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Finding Win-Win Policy in Platform-Price Regulation under Network Externalities, with an Application to Media Advertisement *

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Abstract

This paper studies which side of a two-sided platform should be subject to price regulation when prices on the two sides are jointly determined. We develop a model in which a platform sets the price of a network good and a commission fee charged to participating firms. Cross-side network externalities determine both the platform's unregulated price structure and the welfare effects of regulation. When the network-size externality is stronger than the app-variety externality, the platform charges a positive commission fee. A ceiling on this fee induces a waterbed effect: the platform raises the network-good price, reducing network-good consumer surplus. In contrast, a ceiling on the network-good price expands network participation and increases both network-good consumer surplus and developers' producer surplus. When the relative strength of the externalities is reversed, the policy prescription can also reverse. Thus, the unregulated commission fee provides information about which side should be regulated to generate mutual gains. Numerical simulations show that the qualitative policy results are robust across nonlinear extensions, while the main pricing mechanism extends analytically to general demand and entry functions. We also apply the framework to media platforms, where advertising generates a negative cross-side externality.

Keywords: Digital economy; Platform; Antitrust pricing; Network externality

JEL classification number: F23; L13; D85; K21; D62;

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1 Introduction

The development of technology has enabled firms to conduct business online. However, many online markets are oligopolistic, dominated by a few large firms with substantial price-setting power. Online platforms such as Amazon, eBay, iTunes, and Rakuten[1] have a classic two-sided market structure. Consumers access a platform through a network good, such as the iPhone.[2] At the same time, network users have access to relation-specific goods, for which we use *online apps* as a leading example. Participating firms, which we call *developers*, must pay commission fees to the platform to offer their products, The platform, exercising platform-specific market power, sets these fees. Such fees play a central role in the platform's pricing strategy but have also drawn increasing scrutiny from antitrust authorities concerned with fair pricing.

Price regulation in a two-sided market presents a basic difficulty for competition policy. A platform does not choose prices on the two sides of the market independently. When regulation constrains the price charged on one side, the platform may respond by adjusting the price on the other side. This "waterbed effect" (Belleflamme and Peitz (2021), Chapters 5 and 14) changes participation on both sides through cross-side network externalities and may therefore overturn the intended welfare effect of regulation. The contribution of this paper, however, is beyond the existence of a waterbed effect on two-sided platforms. Instead, we examine the choice of the regulatory target when a platform jointly determines the price charged to network-good consumers and the commission fee imposed on participating developers. This paper asks a simple policy question: *which side of a two-sided platform should be subject to a price ceiling?* We show that the answer depends on the relative strength of the two *cross-side externalities*. Importantly for policy, the platform's unregulated pricing behavior can reveal information about these relative strengths and thereby help identify the appropriate regulatory target.

The issue has become increasingly relevant in digital markets. Recent antitrust interventions against digital platforms illustrate this concern. For example, the U.S. Department of Justice has challenged Apple's conduct in smartphone markets.[3] The European Union introduced the Digital Markets Act in 2023 to promote contestable and fair digital markets. Under the new regulatory framework, Apple announced a reduction in its commission fee on certain in-app purchases from

[1]The leading online shopping platform in Japan.

[2]In another interpretation, consumers purchase standard goods through an online marketplace, while demand for these goods is affected by transactions involving relation-specific goods on the platform. See Section 2.1.

[3]Inspired by the Digital Markets Act, Japan also enacted the so-called "Smartphone Act" to prevent large companies such as Google and Apple from leveraging their market position to give their own products "a competitive advantage" or to "impose disadvantages on business users". See Reuters (2024).

30% to 17%.[4] As emphasized by Rysman (2009), however, pricing that appears monopolistic is not by itself sufficient to establish an antitrust violation. For regulatory policy, the relevant question is therefore not simply whether a platform price is high, but how intervention changes the platform's price structure and, through it, participation and welfare on both sides.

To study this problem, we develop a two-sided platform model building on Armstrong (2006), Anderson and Bedre-Defolie (2025), Hayakawa et al. (2025) and Wu et al. (2023). The platform sets two prices: the price of a network good purchased by consumers and a commission fee charged to firms that participate on the platform. Two cross-side externalities link the two sides. A larger network of consumers makes participation more profitable for app developers (network-size externality), while greater app activity raises consumers' willingness to adopt the network good (app-variety externality). As a preliminary step of the analysis, we show the following. When the network-size externality is stronger than the app-variety externality, the platform charges developers a positive commission fee. When the latter externality is sufficiently strong, the platform instead has an incentive to subsidize developer participation.[5] This link between externalities and the unregulated commission fee has an important policy implication: an observable feature of the platform's price structure reveals information about the underlying force that determines which side should be regulated.

Our first main result concerns a ceiling on the commission fee, as recently proposed by the European Commission, when the network-size externality dominates so that the commission fee prior to the regulation is positive. Although the ceiling directly reduces the amount charged to developers, the platform responds by raising the price of the network good. The resulting contraction of network demand reduces network-good consumer surplus. Thus, regulating the price that appears to be the most direct manifestation of platform market power can have an unintended adverse effect on consumers.

In contrast, regulating the other side can reverse this outcome: A ceiling on the network-good price induces the platform[6] to adjust its commission fee and expands network participation. When

[4]See Apple (2024). The scope of antitrust policy is broad, encompassing measures such as facilitating the entry of alternative firms to handle platform registration — potentially exerting downward pressure on commission fees — and promoting a contestable fee structure. Similar regulations have been enacted for delivery apps during the COVID-19 pandemic, with New York City and other jurisdictions introducing caps of 15% on delivery fees for independent restaurants (Li and Wang (2025), Sullivan (2025)).

[5]A negative commission fee can be interpreted as a non-pecuniary subsidy, such as technical support to app developers by providing a construction kit. For example, Apple allows app developers to apply for an Apple Vision Pro Developer Kit. Rysman (2009) gave another example of Microsoft, as a supplier of a computer operating systems, making it very easy to become a software developer for the Windows operating system and arguably subsidizes this activity with tutorials and supportive websites.

[6]See Remark 1. We consider the case as in the Armstrong (2006) model, the network-good price has a platform-specific component.

the network-size externality dominates, this intervention increases both network-good consumer surplus and app developers' producer surplus. Moreover, for two policies that generate the same loss in platform profit, the network-good price ceiling generates greater total surplus than does the commission-fee ceiling under certain conditions.

The policy prescription reverses when the relative strength of the externalities reverses. If the app-variety externality on network demand dominates so that the unconstrained commission fee is negative, reducing the commission fee can instead expand participation on both sides. Our results therefore identify circumstances under which regulation on either side can generate gains on both sides of the market. Importantly for policy, in our baseline model, the sign of the unregulated commission fee provides a simple indicator of which regulatory intervention can generate mutual gains.

We also examine the robustness of our results beyond the baseline specification. Numerical simulations with nonlinear models show that the qualitative welfare result of the network-good price ceiling remains robust to alternative functional forms. In addition, we analytically generalize the main pricing mechanism to arbitrary demand and entry functions. Thus, the baseline specification provides a tractable framework for identifying the underlying economic mechanism.

Our analysis relates to the literature on pricing and regulation in two-sided markets initiated by Rochet and Tirole (2003) and Armstrong (2006). It is particularly related to work studying fee caps and compensating price responses across the two sides of a platform, including Dubé et al. (2010), Genakos and Valletti (2011), Bedre-Defolie and Calvano (2013), Li and Wang (2016), Rey and Tirole (2019), Gomes and Mantovani (2025). While existing studies show that cross-side network effects can affect pricing and the need for regulatory intervention, and also examine the optimal level of a regulated fee or demonstrate circumstances in which fee regulation can backfire, our focus is on the choice of the side to regulate. Our analysis is closely related to Anderson and Bedre-Defolie (2025), who study a device-selling platform charging commissions on app revenues. In their policy analysis, a commission cap increases developer surplus but reduces consumer participation, whereas a device-fee cap increases consumer participation but reduces developer surplus. In contrast, our framework yields distinct results: a device-price ceiling can increase network participation, app transactions, and both consumer and developer surplus. Conversely, a commission reduction can reduce developer surplus in some parameter regions. Thus, their welfare implications differ from ours. We characterize how the relative strengths of cross-side externalities determine the conditions under which regulation generates mutual gains. Bisceglia and Tirole (2024) examine fair access conditions in digital ecosystems on core-service and app

prices and other equilibrium variables. Our analysis addresses the welfare effects of constrained prices through regulatory problems: how ceilings on the network-good price and the developer transaction fee affect participation and surplus when the platform re-optimizes its price. Wang and Wright (2025) study platforms that help consumers discover and transact with suppliers. Their efficient-fee formula equals the platform's marginal cost plus the difference between suppliers' markups in the direct and platform channels. Our focus is on the choice between two regulatory targets when network-good pricing and developer fees jointly make use of network effects. We identify conditions governing the welfare ranking of the two ceilings.

Finally, we apply the framework to media platforms, where advertising generates a negative rather than positive externality for subscribers. In this setting, regulating advertising fees cannot generally produce mutual gains through the mechanism identified for positive cross-side externalities. A ceiling on the subscription price, however, can raise subscriber surplus without reducing advertiser surplus under the conditions characterized in the model. The application illustrates that the regulatory logic depends on the sign as well as the relative magnitude of cross-side externalities.

The remainder of the paper is organized as follows. Section 2 introduces the model, and Section 3 characterizes the unregulated equilibrium. Section 4 analyzes price-ceiling policies on the commission fee and the network-good price, and their welfare effects. Section 5 examines robustness to alternative specifications, and Section 6 applies the framework to media advertising. Section 7 concludes. The Appendix provides proofs and a generalization of the main pricing result.

2 Basic setup

Consider the following digital economy. A firm, called the *platform*, has its own online technology to operate its platform, such as the App Store, in the home country. Consumers in the home country have access to the platform through a platform-specific *network good*, such as the iPhone, which the platform sells to network users. The platform then facilitates transactions in country H involving a relation-specific good which we call, for an illustration, *online apps*.

We consider the following two-stage game. In Stage 1, the platform sets the price p of the network good and the commission fee R for the entry of the app developers. In Stage 2, consumers, app developers, and app users make their adoption decision simultaneously.

2.1 Demand of the network good

Following Wu et al. (2023), consumers are heterogeneous in their preference for the network good and decide whether to buy one unit of the good or not. The aggregate demand — equivalently, the number of consumers — is determined through a conventional micro-founded model (see Appendix B):

$$q(p) = \nu + \alpha m - p. \tag{1}$$

where ν and p represent, respectively, the maximum willingness to pay for the good and its price.[7] In the second term, m represents the total number of apps on the platforms and α is the degree of such network externality. αm captures the indirect network effect generated by the app transactions in the specific-good market (à la Katz and Shapiro (1985)) — a larger ecosystem (in terms of total transactions) signals platform quality, complementary richness, and future support, raising consumers' willingness to pay for the standard good.[8]

Note that this increase in willingness to pay for the good matters not only in the number of apps sold but also in the speed of the internet connection. Therefore, consumers receive benefits from the network good if they are in a more digitalized environment. The network parameter α reflects the level of digitization, and we interpret an exogenous increase in α as digitization as the introduction of 6G, for example, which enables high-speed data transfer.[9]

The platform provides network goods to consumers at the marginal production cost c.

2.2 App's price and developers' entry

On the online platform, apps are developed by a potentially large pool of app developers N. The app developers are heterogeneous in terms of the popularity of apps, denoted by $h_S \geq \underline{h}$ over $[\underline{h}, \infty)$, which are distributed by a density $g(h_S) = M/(h_S)^3$. As shown below, a demand side can be dependent on the good's price which determines the entry margin endogenously. Apps differ in their popularity across firms, but not in the per-unit utility they generate. Thus, h_S captures heterogeneity in usage rather than in quality.

The apps are demanded by a potentially large pool of app users. For the number of users

[7] We assume that $\nu > p$ holds in the equilibrium throughout the analysis. This implies that some consumers choose to buy the network good even in the absence of additional utility from network effects. This assumption is realistic, as indirect externalities typically arise when a baseline level of consumers exists and app developers find market entry profitable. Following the literature, including Katz and Shapiro (1985) and Wu et al. (2023), we assume that each consumer forms expectations about the volume of app transactions in the market, and in equilibrium, these expectations are fulfilled; that is, $m_e = m$ holds.

[8] This is known as an indirect network externalities.

[9] In the last two decades, the percentage of individuals using the internet has grown: It was 7% in 2000 and 67% in 2023.

m, user i has the utility of consumption as $a_i = \phi q - \eta_i m$, with η_i being uniformly distributed in $[0, \eta]$. Note that q is the number of the network good and, equivalently, the number of consumers participating in the online platform. Following Rasch and Wenzel (2013), the willingness to pay for the app is dependent on a network size, so $\phi > 0$ is regarded as a network-size externality. This network externality captures, for example, improvements in online algorithms or other technologies that better match customers with products based on individual consumption or search histories — features that help predict users' preferences. Therefore, a larger ϕ reflects a different aspect of digitization from α. The term $-\eta m < 0$ is interpreted as a downward-sloping demand. The app's price p_a equals the lowest willingness to pay among active buyers: $p_a = \phi q - \eta m$ (Appendix A). Alternatively, as the number of developers increases, competition is intensified, and profits per developer decrease. Such congestion is illustrated by $-\eta < 0$. For example, each developer finds it more difficult to acquire customers when m increases (Hagiu (2009) and Belleflamme and Peitz (2019)).[10] A linear version of $a_i(q, m)$ with $\partial a_i / \partial m < 0$ is represented as $a_i = \phi q - \eta m$. We assume that ϕ is sufficiently greater than η to ensure $a_i > 0$.

In addition, app developers incur a fixed cost $\overline{f}$ of developing and maintaining apps, and pay a commission fee, R. The latter is paid to the platform to sell their app on the platform. In the terminology of Rochet and Tirole (2003), R is levied per unit of transaction. We allow R to be positive or negative and regard a negative R as a subsidy from the platform to app developers.[11] Notice that R exhibits standard buyer-seller independence: whether the platform charges the fee R to the developer of the relation-specific good or its consumer is irrelevant to incidence. In the latter case, the developer charges a price of $p_a - R$ for the consumers, who then decides whether to accept the price. The platform's pricing of R is a policy-relevant matter (as mentioned in the Introduction). So an app developer's profit from developing its app is $\pi_S = (\phi q - \eta m - R) h_S - \overline{f}$.

With this specification, app developers' entry decision leads to the following expression for the equilibrium volume of app transactions, m:

$$\pi_S \geq 0 \iff \phi q - \eta m - R \geq \frac{\overline{f}}{h_S} \Rightarrow \quad m = \int_{h_S \geq h_{\overline{S}}} h_S \frac{N}{(h_S)^3} dh_S = \frac{M(\phi q - R)}{\overline{f}}, \tag{2}$$

for $(\phi q - \eta m - R) h_{\overline{S}} = \overline{f}$ and $\dfrac{N}{1 + \eta N / \overline{f}} \equiv M$.

We solve the two-stage game by backward induction. In stage 2, the network users and the app users are simultaneously determined by (1) and (2). We derive the induced demand function and

[10] The latter provides many examples of sellers' congestion problems, such as a reduced price through monopolistic competition with an increasing number of competitors.

[11] See footnote 5.

the number of app transactions as follows:

$$q(p,R) = \frac{\overline{f}(v-p) - \alpha MR}{\overline{f} - \alpha\phi M} \equiv \frac{v - p - \alpha RM/\overline{f}}{\gamma}, \tag{1'}$$

$$m(p,R) = \frac{M\left[\phi(v-p) - R\right]}{\overline{f} - \alpha\phi M} \equiv \frac{M}{\overline{f}}\frac{\phi(v-p) - R}{\gamma}, \tag{2'}$$

Electric devices that get access to online platforms are provided by knowledge-intensive firms, and firms are sometimes criticized for using their high market power and setting high prices. The online platform operates a digital platform, such as the App Store, with zero fixed costs and zero marginal costs of operating the online platform. With the above setup for consumers and app developers, the platform decides the price of the network good and a commission fee. Thus, the profit of the platform is the following:

$$\pi_P = (p-c)q + Rm. \tag{3}$$

2.3 Alternative interpretations

As an alternative specification, (1) above can be interpreted as the demand function for a *standard good* sold on online platforms such as Amazon, eBay, and iTunes, where demand is influenced by m, the number of relation-specific goods as defined in Section 2.2. Regarding its production, the platform either sets the lowest possible entry price — namely, $-c$ — for the seller of this good, based on sales performance, or it outsources production. We refer to this alternative specification as the *two-good model*.

Our basic model has many possible extensions. In Section 5, we conduct numerical simulations across several distinct non-linear extensions of the basic model. In addition, we analytically generalize the main pricing mechanism to arbitrary demand and entry functions.

In Section 6, we discuss the case where one of the network externalities is negative. As an application, q is the number of viewers of a media channel, and the media levies R on the advertisement whose total is m.

3 Unconstrained equilibrium

This stylized model provides a standard price that includes an additional mark-down or mark-up through indirect network effects, and a commission fee whose sign depends on the relative strength of the two network externalities discussed in the previous section. Clarifying the structure of the

platform's optimization here is essential for Section 4, where the direction of each price-ceiling policy's welfare effect turns out to hinge precisely on this sign.

3.1 Price and commission fee

Differentiating (3) with respect to p and R, and taking into account (1') and (2'), the first-order condition (FOC) $\frac{\partial \pi_P}{\partial p} = 0$ yields the following. From (1'), we have $\frac{\partial q}{\partial p} = \frac{-\overline{f}}{\overline{f}-\alpha\phi M} \equiv -\frac{1}{\gamma} < 0$, $q = \frac{1}{\gamma}\left(\nu - p - \frac{\alpha R M}{\overline{f}}\right)$ and from (2), we have $\frac{\partial m}{\partial p} = \phi\frac{-1}{\gamma}\frac{M}{\overline{f}}$. Therefore, the FOC for p in the platform's problem is written as:

$$\frac{\partial\Big((p-c)q + Rm\Big)}{\partial p}\gamma = -(p-c) + \left(\nu - p - \frac{\alpha R M}{\overline{f}}\right) - R\frac{\phi M}{\overline{f}} = 0, \tag{4A}$$

which derives

$$p^* = c + \frac{\nu - c}{2} - \frac{\{\alpha + \phi\}MR^*}{2\overline{f}} \tag{4}$$

In the above formula, the price is composed of two key components: market power and network externality. In the absence of network externalities, the platform sets the price above marginal cost by a monopoly markup, reflected in the second term on the right-hand side. With network externalities, if the commission fee R is positive, the last term in p^* lowers the equilibrium price, and the demand for the network good is stimulated, which has a further effect on the other market. It should be noted that the price in (4) is below the standard monopoly markup $p^* - c = \frac{\nu - c}{2}$. As Rysman (2009) suggested, monopoly power does not necessarily imply inefficiently high prices. In contrast, if the entries of more developers would scale up the second term on the right-hand side, αm^*, in (1), we show below that $R^* < 0$ can happen, and the platform receive revenues from the increase in p^*. The platform still benefits from increased entry and spillovers from app-side activity.

Two aspects further influence the pricing outcome: (i) $\frac{\partial m}{\partial q} = \frac{M}{\overline{f}}\phi$ in (2), which increases in the responsiveness of developers' entry to the number of users, increases in M and decreases in $\overline{f}$. (ii) Effect of app's variety on demand ($\alpha\frac{M}{\overline{f}}$). For clarity, hereafter we refer to the corresponding parameter $\frac{M}{\overline{f}}$ as *entry responsiveness*.

Turning to the first-order condition for the commission fee R, we have $m \propto \phi(\nu - p) - R$, $R\frac{\partial m}{\partial R} =$

$-R\frac{M}{\overline{f}}\frac{1}{\gamma}$ and $(p-c)\frac{\partial q}{\partial R} = (p-c)\alpha\frac{M}{\overline{f}}\frac{-1}{\gamma}$, so that:

$$\frac{\partial\Big((p-c)q + Rm\Big)}{\partial R}\gamma\frac{\overline{f}}{M} = \phi(\nu - p) - 2R - \alpha(p-c) = 0, \Rightarrow \tag{5A}$$

$$R^* = \frac{\overline{f}(\nu-c)(\phi-\alpha)}{4\overline{f} - M(\alpha+\phi)^2} \gtreqless 0 \quad \Leftrightarrow \quad \phi \gtreqless \alpha \tag{5}$$

The denominator of R^* is positive due to the second-order condition (see Appendix C). This assumption warrants that the induced demand function (1') is downward-sloping in p.

As in Author (2026), whether the commission fee is positive or negative depends on the size of network externalities. If the network-size externality is stronger and $\phi > \alpha$, the motive to charge app developers determines R^*, resulting in a positive equilibrium commission fee $R^* > 0$. Conversely, when the app-variety externality is stronger and $\alpha > \phi$, the platform subsidizes app developers to expand the demand for the network good through increased transactions of the apps and increases demand for the network good. The decision in (5) then feeds back into the pricing channel in (4). When the network-size externality is stronger ($\phi > \alpha$), the platform decreases the equilibrium price to stimulate consumption. Otherwise, the equilibrium price for the good exceeds the standard markup but the entry of app developers and the consumption externality still allow the expansion of the network. See also a general formula (10) in Section 5.

Lemma 1. The monopoly platform sets the commission fee R^* to be positive if network-size externality ϕ is greater than app-variety externality α, and sets R^* to be negative otherwise.[12]

Substituting (4) and (5) into the induced demand (1'), we have:

$$q^* = \frac{2\overline{f}(\nu-c)}{4\overline{f} - M(\alpha+\phi)^2} \quad \text{and} \quad m^* = \frac{M(\nu-c)(\alpha+\phi)}{4\overline{f} - M(\alpha+\phi)^2} < \frac{M}{\underline{h}} \tag{6}$$

q^* increases with both α and ϕ, so regardless of the sign of $\alpha \gtrless \phi$, we see that the role of the commission fee is to increase the number of network users and the number of apps sold. The last part, $m^* < M/\underline{h}$, is equivalent to $4\frac{\overline{f}}{M} > (\alpha+\phi)[(\alpha+\phi) + \frac{(\nu-c)\underline{h}}{M}]$. That is, we assume that entry responsiveness $\frac{M}{\overline{f}}$ is sufficiently low, which also implies that the second-order condition is satisfied (see Appendix C).

[12]The two features so far — the price's markdown by R^* in (4) and the effects of the two externalities in (5) remain valid under competing platforms. Details are available upon request to the author.

3.2 Consumer surplus and producer surplus

Turning to the welfare effects, our specification yields the following consumer surplus (CS_q and CS_m for the network good and apps, respectively) and producer surplus, which increase in the number of network users q^* and the number of apps m^*, respectively. For CS_q, see Appendix B. For CS_m, the units m^* are purchased, and the aggregated surplus is $\eta m^* \cdot m^*/2$:

$$CS_q = \frac{q^{*2}}{2}, \quad CS_m = \eta\frac{m^{*2}}{2}, \quad PS = \int_{h_S \geq h_{\overline{S}}} \{\phi q - \eta m - R - \frac{\overline{f}}{h_S}\}\frac{N}{(h_S)^2}dh_S = m^{*2}\frac{\overline{f}}{2N} \tag{7}$$

4 Price ceiling: to the app store or iPhone?

In the present model, network-good consumers receive direct benefits from the network good, while apps provide indirect benefits. Producers receive indirect benefits from consumers and pay R to the platform. In this section, we discuss the following novel feature: the ceiling policy that benefits a party *in*directly (R for the consumer and p for the developer) will improve both consumer and producer surplus (Proposition 1). As briefly noted in Appendix B, imposing a price ceiling on app prices does not appear to be a desirable policy.[13] Our model generates a typical trade-off by innovators — an internalization of (network) externalities and the rent by Intellectual-Property owners. Therefore, the following analysis focuses on regulating the commission fee and the price of the network good.

We first consider the ceiling policy $\hat{R}$ in the neighborhood of R^*:

$$R^* - \hat{R} \to d\hat{R} = \epsilon,$$

with $\epsilon > 0$, and the platform's commission fee is constrained by $R \leq \hat{R}$. In the following, we deal with the case of $\phi > \alpha$ (so that the unregulated commission fee is $R^* > 0$), since the analysis up to Proposition 1 is symmetric in the reverse case.

4.1 Commission-fee Ceiling

The first key point to understand is that the platform re-optimizes against the commission-fee ceiling. We denote all variables under the ceiling policy with a hat and use the superscript $*$ to represent the benchmark (unregulated) values. The new price $\hat{p}$ has its feature for $\hat{p} - p^* \to d\hat{p}$

[13] First, setting app prices below $p_a = \phi q$ leads to developer exit and results in a total surplus that is lower than in the unconstrained equilibrium. Second, when R^* is adjusted by the platform as the upstream firm, deviating from the competitive app price creates a multiple marginalization problem.

from (4A) and (4):

$$-(\hat{p}-c)-\frac{\nu-c}{2}+\frac{\{\alpha+\phi\}M(R^*-\epsilon)}{2\overline{f}}=0 \quad \Rightarrow \quad d\hat{p}=\epsilon(\alpha+\phi)\frac{M}{2\overline{f}}$$

With $\hat{R} < R^*$, the platform re-optimizes by increasing the product's price. This feature is an application of Upward Pricing Pressure in merger analysis (Cosnita-Langlais et al. (2021)), specifically accounting for network externalities and entry responsiveness.

The second key point goes beyond this price effect for welfare evaluation. The following cross-market effect occurs: substituting these values into the numerator of $q(\hat{p},\hat{R}) = \frac{1}{\gamma}(\nu-\hat{p}-\alpha\hat{R}\frac{M}{\overline{f}})$ in (1') and subtracting it from the unconstrained equilibrium $q=q^*$:

$$dq(\hat{p},\hat{R})\gamma=-d\hat{p}-\alpha d\hat{R}\frac{M}{\overline{f}}<0 \quad \text{if} \quad \{-\alpha+\phi\}(-\epsilon)\frac{M}{2\overline{f}}<0 \quad (\ R^*>0\ ) \tag{8.1}$$

Quite surprisingly, when the unconstrained commission fee R^* is positive, the commission-fee ceiling decreases the consumption of the network good and the consumer surplus of the network good. This result is because, in the induced demand $q(p,R)$, the reduction in the commission fee counts by the order of $\alpha\frac{M}{\overline{f}}$, while this policy raises p by the order of $(\alpha+\phi)\frac{M}{2\overline{f}}$ and lowers the number of network users.

For developers, substitute these values for the numerator of the expression of $m = \frac{M}{\overline{f}\gamma}\big[\phi(\nu-\hat{p})-\hat{R}\big]$,

$$dm(\hat{p},\hat{R})\gamma=\frac{M}{\overline{f}}\big(-\phi\{\alpha+\phi\}\frac{M}{2\overline{f}}+1\big)\epsilon, \tag{8.2}$$

which is positive if entry responsiveness $\frac{M}{\overline{f}}$ is sufficiently low, or, from (6), if the net market size $\nu-c$ is sufficiently high relative to ϕ.[14]

In Section 4.4, we show that the platform's loss is in the order of ϵ^2 and δ^2, whereas the gains in the private sector in (8.1) and (8.2) are in the order of ϵ and δ. As in conventional price-regulation analysis, for sufficiently small ϵ and δ, the latter gains dominate the former loss. Here, we focus on dq and dm from the price-ceiling policies.

[14]However, contrary to (8.4) for the consumer surplus of another price ceiling policy, (6) and the second-order condition (Appendix C) do not warrant (8.2) to be positive. The difference of (8.2) and (8.4) becomes relevant in comparing the total surplus of alternative price-ceiling policies in Proposition 2 below.

4.2 Indirect and direct benefits of policies

The fact that the commission-fee ceiling may not work for the consumers' welfare motivates us to examine another policy, which is *a price cap on p*.

Remark 1. The feasibility of a p-ceiling policy depends on whether the price by the network good (or standard good) seller or the outsourced producer is network-specific. As discussed, the network-specific price p^* has a markdown from the standard monopoly price, and this markdown is platform-specific, depending on the network externalities that the platform can generate. Retailers such as Amazon, eBay and Rakuten routinely set channel-specific prices for the same underlying good. This makes a platform-level price ceiling a policy-relevant instrument.

If a price cap on p, denoted by "bar", is imposed as $p^* - \bar{p} \to d\bar{p} = \delta > 0$, the platform re-optimizes the commission fee through the first-order condition on R: Again, another variable increases. Taking the platform's revised commission fee in (5A), $\phi(-d\bar{p}) - 2d\bar{R} - \alpha d\bar{p} = 0$ into account so that $d\bar{R} = -\{0.5\phi + 0.5\alpha\}d\bar{p}$. Substituting them into the numerator of $m = \frac{M}{\overline{f}\gamma}[\phi(v - \bar{p}) - \bar{R}]$ in (2'), we see whether m increases or decreases under the price ceiling as follows:

$$dm(\bar{p}, \bar{R})\gamma = \frac{M}{\overline{f}}(\phi(-d\bar{p}) - d\bar{R}) > 0 \quad \text{if} \quad \phi - 0.5\phi - 0.5\alpha > 0 \tag{8.3}$$

Interestingly and quite symmetrically with the case of the consumer surplus of the network good, both the total number of apps sold and *the producer surplus* increase by the price ceiling when $\phi - \alpha > 0$. This in turn implies that *the increase in m* from the price ceiling warrants the increase in consumption by $q = v + \alpha m - p$. To be on a scale comparable to (8.2),

$$dq(\bar{p}, \bar{R})\gamma = \big(- \alpha\{\alpha + \phi\}\frac{M}{2\overline{f}} + 1\big)\delta > 0 \tag{8.4}$$

which is positive under the second-order condition in Appendix C when $\phi > \alpha$.

4.3 Welfare evaluation of the two policies (1): win-win policy

The analysis so far is summarized in Table 1. The description is symmetric when $\phi < \alpha$.

$\phi > \alpha$	CS_q	PS
$\hat{R} < R^*$	Eq.(8.1); $(-)$ by the order of $\alpha - \phi$	Eq.(8.2)
$\bar{p} < p^*$	increase (by $p \downarrow$ and $m \uparrow$)	Eq.(8.3); $(+)$ by the order of $\phi - \alpha$

Table 1: Welfare effects of price ceiling

The app-consumers' surplus is $CS_m = \eta \cdot m^2/2$. Then the following conclusions are immediate:

Proposition 1. Suppose that the unregulated commission fee satisfies $R^* > 0$.

(i) The price ceiling on the commission fee on the app developers (R) raises the price of the network good (or the standard good in the two-good model) and reduces the price of the relation-specific good. This price ceiling reduces the consumer surplus of the network good (the buyers of the standard good).

(ii) The price ceiling on the network good (p) brings the following welfare consequences. First, the policy increases the producer surplus. Second, the consumer surplus for the network good increases.

This proposition is contrary to the initial impression. In our typical Armstrong (2006)-type model, *either* a price ceiling on R *or* p is expected to improve consumer and producer surplus (Section 4.4 considers the welfare analysis including the loss of the platform). However, the commission-fee ceiling does not work for the network-good consumers' welfare when $R^* > 0$ (in general, when the unconstrained commission fee is high, reflecting the platform's charging motive from developers).[15]

To the best of our knowledge, the possibility of a welfare improvement for both sides of the market through price ceiling policies has not yet been thoroughly examined and applied to current policy discussions or public debate. Our novel approach begins by focusing on the variables that affect each market participant indirectly — the commission fee R for consumers and the network good price p for developers. The interplay between these variables and consumption (or app production, in the case of developers) determines consumer and producer surplus under various policy regimes.

The profit of app developers under a price ceiling on p as shown in (8.3) depends critically on the relative strength of two externalities. In the case opposite to our baseline, when $\alpha > \phi$, we refer

[15]Even when regulation of the price of the network good may not be plausible, our first result on the commission-fee ceiling retains independent policy relevance. However, see Remark 1 in Section 4.2 for a practical implication of the price ceiling on the network good.

to (8.1) to find that a reduction in R ($-d\hat{R} > 0$) leads to an increase in both $\hat{q} = q(\hat{p}, \hat{R})$ and $m(\hat{q}, \hat{R})$. This implies that, once policymakers identify the relative magnitudes of the underlying parameters α and ϕ, which determine the sign of the unregulated commission fee R^*, within the baseline model, an appropriately targeted price ceiling can generate mutual gains for consumers and producers.

4.4 Welfare evaluation of the two policies (2): equal platform-loss comparison

To account for the *platform's loss*, one way to set the units of $R^* - \hat{R}$ and $p^* - \bar{p}$ is to set them so that they generate the same loss in platform profit. For $\hat{R}$, we have, since $(\hat{R}, \hat{p})$ satisfies $\frac{\partial \pi_P}{\partial p} = 0$ (that is, (4A)), so that:

$$d\pi_P = \frac{\partial \pi_P}{\partial R}\Big|_{R=R^*-\epsilon,\ p=p^*+\epsilon(\alpha+\phi)\frac{M}{2\bar{f}}} d\hat{R} = -(2-(\phi+\alpha)^2\frac{M}{2\bar{f}})(\epsilon)^2\frac{M}{\bar{f}\gamma} < 0. \tag{9.1}$$

See Appendix C for the above formula to be negative. In the same way, the p-ceiling policy generates:

$$d\pi_P = \frac{\partial \pi_P}{\partial p}\Big|_{p=p^*-\delta,\ R=R^*+0.5(\alpha+\phi)\delta} d\bar{p} = -(2-(\phi+\alpha)^2\frac{M}{2\bar{f}})(\delta)^2\frac{1}{\gamma} < 0. \tag{9.2}$$

Therefore, $\delta = \sqrt{\frac{M}{\bar{f}}}\epsilon$ gives the equivalent profit loss of the platform.

Then, we have the following proposition on $dCS_q + dCS_m + dPS + d\pi_P$, via two price-ceiling policies when $\phi > \alpha$ $(R^* > 0)$. It shows again the limit of the R-ceiling policy.

Proposition 2. Suppose that $R^* > 0$. For policies that yield equivalent decreases $d\pi_P < 0$, the price-ceiling policy on the standard goods gives a higher total surplus than the ceiling policy on the commission-fee.

See Appendix D. The conclusion of Proposition 2 may not be completely symmetric; when α is sufficiently close to ϕ, the p-ceiling policy is still better in terms of total welfare than the R-ceiling policy even if $R^* < 0$.

5 Robustness checks: numerical simulations

To assess the theoretical validity and robustness of our analytical findings—specifically the welfare properties of the network-good price ceiling formulated in Proposition 1—we conduct numerical simulations across three distinct non-linear extensions. While our baseline model relies on linear demand and entry structures to derive clear closed-form solutions, real-world platform markets may

exhibit diminishing marginal network effects, different developer distributions, or interdependent price elasticities.

We evaluate the impact of an approximately 10% reduction in the network-good price p (simulating a price ceiling $p^* - \bar{p} = 0.1$ when p^* is around 1) starting from the unconstrained platform equilibrium. Across all specifications, we set the baseline parameters as $\nu = 2$, $\alpha = 0.25$, $M = 1$, $\overline{f} = 0.33$, and $c = 0$, adjusting $\phi \in [0.25, 0.40]$.

Table 2 summarizes the formulation, parameter configurations, and numerical results for the three non-linear cases:

1. *Case 1 (Diminishing Marginal Returns to Network Size):* The developer entry function incorporates $p_a = \phi q^{0.8}$, representing saturation in the network-size externality.

2. *Case 2 (Different Developer Distribution):* The entry exponent is set to $g(h_S) = M/(h_S)^{2.5}$,[16] corresponding to a Pareto distribution with a thicker tail, which captures higher inequality/heterogeneity among developers. Note that this is similar to the scenario with $q = \nu - p + \alpha m^{0.5}$.[17]

3. *Case 3 (Endogenous Price Sensitivity):* The demand function includes an interaction term $q = \nu - (1 - 0.03m)p + \alpha m$, where an increase in app' transactions m modifies network-good consumers' price sensitivity ($\frac{\partial^2 q}{\partial p \partial m} > 0$).

Below, we explain how we additionally adjust ϕ in each case.

Case	Modified formulation	Adjusting parameter ($\alpha = 0.25$)	Baseline	Impact of price ceiling ($p^* - \bar{p} = 0.1$)
Case 1	$m = \frac{M}{\overline{f}}\left[\phi(\nu - p + \alpha m)^{0.8} - R\right]$	$\phi = 0.35$	$p^* = 0.991,\ R^* = 0.0533$	$\Delta q = 0.100$, $\Delta m = 0.0005$
Case 2	$g(h_S) = M/(h_S)^{2.5}$ (Thicker Pareto tail)	$\phi = 0.25$	$p^* = 1,\ R^* = 0.119$	$\Delta q = 0.100$, $\Delta m \approx 0$
Case 3	$q = \nu - (1 - 0.03m)p + \alpha m$	$\phi = 0.40$	$p^* = 0.941,\ R^* = 0.080$	$\Delta q = 0.099$, $\Delta m = 0.013$

Table 2: Robustness Analysis of Price-Ceiling Policy under Non-linear Extensions

As shown in Table 2, three key policy insights emerge from these numerical experiments. First, in Case 1, when the app prices are less sensitive to the market size, one has $R^* < 0$ when $\phi = \alpha$. So we therefore raise ϕ to ensure $R^* > 0$. In the table, we show that the p–ceiling raises both q and m from the unconstrained equilibrium (For comparison, when the commission fee is reduced by around 2%: $R^* - \hat{R} > 0$, we obtain $\Delta m > 0$ but $\Delta q < 0$ in all cases). Second, in

[16]Here, we set $N = M$ so that $\eta = 0$.

[17]This is because $m = (\phi q - R)^{0.5} \frac{M}{0.5(\overline{f})^{0.5}}$ in Case 2: a non-linear effect of the network-size externality has an effect similar to changing the firm distribution.

Case 2, when the developer distribution exhibits a thicker Pareto tail, the platform's incentive to extract monopoly rents from highly productive developers dominates, rendering $R^* > 0$ even under symmetric externality parameters ($\phi = \alpha$, which implies $R^* = 0$ in the linear case). In this case, enforcing a price ceiling on p successfully stimulates user demand ($\Delta q > 0$) without deterring developer entry ($\Delta m \approx 0$).

Third, Case 3 demonstrates that when app transactions reduce price sensitivity, lowering p by 0.1 leads to a mutual improvement in consumer and producer surplus, generating a higher network-good consumer surplus and greater entry of developers without disrupting market participation.

In conclusion, these numerical exercises show that the qualitative policy result — that when the underlying condition indicates $R^* > 0$, the p-ceiling improves consumer and producer welfare—survives across the three nonlinear extensions considered here.

In Appendix E, we show that Lemma 1 extends to a more general setting. When $m = m(\phi q, R)$ and $q = q_1(p) + \alpha m$, we show that, with $\Phi = \phi q$,

$$R^* > 0 \iff \phi \frac{\partial m}{\partial \Phi} \frac{-\partial q_1}{\partial p} > \alpha \left(\frac{-\partial m}{\partial R}\right). \tag{10}$$

Namely, if the externality channel through p is stronger than that through R, the regulator observes a positive commission fee, and the above analysis shows that the p–ceiling is likely to be welfare improving.

6 Application: the case of negative externality

Consider a case in which $\alpha m < 0$ in (1). That is, there is a negative externality of the network-specific good m on the demand of q. As an example, suppose that q is the number of viewers of a media channel, which is decreasing in the volume of advertisement m. We now suppose that $\phi q > 0$ represents the advertisers' unit profit from advertising to an audience of size q, and that π_S in (2) is the net profit of an advertiser that broadcasts/posts h_S units of advertising. For the interior solution of m^* in (6), we suppose $\phi > -\alpha$.

Complementing Anderson and Jullien (2015) and González-Maestre and Martínez-Sánchez (2015),[18] we examine the effect of a government-imposed price ceiling on the advertising fee R charged by the media platform. We see that the advertisement-fee ceiling *harms* the consumer since

[18]See also Kind and Schjelderup (2025), Section 4, for a media-platform model in which subscriber numbers generate positive externalities on advertising revenues.

the media-viewers' fee p increases (notice that $\alpha - \phi < 0$ in Table 1). Assuming that (8.2) is positive (when entry responsiveness is low), even the *price floor* in R would not improve both the consumer surplus of the media subscribers and the producer surplus of advertisers. In contrast, as in our main analysis, the *price ceiling on the media subscription* increases both q and m, as shown in (8.3) and (8.4).[19]

Proposition 3. Suppose that $\alpha < 0$ and $\phi > 0$. Suppose further that $\phi > -\alpha$ (in the media example, the advertisers' benefit ϕ exceeds $-\alpha$ to warrant the equilibrium volume of advertisement to be positive).

(i) The price ceiling on R (the advertising fee that the media charges in the media example) reduces the consumer surplus of the media subscribers. When (8.2) is positive, the price floor reduces the producer surplus.

(ii) The price ceiling on p (the media subscription fee) has the following welfare consequences. First, the policy increases advertising and producer surplus. Second, the media subscribers' consumer surplus increases.

7 Conclusion

Despite increasing globalization, a substantial share of consumption takes place multi-sided markets for digital content studied in this paper, which are dominated by a few large firms. Rysman (2009) raised an important concern about antitrust intervention in two-sided markets. Although intervention by antitrust authorities seems appealing at first sight, it requires caution because our model provides clear real-world explanation for incentive for the monopolist to adjust prices under price-ceiling policies. The monopolist adjusts prices to stimulate participation on both sides of the market. This process is consistent with Rysman's (2009) observation that market power on one side of a two-sided market does not constitute an antitrust violation by itself. Indeed, imposing a price ceiling on the wrong side of the market can hurt consumers or developers. Nevertheless, our results show that the sign of the unconstrained commission fee provides a simple indicator of which side of the two-sided market is the more promising target for a welfare-improving price ceiling. Our analysis provides clear analytical results on the waterbed effect, which is often context-dependent and therefore studied quantitatively.

As mentioned in Section 2.3, our analysis remains valid when congestion (negative externalities)

[19] m increases due to the rise in the network benefit to the advertisement supplier ϕq and the increase in the view from $\frac{M}{\overline{f}} \frac{0.5\alpha}{1-\alpha M\phi/\overline{f}}(-d\bar{R}) > 0$ appearing in (1'). For (8.4), $1 - 0.5\alpha M(\phi + \alpha)/\overline{f} > 0$ since $\phi > -\alpha$.

affects apps' pricing. Another issue is that, in practice, estimating the magnitude of network externalities is essential for identifying appropriate policies. As illustrated by Birke (2009) and Farronato et al. (2024), empirically identifying the magnitudes of network externalities remains an important direction for future research, not only for academics but also for policymakers.

Appendix

A. App's price and congestion

The following derivation follows from Author (2026).

Developers have technology that activates user i's utility of consumption $a_i > 0$ through better online algorithms or technology to match consumers and apps well based on individuals' consumption or search history, which is beneficial to predict their preferences on apps. On the platform, a potential user is matched with an app developer. The user i who wants to have specific good has the utility function $v_i = a_i - p_a$ with p_a being the price of the app, and for the units of apps sold to be m, a_i is uniformly distributed on $a_i \in [\phi q - \eta m, \phi q]$. The lowest willingness to pay among active participants becomes the price of the app, i.e., $p_a = \phi q - \eta m$.

Alternatively, as the number of developers increases, competition is intensified, and profits per developer decrease, following Hagiu (2009) and Belleflamme and Peitz (2019) (see footnote 10).

B. Derivation of (1), consumer surplus and regulation of apps' price

Consumers of the network good are heterogeneous in their preferences for the network good. Specifically, consumer k has the following net utility from purchasing the good:

$$u_k = b_k + \alpha m - p,$$

where b_k is a fundamental willingness to pay for the good and is assumed to be uniformly distributed, $b_k \in (0, \nu]$ while the last term p is the price of the good. We assume that consumers decide whether to buy one unit of the good or not. This means that consumer k buys the good if $u_k \geq 0$, or equivalently, if $b_k \geq p - \alpha m \equiv \underline{\nu}$ holds. Thus, consumers whose fundamental willingness to pay exceeds $\underline{\nu}$ buy the good, leading to (1):

$$q(p) = \int_{\underline{\nu}}^{\nu} 1 db_k = \nu - \underline{\nu} = \nu + \alpha m - p.$$

With the above features, we have the following formulation about the consumer surplus of the network good:

$$CS_q = \int_{\underline{\nu}}^{\nu} (b_k + \alpha m - p)\, db_k = \frac{1}{2}(\nu^2 - \underline{\nu}^2) - \underline{\nu}(\nu - \underline{\nu}) = \frac{1}{2}(\nu - \underline{\nu})^2 = \frac{q^2}{2}.$$

On the platform, each app user is matched with an app developer who has a relation-specific technology that generates a gross benefit to the consumer a through computer algorithms or other technologies. Following Appendix A, we have $p_a = \phi q - \eta m$. The price ceiling $\bar{p}_a = p_a - \delta_a$ merely distorts the entry decision of app developers, thereby reducing entry in (2) to $\bar{m} = \frac{M(\phi q^* - \delta_a - R^*)}{\overline{f}}$, and decreasing the network users in (1) to $q(p) = \nu + \alpha\bar{m} - p$.

C. Meaning of the SOCs

For the second-order conditions, we have $\frac{\partial^2 \pi_P}{\partial p \partial p}\gamma^{-1} = -2 < 0$, $\frac{\partial^2 \pi_P}{\partial R \partial R}\gamma^{-1} = -2\frac{M}{\overline{f}} < 0$, $\frac{\partial^2 \pi_P}{\partial p \partial R}\gamma^{-1} = -\frac{\alpha M}{\overline{f}} - \frac{\phi M}{\overline{f}}$. Therefore,

$$4\overline{f} - M\left(\alpha + \phi\right)^2 > 0$$

has to be satisfied, which is implied by $m^* \leq M/\underline{h}$ in (6).

As a benchmark, consider the case of $\phi = \alpha$ where $\nu - p^*|_{R^*=0} = \frac{\nu - c}{2}$ in (4). One can equate q^* in (6) with the induced demand in (1') as: $q^* = \frac{\overline{f}(\nu - c)/2}{\overline{f} - M\frac{\{\alpha+\phi\}^2}{4}} = \frac{\overline{f}(\nu - p) - \alpha M R}{\overline{f} - \alpha M \phi}$. Intuitively, when the platform utilizes externalities in its pricing, the multiplier (self-enforcing) effect of the retained benefit $\nu - p$ or $\frac{\nu - c}{2}$ on demand for the product is stronger in optimized demand than in induced demand, as reflected by $\frac{\{\alpha+\phi\}^2}{4} > \alpha\phi$ in their respective denominators when $R^* \neq 0$.

D Proof of Proposition 2

From (6) and (7) and $\dfrac{N}{1 + \eta N/\overline{f}} = M$:

$$CS_m + PS = (\eta + \frac{\overline{f}}{N})\frac{m^2}{2} = \frac{\overline{f}}{M}\frac{m^2}{2}.$$

Also, $m^* = 0.5(\alpha + \phi)\frac{M}{\overline{f}}q^*$. Setting $\delta = \sqrt{\frac{M}{\overline{f}}}\epsilon$, the losses in platform profit in (9.1) and that in (9.2) are equalized. In (8.1)-(8.4), $-dCS_q$ by the R-ceiling policy and $dPS + dCS_m$ by the p-ceiling policy

are positive. Therefore, (7) and $\sqrt{\frac{\overline{f}}{M}} > 0.5(\alpha+\phi)$ in Appendix C imply that:

$$
\begin{aligned}
&(dCS_q + dCS_m + dPS + d\pi_P)|_{d\bar{p}=-\delta,\ d\bar{R}=0.5(\alpha+\phi)\delta} - (dCS_q + dCS_m + dPS + d\pi_P)|_{d\hat{R}=-\epsilon,\ d\hat{p}=\epsilon(\alpha+\phi)\frac{M}{2\overline{f}}} \\
&> dq(\bar{p},\bar{R})\frac{M}{\overline{f}}\sqrt{\frac{\overline{f}}{M}}\epsilon q^* - \frac{\overline{f}}{M}dm(\hat{p},\hat{R})0.5\frac{M}{\overline{f}}(\alpha+\phi)\epsilon q^* \\
&> \frac{M}{\gamma\overline{f}}\epsilon q^* 0.5(\alpha+\phi)\{(1-\alpha\{\alpha+\phi\}\frac{M}{2\overline{f}}) - (1-\phi\{\alpha+\phi\}\frac{M}{2\overline{f}})\} > 0.
\end{aligned}
$$

We obtain the desired result. *Q.E.D.*

The above analysis shows that the argument is not necessarily symmetric when $R^* < 0$. If α is sufficiently close to ϕ, continuity implies that $dq(\bar{p},\bar{R})\frac{M}{\overline{f}}\sqrt{\frac{\overline{f}}{M}}\epsilon q^* - dm(\hat{p},\hat{R})0.5\frac{M}{\overline{f}}(\alpha+\phi)\epsilon q^*$ in the above formula remains positive (i.e., the p-ceiling may be better than the R-ceiling, given that $\sqrt{\frac{\overline{f}}{M}} > 0.5(\alpha+\phi)$).

E The general model

For $\Phi \equiv \phi q$, denote the number of apps sold by the general function $m = m(\Phi, R)$, and rewrite it as $m = \phi q\frac{\partial m}{\partial \Phi} - m_2 R$. m_2 is defined as the residual, which is approximated by $-\frac{\partial m}{\partial R}$ by the mean value theorem.

Substituting this relation into the demand function $q = q_1(p) + \alpha m = q_a + \frac{\partial q_1}{\partial p}p + \alpha m$, where q^a is the residual from the first-order approximation of $q_1(p)$, we obtain $q\{1-\alpha\phi\frac{\partial m}{\partial \Phi}\} \equiv q\gamma = q_a + \frac{\partial q_1}{\partial p}p - \alpha m_2 R$ and $dq\gamma = \frac{\partial q_1}{\partial p}dp + \alpha\frac{\partial m}{\partial R}dR$ to obtain $\frac{\partial q}{\partial p}$ and $\frac{\partial q}{\partial R}$. In line with our linear model, we define $\nu = \frac{q^a}{-\partial q_1/\partial p}$. The demand function $q(p, \alpha m)$ allows for a wide range of specifications, including the widely used logit model as a special case.

To maximize $\pi_P^* = (p-c)q^* + Rm^*$, with $n \equiv \frac{\partial m/\partial \Phi}{m_2}\frac{-\partial q_1}{\partial p}$ and $\epsilon_{m_2 R} = \frac{\partial(\phi q\frac{\partial m}{\partial \Phi})/\partial R}{m_2} - \frac{\partial m_2/\partial R}{m_2}R$ that incorporates the elasticity of m_2 with respect to R:[20]

$$
\begin{aligned}
&\frac{\partial \pi_P^*}{\partial R}|_{\frac{\partial \pi_P^*}{\partial p}=0}\frac{\gamma}{m_2} = -R^*\left\{2 - \epsilon_{m_2 R} - \left(\frac{\alpha m_2/2}{-\partial q_1/\partial p} + \phi\frac{\partial m/\partial \Phi}{2}\right)\left(\alpha\left(-\frac{\partial m/\partial R}{m_2}\right) + \phi n\right)\right\} \\
&+ [\phi n - \alpha(-\frac{\partial m/\partial R}{m_2})]\frac{\nu - c}{2} = 0.
\end{aligned}
$$

[20]We use the following two formulas:

$$
\begin{aligned}
&\frac{\partial\left((p-c)q^* + Rm^*\right)}{\partial p} = -p(-2\frac{\partial q}{\partial p}) + c(-\frac{\partial q}{\partial p}) + \frac{q_a}{\gamma} - R(\alpha\frac{m_2}{\gamma} - \phi\frac{\partial m}{\partial \Phi}\frac{\partial q}{\partial p}) = 0 \\
&\frac{\partial\left((p-c)q^* + Rm^*\right)}{\partial R}\frac{\gamma}{m_2} = \phi(\nu - p)n - 2R^* + \epsilon_{m_2 R}R^* + \frac{\gamma}{m_2}\frac{\partial q}{\partial R}(p-c) = 0
\end{aligned}
$$

The term in the curly brackets of the above first-order condition corresponds to $2 - M(\alpha + \phi)^2 / (2\overline{f})$ in our basic model, which is assumed to be positive.

Author Contribution information

This is a single-authored paper. Yukihiro Nishimura is the only on who fulfills Authorship definition.

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