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Abstract

The nature of optimal commodity taxes and tax reform are not well understood in the literature under the separability of subset of consumption goods. We show in this paper that, contrary to the claims of previous researches, under the combination of non-linear income tax and linear commodity taxes, a tax reform towards uniform taxes of the subset of commodities may not be welfare-improving. Furthermore, a distortion of production that alters the relative wages has additional benefit through mimickers' consumption choices. We also newly characterize the optimal commodity taxes in this setting to show that, to quench possible heterogeneous impacts to the cross-substitution effects on the other commodities, optimally uniform tax towards the separated goods requires uniform income effects across income classes, so that the constraint on the Engel curves in the corresponding optimally linear income and commodity taxes cannot be fully omitted.

Keywords: Uniform commodity taxes, Tax reform, Income tax

JEL Classification: H21, H24

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1. Introduction

We point out in this paper that the consequences of optimal commodity taxes and tax reform are not well understood in the literature under the separability of subset of consumption goods. Specifically, we show that Deaton (1979) type of optimal uniform taxes *over the subset of commodities* and *their tax-reform extensions* in Kaplow (2006)-Laroque (2005) can*not* hold under the nonlinear income taxes unless the preferences have some restrictions.

Consider a Mirrees' (1976) multi-commodity economy when agents' utility is $u(x, l, n)$ with respect to a vector of commodities x and labor l and ability n (the last argument n is from Mirrlees (1976) and it is optional). Laroque (2005), for example, said (without a proof) as follows (p.143, his Remark 3): for $x = \left(x^a, x^b\right)$, if the utility function is written in the form $u\left(h^a(x^a), h^b(x^b), l, n\right)$, where the sub-utilities $h^a(\cdot)$ and $h^b(\cdot)$ are identical across agents, then a Pareto-improvement is possible where the relative consumption and production prices within groups coincide, but not necessarily between groups. However, a previous result of Mirrlees (1976, p. 338), which Laroque (2005) quoted for the ground of his claim, holds only under the fully non-linear taxation of incomes and commodities. Doligalski et al. (2025) made a similar claim that Pareto improvement is possible by allowing agents to decentralize the purchase of the separated group of commodities through linear and undistorted prices. In this paper, we prove that this claim is not correct in general, either. Our argument is consistent with Bastani et al. (2014).

2. Previewing Results

Allowing the form of separability $u\big(h^a(x^a), x^b, l, n\big)$ (which subsumes Laroque's $u\big(h^a(x^a), h^b(x^b), l, n\big)$, we show two results.

1. First, when groupwise uniform commodity taxes, either linearly or nonlinearly, hold at the optimum, can we conduct a tax reform towards groupwise uniformity? We show that the answer is no: if one tries to change the commodity taxes and post-tax incomes without changing labor supplies (as in Kaplow-Laroque), groupwise unform-tax reforms that aim Pareto improvement may not succeed due to the violation of incentive constraints. As a Corollary, Naito (1999)-type production distortion additionally finds its social benefit by striking the wage-oriented preference changes. This way of highlighting a *violation* of production efficiency contrasts with the emphasis of production efficiency by Doligalski et al.'s (2025).

2. We further confirm additional conditions required for uniform commodity taxes on the subset of commodities. Adopting the linear commodity tax under the non-linear income taxation, Deaton (1979)[1] type of uniform taxes in the subset of commodities requires that the uniformly taxed goods must have the parallel income effects. To state specifically, for example, in the Laroque-case of $u\big(h^a(x^a), h^b(x^b), l, n\big)$ which satisfies Doligalski et al.'s (2025) incentive separability, the optimal uniform taxes within *a*-group and *b*-group require that the sub-utility functions must satisfy the parallel income effects within each group of commodities. Intuitively, the obtained first-order

[1] Besley-Jewitt (1992) enlarges a sufficient condition for fully uniform commodity taxes. Their condition does not apply to uniform taxes on subset of commodities.

condition needs uniformity of income effects of each good across income classes, so Deaton's (1979) additional condition akin to linear Engel curve cannot be omitted even under non-linear income tax as long as consumption taxes are linear.

Our argument is consistent with Bastani et al.'s (2014). Suppose that the utility $u(h^a(x^a), x^b, l, n)$ has x^b which are labor substitutes (child care and elderly care). Bastani et al. (2014) showed that a uniform tax on x^a is not derived at the second-best optimum. We go further to confirm the additional conditions required for optimally partial-uniform commodity taxes.

3. Tax Reform for Partial Uniformity Is Not Welfare-improving

Suppose that the goods, indexed by 0,1,...I, are divided by group a with a set A={1,⋯,I^a} and group b for the other goods, B={k^a+1,⋯,I,0}, forming consumption vectors x^a and x^b. Good 0 is normalized to be untaxed. The commodities are produced by labor as the input with a linear technology whose marginal cost is normalized to 1. The case of concave production process with endogenous wages is discussed at the end of this section.

The government levies a commodity tax rate t_j (a subsidy when it is negative) per unit of good j. The utility function takes a separable form with respect to x^a, $u(h(x^a), x^b, l, n)$. The vector of after-tax commodity prices, $1 + t_j = q_j$ for good j, is written as q^a and q^b for each group.

We begin our main analysis with Kaplow (2006)-Laroque (2005) tax reform, since the analysis is easier and the intuition is more straightforward than the optimal taxation. Take Laroque's (2005) utility function $u(h^a(x^a), h^b(x^b), l, n)$ for illustration. When groupwise uniform commodity taxes, either linearly or nonlinearly, hold at the optimum, can we conduct a tax reform towards groupwise uniformity?

Since the matter is invariant with a (general) non-linear tax reform, and the optimal non-linear schedule for pairwise shadow-price for each set of commodities exists, we discuss this case first. Suppose that the policy-maker sets a reform schedule $({x_n}^{*a}, {x_n}^{*b})$ without changing labor supplies (as in Kaplow-Laroque-Doligalski et al. (2025)). $({x_n}^{*a}, {x_n}^{*b})$ is constructed from the expenditure function $e(q^*, l_n, v_n, n) \equiv arg\, min\{ q^* \cdot x | u(h^a(x^a), h^b(x^b), l_n, n) \geq v_n\}$ which linearly approximate the reform schedule with the targeted (shadow) price q^*. For a linear reform, the disposable income $y_n^o \equiv q^* \cdot (x^a, x^b) = ax^a + x^b$ for a scaler $a = 1 + t_i > 0$ in group a (recall that the non-distortionary price is 1, and group b includes good 0 with $t_0 = 0$) is assigned to induce type n to select the Hicksian demand under q^*. Denoting the status quo allocation as $({x_n}^{sa}, {x_n}^{sb})$ that is incentive compatible, for tax implementability, the reform schedule has to satisfy:

$$u(h^a({x_n}^{sa}), h^b({x_n}^{sb}), l_n, n) = u(h^a({x_n}^{*a}), h^b({x_n}^{*b}), l_n, n) \ \ and$$

(1) $$u(h^a({x_m}^{*a}), h^b({x_m}^{*b}), l_m, m) \geq u(h^a({x_n}^{*a}), h^b({x_n}^{*b}), nl_n/m, m).$$

For type m, the reform satisfies $u(h^a({x_m}^{*a}), h^b({x_m}^{*b}), l_m, m) = u(h^a({x_m}^{sa}), h^b({x_m}^{sb}), l_m, n)$. Under weak separability $u = u\left(g\left(h^a(x^a), h^b(x^b)\right), l, n\right)$, type $m \neq n$, as a potential mimicker of type n, from the Hicksian expenditure y_n^o, chooses the same consumption as (true) type n, so we have $u(h^a({x_n}^{*a}), h^b({x_n}^{*b}), nl_n/m, m) = u(h^a({x_n}^{sa}), h^b({x_n}^{sb}), nl_n/m, m)$. As a result, (1) for type m holds since the status-quo allocation is incentive-compatible (Kaplow (2006, Lemma 1)).

However, denoting the utility by $u = u(h^a, h^b, l, n)$, the matter is very different. The preference between $\left(h^a({x_n}^{sa}), h^b({x_n}^{sb})\right)$ and $\left(h^a({x_n}^{*a}), h^b({x_n}^{*b})\right)$ is dependent on l and n. If group a as a whole is more complementary to leisure than group b, then type $m > n$ will prefer the bundle that gives higher h^a to the other. Denote the partial derivatives with the subscript, e.g., $u_a = \frac{\partial u}{\partial h^a}$. Given that type n is indifferent between the status-quo consumption $\left(h^a({x_n}^{sa}), h^b({x_n}^{sb})\right)$ and the pairwise-uniform price allocation $\left(h^a({x_n}^{*a}), h^b({x_n}^{*b})\right)$, the sign of

$$\frac{\partial ln(u_a/u_b)}{\partial d} = \frac{u_{ad}}{u_a} - \frac{u_{bd}}{u_b},\ d = l_m, m$$

will indicate type m's (potential mimicker's) preferences between these bundles. As $u\left(h^a({x_n}^{*a}), h^b({x_n}^{*b}), nl_n/m, m\right)$ could be higher or lower than $u\left(h^a({x_n}^{sa}), h^b({x_n}^{sb}), nl_n/m, m\right)$ in general, even though the status-quo allocation is incentive compatible, the reformed schedule does not warrant (1).

Also, for a reform with a linear price q^*, bear in mind that individuals choose y_n^o in combination with nl_n/m, and also that allocation of the budget between two groups is a choice variable. Individual m can, for example, tilt the composition of y_n^o towards leisure − complements than n. Removing the distortion of a group of commodities increases the utilities of potential mimickers of true type n for any n. We do not know that if the removal of distortions favors the true type or the mimicker, but at least the consumption efficiency and incentive issue are not separable issues.

The Kaplow-Laroque argument is valid only if all the commodities move towards the uniform taxes at the same time under the weak separability.

Furthermore, this construction makes clear that Naito (1999)-type of distortion which affects relative wages would, in addition to Naito's effect, it alters the proportion of h^a and h^b in a different way between the true type n and a potential mimicker, so a distortion striking a biding incentive constraint is welfare-improving.

Proposition 1 Even if the utility function satisfies pairwise separability $u(h^a(x^a), h^b(x^b), l_n, n)$, and the government uses non-linear tax reform $(x_n{}^{*a}, x_n{}^{*b})$ for each type, a reform that assigns Hicksian demands of the status-quo allocation to each type generally would not warrant incentive compatibility. Furthermore, a distortion of production efficiency affects the binding incentive constraint by altering the proportion of the two group of commodities differently between the true type and a potential mimicker, which gives a scope of welfare-improvement.

4. Conditions for Optimal Partial Uniformity

To facilitate the uncompensated and compensated demand functions toward the optimal-tax analysis with partial separability, it is convenient to divide the conventional two-stage optimization of the labor supply and consumption into three stages. In the first stage, type-n person with labor supply l_n earns the labor income $z_n = nl_n$ and the after-tax income $y_n \equiv z_n - T(z_n)$. In the consumption stage, (s)he first chooses y_n^a and $y_n^b \equiv y_n - y_n^a$ for the consumption of x^a and x^b respectively. Then the consumption is decided in respective groups.

In the third stage, type n maximizes $u(h(x_n^a), x_n^b, l_n, n)$ given l_n, y_n and y_n^a, subject to

$$q^a \cdot x_n^a \leq y_n^a \text{ and } q^b \cdot x_n^b \leq y_n - y_n^a.$$

For group *a*, the indirect utility determined at this stage is written as $k(q^a, y_n^a) \equiv arg\,max\{h(x_n^a)|q^a \cdot x_n^a \leq y_n^a\}$. For group *b* individuals solve:

$$max_{x_n^b} \quad u(k(q^a, y_n^a), x_n^b, l_n, n) \quad s.t.\ q^b \cdot x_n^b \leq y_n - y_n^a$$

x_n^b may, as in the case of child care, depend on labor supply, so the demand is written as $\hat{x}_n^j(q^a, q^b, y_n^b, l_n, n)$ for each good $j \in B$.

In our second stage, the consumer decides y_n^a:

(2) $\quad \max_{y_n^a} \quad u(k(q^a, y^a), \hat{x}_n^b(q^a, q^b, y_n - y_n^a, l_n, n), l_n, n)$

Dividing the consumer's decision this way will not lose its subjective optimality.

For group a, we have, equating the conventional two-stage conditional demand and our stage-three demand, $x_n^i(q^a, q^b, l_n, y_n) = \tilde{x}_n^i(q^a, y_n^a(\cdot))$ where y_n^a is determined in (2),

(3) $$\frac{\partial x_n^i}{\partial l_n} = \frac{\partial \tilde{x}_n^i}{\partial y_n^a}\frac{\partial y_n^a}{\partial l_n} \qquad i \in A$$

In (2), the choice of labor efforts changes the expenditure share on group *a*, so the uncompensated derivative of goods demands relate to its (third-stage) income effects.

Denote the expenditure to group a as $e_n^a(q, l_n, v_n, n) \equiv arg\,min\{q^a \cdot x_n^a | u(h(x_n^a), x_n^b, l_n, n) \geq v_n\}$ where the utility level to maintain is the total utility (not the sub-utility). Define

(4) $$e_n^{a,j} \equiv \frac{\partial\, e_n^a(q, l_n, v_n, n)}{\partial\, q_j}, \qquad j \in A \cup B$$

so the Hicksian derivative of group *a*'s conditional demand, $x_n^{c*,i}(q, l_n, v_n)$, which is a dual form of

$x_n^i(q, l_n, y_n)$, is:

(5) $$\frac{\partial x_n^{c*,i}(q,l_n,v_n)}{\partial q_j} \equiv s_n^{ij} = \frac{\partial \tilde{x}_n^i(q^a, e_n^a(\cdot))}{\partial y_n^a} e_n^{a,j} \quad i \in A, \; j \in B$$

Cross-derivatives are sufficient to derive the optimal taxes. Suppose that $t_i/(1+t_i) = \tau_a$ holds for all $i \in A$ (uniform tax). In the Appendix we show that:

(6) $$\tau_a = \frac{\int_{m=\underline{n}}^{\bar{n}} \frac{\partial \tilde{x}_m^i}{\partial y_m^a} (\sum_{j \in B} t_j e_m^{a,j} - \frac{\partial y_m^a}{\partial l_n} \frac{W_m}{A_m} l_m) dF(m)}{\int_{m=\underline{n}}^{\bar{n}} \frac{\partial \tilde{x}_m^i}{\partial y_m^a} \sum_{j \in B} q_j e_m^{a,j} \; dF(m)}, \forall i \in A$$

In (6), Similar to Deaton (1979), the cross-substitution effects of t_i to group b in our Equation (5) and the incentive effect from the mimicker's labor-supply adjustment in our Equation (3) appear. The three-step procedure made it clear that both relate to $\frac{\partial \tilde{x}_m^i}{\partial y_m^a}$. Optimizing with group b shows that, for (6) to be independent of i, the preferences must satisfy the parallel income effects (see the Appendix). Intuitively, the inverse of the aggregate Slutsky matrix appears for the optimal t_j's in group b in the numerator, whereas the individual Slutsky term matters for $e_m^{a,j}$ multiplied by t_j. To be consistent with the other terms, $\frac{\partial \tilde{x}_m^i}{\partial y_m^a}$ has to be the same for all m for each i.[2] Such an additional requirement is akin to Deaton's (1979) parallel Engel curves, which cannot be relaxed with nonlinear income tax.

Proposition 2: Suppose that consumer preferences in group a are separable with other goods. Then the uniformity of commodity taxes on these goods is optimal only if the demand function for group a

[2] Otherwise, the term implied by the self-substitution effect (the denominator) and by the cross substitution effect (the numerator) diverge.

have the parallel income effects. Under the fully optimal tax, the income effect $\frac{\partial \tilde{x}_m^i}{\partial y_m^a}$ must be the same for all *m* for each $i \in A$.

Since we have not made any specification on group *b*'s preference, a special case of pairwise uniformity is straightforward. Constancy of $\frac{\partial \tilde{x}_m^i}{\partial y_m^b}$ is also required to derive pairwise uniformity. Therefore, a previous result of Mirrlees (1976, p. 338), which Laroque (2005) quoted for the ground of optimality of pairwise uniform taxes, holds only under special types of preferences or the fully non-linear taxation of incomes and commodities.

Appendix:

Derivation of (5)

With respect to the Hicksian derivative s_n^{ij} of good j's demand with respect to good i's price, the optimal commodity taxes satisfy, for the labor tax wedge $\mathcal{W}_n \equiv \frac{T\prime(z_n)}{1-T\prime(z_n)} + \sum_{i=1\ldots I} t_i \left(\frac{1}{(1-T\prime(z_n))n} \frac{\partial x_n^i}{\partial l_n} + \frac{\partial x_n^i}{\partial y_n} \right)$ (Jacobs and Boadway (2014, eq. (25)), Mirrlees (1976, eq. (96))) which we assume to be positive, and $A_n = 1 + \frac{1+(1-T\prime(z_n))\frac{\partial z_n}{\partial y_n}}{\frac{\partial \ln z_n^{C*}}{\partial \ln\left(1-T'(z_n)\right)}} > 0$ (Seade (1982)),

$$(A.1) \qquad \int_{m=\underline{n}}^{\bar{n}} \sum_{j=1\ldots I} t_j s_n^{ij} \, dF(m) = \int_{m=n_0}^{\bar{n}} \frac{\mathcal{W}_m}{A_m} \frac{\partial x_m^i}{\partial l_m} l_m dF(m) \quad :$$

(Jacobs and Boadway (2014, eq. (31))).

From (4), we have:

$(A.2) \qquad \sum_{k\in B} t_k s_n^{ik} = \frac{\partial \tilde{x}_n^i}{\partial y_n^a} \sum_{k\in B} t_k e_n^{a,i}.$

From (3), the RHS of (A.1) is

$(A.3) \quad \int_{m=n_0}^{\bar{n}} \frac{W_m}{A_m}\frac{\partial x_m^i}{\partial l_m} l_m dF(m) = \int_{m=n_0}^{\bar{n}} \frac{\partial \tilde{x}_m^i}{\partial y_m^a}\frac{\partial y_m^a}{\partial l_n}\frac{W_m}{A_m} l_m dF(m)$

To see if uniform tax is optimal in group *a*, impose the condition of $t_i/(1+t_i) = \tau_a$ into (*). By using a property of the Slutsky matrix,

$(A.4) \qquad \sum_{j\in A} \frac{t_a}{q_a} q_a s_n^{ij} = -\sum_{j\in B} \tau_a q_j s_n^{ij} = \tau_a \frac{\partial \tilde{x}_n^i}{\partial y_n^a} \sum_{j\in B} q_j e_n^{a,j},$

Substituting (A.2)-(A.4) into (A.1), we have

$$\tau_a = \frac{\int_{m=\underline{n}}^{\bar{n}} \frac{\partial \tilde{x}_m^i}{\partial y_m^a} (\sum_{j\in B} t_j e_m^{a,j} - \frac{\partial y_m^a}{\partial l_n}\frac{W_m}{A_m} l_m) dF(m)}{\int_{m=\underline{n}}^{\bar{n}} \frac{\partial \tilde{x}_m^i}{\partial y_m^a} \sum_{j\in B} q_j e_m^{a,j}\, dF(m)}, \forall i \in A$$

which is (6). *QED*

Derivation of the constancy of $\frac{\partial \tilde{x}_n^i}{\partial y_n^a}$ *under the optimally groupwise uniform tax:* Now we derive the optimal rate of t_j in group *b*. From the Hicksian homogeneity $q^a \cdot \frac{\partial x_n^{c*,a}}{\partial q_j} + q^b \cdot \frac{\partial x_n^{c*,b}}{\partial q_j} = 0$, (4) is:

$(A.5) \qquad e_n^{a,j} = -q^b \cdot \frac{\partial x_n^{c*,b}}{\partial q_j}, \quad j \in B$

Let s_n^b be individual n's Slutsky matrix on group *b* whose ij's element is s_n^{ij}, and S^b be its aggregated version whose ij's element is $\int_{m=\underline{n}}^{\bar{n}} s_n^{ij}\, dF(m)$. In the matrix form, (A.5) can be written as

$(A.6) \qquad e_n^{a,b} = -q^b \cdot s_n^b$

For the ease of notation, let $\tilde{x}_n^{i,y_a} \equiv \frac{\partial \tilde{x}_n^i}{\partial y_n^a}$. Differentiating the budget constraint $\sum_{i\in A} q^i x_n^i = y_n^a$, we have $\sum_{i\in A} q^a \tilde{x}_n^{i,y_a} = 1$. Using (5), (A.6) and $t_i/(1+t_i) = \tau_a$, the FOC for good $j \in B$ is:

$$\int_{m=\underline{n}}^{\bar{n}} \sum_{k\in B} t_k s_m^{jk}\, dF(m) + \int_{m=\underline{n}}^{\bar{n}} \sum_{i\in A} \tau_a q^a \tilde{x}_n^{i,y_a} e_m^{a,j}\, dF(m) = \int_{m=\underline{n}}^{\bar{n}} \sum_{k\in B} t_k s_m^{jk}\, dF(m) + \tau_a \int_{m=\underline{n}}^{\bar{n}} e_m^{a,j}\, dF(m)$$

$$= \int_{m=\underline{n}}^{\bar{n}} \sum_{k \in B} t_k s_m^{jk} \, dF(m) - \tau_a \int_{m=\underline{n}}^{\bar{n}} q^b \cdot s_m^j \, dF(m) \quad = \int_{m=n_0}^{\bar{n}} \frac{\mathcal{W}_m}{A_m} \frac{\partial x_m^j}{\partial l_m} l_m dF(m) \qquad \forall j \in B$$

So we have the optimal formula as:[3]

$$(A.7) \qquad (t_k)_{k \in B} = \left(S^b\right)^{-1} \int_{m=n_0}^{\bar{n}} \frac{\mathcal{W}_m}{A_m} \frac{\partial x_m^b}{\partial l_m} l_m dF(m) + \tau_a q^b$$

Differentiating the budget constraint $\sum_{j \in B} q^j x_n^j = y - y_n^a$, we have $\frac{\partial y_n^a}{\partial l_n} = -q^b \cdot \frac{\partial x_n^b}{\partial l_n}$. Substituting them into (6), we have:

$$-\int_{n=\underline{n}}^{\bar{n}} \tilde{x}_n^{i,y_a} e_n^{a,b} \left(S^b\right)^{-1} \int_{m=n_0}^{\bar{n}} \frac{\mathcal{W}_m}{A_m} \frac{\partial x_m^b}{\partial l_m} l_m dF(m) \, dF(n)$$

$$(A.8) \qquad = \int_{m=n_0}^{\bar{n}} \tilde{x}_m^{i,y_a} q^b \cdot \frac{\partial x_m^b}{\partial l_m} \frac{\mathcal{W}_m}{A_m} l_m dF(m)$$

The only natural way for the LHS of (A.8) to make sense is that $\tilde{x}_n^{i,y_a}$ is n-independent, in which case

$$-\int_{n=\underline{n}}^{\bar{n}} \tilde{x}_n^{i,y_a} e_n^{a,b} \left(S^b\right)^{-1} \int_{m=n_0}^{\bar{n}} \frac{\mathcal{W}_m}{A_m} \frac{\partial x_m^b}{\partial l_m} l_m dF(m) \, dF(n)$$

$$= \int_{n=\underline{n}}^{\bar{n}} \tilde{x}_n^{i,y_a} q^b \cdot s_n^b \left(S^b\right)^{-1} \int_{m=n_0}^{\bar{n}} \frac{\mathcal{W}_m}{A_m} \frac{\partial x_m^b}{\partial l_m} l_m dF(m) \, dF(n)$$

$$= \tilde{x}_n^{i,y_a} q^b \cdot \int_{n=\underline{n}}^{\bar{n}} s_n^b dF(n) \left(S^b\right)^{-1} \int_{m=n_0}^{\bar{n}} \frac{\mathcal{W}_m}{A_m} \frac{\partial x_m^b}{\partial l_m} l_m dF(m)$$

$$= \int_{m=n_0}^{\bar{n}} \tilde{x}_n^{i,y_a} q^b \cdot \frac{\mathcal{W}_m}{A_m} \frac{\partial x_m^b}{\partial l_m} l_m dF(m)$$

= RHS of (A.8)

QED

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[3] (A.7) includes the tax rate of the untaxed numeraire, whose FOC holds due to Hicksian homogeneity; the zero-tax rate in that row would determine the tax rates of all the goods.

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