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Abstract

Under a regime of nonlinear income tax and fully linear commodity taxes, we may not be able to construct a welfare-improving tax reform towards uniform taxes on a subset of commodities. Furthermore, distortions of production that alter relative wages can improve welfare by influencing mimickers' consumption choices and thus relaxing incentive compatibility constraints. We then identify conditions for optimal uniform taxation within separated consumption groups. Specifically, to neutralize heterogeneous cross-substitution effects across income classes, uniform commodity taxes require parallel income effects, so the restrictions on the Engel curves cannot be dispensed with even under nonlinear income taxation.

Keywords: Uniform commodity taxes, Tax reform, Income effects

JEL Classification: H21, H24

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1. Introduction

The question of whether commodity taxes should be uniform or differentiated is a practically relevant issue in the optimal tax theory, given that many countries adopt non-uniform consumption taxes (such as Value Added Tax). The related research has been there, dating back to Atkinson and Stiglitz (1976) and Deaton (1979). Consider a multi-commodity economy where each agent's utility is given by $u(x, l, n)$ over a vector of commodities x, labor l and ability n.[1] Among others, Deaton (1979) showed that uniform taxation across a subset of goods can be optimal under three conditions: (i) the utility function takes the form $u(h^a(x^a), x^b, l, n)$, where the sub-utility $h^a(\cdot)$ is identical across agents, (ii) income taxation is linear, and (iii) $h^a(\cdot)$ generates linear Engel curves in the demand function. Importantly, condition (iii) is not needed when there is no x^b (full separability between all the goods and labor efforts) and when nonlinear income taxation is available. In the chapter of Value Added Tax of the Mirrlees Review, Crawford et al. (2011) align with this partial uniformity of commodity taxes with an exception of tax favor on child care as a substitute for labor supply. Another possible situation is the multi-period model $u(h^{a_y}(x^{a_y}), h^{a_o}(x^{a_o}), x^b, l, n)$ in which a subset of young-period consumption a_y and another subset of old-peroid consumption a_o are separable with labor. Are uniform taxes on the subsets of commodities optimal?

In this paper, we demonstrate first that a Pareto-improving tax reform toward uniformity in taxes within group a cannot be constructed if there is group b which is also linearly taxed as in Guesnerie (1995). In response to a tax reform towards partial uniformity, potential mimickers can profitably deviate by reshaping the composition of separated goods x^a and the other goods x^b. We then conduct an optimal-tax analysis, and show a condition that would restore optimality of partially uniform commodity taxes.

[1] The last argument n follows Mirrlees (1976) and is optional.

Our argument is consistent with Bastani et al. (2014),[2] but we extend their analysis by identifying the additional conditions under which partially uniform commodity taxes can be optimal.

Moreover, we show that Naito (1999)-type distortion also alters the proportions of h^a and x^b differently between the true type and a potential mimicker, and therefore, there is a scope for Pareto improvement.

Our results are independent of Laroque (2005) and Doligalski et al. (2025) which argue that a Pareto improvement can be achieved by allowing agents to decentralize the purchase of a separated group of commodities at linear and undistorted prices. We discuss the differences between our results and theirs in Section 3 after our main proposition.

In Section 2, we show an overview of the two main results. Sections 3 and 4 are the main analysis. The proof of the second proposition is in the Appendix.

2. Previewing Results

In the formal analysis, we assume the utility function takes the form $u(h^a(x^a), x^b, l, n)$, which satisfies Doligalski et al.'s (2025) incentive separability. We establish two results.

1. First, when linear commodity taxes are in place, a tax reform toward linear groupwise uniformity cannot generally be Pareto-improving. Attempts to adjust commodity taxes and post-tax incomes without changing labor supplies (as in the Kaplow–Laroque framework) towards partial uniformity may violate incentive constraints. As a corollary, a Naito (1999)-type production distortion

[2] Suppose that x^b is a labor substitute (such as child care and elderly care). Bastani et al. (2014) show that a uniform tax on x^a does not generally arise at the second-best optimum. Our contribution in this paper is to identify the additional conditions in Proposition 2 under which partially uniform commodity taxes can be optimal.

acquires additional social value since the associated changes of labor supply induce preference changes.

2. Second, we identify additional conditions required for uniform commodity taxation on within a subset of goods. When adopting linear commodity taxation alongside nonlinear income taxation, a Deaton (1979)[3]-type uniform tax on a subset of commodities requires that the sub-utility functions exhibit parallel income effects. Intuitively, this means that the income effects of each good in the uniformly taxed group must be proportional across ability types. Hence, a condition akin to Deaton's (1979) linear Engel curve remains indispensable, even under nonlinear income taxation, so long as commodity taxes remain linear.

3. Tax Reform for Partial Uniformity Is Not Welfare-improving

We begin our main analysis with the Kaplow (2006)–Laroque (2005) tax reform, as this framework is more tractable and provides clearer intuition for understanding the nature of optimal taxation in the following section. When linear commodity taxes are in place, can a reform toward a linear and uniform tax on group a generate a Pareto improvement?

3.1 The Model

Suppose that the goods, indexed by 0,1,...I, are divided by group a with a set $\{1,\cdots,I^a\}$ with $I^a \geq 2$ and the other goods, $\{I^a+1,\cdots,I, 0\}$, forming consumption vectors x^a and x^b. Good 0 is normalized to

[3] Besley and Jewitt (1992) enlarges a sufficient condition for fully uniform commodity taxes. Their condition does not apply to uniform taxes on subset of commodities.

be untaxed. The utility function takes a separable form with respect to x^a, $u\left(h(x^a), x^b, l, n\right)$. The population is distributed with the skill level $n \in [\underline{n}, \bar{n}]$ by the density function $f(n)dn = dF(n)$. Commodities are produced by the efficient-unit labor as the input with a linear technology whose marginal costs are normalized to 1: type-n's labor effort l_n generates nl_n unit of the output, and n becomes the competitive wage for type n. The case of a concave production process with endogenous wages is discussed at the end of this section.

The government levies a commodity tax rate t_j (a subsidy when it is negative) per unit of good j's consumption. Besides, nonlinear income taxes take place. The vector of after-tax commodity prices, $1 + t_j = q_j$ for good j, is written as q^a and q^b for each group. We assume that the goods in group b can only be subject to linear taxation. More broadly, group b is understood to include at least one good for which only linear taxation is feasible. An illustrative case is the two-period model with $u\left(h^a(x^a), h^b(x^b), l, n\right)$ where the utility is pairwise-separable a basket of consumption goods in each period.[4]

3.2 Tax Reform toward Partial Uniformity

Given a status-quo allocation as $\left(x_n{}^{sa}, x_n{}^{sb}\right)$ for type n's consumption that is incentive compatible, suppose the policymaker introduces a reform schedule $x_n{}^{*a}$ supported by a partially uniform linear

[4] If a wedge arises between periods (corresponding to capital income taxation) but no wedge appears among goods within each group, the resulting tax pattern resembles the optimal taxes in Golosov et al. (2003). However, we show that such a pairwise uniform tax reform is, in general, not incentive compatible.

commodity taxes, without altering labor supplies, as in Kaplow–Laroque–Doligalski et al.. $({x_n}^{*a}, {x_n}^{*b})$ is constructed from the expenditure function at the after-tax price vector q^*, where $q_i^{a*} = 1 + t_a$ in group a and q^{b*} remains at the status-quo;[5] as in Guesnerie (1995), x^b is also under linear commodity tases. Also, the expenditure is conditional on the status-quo labor supply l_n and utility level v_n for type n: $e(q^*, l_n, v_n, n) \equiv arg\,min\{ q^* \cdot x | u(h^a(x^a), x^b, l_n, n) \geq v_n\} \equiv y_n^o$. This disposable income $y_n^o = q^* \cdot ({x_n}^{*a}, {x_n}^{*b})$ is assigned so as to induce type n to choose the Hicksian demand at prices q^*. The linearity of the commodity taxes also alters ${x_n}^b$ by the reform. As a special case, one can consider Laroque's pairwise separability and a pairwise-uniform reform.

Let $({x_{nm}}^{*a}, {x_{nm}}^{*b})$ denote the consumption choice of type m when mimicking type n under the proposed scheme (q^*, y_n^o) for the proposed price and the disposable income, with the mimicking labor supply nl_n/m. Surrounding the status-quo allocation, tax implementability requires the reform schedule to satisfy:

$$u(h^a({x_n}^{sa}), {x_n}^{sb}, l_n, n) = u(h^a({x_n}^{*a}), {x_n}^{*b}, l_n, n) \;\; and$$

(1) $$u(h^a({x_m}^{*a}), {x_m}^{*b}, l_m, m) \geq u(h^a({x_{nm}}^{*a}), {x_{nm}}^{*b}, nl_n/m, m).$$

For type m, a Pareto-indifferent reform satisfies $u(h^a({x_m}^{*a}), {x_m}^{*b}, l_m, m) = u(h^a({x_m}^{sa}), {x_m}^{sb}, l_m, n)$ where the disposable income after the reform, $e(q^*, l_m, v_m, n)$, is constructed in the same way as above. Under weak separability (no x^b so $u = u(h^a(x), l, n)$), type $m \neq n$, acting as a potential mimicker of type n, chooses the same consumption bundle as the (true) type n from the

[5] Recall that the non-distortionary price is 1. The value of q^{b*} is not important for the proposition.

expenditure y_n^o ; $x_{nm}{}^* = x_n{}^*$ in the above equation, and we have $u(h^a(x_n{}^*), nl_n/m, m) = u(h^a(x_n{}^s), nl_n/m, m)$ if $h^a(x)$ is separable with l and n. As a result, (1) for type m holds since the status-quo allocation is incentive compatible (Kaplow (2006, Lemma 1)).

3.3 Two Alternative Explanations

However, when utility is written as $u = u(h^a, x^b, l, n)$, the situation is very different. There may exist a bundle $\left(h^a(x_{nm}{}^{*a}), x_{nm}{}^{*b}\right)$, feasible under the post-reform budget constraint, which yields higher utility to the mimicker than the status quo bundle $\left(h^a(x_n{}^{sa}), x_n{}^{sb}\right)$, since preferences over the combination of h^a and x^b depend on l and n. If group b contains a good that is substitutable to leisure, then a type $m > n$ with more leisure will prefer the bundle that yields a lower level of x^b provided that this bundle delivers a higher level of h^a.

As an illustration, consider a case in which the utility is separable between consumption in the young period (group *a*) and consumption in the old period (group *b*). The latter has an expenditure category that is substitutable for the young-period leisure, say, health expenditure. In this case, if *income effects* differ across commodities within group *a*, differentiating the taxes on commodities in group *a* away from uniformity will affect the consumption choices of mimickers and the true type differently, unless specific conditions on the income effects of the goods' demand are imposed. This logic is behind the proposition in the next section that the optimum is generally not partially uniform. Once the optimum is not partially uniform, a reform toward partial uniformity does not guarantee an incentive-compatible welfare improvement.

As an alternative explanation, it is convenient to divide the conventional two-stage optimization of labor supply and consumption into three stages, which is an approach we adopt in Section 4. In the first stage, type n with labor supply l_n earns labor income $z_n = nl_n$ and the after-tax income $y_n \equiv z_n - T(z_n)$. In the consumption stage, (s)he first chooses y_n^a and $y_n^b \equiv y_n - y_n^a$ for the consumption of x^a and x^b respectively. Then the consumption is decided in respective groups.

In the third stage, type n maximizes $u\big(h(x_n^a), x_n^b, l_n, n\big)$ given l_n, the expenditure y_n^o, and y_n^a, subject to

$$q^a \cdot x_n^a \leq y_n^a \text{ and } q^b \cdot x_n^b \leq y_n^o - y_n^a.$$

For group a, the indirect utility determined at this stage is written as $k(q^a, y_n^a) \equiv arg\,max\{\,h(x_n^a)|q^a \cdot x_n^a \leq y_n^a\}$. For group b individuals solve:

$$max_{x_n^b} \quad u\big(k(q^a, y_n^a), x_n^b, l_n, n\big) \quad s.t.\ q^b \cdot x_n^b \leq y_n^o - y_n^a$$

x_n^b may, as in the case of child care, depend on labor supply, so the demand is written as $\hat{x}_n^j\big(q^a, q^b, y_n^o - y_n^a, l_n, n\big)$ for each good $j \in \{I^a+1, \cdots, I, 0\}$.

In our second stage, the consumer decides y_n^a:

$$\text{(2)} \qquad \max_{y_n^a} \quad u\Big(k(q^a, y^a), \hat{x}_n^b\big(q^a, q^b, y_n^o - y^a, l_n, n\big), l_n, n\Big)$$

Dividing the consumer's decision this way will not lose its subjective optimality.

For a reform with a linear price vector q^*, it is important to note that a potential mimicker chooses the budget allocation for group a conditional on the labor supply nl_n/m and y_n^o, so that allocation of h^a and $x^b$is type-dependent. As mentioned above, individual m can tilt the composition of y_n^o toward leisure complements. Removing the distortion of a group of commodities may increase the utilities of potential mimickers more than the true type m. Consumption efficiency and incentive compatibility are not separable.

3.4 Proposition and discussion

The above discussion is concluded as follows:

Proposition 1 Suppose that the utility function satisfies partial separability $u\left(h^a(x^a), x^b, l_n, n\right)$, and the government implements a linear tax reform with partially uniform commodity taxes. Then a tax reform that assigns each type the status-quo utility level will generally fail to ensure incentive compatibility.

Now we discuss the implications of Proposition 1 and its relationship with the previous literature. We show that the Kaplow–Laroque argument is valid only if all commodities are weakly separable.

Laroque (2005) argues, though without a formal proof, that a Pareto-improving tax reform toward uniformity in taxes within group a can be constructed, without additional assumptions in the demand function (p. 143, Remark 3). Doligalski et al. (2025, Corollary 1) made a similar argument as Laroque (2005) in that a Pareto improvement can be achieved by allowing agents to decentralize the purchase of a separated group of commodities at linear and undistorted prices.

The independence of our result and the previous papers is as follows. A result of Mirrlees (1976, p. 338), cited by Laroque (2005) as the basis for his claim, holds under fully nonlinear taxation of both income and commodities, in which the government can allocate the budget across x^a and x^b nonlinearly. If x^b is subject to linear commodity taxation, as explained in Section 3.3, the mimicker and the true type choose different compositions of expenditures from the total expenditure y_n^o, which makes partial uniformity insufficient to guarantee welfare improvement.

3.5 Production efficiency

Furthermore, the above discussion clarifies that a Naito (1999)-type distortion, which affects relative wages would, beyond Naito's effect on labor supply, also alter the proportions of h^a and x^b differently between the true type and a potential mimicker. Consequently, addressing a binding incentive constraint can be welfare-improving. That is, omitting the dependence on n in the utility function (since the wage w_n is now endogenous) we have, for $\Omega(P) \equiv \frac{w_m}{w_n}$ which depends on the list of the government's policy instruments P:

$$u\left(h^a({x_m}^{*a}), {x_m}^{*b}, l_m\right) \geq u\left(h^a({x_{nm}}^{*a}), {x_{nm}}^{*b}, \Omega(P) l_n\right)$$

for the self-selection constraint by type m against type n. Taxes and public production can affect, in addition to $\Omega(P)$, the consumption ${x_{nm}}^*$, since the induced changes in the labor supply of the mimicker alter their preferred consumption. In other words, a distortion in production efficiency affects the binding incentive constraint by changing the relative proportions of the two groups of commodities differently between the true type and a potential mimicker, thereby creating a scope for welfare improvement.

This implication of Proposition 1 is independent of Doligalski et al. (2025, Corollary 3) who affirmed the use of tax reform compatible with production efficiency. Dolingaliski et al. (2025) achieved that conclusion since they assume that production decisions of firms do not affect incentive constraints (Dolingaliski et al. (2025), p. 558)). However, we note that the redistributive policies achieve their local optimum through tax reforms when incentive constraints become binding.

4. Conditions for Optimal Partial Uniformity

Having shown that piecewise tax reform towards uniformity cannot maintain incentive compatibility, it is worthwhile to examine the feature of optimal linear commodity taxation.

For the indirect utility $v_m = u(x_m(q, l_m, y_m), l_m)$, the government, by chooses q and (l_m, y_m) for all individuals, maximizes a Bergson-Samuelson Social Welfare Function (SWF), $\int_{m=\underline{n}}^{\bar{n}} \Psi(v_m)\, dF(m)$, $\Psi' > 0$ and $\Psi'' \leq 0$, subject to the resource constraint $\int_{m=\underline{n}}^{\bar{n}} \{ml_m - \sum_{i=0\ldots I} x_m^i(q, y_m, l_m) - R\}\, dF(m)$ for an exogenous revenue requirement $R > 0$, and the conventional incentive constraint $u\big(x_m^i(q, y_m, l_m), l_m\big) \geq u\big(x_n^i(q, y_m, ml_m/n), ml_m/n\big)$ for all m and all n.

To represent the uncompensated and compensated demand functions which are necessary in the optimal-tax analysis with partial separability, we adopt the three-stage framework described above: (i) the choice of labor supply, (ii) the allocation of income to the separated groups of goods, and (iii) consumption.

For group *a*, by equating the conventional conditional demand with our stage-three demand, we obtain $x_n^i\big(q^a, q^b, l_n, y_n\big) = \tilde{x}_n^i(q^a, y_n^a(\cdot))$ where y_n^a is determined in the second stage by (2):

(3) $$\frac{\partial x_n^i}{\partial l_n} = \frac{\partial \tilde{x}_n^i}{\partial y_n^a}\frac{\partial y_n^a}{\partial l_n} \qquad i \in \{1, \dots, I^a\}$$

In (2), the choice of labor effort affects the expenditure share on group *a*; thus, the uncompensated derivatives of goods demands are related to the third-stage income effects.

Let the expenditure allocated to group *a* be denoted as $e_n^a(q, l_n, v_n, n) \equiv arg\,min\{\, q^a \cdot x_n^a \big| u\big(h(x_n^a), x_n^b, l_n, n\big) \geq v_n\big\}$ where the utility level v_n is the total utility (not the sub-utility). Define

(4) $$e_n^{a,j} \equiv \frac{\partial\, e_n^a(q, l_n, v_n, n)}{\partial\, q_j}, \qquad j \in \{1, \dots, I\}$$

so the Hicksian derivative of group *a* 's conditional demand, $x_n^{c*,i}(q, l_n, v_n)$, as a dual form of $x_n^i(q, l_n, y_n)$, satisfies:

(5) $$\frac{\partial x_n^{c*,i}(q, l_n, v_n)}{\partial q_j} \equiv s_n^{ij} = \frac{\partial \tilde{x}_n^i(q^a, e_n^a(\cdot))}{\partial y_n^a} e_n^{a,j} \quad i \in \{1, \dots, I^a\},\ \ j \in \{I^a + 1, \dots, I, 0\}$$

Cross-derivatives are sufficient to derive the optimal taxes. Suppose that $t_i/(1 + t_i) = \tau_a$ holds for all $i \in \{1, \dots, I^a\}$ (uniform tax). In the Appendix we show that, for $(v_y^m \theta_m)/\eta$ defined in the Appendix:

(6) $$\tau_a = \frac{\int_{m=\underline{n}}^{\bar{n}} \frac{\partial \tilde{x}_m^i}{\partial y_m^a} \left(\sum_{j \in \{I^a+1,\dots,I\}} t_j e_m^{a,j} - \frac{\partial y_m^a}{\partial l_m} \frac{(v_y^m \theta_m)/\eta}{m f(m)} l_m\right) dF(m)}{\int_{m=\underline{n}}^{\bar{n}} \frac{\partial \tilde{x}_m^i}{\partial y_m^a} \sum_{j \in \{I^a+1,\dots,I\}} q_j e_m^{a,j}\ dF(m)}, \forall i \in \{1, \dots, I^a\}$$

In (6), Similar to Deaton (1979), the compensated substitution effects of t_i in (5) and the incentive effects that represent the difference of the consumption choices between the true type and the mimicker[6] in (3) appear. Optimizing with group b shows that, for (6) to be independent of i, preferences must satisfy a condition of parallel income effects (see the Appendix).[7] Such an additional requirement is analogous to Deaton's (1979) condition of parallel Engel curves, and it cannot be relaxed even in the presence of nonlinear income taxation.

Proposition 2: Suppose that consumer preferences for goods in group a are separable from the other goods. Then, uniform commodity taxation within group a is optimal only if the demand functions for these goods exhibit parallel income effects. Under the optimal tax system, the income effect, $\frac{\partial \tilde{x}_m^i}{\partial y_m^a}$, must be identical across $i \in \{1, \ldots, I^a\}$ for all $m \in [\underline{n}, \bar{n}]$.

Although we have not imposed any specific structure on preferences in group b, the special case of pairwise uniformity follows directly. In particular, constancy of $\frac{\partial \tilde{x}_m^i}{\partial y_m^b}$ in group b is also required to derive pairwise uniformity.

5 Conclusion

[6] The term $v_y^m \theta_m$ is related to the marginal income tax rate that type m faces. (6) shows that, the larger are the optimal marginal labor income taxes, the more are the goods-market distortions by commodity taxes (Jacobs and Boadway (2014, p. 206)).

[7] Intuitively, when solving the simultaneous system t_j ($j \in \{I^a + 1, \ldots, I, 0\}$) involving Mirrlees' (1976) index of discouragement, the inverse of the aggregate Slutsky matrix appears in the numerator for the optimal t_j's in group b, whereas the individual Slutsky term matters in $e_m^{a,j}$ multiplied by t_j. To be consistent with the other terms, $\frac{\partial \tilde{x}_m^i}{\partial y_m^a}$ must be identical across i for all m.

In this paper, we show that the result of the optimality of partially uniform taxation cannot proceed along the lines of tax reform by Kaplow (2006) and Laroque (2005) if the commodity taxes are linear for goods that are not separable from labor. This is because a potential mimicker would deviate by choosing a different consumption bundle than the true type. We demonstrate that the income effects of commodities are crucial for assessing the desirability of tax reforms toward uniformity. Moreover, a Naito (1999)-type distortion of production possibilities alters the relative proportions of h^a and x^b for a potential mimicker, so striking a binding incentive constraint can be welfare-improving. Our results are independent of Laroque (2005) and Doligalski et al. (2025) where the feasibility of nonlinear budget sets on x^b matters for the desirability of incentive-separable uniform taxation.

Appendix

Derivation of (6)

Let η be the Lagrange multiplier of the resource constraint. The first-order approach for the self-selection problem is $\frac{\partial u(x_m^i(q,z_m-T(z_m),z_m/m),z_m/m)}{\partial m} = -\frac{\partial u(x_m^i(q,y_m,z_m/m),z_m/m)}{\partial l}\frac{l}{m}$, and the Lagrangian for the optimal control problem is written as (Jacobs and Boadway (2014), Eq. (20));

$$L \equiv \int_{m=\underline{n}}^{\bar{n}} \{\Psi(v_m) + \eta(ml_m - \sum_{i=0\ldots I} x_m^{c*,i}(q, l_m, v_m) - R)\}\, dF(m) + \int_{m=\underline{n}}^{\bar{n}} \theta_m \frac{\partial u/\partial l}{m} l_m - \frac{d\theta_m}{dm} v_m\, dm$$
$$+\theta_{\bar{n}} u_{\bar{n}} - \theta_{\underline{n}} u_{\underline{n}},$$

The first-order conditions (FOC) for q_i are: for s_n^{ij} be a derivative of good j's Hicksian demand with respect to good i's price and $v_y^m = \frac{\partial v_m}{\partial y_m}$,

$$(A.1)\ \frac{\partial L}{\partial q^i} = 0 \ \leftrightarrow\ \int_{m=\underline{n}}^{\bar{n}} \sum_{j=1\ldots I} t_j s_n^{ij}\, dF(m) = \int_{m=n_0}^{\bar{n}} \frac{(v_y^m \theta_m)/\eta}{mf(m)} \frac{\partial x_m^i}{\partial l_m} l_m dF(m)$$

From (5), we have:

$$(A.2) \qquad \sum_{k\in B} t_k s_n^{ik} = \frac{\partial \tilde{x}_n^i}{\partial y_n^a} \sum_{k\in B} t_k e_n^{a,k}.$$

From (3), the RHS of (A.1) is

$$(A.3) \quad \int_{m=n_0}^{\bar{n}} \frac{(v_y^m \theta_m)/\eta}{mf(m)} \frac{\partial x_m^i}{\partial l_m} l_m dF(m) = \int_{m=n_0}^{\bar{n}} \frac{\partial \tilde{x}_m^i}{\partial y_m^a} \frac{\partial y_m^a}{\partial l_n} \frac{(v_y^m \theta_m)/\eta}{mf(m)} l_m dF(m)$$

To examine the optimality of uniform taxation within group *a*, impose the condition $t_i/(1+t_i) = \tau_a$ into equation (A.1). By exploiting a property of the Slutsky matrix,

$$(A.4) \qquad \sum_{j\in\{1,\dots,I^a\}} \frac{t_a}{q_a} q_a s_n^{ij} = -\sum_{j\in\{I^a+1,\dots,I\}} \tau_a q_j s_n^{ij} = \tau_a \frac{\partial \tilde{x}_n^i}{\partial y_n^a} \sum_{j\in\{I^a+1,\dots,I\}} q_j e_n^{a,j},$$

Substituting (A.2)-(A.4) into (A.1), we have

$$\tau_a = \frac{\int_{m=\underline{n}}^{\bar{n}} \frac{\partial \tilde{x}_m^i}{\partial y_m^a} \left(\sum_{j\in\{I^a+1,\dots,I\}} t_j e_m^{a,j} - \frac{\partial y_m^a}{\partial l_n} \frac{(v_y^m \theta_m)/\eta}{mf(m)} l_m\right) dF(m)}{\int_{m=\underline{n}}^{\bar{n}} \frac{\partial \tilde{x}_m^i}{\partial y_m^a} \sum_{j\in\{I^a+1,\dots,I\}} q_j e_m^{a,j}\, dF(m)}, \forall i \in \{1,\dots,I^a\}$$

which is (6). *QED*

Derivation of the constancy of $\frac{\partial \tilde{x}_n^i}{\partial y_n^a}$ ***under the optimally groupwise uniform tax:*** We now derive the optimal rate of t_j in group *b*. Using the Hicksian homogeneity $q^a \cdot \frac{\partial x_n^{c*,a}}{\partial q_j} + q^b \cdot \frac{\partial x_n^{c*,b}}{\partial q_j} = 0$, (4) is:

$$(A.5) \qquad e_n^{a,j} = -q^b \cdot \frac{\partial x_n^{c*,b}}{\partial q_j}, \quad j \in \{I^a+1,\dots,I\}$$

Let s_n^b be individual n's Slutsky matrix on group *b* whose (*i, j*) element is s_n^{ij}, and S^b be the corresponding aggregate matrix whose (*i, j*) element is $\int_{m=\underline{n}}^{\bar{n}} s_n^{ij}\, dF(m)$. In the matrix form, (A.5) can be written as

$$(A.6) \qquad e_n^{a,b} = -q^b \cdot s_n^b$$

For the ease of notation, let $\tilde{x}_n^{i,y_a} \equiv \frac{\partial \tilde{x}_n^i}{\partial y_n^a}$. Differentiating the budget constraint $\sum_{i\in\{1,\dots,I^a\}} q^i x_n^i = y_n^a$, we have $\sum_{i\in\{1,\dots,I^a\}} q^a \tilde{x}_n^{i,y_a}=1$. Using (6), (A.6) and $t_i/(1+t_i)=\tau_a$, the FOC for good $j\in\{I^a+1,\dots,I\}$ is:

$$\int_{m=\underline{n}}^{\bar{n}} \sum_{k\in\{I^a+1,\dots,I\}} t_k s_m^{jk}\, dF(m) + \int_{m=\underline{n}}^{\bar{n}} \sum_{i\in\{1,\dots,I^a\}} \tau_a q^a \tilde{x}_n^{i,y_a} e_m^{a,j}\, dF(m)$$

$$= \int_{m=\underline{n}}^{\bar{n}} \sum_{k\in\{I^a+1,\dots,I\}} t_k s_m^{jk}\, dF(m) + \tau_a \int_{m=\underline{n}}^{\bar{n}} e_m^{a,j}\, dF(m)$$

$$= \int_{m=\underline{n}}^{\bar{n}} \sum_{k\in\{I^a+1,\dots,I\}} t_k s_m^{jk}\, dF(m) - \tau_a \int_{m=\underline{n}}^{\bar{n}} q^b \cdot s_m^{bj}\, dF(m)$$

$$= \int_{m=n_0}^{\bar{n}} \frac{(v_y^m \theta_m)/\eta}{mf(m)} \frac{\partial x_m^j}{\partial l_m} l_m dF(m) \quad \forall j\in B$$

So we have the optimal formula as:[8]

$$(A.7) \qquad (t_k)_{k\in\{I^a+1,\dots,I\}} = \left(S^b\right)^{-1} \int_{m=n_0}^{\bar{n}} \frac{(v_y^m \theta_m)/\eta}{mf(m)} \frac{\partial x_m^b}{\partial l_m} l_m dF(m) + \tau_a q^b$$

Differentiating the budget constraint $\sum_{j\in B} q^j x_n^j = y - y_n^a$, we have $\frac{\partial y_n^a}{\partial l_n} = -q^b \cdot \frac{\partial x_n^b}{\partial l_n}$. Multiplying both sides of $\int_{m=\underline{n}}^{\bar{n}} \frac{\partial \tilde{x}_m^i}{\partial y_m^a} \sum_{j\in B} q_j e_m^{a,j}\, dF(m)$ and substituting (A.7), we obtain:

$$(A.8) \quad -\int_{n=\underline{n}}^{\bar{n}} \tilde{x}_n^{i,y_a} e_n^{a,b} \,(S^b)^{-1} \int_{m=n_0}^{\bar{n}} \frac{(v_y^m \theta_m)/\eta}{mf(m)} \frac{\partial x_m^b}{\partial l_m} l_m dF(m)\, dF(n)$$

$$= \int_{m=n_0}^{\bar{n}} \tilde{x}_m^{i,y_a} q^b \cdot \frac{\partial x_m^b}{\partial l_m} \frac{(v_y^m \theta_m)/\eta}{mf(m)} l_m dF(m)$$

The only natural way for the LHS of (A.8) to make sense is that $\tilde{x}_n^{i,y_a}$ is n-independent, in which case

$$-\int_{n=\underline{n}}^{\bar{n}} \tilde{x}_n^{i,y_a} e_n^{a,b} \left(S^b\right)^{-1} \int_{m=n_0}^{\bar{n}} \frac{(v_y^m \theta_m)/\eta}{mf(m)} \frac{\partial x_m^b}{\partial l_m} l_m dF(m)\, dF(n)$$

[8] (A.7) includes the formula on the untaxed numeraire, whose first-order condition is satisfied by Hicksian homogeneity. Setting the tax rate in that row to zero resolves the indeterminacy of absolute post-tax prices.

$$= \int_{n=\underline{n}}^{\bar{n}} \tilde{x}_n^{i,y_a} q^b \cdot s_n^b \left(S^b\right)^{-1} \int_{m=n_0}^{\bar{n}} \frac{(v_y^m \theta_m)/\eta}{mf(m)} \frac{\partial x_m^b}{\partial l_m} l_m dF(m)\, dF(n)$$

$$= \tilde{x}_n^{i,y_a} q^b \cdot \int_{n=\underline{n}}^{\bar{n}} s_n^b dF(n) \left(S^b\right)^{-1} \int_{m=n_0}^{\bar{n}} \frac{(v_y^m \theta_m)/\eta}{mf(m)} \frac{\partial x_m^b}{\partial l_m} l_m dF(m)$$

$$= \int_{m=n_0}^{\bar{n}} \tilde{x}_n^{i,y_a} q^b \cdot \frac{(v_y^m \theta_m)/\eta}{mf(m)} \frac{\partial x_m^b}{\partial l_m} l_m dF(m)$$

= RHS of (A.8)

QED

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