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Hayato Kato, Kensuke Suzuki, Motoaki Takahashi

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Graduate School of Economics
The University of Osaka, Toyonaka, Osaka 560-0043, JAPAN

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Hayato Kato[†] Kensuke Suzuki[‡] Motoaki Takahashi[§]

Abstract

We examine how tariffs affect sectoral composition and welfare in an economy with nonhomothetic preferences and sectors being complements—key drivers of structural change. Beyond their conventional role in trade protection, tariffs influence industrial structure by altering relative prices and income levels. We qualitatively characterize these mechanisms and use a quantitative dynamic model to show that a counterfactual 20-percentage-point increase in U.S. manufacturing tariffs since 2001 would have raised the manufacturing value-added share by one percentage point and increased welfare by 0.36 percent. However, if all the U.S. trading partners responded reciprocally, U.S. welfare would have declined by 0.12 percent. (*JEL* F11, F13, O41)

Keywords: Ricardian model of trade; Structural transformation; Nonhomothetic preferences; Capital accumulation; Trade war

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[†]Graduate School of Economics, the University of Osaka; CESifo Research Network. Email: hayato.kato@econ.osaka-u.ac.jp

[‡]Department of Economics, Clark University; Economic Research Center, Nagoya University. Email: KSuzuki@clarku.edu

[§]Graduate School of Economics, the University of Osaka. Email: takahashi.motoaki@gmail.com

1 Introduction

There has been renewed attention paid to understanding trade policies as a means of protecting domestic industries. A notable recent event sparking such interest occurred on February 1, 2025, when the U.S. President signed executive orders imposing a 25-percentage-point additional tariff on most imports from Canada and Mexico, and a 10-percentage-point additional tariff on imports from China.[1] These measures reflect the current U.S. administration's attempts to reshape trade relations in favor of American workers and manufacturers, who have faced growing competition from developing countries in the global market (Goldberg and Reed, 2023).[2]

However, shrinking manufacturing is not unique to the U.S.; it is ubiquitous in major developed countries. From a long-term perspective, the average share of manufacturing in domestic value-added (manufacturing value-added share, henceforth) among G7 nations declined from 20.1 to 14.6 percent between 1965 and 2014, while the service value-added share increased from 59.1 to 78.2 percent over the same period.[3] The shift of resources from manufacturing to services within a country occurs even in the absence of international trade, a process known as structural change (Kuznets, 1973).

Can tariffs effectively mitigate the decline of manufacturing in a country undergoing structural change? In addition to their conventional role in trade protection, how do tariffs interact with key drivers of structural change such as sectoral complementarity (Ngai and Pissarides, 2007) and nonhomothetic preferences (Kongsamut et al., 2001)?[4] Tariffs on manufacturing raise its relative price compared to agriculture and services. If sectoral goods are gross complements in consumption, the higher relative price of manufacturing may bias expenditure toward manufacturing and shift resources away from agriculture and services. On the other hand, tariffs generate government revenue

[1]See the White House statements, "Fact Sheet: President Donald J. Trump Imposes Tariffs on Imports from Canada, Mexico, and China": https://www.whitehouse.gov/fact-sheets/2025/02/fact-sheet-president-donald-j-trump-imposes-tariffs-on-imports-from-canada-mexico-and-china/ (accessed May 13, 2025).

[2]In the context of the U.S., Mexico and China played a major role. For example, Hakobyan and McLaren (2016) find that the North American Free Trade Agreement (NAFTA) significantly reduced wage growth for blue-collar workers in industries and regions most exposed to Mexican import competition from 1990 to 2000 (see Choi et al., 2024 for the negative effect of NAFTA on employment growth). Acemoglu et al. (2016) report that 2.0 to 2.4 million U.S. manufacturing workers lost their jobs due to Chinese import competition from 1999 to 2011 (see also Pierce and Schott, 2016).

[3]Over the period from 1965 to 2014, the average manufacturing value-added share among four major non-resource-dependent emerging economies, China, India, South Korea, and Taiwan, increased from 16.3 to 24.1 percent (the authors' calculations using the World Input-Output Database). The sectoral value-added we refer to throughout this paper is the sectoral nominal GDP adjusted by taxes less subsidies on products (Woltjer et al., 2021, pp.7-8).

[4]Ngai and Pissarides (2007) demonstrate that sectoral complementarity, combined with sector-biased productivity growth, drives resources reallocation away from sectors with relatively higher productivity growth towards those with lower productivity growth. In this paper, we take a stance on exogenous sectoral productivity and focus on the role of tariffs under sectoral complementarity.

and may raise overall income,[5] shifting demand from less income-elastic sectors, such as agriculture and manufacturing, toward more income-elastic sectors like services. The two effects of tariffs on manufacturing—through changes in relative prices (*relative price effect*) and through changes in income (*income effect*)—may work in opposite directions.

Our primary goal is to qualitatively and quantitatively evaluate the relative price and income effects of tariffs on sectoral composition. Our static model for theoretical analysis consists of two countries and three sectors and features trade based on the Ricardian comparative advantage (Eaton and Kortum, 2002) and the (isoelastically) nonhomothetic constant elasticity of substitution (CES) preferences (Matsuyama, 2019; Comin et al., 2021). The nonhomothetic CES neatly delineates the income effect from the relative price effect of tariffs.[6]

A key qualitative finding reveals that, in the presence of sectoral complementarity and nonhomothetic preferences, tariffs on a specific sector may either expand or contract its sectoral output and consumption. This ambiguity arises from the interplay of three distinct effects. First, consider a sector-specific tariff where only the relative price effect operates. In this case, a rise in tariffs increases the domestic consumption expenditure share, and consequently the value-added share, of the protected sectors. In contrast, under nonhomothetic preferences, a hypothetical uniform rise in tariffs across all sectors, where only the income effect is at work, leads to different results. Here, increased tariffs and tariff revenue reduce the expenditure share, and thus the value-added share, of sectors with lower income elasticity. Moreover, the conventional protective role of tariffs, i.e., lower imports in protected sectors improving their net exports, contributes to an increased value-added share. When all these effects are in effect simultaneously, tariffs may ultimately either expand or contract the protected sector.

To quantitatively evaluate which effect dominates, we extend the two-country static model to a multi-country dynamic framework incorporating endogenous capital accumulation and input-output linkages. We bring the model to the data for the world economy, encompassing 24 countries over five decades from 1965 to 2014. We calibrate the model's fundamentals, such as sectoral productivity and non-tariff trade barriers, which allows us to solve for the transition paths of the economy in terms of

[5] Following the current U.S. administration's tariff increases on nearly all products, the nation's gross tariff revenue (including certain other excise tax revenue) reached 68.9 billion USD during the first five months of 2025 (Horsley, 2025). This represents a 78 percent increase from the corresponding period in 2024 and accounts for 13 percent of corporate income tax revenue in the fiscal year 2024 (Hernandez, 2025).

[6] As we will see, nonhomothetic CES preferences allow for sector-specific parameters (ϵ^j) capturing differences in income elasticity across sectors, while treating separately the parameter (σ) that captures the (constant) elasticity of substitution. Another advantage of nonhomothetic CES preferences is that, unlike Stone-Geary preferences, they allow income elasticity differences across sectors to persist even among rich countries or households. This feature helps explain empirical observations (Buera and Kaboski, 2009; Comin et al., 2021; Alder et al., 2022).

levels, not in *relative changes* typically referred to as exact hat-algebra method (Dekle et al., 2008; Caliendo and Parro, 2015; Caliendo et al., 2019). We then conduct a counterfactual experiment of a 20-percentage-point increase in U.S. tariffs applied to manufacturing imports from all countries, effective starting in 2001.[7] We select the year 2001 because the U.S. manufacturing value-added share experienced its largest annual decline in that year over our sampled period, dropping sharply by over two percentage points (from 14.8 percent in 2000 to 12.4 percent in 2001).[8]

We find that tariffs alter sectoral composition in a manner consistent with the standard trade protection argument and the relative price effect. Specifically, compared with the baseline equilibrium, a 20-percentage-point increase in the U.S. manufacturing tariffs since 2001 leads to a 6.4–12.4 percent (0.9–1.3 percentage points) increase in manufacturing value-added share in each year from 2001 to 2014, and service value-added share falls by 0.9–1.2 percent (0.8–1.0 percentage point). Although these results might seem encouraging for trade protectionists, the U.S. welfare increases only by 0.36 percent. The U.S. welfare gains, however, come at the expense of the other countries. Canada experiences the largest loss, with a 1.26 percent decline in welfare. Furthermore, if trading partners retaliate with equally high tariffs on the U.S. manufacturing exports, U.S. welfare drops by 0.12 percent.

We also find the importance of nonhomothetic preferences in evaluating the welfare impact. When conducting the same counterfactual exercise under homothetic CES preferences, the U.S. welfare increases by 0.41 percent, 14 percent (0.05 percentage point) higher than in the benchmark case of nonhomothetic CES. This disparity arises because homothetic CES preferences underestimate the negative effect of a higher aggregate price index following the tariff increase.[9] The quantitative assessment based on widely used homothetic CES preferences may overestimate the gains from unilateral tariff policy.

Our paper relates to two strands of the literature. The first is a growing body of studies on structural change and trade (Alessandria et al., 2023; Gollin and Kaboski, 2023 for recent surveys).[10] These studies offer a number of new insights such as the impact of trade on the skill

[7]This exercise of a permanent tariff increase reflects the persistence of such policies, e.g., tariffs imposed during the first term of the Trump administration (2017–2020) remained in place throughout the Biden administration (2021–2024) (Hu et al., 2025). A counterfactual experiment for a temporary tariff increase is presented in Supplemental Appendix L.

[8]Additionally, 2001 marked China's accession to the World Trade Organization (WTO) and the beginning of George W. Bush's first term.

[9]The aggregate price index is defined as the aggregate expenditure E divided by the real consumption C, i.e., $P = E/C$. Under homothetic CES preferences, P is independent of C. However, under nonhomothetic CES preferences, P may increase with C because a greater C following a tariff hike can increase spending on more (real) income-elastic sectors, thereby increasing the aggregate expenditure E more than proportionally.

[10]Herrendorf et al. (2014); and Donovan and Schoellman (2023) survey the literature on structural change in the closed-economy context. Recent studies highlight the roles of domestic transportation infrastructure (Fajgelbaum and

premium (Cravino and Sotelo, 2019), the deepening intermediate-input intensity as economies develop (Sposi, 2019; Farrokhi and Pellegrina, 2023), the decomposition of different mechanisms for declining manufacturing share (Święcki, 2017; Smitkova, 2024), the interaction between host and home countries of multinationals (Alviarez et al., 2022), the joint evolution of service expenditure and trade openness (Lewis et al., 2022; Bonadio et al., 2025).

Closest to our paper are Matsuyama (2019); and Sposi et al. (2024). Matsuyama (2019) analytically characterizes how trade cost reductions affect sectoral composition and welfare using a two-country model of intra-industry trade. Sposi et al. (2024) develop a quantitative multi-country model embedding capital accumulation, input-output linkages to explore the determinants of the hump-shaped path of manufacturing share during development.[11] While these studies treat tariffs as a part of trade costs, our contribution is to explicitly analyze the role of tariffs distinct from trade costs in general. Unlike trade costs, tariffs bring about government revenue, thereby leading to richer welfare implications. Quantitatively, we use tariff data and separately calibrate sector-specific non-tariff trade barriers using the structural gravity.

Our paper also relates to a large literature that quantitatively evaluates trade policies, import tariffs in particular (Ossa, 2016; Caliendo and Parro, 2022 for surveys). Many studies confirm the possibility of welfare gains from unilaterally increasing tariffs from a low level, unless other countries retaliate (Costinot and Rodríguez-Clare, 2014; Ossa, 2014; Balistreri et al., 2024; Ignatenko et al., 2025).[12] However, most of these studies assume homothetic preferences and/or an elasticity of substitution across sectors being greater than one. Among others, Spearot (2016) highlights the role of nonhomothetic preferences (specifically, linear demand) in evaluating tariff policies. Although his model does not address structural change, we share his insight that overlooking nonhomothetic preferences can lead to different conclusions—particularly, an overestimation of welfare gains from unilateral tariffs in our case.

The remainder of this paper is structured as follows. Section 2 presents a two-country model of Ricardian trade and nonhomothetic preferences. Section 3 extends it to a full-fledged quantitative model. Section 4 explains the calibration of the model, solution algorithm, and the model fit. Section 5 presents the counterfactual results, and the final section 6 concludes.

Redding, 2022; Cheung and Yang, 2024; Kaboski et al., 2024), population aging (Cravino et al., 2022; Caron et al., 2020), schooling (Porzio et al., 2022; Cheung, 2023), and new entrants (Dent et al., 2016).

[11] See also Fujiwara and Matsuyama (2024) for an analytical approach to this issue.

[12] For example, Costinot and Rodríguez-Clare (2014) report in their Table 4.2 that U.S. welfare gains from imposing a 40 percent tariff on all imports in 2008 would be at most 0.63 percent, which is close in magnitude to our result of 0.36 percent, despite differences in model setup.

2 Two-country Model

To highlight the role of tariffs in shaping a country's sectoral composition, we first present a simple trade model *à la* Eaton and Kortum (2002), incorporating essential features for structural change: nonhomothetic CES preferences and a less-than-unity elasticity of substitution across sectors. Consider a static economy with two countries, Home (H) and Foreign (F). There are three tradable sectors, agriculture (a), manufacturing (m), and services (s).[13]

Demand Side: Let C_n be the aggregate consumption of country $n \in \{H, F\}$ and L_n be the mass of workers. The representative household minimizes its expenditure given a certain level of C_n, by choosing consumption for sectoral composite goods, C_n^j for $j \in \{a, m, s\}$:

$$\min_{\{C_n^j\}_j} \sum_{j=a,m,s} P_n^j C_n^j, \qquad \text{s.t.} \sum_{j=a,m,s} \left(\frac{C_n}{L_n}\right)^{\frac{\epsilon^j(1-\sigma)}{\sigma}} \left(\frac{C_n^j}{L_n}\right)^{\frac{\sigma-1}{\sigma}} = 1,$$

where P_n^j is the price index of the sectoral composite good in sector j in country n, and E_n is the aggregate income. The aggregate consumption C_n is *implicitly* defined by the equation in the second line. Two key parameters governing structural change are $\epsilon^j > 0$, capturing the degree of nonhomotheticity, and $\sigma \in (0, 1)$ measuring the elasticity of substitution across sectoral composite goods. We assume the parameter ranges such that $0 < \epsilon^a < \epsilon^m = 1 < \epsilon^s$ and $\sigma \in (0, 1)$ hold. If $\epsilon^j = 1$ for all j, the utility function is boiled down to a standard CES aggregator of sectoral composite goods, $C_n = \left[\sum_j (C_n^j)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$. If we let σ approach one under appropriate normalization, the utility function reduces to the Cobb-Douglas one, $C_n = \Pi_j (C_n^j)^{\frac{1}{3}}$.

Letting $E_n = \sum_j P_n^j C_n^j$ be the (minimized) total expenditure, the Hicksian demand function for the sectoral composite good is obtained as

$$C_n^j = L_n \left(\frac{P_n^j}{E_n/C_n}\right)^{-\sigma} \left(\frac{C_n}{L_n}\right)^{\epsilon^j(1-\sigma)+\sigma}. \tag{1}$$

Substituting this into the budget constraint, we solve for the expenditure function:

$$E_n = L_n \left[\sum_{j=a,m,s} \left\{\left(\frac{C_n}{L_n}\right)^{\epsilon^j} P_n^j\right\}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{2}$$

[13] Sectors and industries are synonymous in this paper. Although we state Propositions in the case of three sectors, the analytical results of this section hold in the case of general J sectors (see the Online Appendices A to C).

Letting $P_n = E_n/C_n$ be an aggregate price index, we have

$$P_n = \left[\sum_{j=a,m,s} \left\{ \left(\frac{C_n}{L_n} \right)^{\epsilon^j - 1} P_n^j \right\}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{3}$$

Using the relation $C_n/L_n = (E_n/L_n)/P_n$, we interpret C_n/L_n as the real per capita consumption, which in equilibrium corresponds to real per capita income. This measure is distinct from the nominal aggregate expenditure E_n.

From these results, we can see the income (expenditure) and the relative price effects. The income effect is seen from the non-constant income (expenditure) elasticity of sectoral demand:[14]

$$\frac{\partial \ln C_n^j}{\partial \ln E_n} = \sigma + (1-\sigma)\frac{\epsilon^j}{\bar{\epsilon}_n} > 0,$$

where $\bar{\epsilon}_n$ is country n's average degree of nonhomotheticity, ϵ^js, weighted by the sectoral expenditure shares, ω_n^js:

$$\bar{\epsilon}_n = \sum_{h=a,m,s} \omega_n^h \epsilon^h, \qquad \omega_n^h = \frac{P_n^h C_n^h}{\sum_k P_n^k C_n^k} = \frac{(P_n^h)^{1-\sigma}(C_n/L_n)^{\epsilon^j(1-\sigma)}}{\sum_k (P_n^k)^{1-\sigma}(C_n/L_n)^{\epsilon^k(1-\sigma)}}. \tag{5}$$

In the case of CES preferences with $\epsilon^j = 1$ for all j, the sectoral demand elasticity is σ across all sectors and $\bar{\epsilon}_n$ reduces to one. As we assume $0 < \epsilon^a < \epsilon^m = 1 < \epsilon^s$, agriculture has the lowest elasticity, and services have the highest one. Therefore, ϵ^j can be interpreted as the degree of income elasticity of demand for sector j.

The relative price effect results from the sectoral demands that are gross complements:[15]

$$\frac{\partial \ln C_n^j}{\partial \ln P_n^h} = -(1-\sigma)\frac{\omega_n^h \epsilon^j}{\bar{\epsilon}_n} < 0, \qquad j \neq h$$

It also reflects the less-than-unity elasticity of substitution across sectors in the absence of the income

[14] We rearrange (1) and take its log to obtain

$$\ln C_n^j = (1-\sigma)(1-\epsilon^j)\ln L_n - \sigma \ln P_n^j + \sigma \ln E_n + (1-\sigma)\epsilon^j \ln C_n. \tag{4}$$

To obtain the income elasticity of sectoral demand, we take the derivative of this with respect to E_n, considering its effect on the real consumption (i.e., utility) by noting $\partial \ln C_n / \partial \ln E_n = 1/\bar{\epsilon}_n$ from (2).

[15] Using $C_n = E_n/P_n$, we have

$$\ln C_n^j = (1-\sigma)(1-\epsilon^j)\ln L_n - \sigma \ln P_n^j + [\sigma + (1-\sigma)\epsilon^j]\ln E_n - (1-\sigma)\epsilon^j \ln P_n.$$

Differentiating this with respect to P_n^h while keeping E_n fixed (i.e., C_n^j here being a Marshallian demand) yields the expression in the text. In doing so, we use $\partial \ln P_n / \partial \ln P_n^h = \omega_n^h / \bar{\epsilon}_n$ from (3).

effect:

$$-\frac{\partial \ln(C_n^j/C_n^h)}{\partial \ln(P_n^j/P_n^h)} = \sigma \in (0,1) \qquad \text{if } \ \epsilon^j = 1 \text{ for all } j.$$

Supply Side: The production side follows Eaton and Kortum (2002). Producers of sectoral composite goods in sector j in country n are perfectly competitive and bundle input varieties $z \in [0,1]$ using a CES technology, with $\eta > 0$ being the elasticity of substitution. While the sectoral composite goods are not tradable, the producers source tradable input varieties from the least expensive country.

The variety producers are also perfectly competitive and produce $y_n^j(z)$ using a linear technology such that $y_n^j(z) = a_n^j(z) l_n^j(z)$, where $a_n^j(z)$ and $l_n^j(z)$ are respectively the labor productivity and the labor input in sector j in country n for producing variety z. The labor productivity follows the Frechét distribution with the cumulative distribution function (CDF): $\Pr[a_n^j \le a] = \exp[-(a/(\widetilde{\gamma} A_n^j))^{-\theta}]$. Here, the shape parameter of $\theta > 1$ governs the dispersion of productivity shocks, and the location parameter A_n^j governs the average productivity. $\widetilde{\gamma} = [\Gamma((\theta+1-\eta)/\theta)]^{-\frac{1}{1-\eta}}$ is a normalizing constant, where $\Gamma(\cdot)$ is the Gamma function. We assume $\theta + 1 - \eta > 0$ to ensure that the expectation of prices is finite. In this section, we assume that the average productivity is the same across sectors, $A_n^j = A_n$ for all j, to highlight the role of sector-specific tariffs.

International Trade: Trade in varieties are subject to tariffs, $\tau_{ni}^j \ge 0$ for $n \ne i \in \{H, F\}$ and $\tau_{nn}^j = 1$, and non-tariff trade barriers, $d_{ni} > 1$ for $n \ne i \in \{H, F\}$ and $d_{nn} = 1$. Thus, the total bilateral trade costs per shipment of a variety in sector j from country i to n are $b_{ni}^j = d_{ni}\left(1 + \tau_{ni}^j\right)$. Thus, letting w_n be the wage in country n, the unit cost of variety z produced in country n and shipped to i is $w_n b_{ni}^j / a_n(z)$. As a result of the cost-minimization of sectoral composite good producers, the price of the variety available in country n becomes $p_n^j(z) = \min_i\{w_i b_{ni}^j / a_i(z)\}$. With these results and the Frechét-distributed productivity shocks, the price of the sectoral composite good is obtained by

$$P_n^j = \left[\sum_{i=H,F} \left(w_i b_{ni}^j / A_i\right)^{-\theta}\right]^{-\frac{1}{\theta}}. \tag{6}$$

Let X_{ni}^j be country n's expenditure on sector j varieties from i. The share of country i's varieties in country n's sectoral expenditure becomes

$$\frac{X_{ni}^j}{P_n^j C_n^j} = \pi_{ni}^j = \frac{\left(w_n b_{ni}^j / A_i\right)^{-\theta}}{\sum_{i'=H,F} \left(w_{i'} b_{ni'}^j / A_{i'}\right)^{-\theta}},$$

which we call the trade share of country i in the market of country n.

Market Clearing: As labor is the only factor of production, the sectoral value added is the sectoral labor income, which comes from the sales from the domestic and the foreign markets:

$$VA_n^j = w_n L_n^j = \sum_{i=H,F} \frac{X_{in}^j}{1+\tau_{in}^j} = \sum_{i=H,F} \frac{\pi_{in}^j P_i^j C_i^j}{1+\tau_{in}^j},$$

where L_n^j is sector j employment in country n, and the export sales are divided by gross tariff rates, $1+\tau_{in}^j$. This is because the price index is tariff inclusive (c.i.f.), while sales for producers are based on the tariff-exclusive (f.o.b.) price. Summing this condition over sectors gives the labor market clearing condition:

$$w_n L_n = \sum_{j=a,m,s} \sum_{i=H,F} \frac{\pi_{in}^j P_i^j C_i^j}{1+\tau_{in}^j}. \tag{7}$$

One can check that this is equivalent to the trade balance condition.[16] The aggregate expenditure must be equal to the aggregate income consisting of labor income and tariff revenues, $\widetilde{T}_n$:

$$E_n = w_n L_n + \widetilde{T}_n, \tag{8}$$

where $\widetilde{T}_n = \sum_j \widetilde{T}_n^j = \sum_j \tau_{ni}^j IM_n^j$, $IM_n^j = X_{ni}^j/(1+\tau_{ni}^j) = \pi_{ni}^j P_n^j C_n^j/(1+\tau_{ni}^j)$ for $i \neq n$, and $\widetilde{T}_n^j$ is country n's tariff revenues in sector j and IM_n^j is country n's imports in sector j. This completes the model. With a choice of numéraire such that $w_F = 1$, the equilibrium wage w_H and the aggregate consumption $\{C_i\}_{i=H,F}$ satisfy equilibrium conditions (1), (2), (6), (7), and (8).

2.1 Tariffs and Real Per Capita Consumption

Before examining the tariff impact on sectoral composition, let us first see how tariffs affect real per capita consumption C_n/L_n.[17] In the following subsections, we assume that existing tariffs

[16] The trade balance condition is $\sum_j \pi_{HF}^j P_H^j C_H^j/(1+\tau_{HF}^j) = \sum_j \pi_{FH}^j P_F^j C_F^j/(1+\tau_{FH}^j)$.

[17] As tariffs do not affect population L_n, the following discussion holds both in terms of aggregate and per capita consumption.

applied by Home to Foreign exports are uniform across sectors (including services, hypothetically), $\tau^j_{HF} = \tau_{HF} \geq 0$ for all $j \in \{a, m, s\}$, and Foreign does not set tariffs, $\tau^j_{FH} = 0$ for all j. We then consider a unilateral increase in Home's tariffs. For analytical convenience, in the case of nonhomothetic CES ($0 < \epsilon^a < \epsilon^m = 1 < \epsilon^s$), the tariff increase is assumed to be uniform across sectors, $d\tau^j_{HF} = d\tau_{HF} > 0$ for all j. In the case of homothetic CES including Cobb-Douglas as a special case ($\epsilon^j = 1$ for all j), however, the tariff increase can vary across sectors, $d\tau^j_{HF} = d\tau_{HF} > 0$ for some j and $d\tau^h_{HF} = 0$ for $h \neq j$. As will be seen clearly in the next subsection, the former highlights the income effect and the latter the relative price effect.

In both cases of nonhomothetic CES and homothetic CES, the real per capita consumption in Home is expressed as $C_H/L_H = (1 + \mu_H)w_H/P_H$, and its logarithmic change is

$$d \ln \left(\frac{C_H}{L_H}\right) = \underbrace{d \ln w_H}_{>0} + \underbrace{\frac{\mu_H d \ln \mu_H}{1 + \mu_H}}_{\gtreqless 0} \underbrace{- d \ln P_H}_{\gtreqless 0}, \tag{9}$$

where μ_H denotes the ratio of tariff revenue to labor income and is given by $\mu_H = \widetilde{T}_H/(w_H L_H)$.[18] An increase in tariffs in Home reduces import demand there and raises Home's export price relative to its import price to maintain balanced trade. This leads to a higher relative wage in Home, which Matsuyama (2019) refers to as the terms-of-trade effect in terms of production factors. This mechanism ensures that the first term in (9) is positive, $d \ln w_H > 0$.

In addition, if the tariff rises from a sufficiently low level, it increases the ratio of tariff revenue to labor income and makes the second term in (9) positive, $\mu_H d \ln \mu_H/(1 + \mu_H) > 0$. A further tariff increase, however, reduces Home's imports significantly, so that the second term turns from positive to negative at some point, $\mu_H d \ln \mu_H/(1 + \mu_H) < 0$.

In the case of homothetic CES, the aggregate price index always increases with tariffs and thus the third term in (9) is always negative: $-d \ln P_H < 0$, for the obvious reason that tariffs make the imported varieties more expensive and thus raise the sectoral price indices constituting the aggregate price index. In the case of nonhomothetic CES, however, the aggregate price index may fall with tariffs and the third term in (9) can be positive: $-d \ln P_H > 0$, if the income elasticity of manufacturing demand is sufficiently high such that $\epsilon^m = 1 > \bar{\epsilon}_H$.[19]

[18] Strictly speaking, the second term in (9), $d \ln \mu_H = d\mu_H/\mu_H$, is not defined at zero tariff (zero revenue). However, the discussion in the text also goes through in this case.

[19] The change in the aggregate price index in Home is given by

$$d \ln P_H = d \ln E_H - d \ln (C_H/L_H) = \sum\nolimits_j \omega^j_H d \ln P^j_H + (\bar{\epsilon}_H - 1) d \ln (C_H/L_H),$$

As in the tariff-revenue-to-income ratio μ_H, we can show that rising tariffs first increase and then decrease the real per capita consumption C_H/L_H, whether preferences are nonhomothetic CES or homothetic CES including Cobb-Douglas as a special case.[20] However, an important difference between the two types of preferences lies in the magnitude of the effects of tariffs. Tariffs influence real per capita consumption through the channels we discussed. And this real per capita consumption in turn affects into the aggregate price index. Under nonhomothetic CES preferences, if the income elasticity of manufacturing demand is sufficiently low ($\epsilon^m = 1 < \bar{\epsilon}_H$), the rise in consumption driven by tariffs disproportionately expands the high-income-elastic service demand. This pushes up the aggregate expenditure and thus the aggregate price index (i.e., the minimum expenditure per unit of real per capita consumption $P_H = E_H/(C_H/L_H)$; see footnote 19). The higher aggregate price index lowers welfare gains from tariffs. Therefore, if $\epsilon^m = 1 < \bar{\epsilon}_H$, the magnitude of the effect of tariffs on real per capita consumption is smaller under nonhomothetic CES than under homothetic CES.[21]

The discussion is summarized in the following proposition. The proofs of this and the subsequent propositions, along with other results, are provided in Supplemental Appendices A to C. In particular, we can also show that Foreign is always worse off, and this negative welfare impact is more pronounced under nonhomothetic CES than under homothetic CES if $\epsilon^m = 1 < \bar{\epsilon}_F = \sum_j \epsilon^j \omega_F^j$.

Proposition 1. Tariffs and Real Per Capita Consumption

We consider the effect of an increase in Home's tariffs in a model with two asymmetric countries and three symmetric sectors. If preferences are nonhomothetic CES, the tariff increase is assumed to be uniform across all sectors; while if preferences are either homothetic CES or Cobb-Douglas, the tariff increase can be sector specific. Then the following holds:

(i) Regardless of the types of preferences, the real per capita consumption in Home first rises and then falls; there exists a tariff level τ_{HF}^* below which $d(C_H/L_H)/d\tau_{HF} > 0$, and above which $d(C_H/L_H)/d\tau_{HF} < 0$.

(ii) If the income elasticity of manufacturing demand is sufficiently low ($\epsilon^m = 1 < \bar{\epsilon}_H$), the

where $\bar{\epsilon}_H = \sum_j \omega_H^j \epsilon^j$ is the expenditure-weighted average of nonhomotheticity parameters. In the case of homothetic CES with $\epsilon^j = 1$ for all j, the second term vanishes. If $\bar{\epsilon}_H < 1 = \epsilon^m$ holds, a rise in real per capita consumption due to tariffs ($d\ln(C_H/L_H) > 0$) may decrease the aggregate price index ($d\ln P_H < 0$).

[20] This implies the optimal level of tariff that maximizes welfare. We can analytically derive the optimal uniform tariffs across sectors and show that they are identical regardless of preferences. See Supplemental Appendix A for details.

[21] If the opposite $\epsilon^m = 1 > \bar{\epsilon}_H$ holds, in response to the rise in consumption due to tariffs, the low-income-elastic agricultural demand increases disproportionately, pushing down the aggregate expenditure and thus the aggregate price index. Therefore, the magnitude of the effect of tariffs is larger under nonhomothetic CES than under homothetic CES.

magnitude of the effect of increasing tariffs across all sectors, as described in (i), is smaller under nonhomothetic CES preferences than under homothetic CES or Cobb-Douglas preferences. The opposite holds if $\epsilon^m = 1 > \bar{\epsilon}_H$.

2.2 Tariffs and Expenditure Shares

Let us turn to the impact of tariffs on the sectoral expenditure share given in (5). The logarithmic change in Home's expenditure share of sector $j \in \{a, m, s\}$ is decomposed as

$$d \ln \omega_H^j = \underbrace{(1-\sigma)\left(d \ln P_H^j - \sum_{h=a,m,s} \omega_H^h d \ln P_H^h\right)}_{\text{Relative price effect}} + \underbrace{(1-\sigma)\left(\epsilon^j - \bar{\epsilon}_H\right) d \ln\left(\frac{C_H}{L_H}\right)}_{\text{Income effect}}, \tag{10}$$

where we recall that $\bar{\epsilon}_H = \sum_h \omega_H^h \epsilon^h$.

The first set of terms in (10) is the relative price effect through changes in the relative price of the sector j composite good. To highlight this, we assume homothetic CES preferences by setting $\epsilon^j = 1$ for all j, and consider an increase in the sector j tariff only, $d\tau_{HF}^j = d\tau_{HF} > 0$. The tariff increase in a sector raises its price index relative to those in the other sectors by making the imported intermediate varieties more expensive to produce the sectoral composite good. With a less-than-unity elasticity of substitution across sectors, $\sigma \in (0, 1)$, the higher relative price of the sector's composite good leads to a higher expenditure share in that sector and lowers shares in the other sectors.

The second set of terms in (10) is the income effect through changes in the real per capita consumption. The income effect is absent in the case of homothetic CES with $\epsilon^j = 1$ for all j. To see this, we assume nonhomothetic CES preferences with $\epsilon^a < \epsilon^m = 1 < \epsilon^s$ and consider a uniform increase in tariffs across all sectors j, $d\tau_{HF}^j = d\tau_{HF} > 0$. This raises the price indices of all sectors proportionally, shutting down the relative price effect. The tariff increase, starting from a sufficiently low level, raises the real per capita consumption in Home (Proposition 1). Home then shifts expenditure away from the sectors with a lower income elasticity (smaller ϵ^j) toward the sectors with a higher income elasticity (greater ϵ^j). In our model of three sectors, we can show that the income effect is negative in agriculture, positive in services, and it can be negative or positive in manufacturing, depending on the value of ϵ^m relative to $\bar{\epsilon}_H$. Letting $\sigma = 1$, preferences reduce to Cobb-Douglas, and the sectoral expenditures are fixed. Therefore, tariffs have no effect on the sectoral expenditure shares; (10) is always zero.

Focusing on the case where tariffs increase real per capita consumption ($\tau_{HF} \in [0, \tau_{HF}^*)$ from

Proposition 1), we formally state the following proposition.

Proposition 2. Tariffs and Expenditure Shares

We consider the effect of an increase in Home's tariffs in a model with two asymmetric countries and three symmetric sectors.

(i) Assume preferences are nonhomothetic CES and the tariffs uniformly increase from $\tau_{HF} \in [0, \tau^*_{HF})$ *in all sectors. Then the agricultural expenditure share in Home falls* ($d\omega^a_H/d\tau_{HF} < 0$)*; the service expenditure share rises* ($d\omega^s_H/d\tau_{HF} > 0$)*; the manufacturing expenditure share falls if the income elasticity of manufacturing demand is sufficiently low (i.e.,* $d\omega^m_H/d\tau_{HF} < 0$ *if* $\epsilon^m = 1 < \bar{\epsilon}_H$*), while it rises otherwise (i.e.,* $d\omega^m_H/d\tau_{HF} > 0$ *if* $\epsilon^m = 1 > \bar{\epsilon}_H$*).*

(ii) Assume preferences are homothetic CES and the tariffs uniformly increases from $\tau_{HF} \in [0, \tau^*_{HF})$ *in some sectors. Then the sectoral expenditure share in Home rises in the sectors where tariffs increase* ($d\omega^j_H/d\tau^j_{HF} > 0$ *for some* j *with* $d\tau^j_{HF} > 0$*), while it falls in the sectors where tariffs remain unchanged* ($d\omega^h_H/d\tau^j_{HF} < 0$ *for* $h \neq j$ *with* $d\tau^h_{HF} = 0$*).*

(iii) Assume preferences are Cobb-Douglas. Changes in tariffs have no effect on the sectoral expenditure shares.

From Proposition 2, we can infer (not formally prove) how the manufacturing tariff affects the manufacturing expenditure shares in the presence of both nonhomothetic CES preferences and sector-specific tariffs. Suppose Home increases the manufacturing tariff from a sufficiently low level and that $\epsilon^m = 1 < \bar{\epsilon}_H$ holds, which is true for developed countries such as the U.S.[22] As the relative price of manufacturing rises, Home shifts expenditure from agriculture and services toward manufacturing. However, the tariff is also likely to increase Home's real per capita consumption, shifting expenditure from agriculture and manufacturing toward services.

In summary, the manufacturing expenditure share is positively affected by the manufacturing tariff through the relative price effect, but negatively affected through the income effect. The dominance of either effect is left to the full quantitative analysis in the later sections.

[22] The average U.S. tariff on manufacturing products ranged from three to five percent between 1990 and 2014 (see "inward" tariffs in Figure A2 in Supplemental Appendix J for the U.S. and other developed countries). Given our calibrated values in Section 4 ($(\epsilon^a, \epsilon^m, \epsilon^s) = (0.05, 1, 1.2)$ as in Comin et al., 2021), the expenditure-weighted average of nonhomotheticity parameters in the U.S., $\bar{\epsilon}_{US}$, consistently exceeded one and increased in most years from 1965 to 2014 (from 1.06 to 1.13).

2.3 Tariffs and Value-added Shares

The value added of sector $j \in \{a, m, s\}$ in the tariff-imposing Home is the labor income in that sector, which is earned from sales in the domestic and foreign markets:

$$\begin{aligned} VA_H^j = w_H L_H^j &= \sum_{i=H,F} \pi_{iH}^j P_i^j C_i^j \\ &= P_H^j C_H^j + NX_H^j - \widetilde{T}_H^j, \qquad NX_H^j = EX_H^j - IM_H^j = \pi_{FH}^j P_F^j C_F^j - \frac{\pi_{HF}^j P_H^j C_H^j}{1+\tau_{HF}^j}, \end{aligned}$$

where NX_H^j is Home's net exports in sector j, defined as gross exports minus gross imports. The sectoral value added in Home increases with domestic expenditure and net exports, but decreases with tariff revenue, since the revenue accrues from the value added generated by workers in Foreign.

The value-added share of sector j in Home is then $va_H^j = VA_H^j/(w_H L_H)$. Its logarithmic change is given by[23]

$$d\ln va_H^j = \underbrace{\frac{P_H^j C_H^j}{w_H L_H^j}\left(d\ln \omega_H^j + \frac{\mu_H d\ln \mu_H}{1+\mu_H}\right)}_{\text{(a) Expenditure adj. by tariff revenue}} + \underbrace{\frac{NX_H^j}{w_H L_H^j} d\ln\left(\frac{NX_H^j}{w_H L_H}\right)}_{\text{(b) Net exports}} - \underbrace{\frac{\widetilde{T}_H^j}{w_H L_H^j} d\ln\left(\frac{\widetilde{T}_H^j}{w_H L_H}\right)}_{\text{(c) Tariff revenue}}. \tag{11}$$

Under autarky, the value-added shares and the expenditure shares always coincide: $va_H^j = \omega_H^j$. With international trade, however, they may differ.

The contribution of sectoral expenditure shares to value-added shares is captured by the first term (a) in (11). This expenditure channel also includes changes in aggregate tariff revenue relative to labor income ($\mu_H = \widetilde{T}_H/(w_H L_H)$)—the second term within (a)—since higher revenue scales up consumption across all sectors. How the sectoral expenditure shares respond to tariffs—the first term within (a)—is already discussed in Proposition 2. If only the relative price effect operates in sector j under homothetic CES preferences and sector-specific tariffs, then $d\ln \omega_H^j > 0$ holds, contributing positively to $d\ln va_H^j$. In contrast, if only the income effect operates under nonhomothetic CES preferences with uniform tariffs across sectors, then $d\ln \omega_H^j < 0$ holds for a lower-income-elastic sector j, contributing negatively to $d\ln va_H^j$.

The last term (c) in (11) is a mechanical channel from the definition of va_H^j. When tariffs raise

[23]The second term (b) is defined as

$$\frac{NX_H^j}{w_H L_H^j} d\ln\left(\frac{NX_H^j}{w_H L_H}\right) = \frac{EX_H^j}{w_H L_H^j} d\ln\left(\frac{EX_H^j}{w_H L_H}\right) - \frac{IM_H^j}{w_H L_H^j} d\ln\left(\frac{IM_H^j}{w_H L_H}\right).$$

tariff revenue in a sector more than labor income, the share of contribution by Home's workers in that sector falls.

The second term (b) in (11) captures the standard protective role of tariffs. As Home increases tariffs on a sector, imports in that sector decline and net exports improve, thereby expanding its value-added share. This net export channel (b) operates under Cobb-Douglas preferences, where neither the relative price effect nor the income effect is present.

To determine the sign of (11), we highlight the polar cases where either one of the income effect via nonhomothetic preferences or the relative price effect via sector-specific tariffs operates, as in Proposition 2. Under slightly more restrictive conditions (an increase in tariffs from zero), we can formally show the following proposition.

Proposition 3. Tariffs and Value-added Shares

We consider the effect of an increase in Home's tariffs in a model with two countries and three symmetric sectors.

(i) Assume preferences are nonhomothetic CES; the two countries are symmetric; the income elasticity of manufacturing demand is low enough; and the tariff uniformly increases from zero in all sectors. Then the sectoral value-added share in agriculture and manufacturing falls ($dva^j_H/d\tau_{HF} < 0$ for $j \in \{a, m\}$), while it rises in services ($dva^s_H/d\tau_{HF} > 0$), and the sectoral expenditure shares exhibit the same pattern.

(ii) Assume preferences are homothetic CES; the two countries are asymmetric; and the tariff uniformly increases from zero in some sectors. Then the sectoral value-added share in Home rises in the sectors where tariffs increase ($dva^j_H/d\tau^j_{HF} > 0$ for some j with $d\tau^j_{HF} > 0$), while it falls in the sectors where tariffs remain unchanged ($dva^h_H/d\tau^j_{HF} < 0$ for $h \neq j$ with $d\tau^h_{HF} = 0$), and the sectoral expenditure shares exhibit the same pattern.

(iii) Assume preferences are Cobb-Douglas and the two countries are asymmetric. The sectoral value-added shares exhibit the same pattern as under homothetic CES, while the sectoral expenditure shares remain unchanged.

From Proposition 3, we can infer (not formally prove) the effect of tariffs on sectoral value-added shares in the presence of both nonhomothetic CES preferences and sector-specific tariffs. Suppose Home increases tariffs from a sufficiently low level only in manufacturing sector with a lower income elasticity, $\epsilon^m = 1 < \bar{\epsilon}_H$, which seems plausible in developed countries today (see footnote 22). Then the effects of the manufacturing tariff on its value-added share mirror those on the manufacturing expenditure share, captured by the expenditure channel (a) in (11): the relative price effect is likely to contribute positively to the manufacturing value-added share, while the income effect is likely to contribute negatively. In addition, more expensive foreign manufacturing varieties will be replaced with domestic varieties, leading to a higher manufacturing value-added share, which is captured by the net export channel in (11). We will quantify the magnitudes of those three channels in later sections.

2.4 Toward a Full Dynamic Model

To bridge to a fully quantitative dynamic model, we introduce capital into the two-country model and extend it to two periods. Using this extended model, we examine how tariff shocks affect the household's consumption-saving decision. Letting $K_{n,t}$ and $I_{n,t}$ denote the capital stock and investment, respectively, in country $n \in \{H, F\}$ and period $t \in \{1, 2\}$, the law of motion of the capital stock is $K_{n,t+1} = I_{n,t}$, where capital in the last period fully depreciates. Capital is owned by the household and rented to domestic variety producers at the rate $r_{n,t}$. Varieties are produced according to a Cobb-Douglas production technology using capital and labor. The investment good is produced using the sectoral composite goods *à la* CES technology with the elasticity parameter $\sigma^K \in (0, 1)$ and is domestically sold at price $P^K_{n,t}$. The consumption-saving decision maximizes the discounted sum of utility derived from the aggregate consumption good: $u(C_{n,1}) + \beta u(C_{n,2})$.

Saving Decisions and a Tariff Shock: In each period $t \in \{1, 2\}$, a representative household in country H earns labor and capital income, $w_{H,t}L_{H,t} + r_{H,t}K_{H,t}$, as well as tariff revenue, $\widetilde{T}_{H,t}$, and spends on consumption, $E_{n,t}$, and investment, $P^K_{H,t}I_{n,t}$:

$$\begin{aligned} E_{H,t} + P^K_{H,t}I_{H,t} &= w_{H,t}L_{H,t} + r_{H,t}K_{H,t} + \widetilde{T}_{H,t} \\ &= (1 + \mu_{H,t})(w_{H,t}L_{H,t} + r_{H,t}K_{H,t}), \end{aligned}$$

where $\mu_{H,t} = \widetilde{T}_{H,t}/(w_{H,t}L_{H,t} + r_{H,t}K_{H,t})$ is the ratio of tariff revenue to factor income. We rearrange the intertemporal budget constraint in Home in period 1 to obtain

$$I_{H,1} = \frac{P_{H,1}}{P^K_{H,1}} \left[\frac{(1 + \mu_{H,1})(w_{H,1}L_{H,1} + r_{H,1}K_{H,1})}{P_{H,1}} - C_{H,1} \right]. \tag{12}$$

Investment occurs ($I_{H,1} > 0$) when part of aggregate real income is saved. This helps us understand how tariffs affect the consumption-saving decision.

Let us consider an increase in the manufacturing tariff that takes effect in both periods 1 and 2. The tariff increase in period 1 is a surprise shock, and a household anticipates that it will remain effective in period 2. In response to the tariff increase from a sufficiently low level, the aggregate real income rises in both periods, which results in higher real consumption ($C_{H,t}$). As implied by (12), investment in period 1 is determined by the relative changes in income and consumption. The household anticipating a higher income in the future has a weaker incentive to save for intertemporal consumption smoothing. Therefore, the aggregate real consumption may increase more than the aggregate real income, leading to a decline in investment in period 1 following the tariff shock, $dI_{H,1} < 0$. We will examine whether this intuition holds in the counterfactual exercise using the full dynamic quantitative model developed below.

3 Quantitative Model

We extend the two-country model to a dynamic multi-country model with capital accumulation and sectoral input-output linkages. Time is discrete: $t = 0, 1, \cdots$. The set of countries is $\{1, \cdots, N\}$, with the number of countries being N. Countries are generically indexed by i or n. We maintain three sectors as in the previous model: agriculture, manufacturing, and services.

The representative household in country n as of period 0 maximizes the lifetime utility function:

$$\sum_{t=0}^{\infty} \beta^t \zeta_{n,t} L_{n,t} \frac{(C_{n,t}/L_{n,t})^{1-\psi}}{1-\psi}, \tag{13}$$

where $\beta \in (0,1)$ is the discount factor, $\psi > 1$ is the inverse of the intertemporal elasticity of substitution, $\zeta_{n,t}$ is the demand shifter in country n and period t, and $L_{n,t}$ is the population of country n in period t. The aggregate consumption in country n and period t, $C_{n,t}$, is *implicitly* defined by

$$\sum_{j=a,m,s} (\Omega^j_{n,t})^{\frac{1}{\sigma}} \left(\frac{C_{n,t}}{L_{n,t}}\right)^{\frac{\epsilon^j(1-\sigma)}{\sigma}} \left(\frac{C^j_{n,t}}{L_{n,t}}\right)^{\frac{\sigma-1}{\sigma}} = 1, \tag{14}$$

where, for $j \in \{a, m, s\}$, $C^j_{n,t}$ is the composite good of sector j which the representative household in country n and period t consumes, $\Omega^j_{n,t}$ is the demand shifter for sector j in country n and period t.[24]

Solving the intratemporal expenditure minimization problem given $C_{n,t}$, the expenditure of

[24] Besides this nonhomothetic CES period utility function, we consider a homothetic CES function (letting $\epsilon^j = 1$ for any j) and a Cobb-Douglas function $\prod_{j=a,m,s}(C^j_{n,t}/\chi^j_{n,t})^{\chi^j_{n,t}}$ with $\sum_{j=a,m,s} \chi^j_{n,t} = 1$.

country n in period t is

$$E_{n,t} = L_{n,t} \left[\sum_{j=a,m,s} \Omega^j_{n,t} \left\{ \left(\frac{C_{n,t}}{L_{n,t}} \right)^{\epsilon^j} P^j_{n,t} \right\}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{15}$$

where $P^j_{n,t}$ is the price of the composite good of sector j in country n and period t. Define $P_{n,t}$ by $P_{n,t} = E_{n,t}/C_{n,t}$. Then we have

$$P_{n,t} = \left[\sum_{j=a,m,s} \Omega^j_{n,t} \left\{ \left(\frac{C_{n,t}}{L_{n,t}} \right)^{\epsilon^j - 1} P^j_{n,t} \right\}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$

The consumption of the composite good of sector j is

$$C^j_{n,t} = L_{n,t} \Omega^j_{n,t} \left(\frac{P^j_{n,t}}{P_{n,t}} \right)^{-\sigma} \left(\frac{C_{n,t}}{L_{n,t}} \right)^{\epsilon^j(1-\sigma)+\sigma}. \tag{16}$$

Let $\omega^j_{n,t} = P^j_{n,t} C^j_{n,t} / E_{n,t}$ be country n's consumption expenditure share on sector j in period t. Then we have

$$\omega^j_{n,t} = \frac{\Omega^j_{n,t} \left\{ \left(\frac{C_{n,t}}{L_{n,t}} \right)^{\epsilon^j} P^j_{n,t} \right\}^{1-\sigma}}{\sum_{j'=a,m,s} \Omega^{j'}_{n,t} \left\{ \left(\frac{C_{n,t}}{L_{n,t}} \right)^{\epsilon^{j'}} P^{j'}_{n,t} \right\}^{1-\sigma}}. \tag{17}$$

By definition, we have $\sum_{j=a,m,s} \omega^j_{n,t} = 1$.

The representative household in country n is the sole owner of labor and capital there. The budget constraint of country n in period t is

$$E_{n,t} + P^K_{n,t} I_{n,t} \leq (1 - \phi_{n,t}) \left(w_{n,t} L_{n,t} + r_{n,t} K_{n,t} + \widetilde{T}_{n,t} \right) + L_{n,t} T^P_t, \tag{18}$$

where $P^K_{n,t}$ is the capital good price index which will be defined later, and $I_{n,t}$ is the quantity of investment. The right-hand side represents the national income, taking into account aggregate trade imbalances. There is a global portfolio that collects an exogenous $\phi_{n,t}$ share of total value added, $w_{n,t} L_{n,t} + r_{n,t} K_{n,t}$, plus tariff revenue, $\widetilde{T}_{n,t}$, and redistributes a per capita transfer, T^P_t, to each country to balance the global portfolio's budget. In this way, we model trade imbalances as transfers, abstracting from cross-border borrowing and lending (Sposi et al., 2024).

Let $K_{n,t}$ be the quantity of capital in country n and period t. Then capital dynamics are

$$K_{n,t+1} = (1-\delta_{n,t})K_{n,t} + I_{n,t}^{\lambda}(\delta_{n,t}K_{n,t})^{1-\lambda}, \tag{19}$$

where $\delta_{n,t}$ is the capital depreciation rate in country n and period t and $\lambda \in [0,1]$ is a parameter governing capital adjustment costs. Solving this for $I_{n,t}$ and viewing it as a function of $K_{n,t}$, $K_{n,t+1}$, and $\delta_{n,t}$, we have

$$I_{n,t} = \Phi(K_{n,t+1}, K_{n,t}; \delta_{n,t}) = \delta_{n,t}^{1-\frac{1}{\lambda}} K_{n,t} \left[\frac{K_{n,t+1}}{K_{n,t}} - (1-\delta_{n,t})\right]^{\frac{1}{\lambda}}.$$

The dynamic optimization problem of the representative household in country n and period 0 is to maximize (13) subject to (15), (18), and (19). Solving this problem, we obtain the Euler equation:

$$\left(\frac{C_{n,t+1}/L_{n,t+1}}{C_{n,t}/L_{n,t}}\right)^{\psi-1} \frac{E_{n,t+1}\bar{\epsilon}_{n,t+1}}{E_{n,t}\bar{\epsilon}_{n,t}} = \beta \underbrace{\frac{\zeta_{n,t+1}}{\zeta_{n,t}}}_{\text{(i)}} \underbrace{\frac{L_{n,t+1}}{L_{n,t}}}_{\text{(ii)}} \underbrace{\frac{(1-\phi_{n,t+1})r_{n,t+1} - P_{n,t+1}^{K}\Phi_2(K_{n,t+2}, K_{n,t+1}; \delta_{n,t+1})}{P_{n,t}^{K}\Phi_1(K_{n,t+1}, K_{n,t}; \delta_{n,t})}}_{\text{(iii)}}, \tag{20}$$

where Φ_l is the derivative of $I_{n,t} = \Phi(K_{n,t+1}, K_{n,t}; \delta_{n,t})$ with respect to its l-th argument,[25] and

$$\bar{\epsilon}_{n,t} = \sum_{j=a,m,s} \omega_{n,t}^{j}\epsilon^{j}. \tag{21}$$

The associated transversality condition is also obtained. Both $E_{n,t+1}/E_{n,t}$ and $\bar{\epsilon}_{n,t+1}/\bar{\epsilon}_{n,t}$ are both increasing in $\frac{C_{n,t+1}/L_{n,t+1}}{C_{n,t}/L_{n,t}}$. Since $\psi > 1$, the left-hand side is just an increasing function of the ratio in per capita consumption between periods $t+1$ and t. Equation (20) tells that this per capita consumption ratio depends on the discount factor β, (i) the ratio in the intertemporal demand shifters, (ii) the ratio in populations, and (iii) the real return to capital.

We move on to producers' behavior. As in Section 2, the sectoral composite good producers bundle input varieties $z \in [0,1]$ using a CES aggregator with elasticity $\eta > 0$. The production

[25]Specifically,

$$\Phi_1(K_{n,t+1}, K_{n,t}) = \frac{\partial\Phi(K_{n,t+1}, K_{n,t})}{\partial K_{n,t+1}} = \frac{1}{\lambda}\delta_{n,t}^{1-\frac{1}{\lambda}} \left[\frac{K_{n,t+1}}{K_{n,t}} - (1-\delta_{n,t})\right]^{\frac{1}{\lambda}-1},$$

$$\Phi_2(K_{n,t+1}, K_{n,t}) = \frac{\partial\Phi(K_{n,t+1}, K_{n,t})}{\partial K_{n,t}} = \Phi_1(K_{n,t+1}, K_{n,t}) \cdot \left[(\lambda-1)\frac{K_{n,t+1}}{K_{n,t}} - \lambda(1-\delta_{n,t})\right].$$

function of variety z of sector j in country n and period t is

$$y^j_{n,t}(z) = a^j_{n,t}(z) \left[\frac{K^j_{n,t}(z)}{\gamma^j_{n,t}\alpha^j_{n,t}}\right]^{\gamma^j_{n,t}\alpha^j_{n,t}} \left[\frac{L^j_{n,t}(z)}{\gamma^j_{n,t}(1-\alpha^j_{n,t})}\right]^{\gamma^j_{n,t}(1-\alpha^j_{n,t})} \left[\frac{M^j_{n,t}(z)}{1-\gamma^j_{n,t}}\right]^{1-\gamma^j_{n,t}}. \tag{22}$$

Here $y^j_{n,t}(z)$ is the quantity of output, $a^j_{n,t}(z)$ is the productivity which will be expressed as a realization of a random variable, $K^j_{n,t}(z)$ is the capital, $L^j_{n,t}(z)$ is the labor, $\gamma^j_{n,t} \in (0,1)$ is the cost share of value added (contribution by primary production factors) in total output, $\alpha^j_{n,t} \in (0,1)$ is the share of capital within value added, $M^j_{n,t}(z)$ is the CES aggregate of sectoral intermediate inputs used for production of variety z, that is,

$$M^j_{n,t}(z) = \left[\sum_{j'=a,m,s} (\kappa^{j,j'}_{n,t})^{\frac{1}{\sigma^j}} (M^{j,j'}_{n,t}(z))^{\frac{\sigma^j-1}{\sigma^j}}\right]^{\frac{\sigma^j}{\sigma^j-1}},$$

where $\kappa^{j,j'}_{n,t}$ is the shifter for sector j's demand for sector j' composite good, $M^{j,j'}_{n,t}(z)$ is the input of sector j' composite good for production of variety z of sector j and is produced using the sectoral composite goods, and σ^j is the elasticity of substitution across sectoral composite goods for production of sector j varieties. In production of varieties in sector j, the cost share of sector j' inputs *within intermediate-input costs* is

$$g^{j,j'}_{n,t} = \frac{P^{j'}_{n,t} M^{j,j'}_{n,t}}{\sum_{j''=a,m,s} P^{j''}_{n,t} M^{j,j''}_{n,t}} = \frac{\kappa^{j,j'}_{n,t} (P^{j'}_{n,t})^{1-\sigma^j}}{\sum_{j''=a,m,s} \kappa^{j,j''}_{n,t} (P^{j''}_{n,t})^{1-\sigma^j}}.$$

The productivity of variety z of sector j in country n and period t, $a^j_{n,t}(z)$, follows the Frechét distribution whose CDF is given by $F^j_{n,t}(a) = \Pr[a^j_{n,t} \leq a] = \exp[-(a/(\widetilde{\gamma}^j A^j_{n,t}))^{-\theta^j}]$. Unlike the two-country model, θ^j varies across sectors, consequently so does $\widetilde{\gamma}^j = [\Gamma((\theta^j + 1 - \eta)/\theta^j)]^{-\frac{1}{1-\eta}}$. Productivity of varieties are independent within and across sectors, countries, and periods.

Cost minimization problem for the production function (22) yields the cost of input bundle:

$$\widetilde{c}^j_{n,t} = (r_{n,t})^{\gamma^j_{n,t}\alpha^j_{n,t}} (w_{n,t})^{\gamma^j_{n,t}(1-\alpha^j_{n,t})} (\xi^j_{n,t})^{1-\gamma^j_{n,t}}, \tag{23}$$

where $\xi^j_{n,t}$ is the price index for the composite intermediate input for production of sector j varieties:

$$\xi^j_{n,t} = \left[\sum_{j'=a,m,s} \kappa^{j,j'}_{n,t} (P^{j'}_{n,t})^{1-\sigma^j}\right]^{\frac{1}{1-\sigma^j}}.$$

The price index of the composite good of sector j in country n and period t is

$$P^j_{n,t} = \left[\sum_{i=1}^{N}\left(\frac{\widetilde{c}^j_{i,t} b^j_{ni,t}}{A^j_{i,t}}\right)^{-\theta^j}\right]^{-1/\theta^j}, \tag{24}$$

where $b^j_{ni,t}$ is the total trade costs including tariffs and non-tariff trade barriers for sector j from country i to n. $b^j_{ni,t}$ is expressed as

$$b^j_{ni,t} = d^j_{ni,t}(1 + \tau^j_{ni,t}),$$

where $d^j_{ni,t}$ is the iceberg trade cost for sector j varieties from country i to n in period t including non-tariff barriers, and $\tau^j_{ni,t}$ is country n's tariff rate on sector j varieties from country i in period t. For later use, define gross tariffs $\widetilde{\tau}^j_{ni,t}$ by $\widetilde{\tau}^j_{ni,t} = 1 + \tau^j_{ni,t}$.

The production function of capital (investment) goods in country n and period t is

$$I_{n,t} = \left[\sum_{j=a,m,s} (\kappa^{K,j}_{n,t})^{\frac{1}{\sigma^K}} (M^{K,j}_{n,t})^{\frac{\sigma^K - 1}{\sigma^K}}\right]^{\frac{\sigma^K}{\sigma^K - 1}},$$

where $M^{K,j}_{n,t}$ is the sector j composite goods used for production of capital goods, and σ^K is the elasticity of substitution across sectoral composite intermediates for production of capital goods. Then the cost share of sector j inputs in capital goods production is

$$g^{K,j}_{n,t} = \frac{P^j_{n,t} M^{K,j}_{n,t}}{\sum_{j'=a,m,s} P^{j'}_{n,t} M^{K,j'}_{n,t}} = \frac{\kappa^{K,j}_{n,t} (P^j_{n,t})^{1-\sigma^K}}{\sum_{j'=a,m,s} \kappa^{K,j'}_{n,t} (P^{j'}_{n,t})^{1-\sigma^K}}.$$

The ideal price index of capital goods is

$$P^K_{n,t} = \left[\sum_{j=s,m,s} \kappa^{K,j}_{n,t} (P^j_{n,t})^{1-\sigma^K}\right]^{\frac{1}{1-\sigma^K}}.$$

Let $X^j_{ni,t}$ be country n's spending on sector j varieties sourced from country i in period t. This includes spending on consumption, investment, and intermediate inputs. Summing $X^j_{ni,t}$ across i, let $X^j_{n,t}$ be country n's spending on sector j varieties in period t. Let $\pi^j_{ni,t} = X^j_{ni,t}/X^j_{n,t}$ be the share of goods sourced from country i within country n's expenditure on sector j varieties in period t, or the trade share of country i in the market of country n. Then we have

$$\pi^j_{ni,t} = \frac{(\widetilde{c}^j_{i,t} b^j_{ni,t}/A^j_{i,t})^{-\theta^j}}{\sum_{i'=1}^N (\widetilde{c}^j_{i',t} b^j_{ni',t}/A^j_{i',t})^{-\theta^j}} = \frac{(\widetilde{c}^j_{i,t} b^j_{ni,t}/A^j_{i,t})^{-\theta^j}}{(P^j_{n,t})^{-\theta^j}}. \tag{25}$$

Let $Y^j_{n,t}$ be the value of gross production of sector j in country n and period t:

$$Y^j_{n,t} = \sum_{i=1}^N \frac{\pi^j_{in,t}}{\widetilde{\tau}^j_{in,t}} X^j_{i,t}, \tag{26}$$

Country n's spending on sector j varieties in period t consists of the final consumption, the input for production of capital goods, and the input for production of goods and services across sectors:

$$X^j_{n,t} = \omega^j_{n,t} E_{n,t} + g^{K,j}_{n,t} P^K_{n,t} I_{n,t} + \sum_{j'=a,m,s} (1-\gamma^{j'}_{n,t}) g^{j',j}_{n,t} Y^{j'}_{n,t}, \tag{27}$$

noting $\omega^j_{n,t} E_{n,t} = P^j_{n,t} C^j_{n,t}$ and $g^{K,j}_{n,t} P^K_{n,t} I_{n,t} = P^j_{n,t} M^{K,j}_n$.

In country n and period t, the aggregate labor income must be equal to the aggregate labor cost:

$$w_{n,t} L_{n,t} = \sum_{j=a,m,s} \gamma^j_{n,t} (1-\alpha^j_{n,t}) Y^j_{n,t}. \tag{28}$$

Similarly, the aggregate capital income must be equal to the aggregate capital cost:

$$r_{n,t} K_{n,t} = \sum_{j=a,m,s} \gamma^j_{n,t} \alpha^j_{n,t} Y^j_{n,t}. \tag{29}$$

The trade deficit, $D_{n,t}$, is the imports minus the exports:

$$D_{n,t} = \underbrace{\sum_{j=a,m,s} \sum_{i=1}^N X^j_{n,t} \frac{\pi^j_{ni,t}}{\widetilde{\tau}^j_{ni,t}}}_{\text{Imports}} - \underbrace{\sum_{j=a,m,s} \sum_{i=1}^N X^j_{i,t} \frac{\pi^j_{in,t}}{\widetilde{\tau}^j_{in,t}}}_{\text{Exports}}.$$

Country n's trade deficit must be equal to its net payment to the global portfolio:

$$D_{n,t} = L_{n,t} T^P_t - \phi_{n,t} \left(w_{n,t} L_{n,t} + r_{n,t} K_{n,t} + \widetilde{T}_{n,t} \right).$$

The budget balance of the global portfolio requires $\sum_{n=1}^N D_{n,t} = 0$. Solving this for the payment from the global portfolio to each individual T_t^P, we have

$$T_t^P = \frac{\sum_{n=1}^{N} \phi_{n,t} \left(w_{n,t} L_{n,t} + r_{n,t} K_{n,t} + \widetilde{T}_{n,t} \right)}{\sum_{n=1}^{N} L_{n,t}}. \tag{30}$$

We now define the equilibrium of our dynamic model. We also define the steady state because we compute transition paths which are equilibria converging to steady states. The definitions of equilibria and steady states are respectively given by

Definition 1 (Equilibrium)**.** Given the capital stocks in the initial period $\{K_{n,0}\}_n$, an equilibrium is a tuple of $\{w_{n,t}\}_{n,t}$, $\{r_{n,t}\}_{n,t}$, $\{E_{n,t}\}_{n,t}$, $\{\widetilde{c}^j_{n,t}\}_{n,t,j}$, $\{P^j_{n,t}\}_{n,t,j}$, $\{\pi^j_{ni,t}\}_{n,i,t,j}$, $\{Y_{n,t}\}_{n,t}$, $\{X^j_{n,t}\}_{n,t,j}$, $\{\bar{\epsilon}_{n,t}\}_{n,t}$, $\{\omega^j_{n,t}\}_{n,t,j}$, $\{C_{n,t}\}_{n,t}$, $\{K_{n,t}\}_{n,t}$, $\{I_{n,t}\}_{n,t}$, $\{T_t^P\}_t$ satisfying a system of equations (15), (17), (18), (19), (20), (21), (23), (24), (25), (26), (27), (28), (29), and (30) for $n, i \in \{1, \ldots, N\}$, $j \in \{a, m, s\}$, and $t \in \{0, 1, \ldots\}$.

Definition 2 (Steady state)**.** A steady state is an equilibrium in which relevant endogenous variables are time-invariant. Specifically, a steady state is a tuple of $\{w_n\}_n$, $\{r_n\}_n$, $\{E_n\}_n$, $\{\widetilde{c}^j_n\}_{n,j}$, $\{P^j_n\}_{n,j}$, $\{\pi^j_{ni}\}_{n,i,j}$, $\{Y_n\}_n$, $\{X^j_n\}_{n,j}$, $\{\omega^j_n\}_{n,j}$, $\{C_n\}_n$, $\{K_n\}_n$ satisfying a system of equations, (15), (17), (23), (24), (25), (26), (27), (28), (30),

$$r_n K_n = \frac{\alpha}{1-\alpha} w_n L_n, \qquad r_n = \frac{1 - \beta(1 - \lambda\delta_n)}{\beta(1 - \phi_n)\lambda} P^K_n,$$

and

$$E_n = (1 - \phi_n) \left(w_n L_n + r_n K_n + \widetilde{T}_n \right) - \delta_n P^K_n K_n + L_n T^P,$$

for $n, i \in \{1, \ldots, N\}$ and $j \in \{a, m, s\}$, where time subscripts t are dropped from all the equations.

In the following quantitative analysis, as in the qualitative analysis of Section 2, we will focus on two measures of sectoral allocation: the sectoral consumption expenditure share $\omega^j_{n,t} = P^j_{n,t} C^j_{n,t} / E_{n,t}$ in (16) and the sectoral value-added share:

$$va^j_{n,t} = \frac{\gamma^j_{n,t} Y^j_{n,t}}{\sum_{j'=a,m,s} \gamma^{j'}_{n,t} Y^{j'}_{n,t}} = \frac{w_{n,t} L^j_{n,t} + r_{n,t} K^j_{n,t}}{\sum_{j'=a,m,s} \left(w_{n,t} L^{j'}_{n,t} + r_{n,t} K^{j'}_{n,t} \right)}, \tag{31}$$

where $\gamma^j_{n,t}$ is the share of contribution by primary production factors (labor and capital) in sector j, $Y^j_{n,t}$ is the gross production of sector j (see (26)), and $L^j_{n,t}$ and $K^j_{n,t}$ are respectively labor and capital employed in sector j. Moreover, we will discuss saving decisions by looking at the saving rate $\rho_{n,t}$ defined as the ratio of capital investment to national income:

$$\rho_{n,t} = \frac{P^K_{n,t} I_{n,t}}{(1-\phi_{n,t})(w_{n,t}L_{n,t} + r_{n,t}K_{n,t} + \widetilde{T}_{n,t}) + L_{n,t}T^P_t}. \tag{32}$$

4 Calibration and Fit of the Model

We bring the model to the data for the global economy. We first describe our main data sources and discuss the calibration of the model. We then present the solution algorithm for computing transition paths. Lastly, we discuss the fit of the model.

4.1 Data

Our primary data source is the World Input-Output Database (WIOD) Release 2016 and the Long-Run WIOD (Woltjer et al., 2021; Timmer et al., 2015), which allows us to observe the intermediate input uses across different countries and sectors of both origins and destinations. By merging the two datasets, we constructed a database that spans half a century, 1965–2014. Our empirical exercise encompasses 24 countries (see Table 1) and the rest of the world (RoW). They are the listed countries in the Long-Run WIOD, and we classify Hong Kong as part of the RoW. We aggregate the ISIC industries into three sectors (see Table A4 in Supplemental Appendix E). We complement the WIOD data with the Penn World Table (PWT) 10.0 (Feenstra et al., 2015) and CEPII Gravity database (Mayer and Zignago, 2011). Bilateral tariffs on goods are sourced from the World Integrated Trade Solution (WITS).[26]

4.2 Calibration of Structural Parameters and Fundamentals

We begin with our calibration of the parameters in preferences. The discount factor β is set at 0.96 following the literature on macroeconomics. We set the inverse of the intertemporal elasticity of substitution $\psi = 2$ following Ravikumar et al. (2019). For parameters in the period utility, we choose the elasticities of substitution across sectors $\sigma = 0.5$, and the degree of nonhomotheticity in three

[26]As the bilateral tariffs are specified at each Harmonized System (HS) product level, we first group the HS products to the ISIC industries using the concordance table provided by the WITS and then compute the simple average for agriculture and manufacturing sectors.

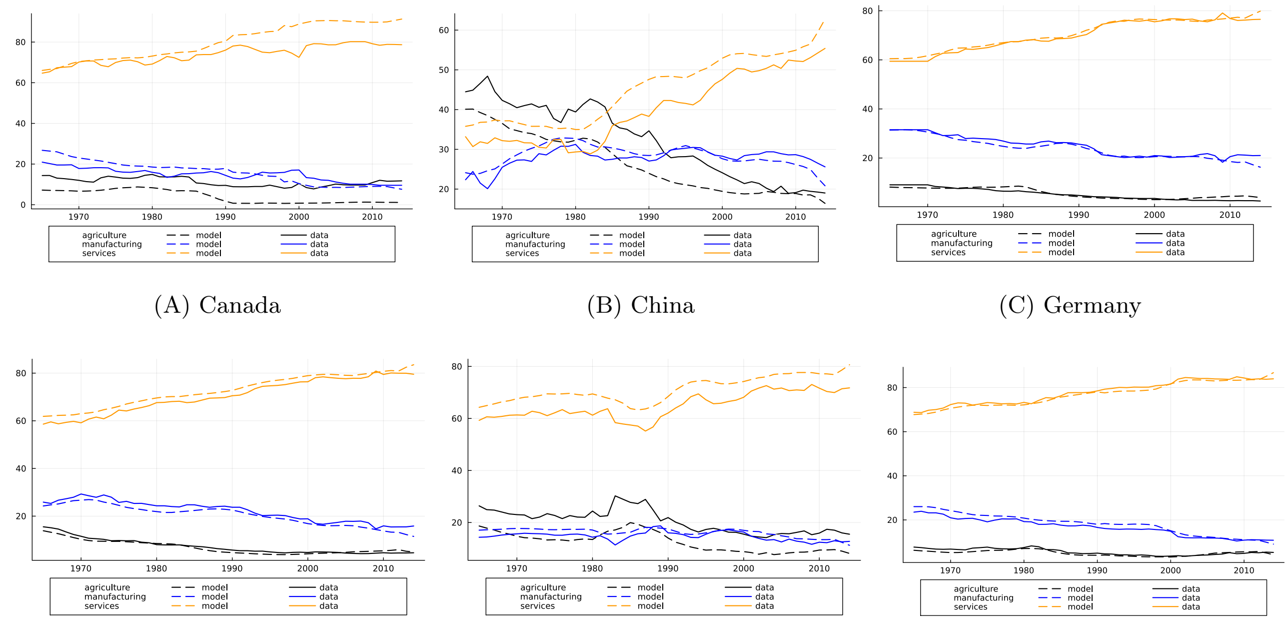
sectors is $(\epsilon^a, \epsilon^m, \epsilon^s) = (0.05, 1, 1.2)$ following Comin et al. (2021). $\sigma < 1$ implies that sectoral goods are complements, and, therefore, the relative price effect is at work. The values of $\{\epsilon^j\}_j$ indicate that the income effect also operates: agriculture is a necessity, services are a luxury, and manufacturing can be either (see (4)).

Shares of primary production factors in the total production cost $\gamma^j_{n,t}$ are directly observed in the IO table. Cost shares of capital within value added $\alpha^j_{n,t}$ are calibrated as one minus labor shares, which are obtained from the PWT. Since the PWT does not provide the sectoral labor share, we apply the common value across sectors for each year and country. We set the elasticity of substitution across intermediate inputs $\sigma^j = 0.38$ for all j following Atalay (2017). For the capital goods production, we set the elasticity of substitution $\sigma^K = 0.29$ following Sposi et al. (2024). Lower-than-one elasticities of substitution indicate that the relative price effect applies to both the production of goods and services and investment goods. Shape parameters of the Fréchet distribution are calibrated based on the estimates of Caliendo and Parro (2015). We choose $\theta^a = 8.11$ and $\theta^m = 4.55$. For the service trade elasticity, we follow Gervais and Jensen (2019) and set $\theta^s = 0.75 \cdot \theta^m$. We set the adjustment cost elasticity in the low of motion for capital $\lambda = 0.75$ following Eaton et al. (2016), and the depreciation rate of capital $\delta_{n,t}$ is obtained from the PWT.

We calibrate the sequences of iceberg trade costs $\{d^j_{ni,t}\}$, productivity $\{A^j_{n,t}\}$, and demand shifters $\{g^{j,j'}_{n,t}\}$ and $\{g^{K,j}_{n,t}\}$ by exploiting the data from the WIOD and the PWT. Demand shifters $\{\Omega^j_{n,t}\}$ in nonhomothetic CES preferences (14) are calibrated such that the consumption expenditure shares implied by the model (given the sectoral price indices) $\{\omega^j_{n,t}\}$ are matched with the data. See Supplemental Appendix F for details.

4.3 Solution Algorithm

We solve the equilibrium transition path backward. We first solve the model for the steady state according to Definition 2, assuming the 2014 fundamentals (e.g., productivity, trade costs, exogenous demand shifters, etc.) remain constant forever. We then suppose that the economy will reach the steady state 450 years after 2014. The solution algorithm for the transition path has two loops: the outer loop finds the sequence of investment (saving) rates, $\{\rho_{n,t}\}_{n,t}$ in (32), that satisfies the dynamic optimality condition governed by the Euler equation (20) and the inner loop solves the intratemporal equilibrium for each period (i.e., solving the sectoral prices and factor prices that satisfy the equilibrium conditions listed in Definition 1). More specifically, for the given sequence of $\{\rho_{n,t}\}_{n,t}$ and the initial period capital stock, we first solve the static equilibrium period-by-period



(A) Canada (B) China (C) Germany

(D) Japan (E) Mexico (F) U.S.

Figure 1: Model fit: Sectoral value-added share

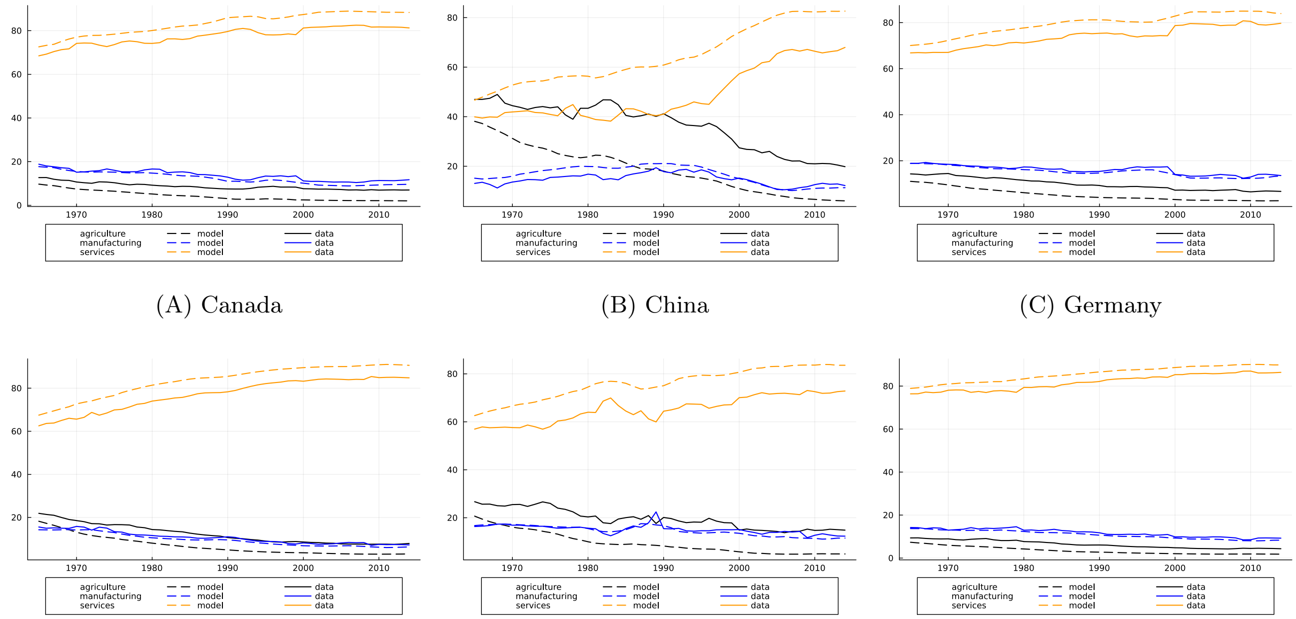
Note: Each panel shows the sectoral value-added shares in the baseline equilibrium (dashed lines) and in the data (solid lines).

sequentially from 1965 to 2464. After we solve the periodic equilibria up to the year 2464, we update $\rho_{n,t}$ backward from 2464 to 1965 according to the Euler equation. More details are in Supplemental Appendix H (see also Ravikumar et al., 2019 for the details of the outer loop iteration).

4.4 Fit of the Baseline Model

To examine the model's ability to match the data, Figure 1 compares the model-implied (dashed lines) value-added shares in three sectors (black for agriculture, blue for manufacturing, and orange for services), $va^j_{n,t}$ in (31), with the data counterparts (solid lines) for six selected countries, Canada, China, Germany, Japan, Mexico, and the U.S. In the six countries, the model captures the overall trend of falling manufacturing and rising services over time. The fit of the model is particularly better for Germany, Japan, and the U.S., while in the other three countries, the model overestimates the service share and underestimates the agricultural share, despite the overall trend being well captured.

Next, Figure 2 demonstrates the model fit of sectoral expenditure shares in final consumption, $\omega^j_{n,t} = P^j_{n,t}C^j_{n,t}/E_{n,t}$. The model-implied expenditure shares are shown in dashed lines, while the data counterparts are shown in solid lines, and the colors are the same as in Figure 1. The model captures the shift of final expenditure from agriculture to manufacturing, and then to services over time. In



(A) Canada (B) China (C) Germany

(D) Japan (E) Mexico (F) U.S.

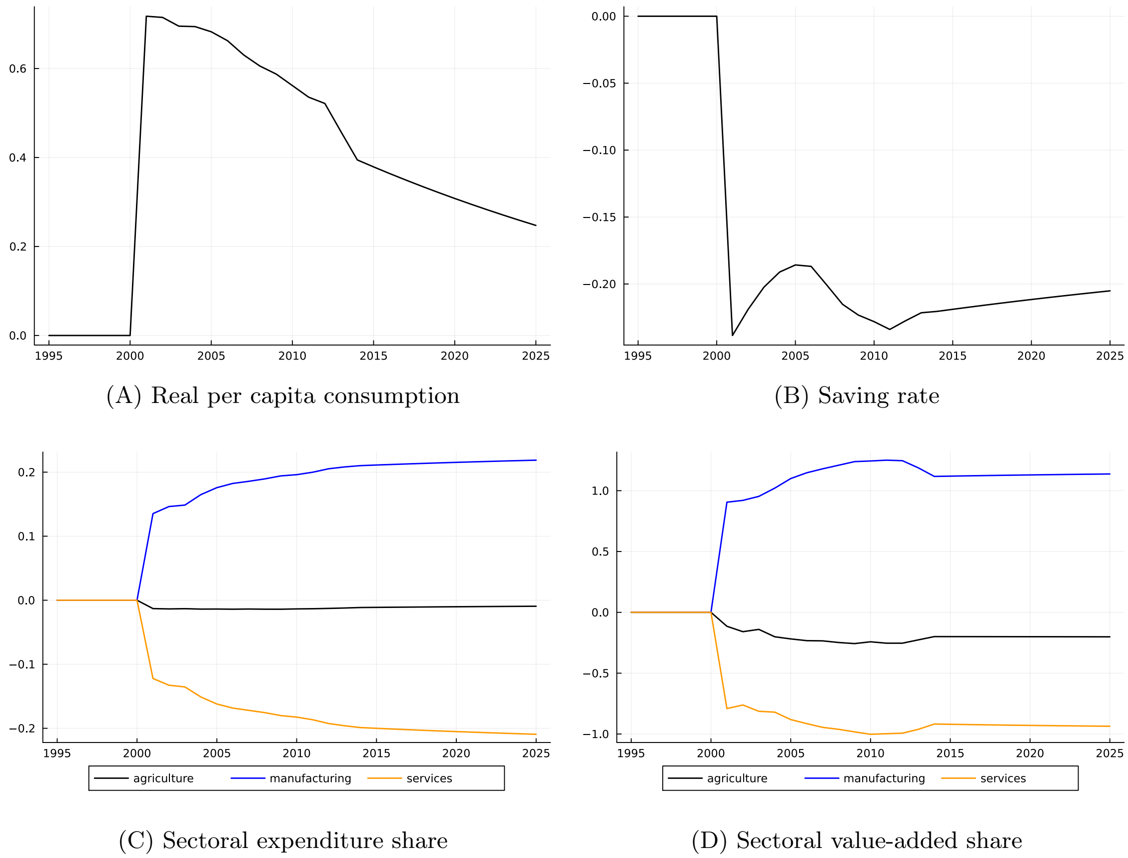
Figure 2: Model Fit: Sectoral Consumption Expenditure Share

Note: Each panel shows the sectoral consumption expenditure shares in the baseline equilibrium (dashed lines) and in the data (solid lines).

all countries, the model overpredicts the service expenditure and underestimates the agriculture expenditure. As in the case of the value-added shares, the fit of the model is better for advanced economies compared to emerging economies. The model fit of saving rates, $\rho_{n,t}$ in (32) is included in Supplemental Appendix K.

5 Counterfactuals

Now, we will use the model to examine the impacts of tariffs on the sectoral allocation of economic activities. We first consider a permanent rise in the U.S. import tariffs on manufacturing by 20 percentage points, effective starting in 2001 (U.S. unilateral tariffs). The year 2001 is chosen given the significant decline in the U.S. manufacturing, which has never recovered since then. We keep all the fundamentals, such as non-tariff barriers and productivity, unchanged from the baseline to isolate the pure effect of the tariff. We compare the results to those under different preferences. We then consider the trade war scenario in which U.S. manufacturing exports are also subject to a 20-percentage-point higher tariff by all other countries, effective starting in 2001 and lasting indefinitely. In what follows, we treat the increase in tariffs as a surprise shock; everyone realizes the tariff hike in 2001, while



(A) Real per capita consumption

(B) Saving rate

(C) Sectoral expenditure share

(D) Sectoral value-added share

Figure 3: Impacts of a U.S. manufacturing tariff increase

Note: Each panel shows the impacts of a counterfactual 20-percentage-point increase in the U.S. manufacturing tariff since 2001 on the transition paths in the U.S. For real per capita consumption, the vertical axis represents the percent change from the baseline to the counterfactual equilibrium. For the other variables, the vertical axis measures the percentage point change from the baseline.

people have perfect foresight regarding the tariff schedule from then onward.

5.1 U.S. Unilateral Tariffs

Figure 3 summarizes the impacts of a U.S. unilateral manufacturing tariff on the four key variables in the U.S., (A) real per capita consumption, (B) saving rate, (C) sectoral expenditure share, and (D) sectoral value-added share. All results are shown relative to the baseline transition path.

Real Per Capita Consumption and Saving: Panel (A) of Figure 3 shows the percent changes in real per capita consumption $C_{US,t}/L_{US,t}$ from the baseline to this counterfactual equilibrium. U.S. real per capita consumption is 0.72 percent higher than the baseline equilibrium in 2001. In relation to Proposition 1(i), this result confirms that the U.S. in 2001 was in the increasing part of the tariff–consumption schedule, and a 20 percentage point increase in the tariff is small enough to raise the real consumption. The figure also shows that the difference between the baseline and counterfactual is largest in the first year after the shock, and it diminishes over time. This is because the household has an incentive to front-load the consumption.

Consistent with the intuition discussed in Section 2.4 with a two-country, two-period model, households anticipating the permanent lump-sum transfer of tariff revenue are disincentivised to save, leading to a lower-than-baseline saving rate in the counterfactual.[27] Panel (B) shows that the saving rate, $\rho_{US,t}$ in (32), is approximately 0.2 percentage points lower than the baseline equilibrium. A lower saving rate implies a lower capital stock, resulting in lower real consumption in the long run. While the real per capita consumption remains above the baseline level for the first 60 years after the shock, it falls below the baseline level afterwards.

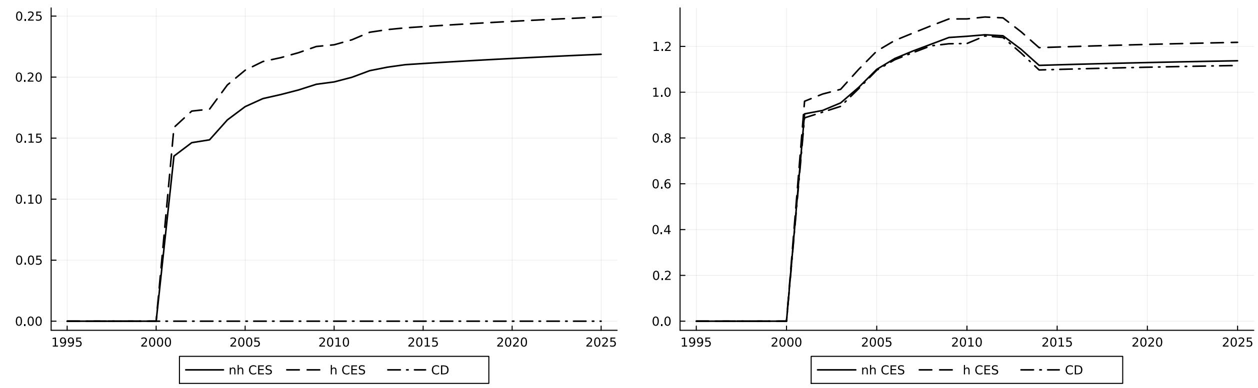
Sectoral Expenditure Share: Panel (C) of Figure 3 shows the impact on sectoral consumption expenditure shares, $\omega^j_{US,t} = P^j_{US,t} C^j_{US,t} / E_{US,t}$, relative to the baseline equilibrium. As we discussed in Section 2.2, with nonhomothetic CES preferences, there are two competing effects of tariffs. On the one hand, a higher relative price of manufacturing shifts the expenditure share to manufacturing due to the relative price effect (Proposition 2(ii)). On the other hand, if tariffs raise U.S. real per capita consumption ($d(C_{US,t}/L_{US,t}) > 0$) and its income elasticity of manufacturing demand is sufficiently low ($\epsilon^m = 1 < \bar{\epsilon}_{US,t} = \sum_j \epsilon^j \omega^j_{US,t}$), the income effect will counteract the first effect (Proposition 2(i)). In our counterfactual scenario where both $d(C_{US,t}/L_{US,t}) > 0$ (panel (A)) and $\bar{\epsilon}_{US,t} > 1$ hold,[28] the two effects indeed move in opposite directions, but the relative price effect dominates: manufacturing expenditure share in the U.S. will rise by approximately 0.2 percentage points compared to the baseline equilibrium.

Sectoral Value-added Share: Panel (D) of Figure 3 represents the impacts on sectoral value-added shares, $va^j_{US,t}$ in (31). A 20-percentage-point additional tariff on U.S. manufacturing imports will lead to approximately a one-percentage-point increase in the manufacturing value-added share, which primarily comes at the expense of a lower share in the service sector. This is in line with our intuition that manufacturing tariffs will have qualitatively similar effects on both sectoral consumption expenditure shares and value-added shares (Proposition 3 and the following discussion).

In terms of magnitude, the sectoral value-added shares respond more to the tariff shock than the sectoral expenditure shares. The manufacturing expenditure share under the counterfactual is

[27] If an increase in tariffs is temporal (e.g., lasting four years during a U.S. presidency), the implication for savings is opposite. Supplemental Appendix L.1 shows that households anticipating no additional tariff revenue in the future will save more during the high tariff period to smooth out consumption over time.

[28] We confirm $\bar{\epsilon}_{US,t}$ has always been greater than one from $t = 1965$ onward.



(A) Manufacturing expenditure share

(B) Manufacturing value-added share

Figure 4: Comparison of Impacts under Different Preferences

Note: Each panel shows the impacts of a counterfactual 20-percentage-point increase in the U.S. manufacturing tariff since 2001 on the transition paths in the U.S. under the three different preferences, nonhomothetic CES (`nh CES`, solid lines), homothetic CES (`h CES`, dashed lines), and Cobb-Douglas (`CD`, dash-dotted lines). The vertical axes represent percentage point change from the baseline.

approximately 0.2 percentage point higher than the baseline, whereas its value-added share is 0.9–1.3 percentage points higher. This discrepancy is due to our quantitative model featuring international trade as well as input-output linkages through intermediate inputs.[29]

Welfare: We evaluate the overall welfare implications of the tariff shock measured by consumption equivalent from the viewpoint as of 2001 (see the Supplemental Appendix D for the formula). The second column of Table 1 shows that the U.S. welfare rises by 0.36 percent while all other countries are worse off.[30] The imposed tariff directly lowers U.S. imports from other countries, thereby reducing their factor prices and welfare due to the terms-of-trade effect.[31] Among others, Canada, India, and Mexico experience the largest welfare losses (1.26, 0.74, and 0.64 percent, respectively), largely due to the U.S. being their primary trading partner and a major importer of their manufacturing goods.

Impacts under Different Preferences: To highlight the role of nonhomotheticity in accounting for the impacts of tariffs on the sectoral allocation, Figure 4 compares the results for manufacturing expenditure shares in panel (A) and value-added shares in panel (B) across three different preferences; nonhomothetic CES, homothetic CES, and Cobb-Douglas.[32]

Under homothetic CES preferences, both the expenditure share and value-added share respond

[29] In other words, if we abstract from these two features, the sectoral expenditure shares and value-added shares would be exactly the same as in Comin et al. (2021).

[30] Real per capita consumption of the U.S. will fall below the baseline level in six decades after the shock. Yet, our welfare measures discount the future real consumption, resulting in a positive welfare implication.

[31] This point is discussed using our two-country model in Section 2.1 and Supplemental Appendix A.

[32] The expressions are given in (14) and footnote 24.

Table 1: Welfare impacts of the U.S. tariffs

	Unilateral U.S. tariffs		Retaliatory tariffs	
	nh CES	h CES	nh CES	h CES
Australia	-0.026	-0.024	-0.048	-0.055
Austria	-0.126	-0.151	-0.092	-0.109
Belgium	-0.095	-0.110	-0.075	-0.088
Brazil	-0.177	-0.195	-0.130	-0.146
Canada	-1.260	-1.605	-1.078	-1.385
China	-0.361	-0.424	-0.220	-0.255
Germany	-0.085	-0.094	-0.060	-0.068
Denmark	-0.048	-0.058	-0.041	-0.050
Spain	-0.096	-0.112	-0.073	-0.086
Finland	-0.141	-0.171	-0.101	-0.122
France	-0.069	-0.080	-0.051	-0.059
U.K.	-0.064	-0.073	-0.044	-0.051
Greece	-0.157	-0.182	-0.126	-0.145
India	-0.744	-0.791	-0.442	-0.465
Ireland	-0.074	-0.088	-0.067	-0.081
Italy	-0.103	-0.120	-0.076	-0.088
Japan	-0.070	-0.083	-0.046	-0.053
Korea	-0.166	-0.197	-0.134	-0.157
Mexico	-0.640	-0.715	-0.716	-0.817
Netherland	-0.089	-0.104	-0.072	-0.084
Portugal	-0.132	-0.156	-0.109	-0.129
Sweeden	-0.108	-0.129	-0.078	-0.093
Taiwan	-0.212	-0.250	-0.150	-0.176
U.S.	0.357	0.408	-0.121	-0.165
Rest of the World	-0.128	-0.144	-0.078	-0.087

Notes: The table shows percent changes in welfare (consumption equivalent, see Supplemental Appendix D) under nonhomothetic CES (nh CES) and homothetic CES preferences (h CES) in the two different counterfactual scenarios, U.S. unilateral tariffs in Section 5.1 and trade war in Section 5.2.

more in magnitude to the tariff shock than under nonhomothetic CES preferences. This is because homothetic CES preferences shut down the negative income effect on manufacturing shares, which counteracts the positive relative price effect. For the manufacturing expenditure shares presented in panel (A), the gap between the dashed and solid lines, 0.03 percentage point, implies the mitigating impacts coming from the income effect. Under Cobb-Douglas preferences, expenditure shares are determined by the parameters, and therefore, tariffs have no impact.

Panel (B) shows that the manufacturing value-added share is 0.06–0.08 percentage point higher under homothetic CES than under nonhomothetic CES preferences, a magnitude similar to that observed for expenditure shares. However, under Cobb-Douglas preferences where the expenditure-share channel is shut off (i.e., term (a) in (11) vanishes), the model replicates the result very close to that obtained under nonhomothetic CES preferences. This suggests that the primary driver of the effect on value-added shares is a change in the sectoral net exports, corresponding to the term (b) in (11).

Regarding the welfare implications, the second column of Table 1 shows that the welfare impacts are larger in magnitude under homothetic CES preferences than under nonhomothetic CES preferences for all countries except Australia. According to Proposition 1(ii), homothetic CES preferences overestimate welfare impacts if the income elasticity of manufacturing demand is sufficiently low, $\epsilon^m = 1 < \bar{\epsilon}_{n,t}$. Consistent with this qualitative result, we confirm that the inequality holds for all countries from 2001 onward.

5.2 Trade War

Next, we consider a trade war scenario: the U.S. imposes an additional 20-percentage-point tariff on all the manufacturing imports, and every other country retaliates against the U.S. by imposing tariffs on U.S. exports with the same magnitude. As before, the trade war is assumed to come as a surprise shock. Results are displayed in Figure A8 in Supplemental Appendix L. Even with retaliatory tariffs from around the world, U.S. real per capita consumption rises for the first 13 years relative to the baseline, but then turns negative. The saving rate is lower than the baseline equilibrium, which follows the same intuition as in the unilateral tariffs. Implications for sectoral expenditure shares and value-added shares remain the same qualitatively, but are smaller in magnitude. The manufacturing value-added share rises by 0.7–0.8 percentage point relative to the baseline, as compared to 0.9–1.3 percentage points in the case of the unilateral tariff. As the retaliatory scenario is more plausible in the real world, our result suggests that the sectoral implications of tariffs on manufacturing will be more marginal in a quantitative sense.

Welfare implications of the trade war are summarized in the fourth and fifth columns of Table 1. We confirm that under the trade war scenario, all countries are worse off compared to the baseline equilibrium. The U.S. welfare loss is 0.12 percent, which is larger than the welfare losses of most European countries and Japan. Canada has the largest welfare loss of 1.08 percent.

In large manufacturers such as China, Germany, and Japan, the welfare losses are larger under U.S. unilateral tariffs than under the trade war. When the U.S. unilaterally imposes tariffs, trade values from China, Germany, and Japan to the U.S. decline. Due to the terms-of-trade effect, factor prices and thus welfare in these countries also fall. If they retaliate against the U.S., however, they will receive tariff revenues, and more importantly, the lower import from the U.S. mitigates the negative terms-of-trade effect. In contrast to those three countries, Mexico, the second-worst-off country under the trade war, experiences welfare losses from the trade war (0.72 percent) that are greater than those under the U.S. unilateral tariff (0.64 percent). Mexico is heavily reliant on the U.S. economy as both the largest export destination and the primary import source; the economy is worse off if all the other countries retaliate against the U.S. The last column of Table 1 confirms that, as in the case of U.S. unilateral tariffs, the welfare impacts are larger in magnitude under homothetic CES than under nonhomothetic CES preferences.[33]

6 Conclusion

This paper examined the effects of tariffs on sectoral composition and welfare, in the presence of two key economic drivers of structural change—sectors being complements and nonhomothetic preferences—as well as the standard protective role of tariffs. We examined whether tariffs could help manufacturing regain its share in sectoral consumption expenditure and value added.

Using a two-country model, we show qualitatively that an increase in tariffs from a low level raises income and shifts demand from manufacturing to services through the income effect driven by nonhomothetic preferences. However, tariffs targeting manufacturing shift demand in the opposite direction via the relative price effect, given an elasticity of substitution less than one. In addition to these structural change-driven effects, tariffs replace foreign manufacturers with domestic producers, thereby increasing the manufacturing value-added share.

To quantitatively assess these effects, we extend the model to a multi-country dynamic framework with capital accumulation and input-output linkages. Using data from 24 countries spanning 1965 to 2014 and calibrating fundamentals such as sectoral productivity and non-tariff trade barriers, we compute transition paths in terms of levels rather than relative changes.

The model is used to simulate a 20-percentage-point increase in U.S. manufacturing tariffs on all trading partners beginning in 2001. We find that the relative price effect and the trade protection effect dominate the income effect. Specifically, U.S. manufacturing value-added share rises by 0.9–1.3 percentage points, while the service sector declines by 0.8–1.0 percentage point between 2001 and 2014. Despite these shifts supporting protectionist objectives, the welfare impact is quantitatively minor. The U.S. is better off only by 0.36 percent at the expense of all other countries. In particular, Canada's welfare falls by 1.3 percent. If the other countries retaliate with 20 percentage-point higher

[33] In the trade war scenario, we can check $\bar{\epsilon}_{n,t} > 1$ for all the countries n and $t = 2001$ onward, as implied by Proposition 1(ii).

tariffs, all countries are worse off, with the U.S. welfare loss being 0.12 percent.

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Supplemental Appendix of
"Trade Policy and Structural Change"

Hayato Kato[*] Kensuke Suzuki[†] Motoaki Takahashi[‡]

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Contents

A Proof of Proposition 1

In the proofs of Propositions 1 to 3, we allow for an arbitrary number of sectors, $j \in \{1, \ldots, J\}$. In the case of nonhomothetic CES, the nonhomotheticity parameters are common in the two countries

[*]Graduate School of Economics, the University of Osaka; CESifo Research Network. Email: hayato.kato@econ.osaka-u.ac.jp

[†]Department of Economics, Clark University; Economic Research Center, Nagoya University. Email: ksuzuki@clarku.edu

[‡]Graduate School of Economics, the University of Osaka. Email: takahashi.motoaki@gmail.com

and ordered in a way that $0 < \epsilon^1 < \cdots < \epsilon^J$ and there exists a sector j such that $\epsilon^j = 1$. In the case of homothetic CES including Cobb-Douglas as a special case, the nonhomotheticity parameters are common in both countries and sectors, $\epsilon^j = 1$ for all $j \in \{1, \ldots, J\}$.

Nonhomothetic CES We assume that preferences are nonhomothetic CES and allow productivity A_n, population L_n, non-tariff trade barriers d_{ni}, and tariffs τ_{ni} to be country (pair) specific, but not to be sector specific. We here look at the effect of tariff applied by Home to Foreign uniform across sectors, $\tau_{HF}^j = \tau_{HF}$ for all j.

We first show that there exists a unique equilibrium wage. The trade balance condition requires

$$\sum_{j=1}^{J} \frac{\pi_{HF}^j \omega_H^j (1+\mu_H) w_H L_H}{1+\tau_{HF}} = \sum_{j=1}^{J} \frac{\pi_{FH}^j \omega_F^j (1+\mu_F) w_F L_F}{1+\tau_{FH}},$$

where

$$\begin{aligned}
\pi_{HF}^j &= \left(\frac{w_F b_{HF}/A_F}{P_H^j}\right)^{-\theta} = \frac{(A_F/b_{HF})^\theta}{(A_F/b_{HF})^\theta + (A_H/w)^\theta} = \frac{(w/b_{HF})^\theta}{(w/b_{HF})^\theta + A^\theta} = \pi_{HF}, && \forall j \\
\pi_{FH}^j &= \left(\frac{w_H b_{FH}/A_H}{P_F^j}\right)^{-\theta} = \frac{(A_H/b_{FH})^\theta w^{-\theta}}{A_F^\theta + (A_H/b_{FH})^\theta w^{-\theta}} = \frac{(A/b_{FH})^\theta}{w^\theta + (A/b_{FH})^\theta} = \pi_{FH}, && \forall j \\
\mu_n^j &= \frac{\tau_{ni}\pi_{ni}^j \omega_n^j}{1+\tau_{ni}} = \frac{\tau_{ni}\pi_{ni}\omega_n^j}{1+\tau_{ni}}, && n \neq i \in \{H, F\},\ \forall j \\
\mu_n &= \frac{\sum_j \mu_n^j}{1 - \sum_j \mu_n^j} = \frac{\frac{\tau_{ni}\pi_{ni}}{1+\tau_{ni}}}{1 - \frac{\tau_{ni}\pi_{ni}}{1+\tau_{ni}}} = \frac{\tau_{ni}\pi_{ni}}{1+\tau_{ni}\pi_{nn}}, && \forall n \\
b_{ni} &= d_{ni}(1+\tau_{ni}), \quad w = w_H/w_F = w_H, \quad A = A_H/A_F,
\end{aligned}$$

noting $\sum_j \omega_n^j = \sum_j P_n^j C_n^j / E_n = 1$ and $w_F = 1$ due to the choice of numéraire; and $\pi_{ni} + \pi_{nn} = 1$. As there are no sectoral heterogeneity except for the nonhomotheticity parameter ϵ^j, the sectoral price indices and the trade shares are all the same across sectors, $\pi_{ni}^j = \pi_{ni}$ and $P_n^j = P_n$ for all j. Using $\sum_j \omega_n^j = 1$, we rearrange the trade balance condition to obtain

$$\begin{aligned}
&\frac{\pi_{HF}(1+\mu_H) w_H L_H}{1+\tau_{HF}} = \frac{\pi_{FH}(1+\mu_F) w_F L_F}{1+\tau_{FH}}, \\
\Leftrightarrow\ &\frac{L_H}{L_F} = \frac{\pi_{FH}}{w\pi_{HF}} \left(\frac{1+\mu_F}{1+\mu_H}\right)\left(\frac{1+\tau_{HF}}{1+\tau_{FH}}\right) = \frac{\pi_{FH}}{w\pi_{HF}} \frac{\frac{1+\tau_{FH}}{1+\tau_{FH}\pi_{FF}}}{\frac{1+\tau_{HF}}{1+\tau_{HF}\pi_{HH}}} \left(\frac{1+\tau_{HF}}{1+\tau_{FH}}\right) = \frac{\pi_{FH}}{w\pi_{HF}} \left(\frac{1+\tau_{HF}\pi_{HH}}{1+\tau_{FH}\pi_{FF}}\right).
\end{aligned}$$

Finally, we have

$$\frac{L_H}{L_F} = \frac{(A/b_{FH})^\theta \left[(w/b_{HF})^\theta + (1+\tau_{HF})A^\theta\right]}{b_{HF}^{-\theta} w^{1+\theta} \left[(1+\tau_{FH})w^\theta + (A/b_{FH})^\theta\right]}.$$

We can conclude that this equation has a unique solution of w by checking that (i) the right-hand side approaches infinity as w goes to zero; (ii) it approaches zero as w goes to infinity; and (iii) it decreases with w.

Two comments are in order. First, the trade balance condition above does not include ϵ^j nor σ, implying that the equilibrium (relative) wage is the same in both the nonhomothetic and homothetic CES cases. Second, the equilibrium (relative) wage in Home increases with its tariff. Taking the derivative of the trade balance condition with respect to τ_{HF} gives

$$d\ln\pi_{HF} + \frac{d\mu_H}{1+\mu_H} + d\ln w - \frac{d\tau_{HF}}{1+\tau_{HF}} = d\ln\pi_{FH} + \frac{d\mu_F}{1+\mu_F} - \frac{d\tau_{FH}}{1+\tau_{FH}},$$

where, for example, $d\ln w = dw/w$ and

$$\begin{aligned}
& d\ln b_{HF} = d\ln d_{HF}(1+\tau_{HF}) = \frac{d\tau_{HF}}{1+\tau_{HF}} > 0, \qquad d\ln b_{FH} = \frac{d\tau_{FH}}{1+\tau_{FH}} = 0, \\
& d\ln P_F^j = \pi_{FH}\left(d\ln b_{FH} + d\ln w\right), \qquad d\ln P_H^j = \pi_{HF} d\ln b_{HF} + \pi_{HH} d\ln w, && \forall j \\
& d\ln\pi_{HF} = -\theta\left(d\ln b_{HF} - d\ln P_H^j\right) = -\theta\pi_{HH}\left(d\ln b_{HF} - d\ln w\right), && \forall j \\
& d\ln\pi_{FH} = -\theta\left(d\ln b_{FH} + d\ln w - d\ln P_F^j\right) = -\theta\pi_{FF}\left(d\ln b_{FH} + d\ln w\right), && \forall j \\
& d\mu_n = \begin{cases} \pi_{ni} d\tau_{ni} & \text{if } \tau_{ni} = 0 \\ \dfrac{\tau_{ni}\pi_{ni}}{(1+\tau_{ni}\pi_{nn})^2}\left[d\ln\tau_{ni} + (1+\tau_{ni}) d\ln\pi_{ni}\right] & \text{if } \tau_{ni} > 0 \end{cases}, && n \neq i \in \{H, F\}
\end{aligned}$$

We set $\tau_{FH} = 0$ in what follows. Solving the equations for $d\ln w$ yields

$$\frac{d\ln w}{d\tau_{HF}} = \Gamma^{-1}(1+\theta)A^\theta\left[w^\theta + (A/b_{FH})^\theta\right] > 0,$$

where $\Gamma > 0$ is a bundle of variables.

Using (1) in the text and $E_H = (1+\mu_H)w_H L_H$, we write the expenditure share on the sector j composite good in Home, $\omega_H^j = P_H^j C_H^j / E_H$, as

$$\omega_H^j = \left(\frac{(1+\mu_H)w_H}{P_H^j}\right)^{\sigma-1}\left(\frac{C_H}{L_H}\right)^{\epsilon^j(1-\sigma)} = (1+\mu_H)^{\sigma-1}\left[(A_F w/b_{HF})^\theta + A_H^\theta\right]^{\frac{\sigma-1}{\theta}}\left(\frac{C_H}{L_H}\right)^{\epsilon^j(1-\sigma)},$$

Summing this over sectors yields

$$1 = \sum_{j=1}^{J}\omega_H^j = \sum_{j=1}^{J} x_H^{\frac{\sigma-1}{\theta}}\left(\frac{C_H}{L_H}\right)^{\epsilon^j(1-\sigma)}, \quad x_H = (1+\mu_H)^\theta\left[(A_F w/b_{HF})^\theta + A_H^\theta\right]. \tag{A1}$$

This equation implicitly defines the real per capita income (or utility), C_H/L_H, as a function of index x_H (see Lemma 1 in Section B of this Supplemental Appendix for details). To highlight this, we write $u(x_H) = C_H/L_H$. With $\sigma \in (0,1)$, we can see that $u(x_H)$ increases with x_H. Therefore, the optimal level of tariff that achieves the highest $u(x_H)$ maximizes x_H. Taking the derivative of

x_H with respect to τ_{HF} gives

$$\frac{d\ln x_H}{d\tau_{HF}} = \Lambda_H, \tag{A2}$$

$$\Lambda_H \equiv \frac{(1+\theta)(\theta w^\theta)^2(A/b_{HF})^\theta}{\Gamma(1+\tau_{HF})[(w/b_{HF})^\theta + A^\theta]}\left(\frac{1}{\theta\pi_{FF}} - \tau_{HF}\right)\begin{cases}\geq 0 & \text{if } \tau_{HF} \leq (\theta\pi_{FF})^{-1} \text{ or } \tau_{HF} \leq \tau^*_{HF} \\ < 0 & \text{if } \tau_{HF} > (\theta\pi_{FF})^{-1} \text{ or } \tau_{HF} > \tau^*_{HF}\end{cases},$$

where $\Gamma > 0$ is the same bundle of variables as the one in $d\ln w/d\tau_{HF}$, and τ^*_{HF} is the solution of $\tau_{HF} = (\theta\pi_{FF})^{-1}$, noting that π_{FF} is a function of τ_{HF}. At $\tau_{HF} = 0$, we have $d\ln x_H/d\tau_{HF} > 0$. Letting $f(\tau_{HF}) = \tau_{HF} - (\theta\tau_{FF})^{-1} = \tau_{HF} - \theta^{-1}\left[(A/(wb_{FH}))^\theta - 1\right]$, the condition $\tau_{HF} \leq (\theta\pi_{FF})^{-1}$ is equal to $f(\tau_{HF}) \leq 0$. We can check that $f(\tau_{HF})$ increases with τ_{HF} and $f(0) < 0$ holds, so that $f(\tau_{HF}) = 0$ holds at some point $\tau_{HF} = \tau^*_{HF}$. This implies that the condition $\tau_{HF} \leq (\theta\pi_{FF})^{-1}$ is equivalent to $\tau_{HF} \leq \tau^*_{HF}$. The index x_H and thus the real per capita income $u(x_H)$ is maximized at $\tau_{HF} = (\theta\pi_{FF})^{-1}$ or $\tau_{HF} = \tau^*_{HF}$.

The optimal tariff is (implicitly) given by

$$\widehat{\tau}_{HF} = \frac{1}{\theta\pi_{FF}}, \quad \pi_{FF} = \left(\frac{w_F/A_F}{P_F}\right)^{-\theta} = \frac{w^\theta}{w^\theta + (A/b_{HF})^\theta},$$

The same expression is obtained in Caliendo and Parro (2020) in a single sector model with homothetic CES preferences. Three points are worth noting here. First, the optimal tariff increases with the trade elasticity θ. This is because imports are less responsive to changes in tariffs when productivity distribution is more dispersed (low θ), which allows Home to set a higher tariff. Second, the optimal tariff decreases with the share of domestic expenditure in Foreign, π_{FF}. If Home is very small relative to Foreign in the world market, π_{FF} approaches one. Smaller countries have less room for manipulating terms-of-trade than larger countries. Third, the degree of nonhomotheticity ϵ^j does not affect τ^*_{HF}, implying that the level of optimal tariff under nonhomothetic CES preferences is the same as the one under homothetic CES preferences, which we will see shortly. The optimal tariff is independent of ϵ^j because we restrict changes in tariffs to be uniform across sectors.

Finally, we can verify that the revenue to labor-income ratio, μ_H, also has an inverted U-shaped relationship with τ_{HF}. We again take the derivative of x_H while expressing explicitly the derivative of μ_H, and use the result of (A2) to obtain

$$\begin{aligned}
&\frac{d\ln x_H}{d\tau_{HF}} = \theta\left(\frac{1}{1+\mu_H}\frac{d\mu_H}{d\tau_{HF}} + \frac{d\ln w}{d\tau_{HF}} - \frac{d\ln P_H^j}{d\tau_{HF}}\right),\\
&\Leftrightarrow \frac{(1+\theta)(\theta w^\theta)^2(A/b_{HF})^\theta}{\Gamma(1+\tau_{HF})[(w/b_{HF})^\theta + A^\theta]}\left(\frac{1}{\theta\pi_{FF}} - \tau_{HF}\right) = \\
&\qquad \frac{\theta}{1+\mu_H}\frac{d\ln\mu_H}{d\tau_{HF}} - \frac{\theta\pi_{HF}w^\theta\left[(1+\theta)(w/b_{HF})^\theta + A^\theta\left(\theta(1+\tau_{HF}) + (b_{HF}b_{FH})^{-\theta}\right)\right]}{\Gamma(1+\tau_{HF})},\\
&\Leftrightarrow \frac{\theta}{1+\mu_H}\frac{d\mu_H}{d\tau_{HF}} = \frac{(1+\theta)(\theta w^\theta)^2(A/b_{HF})^\theta}{\Gamma(1+\tau_{HF})[(w/b_{HF})^\theta + A^\theta]}\left(\frac{1}{\theta\pi_{FF}} + \Gamma' - \tau_{HF}\right)\\
&\qquad = \begin{cases}\geq 0 & \text{if } \tau_{HF} \leq (\theta\pi_{FF})^{-1} + \Gamma' \text{ or } \tau_{HF} \leq \tau_{HF}^{**}\\ < 0 & \text{if } \tau_{HF} > (\theta\pi_{FF})^{-1} + \Gamma' \text{ or } \tau_{HF} > \tau_{HF}^{**}\end{cases},
\end{aligned}$$

where $\Gamma' > 0$ is another bundle of parameters, and τ_{HF}^{**} is the solution of $\tau_{HF} = (\theta\pi_{FF})^{-1} + \Gamma'$, noting that π_{FF} and Γ' are functions of τ_{HF}. Letting $g(\tau_{HF}) = \tau_{HF} - (\theta\pi_{FF})^{-1} - \Gamma'$, the condition $\tau_{HF} \leq (\theta\pi_{FF})^{-1} + \Gamma'$ is equal to $g(\tau_{HF}) \leq 0$. We can check that $g(\tau_{HF})$ increases with τ_{HF} so that $g(\tau_{HF}) = 0$ holds at some point $\tau_{HF} = \tau_{HF}^{**}$. The condition $\tau_{HF} \leq (\theta\pi_{FF})^{-1} + \Gamma'$ is equivalent to $\tau_{HF} \leq \tau_{HF}^{**}$. The revenue to labor-income ratio is maximized at $\tau_{HF} = (\theta\pi_{FF})^{-1} + \Gamma'$ or $\tau_{HF} = \tau_{HF}^{**}$.

Analogously, the real per capita income in Foreign is implicitly defined as

$$\begin{aligned}
&1 = \sum_{j=1}^J \omega_F^j = \sum_{j=1}^J \left(\frac{(1+\mu_F)w_F}{P_F^j}\right)^{\sigma-1}\left(\frac{C_F}{L_F}\right)^{\epsilon^j(1-\sigma)} = \sum_{j=1}^J x_F^{\frac{\sigma-1}{\theta}}\left(\frac{C_F}{L_F}\right)^{\epsilon^j(1-\sigma)},\\
&x_F = (1+\mu_F)^\theta\left[A_F^\theta + (A_H/(wb_{FH}))^\theta\right].
\end{aligned}$$

As in Home, we see that C_F/L_F positively depends on x_F and thus write $u(x_F) = C_F/L_F$. The effect of Home tariff on Foreign real per capita consumption is

$$\frac{d\ln x_F}{d\tau_{HF}} = -\Gamma^{-1}\theta(1+\theta)\left(A^2/b_{FH}\right)^{-\theta} < 0,$$

where $\Gamma > 0$ is the same bundle of variables as the one in $d\ln w/d\tau_{HF}$. □

Homothetic CES We set $\epsilon^j = 1$ for all j and consider Home's unilateral uniform tariff change in some sectors $j \in \mathcal{J}$ with $\mathcal{J}$ being the set of sectors that increase tariffs and $\mathcal{J}^c$ being its complement set, $d\tau_{HF}^j = d\tau_{HF} > 0$ for $j \in \mathcal{J}$ and $d\tau_{HF}^h = 0$ for $h \in \mathcal{J}^c$. This case includes the Cobb-Douglas preferences as a special case at $\sigma = 1$. We keep assuming that initial tariffs before the change are symmetric in sector, $\tau_{HF}^j = \tau_{HF}$ for all j, and Foreign never sets tariffs, $\tau_{FH}^j = 0$ for all j. As in the case of nonhomothetic CES, the change in the (relative) wage in Home is derived by differentiating the change in the trade balance condition:

$$\sum_{j=1}^J \frac{1}{J}\left(d\ln\pi_{FH}^j + d\ln\omega_F^j + \frac{d\mu_F}{1+\mu_F}\right) = \sum_{j=1}^J \frac{1}{J}\left(d\ln\pi_{HF}^j + d\ln\omega_H^j + \frac{d\mu_H}{1+\mu_H} + d\ln w - \frac{d\tau_{HF}^j}{1+\tau_{HF}}\right),$$

where

$$d\ln b_{HF}^j = d\ln d_{HF}\left(1+\tau_{HF}^j\right) = \frac{d\tau_{HF}^j}{1+\tau_{HF}} = \frac{d\tau_{HF}}{1+\tau_{HF}} > 0, \qquad j\in\mathcal{J}$$

$$d\ln b_{HF}^j = \frac{d\tau_{HF}^j}{1+\tau_{HF}} = 0, \qquad j\in\mathcal{J}^c$$

$$d\ln P_F^j = \pi_{FH} d\ln w, \qquad \pi_{FH} = \left(\frac{wb_{FH}/A_H}{P_F}\right)^{-\theta} = \frac{(A/b_{FH})^\theta}{w^\theta + (A/b_{FH})^\theta} = 1-\pi_{FF}, \qquad \forall j$$

$$d\ln P_H^j = \pi_{HF} d\ln b_{HF}^j + \pi_{HH} d\ln w, \qquad \pi_{HF} = \left(\frac{b_{HF}/A_F}{P_H}\right)^{-\theta} = \frac{(w/b_{HF})^\theta}{(w/b_{HF})^\theta + A^\theta} = 1-\pi_{HH}, \qquad \forall j$$

$$d\ln\pi_{HF}^j = -\theta\left(d\ln b_{HF}^j - d\ln P_H^j\right) = -\theta\pi_{HH}\left(d\ln b_{HF}^j - d\ln w\right), \qquad \forall j$$

$$d\ln\pi_{FH}^j = -\theta\left(d\ln w - d\ln P_F^j\right) = -\theta\pi_{FF} d\ln w, \qquad \forall j$$

$$d\ln\omega_n^j = (1-\sigma)\left(d\ln P_n^j - \sum_{h=1}^J \omega_n^h d\ln P_n^h\right), \qquad \omega_n^j = \frac{(P_n^j)^{1-\sigma}}{\sum_{h=1}^J (P_n^h)^{1-\sigma}} = \frac{1}{J}, \qquad \forall(n,j)$$

$$d\mu_F = \sum_{j=1}^J d\mu_F^j = \sum_{j=1}^J \pi_{FH}\omega_F^j d\tau_{FH}^j = 0,$$

$$d\mu_H = \begin{cases} \sum_j^J d\mu_H^j = \sum_j^J \pi_{HF}\omega_H^j d\tau_{HF}^j = \sum_j^J \pi_{HF} J^{-1} d\tau_{HF}^j & \text{if } \tau_{HF}=0 \\ \dfrac{\mu_H(1+\tau_{HF})}{1+\tau_{HF}\pi_{HH}} \sum_j^J \omega_H^j d\ln\mu_H^j = \dfrac{\tau_{HF}\pi_{HF}(1+\tau_{HF})}{J(1+\tau_{FH}\pi_{HF})^2} \sum_j^J \left(\dfrac{d\ln\tau_{HF}^j}{1+\tau_{HF}} + d\ln\pi_{HF}^j + d\ln\omega_H^j\right) & \text{if } \tau_{HF}>0 \end{cases},$$

noting $w = w_H/w_F = w_H$; $A = A_H/A_F$; and $P_n^j = P_n^h$ for $j\neq h$ because of no sectoral asymmetry in parameters. We solve the trade balance condition for $d\ln w$ to obtain

$$\frac{d\ln w}{d\tau_{HF}} = |\mathcal{J}|(J\Gamma)^{-1}(1+\theta)A^\theta\left[w^\theta + (A/b_{FH})^\theta\right] > 0,$$

where $|\mathcal{J}|$ is the number of sectors that increase tariffs, $d\tau_{HF}^j = d\tau_{HF} > 0$, and $\Gamma > 0$ is the same bundle of variables as the one in the nohomothetic CES case.

Similarly, the revenue to wage-income ratio and the real per capita income in Home respond as

$$\frac{d\mu_H}{d\tau_{HF}} = \frac{|\mathcal{J}|(1+\mu_H)(1+\theta)w^{2\theta}(A/b_{HF})^\theta}{J\Gamma(1+\tau_{HF})[(w/b_{HF})^\theta + A^\theta]}\left(\frac{1}{\theta\pi_{FF}} + \Gamma' - \tau_{HF}\right) = \begin{cases} \geq 0 & \text{if } \tau_{HF} \leq (\theta\pi_{FF})^{-1} + \Gamma' \text{ or } \tau_{HF} \leq \tau_{HF}^{**} \\ < 0 & \text{if } \tau_{HF} > (\theta\pi_{FF})^{-1} + \Gamma' \text{ or } \tau_{HF} > \tau_{HF}^{**} \end{cases},$$

$$\frac{d}{d\tau_{HF}}\left(\frac{C_H}{L_H}\right) = \frac{|\mathcal{J}|\theta(1+\theta)w^{2\theta}(A/b_{HF})^\theta}{J\Gamma(1+\tau_{HF})[(w/b_{HF})^\theta + A^\theta]}\left(\frac{1}{\theta\pi_{FF}} - \tau_{HF}\right)\begin{cases} \geq 0 & \text{if } \tau_{HF} \leq (\theta\pi_{FF})^{-1} \text{ or } \tau_{HF} \leq \tau_{HF}^{*} \\ < 0 & \text{if } \tau_{HF} > (\theta\pi_{FF})^{-1} \text{ or } \tau_{HF} > \tau_{HF}^{*} \end{cases},$$

for $j\in\mathcal{J}$, where the parameters such as Γ, Γ', τ_{HF}^* and τ_{HF}^{**} are the same as those in the nohomothetic CES case. □

The effects of tariffs across different models We consider an unilateral increase in Home's tariff applied uniformly across all sectors, $d\tau_{HF}^j = d\tau_{HF} > 0$ for all $j\in\{1,\ldots,J\}$. As in the previous proofs, we also assume $\tau_{HF}^j = \tau_{HF}$ for all j and no tariffs applied by Foreign. We totally differentiate (A1) with respect to

τ_{HF} to obtain

$$
\begin{aligned}
&1 = \sum_{j=1}^{J} x_H^{\frac{\sigma-1}{\theta}} u(x_H)^{\epsilon^j(1-\sigma)}, \\
&\rightarrow\; 0 = \sum_{j=1}^{J} \left[\frac{\sigma-1}{\theta} x_H^{\frac{\sigma-1}{\theta}} u(x_H)^{\epsilon^j(1-\sigma)} d\ln x_H - \theta \epsilon^j x_H^{\frac{\sigma-1}{\theta}} u(x_H)^{\epsilon^j(1-\sigma)} d\ln u(x_H) \right], \\
&\rightarrow\; \frac{d\ln u^{nhCES}(x_H)}{d\ln x_H} = \frac{1}{\Lambda_H} \frac{d\ln u^{nhCES}(x_H)}{d\tau_{HF}} = \frac{1}{\theta \sum_{j=1}^{J} \epsilon^j x_H^{\frac{\sigma-1}{\theta}} u(x_H)^{\epsilon^j(1-\sigma)}}, \qquad \text{(A3)}
\end{aligned}
$$

where $u^{nhCES}(x_H) = (C_H/L_H)^{nhCES}$ is Home's real per capita consumption (or utility) under nonhomothetic CES preferences ($nhCES$), and changes in x_H, $d\ln x_H$, is caused by changes in τ_{HF}, $d\tau_{HF}$, and their relationship is given by (A2): $d\ln x_H/d\tau_{HF} = \Lambda_H$. This represents the semi-elasticity of the real per capita consumption with respect to tariffs. Under homothetic CES preferences ($\epsilon^j = 1$ for all j) including Cobb-Douglas as a special case ($\sigma = 1$), this semi-elasticity becomes $1/\theta$. Comparing the semi-elasticities between the two types of preferences, we see

$$
\frac{d\ln u^{nhCES}(x_H)}{d\tau_{HF}} \Big/ \frac{d\ln u^{CES}(x_H)}{d\tau_{HF}} = \frac{1}{\theta \sum_{j=1}^{J} \epsilon^j x_H^{\frac{\sigma-1}{\theta}} u(x_H)^{\epsilon^j(1-\sigma)}} \Big/ \frac{1}{\theta} = \frac{1}{\sum_{j=1}^{J} \epsilon^j x_H^{\frac{\sigma-1}{\theta}} u(x_H)^{\epsilon^j(1-\sigma)}},
$$

where $u^{CES}(x_H) = (C_H/L_H)^{CES}$ is Home's real per capita consumption under homoethetic CES preferences (CES). This is smaller than one if

$$
1 = \epsilon^{\hat{j}} < \sum_{j=1}^{J} \epsilon^j x_H^{\frac{\sigma-1}{\theta}} u(x_H)^{\epsilon^j(1-\sigma)} = \sum_{j=1}^{J} \epsilon^j \omega_H^j = \bar{\epsilon}_H,
$$

where $\hat{j}$ is the sector with $\epsilon^{\hat{j}} = 1$ among $j \in \{1, \ldots, J\}$ sectors and corresponds to manufacturing in the three-sector case in the text: $\hat{j} = m$. We can show the analogous result for Foreign:

$$
\frac{d\ln u^{nhCES}(x_F)}{d\tau_{HF}} \Big/ \frac{d\ln u^{CES}(x_F)}{d\tau_{HF}},
$$

which is smaller than one if $1 = \epsilon^{\hat{j}} < \bar{\epsilon}_F$ holds. That is, if the income elasticity of manufacturing demand is sufficiently low, the magnitude of the effect of tariffs on real per capita consumption is smaller under nonhomothetic CES than under homothetic CES including Cobb-Douglas. □

B Proof of Proposition 2

For later reference, we first prove the following two lemmas.

Lemma 1: *Define a probability function* $f(j; x) : \{1, .., J\} \rightarrow [0, 1]$ *by*

$$
f(j; x) = \frac{[u(x)]^{\epsilon^j(1-\sigma)}}{\sum_{h=1}^{J} [u(x)]^{\epsilon^h(1-\sigma)}},
$$

where $u(x) > 0$ *is a positive-valued function of* $x \in (0, \infty)$ *and satisfies* $x^{\frac{\sigma-1}{\theta}} [u(x)]^{\epsilon^j(1-\sigma)} = \omega^j \in (0, 1)$ *and* $\sum_{j=1}^{J} \omega^j = 1$; $\{\epsilon^j\}_{j=1}^{J}$ *are parameters such that* $0 < \epsilon^1 < \ldots < \epsilon^J$ *with a sector* j *such that* $\epsilon^j = 1$; *and*

$\sigma \in (0,1)$. *Then, its distribution function, defined by* $F(k;x) = \sum_{j=1}^{k} f(j;x)$, *decreases with* x.

Proof of Lemma 1: Differentiating $F(k;x)$ with respect to x yields

$$\begin{aligned}\frac{d\ln F(k;x)}{d\ln x} &= \frac{(1-\sigma)\xi(x)}{\sum_{j=1}^{k}\omega^j}\left[\sum_{j=1}^{k}\epsilon^j\omega^j - \left(\sum_{j=1}^{J}\epsilon^j\omega^j\right)\left(\sum_{j=1}^{k}\omega^j\right)\right] \\ &= \frac{(1-\sigma)\xi(x)}{\sum_{j=1}^{k}\omega^j}\sum_{h=1}^{k}\omega^h\sum_{j=k+1}^{J}\left(\epsilon^h-\epsilon^j\right)\omega^j, \qquad k\in\{1,\ldots,J-1\}\end{aligned}$$

where $\xi(x) = xu'(x)/u(x)$, which we saw in Section A of this Supplemental Appendix. From the first to the second line, we use the mathematical induction. Specifically, we can check the equality holds at $k=1$; by assuming it holds at $k = k' \in \{2,\ldots,J-1\}$, we can prove it holds at $k = k'+1$.

The function $\xi(x)$ is positive because we rewrite $1 = \sum_j \omega^j = \sum_j x^{\frac{\sigma-1}{\theta}}[u(x)]^{\epsilon^j(1-\sigma)}$ given in (A1) as $x^{\frac{1-\sigma}{\theta}} = \sum_j^J [u(x)]^{\epsilon^j(1-\sigma)}$ and differentiate it with respect to x to obtain

$$\frac{1}{\xi(x)} = \theta\sum_{j=1}^{J}\epsilon^j x^{\frac{\sigma-1}{\theta}}[u(x)]^{\epsilon^j(1-\sigma)} > 0.$$

From this and the assumptions that $1-\sigma>0$ and $\epsilon^h - \epsilon^j < 0$ for $h<j$, we can conclude that the derivative is negative: $d\ln F(k;x)/d\ln x < 0$ for $k\in\{1,\ldots,J\}$. □

Lemma 2: *The function* $\xi(x) = xu'(x)/u(x)$ *defined in Lemma 1 decreases with* x.

Proof of Lemma 2: Again we differentiate $x^{\frac{1-\sigma}{\theta}} = \sum_j^J [u(x)]^{\epsilon^j(1-\sigma)}$ with respect to x to obtain

$$\frac{1}{\xi(x)} = \theta\sum_{j=1}^{J}\epsilon^j\cdot\frac{[u(x)]^{\epsilon^j(1-\sigma)}}{\sum_{h=1}^{J}[u(x)]^{\epsilon^h(1-\sigma)}} = \theta\sum_{j=1}^{J}\epsilon^j f(j;x).$$

What we need to show is that if $x_F < x_H$, $\xi(x_F) > \xi(x_H)$ holds. From Lemma 1, the distribution function $F(j;x)$ exhibits the first-order stochastic dominance in the discrete case (Courtault et al, 2006). That is, if $x_F < x_H$, $F(k;x_H) = \sum_{j=1}^{k} f(j;x_H) < F(k;x_F) = \sum_{j=1}^{k} f(j;x_F)$ holds for all $k\in\{1,2,\ldots,J-1\}$. Then, Proposition 1 in Courtault et al. (2006) implies $1/\xi(x_H) = \theta\sum_{j=1}^{J}\epsilon^j f(j;x_H) > 1/\xi(x_F) = \theta\sum_{j=1}^{J}\epsilon^j f(j;x_F)$, or equivalently $\xi(x_F) > \xi(x_H)$. □

Nonhomothetic CES As in Proposition 1, we assume productivity A_n, population L_n, non-tariff trade barriers d_{ni}, and tariffs τ_{ni} are country (pair) specific, but not sector specific. We consider a uniform increase in Home's tariffs across all sectors, $d\tau_{HF}^j = d\tau_{HF} > 0$ for all j.

We derive the derivative of x_H given in (A1) with respect to τ_{HF}:

$$\begin{aligned}0 = \sum_{j=1}^{J}\frac{d\omega_H^j}{d\tau_{HF}} &= (1-\sigma)\sum_{j=1}^{J}x_H^{\frac{\sigma-1}{\theta}}\,[u(x_H)]^{\epsilon^j(1-\sigma)}\left[\epsilon^j\xi(x_H)-\frac{1}{\theta}\right]\frac{d\ln x_H}{d\tau_{HF}} \\ &= (1-\sigma)\sum_{j=1}^{J}\omega_H^j\left[\epsilon^j\xi(x_H)-\frac{1}{\theta}\right]\frac{d\ln x_H}{d\tau_{HF}},\end{aligned}$$

where $u(x_H) = C_H/L_H$ and $\xi(x_H) = [\theta\sum_j \epsilon^j x_H^{\frac{\sigma-1}{\theta}}(u(x_H))^{\epsilon^j(1-\sigma)}]^{-1}$. Since $d\ln x_H/d\tau_{HF} \neq 0$ in general, the equation above implies $\sum_j^J \omega_H^j \epsilon^j \xi(x_H) = 1/\theta$. Substituting this back into $d\omega_H^j/d\tau_{HF}$ yields

$$\begin{aligned}
\frac{d\omega_H^j}{d\tau_{HF}} &= (1-\sigma)\omega_H^j \left[\epsilon^j \xi(x_H) - \frac{1}{\theta}\right] \frac{d\ln x_H}{d\tau_{HF}} \\
&= (1-\sigma)\omega_H^j \left[\epsilon^j \xi(x_H) - \sum_{h=1}^J \omega_H^h \epsilon^h \xi(x_H)\right] \frac{d\ln x_H}{d\tau_{HF}}, \\
\Leftrightarrow \frac{d\ln \omega_H^j}{d\tau_{HF}} &= (1-\sigma)\left(\epsilon^j - \bar{\epsilon}_H\right) \frac{d\ln u(x_H)}{d\tau_{HF}},
\end{aligned} \tag{B1}$$

noting $d\ln x_H/d\tau_{HF} = \xi(x_H)^{-1} d\ln u(x_H)/d\tau_{HF}$ from (A3) and $\bar{\epsilon}_H = \sum_h^J \omega_H^h \epsilon^h$. The equation in the last line was studied in Proposition 2. An equivalent condition to, for example, $\epsilon^j - \bar{\epsilon}_H > 0$, is $\epsilon^j \xi(x_H) - 1/\theta > 0$.

Suppose that τ_{HF} is in $[0, \tau_{HF}^*)$ so that Home is in the increasing part of the tariff–consumption schedule: $d\ln x_H/d\tau_{HF} > 0$. Given the ordering of $\{\epsilon^j\}_j$, for the above equation to hold, we must have (i) $\epsilon^1 \xi(x_H) - 1/\theta < 0$; (ii)$\epsilon^J \xi(x_H) - 1/\theta > 0$; and (iii) there exists a cutoff sector j_H such that $\epsilon^{j_H} \xi(x_H) - 1/\theta \leq 0$ and $\epsilon^{j_H+1} \xi(x_H) - 1/\theta \geq 0$ hold where the exact equality, if any, only holds in either $\epsilon^{j_H} \xi(x_H) - 1/\theta = 0$ or $\epsilon^{j_H+1} \xi(x_H) - 1/\theta = 0$. The index of the cutoff sector, j_H, depends on x_H and thus on τ_{HF}. These results imply

$$\frac{d\ln \omega_H^j}{d\tau_{HF}} \begin{cases} < 0 & \text{for } j \in \{1, \ldots, j_H\} \\ > 0 & \text{for } j \in \{j_H + 1, \ldots, J\} \end{cases},$$

except for the rare cases in which the exact equality holds, $\epsilon^{j_H} \xi(x_H) - 1/\theta = 0$ or $\epsilon^{j_H+1} \xi(x_H) - 1/\theta = 0$.

For the sake of completeness, supposing that τ_{HF} is in (τ_{HF}^*, ∞) and thus $d\ln x_H/d\tau_{HF} < 0$ holds, we have

$$\frac{d\ln \omega_H^j}{d\tau_{HF}} \begin{cases} > 0 & \text{for } j \in \{1, \ldots, j_H\} \\ < 0 & \text{for } j \in \{j_H + 1, \ldots, J\} \end{cases},$$

except for the rare cases in which the exact equality holds, $\epsilon^{j_H} \xi(x_H) - 1/\theta = 0$ or $\epsilon^{j_H+1} \xi(x_H) - 1/\theta = 0$.

Analogously, we can derive the tariff effect on the sectoral expenditure share in the tariff-imposed Foreign. Considering $d\ln x_F/d\tau_{HF} < 0$ for any $\tau \in [0, \infty)$, we can conclude

$$\frac{d\ln \omega_F^j}{d\tau_{HF}} = \left[\epsilon^j \xi(x_F) - \frac{1}{\theta}\right] \frac{d\ln x_F}{d\tau_{HF}} \begin{cases} > 0 & \text{for } j \in \{1, \ldots, j_F\} \\ < 0 & \text{for } j \in \{j_F + 1, \ldots, J\} \end{cases},$$

where j_F is the index of the cut-off sector and satisfies $\epsilon^{j_F} \xi(x_F) - 1/\theta < 0$ (or equivalently, $\epsilon^{j_F} - \bar{\epsilon}_F < 0$) and $\epsilon^{j_F+1} \xi(x_F) - 1/\theta > 0$ (or equivalently, $\epsilon^{j_F+1} - \bar{\epsilon}_F > 0$) except when either of the two conditions holds with equality. □

Using Lemma 2, we can also see how the sectoral expenditure share evolves as tariff rises. In the low income-elastic sectors with $\epsilon^j \xi(x_H) - 1/\theta < 0$, an increase in τ_{HF} from zero first reduces and then raises ω_H^j. In the high income-elastic sectors with $\epsilon^j \xi(x_H) - 1/\theta > 0$, an increase in τ_{HF} from zero first raises and then reduces ω_H^j. In the middle income-elastic sectors, we see $\epsilon^j \xi(x_H) - 1/\theta \geq 0$ for $\tau_{HF} \in (0, \tau_{HF}^c]$ and $\epsilon^j \xi(x_H) - 1/\theta < 0$ for $\tau_{HF} \in (\tau_{HF}^c, \infty)$, and the tariff increase from a sufficiently low level raises ω_H^j, but the tariff increase from a sufficiently high level reduces ω_H^j.

Homothetic CES We allow for sector-specific tariff changes, while maintaining sectoral symmetry in all the other parameters and shutting down nonhomotheticity, $\epsilon^j = 1$ for all j. We here consider Home's unilateral uniform tariff increase in some sectors $j \in \mathcal{J}$ with $\mathcal{J}$ being the set of sectors whose tariffs are increased, $d\tau^j_{HF} = d\tau_{HF} > 0$ for $j \in \mathcal{J}$, and $\mathcal{J}^c$ being the complement set, $d\tau^j_{HF} = d\tau_{HF} = 0$ for $j \in \mathcal{J}^c$. As in the previous proofs, we also assume $\tau^j_{HF} = \tau_{HF}$ for all j and no tariffs applied by Foreign.

Since the relative wage is determined by the trade balance condition, the change in the (relative) wage in Home is derived by differentiating the change in the trade balance condition:

$$\sum_{j=1}^{J} \frac{1}{J}\left(d\ln \pi^j_{FH} + d\ln \omega^j_F + \frac{d\mu_F}{1+\mu_F}\right) = \sum_{j=1}^{J} \frac{1}{J}\left(d\ln \pi^j_{HF} + d\ln \omega^j_H + \frac{d\mu_H}{1+\mu_H} + d\ln w - \frac{d\tau^j_{HF}}{1+\tau_{HF}}\right),$$

where

$$d\ln b^j_{HF} = d\ln d_{HF}\left(1+\tau^j_{HF}\right) = \frac{d\tau^j_{HF}}{1+\tau_{HF}} = \frac{d\tau_{HF}}{1+\tau_{HF}} > 0, \qquad j \in \mathcal{J}$$

$$d\ln b^j_{HF} = \frac{d\tau^j_{HF}}{1+\tau_{HF}} = 0, \qquad j \in \mathcal{J}^c$$

$$d\ln b^j_{FH} = \frac{d\tau^j_{FH}}{1+\tau_{FH}} = 0, \qquad \forall j$$

$$d\ln P^j_F = \pi_{FH} d\ln w, \qquad \pi_{FH} = \left(\frac{w b_{FH}/A_H}{P_F}\right)^{-\theta} = \frac{(A/b_{FH})^\theta}{w^\theta + (A/b_{FH})^\theta} = 1 - \pi_{FF}, \qquad \forall j$$

$$d\ln P^j_H = \pi_{HF} d\ln b^j_{HF} + \pi_{HH} d\ln w, \qquad \pi_{HF} = \left(\frac{b_{HF}/A_F}{P_H}\right)^{-\theta} = \frac{(w/b_{HF})^\theta}{(w/b_{HF})^\theta + A^\theta} = 1 - \pi_{HH}, \qquad \forall j$$

$$d\ln \pi^j_{HF} = -\theta\left(d\ln b^j_{HF} - d\ln P^j_H\right) = -\theta\pi_{HH}\left(d\ln b^j_{HF} - d\ln w\right), \qquad \forall j$$

$$d\ln \pi^j_{FH} = -\theta\left(d\ln w - d\ln P^j_F\right) = -\theta\pi_{FF} d\ln w, \qquad \forall j$$

$$d\ln \omega^j_n = (1-\sigma)\left(d\ln P^j_n - \sum_{h=1}^{J}\omega^h_n d\ln P^h_n\right), \qquad \omega^j_n = \frac{(P^j_n)^{1-\sigma}}{\sum_{h=1}^{J}(P^h_n)^{1-\sigma}} = \frac{1}{J}, \qquad \forall (n,j)$$

$$d\mu_F = \sum_{j=1}^{J} d\mu^j_F = \sum_{j=1}^{J}\pi_{FH}\omega^j_F d\tau^j_{FH} = 0,$$

$$d\mu_H = \begin{cases} \sum_j^J d\mu^j_H = \sum_j^J \pi_{HF}\omega^j_F d\tau^j_{HF} = \sum_j^J \pi_{HF} J^{-1} d\tau^j_{HF} & \text{if } \tau_{HF} = 0 \\ \dfrac{\mu_H(1+\tau_{HF})}{1+\tau_{HF}\pi_{HH}}\sum_j^J \omega^j_H d\ln\mu^j_H = \dfrac{\tau_{HF}\pi_{HF}(1+\tau_{HF})}{J(1+\tau_{FH}\pi_{HF})^2}\sum_j^J\left(\dfrac{d\ln\tau^j_{HF}}{1+\tau_{HF}} + d\ln\pi^j_{HF} + d\ln\omega^j_H\right) & \text{if } \tau_{HF} > 0 \end{cases},$$

noting $w = w_H/w_F = w_H$; $A = A_H/A_F$; and $P^j_n = P^h_n$ for $j \neq h$ because of no sectoral asymmetry in parameters. We solve the trade balance condition for $d\ln w$ and substitute this back into $d\ln\omega^j_n$ to obtain

$$\frac{d\ln\omega^j_H}{d\tau_{HF}} = \frac{(J - |\mathcal{J}|)(1-\sigma)}{J(1+\tau_{HF})\left[(b_{HF}A/w)^\theta + 1\right]} > 0, \qquad j \in \mathcal{J}$$

$$\frac{d\ln\omega^j_H}{d\tau_{HF}} = -\frac{|\mathcal{J}|(1-\sigma)}{J(1+\tau_{HF})\left[(b_{HF}A/w)^\theta + 1\right]} < 0, \qquad j \in \mathcal{J}^c$$

$$\frac{d\ln\omega^j_F}{d\tau_{HF}} = 0, \qquad \forall j$$

noting that there are $|\mathcal{J}|$ sectors whose tariffs are increased, $d\tau^j_{HF} = d\tau_{HF} > 0$. Due to the relative price effect, Home shifts its expenditure to the sectors with increasing tariff. Foreign does not change their expenditure pattern simply because changes in relative wage due to tariff affect sectoral price indices in Foreign proportionally and thus the relative price effect is not present there. □

C Proof of Proposition 3

We assume here that before tariff changes, the initial tariffs are zero, $\tau^j_{HF} = \tau^j_{FH} = 0$ for all j, and the sectors are symmetric (except for $\{\epsilon^j\}^J_j$ in the case of nonhomothetic CES) in all aspects such as productivity $A^j_n = A_n$ and non-tariff trade barriers $d^j_{ni} = d_{ni}$. The value-added share of sector j in country n is given by

$$\begin{aligned} va^j_n &= \frac{VA^j_n}{\sum_{h=1}^J VA^h_n} = \frac{w_n L^j_n}{\sum_{h=1}^J w_n L^h_n} \\ &= \frac{P^j_n C^j_n}{w_n L_n} + \frac{NX^j_n}{w_n L_n} - \frac{\widetilde{T}_n}{w_n L_n} = \frac{P^j_n C^j_n}{E_n}\frac{E_n}{w_n L_n} + \frac{NX^j_n}{w_n L_n} - \frac{\widetilde{T}_n}{w_n L_n} \\ &= \omega^j_n(1+\mu_n) + \frac{NX^j_n}{w_n L_n} - \frac{\widetilde{T}_n}{w_n L_n}. \end{aligned}$$

As in the text, its marginal change is given by

$$d\ln va^j_n = \underbrace{\frac{P^j_n C^j_n}{w_n L^j_n}\left(d\ln\omega^j_n + \frac{d\mu_n}{1+\mu_n}\right)}_{\text{(a) Expenditure adjusted by tariff revenue}} + \underbrace{\frac{NX^j_n}{w_n L^j_n} d\ln\left(\frac{NX^j_n}{w_n L_n}\right)}_{\text{(b) Net exports}} \underbrace{- \frac{1}{va^j_n} d\left(\frac{\widetilde{T}^j_n}{w_n L_n}\right)}_{\text{(c) Tariff revenue}}, \tag{C1}$$

noting that we write $d\mu_H$ and $d[\widetilde{T}^j_n/(w_n L_n)]$ in terms of changes in level since $\mu_n = \widetilde{T}_n/(w_n L_n)$ and $\widetilde{T}^j_n$ are zero at the initial situation. We can tell from Propositions 1 and 2 how $d\ln\omega^j_n$ and $d\mu_n$ adjust after tariffs change.

In the case of Home, the change in net exports, term (b) in (C1), is further decomposed as

$$\begin{aligned} \frac{NX^j_H}{w_H L^j_H} d\ln\left(\frac{NX^j_H}{w_H L_H}\right) &= \frac{EX^j_H}{w_H L^j_H} d\ln\left(\frac{EX^j_H}{w_H L_H}\right) - \frac{IM^j_H}{w_H L^j_H} d\ln\left(\frac{IM^j_H}{w_H L_H}\right) \\ &= \frac{1}{va^j_H}\frac{EX^j_H}{w_H L_H} d\ln\left(\frac{EX^j_H}{w_H L_H}\right) - \frac{1}{va^j_H}\frac{IM^j_H}{w_H L_H} d\ln\left(\frac{IM^j_H}{w_H L_H}\right) \\ &= \frac{1}{va^j_H}\left[\frac{\pi^j_{FH}\omega^j_F}{wL}\left(d\ln\pi^j_{FH} + d\ln\omega^j_F - d\ln w\right) - \pi^j_{HF}\omega^j_H\left(d\mu_H + d\ln\pi^j_{HF} + d\ln\omega^j_H - d\tau^j_{HF}\right)\right], \end{aligned}$$

noting $w = w_H/w_F$; $L = L_H/L_F$; and $\tau^j_{HF} = \tau^j_{FH} = 0$ for all j (thus $\mu_H = \mu_F = 0$). An analogous expression holds for Foreign. Changes in tariffs by Home affect their net exports to Foreign in three channels. First, tariffs directly make Foreign's composite good more expensive, and hence reduce Home's imports and raise their net exports, which is captured by the very last term in the equation above, $d\tau^j_{HF}$. Second, tariffs affect the demand for sector j composite good in both countries by changing their sectoral expenditure shares and Home's tariff revenue. Finally, tariffs affect the trade shares, $d\ln\pi^j_{ni}$, by changing the relative cost of production, i.e., the comparative advantage of the two countries.

Similarly, the change in tariff revenue, term (c) in (C1), is rewritten as

$$-\frac{1}{va_H^j} d\left(\frac{\widetilde{T}_H^j}{w_H L_H}\right) = -\frac{1}{va_H^j} d\left(\frac{\tau_{HF}^j IM_H^j}{w_H L_H}\right) = -\frac{1}{va_H^j} \pi_{HF}^j \omega_H^j d\tau_{HF}^j,$$

which is always non-positive, as long as $d\tau_{HF}^j \geq 0$. Note again that the total derivative is evaluated at $\tau_{HF}^j = \tau_{FH}^j = 0$ for all j.

Nonhomothetic CES In addition to the symmetry in sectors except for $\{\epsilon^j\}_j^J$, we also assume symmetric countries, $A_H = A_F$, $L_H = L_F$, and $d_{HF} = d_{FH} = d$. We here consider the effect of tariff applied by Home to Foreign uniform across sectors, $d\tau_{HF}^j = d\tau_{HF} > 0$ for all j.

The tariff effects on (a) expenditure, (b) net exports, and (c) tariff revenue are

$$\begin{aligned}
&\text{(a)}\ \frac{P_H^j C_H^j}{w_H L_H^j}\left(d\ln\omega_H^j + \frac{d\mu_H}{1+\mu_H}\right) = \pi_{HF} d\tau_{HF} + d\ln\omega_H^j \begin{cases} \gtrless 0 & \text{if } j \in \{1,\ldots,j_*\} \\ > 0 & \text{if } j \in \{j_*+1,\ldots,J\} \end{cases}, \\
&\text{(b)}\ \frac{NX_H^j}{w_H L_H^j} d\ln\left(\frac{NX_H^j}{w_H L_H}\right) = \pi_{HF}\left(d\ln\omega_F^j - d\ln\omega_H^j\right) \begin{cases} > 0 & \text{if } j \in \{1,\ldots,j_*\} \\ < 0 & \text{if } j \in \{j_*+1,\ldots,J\} \end{cases}, \\
&\text{(c)}\ -\frac{1}{va_H^j} d\left(\frac{\widetilde{T}_H^j}{w_H L_H}\right) = -\pi_{HF} d\tau_{HF} < 0, \qquad\qquad \forall j
\end{aligned}$$

where the symmetry of countries and sectors implies $w_H = w_F$, $x_H = x_F$, $E_H = E_F$, $\pi_{HF}^j = \pi_{HF} = \pi_{FH}^j = \pi_{FH}^j = 1/(1+b^\theta)$ for all j, $\omega_H^j = \omega_F^j$ for all j, and $va_n^j = \omega_n^j$ for all n and j. We tell from Section B of this Supplemental Appendix ((B1) in particular) that $d\ln\omega_H^j$ increases with j, while $d\ln\omega_F^j$ decreases with j. This implies $d\ln\omega_H^j < 0$ for $j \in \{1,\ldots,j_*\}$ and $d\ln\omega_H^j > 0$ for $j \in \{j_*+1,\ldots,J\}$, while $d\ln\omega_F^j > 0$ for $j \in \{1,\ldots,j_*\}$ and $d\ln\omega_F^j < 0$ for $j \in \{j_*+1,\ldots,J\}$, where $j_* = j_H = j_F$ is the cut-off sector common between the two countries such that $\epsilon^{j_*}\xi(x_n) - 1/\theta < 0$ (or equivalently, $\epsilon^{j_*} - \bar{\epsilon}_n < 0$) and $\epsilon^{j_*+1}\xi(x_n) - 1/\theta > 0$ (or $\epsilon^{j_*+1} - \bar{\epsilon}_n > 0$) hold. We can thus conclude that (b) decreases with j and turns negative to positive at j_*+1, while (a) increases with j and turns positive to negative at $j_{**}+1 (< j_*+1)$. In general, the effects on expenditure and net exports go in the opposite way. Home's sectoral expenditure share directly contributes to the sector's value-added share, but raises Home's imports and thus reduces its net exports.

Combining the three effects, we have

$$d\ln va_H^j = \pi_{HF} d\ln\omega_F^j + \pi_{HH} d\ln\omega_H^j = \frac{b^\theta - 1}{1+b^\theta}\left[\epsilon^j\xi(x_H) - \frac{1}{\theta}\right]\frac{d\ln x_H}{d\tau_{HF}} \begin{cases} < 0 & \text{if } j \in \{1,\ldots,j_*\} \\ > 0 & \text{if } j \in \{j_*+1,\ldots,J\} \end{cases},$$

where $b_{HF} = b_{FH} = b = d > 1$, $b^\theta > 1$, and $d\ln x_H/d\tau_{HF} = -d\ln x_F/d\tau_{HF} > 0$. The signs are the same as those of $d\ln\omega_H^j$, meaning that changes in the sectoral value-added shares in a country are largely shaped by those in the domestic sectoral expenditure shares.

Analogously, the tariff effects in Foreign are

$$\text{(a)}\ \frac{P_F^j C_F^j}{w_F L_F^j} d\ln\omega_F^j = d\ln\omega_F^j \begin{cases} >0 & \text{if } j\in\{1,\dots,j_*\} \\ <0 & \text{if } j\in\{j_*+1,\dots,J\} \end{cases},$$

$$\text{(b)}\ \frac{NX_F^j}{w_F L_F^j} d\ln\left(\frac{NX_F^j}{w_F L_F}\right) = \pi_{FH}\left(d\ln\omega_H^j - d\ln\omega_F^j\right)\begin{cases} <0 & \text{if } j\in\{1,\dots,j_*\} \\ >0 & \text{if } j\in\{j_*+1,\dots,J\} \end{cases},$$

$$\text{(c)}\ -\frac{1}{va_F^j} d\left(\frac{\widetilde{T}_F^j}{w_F L_F}\right) = 0, \qquad j\in\{1,\dots,J\}$$

$$d\ln va_F^j = \pi_{FF} d\ln\omega_F^j + \pi_{FH} d\ln\omega_H^j = \frac{1-b^\theta}{1+b^\theta}\left[\epsilon^j\xi(x_F) - \frac{1}{\theta}\right]\frac{d\ln x_F}{d\tau_{HF}}\begin{cases} >0 & \text{if } j\in\{1,\dots,j_*\} \\ <0 & \text{if } j\in\{j_*+1,\dots,J\} \end{cases},$$

where $d\mu_F = 0$ and $d[\widetilde{T}_F^j/(w_F L_F)] = 0$ since Foreign does not change tariffs from zero, $d\tau_{FH} = 0$. The results are a mirror image of those in Home. Tariffs by Home change Foreign's domestic sectoral expenditure and net exports in reverse. The tariff effect on sectoral value-added shares is largely governed by that of the sectoral domestic expenditure shares. The results are summarized in Table A1. □

Table A1: The effect of a unilateral tariff increase under nonhomothetic CES preferences

Country n	Sector j	$d\tau_{ni}$	$d\ln\omega_n^j$	(a) Exp_n^j	(b) NX_n^j	(c) Tariff rev_n^j	$d\ln va_n^j$: (a) + (b) + (c)
H	$\{1,\dots,j_*\}$	+	−	+/−	+	−	−
H	$\{j_*+1,\dots,J\}$	+	+	+	−	−	+
F	$\{1,\dots,j_*\}$	0	+	+	+	−	+
F	$\{j_*+1,\dots,J\}$	0	−	+/−	−	−	−

Notes: This table shows the effects of a unilateral increase in uniform tariffs across sectors by Home ($d\tau_{HF}^j = d\tau_{HF} > 0$ for all j). Except for the degree of nonhomotheticity $\{\epsilon^j\}_j^J$, the two countries and the J sectors are symmetric in all respects at the initial situation: technology, population, non-tariff trade barriers, and (zero) tariffs. The cut-off sector is denoted by j_*, where $\epsilon^{j_*}\xi(x) - 1/\theta < 0$ and $\epsilon^{j_*+1}\xi(x) - 1/\theta > 0$ hold. Each of the three effects, (a) expenditure adjusted by tariff revenue, (b) net exports, and (c) tariff revenue, corresponds to a distinct term in (C1). In the three-sector model in the text, the set of sectors $\{1,2,3\}$ corresponds to $\{a,m,s\}$ and the cut-off sector is $j_* = 2 = m$.

Homothetic CES Unlike the case of nonhomothetic CES, we allow for country asymmetry, but keep sector symmetry and shut down nonhomotheticity by setting $\epsilon^j = 1$ for all j. We here consider the effect of Home's unilateral uniform tariff increase in some sectors $j \in \mathcal{J}$ with $\mathcal{J}$ being the set of sectors whose tariffs are increased and $\mathcal{J}^c$ being its complement set, $d\tau_{HF}^j = d\tau_{HF} > 0$ for $j \in \mathcal{J}$, and $d\tau_{HF}^j = d\tau_{HF} = 0$ for $j \in \mathcal{J}^c$. As in the previous proofs, we also assume $\tau_{HF}^j = \tau_{HF}$ for all j and no tariffs applied by Foreign.

The tariff effects in Home are

$$\text{(a)}\ \frac{P_H^j C_H^j}{w_H L_H^j}\left(d\ln\omega_H^j + \frac{d\mu_H}{1+\mu_H}\right) = \begin{cases} \left[1-\sigma\left(1-\frac{|\mathcal{J}|}{J}\right)\right]\pi_{HF}d\tau_{HF} > 0 & \text{if } j\in\mathcal{J} \\ \frac{|\mathcal{J}|}{J}\sigma\pi_{HF}d\tau_{HF} > 0 & \text{if } j\in\mathcal{J}^c \end{cases},$$

$$\text{(b)}\ \frac{NX_H^j}{w_H L_H^j} d\ln\left(\frac{NX_H^j}{w_H L_H}\right) = \begin{cases} \frac{\pi_{HF}(J-|\mathcal{J}|)[\sigma(w/b_{HF})^\theta + (1+\theta)A^\theta]}{J[(w/b_{HF})^\theta + A^\theta]} d\tau_{HF} > 0 & \text{if } j\in\mathcal{J} \\ -\frac{\pi_{HF}|\mathcal{J}|[\sigma(w/b_{HF})^\theta + (1+\theta)A^\theta]}{J[(w/b_{HF})^\theta + A^\theta]} d\tau_{HF} < 0 & \text{if } j\in\mathcal{J}^c \end{cases},$$

$$\text{(c)}\ -\frac{1}{va_H^j} d\left(\frac{\widetilde{T}_H^j}{w_H L_H}\right) = \begin{cases} -\pi_{HF}d\tau_{HF} < 0 & \text{if } j\in\mathcal{J} \\ 0 & \text{if } j\in\mathcal{J}^c \end{cases},$$

$$d\ln va_H^j = \begin{cases} \frac{\pi_{HF}(J-|\mathcal{J}|)A^\theta(\theta+1-\sigma)}{J[(w/b_{HF})^\theta + A^\theta]} d\tau_{HF} > 0 & \text{if } j\in\mathcal{J} \\ -\frac{\pi_{HF}|\mathcal{J}|A^\theta(\theta+1-\sigma)}{J[(w/b_{HF})^\theta + A^\theta]} d\tau_{HF} < 0 & \text{if } j\in\mathcal{J}^c \end{cases}.$$

Although the expenditure shares in the sectors whose tariffs remain unchanged, $d\ln\omega_H^j < 0$ for $j\in\mathcal{J}^c$, the overall tariff effect on expenditures in those sectors is positive due to an increase in tariff revenues, $d\ln\omega_H^j + d\mu_H/(1+\mu_H) > 0$ for $j\in\mathcal{J}^c$.

Similarly, the tariff effects in Foreign are

$$\text{(a)}\ \frac{P_F^j C_F^j}{w_F L_F^j} d\ln\omega_F^j = 0, \qquad \text{(b)}\ -\frac{1}{va_F^j} d\left(\frac{\widetilde{T}_F^j}{w_F L_F}\right) = 0, \qquad \forall j$$

$$\text{(c)}\ \frac{NX_F^j}{w_F L_F^j} d\ln\left(\frac{NX_F^j}{w_F L_F}\right) = d\ln va_F^j = \begin{cases} -\frac{\pi_{FH}(J-|\mathcal{J}|)[\sigma(w/b_{HF})^\theta + (1+\theta)A^\theta]}{J[(w/b_{HF})^\theta + A^\theta]} d\tau_{HF} < 0 & \text{if } j\in\mathcal{J} \\ \frac{\pi_{FH}|\mathcal{J}|[\sigma(w/b_{HF})^\theta + (1+\theta)A^\theta]}{J[(w/b_{HF})^\theta + A^\theta]} d\tau_{HF} > 0 & \text{if } j\in\mathcal{J}^c \end{cases},$$

where $d\mu_F = 0$ and $d\widetilde{T}_F^j = 0$ since Foreign does not change tariffs from zero, $d\tau_{FH} = 0$. The results are summarized in Table A2.

Table A2: The effect of a unilateral tariff increase under homothetic CES preferences

Country n	Sector j	$d\tau_{ni}^j$	$d\ln\omega_n^j$	(a) Exp$_n^j$	(b) NX$_n^j$	(c) Tariff rev$_n^j$	$d\ln va_n^j$: (a) + (b) + (c)
H	$j\in\mathcal{J}$	+	+	+	+	−	+
H	$j\in\mathcal{J}^c$	0	−	+	−	0	−
F	$j\in\mathcal{J}$	0	0	0	−	0	−
F	$j\in\mathcal{J}^c$	0	0	0	+	0	+

Notes: This table shows the effects of Home's unilateral uniform tariff increase in some sectors $j\in\mathcal{J}$ with $\mathcal{J}$ being the set of sectors that increase tariffs and $\mathcal{J}^c$ being the complement set ($d\tau_{HF}^j = d\tau_{HF} > 0$ for $j\in\mathcal{J}$ and $d\tau_{HF}^j = d\tau_{HF} = 0$ for $j\in\mathcal{J}^c$). Sectors are symmetric in all respects at the initial situation, including the degree of nonhomotheticity, productivity, non-tariff trade barriers, and zero tariffs, while countries are asymmetric in terms of productivity, population, and non-tariff trade barriers. Each of the three, (a) expenditure adjusted by tariff revenue, (b) net exports, and (c) tariff revenue, corresponds to a distinct term in (C1). In the three-sector model in the text, the sets of sectors, $\mathcal{J}$ and $\mathcal{J}^c$, correspond to $\{m\}$ and $\{a, s\}$ respectively.

Cobb-Douglas For the sake of illustration, we provide the results under the Cobb-Douglas preferences. The non/homothetic CES preferences are reduced to the Cobb-Douglas ones if σ is set to one. In this case, neither the income effect nor the relative price effect works, meaning that the sectoral expenditure share does not respond to tariff changes at all, $d\ln\omega_n^j = 0$. Except for this, the tariff effects have the same sign as the homothetic CES case. The results are summarized in Table A3.

Table A3: The effect of a unilateral tariff increase under Cobb-Douglas preferences

Country n	Sector j	$d\tau_{ni}^j$	$d\ln\omega_n^j$	(a) Exp_n^j	(b) NX_n^j	(c) Tariff rev_n^j	$d\ln va_n^j$: (a) + (b) + (c)
H	$j \in \mathcal{J}$	+	0	+	+	+	+
H	$j \in \mathcal{J}^c$	0	0	+	−	0	−
F	$j \in \mathcal{J}$	0	0	0	−	0	−
F	$j \in \mathcal{J}^c$	0	0	0	+	0	+

Notes: This table shows the effects of Home's unilateral uniform tariff increase in some sectors $j \in \mathcal{J}$ with $\mathcal{J}$ being the set of sectors whose tariffs are increased and $\mathcal{J}^c$ being the complement set ($d\tau_{HF}^j = d\tau_{HF} > 0$ for $j \in \mathcal{J}$ and $d\tau_{HF}^j = d\tau_{HF} = 0$ for $j \in \mathcal{J}^c$). Preferences are Cobb-Douglas, $\sigma = 1$. Sectors are symmetric in all respects at the initial situation, including the degree of nonhomotheticity, productivity, non-tariff trade barriers, and zero tariffs, while countries are asymmetric in terms of productivity, population, and non-tariff trade barriers. Each of the three, (a) expenditure adjusted by tariff revenue, (b) net exports, and (c) tariff revenue, corresponds to a distinct term in (C1). In the three-sector model in the text, the sets of sectors, $\mathcal{J}$ and $\mathcal{J}^c$, correspond to $\{m\}$ and $\{a, s\}$ respectively.

D Welfare

To measure changes in welfare moving from the baseline to a counterfactual situation, we calculate a constant fraction λ_n of per capita consumption that would be paid to the country n's representative consumer in each year in the baseline to achieve the same utility in the counterfactual. Letting $\{C_{n,t}\}_t$ and $\{C_{n,t}^*\}_t$ be the consumption streams in country n in the baseline and in the counterfactual respectively, this fraction λ_n is given by

$$\sum_{t=0}^{\infty} \beta^t \zeta_{n,t} L_{n,t} \frac{(C_{n,t}^*/L_{n,t})^{1-\psi}}{1-\psi} = \sum_{t=0}^{\infty} \beta^t \zeta_{n,t} L_{n,t} \frac{\left(\left(1+\frac{\lambda_n}{100}\right) C_{n,t}/L_{n,t}\right)^{1-\psi}}{1-\psi},$$

$$\Leftrightarrow \lambda_n = 100 \times \left[\left\{ \frac{\sum_{t=0}^{\infty} \beta^t \zeta_{n,t} L_t (C_{n,t}^*/L_{n,t})^{1-\psi}}{\sum_{t=0}^{\infty} \beta^t \zeta_{n,t} L_t (C_{n,t}/L_{n,t})^{1-\psi}} \right\}^{\frac{1}{1-\psi}} - 1 \right].$$

In addition, our economy reaches the steady state at $t = T$. Letting the variables without the time subscript represent the steady state values, the formula can be rewritten as

$$\lambda_n = 100 \times \left[\left\{ \frac{\sum_{t=0}^{T} \beta^t \zeta_{n,t} L_{n,t} \left(\frac{C_{n,t}^*}{L_{n,t}}\right)^{1-\psi} + \frac{\beta^{T+1}}{1-\beta} \zeta_n L_n \left(\frac{C_n^*}{L_n}\right)^{1-\psi}}{\sum_{t=0}^{T} \beta^t \zeta_{n,t} L_{n,t} \left(\frac{C_{n,t}}{L_{n,t}}\right)^{1-\psi} + \frac{\beta^{T+1}}{1-\beta} \zeta_n L_n \left(\frac{C_n}{L_n}\right)^{1-\psi}} \right\}^{\frac{1}{1-\psi}} - 1 \right],$$

where $L_{n,t} = L_n$ and $\xi_{n,t} = \xi_n$ are time invariant after T and are common in both the baseline and the counterfactual scenarios. We also note

$$\sum_{t=T+1}^{\infty} \beta^t \zeta_n L_n \left(\frac{C_n^*}{L_n}\right)^{1-\psi} = \beta^{T+1} \zeta_n L_n \left(\frac{C_n^*}{L_n}\right)^{1-\psi} \sum_{t=0}^{\infty} \beta^t = \beta^{T+1} \zeta_n L_n \left(\frac{C_n^*}{L_n}\right)^{1-\psi} \frac{1}{1-\beta}.$$

E List of ISIC3 Industries Included in the Three Sectors

Table A4: Three sectors and corresponding ISIC3 codes

Sector	ISIC3	Description
Agriculture	A to B	Agriculture, Hunting, Forestry and Fishing
	C	Mining and Quarrying
	D15 to 16	Food, Beverages and Tobacco
Manufacturing	D17 to 19	Textiles, Textile, Leather and Footwear
	D21 to 22	Pulp, Paper, Printing and Publishing
	D23	Coke, Refined Petroleum and Nuclear Fuel
	D24	Chemicals and Chemical Products
	D25	Rubber and Plastics
	D26	Other Non-Metallic Mineral
	D27 to 28	Basic Metals and Fabricated Metal
	D29	Machinery, Nec
	D30 to 33	Electrical and Optical Equipment
	D34 to 35	Transport Equipment
	D n.e.c.	Manufacturing, Nec; Recycling
Service	E	Electricity, Gas and Water Supply
	F	Construction
	G	Wholesale and Retail Trade
	H	Hotels and Restaurants
	I60 to 63	Transport and Storage
	I64	Post and Telecommunications
	J	Financial Intermediation
	K	Real Estate, Renting and Business Activities
	L to Q	Community Social and Personal Services

F Calibration of Fundamentals

We calibrate the iceberg trade costs (including tariffs and non-tariff barriers), $b^j_{n,t}$, and average productivity, $A^j_{n,t}$, following Levchenko and Zhang (2016). Apart from Levchenko and Zhag (2016), we use information from price indices on WIOD to make sequences of productivity $A^j_{n,t}$ which are comparable across countries and over time within each sector. To begin with, we express the trade share normalized by its own trade share as follows:

$$\frac{\pi^j_{ni,t}}{\pi^j_{nn,t}} = \frac{\left(\frac{\widetilde{c}^j_{i,t} b^j_{ni,t}}{A^j_{i,t}}\right)^{-\theta^j}}{\left(\frac{\widetilde{c}^j_{n,t}}{A^j_{n,t}}\right)^{-\theta^j}} = \left(\widetilde{c}^j_{i,t}/A^j_{i,t}\right)^{-\theta^j} \times \left(\widetilde{c}^j_{n,t}/A^j_{n,t}\right)^{\theta^j} \times \left(b^j_{ni,t}\right)^{-\theta^j}.$$

Taking the log of both sides yields

$$\ln\left(\frac{\pi^j_{ni,t}}{\pi^j_{nn,t}}\right) = \ln\left(\widetilde{c}^j_{i,t}/A^j_{i,t}\right)^{-\theta^j} + \ln\left(\widetilde{c}^j_{n,t}/A^j_{n,t}\right)^{\theta^j} - \theta^j \ln\left(b^j_{ni,t}\right).$$

The total trade costs $b^j_{ni,t}$ can be decomposed into tariffs and non-tariff barriers:

$$\ln\left(\frac{\pi^j_{ni,t}}{\pi^j_{nn,t}}\right) = \ln\left(\widetilde{c}^j_{i,t}/A^j_{i,t}\right)^{-\theta^j} + \ln\left(\widetilde{c}^j_{n,t}/A^j_{n,t}\right)^{\theta^j} - \theta^j \ln\left(d^j_{ni,t}\right) - \theta^j \ln\left(\widetilde{\tau}^j_{ni,t}\right),$$

where $\widetilde{\tau}^j_{ni,t} = 1 + \tau^j_{ni,t}$. We express the log of non-tariff barriers $d^j_{ni,t}$ with the set of bilateral observables commonly used in the gravity estimation:

$$\ln\left(d^j_{ni,t}\right) = \text{dist}^j_{k(ni)} + \text{CB}^j_{ni,t} + \text{CU}^j_{ni,t} + \text{RTA}^j_{ni,t} + ex^j_{i,t} + \nu^j_{ni,t},$$

where $\text{dist}^j_{k(ni),t}$ is the contribution to trade costs of the distance between n and i being in a certain interval[1], $\text{CB}^j_{ni,t}$ is the indicator if the two countries n and i share the border, $\text{CU}^j_{ni,t}$ indicates if they are in a currency union, $\text{RTA}^j_{ni,t}$ indicates if they are in a regional trade agreement (WTO definition), ex^j_{it} is the exporter fixed effects, and $\nu^j_{ni,t}$ is the bilateral error term. Note that each component in the bilateral trade cost is indexed by t, and we estimate them as the fixed effects interacted with years. This implies that, for instance, the contribution of distance to trade costs can vary over time due to the technological progress of transportation. Exporter fixed effects are included to allow asymmetry in trade costs in the spirit of Waugh (2010). We plug this into the trade share equation (25) in the text and estimate the following using the Pseudo Poisson Maximum Likelihood (PPML) for each sector j while pooling all sampled countries and years:

$$\ln\left(\frac{\widetilde{\pi}^j_{ni,t}}{\pi^j_{nn,t}}\right) = \underbrace{\left(\ln\left(\widetilde{c}^j_{i,t}/A^j_{i,t}\right)^{-\theta^j} - \theta^j ex^j_{it}\right)}_{\text{exporter-year F.E.}} + \underbrace{\ln\left(\widetilde{c}^j_{n,t}/A^j_{n,t}\right)^{\theta^j}}_{\text{importer-year F.F.}}$$
$$\underbrace{-\theta^j\left(\text{dist}^j_{k(ni),t} + \text{CB}^j_{ni,t} + \text{CU}^j_{ni,t} + \text{RTA}^j_{ni,t}\right)}_{\text{bilateral observables}} - \theta^j \nu^j_{ni,t}.$$

where $\ln\left(\frac{\widetilde{\pi}^j_{ni,t}}{\pi^j_{nn,t}}\right) = \ln\left(\frac{\pi^j_{ni,t}}{\pi^j_{nn,t}}\right) + \theta^j \ln\left(\widetilde{\tau}^j_{ni,t}\right)$. Estimating the gravity equation above allows us to identify the technology-cum-unit-cost term, $\ln\left(\widetilde{c}^j_{n,t}/A^j_{i,t}\right)^{\theta^j}$, for each county and year as an importer-year fixed effect, relative to the reference country and year (U.S. in 1965), which we denote by $S^j_{n,t} = \left(\widetilde{c}^j_{n,t}/A^j_{n,t}\right)^{\theta^j} / \left(\widetilde{c}^j_{US,1965}/A^j_{US,1965}\right)^{\theta^j}$. We can then tease out the term $(-\theta^j ex^j_{it})$ from the exporter-year fixed effects. By combining all the terms in the bilateral trade costs, we can recover the asymmetric bilateral trade costs.

To back out productivity, we need a few preliminary steps. First, following Shikher (2012), we recover the sectoral price indices as follows. We define the own trade share relative to that of the reference country and year:

$$\frac{\pi^j_{nn,t}}{\pi^j_{US,US,1965}} = \frac{\left(\widetilde{c}^j_{n,t}/A^j_{n,t}\right)^{-\theta^j}}{\left(\widetilde{c}^j_{US,1965}/A^j_{US,1965}\right)^{-\theta^j}} \left(\frac{P^j_{n,t}}{P^j_{US,1965}}\right)^{\theta^j} = \frac{1}{S^j_{n,t}} \left(\frac{P^j_{n,t}}{P^j_{US,1965}}\right)^{\theta^j}.$$

[1]We follow Eaton and Kortum (2002) and intervals are defined, in miles, [0, 350], [350, 750], [750, 1500], [1500, 3000], [3000, 6000], [6000, max]

Hence, for given trade elasticity θ^j, we have[2]

$$\frac{P^j_{n,t}}{P^j_{US,1965}} = \left(\frac{\pi^j_{nn,t}}{\pi^j_{US,US,1965}} S^j_{n,t}\right)^{1/\theta^j}. \tag{F1}$$

It is important to note that, for each sector and year, $S^j_{n,t}$ is only identified up to normalization. This implies that the sequence of prices given by equation (F1) is only comparable across countries but not over time. To see this point clearly, consider a sequence of any positive scalar $\{a^j_t\}^j_t$. It is easy to show that the two sequences, $\{S^j_{n,t}\}$ and $\{a^j_t S^j_{n,t}\}$, generate the same trade share $\{\pi^j_{ni}\}$. Therefore, we need to rescaling the sequence of $S^j_{n,t}$ by identifying $\{a^j_t\}$ to measure the productivity growth over time.

To identify the shifters $\{a^j_t\}$, we take advantage of the gross output price index provided by the WIOD Socio Economic Accounts. Let $\{P^j_{US,t,Data}\}$ be the gross output price index of sector j in the U.S. and year t. Note that $\{P^j_{US,t,Data}\}_t$ are comparable over time within sector j. Since the sequence $\{a^j_t\}_t$ is defined for each sector, we will use the gross price index for the three sectors in the U.S. and back out $\{a^j_t\}$ according to

$$P^j_{US,t} = (a^j_t)^{1/\theta^j}\left(\sum_{n=1} S^{-1}_{n,t} b^{-\theta^j}_{US,n,t}\right)^{-1/\theta^j}$$
$$\Leftrightarrow a^j_t = \left(P^j_{US,t,Data}\right)^{\theta^j}\left(\sum_{n=1} S^{-1}_{n,t} b^{-\theta^j}_{US,n,t}\right)$$

Now redefine $S^j_{n,t}$ by $a^j_t S^j_{n,t}$. Such redefined $\{S^j_{n,t}\}_{n,t}$ are comparable over time and across countries within sector j.

Being armed with the sectoral price indices after rescaling the sequence of $\{S^j_{n,t}\}$, we next back out the exogenous demand shifters for intermediate inputs, $\kappa^{jh}_{n,t}$, by solving the system of equations for each j, n, and t:

$$g^{j,h}_{n,t} = \frac{\kappa^{j,h}_{n,t}(P^h_{n,t})^{1-\sigma^j}}{\sum_{h'=a,m,s}\kappa^{j,h'}_{n,t}(P^{h'}_{n,t})^{1-\sigma^j}}.$$

by restricting $\sum_{h'}\kappa^{j,h'}_{n,t} = 1$ for each j, n, and t. The left-hand side of the equation, $g^{j,h}_{n,t}$, is the share of expenditure spent on input from sector h in total input costs of j, which is directly observed in the IO table. After obtaining $\kappa^{j,h}_{n,t}$, we can recover the CES price index for the composite intermediate good $\xi^j_{n,t}$ defined in the text.

We analogously back out the exogenous demand shifter in the capital goods production function, $\kappa^{Kh}_{n,t}$, by solving the system of equations for each n and t:

$$g^{K,h}_{n,t} = \frac{\kappa^{K,h}_{n,t}(P^h_{n,t})^{1-\sigma^K}}{\sum_{h'=a,m,s}\kappa^{K,h'}_{n,t}(P^{h'}_{n,t})^{1-\sigma^K}}.$$

by restricting $\sum_{h'}\kappa^{K,h'}_{n,t} = 1$. This gives the price index of investment good $P^K_{n,t}$ defined in the text.

In order to obtain the factor prices, we construct the sequence of capital stock over time for each country. Starting from the initial capital stock in 1965 for each country provided by the PWT, we use the gross fixed capital formation from the WIOD and follow (19) to construct the nationwide capital stock. Since capital stock is measured as the real variable in the model, we need to obtain the initial period capital stock in the

[2] Note that the price indices are recovered relative to the U.S. in 1965 for each sector, implying that the U.S. price index is 1 for all sectors in 1965.

current U.S. Dollars[3] and then divide the nominal value by the price index of the investment good obtained in the previous step.[4] We then compute the real investment in each year by dividing the gross fixed capital formation (in current U.S. Dollars) by the investment good price index and accumulate the capital stock as implied by the model.

Using the value added from the WIOD, we apply the labor share from the PWT to obtain the wage bill and the return to capital. The wage bill and the total number of employment give the wage, $w_{n,t}$, and the return to capital and the capital stock give the rental price of capital, $r_{n,t}$.

Together with the composite intermediate input price index, $\xi^j_{n,t}$, and factor prices, $r_{n,t}, w_{n,t}$, we can compute the cost of the input bundle according to (23). Finally, we can recover the productivity $A^j_{n,t}$ by[5]

$$\frac{A^j_{n,t}}{A^j_{US,1965}} = (S^j_{n,t})^{1/\theta^j} \left(\frac{\widetilde{c}^j_{n,t}}{-\widetilde{c}^j_{US,1965}} \right).$$

Using the sectoral price indices computed above, we calibrated the sectoral demand shifter $\Omega^j_{n,t}$ as follows. First, we guess the vector of $\{\Omega^j_{n,t}\}$. Given the data on consumption expenditure $E_{n,t}$ from the WIOD, population $L_{n,t}$ from the PWT, sectoral prices $P^j_{n,t}$, and guessed values of $\Omega^j_{n,t}$, solve the consumption index $C^j_{n,t}$ according to (16). Using the computed consumption index, we can find the unique vector of $\Omega^j_{n,t}$ (up to normalization for each n and t) by applying the Perron-Frobenius theorem to (17). We then use the value of $\Omega^j_{n,t}$ as the new guess and repeat the steps until we find the fixed points.

The intertemporal demand shifter $\zeta_{n,t}$ is backed out sequentially according to (20). Using the consumption index $C_{n,t}$ obtained above, we can construct the series of $\zeta_{n,t}$ for each country by normalizing the one in the last sample year $\zeta_{n,2014}$ to be unity.

We also calibrated the sectoral demand shifter $\Omega^j_{n,t}$ and the intertemporal demand shifter $\zeta_{n,t}$ under the homothetic CES preference (i.e., $\epsilon^j = 1$ for all j). See Appendix J for such calibrated productivity as well as average tariff rates.

G Computation of Steady States

We compute steady states in the following way. As such, we drop the time subscript from the variables.

1. Guess wages across countries, $\{w_n\}_n \in \mathbb{R}^N$, normalized such that $w_{US} = 1$.
    (a) Compute r_n as follows.
        i. Guess rental rates across countries, $\{r_n\}_n \in \mathbb{R}^N$.
            A. Compute P^j_n as follows.
                - Guess sectoral price indices across countries, $\{P^j_n\}_{n,j} \in \mathbb{R}^{NJ}$.
                    – Compute ξ^j_n using (F1).
                    – Compute $\widetilde{c}^j_n$ using (F2).
                    – Compute P^j_n using (F3).
                - Check if P^j_n obtained in the last step is close to P^j_n initially guessed. If it does, stop. Otherwise, update $\{P^j_n\}_{n,j}$ and return to the first step.

[3]We use the capital stock at current PPP multiplied by the price level of capital stock to obtain the initial capital stock.

[4]The underlying assumption is that the capital stock in period t is priced at $P^K_{n,t}$.

[5]By construction, sectoral productivity takes 1 for the U.S. in 1965 in all sectors.

B. Compute P_n^K using (F4).

C. Compute r_n using (F5).

ii. Check if r_n obtained in the last step is close to r_n initially guessed. If it does, stop. Otherwise, update $\{r_n\}_n$ and return to step i.

(b) Compute π_{ni}^j using (F6).

(c) Compute K_n using (F7).

(d) Compute $g_n^{j,j'}$ using (F8).

(e) Compute $g_n^{K,j}$ using (F9).

(f) Compute X_n^j as follows.

- Guess sectoral spending across countries, $\{X_n^j\}_{n,j} \in \mathbb{R}^{NJ}$.
  - Compute $\widetilde{T}_n$ using (M1).
  - Compute T^P using (M2).
  - Compute national income NI_n using (H1).
  - Compute E_n using (H2).
  - Compute C_n applying the Newton method to (H3).
  - Compute ω_n^j using (H4).
  - Compute F_n^j using (H5).
  - Compute Y_n^j using (M3).
  - Compute X_n^j using (H6).
- Check if X_n^j obtained in the last step is close to X_n^j initially guessed. If it does, stop. Otherwise, update $\{X_n^j\}_{n,j}$ and return to the first step.

(g) Compute w_n using (M4).

2. Check if w_n obtained in the last step is close to w_n initially guessed. If it does, stop and normalize w_{US} to one. Otherwise, update $\{w_n\}_n$ and return to step 1.

Table A5: Equilibrium conditions at steady state

(F1)	$\xi_n^j = \left[\sum_{j'} \kappa_n^{j,j'} (P_n^{j'})^{1-\sigma^j}\right]^{\frac{1}{1-\sigma^j}}$	$\forall(n,j)$
(F2)	$\widetilde{c}_{n,t}^j = (r_n)^{\gamma_n^j \alpha_n} (w_n)^{\gamma_n^j (1-\alpha_n)} (\xi_n^j)^{1-\gamma_n^j}$	$\forall(n,j)$
(F3)	$P_n^j = \left[\sum_i^N \left(\frac{\widetilde{c}_i^j b_{n,i}^j}{A_i^j}\right)^{-\theta^j}\right]^{-\frac{1}{\theta^j}}$	$\forall(n,j)$
(F4)	$P_n^K = \frac{1}{\kappa_n^K}\left[\sum_j \kappa_n^{K,j} (P_n^j)^{1-\sigma^K}\right]^{\frac{1}{1-\sigma^K}}$	$\forall(n)$
(F5)	$r_n = \frac{1-\alpha_n(1-\lambda\delta_n)}{\alpha_n(1-\phi_n)\lambda} P_n^K$	$\forall(n)$
(F6)	$\pi_{ni}^j = \left(\frac{\widetilde{c}_i^j b_{ni}^j}{A_i^j P_n^j}\right)^{-\theta^j}$	$\forall(n,i,j)$
(F7)	$K_n = \frac{\alpha_n}{1-\alpha_n} \frac{w_n L_n}{r_n}$	$\forall(n)$
(F8)	$g_n^{j,j'} = \frac{\kappa_n^{j,j'} (P_n^{j'})^{1-\sigma^j}}{\sum_{j''} \kappa_n^{j,j''} (P_n^{j''})^{1-\sigma^j}}$	$\forall(n,j,j')$
(F9)	$g_n^{K,j} = \frac{\kappa_n^{K,j} (P_n^{K,j})^{1-\sigma^K}}{\sum_{j'} \kappa_n^{K,j'} (P_n^{K,j'})^{1-\sigma^K}}$	$\forall(n,j)$
(H1)	$NI_n = (1-\phi_n)(w_n L_n + r_n K_n + \widetilde{T}_n) + L_n T^P$	$\forall(n)$
(H2)	$E_n = NI_n - P_n^K \delta_n K_n$	$\forall(n)$
(H3)	$E_n = L_n \left[\sum_j \Omega_n^j \left\{\left(\frac{C_n}{L_n}\right)^{\epsilon^j} P_n^j\right\}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$	$\forall(n)$
(H4)	$\omega_n^j = \frac{\Omega_n^j \left\{\left(\frac{C_n}{L_n}\right)^{\epsilon^j} P_n^j\right\}^{1-\sigma}}{\sum_{j'} \Omega_n^{j'} \left\{\left(\frac{C_n}{L_n}\right)^{\epsilon^{j'}} P_n^{j'}\right\}^{1-\sigma}} \quad \left(= \frac{P_n^j C_n^j}{E_n}\right)$	$\forall(n,j)$
(H5)	$F_n^j = \omega_n^j E_n + g_n^{K,j} P_n^K \delta_n K_n$	$\forall(n,j)$
(H6)	$X_n^j = F_n^j + \sum_{j'} (1-\gamma_n^{j'}) g_n^{j',j} Y_n^{j'}$	$\forall(n,j)$
(M1)	$\widetilde{T}_n = \sum_j \sum_i^N \tau_{ni}^j X_{n,t}^j \frac{\pi_{ni}^j}{\widetilde{\tau}_{ni}^j}$	$\forall(n)$
(M2)	$T^P = \sum_i^N \phi_i (w_i L_i + r_i K_i + \widetilde{T}_i) / \sum_i^N L_i$	
(M3)	$Y_n^j = \sum_i^N X_i^j \frac{\pi_{in}^j}{\widetilde{\tau}_{in}^j}$	$\forall(n,j)$
(M4)	$w_n = (1-\alpha_n) \sum_j \gamma_n^j Y_n^j / L_n$	$\forall(n,j)$

Notes: $b_{ni}^j = d_{ni}^j \widetilde{\tau}_{ni}^j$ and $\widetilde{\tau}_{ni}^j = 1 + \tau_{ni}^j$. Roughly, (F*) is a condition for firms/production; (H*) for household; (M*) for market clearing.

H Computation of Transition Paths

We compute transition paths in the following way. Let $T+1$ be the terminal period, N the number of countries, and J the number of sectors.

1. Give pre-determined values (data in our case) to the initial capital stock, $\{K_{n,1}\}_n$, and arbitrary values to the other variables at the initial period 1.

2. Give the steady-state values to variables at the terminal period $T+1$ including $\{K_{n,T+1}, K_{n,T+2}\}_n$, $\{C_{n,T+1}\}_n$, $\{\bar{\epsilon}_{n,T+1}\}_n$, $\{E_{n,T+1}\}_n$, $\{r_{n,T+1}\}_n$, and $\{P^K_{n,T+1}\}_n$. Note that $K_{n,t}$ is a pre-determined variable at period t. Therefore, $K_{n,T+2}$ is determined at period $T+1$ given the steady-state value of $K_{n,T+1}$ at the same period.

3. Guess nominal investment rates across countries and time, $\{\rho_{n,t}\}_{n,t} \in \mathbb{R}^{N(T+1)}$.

4. Compute variables forward in time from $t=1$ including $\{w_{n,t}\}_{n,t} \in \mathbb{R}^{N(T+1)}$, $\{I_{n,t}\}_{n,t} \in \mathbb{R}^{N(T+1)}$, $\{K_{n,t}\}_{n,t} \in \mathbb{R}^{N(T+2)}$, $\{\bar{\epsilon}_{n,t}\}_{n,t} \in \mathbb{R}^{N(T+1)}$, and others in the sub-steps below.

   (a) Compute $w_{n,t}$ in each period t as follows, noting that in period t capital stock across countries, $\{K_{n,t}\}_n \in \mathbb{R}^N$ are predetermined.

      i. Guess wages across countries in period t, $\{w_{n,t}\}_n \in \mathbb{R}^N$, normalized such that $w_{US,t} = 1$.

         A. Compute $r_{n,t}$ using (F5).

         B. Compute $P^j_{n,t}$ as follows.
            - Guess sectoral price indices across countries in period t, $\{P^j_{n,t}\}_{n,j} \in \mathbb{R}^{NJ}$.
               - Compute $\xi^j_{n,t}$ using (F1).
               - Compute $\widetilde{c}^j_{n,t}$ using (F2).
               - Compute $P^j_{n,t}$ using (F3).
            - Check if $P^j_{n,t}$ obtained in the last step is close to $P^j_{n,t}$ initially guessed. If it does, stop. Otherwise, update $\{P^j_{n,t}\}_{n,j}$ and return to the first step.

         C. Compute $P^K_{n,t}$ using (F4).

         D. Compute $\pi^j_{ni,t}$ using (F6).

         E. Compute $g^{j,j'}_{n,t}$ using (F7).

         F. Compute $g^{K,j}_{n,t}$ using (F8).

         G. Compute $X^j_{n,t}$ as follows.
            - Guess sectoral spending across countries in period t, $\{X^j_{n,t}\}_{n,j} \in \mathbb{R}^{NJ}$.
               - Compute $\widetilde{T}_{n,t}$ using (M1).
               - Compute T^P using (M2).
               - Compute national income $NI_{n,t}$ using (H1).
               - Compute $E_{n,t}$ using (H2).
               - Compute $C_{n,t}$ applying the Newton method to (H3).
               - Compute $\omega^j_{n,t}$ using (H4).
               - Compute $Y^j_{n,t}$ using (M3).
               - Compute $X^j_{n,t}$ using (H5).
            - Check if $X^j_{n,t}$ obtained in the last step is close to $X^j_{n,t}$ initially guessed. If it does, stop. Otherwise, update $\{X^j_{n,t}\}_{n,j}$ and return to the first step.

H. Compute $w_{n,t}$ using (M4).

ii. Check if $w_{n,t}$ obtained in the last step is close to $w_{n,t}$ initially guessed. If it does, stop and normalize $w_{US,t}$ to one. Otherwise, update $\{w_{n,t}\}_n$ and return to step i.

(b) Compute $I_{n,t} = \rho_{n,t} NI_{n,t}/P^K_{n,t}$.

(c) Compute $K_{n,t+1} = (1 - \delta_{n,t})K_{n,t} + I^{\lambda}_{n,t}(\delta_{n,t}K_{n,t})^{1-\lambda}$.

(d) Compute $\bar{\epsilon}_{n,t} = \sum_j \omega^j_{n,t}\epsilon^j$.

5. Update $\rho_{n,t}$ backward in time from period $t = T$ as $\rho_{n,t}(1 + \eta Z_{n,t})$ using (H6) "Euler equation residual"$Z_{n,t}$ with a dampening parameter η and associated functions (H7) and (H8). Note that we restrict the updated $\rho_{n,t}$ to be in $(0, 1)$.

6. Check if $\rho_{n,t}$ obtained in step 5 is close to $\rho_{n,t}$ initially guessed. If it does, stop. Otherwise, update $\{\rho_{n,t}\}_{n,t}$ and return to step 3.

Table A6: Equilibrium conditions

(F1)	$\xi^j_{n,t} = \left[\sum_{j'} \kappa^{j,j'}_{n,t} (P^{j'}_{n,t})^{1-\sigma^j}\right]^{\frac{1}{1-\sigma^j}}$	$\forall(n,j,t)$
(F2)	$\widetilde{c}^j_{n,t} = (r_{n,t})^{\gamma^j_{n,t}\alpha_{n,t}} (w_{n,t})^{\gamma^j_{n,t}(1-\alpha_{n,t})} (\xi^j_{n,t})^{1-\gamma^j_{n,t}}$	$\forall(n,j,t)$
(F3)	$P^j_{n,t} = \left[\sum_i^N \left(\frac{\widetilde{c}^j_{i,t} b^j_{ni,t}}{A^j_{i,t}}\right)^{-\theta^j}\right]^{-\frac{1}{\theta^j}}$	$\forall(n,j,t)$
(F4)	$P^K_{n,t} = \frac{1}{\kappa^K_{n,t}} \left[\sum_j \kappa^{K,j}_{n,t} (P^j_{n,t})^{1-\sigma^K}\right]^{\frac{1}{1-\sigma^K}}$	$\forall(n,t)$
(F5)	$r_{n,t} = \frac{\alpha_{n,t}}{1-\alpha_{n,t}} \frac{w_{n,t}L_{n,t}}{K_{n,t}}$	$\forall(n,t)$
(F6)	$\pi^j_{ni,t} = \left(\frac{\widetilde{c}^j_{i,t} b^j_{ni,t}}{A^j_{i,t} P^j_{n,t}}\right)^{-\theta^j}$	$\forall(n,i,j,t)$
(F7)	$g^{j,j'}_{n,t} = \frac{\kappa^{j,j'}_{n,t}(P^{j'}_{n,t})^{1-\sigma^j}}{\sum_{j''}\kappa^{j,j''}_{n}(P^{j''}_{n,t})^{1-\sigma^j}}$	$\forall(n,j,j',t)$
(F8)	$g^{K,j}_{n,t} = \frac{\kappa^{K,j}_{n,t}(P^{K,j}_{n,t})^{1-\sigma^K}}{\sum_{j'}\kappa^{K,j'}_{n,t}(P^{K,j'}_{n,t})^{1-\sigma^K}}$	$\forall(n,j,t)$
(H1)	$NI_{n,t} = (1-\phi_{n,t})(w_{n,t}L_{n,t} + r_{n,t}K_{n,t} + \widetilde{T}_{n,t}) + L_{n,t}T^P_t$	$\forall(n,t)$
(H2)	$E_{n,t} = (1-\rho_{n,t})NI_{n,t}$	$\forall(n,t)$
(H3)	$E_{n,t} = L_{n,t}\left[\sum_j \Omega^j_{n,t}\left\{\left(\frac{C_{n,t}}{L_{n,t}}\right)^{\epsilon^j} P^j_{n,t}\right\}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$	$\forall(n,t)$
(H4)	$\omega^j_{n,t} = \frac{\Omega^j_{n,t}\left\{\left(\frac{C_{n,t}}{L_{n,t}}\right)^{\epsilon^j} P^j_{n,t}\right\}^{1-\sigma}}{\sum_{j'}\Omega^{j'}_{n,t}\left\{\left(\frac{C_{n,t}}{L_{n,t}}\right)^{\epsilon^{j'}} P^{j'}_{n,t}\right\}^{1-\sigma}} \quad \left(= \frac{P^j_{n,t}C^j_{n,t}}{E_{n,t}}\right)$	$\forall(n,j,t)$
(H5)	$X^j_{n,t} = [\omega^j_{n,t}(1-\rho_{n,t}) + g^{K,j}_{n,t}\rho_{n,t}]NI_{n,t} + \sum_{j'}(1-\gamma^{j'}_{n,t})g^{j',j}_{n,t}\sum_i^N \pi^j_{in,t}X^j_{i,t}/\widetilde{\tau}^j_{in,t}$	$\forall(n,j,t)$
(H6)	$Z_{n,t} = \left[\beta\frac{\zeta_{n,t+1}}{\zeta_{n,t}}\frac{L_{n,t+1}}{L_{n,t}}\frac{E_{n,t}}{E_{n,t+1}}\frac{\bar{\epsilon}_{n,t}}{\bar{\epsilon}_{n,t+1}}\frac{\left[(1-\phi_{n,t+1})r_{n,t+1} - P^K_{n,t+1}\Phi_2(K_{n,t+2},K_{n,t+1})\right]}{P^K_{n,t}\Phi_1(K_{n,t+1},K_{n,t})}\right]^{\frac{1}{\psi-1}} - \frac{C_{n,t+1}}{C_{n,t}}\frac{L_{n,t}}{L_{n,t+1}}$	$\forall(n,t)$
(H7)	$\Phi_1(K_{n,t+2},K_{n,t+1}) = \frac{\delta_{n,t+1}^{1-\frac{1}{\lambda}}}{\lambda}\left(\frac{K_{n,t+2}}{K_{n,t+1}} - (1-\delta_{n,t+1})\right)^{\frac{1-\lambda}{\lambda}}$	$\forall(n,t)$
(H8)	$\Phi_2(K_{n,t+2},K_{n,t+1}) = \Phi_1(K_{n,t+2},K_{n,t+1})\left[(\lambda-1)\frac{K_{n,t+2}}{K_{n,t+1}} - \lambda(1-\delta_{n,t+1})\right]$	$\forall(n,t)$
(M1)	$\widetilde{T}_{n,t} = \sum_j\sum_i^N \tau^j_{ni,t}X^j_{n,t}\frac{\pi^j_{ni,t}}{\widetilde{\tau}^j_{ni,t}}$	$\forall(n,t)$
(M2)	$T^P_t = \sum_i^N \phi_{i,t}(w_{i,t}L_{i,t} + r_{i,t}K_{i,t} + \widetilde{T}_{i,t})\big/\sum_i^N L_{i,t}$	$\forall(t)$
(M3)	$Y^j_{n,t} = \sum_i^N X^j_{i,t}\frac{\pi^j_{in,t}}{\widetilde{\tau}^j_{in,t}}$	$\forall(n,j,t)$
(M4)	$w_{n,t} = (1-\alpha_{n,t})\sum_j \gamma^j_{n,t}Y^j_{n,t}/L_{n,t}$	$\forall(n,j,t)$

Notes: $b^j_{ni,t} = d^j_{ni,t}\widetilde{\tau}^j_{ni,t}$ and $\widetilde{\tau}^j_{ni,t} = 1 + \tau^j_{ni,t}$. Roughly, (F*) is a condition for firms/production; (H*) for household; (M*) for market clearing. An alternative expression of (H5) is
$X^j_{n,t} = \omega^j_{n,t}E_{n,t} + g^{K,j}_{n,t}P^K_{n,t}I_{n,t} + \sum_{j'}(1-\gamma^{j'}_{n,t})g^{j',j}_{n,t}\sum_i^N \pi^j_{in,t}X^j_{i,t}/\widetilde{\tau}^j_{in,t}$.

I Applying the Newton Method

In the system of steady state and equilibrium conditions, most of economic variables are *explicitly* expressed. Therefore they are directly computable given parameters and other economic variables. Only one variable that is only *implicitly* expressed is aggregate consumption $C_{n,t}$ (in steady state, C_n). See equation (H3) in Table A6 (in steady state, Table A5). In the following, we describe the computational method for transition paths, but a similar argument applies for steady states). Given parameters σ, ϵ^j, $\Omega^j_{n,t}$, $L_{n,t}$, and economic variables $P^j_{n,t}$, we need to solve this equation for $C_{n,t}$. We apply the Newton–Raphson method to numerically solve this equation.

Observe that equation

$$\left(\frac{E_{n,t}}{L_{n,t}}\right)^{1-\sigma} = \sum_{j=a,m,s} \Omega^j_{n,t} \left\{ \left(\frac{C_{n,t}}{L_{n,t}}\right)^{\epsilon^j} P^j_{n,t} \right\}^{1-\sigma}.$$

is equivalent to (H3). Based on this equation, define real-valued function Δ by

$$\Delta(x_{n,t}) = \sum_{j=a,m,s} \Omega^j_{n,t} \left\{ \left(\frac{x_{n,t}}{L_{n,t}}\right)^{\epsilon^j} P^j_{n,t} \right\}^{1-\sigma} - \left(\frac{E_{n,t}}{L_{n,t}}\right)^{1-\sigma}.$$

$C_{n,t}$ is such that $\Delta(C_{n,t}) = 0$. The derivative of Δ is

$$\Delta'(x_{n,t}) = (1-\sigma) \sum_{j=a,m,s} \Omega^j_{n,t} (L_{n,t})^{-\epsilon^j(1-\sigma)} (P^j_n)^{1-\sigma} \epsilon^j x_n^{\epsilon^j(1-\sigma)-1}.$$

Using these expressions, we compute $C_{n,t}$ in the following iterative way.

Make an initial guess $x^0_{n,t} > 0$. The superscript of x keeps track of the number of updates in iteration. Then compute the updated value as

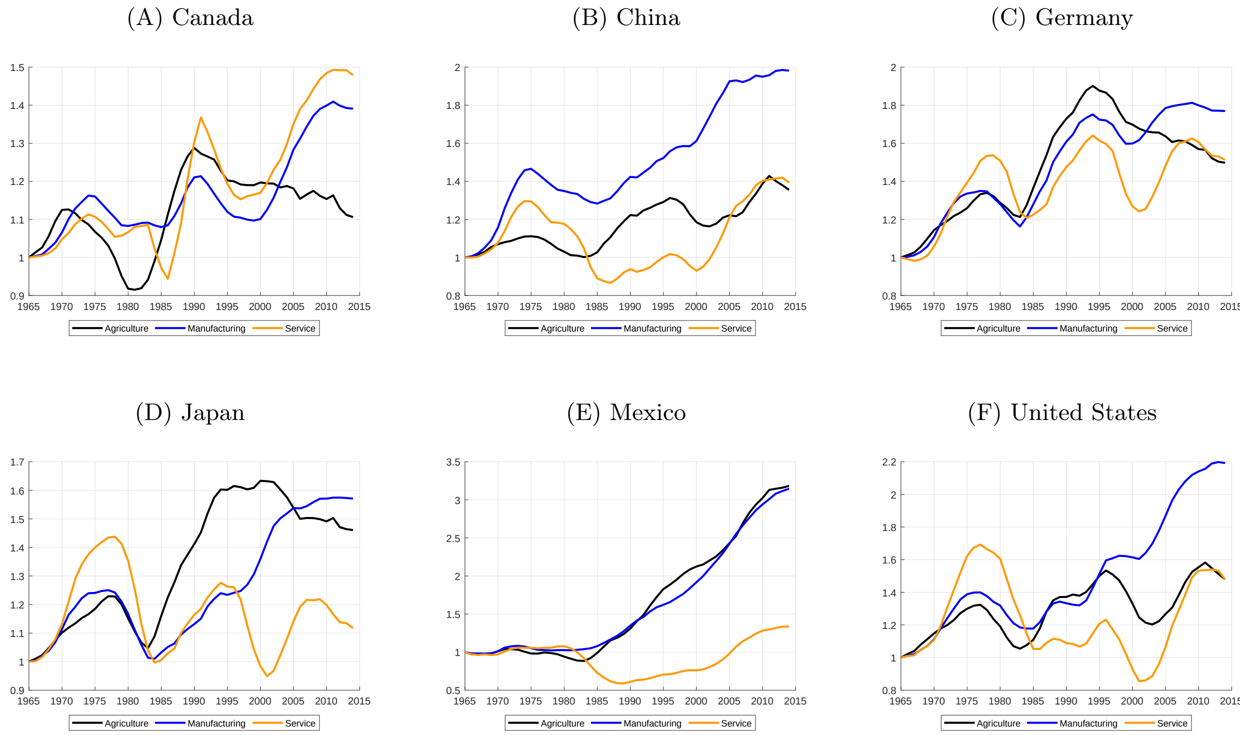
$$x^1_{n,t} = x^0_{n,t} - \frac{\Delta(x^0_{n,t})}{\Delta'(x^0_{n,t})}.$$

If $x^1_{n,t}$ is close enough to $x^0_{n,t}$, we got the solution. Otherwise, use $x^1_{n,t}$ as a new guess, and compute $x^2_{n,t}$ and compare these two. Repeat this process until $x^k_{n,t}$ and $x^{k+1}_{n,t}$ for some k.

J Baseline Parameter Values

We summarize the baseline fundamentals we calibrated in Section F. Figure A1 shows the evolution of sectoral productivity in six countries, Canada, China, Germany, Japan, Mexico, and the U.S. We normalize the productivity in 1965 to be 1 and take the moving average over 5 years to remove the noise. In every country other than Canada, after the 1980s, the productivity of the manufacturing sector grows more than that of the service sector. In the U.S., the manufacturing productivity increased by a factor of 2.2 while the service sector productivity increased by a factor of 1.5. Canada exhibits the opposite pattern, i.e., the productivity of the service sector grows faster than that of the manufacturing sector.

Figure A1: Productivity evolution (1965=1)

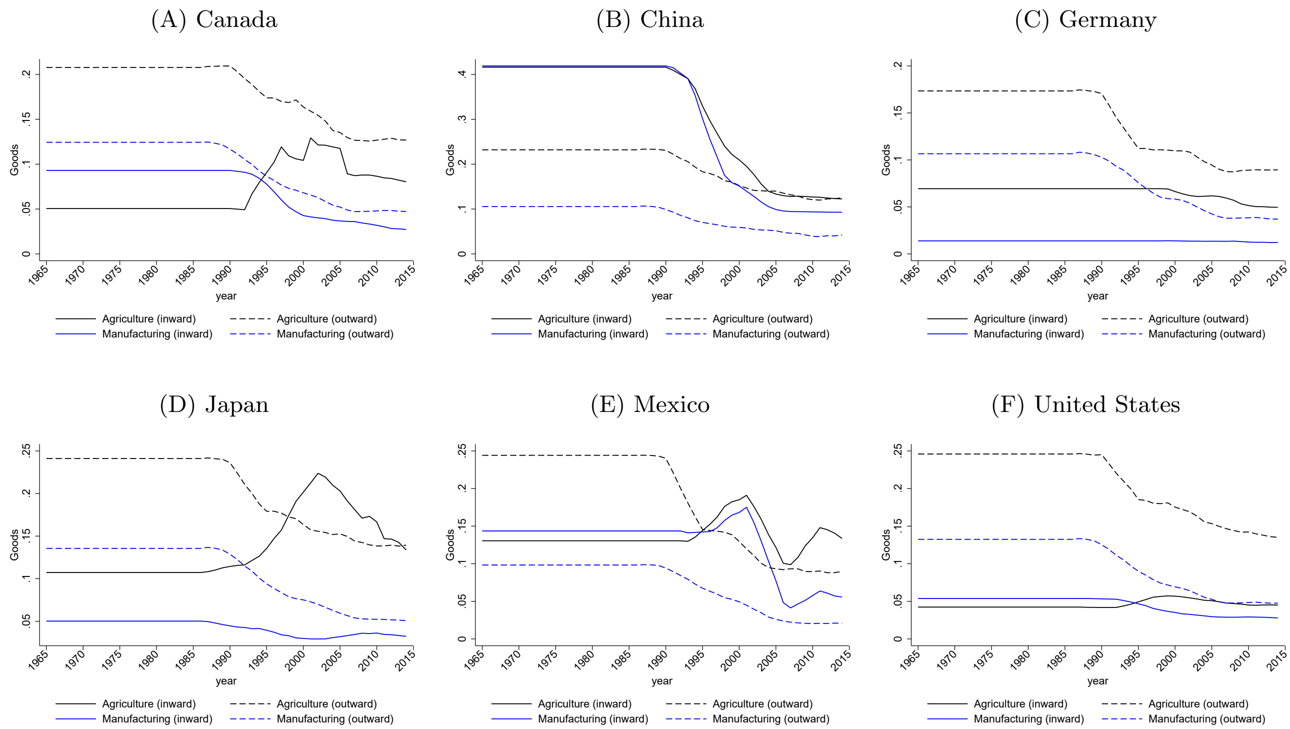


Notes: This table shows the calibrated values of sectoral productivity, $A^j_{n,t}$. We take a 5-year moving average and normalize $A^j_{n,1965} = 1$.

Figure A2 shows the evolution of trade costs in the six countries. For each country and year, we compute the simple arithmetic average of the bilateral tariff rate (inward and outward) with all its trading partners. The source of the tariff data is the WITS, which provides the bilateral tariff after the late 1980s for the major countries. For the period prior to the year when the tariffs are reported for its first time, we apply the tariff rates in the first year. The figure confirms that the tarriffs are continuously falling after 1990s for manufacturing sector in most of the countries. It is also worth noting that China's export tariffs drop more significantly in the late 1990s than in the 2000s when China joined the WTO.

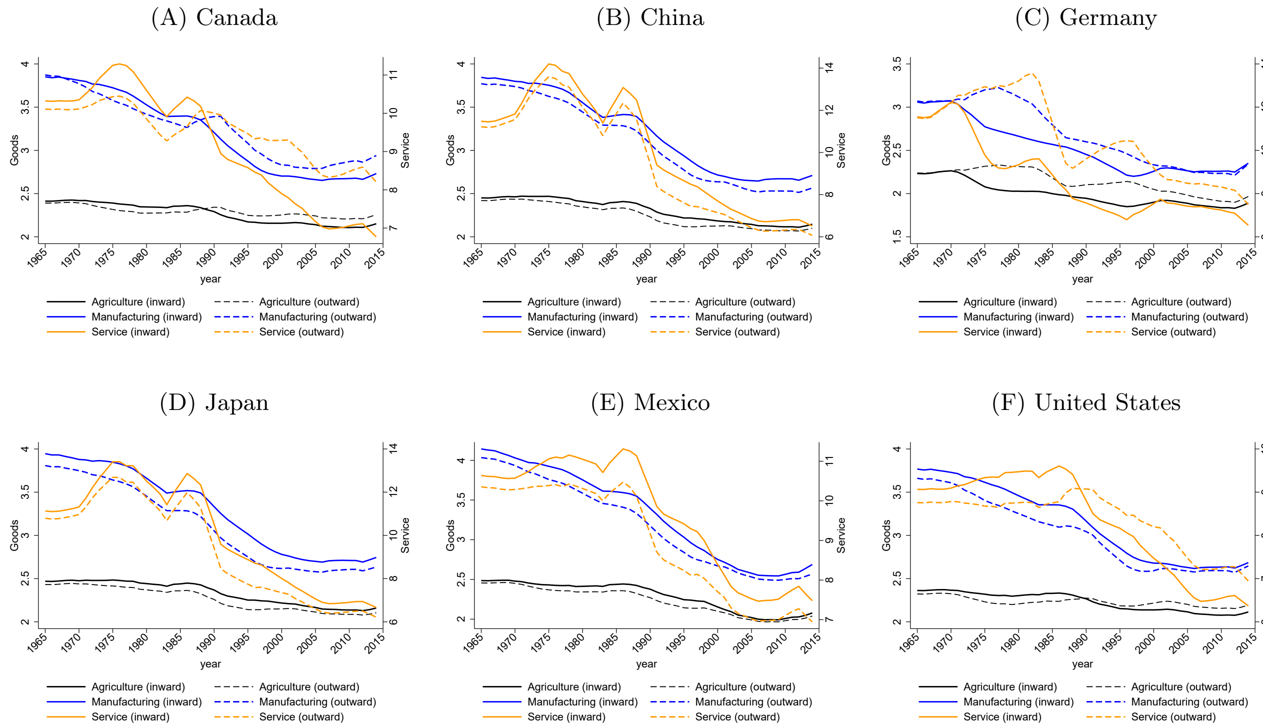
Figure A3 exhibits the evolution of non-tariff barriers in the six countries, measuerd as the simple average of inward and outward iceberg trade costs. First, we see the fall in non-tariff barriers is more significant in magnitude compared to the tariff barriers. For instance, in the U.S., the non-tariff barriers in the U.S. dropped from 400% to 300% over the five decades while the tariff barriers droppped from 5% to 3%. Second, the non-tariff barriers for the service sector are much higher in level than the good sectors, but exhibits a significant drop over time. While the service trade is often overlooked in the quantitative trade analysis, the result suggests that the falling service trade cost is a crucial factor in understanding the sectoral reallocation in the global context.

Figure A2: Evolution of average tariff



Notes: This table shows the average sectoral tariffs a country imposes on the other countries (`inward`, solid lines) and those imposed on the country by the others (`outward`, dashed lines).

Figure A3: Evolution of average non-tariff barrier



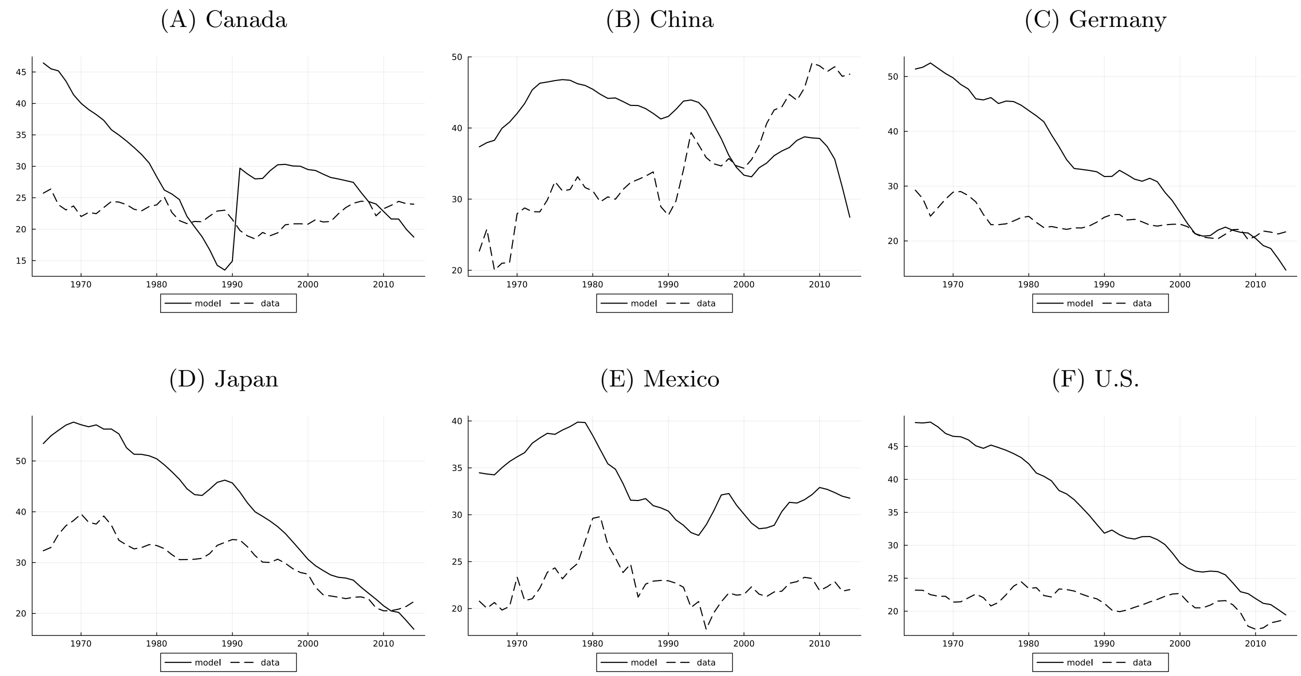
Notes: This table shows the calibrated values of the average sectoral non-tariff barriers a country imposes on the other countries (`inward`, solid lines) and those imposed on the country by the others (`outward`, dashed lines).

K Model Fit and Baseline Results

Figure A4 compares the saving rates in the baseline equilibrium to the data. In all six countries, the model predicts a higher saving rate than the data counterpart in earlier years. The model-implied saving rate gradually falls and converges to levels close to the data.

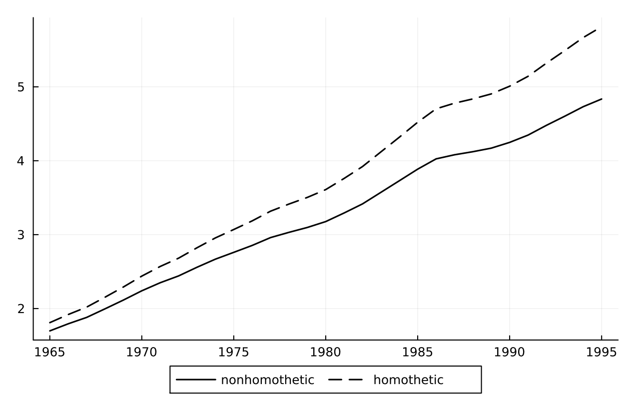
Figure A5 compares the U.S. real per capita consumption in the baseline equilibrium under nonhomothetic CES preferences (solid line) and homothetic CES preferences (dashed line). The real consumption is higher in magnitude under homothetic preferences than under nonhomothetic preferences.

Figure A4: Model fit: Saving rate



Notes: This table shows the saving rate, $\rho_{n,t}$ in (32), in the baseline equilibrium (solid lines) and in the data (dashed lines).

Figure A5: U.S. real per capita consumption



Notes: This figure shows U.S. real per capita consumption in the baseline equilibrium under nonhomothetic CES preferences (solid lines) and homothetic CES preferences (dashed lines).

L More on Counterfactual Results

L.1 Temporary Tariff Shock

In the text, we have considered the tariff, which is in effect indefinitely. To examine the static and dynamic implications of tariff shock separately, we consider here the U.S. manufacturing tariff with the same magnitude (20 percentage points), but lasting only four years from 2001 to 2004. We assume that every agent in the model realizes the trade policy in 2001 by surprise, but anticipates that it will be lifted in four years. The results are summarized in Figures A6 and A7.

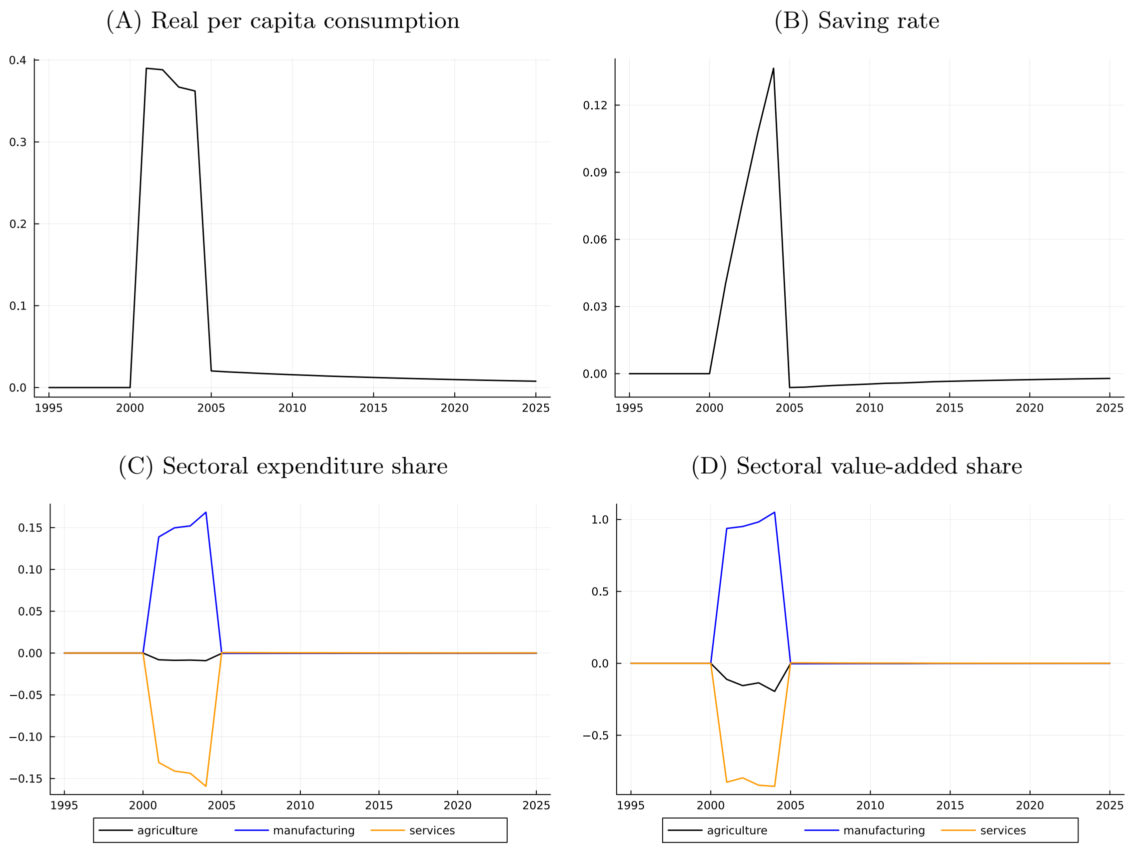
The real per capita consumption is approximately 0.4 percent higher in the counterfactual than in the baseline for the four years. Even after the high tariff period, the consumption in the counterfactual is slightly higher than the baseline. The striking difference from the permanent tariff is the implication on savings. Panel (B) of Figure A6 shows that, during the high tariff period, the saving rate is higher than in the baseline. After the tariff is lifted, the saving rate becomes slightly lower than the baseline rate, and it eventually converges to the baseline level. Since the U.S. households anticipate that the high tariffs will be lifted in four years, they do not fully spend the increased tariff revenue. After four years, the U.S. households dissave the piled-up temporal income.[6]

Panels (C) and (D) of Figure A6 report the impacts on sectoral expenditure shares and value-added shares. As in the permanent shock, the manufacturing shares rise in response to the tariff shock. The magnitude of the impacts is close, too. The key difference from the permanent shock is that it has no long-run implications on sectoral allocation after the high tariff period, i.e., the expenditure and value-added shares return to the baseline level immediately after the tariff is lifted.

The dynamic welfare implication for the U.S. is positive but smaller in magnitude compared to the permanent shock (0.06 percent). This is simply because the time period during which the U.S. receives higher tariff revenue is shorter in the case of the temporary tariff shock. Canada, India, and Mexico remain the three worst-off countries, but under the temporary shock, welfare loss is largest for Mexico (0.27 percent), followed by India (0.23 percent) and Canada (0.19 percent).

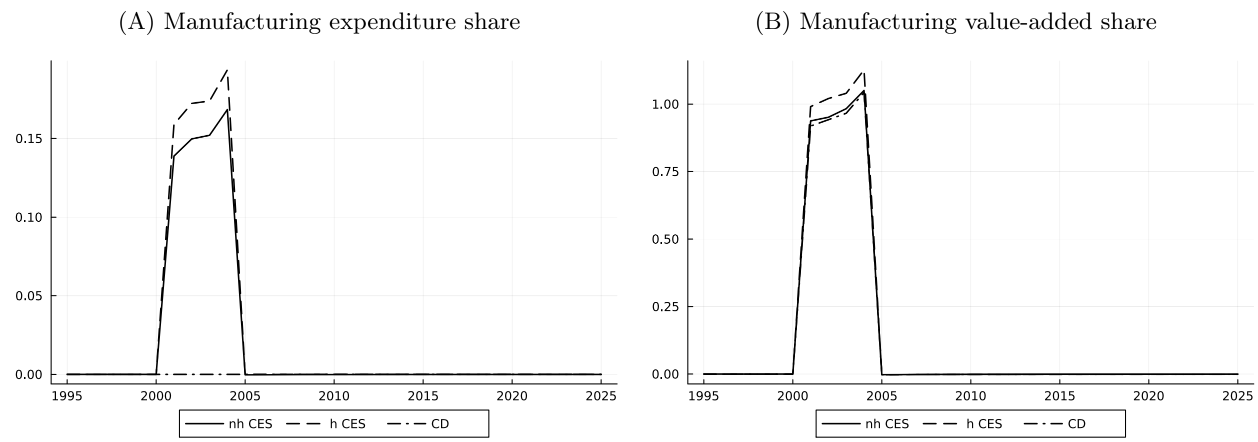
[6] Consumption front-load motive makes the saving rate rise during the high tariff period.

Figure A6: Impacts of a temporary increase in U.S. manufacturing tariffs



Notes: Each panel shows the impacts of a counterfactual 20-percentage-point increase in the U.S. manufacturing tariff in 2001 to 2004 on the transition paths in the U.S. For real per capita consumption, the vertical axis represents the percent change from the baseline to the counterfactual equilibrium. For the other variables, the vertical axis measures the percentage point change from the baseline.

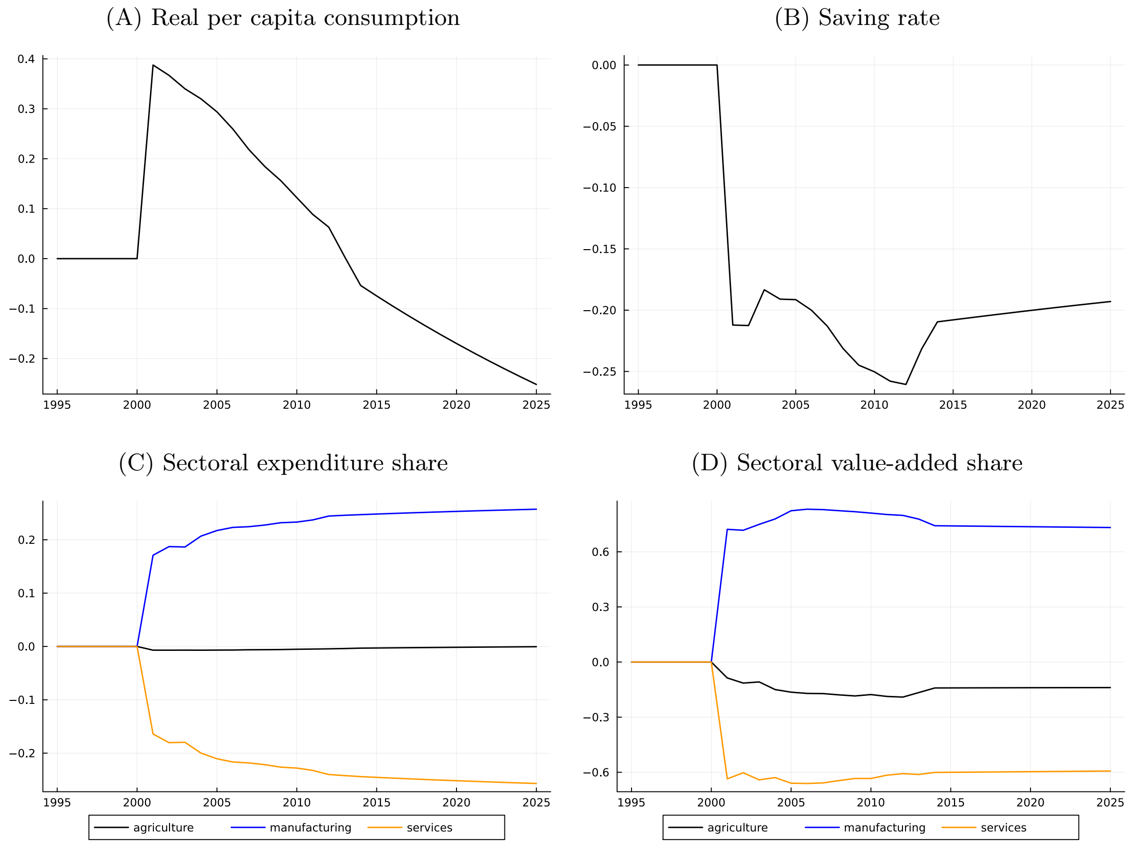
Figure A7: Impacts of a temporary U.S. tariff increase across different preferences



Notes: Each panel shows the impacts of a counterfactual 20-percentage-point increase in the U.S. manufacturing tariff in 2001 to 2004 on the transition paths in the U.S. under the three different preferences, nonhomothetic CES (`nh CES`, solid lines), homothetic CES (`h CES`, dashed lines), and Cobb-Douglas (`CD`, dash-dotted lines). The vertical axes represent the percentage point change from the baseline.

L.2 Trade War

Figure A8: Impacts of a trade war between the U.S. and the rest of the World



Notes: Each panel shows the impacts of a counterfactual trade war between the U.S. and the other countries since 2001 on the transition paths in the U.S. For real per capita consumption, the vertical axis represents the percent change from the baseline to the counterfactual equilibrium. For the other variables, the vertical axis measures the percentage point change from the baseline.

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