P-\Delta)\right)=0,\ \ \Delta\in[\underline{\Delta}_i,\overline{\Delta}_i], \tag{30}$$

where $\underline{\Delta}_i$ and $\overline{\Delta}_i$ denote the lower and upper bounds on the true markup Δ_i^0. In addition, the above moment conditions are satisfied under Δ_i^0:

$$E\left(z_{H,i}^{\top}(P-\Delta_i^0)\right)=0. \tag{31}$$

Combining these two conditions yields the following incomplete moment conditions:

$$E\left(z_{H,i}^{\top}(\Delta_i^0-\Delta)\right)=0,\ \ \Delta\in[\underline{\Delta}_i,\overline{\Delta}_i]. \tag{32}$$

The above conditions indicate that, among the possible implied markups within the interval $[\underline{\Delta}_i,\overline{\Delta}_i]$, only those implied markups Δ whose covariation with $z_{H,i}$ matches the covariation between Δ_i^0 and $z_{H,i}$ are admissible. On the other hand, implied markups that do not exhibit the same covariation are ruled out. Therefore, the above incomplete moment conditions provide falsifying restrictions that can be used to rule out hypotheses about firm conduct, as these hypotheses are represented by markups within $[\underline{\Delta}_i,\overline{\Delta}_i]$.

This role of the supernumerary IVs in this context is analogous to that of excluded IVs in the literature on testing models of firm conduct. Berry and Haile (2014) provide falsifiable restrictions to test oligopolistic models. More recently, Duarte, Magnolfi, Sølvsten, and Sullivan (2024) and Backus, Conlon, and Sinkinson (2021) have derived moment conditions based on the falsifiable restrictions proposed by Berry and Haile (2014). (32) is essentially equivalent to the moment restrictions used in the literature to discriminate between candidate hypotheses or assess the validity of hypotheses.[29]

[29] (32) can be rewritten in the following form under a specific hypothesis Δ^h:

$$E\left(z_{H,i}^{\top}(\Delta_i^0-\Delta_i^h)\right)=E\left(z_{H,i}^{\top}\left(\left(\Delta_i^0-x(z_i)\gamma^0\right)-\left(\Delta_i^h-x(z_i)\gamma^h\right)\right)\right)=E\left(z_{H,i}^{\top}(\tilde{\Delta}_i^0-\tilde{\Delta}_i^h)\right)=0, \tag{33}$$

The main difference is that our approach not only considers a limited number of hypotheses but also examines all possible hypotheses represented by $\Delta \in [\underline{\Delta}_i, \overline{\Delta}_i]$. Consequently, our approach allows many forms of collusion. For instance, it allows for the possibility that firms raise prices gradually instead of hiking them immediately to a certain target level as modeled in analyses of illegal collusion such as the study by Harrington (2004). Alternatively, it allows a situation in which demand and cost fluctuations affect colluding firms' behavior, as studied by Rotemberg and Saloner (1986). For instance, at certain demand and cost states, firms are able to collude perfectly, while at others, they are able to collude only imperfectly and thus fail to achieve (unconstrained) joint-profit maximum. The set $\mathcal{B}^s$ arises as a result of examining such possible hypotheses.

Lastly, because the supernumerary IVs play the same role as excluded IVs in firm conduct tests do, the conditions that the supernumerary IVs should satisfy are identical to those for excluded IVs in the IO literature. As shown by Duarte, Magnolfi, Sølvsten, and Sullivan (2024) in their first lemma, excluded IVs need to be relevant for testing to falsify wrong models. This means that excluded IVs need to be correlated with at least one of Δ_i^0 and Δ_i^h. Candidates for such excluded IVs are demand-side variables, including demand shifters and rotators, and cost-side variables, such as rival firms' cost shifters. These exogenous variables are highly likely to be relevant to firms' markups.[30]

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where $\tilde{\Delta}_i^0$ and $\tilde{\Delta}^h$ represent the residualized markups with respect to $x(z_i)$, $\tilde{\Delta}_i^0 = \Delta_i^0 - x(z_i)\gamma^0 = \Delta_i^0 - x(z_i)E\left(x(z_i)^\top x(z_i)\right)^{-1} E\left(x(z_i)^\top \Delta_i^0\right)$ and $\tilde{\Delta}_i^h = \Delta_i^0 - x(z_i)\gamma^h = \Delta_i^h - x(z_i)E\left(x(z_i)^\top x(z_i)\right)^{-1} E\left(x(z_i)^\top \Delta_i^h\right)$, respectively. As is clear from the way that it is constructed, $z_{H,i}$ is uncorrelated with $x(z_i)$, $E\left(z_{H,i}^\top x(z_i)\right) = 0$. Condition (33) is equivalent to the following condition:

$$E\left(\zeta_i^{s\top}(\tilde{\Delta}_i^0 - \tilde{\Delta}_i^h)\right) = 0, \tag{34}$$

where ζ_i^s is the set of residualized supernumerary IVs (with respect to $x(z_i)$), as defined in (27). This type of moment condition is similar to those provided by excluded IVs in Duarte, Magnolfi, Sølvsten, and Sullivan (2024) and Duarte, Magnolfi, Quint, Sølvsten, and Sullivan (2025). In the case of constant marginal costs, the moment conditions in (34) are exactly the same as those used in Duarte, Magnolfi, Sølvsten, and Sullivan (2024). In this case, the residualized supernumerary IVs and markups are defined as follows:

$$\zeta_i^s = z_i^s - w_i E(w_i^\top w_i)^{-1} E(w_i^\top z_i^s), \quad \tilde{\Delta}_i = \Delta_i - w_i E\left(w_i^\top w_i\right)^{-1} E\left(w_i^\top \Delta_i\right).$$

[30] As is clear from the way that it is constructed, $z_{H,i}$ is obtained after z_i^s is residualized with respect to $x(z_i)$. Therefore, it is necessary for $z_{H,i}$, not z_i^s, to be relevant to firms' markups to

Frontier and Interior Points As explained, $\mathcal{B}$ shrinks to $\mathcal{B}^s$ when there are supernumerary (or excluded) IVs. The question is which points of $\mathcal{B}$ are ruled out and how $\mathcal{B}^s$ is eventually constructed. The key to answering this question is to understand the difference between frontier and interior points. The frontier point of $\mathcal{B}$ in direction q, β_q, is uniquely determined by $E(z_i^\top x_i)^{-1}(x_i^\top y_{i,q})$. Therefore, if $E(z_{H,i}^\top y_{i,q}) \neq 0$, β_q is not contained in $\mathcal{B}^s$.

In contrast to frontier points, the expression for interior points of $\mathcal{B}$ is not unique. For $\beta^{int} \in \mathcal{B}$, there is a set of y that are associated with β^{int}:

$$\mathcal{Y}_{\beta^{int}} = \left\{ y : E(z_i^\top x_i)^{-1} E(x_i^\top y) = \beta^{int}, \;\; y \in [\underline{mc}_i, \overline{mc}_i] \right\}. \tag{35}$$

Therefore, for an interior point to be excluded from $\mathcal{B}^s$, it is necessary that all $y \in \mathcal{Y}_{\beta^{int}}$ are correlated with $z_{H,i}$. On the other hand, if there is even one $y \in \mathcal{Y}_{\beta^{int}}$ that is not correlated with $z_{H,i}$, β^{int} is not excluded from $\mathcal{B}^s$.

The implications of this fact can be explained by using the following examples. The cost parameters implied by the benchmarks are frontier points of $\mathcal{B}$, $\beta_{q^C}, \beta_{q^{JPM}} \in \partial\mathcal{B}$. Therefore, when $E(z_{H,i}^\top \overline{mc}_i) \neq 0$, the hypothesis that the competitive benchmark holds over the observation period is rejected, so β_{q^C} does not belong to $\mathcal{B}^s$. Similarly, when $E(z_{H,i}^\top \underline{mc}_i) \neq 0$, $\beta_{q^{JPM}}$ does not belong to $\mathcal{B}^s$ either.

On the other hand, consider the hypothesis that firms are engaged in a form of imperfect collusion, rather than the benchmark joint-profit maximization. Let $y^{int'}$ denote the marginal cost implied by this behavior, and let $\beta^{int'}$ denote the cost parameters associated with $y^{int'}$. However, $\beta^{int'}$ is an interior point of $\mathcal{B}$ and thus is expressed using different $y \in \mathcal{Y}_{\beta^{int'}}$. Therefore, even if the moment condition $E(z_{H,i}^\top y^{int'}) = 0$ is violated, it is still possible that $\beta^{int'}$ belongs to $\mathcal{B}^s$ when another $y \in \mathcal{Y}_{\beta^{int'}}$ is not correlated with $z_{H,i}$. This y may be implied by another form of collusion because, unlike the frontier points, $\beta^{int'}$ is possibly associated with more than one (collusive) firm conduct hypothesis. Therefore, unless all y in $\mathcal{Y}_{\beta^{int'}}$ are ruled out, $\beta^{int'}$ belongs to $\mathcal{B}^s$.

eliminate false hypotheses about firm conduct and reduce the size of the identified set. In particular, in the case of non-constant marginal costs, x_i includes q_i and thus $x(z_i)$ includes the projection of q_i onto z_i, $\hat{q}_i = z_i E(z_i^\top z_i)^{-1} E(z_i^\top q_i)$. This means that the part of the variation in z_i^s that is already used to explain the endogenous variable q_i is not used to construct $z_{H,i}$.

4.2 Analyzing Collusion Using the Identified Set $\mathcal{B}^s$

The fact that supernumerary IVs reduce the size of the identified set of cost parameters has important implications for analyzing collusion. First, the resulting identified set $\mathcal{B}^s$ can be used for firm conduct testing. Of particular interest to us is whether the cost parameter vector β_{q^C} which is implied by the competitive benchmark belongs to $\mathcal{B}^s$. When $\beta_{q^C} \notin \mathcal{B}^s$, the hypothesis that the observed data were generated from the competitive benchmark is rejected. We interpret the rejection of the competitive benchmark as suggestive evidence of collusion.

Second, $\mathcal{B}^s$ allows us not only just to find suggestive evidence of collusion but also to quantify the welfare effect of collusion. Based on the set identified marginal cost parameters, we simulate a counterfactual scenario where firms compete with each other and compare this against the actual situation. The comparison of this counterfactual firm behavior with the actual firm behavior provides lower and upper bounds on the consumer welfare loss caused by collusion.

Screening for Collusion For testing firm conduct, the following alternative characterization of the identified set is useful:

$$\begin{aligned} \beta \in \mathcal{B}^s \quad &\iff \quad \forall q \in \mathbb{S}^{K-1},\ \ T(q;\beta) = \delta^*\left(q \mid \mathcal{B}^s\right) - q^\top \beta \geq 0 \\ &\iff \quad \min_{q \in \mathbb{S}^{K-1}} T(q;\beta) \geq 0, \end{aligned} \tag{36}$$

where $\delta^*\left(q \mid \mathcal{B}^s\right)$ is the support function of $\mathcal{B}^s$. This expression suggests a testing procedure for examining whether a particular parameter vector belongs to $\mathcal{B}^s$. In the context of screening for cartels, our particular interest lies in testing whether the competitive benchmark behavior is compatible with the data:[31]

$$H_C : \beta_{q^C} \in \mathcal{B}^s. \tag{37}$$

[31] We can use different equilibrium notions as the benchmarks. For instance, perfect competition can be used as the competitive benchmark instead of Cournot competition when we believe that this is a more appropriate notion of competition. In this case, the marginal cost interval changes from $[\underline{mc}_i, \overline{mc}_i]$ to $[\underline{mc}_i, \overline{mc}'_i]$ $(\overline{mc}_i < \overline{mc}'_i)$ and, as a result, the identified sets $\mathcal{B}$ and $\mathcal{B}^s$ become larger. However, the underlying idea of our approach for detecting collusion remains unchanged, regardless of which equilibrium notion is used. The reason is that our test is based on whether the competitive benchmark belongs to $\mathcal{B}^s$, meaning that only the relationship between $\mathcal{B}$ and $\mathcal{B}^s$ matters.

From relationship (36), testing $\beta_{q^C} \in \mathcal{B}^s$ is equivalent to testing the following inequality:

$$T(q^*; \beta_{q^C}) \geq 0, \tag{38}$$

where $T(q^*; \beta_{q^C}) = \min_{q \in \mathbb{S}^{K-1}} T(q; \beta_{q^C})$. We also test $\beta_{q^{JPM}} \in \mathcal{B}^s$ in the same manner.[32]

Quantifying the Effects of Collusion The rejection of the competitive benchmark calls for further analysis of potential collusive behavior. In particular, measuring the potential welfare effect of collusion is a key interest. In our approach, this type of welfare analysis is possible because, even when certain benchmarks and other firm behaviors are rejected, we still have cost parameters that are not rejected by the moment restrictions provided by the supernumerary IVs.[33] Using the identified set of these cost parameters, we derive bounds on the counterfactual consumer welfare under the assumption that firms exhibit competitive behavior. This allows us to quantify the effects of non-competitive, possibly collusive behavior. Under this counterfactual scenario, we obtain the following lower and upper bounds on the consumer surplus:

$$\left[\underline{CS},\ \overline{CS}\right] = \left[\min_{\beta \in \mathcal{B}^s} CS\left(Q^*(\beta)\right),\ \max_{\beta \in \mathcal{B}^s} CS\left(Q^*(\beta)\right)\right], \tag{39}$$

where $Q^*(\beta)$ represents the counterfactual market supply evaluated at $\beta \in \mathcal{B}^s$ and $CS\left(Q^*(\beta)\right)$ is the consumer surplus at $Q^*(\beta)$. These bounds on consumer welfare are used to calculate the magnitude of the effect of potential collusion on consumer welfare.[34]

[32] An alternative and simpler method of testing whether $\beta_{q^C} \in \mathcal{B}^s$ and $\beta_{q^{JPM}} \in \mathcal{B}^s$ involves testing the equalities $E(z_{H,i}^\top \overline{mc}_i) = 0$ and $E(z_{H,i}^\top \underline{mc}_i) = 0$. Testing these equalities is sufficient because β_{q^C} and $\beta_{q^{JPM}}$ are frontier points of $\mathcal{B}$ and are therefore uniquely determined by $\overline{mc}_i$ and $\underline{mc}_i$ respectively. This might suggest that characterizing the identified set of cost parameters is unnecessary. However, because our interest lies not just in testing the benchmarks but also in quantifying and conducting inference on the welfare effects of collusion, the set of cost parameters is required.

[33] In contrast, in the AR test framework, for instance, we can reject the competitive benchmark. However, this type of counterfactual analysis cannot be conducted unless we construct a model (of collusion) that reasonably approximates the true firm behavior.

[34] Note that, when $\beta_{q^C} \in \mathcal{B}^s$, the lower bound on the welfare effect of collusion is trivially zero.

4.3 Estimation and Inference

By applying the analogy principle, we estimate the identified set of cost parameters and its support function. Estimating the support function and (frontier points of) the set of cost parameters is quite straightforward when $M = K$. The estimate of $\delta^*(q \mid \mathcal{B})$ is obtained as the following sample analogue of (18):

$$\hat{\delta}^*(q \mid \mathcal{B}) = q^\top \hat{\beta}_q = q^\top \left(\frac{1}{N} \sum_{i=1}^{N} z_i^\top x_i \right)^{-1} \left(\frac{1}{N} \sum_{i=1}^{N} z_i^\top \hat{y}_{i,q} \right), \tag{40}$$

where $\hat{y}_{i,q} = \mathbf{1}\left\{\hat{z}_{i,q} > 0\right\}(\underline{mc}_i - \overline{mc}_i) + \underline{mc}_i$ and $\hat{z}_{i,q} = q^\top \left(\frac{1}{N} \sum_{i=1}^{N} z_i^\top x_i \right)^{-1} z_i^\top$.

On the other hand, when $M > K$, the estimation of the support function and frontier points of the identified set becomes much more complex because $E(z_i^\top x_i)$ is no longer invertible. Bontemps, Magnac, and Maurin (2012) propose a method to deal with this case. Following their approach, we introduce auxiliary parameters, $\gamma \in \mathbb{R}^{M-K}$, and consider the following incomplete linear moment conditions instead of the original ones:

$$E\left(z_i^\top (y - x_i\beta - z_{H,i}\gamma)\right) = 0, \;\; y \in [\underline{mc}_i, \overline{mc}_i]. \tag{41}$$

Define the set of parameters satisfying the above moment restrictions as follows:

$$\mathcal{B}^U = \left\{(\beta, \gamma) \mid E\left(z_i^\top (y - x_i\beta - z_{H,i}\gamma)\right) = 0, \;\; y \in [\underline{mc}_i, \overline{mc}_i]\right\}. \tag{42}$$

Using (41), we can readily characterize $\mathcal{B}^s$ and its support function. First, the introduction of auxiliary parameters γ ensures there are as many parameters as moment restrictions. Second, $\mathcal{B}^s$ is defined by imposing restrictions on the solution to the above program. That is, $\mathcal{B}^s$ is the set of parameters that satisfy the following restrictions:

$$\begin{aligned} &E\left(z_i^\top (y - x_i\beta - z_{H,i}\gamma)\right) = 0, \; y \in [\underline{mc}_i, \overline{mc}_i], \\ &\gamma = 0. \end{aligned}$$

This implies that $\mathcal{B}^s$ is equal to the intersection of $\mathcal{B}^U$ and the hyperplane, $\gamma = 0$,

both of which are convex sets.[35]

From Proposition 6 in Bontemps, Magnac, and Maurin (2012), the support function of $\mathcal{B}^s$ is obtained as the solution to the following minimization problem:

$$\delta^* \left((q, \lambda) \mid \mathcal{B}^s \right) = \inf_{\lambda} \delta^* \left((q, \lambda) \mid \mathcal{B}^U \right), \tag{43}$$

where $q \in \mathbb{S}^{K-1}$ and $\lambda \in \mathbb{R}^{M-K}$. This characterization suggests the following steps to estimate the support function: first, estimate the support function of $\mathcal{B}^U$; second, minimize the support function with respect to λ while fixing q. In addition to the support function, we can derive the frontier points of $\mathcal{B}^s$ as follows:

$$(\beta_q^\top, 0^\top)^\top = E\left(z_i^\top (x_i, z_{H,i})\right)^{-1} E\left(z_i^\top y_{i,(q,\lambda^*(q))}\right), \tag{44}$$

where $\lambda^*(q)$ is the solution to (21) in direction q.

To estimate $\delta^* \left((q, \lambda) \mid \mathcal{B}^s \right)$, we construct the following sample analogue to (43):

$$\hat{\delta}^*(q \mid \mathcal{B}^s) = \inf_{\lambda} (q^\top, \lambda^\top) \left(\frac{1}{N} \sum_{i=1}^N z_i^\top (x_i, z_{H,i}) \right)^{-1} \left(\frac{1}{N} \sum_{i=1}^N z_i^\top y_{i,(q,\lambda)} \right). \tag{45}$$

Similarly, the estimate of the frontier point in direction q is obtained as follows:

$$(\hat{\beta}_q^\top, 0^\top)^\top = \left(\frac{1}{N} \sum_{i=1}^N z_i^\top (x_i, z_{H,i}) \right)^{-1} \left(\frac{1}{N} \sum_{i=1}^N z_i^\top y_{i,(q,\lambda_N^*(q))} \right), \tag{46}$$

where $\lambda_N^*(q)$ is the solution to (45).

In addition to the estimation challenge, there is also an inference challenge. Marginal cost function (1) includes both discrete and continuous variables. This means that conducting inference on the identified set and its support function is not straightforward due to the presence of kinks and flat faces in the identified set (Molchanov and Molinari (2014) and Bontemps and Magnac (2017)). To address this issue, we employ the simple data-jittering method proposed by Chandrasekhar, Chernozhukov, Molinari, and Schrimpf (2019). We add continuously distributed random variables to the discrete variables to eliminate flat faces on the identified set and construct confidence

[35] Rockafellar (1970) provides results for the support function of intersections of convex sets, which are used by Bontemps, Magnac, and Maurin (2012) to characterize the identified set in the context of supernumerary moment restrictions.

intervals for the cost parameters. In constructing these confidence intervals, we use a bootstrap technique to take into account the estimation error in the demand function estimation, as explained in Footnote 18 in Section 3.

4.4 Test

As previously introduced in (38), we use the support function of $\mathcal{B}^s$ to test hypotheses regarding competition between firms. Although the testing procedure can be applied to virtually any hypothesis represented by $[\underline{mc}_i, \overline{mc}_i]$, our primary interest lies in determining whether the benchmark behaviors are consistent with the observed firm behavior, $\beta_{q^C}, \beta_{q^{JPM}} \in \mathcal{B}^s$.

To test these hypotheses, the test statistic is defined as follows:

$$\frac{\sqrt{N}\hat{T}(q^* \mid \hat{\beta}^h)}{\sqrt{\hat{V}_{q^*}}} \quad (\hat{\beta}^h = \hat{\beta}_{q^C}, \hat{\beta}_{q^{JPM}}), \tag{47}$$

where $\hat{T}(q^* \mid \hat{\beta}^h) = \hat{\delta}^*(q^* \mid \mathcal{B}^s) - q^{*\top}\hat{\beta}^h$ and $\hat{V}_{q^*}$ is an estimate of the variance of $\sqrt{N}\hat{T}(q^* \mid \hat{\beta}^h)$. The statistic is asymptotically normally distributed under the null hypothesis $\beta^h \in \partial\mathcal{B}^s$.

5 Empirical Analysis of the Cartel

In this section, we conduct a retrospective analysis of the cement cartel in the Chugoku region using our approach and present the results. First, we present the estimation results for the identified set of cost parameters and demonstrate how it shrinks as supernumerary IVs are added. Next, we perform inference on the effects of the cartel's impact on prices and consumer welfare.

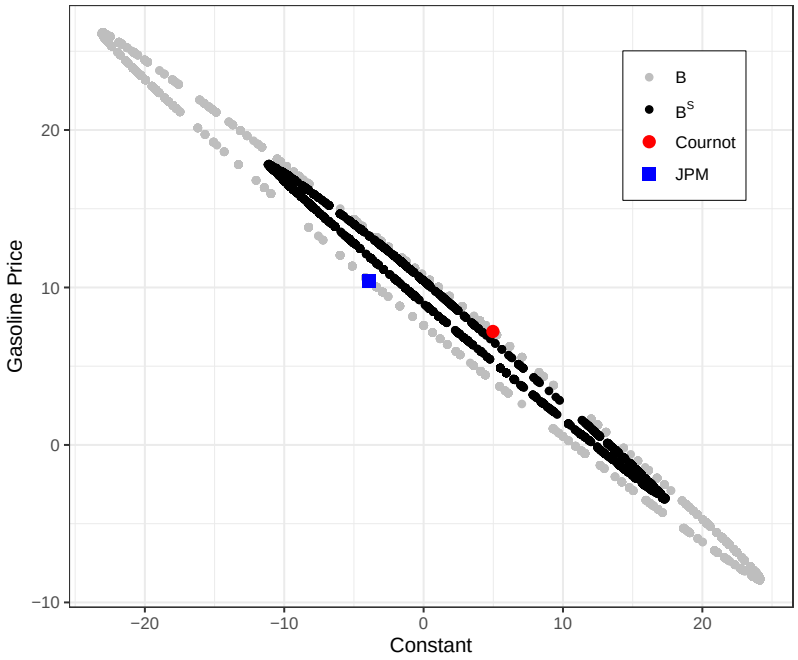
5.1 The Identified Set of Cost Parameters

To illustrate our approach, we first present the results obtained for a marginal cost function with only two parameters, the intercept and the coefficient on the gasoline price. Figure 1 presents the set of cost parameters estimated both without and with the use of supernumerary IVs.[36] The figure shows that the set shrinks when

[36]The variables used as supernumerary IVs are the same as those used in (3) below.

supernumerary IVs are employed. Moreover, importantly, the two frontier points implied by the competitive and collusive benchmarks are not included in the estimate of $\mathcal{B}^s$. This means that the cost parameters associated with these benchmarks do not satisfy the supernumerary moment restrictions.[37]

Figure 1: The Estimated Identified Set



Note: The gray and black points represent the frontier points of the estimated identified sets, where their support functions are evaluated.

Although the simple cost function specification above helps clarify what we do in this paper, it may not be sufficient to fully explain marginal costs. Therefore, in the main specification, we include other important cost shifters to determine whether the same kind of shrinkage occurs in the identified set. We estimate the identified set using (1) no supernumerary IVs (No IV), (2) the amount of construction investment and number of rival firms' SSs (IV1), and (3) these two IVs along with their squared terms (IV2). In the last specification, the use of the squared terms aims to capture the (possibly) non-linear relationship between implied markups and IVs, a point highlighted by Backus, Conlon, and Sinkinson (2021) in the context of firm conduct

[37]To obtain the frontier points of the identified sets in Figure 1 (as well as Figure 2), we first draw a random variable q' 5,000 times from the distribution of $N(0, \frac{1}{N}\sum_{i=1}^{N} x_i^\top x_i)$. Next, we then normalize q' by dividing it by its norm, yielding $q = q'/\|q'\|$, and we calculate β_q at these points.

testing.

Table 7 provides the main results of the identified set estimation, including estimates of the lower and upper bounds on each cost parameter and the 95% confidence interval. The lower and upper bounds on the kth parameter can be represented using the estimated support function of $\mathcal{B}$:

$$[\min_{\beta \in \hat{\mathcal{B}}} \beta_k, \ \max_{\beta \in \hat{\mathcal{B}}} \beta_k] = [-\hat{\delta}(-e_k \mid \mathcal{B}), \hat{\delta}(e_k \mid \mathcal{B})], \tag{48}$$

where e_k is the unit vector whose kth element is 1 and all other elements are 0. To construct the confidence interval for β_k, we use 250 (non-parametric) bootstrap samples, where discrete variables are jittered.

Table 7: Projections of the Estimated Identified Set

	(1) No IV	(2) IV1	(3) IV2
Constant	[−38.663, 38.381]	[−31.479, 33.905]	[−27.199, 29.318]
	(−70.537, 56.745)	(−60.645, 51.258)	(−56.960, 47.290)
Gasoline Price	[−15.170, 35.339]	[−11.963, 30.310]	[−9.139, 27.747]
	(−27.762, 56.023)	(−23.732, 49.005)	(−21.255, 47.008)
No.SS	[−3.255, 0.783]	[−2.921, 0.311]	[−2.781, 0.199]
	(−5.517, 1.214)	(−5.002, 0.579)	(−4.943, 0.517)

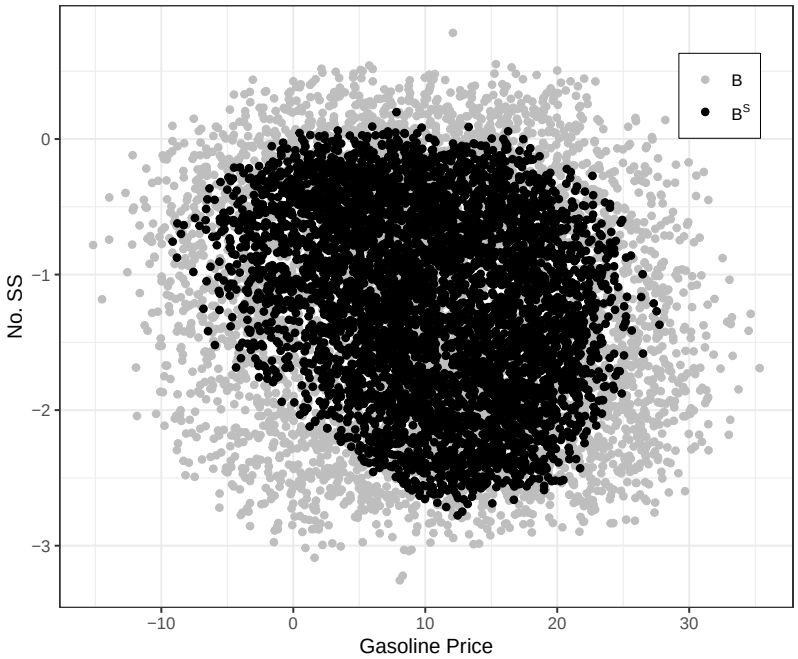
Note: Interval estimates for each cost parameter are presented in brackets, while the 95% bootstrapped confidence intervals are presented in parentheses. Year fixed effects are included but not reported.

The results in Table 7 show that, when we use supernumerary IVs, the identified set indeed shrinks, as measured by the projections for each parameter.[38] First, compared to the set estimated without IVs, all of the intervals and 95% confidence intervals are shorter both when we use IV1 and when we use IV2. Second, when we use IV2, the interval estimates and 95% confidence intervals are shorter than when we use IV1. This result implies that the correlation between markups and supernumerary IVs is not linear and thus adding squared IVs provides additional power to rule out false markup hypotheses.

Next, Figure 2 provides a graphical illustration of the shrinkage of the identified set. In this figure, we project the identified set estimated without using IVs and that estimated using IVs (IV2) onto the space of coefficients for the gasoline price and the

[38] We relegate the estimation results in the case of non-constant marginal costs to the Supplemental Appendix. The results are essentially the same as those presented in Tables 7 and 9.

Figure 2: Projections of the Estimated Identified Set



Note: The gray and black points represent the frontier points of the estimated identified sets $\mathcal{B}$ (No IV) and $\mathcal{B}^s$ (IV2), where their support functions are evaluated.

number of SSs. In Figure 2, we observe, as in Figure 1, but now more strikingly, that the identified set shrinks.

Table 8: Marginal Cost Interval

	(1) No IV	(2) IV1	(3) IV2
Interval Estimate	[9.795, 14.455]	[10.923, 14.096]	[11.270, 13.953]
95% Bootstrapped CI	(6.469, 14.874)	(8.180, 14.565)	(8.576, 14.363)

Note: The interval estimates are estimates of marginal costs evaluated at the mean of the cost variables.

To examine how the marginal cost intervals themselves become shorter with supernumerary IVs, we present interval estimates of marginal costs evaluated at the mean of the cost variables in Table 8. The interval estimate shrinks from [9.795, 14.455] to [10.923, 14.096] (IV1) to [11.270, 13.953] (IV2). This result mirrors the findings presented in Table 7.

One interesting point is that the impact of the IVs is not symmetric for the lower and upper bounds: the increase in the lower bound is larger than the decrease in

the upper bound. This suggests that smaller values in the interval $[\underline{mc}_i, \overline{mc}_i]$ tend to be incompatible with the data, making the cost parameters associated with these values not admissible. Conversely, higher values within $[\underline{mc}_i, \overline{mc}_i]$ are more likely to be admissible and the cost parameters associated with these higher values are more likely to belong to $\hat{\mathcal{B}}^s$. That is, the cost parameters associated with implied marginal costs closer to the competitive benchmark $\overline{mc}_i$ are more likely to be contained in $\hat{\mathcal{B}}^s$.

It is worth noting that this shrinkage pattern is in line with the results in Section 3. In the RV test, the Cournot hypothesis is favored, meaning that the true behavior is closer to Cournot behavior, while, in the AR test, the lack-of-fit measure is smaller under the Cournot competition hypothesis. These findings imply that implied marginal costs closer to $\overline{mc}_i$ are closer to the true marginal costs than those closer to $\underline{mc}_i$. Therefore, higher values in $[\underline{mc}_i, \overline{mc}_i]$ tend to be admissible, while smaller values in $[\underline{mc}_i, \overline{mc}_i]$ are more likely to be inadmissible. Consequently, a larger portion of the shrinkage occurs on the side of the lower bound.

5.2 Screening for Collusion

Next, we conduct statistical tests using the estimated set of cost parameters, $\hat{\mathcal{B}}^s$. Although tests can be conducted on any parameter value, it is particularly important to assess whether the cost parameters implied by the competitive and collusive benchmarks belong to $\mathcal{B}^s$.

Table 9: Testing Firm Conduct Hypotheses

	(1) No IV	(2) IV1	(3) IV2
Cournot: $\beta_{q^C} \in \mathcal{B}^s$	0	−7.934	−7.950
JPM: $\beta_{q^{JPM}} \in \mathcal{B}^s$	0	−3.095	−3.592

The results of the firm conduct tests based on the test statistic (47) are provided in Table 9. The test statistic follows the standard normal distribution at the null hypothesis $\beta^h \in \partial\mathcal{B}^s$ ($\beta^h = \beta_{q^C}, \beta_{q^{JPM}}$). A negative sign on the test statistic indicates that a tested point is outside the identified set. The findings are clear: the cost parameters implied by the competitive and collusive benchmarks are not contained in $\mathcal{B}^s$. Neither the Cournot competition nor the joint-profit maximization hypothesis provides an appropriate description of the true firm conduct. The test results

imply that the true firm conduct is more competitive than perfect collusion and less competitive than Cournot competition.

5.3 Cartel Price Overcharge and Consumer Welfare Loss

Since the cost parameters associated with the competitive benchmark are rejected, the next task is to quantify the extent of market distortions and welfare losses caused by the non-competitive firm behavior.

To this end, we consider a counterfactual scenario where firms adopt the competitive benchmark behavior. Under this scenario, we derive the lower and upper bounds on cement prices and the consumer surplus (CS) and estimate the cartel price overcharge and CS loss. Table 10 presents the results of this counterfactual exercise. The "Price" row shows the lower and upper bounds on the mean cement price obtained using different sets of cost parameters, while the "Consumer Surplus" row presents the lower and upper bounds on the mean consumer surplus. The lengths of these intervals stem from those of the marginal cost intervals in Table 8.

Table 10: Cartel Price Overcharge and Consumer Welfare Loss

	(1) No IV	(2) IV1	(3) IV2
Price	[10.978, 15.121]	[11.981, 14.802]	[12.289, 14.675]
	(8.395, 15.186)	(9.915, 14.892)	(10.266, 14.802)
CS	[235.218, 743.403]	[264.226, 594.223]	[276.288, 551.778]
	(80.182, 1230.322)	(104.734, 998.941)	(122.998, 946.767)
Cartel Effect			
Price Overcharge	[0.000, 4.143]	[0.319, 3.139]	[0.446, 2.831]
	(0.000, 6.730)	(0.252, 5.210)	(0.344, 4.858)
CS Loss	[0.000, 508.186]	[29.008, 359.005]	[41.071, 316.560]
	(0.000, 840.155)	(22.560, 608.913)	(31.278, 557.007)

Note: Interval estimates of the mean price and the consumer surplus (CS) evaluated at the means of the demand and cost variables are presented in brackets, while the 95% bootstrapped confidence intervals are presented in parentheses. The unit is 1,000 JPY.

In the first column, we present the upper and lower bounds on the cement price and CS using the set estimated without IVs. In this case, the upper bound on the marginal cost, which is equivalent to the mean of $\overline{mc}_i$, is compatible with the data (because it cannot be rejected), which implies that the Cournot hypothesis is compatible with the data. This means that the cement price and CS obtained using the mean of $\overline{mc}_i$

under the Cournot assumption correspond to the means of the actual cement price and CS. Therefore, in order to quantify the cartel price overcharge and consumer welfare loss, we evaluate the counterfactual prices and CSs that are obtained under the Cournot assumption using $\hat{\mathcal{B}}^s$ relative to those obtained using $\hat{\mathcal{B}}$.

In Table 10, "Price Overcharge" refers to the price difference relative to the upper bound on the price in (1), 15.121, while "CS Loss" refers to the CS difference relative to the lower bound on CS in (1), 235.218. We focus on the third column, which shows the tightest intervals. In this case, the interval estimate for the cartel price overcharge (per month) is [0.446, 2.831] and the 95% confidence interval is (0.344, 48.858), where the unit is 1,000 JPY. This interval estimate of the price overcharge translates to a price increase due to the existence of the cartel of around 3% to 18%.[39] The estimated CS loss is [41.071, 316.560], with a 95% confidence interval of (31.278, 557.007). This interval estimate means that around 15% to 57% of the consumer surplus was lost during the cartel period. Even the lower bound indicates a sizable consumer surplus loss.

Lastly, although our analysis focuses on a detected cartel, we re-emphasize that our approach can be used to analyze markets where firms are suspected of colluding. By screening observed firm behavior and inferring the effects of non-competitive firm behavior, our approach provides practically useful metrics. For instance, an estimate of the lower bound on the consumer surplus loss indicates the minimum consumer loss that could be eliminated through detection.

6 Relation to Conduct Parameter Methods

Before concluding this paper, we clarify the relationship between traditional methods for measuring firm conduct and our approach. Our approach is closely related to the conduct parameter methods (CPM) and internalization parameter approach (see, e.g., Porter (1983), Ellison (1994), Ciliberto and Williams (2014), and Miller and Weinberg (2017)). Here, we highlight the differences between these methods and ours. In conduct and internalization parameter methods, researchers impose strong restrictions on parameters representing the form of competition between firms to achieve

[39] Estimates of the cartel's price overcharges using information on the workings of the cartel are provided in Nishiwaki (2024). The estimates range from about 0.8 in year 1989 to 3.5 in year 1987, where the unit is 1,000 JPY.

point identification. For instance, Porter (1983) expresses the conduct parameter as a time-invariant parameter, while Ciliberto and Williams (2014) express conduct parameters using a function of observed variables including the degree of multi-market contact between firms. However, these methods assume that other observed and unobserved variables do not affect firm conduct and impose a specific functional form. Meanwhile, Miller and Weinberg (2017) allow the price internalization parameters to be time-varying, but only in a limited manner (i.e., parameters vary only across years, not within the same year).

Our approach can be viewed as an approach introducing a conduct or internalization parameter in a very flexible manner. For instance, following the formulation of Rosen (2006), the conduct parameter is introduced as follows:

$$P - mc_i = -(1+\lambda)\frac{\partial P(Q)}{\partial Q}q_i, \quad \lambda \in [\underline{\lambda}_i, \overline{\lambda}_i], \tag{49}$$

where $\underline{\lambda}_i$ denotes the most competitive firm conduct and $\overline{\lambda}_i$ denotes the least competitive conduct. λ is an unknown function of all observed and unobserved variables in the market, $\lambda(w, v, \epsilon)$. That is, we do not impose any unnecessary or strong restrictions on λ in our approach, although the function is bounded by $\underline{\lambda}_i$ and $\overline{\lambda}_i$. On the other hand, an obvious disadvantage of this flexible specification is that we lose point identification of both marginal cost and firm conduct parameters.[40]

Unlike the conduct parameter methods, our approach requires setting the competitive and collusive benchmarks in bounding firms' marginal costs. Pre-defining the least and most competitive behaviors may seem like a restrictive assumption, one that is absent in conduct parameter methods. However, setting these benchmarks is similar to setting a baseline model in the conduct parameter methods, where it is necessary to set a benchmark equilibrium behavior, such as Cournot, Betrand, or other equilibrium behavior, so that conduct parameters at best are identified as deviations from this benchmark behavior. In other words, conduct parameters are identified relative to a benchmark equilibrium concept.[41] This type of normalization is required to address Corts' critique (Corts (1999)), as highlighted by Ciliberto and

[40]This trade-off is akin to that between point and partial identification of the parameters of the payoff functions in games with discrete actions. It is well known that the presence of multiple equilibria in these games, in general, leads to partial identification of the model parameters. To achieve point identification, strong assumptions are needed.

[41]This identification issue is pointed out and examined in Miller and Weinberg (2017), p.1784.

Williams (2014).

Our identified set depends on the competitive and collusive benchmarks that we specify. With supernumerary IVs, the identified set $\mathcal{B}$ is reduced to $\mathcal{B}^s$. The distance between $\mathcal{B}$ and $\mathcal{B}^s$ in direction q^C (or q^M) measures the difference between the benchmark competitive (or collusive) behavior and the most (or least) competitive behavior not falsified by the supernumerary IVs. By examining the distance between $\mathcal{B}$ and $\mathcal{B}^s$ in direction q^C, we determine whether the benchmark is compatible with the observed data. This way of testing firm conduct conditional on the baseline competitive behavior is quite similar to identifying conduct parameters relative to the baseline, typically competitive, equilibrium behavior in the conduct parameter methods.

7 Conclusion

In this paper, we proposed an empirical approach for analyzing markets where firms are (suspected of) colluding. Our approach involves testing firm conduct and estimating welfare losses when collusion is present. One of the key benefits of our approach is that it does not require complex models of collusion to explain observed firm behavior. Instead, it relies on two simple models of firm conduct, both of which are commonly used in current empirical practice for testing firm conduct.

Finally, some shortcomings and potential areas for future research should be noted. First, admittedly, our approach provides only a partial resolution to the challenges in existing testing practices. The reason is that we forgo developing models that describe the observed behavior of (colluding) firms. An ideal direction for future research might involve considering more realistic candidate models, testing them, and finding the best approximation to reality. Second, taking possible misspecifications into account is important. Misspecifications can arise in both demand and cost functions and affect the construction of the identified set of cost parameters. We leave these issues for future research.

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Supplemental Appendix for "Analyzing Collusion Using Set-Identified Marginal Cost Functions"

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A Firm Conduct Tests under Non-Constant Marginal Costs

We examine whether allowing for scale economies affects the firm conduct test results obtained in the main text presented in Table 6.[1] Unlike the case of constant marginal costs, estimating a non-constant marginal cost function and testing firm conduct require multiple sources of variation that are economically distinct, as pointed out by Duarte, Magnolfi, Quint, Sølvsten, and Sullivan (2025). In the main text, we used multiple (excluded) instrumental variables for testing firm conduct–the amount of construction investment and the number of rival firms' service stations (SS). In this appendix, we continue to use these variables to estimate the marginal cost function in (1) and test firm conduct, because they can be considered economically distinct IVs.[2] The amount of construction investment rotates the demand curve (as indicated in Table 4 in the main text), while the number of rival firms' SSs shifts rival firms' marginal costs and thus influences the residual demand curve faced by a cement firm. Therefore, these two sources of variation can be considered economically distinct so

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[1]Duarte, Magnolfi, Quint, Sølvsten, and Sullivan (2025) examine how misspecifying marginal costs affects testing outcomes. When the true specification is a non-constant marginal cost function, ignoring this leads to incorrect test results: false hypotheses may not be rejected, while the correct hypothesis may be incorrectly rejected.

[2]We use the demand function defined in (3) in the main text throughout this supplemental appendix. To estimate the demand function, we use the (prefecture-level) price of gasoline, along with its squared and cubed terms, as IVs, as explained in the main text.

that they enable us to identify the marginal cost function and distinguish different models of firm conduct under non-constant marginal costs.

Table A1: RV and AR Tests

	RV Test		
	(1)	(2)	(3)
Hypothesis	Cournot vs. JPM	Cournot vs. JPM	Cournot vs. JPM
Test Statistic	−2.799	−2.258	−2.581
Construction Inv.	Yes	No	Yes
Rival No. SS	No	Yes	Yes

	AR Test					
	(1)	(2)	(3)	(4)	(5)	(6)
Hypothesis	Cournot	JPM	Cournot	JPM	Cournot	JPM
Construction Inv.	−0.0020	0.0081			−0.0004	0.0056
	(0.0005)	(0.0012)			(0.0005)	(0.0010)
Rival No. SS			−0.1351	0.3416	−0.1254	0.2022
			(0.0191)	(0.0581)	(0.0184)	(0.0511)

Note: The panel labeled "RV Test" presents the RV test statistic. The panel labeled "AR Test" presents the estimated values of π ($h \in \{h_C, h_{JPM}\}$), with the standard errors in parenthesis. Construction Inv. is the amount of construction investment, while Rival No. SS is the sum of other firms' No. SS.

The results of the firm conduct tests are presented in Table A1, while the estimates of the marginal cost parameters under the hypothesis of Cournot competition and that of (unconstrained) joint-profit maximization are presented in Table A2.[3] The results in Table A1 are qualitatively similar to those in Table 6 in the main text. The RV tests do not detect the presence of the cartel, as in the main text. The Cournot competition hypothesis is favored over the joint-profit maximization hypothesis. In addition, the Cournot hypothesis is selected at the 5% significance level.[4] The AR tests reject both hypotheses, as in the case of constant marginal costs. Therefore, allowing for scale economies does not change the conclusions obtained in the main text.

[3]To estimate the variances, we need to take the effect of demand uncertainty into account. To do so, we use bootstrapping and estimate these as the variances of 250 bootstrap samples.

[4]The estimation error in the demand function estimation substantially affects the statistical significance. For instance, when this estimation error is ignored, the Cournot competition hypothesis is selected at (at least) the 1% significance level in all specifications.

Table A2: Cost Function Estimation

	(1) Cournot	(2) JPM
Constant	14.777	14.268
	(0.476)	(0.731)
Gasoline Price	1.327	1.141
	(0.302)	(0.469)
No. SS	0.555	0.480
	(0.037)	(0.047)
Quantity	−0.156	−0.496
	(0.023)	(0.176)
Year 1986	−0.181	−0.278
	(0.063)	(0.109)
Year 1987	−0.566	−0.614
	(0.075)	(0.1303)
Year 1988	−1.335	−1.348
	(0.1065)	(0.157)
Year 1989	−2.297	−2.128
	(0.100)	(0.175)
Year 1990	−2.722	−2.427
	(0.144)	(0.255)
No. Obs.	2280	

Note: The column labeled "Cournot" presents the estimation results under the assumption of Cournot competition while the column labeled "JPM" presents the results under the assumption of (unconstrained) joint-profit maximization.

B Identified Set of Cost Parameters under Non-Constant Marginal Costs

We estimate (the projections of) the identified set of cost parameters under the assumption of non-constant marginal costs for a sensitivity analysis. As explained in the main text, we use the following two sets of moment conditions to construct the identified set:

$$E\left[z_{F,i}^{\top}(y - x_i\beta)\right] = 0, \tag{A1}$$

$$E\left[z_{H,i}^{\top}(y - x_i\beta)\right] = E\left[z_{H,i}^{\top}y\right] = 0, \tag{A2}$$

where $y \in [\underline{mc}_i, \overline{mc}_i]$. x_i includes w_i and q_i, $x_i = (w_i, q_i)$. $z_{F,i}$, whose dimension is K, is defined as the orthonormalization of $x(z_i)$, which is the linear projection of x_i onto

z_i, $x(z_i) = z_i E(z_i^\top z_i)^{-1} E(z_i^\top x_i)$:

$$z_{F,i} = x(z_i) E\left(x(z_i)^\top x(z_i)\right)^{-\frac{1}{2}}, \tag{A3}$$

where z_i is a full set of exogenous variables, $z_i = (w_i, z_i^{K+0}, z_i^{K+1}, \cdots, z_i^M)$. $z_{H,i}$, with a dimension of $M-K$, is the orthonormalization of ζ_i^s, which is the residual from the linear projection of $x(z_i)$ onto z_i^s:

$$z_{H,i} = \zeta_i^s E(\zeta_i^{s\top} \zeta_i^s)^{-\frac{1}{2}}, \quad \zeta_i^s = z_i^s - x(z_i) E\left(x(z_i)^\top x(z_i)\right)^{-1} E\left(x(z_i)^\top z_i^s\right), \tag{A4}$$

where z_i^s is a set of supernumeray IVs, $z_i^s = (z_i^{K+1}, \cdots, z_i^M)$.

We use the amount of construction investment, the number of rival firms' SSs, and their squared terms as IVs, as in the main text. In this case, there are four IVs, while there is one endogenous variable, q_i. Therefore, we have three surplus IVs, that is $M - K = 3$. This means that we choose three IVs among the four IVs as supernumerary IVs, z_i^s.[5] One potential concern is that the identified set depends on the choice of z_i^s – different choices of supernumerary IVs may lead to different identified sets. Fortunately, however, as proved in Appendices B.3 and B.5 in Bontemps, Magnac, and Maurin (2012), the choice of z_i^s has no influence on the identified set. That is, regardless of which three variables among the four IVs are chosen as z_i^s, the identified set is unchanged. Therefore, with a set of three arbitrarily chosen supernumerary IVs, we estimate the identified set following the estimation procedures explained in Section 4.3 of the main text.

Before explaining the estimation results, it is worth noting how $z_{H,i}$ is constructed. As is clear from the way that it is constructed in (A4), $z_{H,i}$ is obtained after z_i^s is residualized with respect to $x(z_i)$. Therefore, it is necessary for $z_{H,i}$, not z_i^s, to be relevant to firms' markups to eliminate false hypotheses about firm conduct and reduce the size of the identified set. An important fact here is that $x(z_i)$ includes $\hat{q}_i$, which is a function of all exogenous variables, including z_i^s.[6] This means that the part of the variation in z_i^s that is already used to explain the endogenous variable q_i is not used to construct $z_{H,i}$.

[5] In the case of constant marginal costs, we use all these IVs as z_i^s to construct the identified set, because the marginal cost function has no endogenous variable and thus all these IVs are supernumerary by definition.

[6] As explained, $x(z_i)$ is the linear projection of x_i onto z_i. In the case of non-constant marginal costs, x_i includes q_i and thus $x(z_i)$ includes the projection of q_i onto z_i, $\hat{q}_i = z_i E(z_i^\top z_i)^{-1} E(z_i^\top q_i)$.

Table A3: Projections of the Estimated Identified Set

	(1) IV0	(2) IV1	(3) IV2
Constant	[−30.047, 59.092]	[−29.194, 58.845]	[−28.108, 58.568]
	(−59.882, 89.191)	(−69.805, 99.805)	(−61.571, 92.505)
Gasoline Price	[−27.220, 29.688]	[−27.064, 29.278]	[−26.857, 28.754]
	(−46.464, 48.818)	(−53.346, 55.456)	(−48.609, 50.351)
No. SS	[−2.853, 4.029]	[−2.785, 3.989]	[−2.783, 3.821]
	(−5.090, 6.337)	(−5.848, 7.120)	(−5.373, 6.258)
Quantity	[−0.823, 0.171]	[−0.822, 0.161]	[−0.821, 0.138]
	(−1.325, 0.372)	(−1.423, 0.494)	(−1.359, 0.382)
Year 1986	[−7.442, 6.984]	[−7.405, 6.919]	[−7.352, 6.830]
	(−12.155, 11.651)	(−13.868, 13.546)	(−12.738, 12.095)
Year 1987	[−10.062, 8.882]	[−10.043, 8.742]	[−9.949, 8.675]
	(−16.107, 14.846)	(−18.356, 17.318)	(−16.890, 15.520)
Year 1988	[−12.391, 9.708]	[−12.300, 9.640]	[−12.259, 9.272]
	(−19.501, 16.745)	(−22.212, 19.596)	(−20.397, 17.192)
Year 1989	[−12.282, 7.858]	[−12.230, 7.727]	[−12.157, 7.535]
	(−18.731, 14.206)	(−21.236, 16.592)	(−19.619, 14.627)
Year 1990	[−13.366, 8.217]	[−13.324, 8.075]	[−13.253, 7.903]
	(−20.497, 15.286)	(−-23.048, 17.925)	(−21.332, 15.870)

Note: Interval estimates of each cost parameter are presented in brackets, while the 95% bootstrapped confidence intervals are presented in parentheses.

The projections of the estimated identified set are presented in Table A3. The column labeled "IV0" shows the result obtained using only the first set of moment conditions (A1). The identified set obtained from this condition is equivalent to that obtained using the following incomplete moment conditions:

$$E\left[x(z_i)^\top (y - x_i\beta)\right] = 0, \quad y \in [\underline{mc}_i, \overline{mc}_i]\,. \tag{A5}$$

On the other hand, the column labeled "IV2" presents the result obtained using both (24) and (25). In this case, $z_{H,i}$ consists of a full set of supernumerary IVs. The column labeled "IV1" presents the result obtained using only a subset of supernumerary IVs. Specifically, to construct $z_{H,i}$, we use only the amount of construction investment and the number of rival firms' SSs as z_i^s. The reason for excluding the squared terms is that, by comparing IV1 with IV2, we highlight the importance of capturing the non-linear relationship between markups and IVs.

The results in Table A3 are essentially the same as those in Table 7 in the main text. First, when we use supernumerary IVs, the identified set shrinks. Second, the more supernumerary IVs we use, the shorter the interval estimates become.

Lastly, in Table A4 we present the results of testing $\beta_{q^C} \in \mathcal{B}^s$ and $\beta_{q^{JPM}} \in \mathcal{B}^s$. The test statistic, defined in (47) in the main text, follows the standard normal distribution at the null hypothesis $\beta^h \in \partial\mathcal{B}^s$ ($\beta^h = \beta_{q^C}, \beta_{q^{JPM}}$). A negative sign on the test statistic indicates that a tested point is outside the identified set. Again, these results are essentially the same as those presented in Table 9 in the main text: neither β_{q^C} nor $\beta_{q^{JPM}}$ is contained in $\mathcal{B}^s$.

Table A4: Testing Firm Conduct Hypotheses

	(1) IV0	(2) IV1	(3) IV2
Cournot: $\beta_{q^C} \in \mathcal{B}^s$	0	−5.421	−7.174
JPM: $\beta_{q^{JPM}} \in \mathcal{B}^s$	0	−4.889	−5.714

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