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Yukihiro Nishimura

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Graduate School of Economics
The University of Osaka, Toyonaka, Osaka 560-0043, JAPAN

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Yukihiro Nishimura*

Abstract

This paper develops a Kantian equilibrium framework, subsuming the global pollution model with private ownership, wherein agents condition their contributions on a universalizable moral imperative reflecting income and preference heterogeneity. After showing a specific proportionality assumption linking Kantian reasoning to other agents' behavior that must make the Kantian equilibrium coincide with the Lindahl equilibrium, we show that the level of the public good may or may not increase with income inequality. Inequality invariance is observed in some solution. Applying this model to a global pollution context, we demonstrate that the Lindahl allocation may fail to Pareto dominate the voluntary contribution (disagreement) equilibrium. We compare the Lindahl outcome with other proposed solutions to global public good provision, focusing in particular on the role of international income transfers and their ability to achieve Pareto improvements over the voluntary contribution (disagreement) equilibrium. Our analysis contributes to a reinterpretation of morally grounded mechanisms for global public good provision, bridging normative ethics with economic design.

Keywords: Global externalities, Kantian equilibrium, Income inequality, International emissions trading

JEL classification number: H41, D63, Q54

*Osaka University and CESifo; E-mail: ynishimu@econ.osaka-u.ac.jp

1 Introduction

The provision of global public goods — particularly climate stability — poses persistent challenges at the intersection of equity, efficiency, and international cooperation. Classical approaches such as Lindahl (1919) pricing, voluntary contribution mechanisms (Warr (1982), Warr (1983), Bergstrom et al. (1986), “disagreement equilibrium" in the pollution game), and Pigouvian compensation have each provided partial solutions. Yet the deeper ethical and strategic dimensions of global public good provision remain under-explored, particularly in light of international income inequality and heterogeneity in preferences.

Uzawa (2003) introduced the Lindahl solution in interpreting the allocation of pollution permits where its Lindahlian determination results in unanimous agreement. Nishimura (2008) reformulated this outcome as an equilibrium of a game of international negotiations on the level of global pollution. Nishimura (2008) also clarifies that Uzawa’s determination corresponds to the Pigouvian compensation as well as a matching grant familiar in the public economics literature.

This paper contributes by embedding the Kantian equilibrium framework by Bordignon (1990), Roemer (2010) and Roemer and Silvestre (2023), where universalizable moral imperative is introduced into strategic behavior by positing that agents choose their contributions. As such, our framework generalizes standard public good models by nesting the Lindahl equilibrium within a Kantian motive. We further evaluate our formulation in the Global Public Good Purchasing (GPGP) framework proposed by Bradford (2007) and Guesnerie (2007), and we link the features of international transfers to international environmental agreements to be ethically valid and politically feasible.

As a natural extension of the Warlasian equilibrium in the private-good economy, we design the global public-good mechanisms based on following candidate properties for the resource allocation:

- Benefit principle: compensation against bilateral externalities (Nishimura’s (2008) interpretation of Uzawa (2003); Chander and Tulkens’ (1997) ratio equilibrium)
- Initial endowment (entitlement) on your country’s production as the initial income (Uzawa (2003)) or the disagreement level of individual production (Chander and Tulkens (1997)).

Adding “no inter-country transfer" to the second principle would either imply the disagreement equilibrium itself or the no-transfer-efficient equilibrium (Shiell (2003) for the latter).

We show that Kantian implementation of Lindahl equilibrium requires agents to adjust their moral expectations based on both income differences and preference heterogeneity — the latter is inequality sensitive even when agents have identical utility functions. This links ethical reasoning to economic structure and establishes a micro-foundation for cooperative behavior absent centralized enforcement.

However, the Kantian imperative is, in Edgeworth's (1897) terminology, according to post-contribution outcome. To reconcile this with Lindahl pricing, the clean air benefit must be treated as a virtual income, and no-transfer solutions like that in Shiell (2003) require a two-part tariff structure (Proposition 1). This highlights that initial endowments critically affect the Kantian logic of proportionality.

Moreover, through the equivalence with Kantian is observed, we show counterintuitive implications: due to the strategic structure of proportional contributions, the provision of public good increases with income inequality — a result that does not have an immediate intuition from Kant (Proposition 2). However, the result is ambiguous for the ratio equiumrium, and in the no-transfer allocation, Warr's (1982, 1983) type of neutrality holds (the last two properties are independent of Buchholz and Rübbelke (2023)).

Additionally we demonstrate several key findings. First, contrary to Uzawa (1999), his Lindahl does not Pareto dominate the disagreement equilibrium, echoing results in Nishimura (2008). Second, we confirm that even Pareto-efficient allocations without monetary transfers may leave some countries worse off compared to disagreement, suggesting fundamental limits to decentralized solutions based on tradable permits (Propositions 3 and 4).

2 Model

Consider a simple economy consisting of two groups of countries, —South (group 1) and North (group 2)— where the North is both wealthier and places a higher valuation on environmental quality as a global public good. Intrinsically, a representative citizen in each group of the countries has identical utility functions, but the preference towards the global public good is contingent on the circumstance — which is shaped by income level. The total number of the countries is $n > 1$ with n_1 northern countries and $n_2 = n - n_1$ southern countries.

Suppose that, with respect to private good x_i and public good G, the utility function of a citizen in country i is $u_i = u(x_i, G)$ which is increasing in both arguments and quasi-concave. The countries involve in production defined as a concave production function $f_i(y_i)$, $f_i'(y_i) \geq 0$ and $f_i''(y_i) \leq 0$, where air pollution y_i as an input. The initial level of the public good is $\widetilde{G}$, say, the level of clean air before industrialization, and $G = \widetilde{G} - n_1 y_1 - n_2 y_2$. The resource constraint is:

$$n_1 x_1 + n_2 x_2 = n_1 f_1(y_1) + n_2 f_2(y_2). \tag{1}$$

The Samuelson condition is:

$$n_1 \frac{\partial u_1/\partial G}{\partial u_1/\partial x_1} + n_2 \frac{\partial u_2/\partial G}{\partial u_2/\partial x_2} = f_1'(y_1^*) = f_2'(y_2^*), \tag{2}$$

where we denote the notation y_i^* when the marginal product of the pollution becomes equalized at the allocation under consideration.

In contrast, the disagreement equilibrium is determined by a Nash equilibrium where each agent maximizes $u_i(f_i(y_i), \widetilde{G} - Y_{-i} - y_i)$ given Y_{-i} and $Y = Y_{-1} + y_i$. Let the corresponding Nash equilibrium level of pollution be $\hat{y}_i$ by each agent in Northern or Southern country.

3 Global public good

We first introduce Lindahl equilibrium by Uzawa (2003): $x_i = f_i(y_i^*) + p(\theta_i Y - y_i^*)$, where p is the price of the right to pollute, and θ_i is a variable such that $n_1\theta_1 + n_2\theta_2 = 1$ which is an ownership of the polluting right $Y = n_1 y_1 + n_2 y_2$. Optimization in production results in $f_i'(y_i^*) = p$.

Definition 1. Uzawa's *Lindahl equilibrium with tradable permits*: (θ_1, θ_2) constitutes a Lindahl equilibrium if the representative citizens maximize utility subject to $x_i = f_i(y_i^*) + p(\theta_i Y - y_i^*)$ that generates the utility-maximizing level of the pollution $Y_1^L = Y_2^L$.

Uzawa gave the meaning of the share of the permits θ_i as the willingness to pay for the pollution reduction. The Lindahlian optimization, $\frac{\partial u_i/\partial G}{\partial u_i/\partial x_i} = \theta_i p$, combined with $n_1\theta_1 + n_2\theta_2 = 1$, generates the unanimity on the level of desired pollution constitutes the Lindahl equilibrium which satisfies the Samuelson condition. Uzawa's introduction of this definition is rather mechanical, but Nishimura (2008) gave a meaning as a bilateral price that each pollution firm should pay at the margin to agent i as a victim of multilateral pollution.[1]

As such, this Lindahl allocation generates the transfer $p(\theta_i Y^L - y_i^*)$ by permit trading. In the global climate model, Bradford (2007) and Guesnerie (2007) investigate mechanisms for the

[1]For the case of $n_1 = 1 = n_2$ where $x_i = f_i(y_i^*) + p(\zeta_i s_i + \zeta_j s_i - y_i^*)$, and $\zeta_1 + \zeta_2 = 1$ and $s_1 + s_2 = y_1^* + y_2^*$. This is the public-bad version of the matching contribution system by Guttman (1978) and Danziger and Schnytzer (1991) where citizens announce the tolerable pollution and the other citizen compensates. For $n > 2$, Nishimura (2008) also showed that the application of Walker (1981) mechanism works where, renumbering agents with $k = 1, 2, ..., n$, $x_i = f_i(y_i^*) + p(\zeta_i \sum_{i=1}^n s_i - y_i^*)$ $\zeta_i = (1/n) + s_{i+1} - s_{i+2}$ with the conventions of $n+1 \to 1$ and $n+2 \to 2$.

provision and financing of global public goods (GPGs), such as climate stability. Their proposal on Global Public Good Purchasing (GPGP) lays foundational insights that closely relate to our model and evaluates the Lindahl solution. Their discussion surrounds the following two points:

1. Redistribution for Participation and Equity:

   Their central theme is the need for redistribution to render participation in international environmental agreements politically and economically acceptable for all countries. Refinement of this argument by Murty (2007) is particularly relevant. She distinguishes two components: Pigouvian pricing (to internalize externalities) and equity (income redistribution).

2. Voluntary Cooperation Under Political Constraints:

   The works by Buchholz and Rübbelke (2023) and others are relevant which point out that voluntary contribution equilibrium (referred to as the disagreement equilibrium in the global climate model) may provide more public good than in the Lindahl=Kant equilibrium which increases the South's free-riding benefit. Income rectification matters for the inclusion of the South out of the disagreement equilibrium. It may be worthwhile to mention that, since we incorporated the global-pollution model in the nested structure of the current class of the models, the conclusion by Buchholz and Rübbelke (2023) is applicable to Uzawa's (2003) Lindahl mechanism which does *not*, contrary to Uzawa (1999), Pareto dominate the disagreement equilibrium (Proposition 4 below).

Independently, Chander and Tulkens (1997) and Eyckmans (1997) consider core allocations immune to coalitional deviations. Chander and Tulkens' (1997) core subsumes our focus of Pareto domination of the disagreement equilibrium (as a minimum requirement for the proposed allocation). Also, Shiell (2003) paid attention to an efficient allocation under the constraint that no net monetary transfers occur between countries. Their proposals relate to the two points of GPGP mentioned above, which we elaborate in the following.

3.1 Decentralized ownership and Pigouvian cost share

An allocation proposed by Chander and Tulkens (1997) and Eyckmans (1997) is as follows. The disagreement equilibrium is determined by a Nash equilibrium where each agent maximizes $u_i(f_i(y_i), \widetilde{G} - Y_{-i} - y_i)$ given Y_{-i} and $Y = Y_{-1} + y_i$. Let the corresponding Nash equilibrium level of pollution be $\hat{y}_i$ by each agent in Northern or Southern country. Each country adjusts y_i to y_i^*, and

x_i is determined with $MRS_i \equiv \frac{\partial u_i/\partial G}{\partial u_i/\partial x_i}$ and $r_i \equiv \frac{MRS_i}{n_1 MRS_1 + n_2 MRS_2}$ as follows:

$$x_i = f_i(\hat{y}_i) - r_j \sum_{j=1,2} n_j(f_j(\hat{y}_j) - f_j(y_j^*)) \tag{3}$$

The allocation of y_i^* and $G^* = \widetilde{G} - n_1 y_1^* - n_2 y_2^*$ where each agent maximizes the utility in (3) resembles the classic ratio equilibrium by Kaneko (1977). Also, such allocation satisfies the following *production efficiency*: for $Y = n_1 y_1 + n_2 y_2$,

$$(y_1^*, y_2^*) \text{ minimizes } -\sum_{i=1,2} n_i(f_i(\hat{y}_i) - f_i(y_i)) \ s.t. \sum_{i=1,2} n_i y_i = Y \tag{4}$$

so Eyckmans (1997) defines the allocation by, denoting $y_i(Y)$ as the production-efficient y_i given Y, $-\sum_{i=1,2} n_i(f_i(\hat{y}_i) - f_i(y_i(Y))) \equiv C(Y)$. Hereafter, we call this allocation as the equilibrium *with public ownership of firms*, in contrast to Uzawa (2003) to the equilibrium *with private ownership*. This distinction is an issue in Roemer and Silvestre (2023). Indeed, plugging the minimized cost in (4) into (3) with $C'(Y) = 1$ through the envelope theorem, the ratio-type utility maximization under (3) given r_i yields $MRS_i = r_i$ for all i so the Samuelson condition is satisfied. Also, in the terminology of Mas-Colell and Silvestre (1989), this public-ownership ratio equilibrium is a special form of a cost share equilibrium: consider a class of allocations where agents maximize utilities from the budget constraint of

$$x_i + p_i(\hat{Y} - Y) = f_i(\widetilde{y}_i) + \eta_i^1(p(\hat{Y} - Y) - C(Y)) - \eta_i^2, \quad \widetilde{y}_i = \hat{y}_i \text{ or } \widetilde{y}_i = y_i^* \tag{5}$$

to select x_i and preferred Y consistent with unanimity, where η_i^1 is the Arrow-Debreu shareholding of the centralized firm and $p = n_1 p_1 + n_2 p_2$. If the profit share is $\eta_i^1 = r_i$, then this cost-share equilibrium yields the equilibrium individual price $p_i = r_i p$ and the profit-maximizing production $p = C'(Y)$ and each private firm in country j pays $r_i f_j(y_j^*)$ to agent i (Pigouvian compensation).

In the two-type economy, this allocation satisfies both Pareto efficiency and Pareto domination of the disagreement equilibrium. Nishimura (2008), however, showed that $\hat{y}_i - y_i^*$ may be negative for some country, even though the pollution is reduced in Pareto efficient allocation.

It is also clear that, once we try to correspond to the economy with private ownership with, for example, tradable permit, by $f_i(\hat{y}_i) - r_i \sum_i n_i(f_i(\hat{y}_i) - f_i(y_i^*)) = f_i(y_i^*) + p(\theta_i^{CT} Y - y_i^*)$, then no natural restriction allows θ_i^{CT} to be a normal range of $0 < \theta_1^{CT} < 1$. Though Chander and Tulkens (2011) mentions that the tradable permits can be assigned compatible with the proposed allocation, we show in Section 5 that it is not possible to assign an interior value of initial allocation of tradable permits compatible with the ratio equilibrium.

3.2 Transfers for voluntary cooperation

Shiell's (2003) allocation of no transfer can be, in the terminology of Arrow-Debreu-Foley in (5), $\eta_i^1 = r_i$ and the private ownership of the firms, $f_i(\widetilde{y}_i) - \eta_i^2 = f_i(y_i^*) + r_i C(Y)$. It departs from the standard Lindahl allocation, which typically involves redistribution (by $p(\theta_i Y - y_i)$) through reallocation of production resources. A less widely noted feature of Shiell's allocation is that it does *not* Pareto dominate the disagreement (voluntary contribution) equilibrium, thereby violating this principle of the GPGP framework — namely, the feasibility and desirability of cooperative outcomes (Nishimura (2008)).[2] But in the next section, we show that this conclusion is even stronger.

3.3 Incentive compatibility, fairness and summary

Bradford (2007) and Guesnerie (2007) also proposed the third principle of implementation, inspired from the Clarke-Groves-Vickrey framework. We have discussed above that a natural extension of the standard mechanism-design theory works for Uzawa's (1997) allocation. Eyckmans (1997) showed that the ratio equilibrium is implementable in the Nash equilibrium.

allocations	private ownership	public ownership	no transfers
P.d. of disagreement equil.	No*	Yes	No*
If $x_i = f_i(y_i^*) + p(\theta_i Y - y_i^*)$	$0 < \theta_i < 1$	θ_i may not be in $(0,\ 1)$	$\theta_i = \frac{y_i^*}{n_1 y_1^* + n_2 y_2^*} \in (0,1)$
η_i^1	0	r_i	r_i
$f_i(\widetilde{y}_i) - \eta_i^2$	$f_i(y_i^*) - py_i + p_i\hat{Y}$	$f_i(\hat{y}_i)$	$f_i(y_i^*) - r_i C(Y)$
Nash implementation	Yes (Nishimura (2008))	Yes (Eyckmans (1997))	n.a.

Table 1: Lindahl equilibrium and other solutions.
Uzawa (2003, Definition 1) for the private ownership of the firms, and the ratio equilibrium (3) for the public ownership of the firms.
$*$: "P.d."means "Pareto domination". Uzawa (1999) and Shiell (2003) respectively said "Yes", but Propositions 3 and 4 amend to conclude "No".

The way which the fairness principle applies in the public-good economy is distinct from the

[2]"In the absence of regulation, the countries will be located below the [Pareto] frontier, ... [T]here is one Pareto-efficient allocation associated with zero transfers ... This point is located to the north-east of [the disagreement equilibrium], since both countries benefit from some level of pollution control, even given the existing imbalance in the distribution of income." (Shiell (2003, pp. 42-43)).

private-good economy. As Diamantaras (1991) shows, a Lindahl equilibrium from equal income is *not* envy-free. Instead, envy-free and efficient allocation is generated by a public competitive equilibrium (Foley (1967)) from equal income.

4 Can they be supported by Kantian concept?

In this section, we consider the following nested model, where the correspondence with the standard Lindahl-ratio equilibrium is easier to understand. Suppose that, with respect to private good x_i and public good G, the utility function of a citizen in country i is $u_i = \min\{(1-\alpha_1)\ln x_i + \alpha_1 \ln G,\ \ (1-\alpha_2)\ln x_i + \alpha_2 \ln G\}$ with $0 < \alpha_1 < 0.5 < \alpha_2 < 1$. In other words, $u_i = (1-\alpha_1)\ln x_i + \alpha_1 \ln G$ when $x_i < G$, whereas $u_i = (1-\alpha_2)\ln x_i + \alpha_2 \ln G$ with greater marginal rate of substitution between G and x_i if $G < x_i$. We assume that

$$n\alpha_1 < 1 - \alpha_1 \tag{6}$$

Namely, once an agent is sufficiently poor to choose $x_i < G$ in the Kantian scheme we describe below, then the valuation of the public by such agents is very low.

The countries involve in production defined as $f_i(y_i) = y_i + m_i$ if $y_i \leq \overline{y}_i$ and $f_i(y_i) = \overline{y}_i + m_i$ if $y_i \leq \overline{y}_i$, with $\overline{y}_2 = \infty$. $m_i > 0$ represents rent (excess profit) from production.

A citizen in each country in Southern countries has income $W - \Delta \equiv W_1 > 0$, and a citizen in Northern countries has income $W + \frac{n_1}{n_2}\Delta \equiv W_2$ for $\Delta > 0$. The resource constraint is:

$$n_1 x_1 + n_2 x_2 - Y + Z = n_1 m_1 + n_2 m_2 + Z \equiv nW. \tag{7}$$

where we set $W_i = \hat{y}_i + m_i$ (the disagreement equilibrium) to describe the public-ownership Lindahl allocation, and $W_i = m_i + r_i \widetilde{G}$ for private-ownership Lindahl allocation, and $W_i = m_i$ for no-transfer-efficient allocation.

We also assume that $\overline{y}_1 + m_1$ is sufficiently small so that the following holds at the disagreement equilibrium. Applying $MRS_i(x, x) = 1$ at non-differential points, $\hat{x}_1 = \overline{y}_1 + m_1$ and:

$$MRS_1(\overline{y}_1 + m_1, \widetilde{G} - \hat{Y}) < 1 = MRS_2(\hat{x}_2, \widetilde{G} - \hat{Y}) \text{ and (7).} \tag{8}$$

Namely, the Southern citizens' pollution reaches its maximum. Increasing W_2 and reducing W_1 in this range increases the level of public good (in other words, it reduces $\hat{Y}$) so that $n_1 m_1 + n_2 m_2 + \hat{Y}$, the way to define the RHS of the resource constraint (7) with $Z = \hat{Y}$, decreases.

Lemma 1. The Lindahl equilibrium by Uzawa (2003) and the ratio ewuilibrium by Chander and Tulkens (1997) correspond to the respective Lindahl equilibrium for public-good problem: the representative citizens maximize the utility subject to $x_i + p_i G = W_i$ and (p_1, p_2) such that $p_i = r_i p$ generates the utility-maximizing level $G_1^L = G_2^L$.

4.1 Kantian structure with North-South economy

In contrast, we now consider the Kantian equilibrium that applies to the individual budget constraint $x_i = W_i - g_i$. g_i represents the abatement from W_i =the disagreement level in the public-ownership Lindahl, and $W_i = m_i + r_i \widetilde{G}$ for private-ownership Lindahl allocation.

Definition 2. Given β_1 and β_2, consider the following utility $V_i(g_i) \equiv u_i(W_i - g_i, n_i g_i + n_j(\beta_i g_i))$ for $j \neq i$. A strategy (g_1^K, g_2^K) consists of Kantian equilibrium with (β_1, β_2) if g_i^K maximizes $V_i(g_i)$ and $\beta_i g_i = g_j$ for $i = 1,\ 2$ and $j \neq i$.

Here we formulate Kantian moral reasoning, i.e., "what if everyone acted as I do" as follows: (i) every Southern (Northern) citizen behaves symmetrically; (ii) every Southern citizen has "as if" behavior for Northern citizen based on different conditional preference for the public good and different income; every Northern citizen has similar "as if" behavior for Southern citizens; (iii) the Kantian type of cooperative behavior is formulated as the utility-maximizing contribution to the public good.

For the private-ownership Lindahl, rom W_i =the disagreement emission level $f_i(\hat{y}_i)$).

The first-order condition of optimality given β_i is as follows:

$$\frac{dV_i}{dg_i} \propto -G + n_i \frac{\alpha_i}{1-\alpha_i} x_i + n_j \frac{\alpha_i}{1-\alpha_i} x_i \beta_j = 0. \tag{9}$$

The second term in the middle corresponds to the gain through coordinated contribution of g_i among citizens with the same circumstance, and in the last part, Southern citizens think of the suitable proportional behavior to the wealthier and a more public-good preferring citizens ($\alpha_2 > \alpha_1$), conditional on their wealth. A symmetric explanation applies to β_2.

Proposition 1. In this economy,

(i) if we set the coefficient $\beta_i = \frac{\alpha_j}{\alpha_i} \frac{W_j}{W_i}$ and hence the agent i expect an agent with different circumstance to behave as: $g_j = \frac{\alpha_j}{\alpha_i} \frac{W_j}{W_i} g_i$ as Kantian imperative for the public-ownership Lindahl equilibrium and the private-ownership Lindahl equilibrium with the restrictive definition of W_i above,

(ii) For the maximization of $\Upsilon_i(y_i) \equiv u_i(f_i(y_i), \widetilde{G} - n_i y_i - n_j(\beta_i^a y_i + \beta_i^b))$, if we set the coefficient $\beta_i^a = \frac{\alpha_j/(1-\alpha_j)}{\alpha_i/(1-\alpha_i)} \frac{x_j^*}{x_i^*}$ and $\beta_i^a y_i + \beta_i^b = y_j$,

then the Kantian equilibrium coincides with the Lindahl equilibrium. Namely, the assumption is that each agent contributes according to income and the preference, with the latter being income-dependent in the present scenario.

In a Cobb–Douglas economy used for illustration, we impose a Kantian structure of moral universalizability. We do not insist on these particular forms but our purpose is to show the structure that the resulting allocation is Pareto efficient. The form written as $\beta_i = \frac{\alpha_j/(1-\alpha_j)}{\alpha_i/(1-\alpha_i)} \frac{x_j^*}{x_i^*}$ applies both to the Lindahl and the ratio equilibrium. In the terminology by Edgeworth (1897), the Kantian proportionality is applied to the post-contribution income x_i. For the Lindahl and the ratio allocations, contributions are proportional to income and a circumstance-contingent preference. For the private ownership, the imputed value of the clean air ($p_j \widetilde{G}$) is a part of virtual income W_j, and p_j is dependent on j's preference. For no-transfer allocation, the Kantian coefficient has to be proportional to the endogenously determined consumption level. Our result shows that private ownership weakens the ground for the correspondence between the Kantian and the Lindahl.

4.2 Inequality and the provision of public good

Suppose that Δ is sufficiently high and we suppose that the Lindahl equilibirum with superscript L satisfies $x_1^L < G^L < x_2^L$ (see the complete of description in the Appendix B).

The assumption of $x_1^L < G^L < x_2^L$ generates $x_i^L = (1-\alpha_i)W_i$ so (2) is rearranged to, for $n_1\alpha_1 + n_2\alpha_2 = \overline{\alpha}$:

$$n_1\alpha_1(W - \Delta) + n_2\alpha_2(W + \frac{n_1}{n_2}\Delta) = \overline{\alpha}W + n_1(\alpha_2 - \alpha_1)\Delta = G^L, \tag{10}$$

which also satisfies the resource constraint (7) and we assume such G^L is on $[(1-\alpha_1)(W-\Delta), (1-\alpha_2)(W + \frac{n_1}{n_2}\Delta)]$. Notably, this assumption is satisfied when α_1 and α_2 are not different, n_1/n_2 is large, and Δ is large.

Proposition 2. (i) For the Lindahl allocation, the level of the public good is greater when the inequality of initial endowment represented by Δ is greater.

(ii) For the ratio allocation, whether the level of the public good is greater with inequality is ambiguous.

(iii) If the inequality of initial endowment is moderate, then if (y_1^*, y_2^*) generates a no-transfer efficient solution (x_1^*, x_2^*, G^*), reducing $\overline{y}_1 + m_1$ by Δ that makes $\overline{y}_1 + m_1 - \Delta \geq x_1^*$ in exchange of the increase of the increase of m_2 by $\frac{n_1}{n_2}\Delta$ such that $m_2 + \frac{n_1}{n_2}\Delta \leq x_2^*$, then (x_1^*, x_2^*, G^*) is a non-transfer efficient solution after the increased inequality.

The provision of the public good declines as income becomes more equally distributed. In our formulation of Kantian ethics, the emergence of this paradox is explained as follows: if richer countries are more concerned about the global environment, then, though equality invites by contribution by poorer, less environmentally concerned countries (with this concern arising solely from income differences, as agents share identical preferences), the outcome may ultimately harm the environment more. This is because redistribution in such a setting can reduce the aggregate level of public good provision due to the strategic substitutability of individual contributions.

But the purpose of the present analysis is that this conclusion similar to that of Buchholz and Rübbelke (2023) may or may not hold. For the private-ownership Lindahl solution, the increase of Δ decreases r_1 and increases r_2, so the difference of virtual income $m_i + r_i\widetilde{G}$ increases. However, for the public-ownership Lindahl solution, there is ambiguity since, as we argued above, $W = \frac{n_1}{n}m_1 + \frac{n_2}{n}m_2 + \hat{Y}$ decreases as income becomes more unequal.

The no-transfer solution is characterized by $n_1 r_1(x_1, \widetilde{G} - n_1x_1 - n_2x_2) + n_2 r_2(x_2, \widetilde{G} - n_1x_1 - n_2x_2) = 1$ and $x_1 = y_1^* + m_1 \leq \overline{y}_1 + m_1$ and $x_2 \geq m_2$. Since the marginal pollution is constant for a wide range of allocations, there are multiple solutions without transfers. If the inequality is represented by the decrease of $\overline{y}_1 + m_1$, then, as long as that decrease is made up with the increase of y_1^* and the decrease of y_2^*, then this solution is invariant with respect to the inequality of incomes. This is Warr's (1982, 1983) type of neutrality with respect to endowment inequality.

5 Voluntary cooperation

In the absence of coercive supreme body, the move towards cooperation should include unanimous agreement — namely, Pareto domination — from the disagreement equilibrium. However, we can show the following:

Proposition 3. In the ratio equilibrium associated with (3), defined by Chander and Tulkens (1997):

(i) some countries may emit more pollution than under disagreement equilibrium.

(ii) if we consider the tradable-permit scheme, no natural restriction allows θ_i^{CT} to be in $0 < \theta_1^{CT} < 1$.

Under quasi-linear utility with respevt to x_i, Chander and Tulkens (1997) showed that the ratio allocation belongs to "γ-core", allocations which are immune to coalitional deviation, with Pareto improvement from disagreement equilibrium as a necessary condition.

Proposition 4. There exists an economy such that no Pareto efficient allocation with tradable permits with $\theta_i \in [0,\ 1]$ Pareto dominates the disagreement equilibrium.

5.1 Illustration

To prove Propositions 3 and 4, we use another numerical example that belongs to the global-public good model:

$$n_1 = 1 = n_2,\ u_i = x_i - \frac{0.9}{2}Y^2 \text{ when } x_i \geq 0.5 \text{ or } x_i \geq -\frac{3}{11}Y + 0.5,\ u_i = x_i - \frac{0.1}{2}Y^2 \text{ otherwise}$$
$$f_1(y_1) = 0.2(y_1)^{0.5},\ f_2(y_2) = (y_2)^{0.5} \tag{11}$$

This example amends that of Nishimura (2008) where agents' preference on clean environment is a luxury, and the preference is identical and dependent on income level.

In the disagreement equilibrium, agents equate their marginal disutility of pollution to the marginal productivity of pollution; the pollution-avert Northern citizen ends up with $\hat{y}_2 = (f_2')^{-1}(-\partial u/\partial Y) = 0.258$ whereas the consumption-oriented South has (circumstance-oriented) high willingness to pollute despite low fuel efficiency: $\hat{y}_1 = 0.836$. In the Pareto efficient allocation, marginal cost of pollution is equated; the high productive Northern technology is utilized to the greater extent than at the disagreement equilibrium so that $y_2^* = 0.614$ and $y_1^* = 0.025$ (where $f_2'(y_2^*) = f_1'(y_1^*)$ and the reduction of the pollution at Pareto efficient allocations is made by Southern country).[3] This is Proposition 3(i).

The conclusions corresponding to Buchholz and Rübbelke (2023) and others hold in that Southern citizens become worse-off at the Lindahl equilibrium than at the disagreement point ($u_1^L < \hat{u}_1$; the current numerical example is the independent proof needed in this particular circumstance). However, the conclusion related to tradable permits is sharper. Here, Southern citizens' utility through tradable permit trading becomes the highest when $\theta_1 = 1$ and $\theta_2 = 0$ in the tradable-permit allocation in Section 3. However, this maximal utility by Southern citizens is below the utility level of the disagreement equilibrium since the agent who minds pollution less damages the global climate in the disagreement equilibrium. This confirms Proposition 4. Since

[3] Since the utility is quasi-linear with respect to x_i, the Pareto efficient level of pollution is unique.

$u_1^{CT} > \hat{u}_1 > u_1^L$ by representing the transfer that Southern citizens receive at the public-ownership Lindahl allocation as $p(\theta_1^{CT} Y^* - y_1^*)$, $\theta_1^{CT} > 1$ has to be the case.

Yang (2008) made "reverse-engineering" argument further implies that the implementation of any Pareto-efficient allocation based on a specified Negishi weighting scheme requires explicit redistribution of resources across countries.[4]

6 Conclusion

We suggest a reinterpretation of Lindahl's (1919) justice notion and Roemer's (2010) reciprocity principle in international public finance. However, by integrating ethical reasoning with income and preference heterogeneity, decentralized ownership of firms or the disagreement level as an initial endowment rather weakens the reasons to be compatible with universalizable ethics. We also showed that delivering equitable and cooperative outcomes without explicit redistribution. This issue makes the Southern countries worse-off than the disagreement equilibruium in both Uzawa (2003) and Shiell (2003). It seems that this feature is not known in the literature, but this is in line with Buchholz and Rübbelke (2023) in the pure public-good model.

Appendix

Along the utility possibility frontier, we consider the following way. Consider first $x_1 = x_2 = x^*$ and we consider $\max u_i(x^*, n(W - x^*))$. If $x^* > n(W - x^*)$, agent prefers $\frac{x^*/W}{1-\alpha_2} = \frac{1-\frac{x^*}{W}}{\alpha_2}$ so $x^*/W = 1 - \alpha_2$, which is violated since $\alpha_2 > 0.5$. If $x^* < n(W - x^*)$, agent prefers $\frac{x^*/W}{1-\alpha_1} = \frac{1-\frac{x^*}{W}}{\alpha_1}$ so $x^*/W = 1 - \alpha_1$, which holds if $n\alpha_1 > 1 - \alpha_1$ which we exclude by assumption.

Therefore, the only possibility is $x^* = n(W - x^*)$. If one changes x^* by dx, the utility changes by $(\frac{1-\alpha_i}{x^*} - \frac{\alpha_i}{x^*} n)dx$ with $i = 1$ if $dx < 0$ and $i = 2$ if $dx > 0$. The utility increases in neither direction. This allocation is the Lindahl allocation with $p_i = \frac{1}{n}$ and $W_i = W$, which is Pareto efficient.

Then consider a Pareto efficient allocation with $\tilde{x}_1$ in the neighborhood of x^* but $u_1(\tilde{x}_1, \tilde{G}) < u_1(x^*, x^*)$. Clearly we have $\tilde{x}_1 < \tilde{G}$ so one can reduce $\tilde{x}_1$ from x^* and there exists $(\tilde{x}_2, \tilde{G})$ that satisfies $u_2(\tilde{x}_2, \tilde{G}) > u_2(x^*, x^*)$. $\tilde{x}_2 < \tilde{G}$ would not fit the Samuelson condition so $\tilde{x}_2 \geq \tilde{G}$. If $\tilde{x}_2 > \tilde{G}$, then $\tilde{x}_2$ and $\tilde{G}$ corresponds to x_2^L and G^L in the text, and otherwise the allocation is determined by $\tilde{x}_2 = \tilde{G} > x^*$, $\frac{\alpha_1}{1-\alpha_1} = \frac{\tilde{G}}{\tilde{x}_1}$ and the resource constraint. In both cases, the level of the public good increases when $\tilde{x}_1$ is reduced.

[4] His usage of "Lindahl principle" is different from conventional Lindahl framework since the latter, as shown by Buchholz and Rübbelke (2023) and others, does not necessary Pareto dominate the disagreement equilibrium.

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