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Tax Incidence of VAT Enforcement Reform for Foreign Services and Small Businesses in Two-sided Markets*

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Abstract

Value-added tax (VAT) has two major problems in its enforcement: taxing foreign vendors which do not have a business entity in the destination country, and taxing small and medium-sized enterprises (SMEs) with small tax bases. As a solution of these problems, some countries attempt to utilize online platform, to let the platformer pay the foreign firms' and SMEs' VAT according to the sales each firm made (platform tax for the destination principle and formalization of informal sector). In the monopolistic market where the platformer determines the fees for the network entry and the commission fee of the platform services, standard-good's price, we show that taxing foreign developers increases the tax burden laid on the standard good, and we show that the increased tax burden is born 100% by the domestic standard-good's consumers. We also investigate whether or not the prevention of tax leaks by platform taxes improves the vendors' entry and tax revenue of the destination country. The effect of the tax reform on home developers crucially depends on the responsiveness of the developers' entry to the number of network users, which is decreasing in the VAT rate. The derived formula of marginal value of public funds suggests that, due to the simultaneity of price and quantity, more fully fledged structural analysis may be necessary. Additionally, we show that the VAT serves as a Pigouvian tax to ease congestion externalities, and our results are robust with platform competition. In the context of formalization of SMEs, the strength of network externalities matters to see if the existing formal sector receives windfall gains or losses.

Keywords: Digital economy; Platform; Network externality

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1 Introduction

Fiscal capacity in modern economies often hinges not only on statutory tax rates but also on the effectiveness of enforcement and compliance mechanisms. When increases in statutory tax rates are constrained by political considerations, strengthening tax capacity through enforcement reform becomes a key policy margin. Data from the Japanese Ministry of Finance indicate that revenue collected per 1 percentage point of the value-added tax (VAT) rose by 10% over the past two decades,[1] reflecting improvements in enforcement.

Yet enforcing VAT presents persistent challenges. In particular, VAT has two major problems: taxing foreign vendors that operate without local subsidiaries; and taxing small and medium-sized enterprises (SMEs) with limited reporting obligations.[2] In these cases, domestic tax authorities face difficulties to collect VAT from them effectively. Japan introduced VAT in 1989 amid strong political opposition. As a result, the system initially featured weak enforcement for cross-border and small business transactions, leading to an unfair but lower tax burden on foreign firms and SMEs. The "border tax" argument (Keen, 2007, 2008) becomes less effective when transactions are intangible and difficult to track for tax purposes.[3] As a related issue, VAT collection becomes challenging when consumption involves SMEs, or when the new type of business models emerge, such as recent cases of Uber and Air Airbnb in their beginning period.

To address the issue of tax leak, several new tax reforms have recently been implemented. For example, a key trend involves shifting VAT collection responsibilities to intermediaries (platform-based collection). Starting in 2025, Japan will require platforms like Apple to collect and remit VAT on behalf of foreign developers. Similar measures have been implemented in Canada to enhance compliance without obligating foreign firms to establish local entities. This approach reinforces the destination principle and is expected to boost Japan's tax revenues. In the context of the new economy, which often gives rise to unregistered firms, the Japanese government sought to leverage information from platforms handling Airbnb bookings and payments to enhance regulatory oversight. Also, the registration of businesses through digital platforms is of particular

[1] Although the revenue from a one-percentage-point increase in the VAT rate declined due to preferential treatment for certain categories such as food starting from 2019, the underlying revenue gain remains substantial after adjusting for these policy-induced reductions.

[2] A well-known example of tax leak under the formerly adopted *source principle* is the case of e-books and digital music content provided by Amazon.jp. Because these services were delivered through the American entity Amazon.com Int'l Sales, Inc., Amazon.jp was exempted from paying VAT on purchases made by Japanese customers. This unfair tax treatment of domestic content producers was addressed in 2015, when Japan changed the digital VAT nexus from the seller to the customer, thereby applying the destination principle to digital services.

[3] Keen (2007) and Keen (2008) observed that, in developing countries, a substantial portion of VAT is collected at the national border, and that the administrative costs of collecting consumption taxes on imported goods are comparable to those of import tariffs. However, a critical omission in this argument is the treatment of services — especially online services — whose provision is not constrained by national borders.

interest to government officials in developing countries (e.g., Bussolo et al. (2025)).[4] Moreover, Agrawal and Wildasin (2020) argued that taxing e-commerce helps fulfill the destination principle. In this context, the new rules aimed at preventing foreign content developers from avoiding tax through platform intermediaries promote fair tax burdens between domestic and foreign developers, and appear desirable for the countries that implement them.

Online markets are often characterized by oligopolistic structures, dominated by a few large firms with price-setting power. Against this backdrop, this paper addresses the following key questions: To what extent is the tax burden from a VAT reform shifted to consumers? And how can the trade-off between the welfare loss by market participants and increased tax revenue be formalized? And how does the reform affect the entry and the revenue of VAT-registered firms prior to the reform?

To address the challenges of VAT enforcement in the digital economy, [5] we develop a two-sided market model inspired á la Armstrong (2006) and and Hagiu (2009) by Rasch and Wenzel (2013), Hayakawa et al. (2025) and Wu et al. (2023), incorporating both the standard goods and developers of relation-specific goods, such as online apps, from home and foreign countries. Since we examine the enforcement reform, we apply enforcement elasticity by Keen and Slemrod (2017) in the analysis. Our model focuses on a foreign monopoly platformer (e.g., Apple) operating an online marketplace (App Store) that serves home-country consumers. Both domestic and foreign developers participate in the platform, paying a commission fee set by the platformer. In contrast to prior models, we explicitly integrate the dynamics of platform-based VAT collection and its regulatory implications, which have drawn increasing scrutiny from antitrust authorities concerned with fair pricing. Here, consumers' tax burden arises due to cross-market pass-through. As in Copestake and Bellon (2022), the burden of tax changes (specifically, the increased burden on foreign firms in the current context) is passed on to prices, and these effects depend on the market structure. In the present context, network externalities influence platform pricing and firms' participation decisions. After showing that that ad valorem taxes are superior to unit taxes in two-sided markets (consistent with Kind et al. (2008), Belleflamme and Toulemonde (2018) and Kind and Koethenbuerger (2018)), we show that taxing foreign firms through the proposed tax reform leads to a reduction in the commission

[4]Beyond VAT registration, the visibility of business activity through online transactions has broader regulatory implications. For instance, the Ministry of Labour can monitor working hours and employment status in such firms, while the Fair Trade Commission can assess whether underlying business practices are fair — for example, in the case of Uber Rides versus the traditional taxi industry.

[5]Our related work, Morita and Nishimura (2025), showed that he tax reform increases the price of the standard good and reduces commission fee (Lemma 1 of this paper). Also, the VAT reform increases the number of domestic (home) app developers if the entry responsiveness is low (our Equation (10)). Other than thsese, the propositions we obtained in this paper are independent of their paper.

fees set by the monopolistic platformer. However, since the platformer uses these commission fees to subsidize the price of the standard good, the reform effectively shifts the tax burden onto the standard good. We find that this increased burden is fully passed on to consumers of the standard good. This result is derived by examining changes in tax incidence in a multi-sided monopolistic market with ad valorem taxation, offering a complementary perspective to Weyl and Fabinger (2013) and Adachi and Fabinger (2022)'s quantitative assessment of tax burdens under imperfect competition.

To examine how the tax reform affects the home country, and formalize the trade-off between the welfare loss by market participants and increased tax revenue, by using the Marginal Value of Public Funds (MVPF). The current international tax reform includes the tax-on-non-residents effects as in Wildasin (1987) and Dahlby (2008). We also show when this figure becomes less than 1, and also when this figure is not increasing in the tax rates. Our formula suggests that, due to the simultaneity of price, quantity, and firm entry decisions, instead of recent parsimonious reduced-form methods, more fully fledged structural analysis, such as multi-stage least squares, may be necessary. Additionally, we show that the VAT serves as a Pigouvian tax to ease congestion externalities (the examples in two-sided markets are provided in Belleflamme and Peitz (2019)), and our results are robust with platform competition.

We also demonstrate that our analysis and conclusions extend to cases where the platform-based tax is applied as a tool for VAT enforcement on SMEs — a policy-relevant scenario for developing countries. A common concern regarding such reforms is that new tax burdens on SMEs may lead to windfall gains for VAT-registered firms that existed prior to the reform. In response to this, we show that, if we observe increase in profits for formerly registered firms after the reform, it implies a modest contraction in the number of network users and a limited exit of SMEs. However, if the responsiveness of developers' market entry to the number of users (which we refer to as *entry responsiveness*) is high, then these outcomes are reversed, leading to greater market shrinkage and higher SME exit rates.

The remainder of the paper is organized as follows. Section 2 introduces the model; Section 3 derives the equilibrium. Section 4 examines tax incidence. The final section concludes. The proofs are provided in the Appendix.

2 Basic setup

Consider the following digital economy. A large foreign firm, referred to as the *platformer*, has its own online technology to operate its platform, such as App Store, in the home country (country H). The platformer facilitates in transactions in country H involving two types of goods: a standard good and a relation-specific good.

We consider the following two-stage game. In Stage 1, the platformer sets the price p of the standard good and the commission fee R for the specific-goods' developers. In Stage 2, consumers, specific-goods' developers, and the specific-good users make their adoption decision simultaneously. When variables associated with network externalities are simultaneously determined, each party forms rational expectations about the others' behavior.

The tax liability varies depending on the location of the specific-goods' developers. For example, in 2022 of Japan, more than half of the top 100 smartphone games were developed and released by foreign firms, yet many of these companies avoided paying VAT to Japan.[6] As mentioned in the Introduction, in 2025 Tax Reform, platform-based taxation is introduced to eliminate such location advantage. We introduce the formal model that captures the changes of equilibrium transactions and the Marginal Value of Public Finance as a result of the welfare changes.

2.1 Demand for the standard good

Following Wu et al. (2023), consumers for the standard goods have the following heterogeneous preference. Consumer k has the following net utility from purchasing the good,

$$u_k = b_k + \alpha m - p,$$

where b_k is fundamental willingness to pay for the good and is assumed to be uniformly distributed, $b_k \in (-\infty, \nu]$ whereas the last term p is the price for the good. We assume that consumers decide to buy one unit of the good or not. This means that consumer k buys the good if $u_k \geq 0$ or equivalently $b_k \geq p - \alpha m \equiv \underline{\nu}$ holds, which implies that consumers having higher fundamental willingness to pay than $\underline{\nu}$ buy the good, leading tothe following aggregate demand:

$$q(p) = \nu + \alpha m - p. \tag{1}$$

[6] See `https://www.nikkei.com/article/DGKKZO67632310X10C23A1MM8000/`, accessed 2025 February 5th.

where ν and p represent, respectively, the maximum willingness to pay for the good and its price, and the second term, αm, captures the network effects from specific-goods' variety.[7] m represents the total number of the specific goods sold in the platforms and α is the degree of such network externality.

This consumer-side network externality increases the willingness to pay for the standard good, because more specific goods provide consumers of the good with a wider variety of options and allow them to use their favorite specific goods. Note that such an increase in willingness to pay for the good depends not only on the number of specific goods but also on the speed of internet connection. The network parameter α reflects the level of digitization, and we interpret an exogenous increase in α — for example, the introduction of 6G, which allows high-speed data transfer.[8] Such network externalities generate extra surplus specific to this platform.

As for the production, we assume that the standard good is produced with a constant marginal cost c. The platformer either sets the maximally negative entry price as $-c$ to the standard-good's seller that the platformer can charge according to the sales, or it outsources its production.

2.2 The specific good's price and developers' entry

The relation-specific goods (or "specific goods" in short) sold in the platform are developed by a potentially large pool of developers, M, in country H and and an equally sized pool M in a foreign country, labeled country F. These developers are assumed to supply one product each and are heterogeneous in terms of the fixed costs of developing the product, denoted by f_S, which is uniformly distributed over $[0, \overline{f}]$.

The products are demanded by a potentially large pool of users greater than $2M$ in the home country. User i has the utility function $v_i = \{a_i - p_{ai}\}A_i$ with p_{ai} being the relation-specific price of the product that a developer sets for user i, and $A_i = 0$ or 1, indicating whether consumer i buys the product. Online products such as apps would be examples that entail the network-size externalities as mentioned below. Consumer i is matched with a domestic or a foreign developer. Since the potential users' population is greater than the number of developers, there are people who cannot obtain the relation-specific product. Both domestic and foreign developers have relation-specific technology that activates gross benefit of consumer a_i through computer algorithms or technologies.[9] Following Rasch and Wenzel (2013) and Wu et al. (2023), the network externality

[7] We assume that $\nu > p$ holds in the equilibrium throughout the analysis. This implies that some consumers will still demand the standard good even in the absence of additional utility from network effects. Also,

[8] In the last two decades, the percentage of individuals using the internet has grown: It was 7% in 2000 and 67% in 2023. See `https://data.worldbank.org/indicator/IT.NET.USER.ZS`, accessed 2025 February 5th.

[9] Such technology allows, for example, the developers to match consumers and products well based on individuals'

generated by the users of size q gives the gross benefit of $a_i = \phi q$,[10] and note that the user's benefit is realized only upon the developer's technology; that is, $a_i = 0$ when developers do not give a product for user i. With the fallback option's utility being 0, the buyer of the product has no other choice but to accept the maximal acceptable price:

$$p_{ai} = \phi q \quad \text{for all } i.$$

The parameter $\phi > 0$ represents the network-size externality. This network externality captures, for example, improvements in online algorithms or technologies that better match consumers with products based on individual consumption or search histories — features that help predict users' preferences. Therefore, a larger ϕ reflects a different aspect of digitization than α.

In addition, developers must pay a commission fee, R, to the platformer to sell their product on the platform. We allow R to be either positive or negative, where a negative R is interpreted as a subsidy from the platformer to the developers.[11] The platformer's choice of R has become under increased scrutiny by antitrust authorities due to concerns over fair pricing.[12]

We follow the structure introduced by Hayakawa et al. (2025). Taxation reduces the after-tax value of product production externalities by increasing the cost of providing online services. In our model, $p_{ai} = \phi q$ is subject to a VAT at rate τ and it generates tax revenues. Prior to the tax reform, only domestic developers in country H are liable to pay the VAT. In contrast, foreign developers located in country F are able to avoid such taxation, as the tax authority in country H lacks the jurisdiction to enforce tax collection or impose compliance costs on those foreign firms. After the tax reform is in force, domestic (home) developers remain subject to VAT, whereas foreign developers pay a VAT-inclusive commission fee to the platformer. Let τ denote the VAT rate in country H, and let η be a binary indicator equal to zero before the tax reform and one after its implementation. Then, the entry conditions for specific-good developers from each country based on their (expected) post-tax profits, π_{SH} and π_{SF} for developer S in each country, and the number

consumption or search histories which are beneficial to predict their preference on products.

[10]Strictly speaking, as other externalities mentioned in the text, the right-hand side is the rational expectation value $\phi q_e = \phi q$, which each market participant regards to be given (Katz and Shapiro (1985) and Wu et al. (2023)).

[11]A negative R can be interpreted as a non-pecuniary subsidy, such as technical support offered to developers — for instance, by providing development toolkits. Apple, for example, allows developers to apply for an Apple Vision Pro Developer Kit (see `https://developer.apple.com/visionos/developer-kit/`, accessed 2025 February 5th). Rysman (2009) provides another example: Microsoft, as a supplier of computer operating systems, makes it very easy to become a software developer for the Windows OS and arguably subsidizes this activity through tutorials and dedicated support websites.

[12]For example, in 2024, the European Commission found that the fees charged by Apple to developers exceeded fair levels. See `https://ec.europa.eu/commission/presscorner/detail/en/ip\_24\_3433`, accessed 2025 February 13th. In response to the Commission's recommendations, Apple announced a policy to reduce its fees. See `https://developer.apple.com/jp/support/dma-and-apps-in-the-eu/\#app-analytics`, accessed 2025 February 13th.

of entered developers are given by:

$$\pi_{SH} = \phi q - R - \tau\phi q \geq f_S \equiv \overline{f}_{SH}, \rightarrow \ m_H = \left(\frac{\overline{f}_{SH}}{\overline{f}}\right) M = \frac{M[(1-\tau)\phi q - R]}{\overline{f}}, \tag{2}$$

$$\pi_{SF} = \phi q - R - \eta\tau\phi q \geq f_S \equiv \overline{f}_{SF}, \rightarrow \ m_F = \left(\frac{\overline{f}_{SF}}{\overline{f}}\right) M = \frac{M\left[(1-\eta\tau)\,\phi q - R\right]}{\overline{f}}, \tag{3}$$

where f_S is assumed to be uniformly distributed over the interval $[0, \overline{f}]$, so that the first parenthesis represents the entry rate. As for the network-entry fee, R exhibits standard buyer-seller independence: whether the platform charges the fee R to the developer of the relation-specific good or to its consumer is irrelevant for incidence. In the latter case, the developer sets a price of $(1-\eta\tau)p_{ai} - R$ for consumer i, who then decides whether to accept the price. By summing up the entries from domestic and foreign developers, we obtain the total number of relation-specific products on the platform, $m = m_H + m_F$. The number of users, as previously formulated, satisfies $\int_{A_i=1} 1di = m_H + m_F$.

2.3 Induced demand and platformer's profit

The two-stage game is solved backwards. In Stage 2, the standard-good and the specific-good markets are mutually dependent, so (1), (2) and (3) solve three unknowns q, m_H and m_F, with $m_e = m_H + m_F$ by consumers and $q_e = q$ by the specific-good users and the developers. We then derive them as functions of p and R (determined in Stage 1) and τ (given by the government) as follows: in

$$q(p,R) = \frac{\overline{f}(\nu - p) - 2\alpha MR}{\overline{f} - \alpha M\phi\{2-(1+\eta)\tau\}}, \tag{1'}$$

$$m_H(p,R) = \frac{M\left[\{1-\tau\}\,\phi q(p,R) - R\right]}{\overline{f}}, \tag{2'}$$

$$m_F(p,R) = \frac{M\left[\{1-\eta\tau\}\,\phi q(p,R) - R\right]}{\overline{f}}, \tag{3'}$$

We assume the consumption of q is perfectly taxed (at the end of Section 5, we briefly comment when some q-producers are unregistered before the tax reform).[13] In addition, the platform itself operates with zero marginal cost. Let t be VAT rate in country H on the standard good where we have $t = \tau$ since we consider the general consumption tax. We distinguish the notation for

[13]The standard good is produced by either domestic (and large) firms, or the platformer outsources the production and complies the tax on country H.

clarification in the following analysis. The platformer's post-tax profit is then given by:

$$\pi_P = (p(1-t) - c)q + R(m_H + m_F) \tag{4}$$

where q, m_H and m_F follow (1), (2′) and (3′). The tax obligation to the standard good is explicit in the first term, whereas for the specific good, the tax obligation is implicit in (2) and in (3). In Regime T, the platformer collects the VAT from the foreign developer $\tau\phi q m_F$ and remits this to the home tax authority. The revenue to receive from the developers are $R(m_H + m_F)$, where R, m_H and m_F depend on the tax regime.

Since our primary focus is on tax reform, it is useful to revisit the principle of physical-incidence neutrality (e.g., Weyl and Fabinger (2013)), which we confirm in Appendix A.

To ensure an interior solution throughout the analysis, we adopt the following assumption. For the tilde being the value divided by $1-t$, e.g., $\widetilde{\phi} \equiv \frac{\phi}{1-t}$:

Assumption 1. $\frac{\overline{f}}{M}$ is sufficiently high to satisfy

$$\frac{\overline{f}}{(1-t)M} \geq \frac{\{2\alpha + \widetilde{\phi}(2-\tau)\}^2}{8} + \frac{(2\alpha + (2+\tau)\widetilde{\phi})(\nu - \widetilde{c})}{8M} \tag{5}$$

The first component of condition (5), $\frac{\overline{f}}{(1-t)M} > \frac{\{2\alpha+\widetilde{\phi}(2-\tau)\}^2}{8}$, ensures that the induced demand function (1′) is downward-sloping in p. The second part, involving $\frac{\nu-\widetilde{c}}{M}$, is that the potential number of app developers M is sufficient to absorb the market demand arising from the net market size $\nu - \widetilde{c}$. In the following section, we solve for the platformer's optimal pricing strategy, (p, R), under both the pre- and post-tax reform regimes.

3 Equilibrium

3.1 Entry responsiveness and equilibrium price of the standard good

The case before the tax reform is denoted by the superscript "O", and we use the superscript "T" for the case after the introduction of the platform tax. By differentiating $\pi_P^* = (p-c)q^* + R(m_H^* + m_F^*)$ for $* = O$ and T with respect to p and R, and taking into account (1′), (2′) and (3′), the first-order condition (FOC) $\frac{\partial \pi_P^O}{\partial p} = 0$ yields the following. From (1′), we have $\frac{\partial q}{\partial p} = \frac{-\overline{f}}{\overline{f} - \alpha M\phi\{2-(1+\eta)\tau\}} \equiv -\frac{1}{\gamma} < 0$,

$q = \frac{1}{\gamma}\left(\nu - p - \frac{2\alpha RM}{\overline{f}}\right)$ and from (2′) and (3′), we have $\frac{\partial m}{\partial p} = \{2-(1+\eta)\tau\}\phi\frac{-1}{\gamma}\frac{M}{\overline{f}}$. Therefore:

$$\frac{\partial \pi_P}{\partial p}\left(-\frac{\partial q}{\partial p}\right)^{-1} = -\big(p(1-t)-c\big) + (1-t)(\nu - p) - \frac{2\alpha R(1-t)M}{\overline{f}} - R\frac{[2-(1+\eta)\tau]\widetilde{\phi}(1-t)M}{\overline{f}} = 0, \tag{6A}$$

which derives that taxation of standard goods distorts marginal benefit alignment among users, for $\nu - p^*$:[14]

$$\nu - p^* = \frac{\nu - \widetilde{c}}{2} + \{\alpha + (1 - 0.5(1+\eta)\tau)\widetilde{\phi}\}\frac{M}{\overline{f}}R^* \tag{6}$$

(6) assesses the buyers' burden from the vertical axis of p-q space with respect to the willingness to pay. As we see, the price p^* has $p^T > p^O$ so the tax burden increases.[15] The right-hand side of (6) is composed of the after-tax gains of trade $\nu - \widetilde{c}$ and the cross-market link by the platformer's pricing in the last term of the right-hand side. If the commission fee is positive, the price of the standard good reduces as the commission fee revenue can be used for increasing the network size. On the other hand, if $R^* < 0$, then the increase of p^* occurs since entries of more developers would scale up the consumption externality αm. Later we show that the network externalities from the specific-goods' variety αm^* in (1) decreases, so the tax reform reduces the hight of the demand function, too.

In general, the last term of consdumers' net gain consists of (i) m's sensitivity to q in (2′) and (3′), $\frac{\partial m}{\partial q} = \frac{M}{\overline{f}}(2-(1+\eta)\tau)\phi$, which is increasing in responsiveness of the specific-good developers from the number of users and decreasing in firm heterogeneity $\overline{f}$, matters for production side, and (ii) the number of specific-good developers ($\alpha\frac{M}{\overline{f}}$ increasing in M) matters for consumption side. From this perspective, hereafter we call the corresponding parameter $\frac{(1-t)M}{\overline{f}}$ as *entry responsiveness*.

Before we move on, we see one feature which is consistent with the previous literature (Kind et al. (2008), Belleflamme and Toulemonde (2018) and Kind and Koethenbuerger (2018)). In Appendix B, we show that the unit tax linearly separates the tax burden so that the alleviated burden from ad-valorem tax when $R^* > 0$, $\widetilde{\phi} > \phi$ in (6), does not appear in the corresponding unit-tax case.

[14]If we have the aggregate demand function as $q = \mu + \alpha m - dp$ with d representing the slope of the demand, then $\nu = \frac{\mu}{d}$, and the third term of the right hand side of (6) becomes $\frac{\{2\alpha/d + (2-(1+\eta)\tau)\widetilde{\phi}\}MR^*}{2\overline{f}}$.

[15]Formally, it can be written as $b_k - p^*$ for the willingness of pay for consumer k with $b_k + \alpha m - p \geq 0$. As a related study, Weyl and Fabinger (2013, Section III) and Adachi and Fabinger (2022) showed that the sum of buyers' and the (monopolistic) seller's burdens from the consumption tax is, as the product of the horizontal coordinate q^* and the incidence on the product's price.

3.2 Price incidence of tax reform ($\eta = 0$ to 1)

Turning to the first-order condition for the commission fee R, let $den^* = 2 - (\alpha + (1 - 0.5(1 + \eta)\tau)\phi)^2 \frac{(1-t)M}{\overline{f}} > 0$ ($den^T > den^O$) be the value that appears in the denominator of R^*. We show the following in Appendix C:

$$R^O - R^T = \frac{1}{den^T}\{\frac{((1-t)\nu - c)}{4}\tau\tilde{\phi} + R^O(den^T - den^O)\} \tag{7}$$

Tax reform deters some foreign developers from entering the platform so the platformer lowers commission fee ($R^T < R^O$). This occurs when $R^O \geq 0$. Also, when the entry responsiveness $\frac{(1-t)M}{\overline{f}}$ is low, the first term in the right-hand side of (7) exceeds the second term even if the latter is negative. Consequently, after the reform, the platformer reduces R. Appendix D further shows that a similar property holds for the standard-good's price, i.e., $p^T - p^O > 0$. Lemma 1 is due to Morita and Nishimura (2025) which assumes $t = 0$ and focuses on the tax incidence of τ on the market. The novelty lies in the use of this lemma in our Proposition 1 below.

Lemma 1. *Under any of the following conditions: (a) The pre-reform commission fee (R) is nonnegative; or (b) entry responsiveness ($\frac{(1-t)M}{\overline{f}}$) is sufficiently low (i.e., developers' are not sensitive for entry to prices); or (c) the net market size ($\nu - \frac{c}{1-t}$) is sufficiently large. The tax reform increases the price of the standard good and reduces commission fee.*

4 Tax incidence

4.1 The incidence of tax reform on the standard good's consumers

Substituting (6) and R^* in Appendix C into (1')-(3'), we have

$$q^* = \frac{\overline{f}(\nu - \frac{c}{1-t})}{2\overline{f} - (1-t)M\{\alpha + (1 - 0.5(1+\eta)\tau)\frac{\phi}{1-t}\}^2}, \quad m^* = \frac{\{(1 - 0.5(1+\eta)\tau)\tilde{\phi} + \alpha\}(\nu - \frac{c}{1-t})(1-t)M}{2\overline{f} - (1-t)M\{\alpha + (1 - 0.5(1+\eta)\tau)\frac{\phi}{1-t}\}^2}. \tag{8}$$

In a different context from Etro (2023), the above formula shows that the higher entry response is beneficial for the market size. However, what matters here is its *decline after the tax reform* which becomes greater. Later we show that entry responsiveness crucially matters for the domestic firms' entry.

Electric devices getting access to online platform require are often characterized as a knowledge intensive industry, firms are sometimes criticized of using its high market power and setting their high prices.[16] In the present context, the platformer's sales corresponding to the extra part in pricing in (6) would cancel out with the commission fee. Namely, from (8), $R^*(m_H^* + m_F^*) = R^* \frac{(1-t)M}{\overline{f}}\{(1 - 0.5(1+\eta)\tau)\widetilde{\phi} + \alpha\}q^*$, and thus:

$$\pi_P^* = (\frac{(1-t)\nu - c}{2} - \frac{\{2\alpha + (2-(1+\eta)\tau)\widetilde{\phi}\}(1-t)MR^*}{2\overline{f}})q^* + \frac{(1-t)M}{\overline{f}}\{(1-0.5(1+\eta)\tau)\widetilde{\phi} + \alpha\}R^* q^*$$
$$= \frac{\nu(1-t)-c}{2}q^*. \tag{9}$$

The correct reference of the real burden of the standard-good supplier is the reduction of π_P^* from the case without tax on q. The gains of trade $\nu - c = \nu - p^* + p^*(1-t) - c + tp^*$ consists of: (i) $\nu - p^*$ which decreases after the tax reform in (6), (ii) $p^*(1-t) - c = \frac{\nu(1-t)-c}{2} - \{\alpha + (1 - 0.5(1+\eta)t)\widetilde{\phi}\}\frac{(1-t)M}{\overline{f}}R^*$ out of which, from (9), only the amount $\frac{t\nu}{2}$ is the platformer's loss, and (iii) tp^* which increases by Lemma 1.

Lemma 2. Tax reform does not change the platformer's per-unit burden of the VAT as $\frac{t\nu}{2}$.

Importantly, tax reform does *not* increase the supplier's tax burden per unit of the good, even though the per-unit tax tp^* *increases* by $p^T > p^O$. Furthermore, this amount appears as the burden to consumers even though the platformer does not have an increase of per-unit burden. As a result, *consumers entirely bear the increase of per-unit tax as a result of tax reform.*

Going back to the expression of consumers' willingness to pay in (1), we see that, on the top of $p^T > p^O$, since we have shown above that $m^T < m^O$ happens, we conclude that consumers get worse off as a result of the tax reform by lower consumption, higher price and lower consumption externalities.

Proposition 1. *(i) The standard-good's consumption q^* increase in entry responsiveness. However, high entry responsiveness also brings greater decline of $q^O - q^T$ and discourages domestic firms' entry.*

(ii) The monopolistic supplier only partially bears the burden of the VAT imposed on the standard good. Furthermore, the platformer's effective tax burden does not increase under the VAT reform; instead, the entire increase in the unit tax burden $tp^T - tp^O > 0$ is passed on to consumers of the standard good.

[16] For example, the U.S. department of Justice accused Apple of monopolizing Smartphone market. See `https://www.reuters.com/legal/us-takes-apple-antitrust-lawsuit-2024-03-21/`, accessed 2025 February 5th. European Union has the digital market act in 2023 to ensure for all businesses, contestable and fair markets in the digital sector. Inspired by the digital market act, Japan also enacted so called the Smartphone act to stop large companies such as Google and Apple from taking advantage of their position to give their own products "a competitive advantage" and from "imposing disadvantages on business users". See `https://eu-renew.eu/is-the-eus-digital-markets-act-going-global-how-japan-is-crafting-its-own-version-of-digital-regulation-with-the-smartphone-act/`, accessed 2025 February 5th.

Part (i) of the proposition follows (10) in the text and (A-6) in the Appendix. Part (ii) is shown above.

Often, the MNEs which conduct multiproducts' monopoly voluntarily offer the consumption tax payments to the destination countries according to the destination principle. However, the platformer is willing to do this offer since, through its price setting power and interdependence in the multi-sided market, it makes *consumers* bear the increased tax duties.

4.2 Domestic firms' profits and tax revenue

Let us turn our analysis to the firms' behavior. We have $m_H^* = (\alpha + \phi - (1.5 - 0.5\eta)\tau\frac{\phi}{1-t})\frac{(1-t)M}{2\overline{f}}q^*$, and entry responsiveness $\frac{(1-t)M}{\overline{f}}$ relates to both the domestic entrants' profits π_{SH} in (2) and TR^* below. We have:

$$\pi_{SH}^T - \pi_{SH}^O \propto \frac{0.5\tau\widetilde{\phi}}{2}q^T - \frac{(\alpha + \widetilde{\phi} - 1.5\tau\widetilde{\phi})}{2}(q^O - q^T) \gtreqless 0$$
$$\Leftrightarrow \frac{\overline{f}}{M} \gtreqless (1-t)\frac{\{2\alpha + (2-\tau)\widetilde{\phi}\}^2}{8} + (1-t)\frac{(2\alpha + 2\widetilde{\phi} - 3\tau\widetilde{\phi})(4\alpha + 4\widetilde{\phi} - 3\tau\widetilde{\phi})}{8} \equiv (1-t)K(\alpha, \widetilde{\phi}, \tau), \tag{10}$$

with $t = \tau$ for the general consumption tax. The interpretation of (10) follows Morita and Nishimura (2025) which considered the case of $t = 0$. The tax reform causes the replacement of domestic developers with the exited developers in country F (the first term of the first line of (10)). By the specific good's price in Section 2.2, this effect is proportional to p_{ai} in the post-reform regime. However, the tax reform also reduces consumption of the standard good (the second term of the first line of (10)). So the tax reform brings offsetting positive and negative effects on tax revenue and the domestic developers' profits π_{SH}. See also Online Appendix B.2.

The tax revenues in each scheme are formulated as $TR^O = \tau\phi m_H^O q^O$ and $TR^T = \tau\phi(m_H^T + m_F^T)q^T$, and we have the revenue from the network as tp^*q^*, we can compute the effects of the tax reform on tax revenue in country H is

$$TR^T - TR^O = \tau\phi(2m_H^T q^T - m_H^O q^O) + t(p^T q^T - p^O q^O).$$

Substituting the equilibrium price of the good and commission fee, we have $q^T < q^O$.

A feature particular to the imperfect competition with cross market pass through is the following: even with the reduced opportunity of earning profits, the taxable sales p^*q^* does not necessarily decrease after the tax reform since $p^T q^T(p^T, R^T) > p^O q^T(p^O, R^O)$.[17] We show the following in Appendix E:

Proposition 2. *Tax reform increases the tax revenue if: (i)* $R^O \geq 0$ *(the network-size externalities are dominant for the commission fee), (ii)* $0.30 > t$*, (iii)* $\frac{\bar{f}}{M}$ *is sufficiently large so that* $q^T/q^O \geq 0.75$ *or* $q^T/q^O \leq 0.5$*, and (iv) the market size with distorted marginal cost is still large:* $(1-t)(\nu - \frac{c}{1-t})/(2M) \geq \phi$ *and* $0.5(\nu - \frac{c}{1-t}) > \frac{c}{1-t}$*.*

4.3 Marginal value of public funds

The tax reform brings both positive and negative effects in country H. It is fairly clear from Section 4.1 that consumer surplus decreases ($\Delta CS < 0$) by the tax reform of taxing foreign developers (formally, see (A-6) and footnote 18). For the domestic producer surplus (PS_H), Online Appendix B.1 shows that it is $\frac{\pi_{SH}^2 M}{2\bar{f}}$. We show Marginal Value of Public Funds (MVPF) as the ratio of the burden or the loss of the private sector in country H (the sum of the decrease of consumer surplus and the decrease of domestic producer surplus) to the tax revenue TR: MVPF $= \frac{-\Delta CS - \Delta PS_H}{\Delta TR}$.[18] The denominator is decomposed as $TR^T \equiv TR^O + \tau\phi m_F^T q^T + BE(q^O)^2$ for to separate, as a conventional exercise, the tax-base expansion and the behavioral effects BE:

Proposition 3. *The Marginal Value of Public Funds (MVPF) is described as follows: for* $b = q^T/q^O < 1$*,*

$$MVPF = \frac{\dfrac{1-b^2}{2} - \dfrac{\left\{\left((\alpha+\phi)\frac{\sqrt{1-t}}{2}b\right)^2 - \left(((\alpha+\phi-1.5\tau\frac{\phi}{1-t})\frac{\sqrt{1-t}}{2}\right)^2\right\}(1-t)M}{2\bar{f}}}{\tau\phi(\alpha+\phi)\frac{(1-t)M}{2\bar{f}}b^2 + BE} \tag{11}$$

(10) suggests if the second term in the numerator is positive (and thus it reduces the MVPF) or not.

[17] Let $q^T(p,R)$ be the value of q that satisfies (1′) when $\eta = 1$. From the first-order condition of the platformer $\frac{\partial m^T}{\partial p} = (2-2\tau)\phi\frac{M}{\bar{f}}\frac{\partial q^T}{\partial p}$ and $-\frac{\partial q^T}{\partial p} > 0$, we have $\frac{\partial p q^T}{\partial p} = \left(R^T(2-2\tau)\phi\frac{M}{\bar{f}} - c\right)\left(-\frac{\partial q^T}{\partial p}\right)$. The term of $R^T(2-2\tau)\phi\frac{M}{\bar{f}}$ is per the digit of the number of developers m in (2) and (3) which may well be less than the marginal cost of the standard good. Then p^T is below the revenue-maximizing level of $pq^T(p,R^T)$, and $p^T q^T(p^T,R^T) > p^O q^T(p^O,R^T) > p^O q^T(p^O,R^O)$ since $p^T > p^O$ and $R^O > R^T$ if $\bar{f}/M$ is sufficiently large.

[18] Conceivably, the host government does not take into account the foreign firms' profits or the profits of the foreign multinational enterprise (π_P) and the foreign entrants' profit (π_{SF}). The former is typically a foreign multinational enterprise which does not bring benefits apart from the platform services. To analyze the tax incidence, it is sufficient to examine the developers' side, as the specific-good users' net benefit v_i in Section 2.2 remains unchanged — specifically, zero — both before and after the tax reform. Similarly, in (4), the location of the production of the standard good, either in country H or by the platformer which is a foreign firm, does not matter. With the above features, we have the following formulation about the consumer surplus:

$$CS = \int_{\underline{\nu}}^{\nu} (b_k + \alpha m - p)\, db_k = \frac{1}{2}(\nu^2 - \underline{\nu}^2) - \underline{\nu}(\nu - \underline{\nu}) = \frac{1}{2}(\nu - \underline{\nu})^2 = \frac{q^2}{2}.$$

The numerator and the denominator are expressed in discrete form, reflecting the nature of the tax reform, which is conceptually modeled as a change in η from 0 to 1. Notice that (A-6) and (10) show that the numerator of (11) is proportional to $\tau\phi$, as is the first term of the denominator. As shown in in footnote 14, Proposition 1 and Proposition 2, the behavioral effects on demand and supply involve endogenized prices and external effects. The domestic total surplus ($CS + PS_H + TR$) increases if and only if MVPF is less than 1.

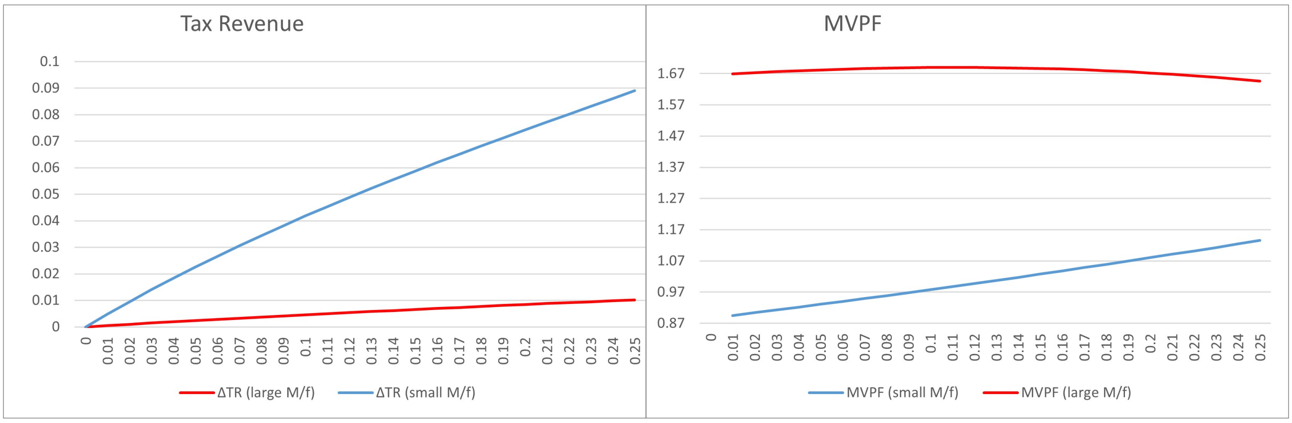

Figure 1: Effects of tax reform on TR and MVPF under high and low entry responsiveness: $\nu = 2$, $c = 0$, $M = 1$ and $\alpha = \phi = 0.25$. $\overline{f} = 0.33$ for the large entry responsiveness, $\overline{f} = 1.5$ for the small entry responsiveness. The horizontal values are $\tau = t = 0$ to 0.25.

As usual, the second term in the numerator, $\pi_{SH} = ((\alpha + \phi - (1.5 - 0.5\eta)\tau\frac{\phi}{1-t})\frac{(1-t)}{2}q$, appears when the developers' profits are not (100%) taxed. Notice that the loss of surplus by the platformer, as, a foreign multinational enterprise, is not included here (see footnote 18). (10) shows that the second term in the numerator is negative if entry responsiveness is sufficiently low, and vice versa.[19]

The denominator (which is positive) corresponds to enforcement elasticity by Keen and Slemrod (2017). The mechanical effect of the tax-base expansion by tax reform appears in the first term, which is a tax on non-residents as in Wildasin (1987) and Dahlby (2008). In addition, the denominator has behavioral responses due to increased tax burden. As such, unlike Proposition 3 by Adachi and Fabinger (2022), the instrument for tax reform is not the tax rates but tax enforcement. Laffont (2001) assumes that MVPF is between 1.1 and 1.3, and Dahlby's (2008) estimates are even greater in some cases.

This feature in the part related to the loss of developers' producer surplus and the dead-weight loss is different from the usual expression in the excess-burden analysis. Typically, the behavioral effects are captured by supply elasticity. However, in the present analysis, we do not model elasticity as a sufficient statistic because the prices are endogenously determined. Instead, entry responses

[19]The values of the consumer surplus and the producer surplus are weighted by entry responsiveness. We should consider this weight as a relative value to the slope of demand d in footnote 14 which is (normalized to) unity.

ultimately depend on the extent of network contraction and the strength of network externalities. The endogenous response of prices to taxation in the analysis of excess burden remains unexplored in the existing literature. The numerator of (11) captures it: Although the present paper does not propose a specific empirical methodology, it is worth pointing out that, due to the simultaneity of price and firm entry decisions and network effects, recent parsimonious reduced-form methods may prove insufficient.[20]

Combining (10) with a sufficient condition (iii) of Proposition 2, if entry responsiveness is low, the fact of $\pi_{SH}^T > \pi_{SH}^O$ ($PS_H^T > PS_H^O$) and the increase of TR are dominant for the increase of domestic welfare and therefore the MVPF becomes less than 1. The MVPF becomes greater than 1 if entry responsiveness is high, in which case, the comparison with the MVPF determined by other tax instruments serves as a cost-benefit criterion of the present enforcement reform.

4.4 Negative externality of entry

As the number of developers increases, competition is intensified and profits per developer decreases. Each developer, for example, finds it more difficult to acquire customers when m is increased. Following Hagiu (2009) and Belleflamme and Peitz (2019) (the latter provides many examples of sellers' congestion problems, such as a reduced price through monopolistic competition with increasing number of competitors), we consider the following formulation:

$$m = \frac{M}{\overline{f}}[\{2 - (1+\eta)\tau\}(\phi_0 q - \phi_1 m_e) - 2R].$$

Namely, we take into account that firm's profit π_{SH} and π_{SF} are now decreasing in the expected numbers of developers in the equilibrium $m_e (= m)$ by the factor $\phi_1 > 0$. The externality here, which is a unit price of the specific goods, is multiplied by $1 - \tau$ for domestic developers and $1 - \eta\tau$ for foreign developers.

This formulation generates $m^* = \frac{M/\overline{f}}{1+\{2-(1+\eta)\tau\}\phi_1 M/\overline{f}}[\{2 - (1+\eta)\tau\}\phi_0 q^* - 2R^*] \equiv \frac{M^*(\tau)}{\overline{f}}[\{2 - (1+\eta)\tau\}\phi_0 q - 2R]$. Notably, the post-reform regime ($\eta = 1$) mitigates this negative externality ($M^T(\tau) > M^O(\tau)$). This effect mitigates the negative the loss of producer surplus.

[20] This is because the analogue of the supply elasticities in competitive market corresponds to entryways responses of a finite number of firms equipped with distinct technologies. More fully fledged structural analysis, such as multi-stage least squares, may therefore be necessary.

4.5 Pricing structure and tax incidence with competing platforms

Our results are robust with platform competition. Suppose now that the consumers on the standard good are differentiated by the attachment parameter θ which is uniformly distributed on a segment $[0,1]$. The marginal rate of substitution between the platform attachment and the utility from consumption is z. The utility of type θ from adopting platform 1 is given by $\nu + \alpha m_1 - p_1 - z\theta + \int\{a_{S_i} - p_{ai}\}dS$ where S_i is a vendor which invents and sells product i to this consumer, whereas that from adopting platform 2 is $\nu + \alpha m_2 - p_2 - z(1-\theta) + \int\{a_{S_i} - p_{ai}\}dS$. Following Hagiu (2009), we restrict to the case when consumers always coordinate on the equilibria in which no consumer multihomes. $p_{ai} = a_{S_i} = \phi q$ holds as in the basic section.

Denote by q_j consumer demand for the standard good in platform j. A foreign developer of the specialized good with fixed cost f_S makes profits $(1-\eta\tau)\phi q_j - R_j$ in platform j as in (3) and the same formulation in (2) applies to the domestic firm. They will generate $m_j = \frac{M\left[(2-(1+\eta)\tau)\phi q_j - 2R_j\right]}{\overline{f}}$. ν is assumed to be large enough so that the consumer market is always entirely covered. With the unit mass of the consumers, we have $q_1 = 0.5 + \frac{1}{2z}(\alpha(m_1 - m_2) - (p_1 - p_2))$ as in Hagiu (2009), and following (1') for a self-enforcing system of the standard good's demand yields:

$$q_i[1-(2-(1+\eta)\tau)\frac{\alpha\phi}{z}\frac{M}{\overline{f}}] = 0.5 - \frac{1}{2z}p_i - \frac{1}{2z}\frac{2\alpha M}{\overline{f}}R_i - \frac{\alpha\{(1-0.5(1+\eta)\tau) - 2R_j\}M/\overline{f} - p_j}{2z} \tag{12}$$

The platformer i maximizes $= (p_i(1-t) - c)q_i + R_i m_i$. For $\gamma \equiv 1-(2-(1+\eta)\tau)\frac{\alpha\phi}{z}\frac{M}{\overline{f}}$, we have $\frac{\partial q_i}{\partial p_i} = -\frac{1}{2z\gamma} < 0$, $q = \frac{1}{2}$ in the symmetric equilibrium where p and R are equalized, and from (2') and (3'), we have $\frac{\partial m_i}{\partial p_i} = \{1-0.5(1+\eta)\tau\}\phi\frac{-1}{z\gamma}\frac{M}{\overline{f}}$. Therefore, we have:

$$\frac{\partial \pi_P}{\partial p} = -\big(p(1-t)-c\big)\frac{1}{2z\gamma} + (1-t)\frac{1}{2} - R\frac{[2-(1+\eta)\tau]\phi M}{2z\gamma\overline{f}} = 0,$$

so that:

$$p^*(1-t) - c = (1-t)\gamma z - R^*\frac{M}{\overline{f}}(2-(1+\eta)\tau)\phi \tag{13}$$

which has, similar to (6) in the benchmark model, a mark-down increasing in the network externalities and in R. Additionally, the mark-up is decreasing in the platform mobility $1/z$. As to the commission fee:

$$R^* = \frac{\{1-0.5(1+\eta)\tau\}\phi - (1-t)\alpha}{2(1+\frac{1}{\gamma})} \tag{14}$$

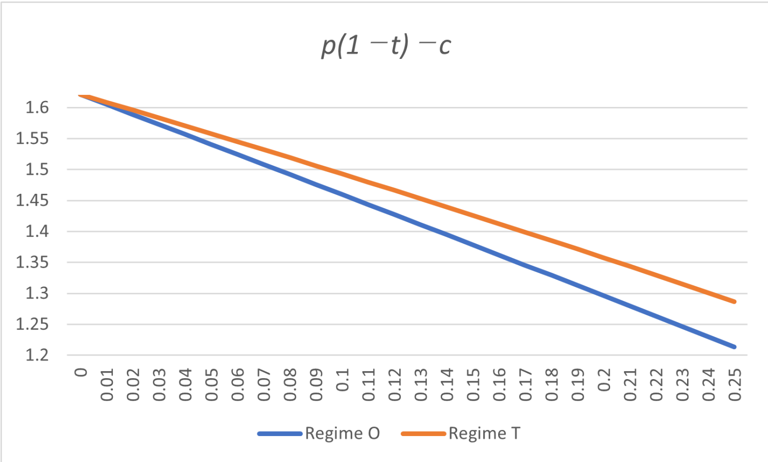

Figure 2: Effects of tax reform on p with competing platforms: $\nu = 2$, $c = 0$, $M = 1$ and $\alpha = \phi = 0.25$. $\overline{f} = 0.33$ and $z = 2$. The horizontal values are $\tau = t = 0$ to 0.25.

Figure 2 illustrates the effects on p^* with competing platform. Since R^*m^* is used to reduce p^* and such effect becomes weakened after tax reform, we have $p^T > p^O$ as tax incidence.

5 Taxing SMEs through platform

Here, as a related formulation and issue, we examine the case where the government captures SMEs, another form of unregistered transactions as a relevant issue in developing countries, through the platform. The formulation of (2) and (3) is mostly equivalent to the case when a country deals with the unfair treatment of the SMEs whose revenue is below the VAT threshold.

Suppose that the number of potential entrants of the specific good is $2M$. Before tax reform, VAT-registered firms (group r) need to earn $v > 0$ as the revenue (net of f_S) so that $m_r = \left(\frac{[(1-\tau)\phi q - R]}{\overline{f}}\right) 2M - \frac{2Mv}{\overline{f}}$.[21] The other group, group u, does not have such a requirement. $m_u = \frac{2M[(1-\eta\tau)\phi q - R]}{\overline{f}} - m_r$. With the notation of (3′), we have $m_r^{Vt}(p, R) + m_u^{Vt}(p, R) = 2m_F^O(p, R)$ for the regime with VAT threshold Vt. The tax reform removes this VAT threshold and every firm pays the VAT.

After the elimination of the threshold, $(1 - \tau)\phi q - R - f_S \geq 0$ for the entry of firm S. To keep track of large (formerly registered) firms, we have $m_r^T = \left(\frac{[(1-\tau)\phi q^T - R^T]}{\overline{f}}\right) 2M - \frac{2Mv}{\overline{f}}$. The decrease of $(1 - \tau)\phi q^* - R^*$ of the formerly-registered firm means that the profits of some inefficient firms becomes less than v, whereas its increase means that the profits of efficient unregistered firms

[21]Since our analysis is the effect of eliminating the VAT threshold, the firm distribution in Onji (2009) and Saez (2010) before tax reform is treated as given. The firms' reaction after reform does not accompany the choice of the firm size so the firm distribution is treated invariant in the present analysis. Incorporating non-uniformity of the firm distribution in practice does not affect the present analysis.

becomes greater than v. In either case, the total number of firms equals to $\left(\frac{\bar{f}_{Su}}{\bar{f}}\right)2M = \frac{2M[(1-\tau)\phi q - R]}{\bar{f}}$ $= 2m_F^T$. We continue to assume that the taxpayers of the standard good q fully comply their tax duties before and after the tax reform. We briefly comment when some q–producers are unregistered before the tax reform.

Solving the equilibrium prices from $m_r^{Vt}(p,R) + m_u^{Vt}(p,R) = \frac{2M[\phi q - R]}{\bar{f}}$ for maximization of $((1-t)p-c)q + R(m_r^{Vt} + m_u^{Vt})$ is a modification of the maximization of (4) above. As in the above analysis, the equilibrium number of the network users becomes $q^{Vt} > q^T$ and the number of developers becomes $m_r^{Vt} + m_u^{Vt} > m^T$. From $m_r^T + m_u^T = 2m_F^T$, the number of developers after reform $m_r^T + m_u^T = m^T$ is invariant from the baseline model as well as q^T.

Since we obtain a formula similar to (10) for the formerly registered firms (group r) in terms of their entry and producer surplus, we apply observations (i) and (ii) mentioned after Proposition 2 in Section 4.2. We then can conclude the following proposition:

Proposition 4. *Consider a model where a country eliminates the VAT threshold through a platform-based tax, where network externalities are present. The increase in profits for formerly registered firms after the tax reform indicates a low proportional shrinkage of network users and a low proportional exit of SMEs. High entry responsiveness reverses these effects.*

Regarding new tax burdens on the SMEs, critics often show concerns on the exits of the SMEs and the windfall gain on the VAT-registered firms prior to the reform. Our model enables to quantify the magnitude of these effects by entry responsiveness.[22]

If the sales instead of the profit is the criterion for the VAT registration, then one can formulate firm heterogeneity by its sales $h\phi q$ with $h \in [0, \bar{h}]$ to have $(1-\eta\tau)h_S\phi q^* - h_S R^* - f$ with[23] $\eta = 1$ when $h_S \geq h_{\bar{S}}$ with a VAT threshold, or when $* = T$. We can show the same results under this formulation. Also, when some of the standard good producers are either foreign untaxed producers or the unregistered producers before the reform,[24] the network shrinkage after the tax reform becomes larger, and both the numerator and the denominator of MVPF increases.

[22] In this setup, the decrease of the profit $(1-\tau)\phi q^* - R^*$ of the formerly-registered firms means that the profit of some inefficient firms becomes less than v, whereas its increase means that the profit of efficient unregistered firms becomes greater than v. The emergence (non-emergence) of this windfall gains to the formerly-registered firms reveals low (high) proportional shrinkage of the network users and the exit of the SMEs.

[23] In the distinction of Rochet and Tirole (2003), R is levied per transaction.

[24] "Tax Surprise" Approach by Einav et al. (2014) is useful: online shoppers click on an item before knowing if the good is taxed — and consequently, its VAT is determined by the status of the seller. One can model the transactions with "home bias" or a bias towards registered firms by the buyers and/or the platformer. (6) applies to the expected post-tax price. All these occur before the reform, and after the reform, an inertia remains so either buyer or the platformer is not ready to change the proportion of foreign (small) sellers of the standard good before the reform.

6 Conclusion

Value-added tax (VAT) has become a major revenue source in many countries, while the forms of consumption have grown increasingly complex. A growing concern in international taxation is how to collect consumption taxes such as VAT. In particular, countries face significant challenges in taxing foreign developers who are located outside the destination country and provide services remotely — such as digital content — through online platforms. This paper theoretically investigates the impact of a tax reform that enables countries to collect VAT from an online platform operator on behalf of foreign content developers.

We first provide clear and realistic explanations of the platformer's behavior in reducing the product price or stimulating supply and demand in multi-sided markets. We also explain how the relation-specific good's price relates to the buyer-side's network externalities. Nevertheless, we highlight the changes in prices before and after the tax reform. Our model finds that the tax reform, while eliminating the unfair tax burden between domestic and foreign developers, also leads to the increased tax burden on the standard good — despite no change in the VAT rate — which is entirely borne by consumers. This outcome reflects a strong cross-market pass-through effect, even under a general consumption tax (rather than a specific commodity tax) in a monopolistic two-sided market.

Moreover, the reform introduces a trade-off: it increases tax revenue in the host country but decreases surplus for consumers and developers. However, we demonstrate that the positive effects of a fairer tax burden and higher revenue outweigh the negative effects associated with network shrinkage. These positive outcomes are more pronounced when the VAT rate is higher and when developers' entry responsiveness to changes in the standard good price and commission fee is low. Notably, we also find that entry responsiveness declines as the VAT rate on the standard good increases. These factors determine the Marginal Value of Public Finance as a cost-benefit index of the tax reform, which could be less than 1 when entry responsiveness is low, and which may decrease in the tax rate.

We also showed that a stylized framework of taxing e-commerce is applicable to the case of taxing SMEs through platform registration, which is an issue of high interest particularly in developing countries. Here we include a broad category of providers of goods and services such as small businesses by family members and those with a few external employees and physical or intangible assets. Large registered firms compete with small and medium providers in each category. If the network externalities and entry responsiveness are significant in taxing SMEs on the platform, then taxing SMEs brings the shrinkage of the market so that formerly registered developers are made worse off from the inclusion of the SMEs in the VAT registration.

Appendix

A. Physical-incidence neutrality of post-reform regime

Consider the following scenario. Regime O is the same as our benchmark case. In Regime T', the platformer collects *tax-inclusive commission fee from both domestic and foreign developers*. Namely, we assign the physical taxpayer in an alternative Regime T' to the platformer, for both domestic and foreign VATs.

$$\begin{aligned}
\pi_{SH} &= \phi q - R - f_S - (1-\lambda)\tau\phi q,\ \lambda = 0 \text{ under Regime } O \textit{ and } \lambda = 1 \text{ under Regime } T' \\
\pi_{SF} &= \phi q - R - f_S, \\
\pi_P^* &= (p(1-t) - c)q^* + (R - \lambda\tau\phi q^*)(m_H^* + m_F^*) \ \text{ for } \ * = O,\ T'.
\end{aligned}$$

A calculation derives $R^{T'} - \tau\phi q^{T'} = R^T$ where the right-hand side is given by (7), and all the real variables in Regime T' are identical to those in Regime T. Namely, it is profit-maximizing for the platformer, as the physical taxpayer, to collect the tax-inclusive commission fee, and the equal-footed domestic and foreign developers have the same burden as in Regime T. Notice that Physical-incidence neutrality of the tax on standard good, t, is synonymous to Weyl and Fabinger (2013).

B. Unit tax

With the unit tax $\hat{t}$ on q (and with a usual equivalent scaling on τ if one also considers the unit tax on ϕq), we have:

$$\nu - p^* = \frac{\nu - \hat{t} - c}{2} + \frac{[\alpha + \{1 - 0.5(1+\eta)\tau\}\phi]M}{\overline{f}} R^*.$$

The unit tax linearly separates the tax burden so that the alleviated burden from ad-valorem tax when $R^* > 0$, $\widetilde{\phi} > \phi$ in (6), does not appear in the corresponding unit-tax case.

C. Derivation of (7)

$\frac{\partial q}{\partial R} = \alpha \frac{M}{\overline{f}} \frac{-1}{\gamma}$, $m = \frac{M}{\overline{f}} \Big[\{2 - (1+\eta)\tau\} \phi q(p, R) - 2R \Big] = \frac{M}{\overline{f}} \frac{\{2-(1+\eta)\tau\}\phi\overline{f}(\nu - p) - 2R\overline{f}}{\overline{f} - \alpha M\phi(2-(1+\eta)\tau)}$ and $\frac{\partial m}{\partial R} = -\frac{M(2-(1+\eta)\tau)\phi}{\overline{f}} \frac{2\alpha M}{\overline{f} - \alpha M\phi(2-(1+\eta)\tau)} + \frac{M(-2)}{\overline{f}} = -\frac{M}{\overline{f}} \frac{2\overline{f}}{\overline{f} - \alpha M\phi(2-(1+\eta)\tau)}$, so that, applying (6) and $p^*(1-t) -$

$c = \frac{\nu(1-t)-c}{2}$ $-\{\alpha + (1-0.5(1+\eta)t)\widetilde{\phi}\}\frac{(1-t)M}{\overline{f}}R^*$ shows:[25]

$$\frac{\partial \pi_P}{\partial R}\left(-\frac{\partial q}{\partial p}\right)^{-1}\frac{\overline{f}}{M} = (2-(1+\eta)\tau)\widetilde{\phi}(1-t)(\nu-p) - 4R - 2\alpha(p(1-t)-c) = 0,$$

which derives:

$$R^* = \frac{(\nu(1-t)-c)\big((2-(1+\eta)\tau)\widetilde{\phi} - 2\alpha\big)}{8-(2\alpha+(2-(1+\eta)\tau)\widetilde{\phi})^2\frac{(1-t)M}{\overline{f}}} \tag{A-2}$$

D. Proof of Lemma 1

The second term of (7) is:

$$\begin{aligned}&\frac{((1-0.5\tau)\widetilde{\phi}-\alpha)((1-\tau)\nu-c)(den^T-den^O)}{2den^O}\\ &= \frac{\tau\phi((1-\tau)\nu-c)}{2den^O}\frac{\left\{(2-\tau)\widetilde{\phi}-2\alpha\right\}\left\{4\alpha+(4-3\tau)\widetilde{\phi}\right\}(1-t)M}{8\overline{f}},\end{aligned}$$

which relates to entry responsiveness $\frac{(1-t)M}{\overline{f}}$ defined in the previous section. The above value is positive when $R^O \geq 0$ as shown in the text, and (7) is positive when $\frac{(1-t)M}{\overline{f}}$ satisfies the following (A-4).

With $den^* = 2-(\alpha+\phi-0.5(1+\eta)\tau\phi)^2\frac{M}{\overline{f}} > 0$ for $\eta = 0,\ 1$, we have:

$$\begin{aligned}p^T - p^O = &\frac{(\alpha+\phi)((1-t)\nu-c)M}{4den^T\overline{f}}\tau\widetilde{\phi}\left\{1+\frac{((2-\tau)\widetilde{\phi}-2\alpha)(\alpha+(1-0.75\tau)\widetilde{\phi})(1-t)M}{den^O\overline{f}}\right\}\\ &+0.5\tau\widetilde{\phi}\frac{((1-t)\nu-c)((1-0.5\tau)\widetilde{\phi}-\alpha)(1-t)M}{2den^O\overline{f}}\end{aligned} \tag{A-3}$$

The sign of (A-3) is positive if $R^O \geq 0$ or

$$\frac{\overline{f}}{(1-t)M}\frac{den^O}{2} \geq \frac{(4\alpha+4\widetilde{\phi}-3\tau\widetilde{\phi})((2\alpha)^2-(2-\tau)^2(\widetilde{\phi})^2)}{8\{4-3\tau\}\widetilde{\phi}} \quad \text{when } 2\alpha-(2-\tau)\widetilde{\phi} \geq 0 \tag{A-4}$$

holds. *Q.E.D.*

[25] For the second-order condition, we have $\frac{\partial^2 \pi_P}{\partial p \partial p}\gamma^{-1} = -2(1-t)$, $\frac{\partial^2 \pi_P}{\partial R \partial R}\gamma^{-1} = -4\frac{M}{\overline{f}}$, $\frac{\partial^2 \pi_P}{\partial p \partial R}\gamma^{-1} = -\frac{2\alpha(1-t)M}{\overline{f}} - \frac{(2-(1+\eta)\tau)\widetilde{\phi}(1-t)M}{\overline{f}}$. Therefore,

$$8-\left(2\alpha+(2-(1+\eta)\tau)\widetilde{\phi}\right)^2\frac{(1-t)M}{\overline{f}} > 0.$$

E. Proof of Proposition 2

A comparison of TR^O and TR^T is done in the following way.

$$TR^T - TR^O = \tau\phi(2m_H^T q^T - m_H^O q^O) + tp^O(q^T - q^O) + t(p^T - p^O)q^T$$

When $R^O \geq 0$, a variant of (A-3) holds to induce $p^T > p^O$ and $R^T < R^O$. For the difference in the VAT revenue from the standard good, we broke it down into parts. The last term is positive. Now we show that the sum of the first and second terms is positive.

We first notice, from (8),

$$m_F^O \leq M \Leftrightarrow \frac{\overline{f}}{M} \geq (1-t)\{\frac{\{2\alpha + \frac{\phi}{1-t}(2-\tau)\}^2}{8} + \frac{(2\alpha + (2+\tau)\frac{\phi}{1-t})(\nu - \frac{c}{1-t})}{8M}\}, \quad \text{(A-5)}$$

$$m_H^* = \frac{M(\nu - \frac{c}{1-t})\{(1-t)\alpha + \{1-(1.5-0.5\eta)\tau\}\phi\}}{4\overline{f} - 2(1-t)M\{\alpha + (1-0.5(1+\eta)\tau)\frac{\phi}{1-t}\}^2},$$

$$q^O - q^T = \frac{\tau\phi(\nu - \frac{c}{1-t})(\alpha + (1-0.75\tau)\frac{\phi}{1-t})(1-t)M/\overline{f}}{\left\{2 - \{\alpha+\phi\}^2(1-t)M/\overline{f}\right\}\left\{2 - \{\alpha + (1-0.5\tau)\frac{\phi}{1-\tau}\}^2(1-t)M/\overline{f}\right\}} \quad \text{(A-6)}$$

From the above formula and $m_F^T = m_H^T$ we have:

$$(m_H^T + m_F^T)q^T - m_H^O q^O + p^O(q^T - q^O)/\phi \propto$$

$$\begin{aligned}
&(\alpha+\phi)\left(1.7[2 - \{(1-0.5\tau)\frac{\phi}{1-t} + \alpha\}^2(1-t)\frac{M}{\overline{f}}]^2 - [2 - \{\phi+\alpha\}^2(1-t)\frac{M}{\overline{f}}]^2\right) \\
&+0.3(\alpha+\phi)[2 - \{(1-0.5\tau)\frac{\phi}{1-t} + \alpha\}^2(1-t)\frac{M}{\overline{f}}]^2 + 0.5\tau\frac{\phi}{1-t}[2 - \{\phi+\alpha\}^2(1-t)\frac{M}{\overline{f}}]^2 \\
&-tp^O(\nu - \frac{c}{1-t})(\alpha + (1-0.75\tau)\frac{\phi}{1-t})\times \\
&\quad [2 - \{\alpha+\phi\}^2(1-t)\frac{M}{\overline{f}}][2 - \{\alpha + (1-0.5\tau)\frac{\phi}{1-t}\}^2(1-t)\frac{M}{\overline{f}}]/(\nu - \frac{c}{1-t})^2
\end{aligned}$$

The first term in the RHS of the above formula is positive if $\sqrt{1.7} - 1 \approx 0.30 > t$ and $\frac{\overline{f}}{M}$ or $\nu - \frac{c}{1-t}$ is sufficiently large ($2\frac{\overline{f}}{(1-t)M} - ((1-0.5\tau)\frac{\phi}{1-t} + \alpha)^2 \geq \frac{\phi}{1-t}(\alpha + (1-0.75\tau)\frac{\phi}{1-t})$). For $b = q^T/q^O$ and $p^O \leq \frac{c}{1-t} + 0.5(\nu - \frac{c}{1-t})$ under $R^O \geq 0$, the rest of the term is positive if the following expression is positive when $t = \tau$:

$$
\begin{aligned}
&(\alpha+\phi)b^2 + 0.5\frac{\phi}{1-t} - p^O\left(\nu - \frac{c}{1-t}\right)^{-1}\left(\alpha + (1-0.75\tau)\frac{\phi}{1-t}\right)b \\
\geq &(\alpha+\phi)(b^2+0.25) + 0.25\left(\frac{1+1.5\tau}{1-t}\phi - \alpha\right) - \left\{0.5 + \frac{c}{1-t}\left(\nu - \frac{c}{1-t}\right)^{-1}\right\} b\,(\alpha+\phi) \qquad \text{(A-7)}
\end{aligned}
$$

(A-6) shows that b can be arbitrarily close to 1 by $\frac{\bar{f}}{M}$ being sufficiently large. If $b^2 + 0.25 \geq b$ (say $b \geq 0.75$ or $b \leq 0.5$) and $\frac{c}{1-t} < 0.5(\nu - \frac{c}{1-t})$, then (A-7) is positive. *Q.E.D.*

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Online Appendix for
"Tax Incidence of VAT Enforcement Reform for Foreign Services and Small Businesses in Two-sided Markets"

Yukihiro Nishimura

B.1. On domestic producer surplus

Morita and Nishimura (2025) showed the following:

$$PS_H = \int_0^{\overline{f}_{SH}} \{(1-\tau)\phi q - R - f_S\} \frac{M}{\overline{f}} df_S = \frac{m_H^2 \overline{f}}{2M}$$

Recall that $m_H = \frac{M\pi_{SH}}{\overline{f}}$.

Morita and Nishimura (2025) showed (10) with $t = 0$; The argument is valid in the present paper by replacing ϕ to $\widetilde{\phi}$ and $\frac{M}{\overline{f}}$ to $\frac{(1-t)M}{\overline{f}}$. The fact of $t > 0$ (and $t = \tau$ under the general consumption tax) decreases the entry responsiveness $\frac{(1-t)M}{\overline{f}}$ in t so that high VAT rate t makes (10) more likely to be positive. As well, the increase of the entry more likely to occur. This is because $\frac{d\{(1-\tau)K\left(\alpha, \frac{\phi}{1-\tau}, \tau\right)\}}{d\tau} = (1-\tau)\frac{\partial K\left(\alpha, \frac{\phi}{1-\tau}, t\right)}{\partial \tau} - K\left(\alpha, \frac{\phi}{1-\tau}, \tau\right) + k\left(\alpha, \frac{\phi}{1-\tau}, \tau\right)$, for a function k such that $k\left(\alpha, \frac{\phi}{1-\tau}, \tau\right) < K\left(\alpha, \frac{\phi}{1-\tau}, \tau\right)$ and $\frac{\partial K(\alpha, \widetilde{\phi}, \tau)}{\partial \tau}\Big|_{d\widetilde{\phi}=0} = -\frac{(11\alpha + 11\widetilde{\phi} - 10\tau\widetilde{\phi})\widetilde{\phi}}{4} < 0$, so we have $\frac{d\{(1-\tau)K\left(\alpha, \frac{\phi}{1-\tau}, \tau\right)\}}{d\tau} < 0$.

B.2 Entry and exit in the general model

Suppose that participating developers earn the amount proportional $m_{pr,J}$ $(J = H, F)$ of the aggregate gross *pr*ofits in H and F respectively, and suppose that the number of apps is written as, for $\Phi \equiv \phi q$, $m_{pr,J}\left((1-\eta_J\tau)\phi q, R\right) - R m_{en,J}\left((1-\eta_J\tau)\phi q, R\right) \equiv (1-\eta_J\tau)\phi q \frac{\partial m_J}{\partial \Phi} - m_{2,J}R$, with $\eta_J = 0$ only under Regime O with $J = F$ and $\eta_J = 1$ otherwise. $m_{en,J}$ is the value proportional to the number of participating (*en*tering) firms. We assume that $m_2 \geq 0$.

The motivation to this generalization is as follows. App developers' heterogeneity may come from different reasons in practice. For example, with heterogeneity of apps' popularity, the developers provide different units of apps $h_{SJ} \in (0, \overline{h}]$ with fixed costs $f > 0$. In this alternative case, developer S's profit is $\pi_{SJ} = h_{SJ}\{(1-\eta_J\tau)\phi q - R\} - f$.[26] The critical entrant $\overline{S}$ which is $h_{\overline{S}J} = \frac{f}{(1-\eta_J\tau)\phi q - R}$ for domestic and foreign entering firms, the number of apps provided is

[26] In the distinction by Rochet and Tirole (2003), R is levied per unit of transaction.

$m_J \equiv \int_{h_{SJ} \leq h_{\bar{S}J}} h_{SJ} dS$ for $J = H,\ F$.

Substituting this relation into the demand function $q = q_1(p) + \alpha m$ to have $q\{1 - \alpha \sum_{J=H,F}(1 - \eta_J \tau)\phi \frac{\partial m_J}{\partial \Phi}\} \equiv q\gamma = q_a + \frac{\partial q_1}{\partial p} p - \alpha m_2 R$ for $m_2 \equiv m_{2,H} + m_{2,F}$. Corresponding to our linear model, we define $\nu = \frac{q^a}{-\partial q_1/\partial p}$.

For maximization of $\pi_P^* = ((1-t)p - c)q^* + Rm^*$, with $n(\tau) \equiv \frac{(1-\tau)\frac{\partial m_H}{\partial \Phi} + (1-\eta\tau)\frac{\partial m_F}{\partial \Phi}}{m_2/(-\partial q_1/\partial p)}$ and $\epsilon_{m_2R} = \frac{\partial(\sum_{J=H,F}(1-\eta_J\tau)\phi q \frac{\partial m_J}{\partial \Phi})/\partial R}{m_2} - \frac{\partial m_2/\partial R}{m_2} R$ that includes the elasticity of m_2 with respect to R:[27]

$$\frac{\partial \pi_P^*}{\partial R}\Big|_{\frac{\partial \pi_P^*}{\partial p}=0} \frac{\gamma}{m_2} = -R^* \left\{ 2 - \epsilon_{m_2R} - \left(\frac{(1-t)\alpha m_2/2}{-\partial q_1/\partial p} + \sum_{J=H,F}(1-\eta_J\tau)\phi \frac{\partial m_J/\partial \Phi}{2}\right)(\alpha(1-\epsilon_{m_2R}) + \tilde{\phi} n(\tau)) \right\}$$
$$+ [\tilde{\phi} n(\tau) - \alpha(1-\epsilon_{m_2R})] \frac{\nu(1-t) - c}{2} = 0.$$

The term in the curly bracket of the above formula is referred to as $den^{*,g}$, which is assumed to be positive.

The general expression of (10) for $m_H(q, R)$ is:[28]

$$m_H^T - m_H^O = \frac{\partial m_H}{\partial R}(R^T - R^O) - \frac{\partial m_H}{\partial q}(q^O - q^T) \gtreqless 0 \quad \text{with weak/strong entry responsiveness,} \tag{10B}$$

where the derivative is identified by the mean value theorem of differentiation. For $m_H^T > m_H^O$, $R^T - R^O < 0$ is necessary. In our model in the text, (7) decomposes $(R^T - R^O) \cdot den^T$ between $-(\nu - \tilde{c})\frac{\tau\phi}{4} \equiv \Delta r_a$ and $\Delta r_b \equiv (R^T - R^O) \cdot den^T - \Delta r_a$. In the general model, the term corresponding to Δr_a is $-\frac{\tau\phi \frac{\partial m_F}{\partial \Phi}}{m_2/(-\partial q_1/\partial p)} \frac{\nu - \tilde{c}}{2} \equiv \Delta r_a^g$ and $\Delta r_b^g \equiv (R^T - R^O) \cdot den^{T,g} - \Delta r_a^g$. If Δr_a^g is large in negative, we have $R^T - R^O < 0$.

For completion, we show that q decreases after tax reform, and the market shrinkage is greater with greater entry responsiveness. We have $m^T - m^O = (-\tau\phi q^T \frac{\partial m_F}{\partial \Phi} + \frac{\partial m}{\partial R}\Delta R_a) - \{\phi((1-\tau)\frac{\partial m_H}{\partial \Phi} +$

[27]We use the following two formula:

$$\frac{\partial\Big((p(1-t) - c)q^* + Rm^*\Big)}{\partial p} = -(1-t)p(-2\frac{\partial q}{\partial p}) + c(-\frac{\partial q}{\partial p}) + (1-t)\frac{q_a}{\gamma} - R(\alpha \frac{(1-t)m_2}{\gamma} - \sum_{J=H,F}(1-\eta_J\tau)\phi \frac{\partial m_J}{\partial \Phi}\frac{\partial q}{\partial p}) = 0$$

$$\frac{\partial\Big(((1-t)p - c)q^* + Rm^*\Big)}{\partial R} \frac{\gamma}{m_2} = \phi(\nu - p)n(\tau) - 2R^* + \epsilon_{m_2R}R^* + \gamma \frac{\partial q}{\partial R}(p(1-t) - c) = 0$$

[28]The exact expression corresponding to (10) is:

$$m_H^T - m_H^O = \frac{\partial m_H}{\partial R}\Delta R_a - \{\frac{\partial m_H}{\partial q}(q^O - q^T) - \frac{\partial m_H}{\partial R}\Delta R_b\} \gtreqless 0 \quad \text{with weak/strong entry responsiveness.}$$

The first term is generally positive since reduction of the commission fee encourages the entry. In (10), the effect of ΔR_b is, whether it is positive or negative, weaker than that of production, so the second term is negative in general.

$(1-\eta\tau)\frac{\partial m_F}{\partial \Phi})(q^O - q^T) - \frac{\partial m}{\partial R}\Delta R_b\} \equiv \Delta m_a - \{\phi((1-\tau)\frac{\partial m_H}{\partial \Phi} + (1-\eta\tau)\frac{\partial m_F}{\partial \Phi})(q^O - q^T) - \frac{\partial m}{\partial R}\Delta R_b\}$ with $\eta \in [0,1]$.[29] For robustness, one can express $q = q_1(p) + \alpha m$ as:

$$(q^T - q^O)\{1 - \alpha \sum_{J=H,F}(1-\eta_J\tau)\phi\frac{\partial m_J}{\partial \Phi} + (1-t)\alpha\frac{\partial m}{\partial R}\frac{\Delta R_b}{q^T - q^O}\} = \frac{\partial q_1}{\partial p}\Delta p + \alpha\Delta m_a.$$

$$(q^T - q^O)\{1 - \alpha \sum_{J=H,F}(1-\eta_J\tau)\phi\frac{\partial m_J}{\partial \Phi}\} = \frac{\partial q_1}{\partial p}\Delta p + \alpha\Delta m_a - (1-t)\alpha\frac{\partial m}{\partial R}\Delta R_b.$$

When ΔR_b is negative (positive), we apply the first (second) formula. Within the range of $p^T - p^O > 0$, the right-hand side of each formula is positive (footnote 29) and decreasing in entry responsiveness, and the term in the curly bracket in the left-hand side of each formula is positive and decreasing in entry responsiveness that depends on $\frac{\partial m_J}{\partial \Phi}$ and $-\frac{\partial m}{\partial R}$.

[29]In the linear model of the text, we have

$$m^T - m^O = -0.5\tau\phi q^T\frac{M}{\overline{f}} + \{(1-0.5\tau)\phi + (1-t)\alpha\}(q^T - q^O)\frac{M}{\overline{f}} < 0.$$

$\Delta m_a = (-\tau\phi q^T\frac{\partial m_F}{\partial \Phi} + \frac{\partial m}{\partial R}\Delta R_a)$ is negative as it is in the first term of the above formula. In the first term of the above formula, the decline of $\frac{\partial m}{\partial q}$ after the tax reform dominates the positive part in (10) accruing in H and F $(-0.5 = -1 + 0.5)$. The second term is negative as in the text. The domestic developers do not have the first term since they pay VAT even before tax reform.