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Abstract

In this paper, we explore the impacts of government policies on endogenous fertility and economic growth in a full-fledged framework with private and public spending on both health and education. First, we show that excessive government expenditure on education and insufficient government expenditure on health may lead to zero or even negative economic growth. Second, in our specified model, the income tax rate is shown to have no impact on fertility. Third, we determine the income tax rate and the fraction of revenue used for education expenditure to maximize the balanced growth rate. Furthermore, it is demonstrated that larger public health expenditure on children leads to greater economic growth. Moreover, the welfare-maximizing tax rate and shares of public expenditure allocated to children's health and education are found to be smaller than the growth-maximizing counterparts. Finally, the welfare-maximizing tax rate exhibits a non-monotonic relationship with social discount factor.

Keywords *human capital, fertility, growth, health, education*

JEL Classification I10, I20, O40

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1 Introduction

Nowadays, many countries around the world are facing increasingly severe challenges of demographic transition. According to empirical research by Bloom et al. (2010), they find that the declining fertility rate is leading to a significant issue of population aging, resulting in problems such as labor shortages and reduced capital savings, particularly in OECD countries. They discuss the viewpoint that population aging leads to a decline in per capita income and economic growth. Furthermore, Bloom et al. (2010) emphasize the crucial role of human capital and government policies in dealing with the decreasing economic growth rate caused by population aging. For example, raising the retirement age, establishing flexible pension systems, investing in education and health improvement are mentioned.

Until now, numerous studies have examined how government policies affect fertility and economic growth. By outlining a theoretical framework intended to be useful in data analysis and presenting the empirical results, Romer (1989) argues about the importance of initial literacy level in understanding subsequent long-run growth. The proposition is consistent with the empirical study in Barro (2001).[1] Other studies (Glomm and Ravikumar, 1997; Zhang and Casagrande, 1998; Braeuninger and Vidal, 2000; Teles and Andrade, 2008) theoretically analyze the impacts of public education expenditure on fertility or economic growth. On the other hand, there also exist a lot of studies (Chakaraborty, 2004; Bhattacharya and Qiao, 2007; Varvarigos and Zakaria, 2013) focusing on the effects of public health expenditure. Although few in number, still others (Osang and Sarkar, 2008; Dioikitopoulos, 2014) consider both public education and health expenditures simultaneously. We summarize the main features of existing studies in Table 1.

However, none of the aforementioned papers simultaneously consider government public health expenditure, government public education expenditure, private health investment, private education investment, and endogenous fertility in one model. Zhang and Casagrande (1998) show that flat-rate tax based education subsidy enhances economic growth, while Dioikitopoulos

[1]Barro (2001) uses panel data to empirically show that economic growth exhibits a positive correlation with the initial average years of schooling attained by adult males, particularly at the secondary and higher education levels.

Table 1: Main features of existing studies

	pri.e	pub.e	pri.h	pub.h	endo.f
Glomm and Ravikumar (1997)	×	○	×	×	×
Zhang and Casagrande (1998)	○	○	×	×	○
Braeuninger and Vidal (2000)	○	○	×	×	×
Chakaraborty (2004)	○	×	×	○	×
Bhattacharya and Qiao (2007)	×	×	○	○	×
Osang and Sarkar (2008)	○	○	×	○	×
Teles and Andrade (2008)	○	○	×	×	×
Varvarigos and Zakaria (2013)	×	×	○	○	○
Dioikitopoulos (2014)	○	○	×	○	○
this paper	○	○	○	○	○

Note: 'pri', 'pub', 'e', 'h' and 'endo.f' denote the abbreviation of private, public, education, health and endogenous fertility, respectively.

(2014), further incorporates public health spending, states that there exists an optimal level education subsidy, beyond which will reversely hamper economic growth. Different results imply that the impacts of government policies might be misjudged when education and health spending are analyzed separately. Osang and Sarkar (2008) compare models with and without private education investment, and conclude that those including private education investment are growth superior, suggesting that private behavior can greatly change the effectiveness and implications of public interventions. Empirically, Schultz (1999) provides evidence that both education and health have statistically significant impacts on improving effective productivity and wage level in Africa. He also argues that public education spending stimulates private education investment, while public health spending affects the relative price of private health inputs. Based on these findings, he calls for integrated models that can capture household demand and investment to evaluate policies. By simultaneously considering both private and public investments in health and education, a more comprehensive analysis of private actions and government policies may be conducted, contributing to the formulation of plausible and effective policies.

Our theoretical framework is based on Dioikitopoulos (2014). Nevertheless, there are two key differences. First, unlike Dioikitopoulos (2014), we permit individuals to make private investments not only in the education of their children, but also in their own health status, as private health investment accounts for a large portion of total health expenditure (e.g., 42.01% in Africa, 39.31% in Americas, 31.99% in Europe, World Health Organization 2021).[2] In specific, following Varvarigos and Zakaria (2013), we model health status in a way that allows individuals to invest in it and directly derive utility from it. Second, public health expenditure contributes to both the formation of human capital and health status in our model. Although seminal empirical studies such as Schultz (1999) and Bloom et al. (2004), model human capital as a combination of education and health, and conclude that health has a statistically positive effect on labor productivity and economic growth, to the best of our knowledge, there is no existing theoretical study that explicitly treats health as a component of human capital function. Instead, we assume that public health expenditure participates in determining human capital. Furthermore, inspired by Varvarigos and Zakaria (2013), we discuss two scenarios for the timing of private and public health investments, helping us to compare and reveal the quantitatively different impacts of health investment in different periods on fertility and economic growth.

We examine that in both two scenarios, there exists a unique and stable BGP equilibrium. Then, in our specified model, we show that neither the income tax rate nor the public health and education expenditures affect the fertility due to the opposing impacts of changes in the opportunity cost of raising a child, and changes in individuals' willingness to provide labor time. We also argue that an extremely imbalanced allocation of revenue favoring public education over public health may undermine the increase in human capital stock, accompanied by a decrease in savings and physical capital stock, leading to zero or even negative economic growth. Moreover, the public health expenditure on children should be large enough to avoid

[2] In conventional literature, health is typically treated as a separate factor linked to life expectancy, which then affects old age utility as a discount weight. Under this setting, it is difficult to incorporate private health investment. The use of logarithmic utility function for tractability often leads to a counterfactual result, that is, increasing private health investment raises life expectancy but may lower overall lifetime utility. While this can be avoided by using other utility functions like the CRRA type (e.g., Bhattacharya and Qiao, 2007), it makes the model too complicated to be solved analytically.

the economic contraction caused by the decrease in private education investment and human capital stock.

Besides, we provide a comparison of growth-maximizing policies and welfare-maximizing policies, a perspective that has received little attention in the literature. We show that the welfare-maximizing tax rate and shares of public expenditure allocated to children's education and health are all lower than the growth-maximizing counterparts, implying the sacrifice of elderly people in terms of welfare when pursuing higher economic growth. We also find that the welfare-maximizing tax rate initially falls and then rises with a continuous increase in social discount factor, reflecting the consideration of intergenerational tradeoff and the changing balance between private and public investments in human capital. Furthermore, welfare-maximizing polices are found to be responsive to individual preference for children because such preference directly affects the utility of individuals.

The rest of this paper is arranged as follows. Section 2 establishes the benchmark scenario, determines the number and stability of BGP equilibrium, and discusses the growth-maximizing policies. Section 3 proposes the alternative scenario. Section 4 compares growth-maximizing policies with welfare-maximizing policies. Section 5 concludes.

2 The benchmark economy

We consider an overlapping generations economy with identical individuals who live for three periods and consume a homogeneous good. Individuals study in order to accumulate human capital in childhood. When they grow up to adulthood, they work to earn income while spending part of their time to raise children at the same time. They arrange their after-tax income for consumption, savings and investment in their children's education. Finally, they use the savings of previous period to consume and invest in their own health in old age.

2.1 Households

The lifetime utility of an agent depends on her consumption in adulthood c_t, consumption in old age d_{t+1}, the health status when she gets old v_{t+1}, the number of children n_t and the human capital stock of a child h_{t+1}. Then the lifetime utility function is given by:

$$U_t = \ln c_t + \beta \ln d_{t+1} v_{t+1} + \rho \ln n_t h_{t+1}, \tag{1}$$

where parameters $\beta \in (0, 1)$ and $\rho > 0$ correspond to the rate of time preference and the taste for the number of children as well as children's human capital stock, respectively.

The agent is endowed with one unit of time when young, and she supplies $q \in (0, 1)$ fraction of time to raise each child. The remaining time $1 - qn_t \in (0, 1)$ is used for working to earn income, while $\tau_1 \in (0, 1)$ portion of the income has to be paid to the government. She allocates disposable income to consumption c_t, private investment in education for each child e_t, and savings s_t. When she gets old, she uses the savings of previous period for consumption d_{t+1} and private health investment x_{t+1}. Thus, the young and old budget constraints are given by:

$$c_t + s_t + e_t n_t = (1 - \tau_1) w_t h_t (1 - qn_t), \tag{2a}$$

$$d_{t+1} + x_{t+1} = (1 + r_{t+1}) s_t, \tag{2b}$$

respectively, where w_t refers to the wage rate, while r_{t+1} refers to the net interest rate.

We assume that health status v_{t+1} is positively related to the private investment in health during old age x_{t+1} and the government health expenditure on the old age per capita $\overline{G}_{t+1}^{v,o} \equiv G_{t+1}^{v,o}/N_t$, where $G_{t+1}^{v,o}$ and N_t denote the aggregate government health expenditure on the old ages in period $t + 1$ and the number of generation t, respectively. Then the health status is given by:

$$v_{t+1} = B x_{t+1}{}^{\gamma_1} (\overline{G}_{t+1}^{v,o})^{1-\gamma_1}, \tag{3}$$

where $B > 0$ is an exogenous parameter which denotes the productivity of health status, and $\gamma_1 \in (0, 1)$ represents the impacts of private health investment on one's health status.

The human capital stock of an adult in period $t + 1$ is supposed to be positively related to e_t, the education expenditure of government for children per capita $\overline{G}_t^{e,c} \equiv G_t^{e,c}/N_{t+1}$, the government health expenditure on children per capita $\overline{G}_t^{v,c} \equiv G_t^{v,c}/N_{t+1}$, and the human capital stock of her parent h_t, where $G_t^{e,c}$ and $G_t^{v,c}$ denote the aggregate government education and health expenditures on children in period t, respectively. Thus, the human capital stock of generation $t + 1$ is given by:

$$h_{t+1} = \theta e_t{}^{\phi} (\overline{G}_t^{e,c})^{\eta} (\overline{G}_t^{v,c})^{\mu} h_t{}^{1-\phi-\eta-\mu}, \tag{4}$$

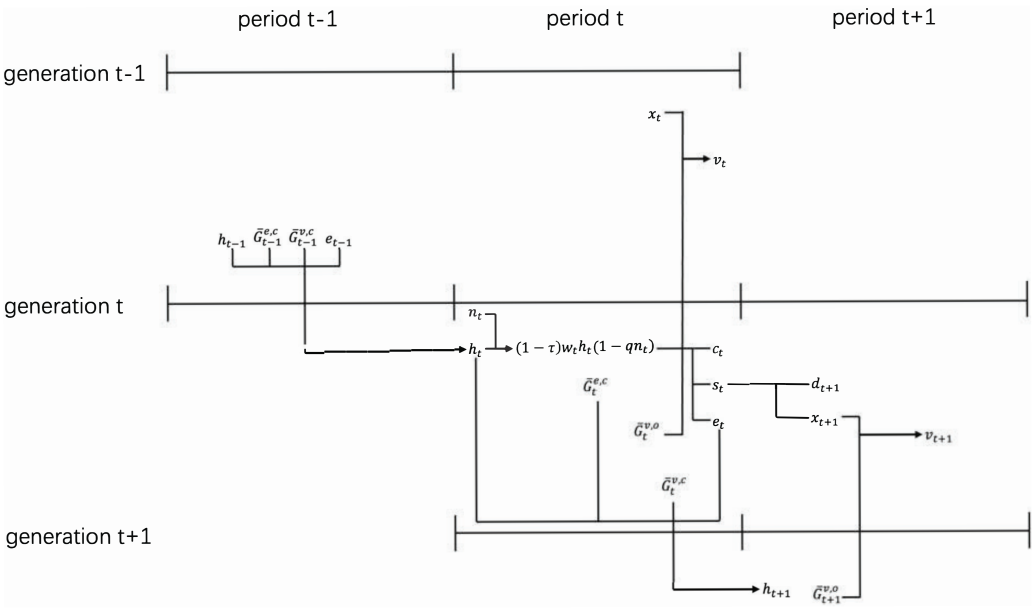

Figure 1: Time structure of benchmark scenario

where the exogenous parameter $\theta > 0$ denotes the productivity of human capital, and $\phi \in (0, 1)$, $\eta \in (0, 1)$, $\mu \in (0, 1)$ capture the elasticity of private education investment, the education expenditure of government and the health expenditure of government for children per capita, respectively. Furthermore, $\phi + \eta + \mu \in (0, 1)$. The time structure of benchmark scenario is as shown in Figure 1.

Individuals choose the optimal c_t, d_{t+1}, x_{t+1}, n_t, e_t, s_t to maximize the lifetime utility subject to the lifetime budget constraint, the health status function and the human capital stock function. Therefore, we set the following Lagrange function:

$$\begin{aligned}\mathcal{L} =& \ln c_t + \beta \ln d_{t+1} v_{t+1} + \rho \ln n_t h_{t+1} \\ &+ \lambda[(1-\tau_1) w_t h_t (1 - q n_t) - c_t - \frac{1}{1 + r_{t+1}}(d_{t+1} + x_{t+1}) - e_t n_t] \\ &+ \sigma[B x_{t+1}{}^{\gamma_1} (\overline{G}^{v,o}_{t+1})^{1-\gamma_1} - v_{t+1}] \\ &+ \varphi[\theta e_t{}^{\phi} (\overline{G}^{e,c}_t)^{\eta} (\overline{G}^{v,c}_t)^{\mu} h_t{}^{1-\phi-\eta-\mu} - h_{t+1}].\end{aligned}$$

By solving the maximization problem, we obtain the optimal allocations[3]:

$$c_t = \frac{(1-\tau_1) w_t h_t (1 - q n_t)}{1 + \beta + \beta\gamma_1 + \rho\phi}, \tag{5}$$

[3]In present model, the fertility rate is shown to be constant without a transition process. However, in other models under different settings, the transition process of fertility rate might exist, and is worth discussing.

$$d_{t+1} = \frac{(1-\tau_1)w_t h_t(1-qn_t)\beta(1+r_{t+1})}{1+\beta+\beta\gamma_1+\rho\phi}, \tag{6}$$

$$x_{t+1} = \frac{(1-\tau_1)w_t h_t(1-qn_t)\beta\gamma_1(1+r_{t+1})}{1+\beta+\beta\gamma_1+\rho\phi}, \tag{7}$$

$$n_t = \frac{\rho-\rho\phi}{(1+\beta+\beta\gamma_1+\rho)q} \equiv {n_1}^*, \tag{8}$$

$$e_t = \frac{\rho\phi(1-\tau_1)w_t h_t(1-qn_t)}{(1+\beta+\beta\gamma_1+\rho\phi)n_t}, \tag{9}$$

$$s_t = \frac{(1-\tau_1)w_t h_t(1-qn_t)(\beta+\beta\gamma_1)}{1+\beta+\beta\gamma_1+\rho\phi}. \tag{10}$$

2.2 Firms

We assume a Cobb-Douglas production function:

$$Y_t = A{K_t}^{\alpha}{H_t}^{1-\alpha}, \tag{11}$$

where Y_t, K_t, and H_t denote the aggregate production, physical and human capital, respectively. Parameter $A > 0$ denotes the total factor productivity, and $\alpha \in (0,1)$ represents the share of physical capital. Under perfect competition in factor markets, the first-order conditions are given by:

$$R_t = A\alpha{K_t}^{\alpha-1}{H_t}^{1-\alpha}, \tag{12}$$

$$w_t = A(1-\alpha){K_t}^{\alpha}{H_t}^{-\alpha}, \tag{13}$$

where R_t stands for rental rate (physical capital is supposed to be fully depreciated, which means $r_t = R_t - 1$), and w_t stands for wage rate.

2.3 Government

Government spends on both education and health to enhance the accumulation of human capital and health status. The fund is financed through the income tax. A portion of the revenue $\pi_1 \in (0,1)$ is dedicated to investing in children's education through the form such as providing compulsory education and building schools. The remaining portion $1-\pi_1$ is allocated to health expenditures for both children and the old ages. Moreover, a fraction of health expenditures represented by $\epsilon_1 \in (0,1)$ is used for children, such as constructing children's hospitals and providing free vaccinations. While the remaining portion $1-\epsilon_1$ is used for the old ages, which involves the construction of nursing homes, training caregivers and supporting research for common diseases affecting the old ages. Hence, the budget constraints of government are given by:

$$T_t = \tau_1 w_t h_t (1 - qn_t) N_t = G_t^{e,c} + G_t^{v,c} + G_t^{v,o}, \tag{14}$$

where $G_t^{e,c} = \pi_1 T_t$, $G_t^{v,c} = \epsilon_1 (1 - \pi_1) T_t$, and $G_t^{v,o} = (1 - \epsilon_1)(1 - \pi_1) T_t$.

2.4 Markets

An individual is endowed with 1 unit of time, and she allocates q unit of time to raise each of her children, which means her remaining time is used for work. Then the labor supply is governed by[4]:

$$L_t = (1 - qn_t) N_t. \tag{15}$$

The capital market clears, thus the relation between capital and savings is given by:

$$s_t = \frac{K_{t+1}}{N_t}. \tag{16}$$

2.5 Balanced Growth Path (BGP) equilibrium and stability

The aggregate physical to human capital ratio and the human capital stock per unit of effective labor are defined as $k_t \equiv K_t / H_t$ and $h_t \equiv H_t / L_t$, respectively. Therefore, wage rate Eq. (13) leads to the following equation:

$$w_t = A(1 - \alpha) {k_t}^{\alpha}, \tag{17}$$

then using Eqs. (8), (10), (16) and (17), we get the following equation:

$$k_{t+1} h_{t+1} = \frac{(1 - \tau_1)(\beta + \beta\gamma_1)}{1 + \beta + \beta\gamma_1 + \rho\phi} A(1 - \alpha) {k_t}^{\alpha} h_t \frac{1}{n_t}, \tag{18}$$

[4]Note that by using Eq. (8), we confirm the labor supply per capita is in the range from 0 to 1:

$1 - qn_t = \frac{1+\beta+\beta\gamma_1+\rho\phi}{1+\beta+\beta\gamma_1+\rho} \in (0, 1)$.

meanwhile, using Eqs. (4) and (9), we get:

$$\frac{h_{t+1}}{h_t} = \theta\left[\frac{1}{n_t}A(1-\alpha)(1-qn_t)\right]^{\phi+\eta+\mu}[\rho\phi(1-\tau_1)]^{\phi}\left(\frac{1}{1+\beta+\beta\gamma_1+\rho\phi}\right)^{\phi} \times {\epsilon_1}^{\mu}{\pi_1}^{\eta}(1-\pi_1)^{\mu}{\tau_1}^{\eta+\mu}{k_t}^{\alpha(\phi+\eta+\mu)}. \tag{19}$$

Substituting Eq. (19) into Eq. (18), the dynamic equation of k is given by:

$$k_{t+1} = \theta^{-1}\left[\frac{1}{n_t}A(1-\alpha)\right]^{1-\phi-\eta-\mu}(1-qn_t)^{-(\phi+\eta+\mu)}(\rho\phi)^{-\phi}(1-\tau_1)^{1-\phi} \times \left(\frac{1}{1+\beta+\beta\gamma_1+\rho\phi}\right)^{1-\phi}(\beta+\beta\gamma_1){\epsilon_1}^{-\mu}{\pi_1}^{-\eta}(1-\pi_1)^{-\mu}{\tau_1}^{-(\eta+\mu)}{k_t}^{\alpha(1-\phi-\eta-\mu)}. \tag{20}$$

According to Eq. (20), $\partial k_{t+1}/\partial k_t > 0$ and $\partial^2 k_{t+1}/\partial {k_t}^2 < 0$ are derived, which means k_{t+1} is a monotonically increasing function of k_t with a decreasing slope[5]. Assuming that $k_{t+1} = k_t = {k_1}^*$ at the steady state, through Eq. (20) we get the following equation:

$${k_1}^* = \left\{\theta^{-1}\left[\frac{1}{{n_1}^*}A(1-\alpha)\right]^{1-\phi-\eta-\mu}(1-q{n_1}^*)^{-(\phi+\eta+\mu)}(\rho\phi)^{-\phi}(1-\tau_1)^{1-\phi} \times \left(\frac{1}{1+\beta+\beta\gamma_1+\rho\phi}\right)^{1-\phi}(\beta+\beta\gamma_1){\epsilon_1}^{-\mu}{\pi_1}^{-\eta}(1-\pi_1)^{-\mu}{\tau_1}^{-(\eta+\mu)}\right\}^{\frac{1}{1-\alpha(1-\phi-\eta-\mu)}}, \tag{21}$$

where ${n_1}^*$ is given by Eq. (8). Then the slope of function $k_{t+1} = f(k_t)$ when k_t takes on ${k_1}^*$ is the following:

$$\left.\frac{\partial k_{t+1}}{\partial k_t}\right|_{k_t={k_1}^*} = \alpha(1-\phi-\eta-\mu) < 1.$$

The growth rate of aggregate physical to human capital ratio and the growth rate of human capital stock per unit of effective labor can be written as $g_k \equiv (k_{t+1}/k_t)-1$ and $g_h \equiv (h_{t+1}/h_t)-1$, respectively. In BGP equilibrium, the growth rates of production per capita ${g_y}^*$ and human capital per unit of effective labor ${g_h}^*$must be the same. We define them as ${g_y}^* = {g_h}^* = {g_1}^*$. [6] Then, the balanced growth rate is given by:

$${g_1}^* = \left\{\theta^{-1}(1-q{n_1}^*)^{-(\phi+\eta+\mu)}(\rho\phi)^{-\phi}{\epsilon_1}^{-\mu}{\pi_1}^{-\eta}(1-\pi_1)^{-\mu}\right\}^{\frac{\alpha-1}{1-\alpha(1-\phi-\eta-\mu)}} \times \left[\frac{1}{{n_1}^*}A(1-\alpha)\right]^{\frac{\phi+\eta+\mu}{1-\alpha(1-\phi-\eta-\mu)}}\left(\frac{1}{1+\beta+\beta\gamma_1+\rho\phi}\right)^{\frac{\phi+\alpha(\eta+\mu)}{1-\alpha(1-\phi-\eta-\mu)}} \times (\beta+\beta\gamma_1)^{\frac{\alpha(\phi+\eta+\mu)}{1-\alpha(1-\phi-\eta-\mu)}}(1-\tau_1)^{\frac{\phi+\alpha(\eta+\mu)}{1-\alpha(1-\phi-\eta-\mu)}}{\tau_1}^{\frac{(\eta+\mu)(1-\alpha)}{1-\alpha(1-\phi-\eta-\mu)}} - 1. \tag{22}$$

[5]Concerning the partial derivatives of the dynamic equation of k, see Appendix A.

[6]Concerning the reason why ${g_y}^* = {g_h}^*$, see Appendix B.

Proposition 1. *There exists a unique and positive balanced growth rate* ${g_1}^* > 0$ *if and only if:*

$$\left\{\theta^{-1}(1-q{n_1}^*)^{-(\phi+\eta+\mu)}(\rho\phi)^{-\phi}{\epsilon_1}^{-\mu}{\pi_1}^{-\eta}(1-\pi_1)^{-\mu}\right\}^{\frac{\alpha-1}{1-\alpha(1-\phi-\eta-\mu)}}$$
$$\times\left[\frac{1}{{n_1}^*}A(1-\alpha)\right]^{\frac{\phi+\eta+\mu}{1-\alpha(1-\phi-\eta-\mu)}}\left(\frac{1}{1+\beta+\beta\gamma_1+\rho\phi}\right)^{\frac{\phi+\alpha(\eta+\mu)}{1-\alpha(1-\phi-\eta-\mu)}}$$
$$\times(\beta+\beta\gamma_1)^{\frac{\alpha(\phi+\eta+\mu)}{1-\alpha(1-\phi-\eta-\mu)}}(1-\tau_1)^{\frac{\phi+\alpha(\eta+\mu)}{1-\alpha(1-\phi-\eta-\mu)}}{\tau_1}^{\frac{(\eta+\mu)(1-\alpha)}{1-\alpha(1-\phi-\eta-\mu)}}$$
$$>1.$$

And the BGP equilibrium is stable.

Proposition 1 gives the condition which is required for the existence of a positive balanced growth rate. We notice that if the condition of Proposition 1 is not satisfied, the economy will experience zero or even minus growth. To guarantee positive and sustainable growth, the government has to establish a proper income tax rate, allocate public expenditure to health and education services in an appropriate proportion or distribute an enough amount of public health expenditures to children. Intuitively, a high tax rate indeed increases government revenue, leading to an increase in public expenditures. Consequently, the human capital stock which is influenced by government expenditures on health and education, also increases, thereby raising output. However, at the same time, due to the increasing tax rate, individuals' disposable income and savings decrease. As a result, physical capital stock decreases, subsequently leading to a reduction in output. Thus, the effects of the former may be offset, or even surpassed by the latter. Finally, it results in zero or negative growth and vice versa. When it comes to the allocation of revenue for public health and education expenditures, for example, an excessively large amount of public investment in children's education will raise the effectiveness of private investment in children's education for the formation of human capital stock relative to investing privately in health status after retirement. This will lead to the increase in human capital stock and output. Nevertheless, individuals will thus cut the size of savings with the consequent of a decreasing physical capital stock. Meanwhile, the proportion of government expenditure on children's health also decreases, subsequently leading to a decrease in human capital accumulation. In general, an excessive allocation to public education expenditure and an insufficient allocation to public health expenditure may lead to the increased accumulation of human capital stock being absorbed, while physical capital savings also decrease.[7] Moreover, when the income tax rate, the allocation of revenue for public health and education expenditures remain constant, an increasing public expenditure on the health status of the old ages will stimulate individuals to reduce their private investment in children's education so that they can save more and invest more in their own health status during their retirement period, and hence the process will enhance the physical capital stock, while weakening the human capital stock. If the public health expenditure on children maintains at a low level, the effects of declining human capital stock may outweigh the effects of increasing physical capital, resulting in economic contraction. Therefore, the public health expenditure on children has to be larger than the threshold to guarantee the positive growth.

2.6 Government policy

We focus on the income tax rate τ_1, the distribution of public expenditures for education π_1, and the distribution of public health expenditures for children ϵ_1, discussing their impacts on fertility and economic growth.

2.6.1 The effects of policy parameters on fertility

The fertility in BGP equilibrium of benchmark scenario is expressed by Eq. (8). This equation presents a constant value determined by exogenous parameters, explaining the relationships among fertility and these parameters.

Proposition 2. *Public health and education expenditures financed by income taxes have no effects on optimal fertility.*

The reason for Proposition 2 is the following. In benchmark scenario, the opportunity cost

[7]The result is consistent with the analysis by Dioikitopoulos (2014) in which they show that government should set a threshold level of tax rate in order to drive growth. Besides, they also find that insufficient public health expenditure diminishes the positive impacts of public investment in education. Thus government has to establish a baseline for public health spending to ensure that the positive impacts of public education investment on productivity are not nullified by low life expectancy.

of raising a child is the after-tax income. Thus, while imposing an individual income tax and increasing the tax rate, the opportunity cost of raising a child will decrease. Consequently, people will be more willing to raise children, which means the fertility rate will increase. However, a higher tax rate increases the revenues of government. Due to the increasing revenues, the government is able to expend more funds on public health and education services which are related to health status and human capital accumulation. This will lead to a higher labor income per unit time. As a result, agents desire to supply more labor time in order to earn more labor income. In other words, they would like to reduce the time spent raising children by giving birth to fewer offspring, thus obtaining more working hours, and hence it will reduce fertility. To sum up, these two conflicting forces offset each other, resulting in fertility not responding to the changes in the government's allocation ratio for taxation and income tax rates.[8] This result might arise from model specification, but the underlying offset mechanism between childcare opportunity cost and labor supply willingness is non-negligible and intriguing.

Furthermore, Eq. (9) shows that the private investment in education for each child increases with the elasticity of private education investment ϕ and the stock of human capital per capita h_t, while decreases with the fertility rate n_t and the fraction of time endowment to raise a child q. Finally, Eq. (10) shows that the savings are increasing with the wage rate w_t and the stock of human capital per capita h_t. An increase in the stock of human capital per capita positively affects both the private investment in education for each child and the economy's savings because an increasing accumulation of human capital enhances disposable income used for private investments and savings. Meanwhile, an increasing human capital also enhances government revenue, enabling higher public investments in education and healthcare services. Thus, private investment in education becomes more effective, motivating young individuals to invest more in their children's education. Individuals will also intend to make larger private investment in their health status when they get old since the effectiveness of private investment

[8]The results are similar to Zhang and Casagrande (1998), in which they use a logarithmic utility function and show that when the increase in the ratio of education subsidies to income are funded through flat-rate consumption and income taxes, their contradictory effects on fertility are precisely balanced. However, we must acknowledge that the constant fertility is driven by model specification. For example, if we treat childcare as a consumption rather than a time cost, or if we use a CRRA type utility function, demographic change may occur.

in health status increases, which means they are willing to save more income.

2.6.2 The effects of policy parameters on economic growth

First, we calculate the first-order and second-order partial derivatives of ${g_1}^*$ with respect to τ_1 and get[9]:

$$\frac{\partial {g_1}^*}{\partial \tau_1} = 0, \quad \text{if} \quad \tau_1 = \frac{(1-\alpha)(\eta+\mu)}{\phi+\eta+\mu}; \qquad \frac{\partial^2 {g_1}^*}{\partial {\tau_1}^2} < 0.$$

Similarly, we calculate the first-order and second-order partial derivatives of ${g_1}^*$ with respect to π_1 and get:

$$\frac{\partial {g_1}^*}{\partial \pi_1} = 0, \quad \text{if} \quad \pi_1 = \frac{\eta}{\eta+\mu}; \qquad \frac{\partial^2 {g_1}^*}{\partial {\pi_1}^2} < 0.$$

Lastly, we calculate the first-order and second-order partial derivatives of ${g_1}^*$ with respect to ϵ_1 and get:

$$\frac{\partial {g_1}^*}{\partial \epsilon_1} > 0; \qquad \frac{\partial^2 {g_1}^*}{\partial {\epsilon_1}^2} < 0.$$

From these partial derivatives, we find that there is an inverted U-shaped functional relationship between ${g_1}^*$ and τ_1. So as ${g_1}^*$ and π_1. In addition, the relationship between ${g_1}^*$ and ϵ_1 exhibits a continuously increasing function curve with a gradually decreasing slope.

Proposition 3. *The balanced growth rate is maximized when the income tax rate τ_1, and the fraction of revenue used for education expenditure π_1 are given by the following:*

$${\tau_1}^* = \frac{(1-\alpha)(\eta+\mu)}{\phi+\eta+\mu}, \tag{23}$$

$${\pi_1}^* = \frac{\eta}{\eta+\mu}. \tag{24}$$

In addition, ${g_1}^*$ rises as ϵ_1 approaches to 1.

Notice that the domain of τ_1, π_1 and ϵ_1 are given by $\tau_1 \in (0,1)$, $\pi_1 \in (0,1)$ and $\epsilon_1 \in (0,1)$. Since $\{[(1-\alpha)(\eta+\mu)]/(\phi+\eta+\mu)\} \in (0,1)$ and $\{\eta/(\eta+\mu)\} \in (0,1)$, ${\tau_1}^*$ and ${\pi_1}^*$ are

[9] The expressions for partial derivatives are provided upon request.

available to maximize growth rate. However, $g_1{}^*$ with respect to ϵ_1 is an increasing function without a vertex, thereby the balanced growth rate rises as ϵ_1 approaches to 1.

Proposition 3 states the growth maximizing $\tau_1{}^*$, $\pi_1{}^*$ and $\epsilon_1{}^*$, represented by exogenous parameters. Specifically, Eq. (23) gives the growth-maximizing income tax rate. We observe a positive relationship among the growth-maximizing income tax and the elasticities of public expenditures on education η and health μ for children in the accumulation of human capital. As the elasticities of public expenditures on education and health for children rise, it is more efficient for government to invest in children's education and health, which means government will increase income tax rate to boost revenue. Oppositely, the relationship between the growth-maximizing tax rate and the elasticity of private investment in education ϕ is negative. If the elasticity of private investment in education rises, it implies that the elasticities of public expenditures on education and health for children are relatively low. Therefore, it is more efficient to invest privately in education rather than public expenditures on children's education and health when accumulating human capital stock. As a result, government will lower the income tax rate. Eq. (24) gives the growth-maximizing tax allocation rate for public education expenditure on children. Evidently, the elasticity of public expenditure on education for children η positively affects the growth-maximizing allocation rate for public education expenditure on children, while the elasticity of public expenditure on health for children μ negatively affects it. The reasons are as follows. When government levies income tax at a fixed rate, an increase in the elasticity of public expenditure on education for children will encourage government to invest more in children's education, thereby increasing the tax allocation rate for public education expenditure on children. On the other hand, an increase in the elasticity of public expenditure on health for children will encourage government to invest more in children's health, thereby decreasing the tax allocation rate for public education expenditure on children. Proposition 3 also suggests that, in order to maximize economic growth, public health expenditures should be allocated to children as much as possible. The intuition, as mentioned below Proposition 1, is that the public health expenditure on children should be large enough to avoid the economic contraction caused by the decrease of private education investment and human capital stock. Additionally, the more public health expenditures are allocated to children, the greater the economic growth.

3 Another scenario

Consider a modified version of the model in Section 2, which differs only in the timing of private and public health spending.[10] This scenario leads to the following equilibrium for fertility, and balanced growth rate:

$$n_t = \frac{\rho - \rho\phi}{(1+\beta+\gamma_2+\rho)q} \equiv {n_2}^*, \tag{25}$$

$$\begin{aligned} {g_2}^* = &\left\{\theta^{-1}(1-q{n_2}^*)^{-(\phi+\eta+\mu)}(\rho\phi)^{-\phi}{\epsilon_2}^{-\mu}{\pi_2}^{-\eta}(1-\pi_2)^{-\mu}\right\}^{\frac{\alpha-1}{1-\alpha(1-\phi-\eta-\mu)}} \\ &\times\left[\frac{1}{{n_2}^*}A(1-\alpha)\right]^{\frac{\phi+\eta+\mu}{1-\alpha(1-\phi-\eta-\mu)}}\left(\frac{1}{1+\beta+\gamma_2+\rho\phi}\right)^{\frac{\phi+\alpha(\eta+\mu)}{1-\alpha(1-\phi-\eta-\mu)}} \\ &\times\beta^{\frac{\alpha(\phi+\eta+\mu)}{1-\alpha(1-\phi-\eta-\mu)}}(1-\tau_2)^{\frac{\phi+\alpha(\eta+\mu)}{1-\alpha(1-\phi-\eta-\mu)}}{\tau_2}^{\frac{(\eta+\mu)(1-\alpha)}{1-\alpha(1-\phi-\eta-\mu)}} \\ &-1. \end{aligned} \tag{26}$$

Comparing Eq. (25) with Eq. (8), we find that in both scenarios, the fertility in the BGP equilibrium is constant, but their values are different.

Proposition 4. *The optimal fertility of alternative scenario (individuals invest in their own health during the young age, and government also spends health expenditure for young people) is smaller than that of benchmark scenario (individuals invest in their own health during their old age, and government also spends health expenditure for elderly people).*

Proof. See Appendix D. □

This result is caused by the setup of two scenarios. The utility derived from an individual's health status does not have to consider time preference β in alternative scenario because it is determined in young age, while in benchmark scenario, the utility derived from an individual's health status is discounted by time preference β because it is determined in old age. That is the reason why, compared to Eq. (25), in the denominator of Eq. (8), γ is additionally multiplied by β.

[10] Details of changes are provided in Appendix C.

Additionally, based on Eq. (26), we obtain the completely same growth-maximizing policies as in Section 2, that is $\tau_2{}^* = \tau_1{}^* = \frac{(1-\alpha)(\eta+\mu)}{\phi+\eta+\mu}$, $\pi_2{}^* = \pi_1{}^* = \frac{\eta}{\eta+\mu}$, and $g_2{}^*$ rises as ϵ_2 approaches to 1. The result stems from the fact that the government simply needs to adjust the timing of public health intervention – either at an earlier or later stage – to align with private health investment behavior.

Next, we compare Eqs. (22) and (26) in an attempt to investigate which scenario yields a higher balanced growth rate. We assume that all parameters in two scenarios are identical, which means $\tau_1 = \tau_2$, $\pi_1 = \pi_2$, $\epsilon_1 = \epsilon_2$ and $\gamma_1 = \gamma_2 (\equiv \gamma)$. Using Eqs. (8), (22), (25) and (26), we get:

$$\frac{1+g_1{}^*}{1+g_2{}^*} = \left\{ \left[1 + \frac{\gamma(2\beta+\beta\gamma+\rho)}{1+\beta+\gamma+\rho} \right]^{\alpha(\phi+\eta+\mu)} \times \left(\frac{1+\beta+\gamma+\rho\phi}{1+\beta+\beta\gamma+\rho\phi} \right)^{\alpha\phi-(1-2\alpha)(\eta+\mu)} \right\}^{\frac{1}{1-\alpha(1-\phi-\eta-\mu)}}.$$

It is apparent that $g_1{}^* > g_2{}^*$ must hold when $\alpha\phi - (1-2\alpha)(\eta+\mu) \geq 0$.[11]

In what follows, we also numerically compare Eqs. (22) and (26). Since nowadays people are becoming more and more concerned about their health status,[12] we examine the magnitude relationship between $g_1{}^*$ and $g_2{}^*$ in terms of the private health investment propensity (γ_1 and γ_2).[13] To give an example, we visualize one numerical result in Figure 2. Since the framework of this study is based on three-period overlapping generations models, one period is roughly equivalent to 30 years. Therefore, when we assume that the annual output growth rate is 2%, the gross output growth after 30 years is given by: $(1+2\%)^{30} \approx 1.81$. And the net growth rate of one period (30 years) is derived by: $1.81 - 1 = 0.81$. This is the reason why we try to calibrate $g_1{}^*$ and $g_2{}^*$ around 0.8. We set the physical capital share as $\alpha = 0.3$, which is widely used in the literature. In terms of time preference β, we refer to the seminal Kydland and Prescott (1982), and assume that the value of discounted preference per quarter is equal to 0.99,

[11]If β approaches to 1, then irrespective of the sign of $\alpha\phi - (1-2\alpha)(\eta+\mu)$, $g_1{}^* > g_2{}^*$ may hold.

[12]Governments worldwide and public institutions like WHO (World Health Organization) have been making great efforts to raise awareness and understanding about health issues through holding variety of campaigns such as World Health Day.

[13]We focus on the fertility rate ($n_1{}^*$ and $n_2{}^*$) as well. In addition, the private health investment propensity implies the elasticity of private health investment.

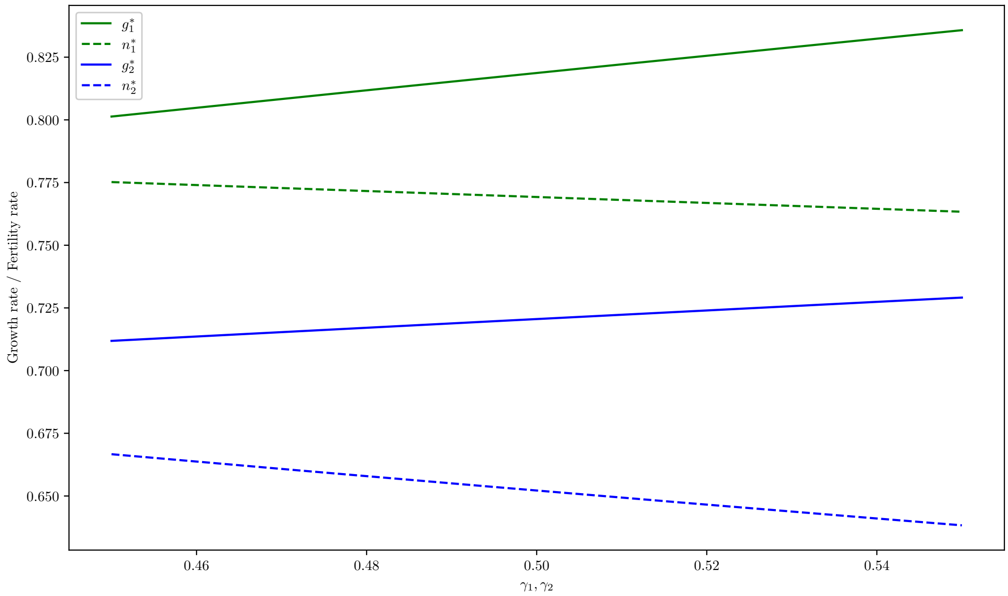

Figure 2: Trend of $n_1{}^*$, $n_2{}^*$, and $g_1{}^*$, $g_2{}^*$ along with the change of γ_1 and γ_2

so that $\beta = (0.99)^{120} \approx 0.3$. Also we set the taste for the number of children as $\rho = 0.5$ and the time cost needed for raising one child as $q = 0.2$ to calibrate the fertility rate around 0.7 in the framework of an overlapping generation model, which means the total fertility rate is around 1.4. Additionally, in order to get net output growth rate around 2% annually in benchmark scenario, we set $\theta = 1$, $\phi = 0.4$, $\eta = 0.15$, $\mu = 0.15$, $A = 21$, $\tau_1 = \tau_2 = 0.3$, $\pi_1 = \pi_2 = 0.50$ and $\epsilon_1 = \epsilon_2 = 0.98$. Under these settings, it is a special case where $\alpha\phi - (1 - 2\alpha)(\eta + \mu) = 0$. The parameter to form health status γ_1 and γ_2 range from 0.45 to 0.55.

Figure 2 demonstrates that when all other parameter values are held constant, in both two scenarios, an increase in the private health investment propensity (γ_1 and γ_2) leads to the decline in the fertility rate, causing population aging. On the other hand, $g_1{}^*$ and $g_2{}^*$ increase as γ_1 and γ_2 increase. Intuitively, either in alternative scenario or in benchmark scenario, individuals are willing to invest more in health since the elasticity of private health investment increases. Thus, individuals tend to provide more labor supply to earn more income, which means they have to reduce time spent on child parenting. Consequently, the increase in γ_1 and γ_2 results in the decline in the fertility rate as Eqs. (8) and (25) show. When it comes to economic growth rate, in both scenarios, lower fertility rate pushes up labor supply, which in turn promotes production and raises economic growth. Meanwhile, in benchmark scenario, the increasing private investment in health leads to higher savings and greater physical capital stock, further reinforcing economic growth. However, the private investment in children's education decreases simultaneously, thereby weakening the human capital accumulation and economic growth. Overall, while γ_1 increases within the range given above, the positive effects on economic growth surpass the negative ones, leading to an increase in economic growth rate. By contrast, in alternative scenario, the increasing private investment in health leads to decrease in both savings and private education investment in children, which means it weakens both the physical and human capital stock accumulation as well as the economic growth. Still, while γ_2 increases within the range given above, the production increasing effects dominate the physical and human capital decreasing effects, finally leading to an increase in economic growth. Moreover, as physical capital stock rises in benchmark scenario but declines in alternative scenario, the economic growth rate is relatively higher in benchmark scenario.

4 Welfare analysis

We turn to the analysis of long-run welfare-maximizing policies in this section. Specifically, the welfare analysis is based on BGP, assuming that the economy has already reached a steady-state. This approach is widely used for long-run welfare analysis (e.g., Fanti and Gori, 2007; Jouvet PA, et al., 2010).

Following Atkinson and Sandmo (1980), we consider a benevolent government that aims to maximize the sum of discounted welfare of all generations, namely, social welfare:

$$SW = \sum_{t=-1}^{\infty} \kappa^t W_t = \kappa^{-1} W_{-1} + \sum_{t=0}^{\infty} \kappa^t W_t, \tag{27}$$

where $\kappa \in (0, 1)$ denotes the social discount factor. The indirect utility functions of generation $t \geq 0$ and initial old are derived as follows:

$$W_t = lnc_t{}^* + \beta lnd_{t+1}{}^* v_{t+1}{}^* + \rho n_1{}^* h_{t+1}, \tag{28}$$

$$W_{-1} = \beta lnd_0{}^* v_0{}^* + \rho lnn_1{}^* h_0. \tag{29}$$

Using Eqs. (8),(21), (22) and

$$c_t{}^* = \frac{(1-\tau_1)w^* h_t(1-qn_1{}^*)}{1+\beta+\beta\gamma_1+\rho\phi}, \qquad d_{t+1}{}^* = \beta(1+r)^* c_t{}^*, \qquad x_{t+1}{}^* = \beta\gamma_1(1+r)^* c_t{}^*,$$

$$v_{t+1}{}^* = Bx_{t+1}{}^{*\gamma_1}(\overline{G}_{t+1}^{v,o\,*})^{1-\gamma_1}, \qquad \overline{G}_{t+1}^{v,o\,*} = (1-\epsilon_1)(1-\pi_1)\tau_1 w^* h_{t+1}(1-qn_1{}^*)n_1{}^*,$$

$$w^* = A(1-\alpha)k_1{}^{*\alpha}, \qquad (1+r)^* = A\alpha k_1{}^{*\alpha-1}, \qquad h_t = (1+g_1{}^*)^t h_0,$$

we finally arrive at the following equation:

$$\begin{aligned} SW = \frac{1}{\kappa(1-\kappa)} \times \Big[& (\kappa+\beta+\beta\gamma_1) ln(1-\tau_1) + \beta(1-\gamma_1) ln(1-\epsilon_1)(1-\pi_1)\tau_1 \\ & + (\kappa\alpha + 3\alpha\beta + \alpha\beta\gamma_1 - \beta - \beta\gamma_1) ln k_1{}^* \\ & + \frac{\kappa^2 + 3\kappa\beta + \kappa\beta\gamma_1 + \kappa\rho - \beta - \beta\gamma_1}{1-\kappa} ln(1+g_1{}^*) \\ & + (\kappa+\beta+\beta\gamma_1) ln \frac{A(1-\alpha)(1-qn_1{}^*)h_0}{1+\beta+\beta\gamma_1+\rho\phi} \\ & + \beta(1-\gamma_1) ln A(1-\alpha)(1-qn_1{}^*)n_1{}^* h_0 \\ & + \beta ln \alpha\beta AB + \beta\gamma_1 ln \alpha\beta A\gamma_1 + \rho ln n_1{}^* h_0 \Big], \end{aligned} \tag{30}$$

where h_0 is given. Since Eq. (30) is in a quite complicated and intractable form, we perform numerical simulations to analyze the welfare-maximizing policies and make a comparison with growth-maximizing policies. When we focus on the effect of one specific policy parameter, then its value is allowed to vary within the interval $(0, 1)$, with all other parameters remaining at their values defined in Section 3. Moreover, we assume that the social discount factor is equal to the individual time preference, so that $\kappa = \beta = (0.99)^{120} \approx 0.3$, as is commonly adopted in the literature (e.g., Uchida and Ono, 2021; Wang, 2023). In addition, the initial value of human capital stock is normalized to $h_0 = 1$. We also experiment with alternative values of social discount factor, namely $\kappa = (0.985)^{120} \approx 0.16$ and $\kappa = (0.995)^{120} \approx 0.55$, to examine the robustness of welfare effects.

Fig. 3 illustrates how the balanced growth rate changes with policy parameters, which is consistent with Proposition 3. Comparing Fig. 4 to Fig. 3, it is apparent that regardless of whether the social discount factor is relatively higher than, lower than, or equal to individual time preference, the values of all welfare-maximizing policy parameters are smaller than those of growth-maximizing counterparts.[14] The results imply that when the objective is to maximize economic growth, government will tend to levy higher tax, and increase the allocation of revenue towards children's education and health, so that it can enhance the accumulation of human capital stock, which is the source of economic growth in our model. However, such growth is achieved by sacrificing the welfare of elderly people, because their consumption and private health

[14]Since the policy parameters lie within the interval $(0, 1)$, and cannot take the values of 0 and 1, we set the starting point and ending point in the numerical simulation as 0.01 and 0.99, respectively.

investment are reduced, along with a decline in the health support from government.

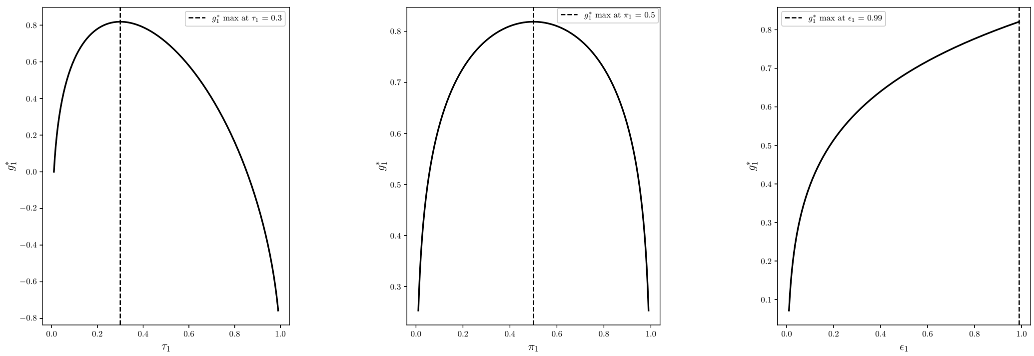

Figure 3: The relationship between balanced growth rate ${g_1}^*$ and policy parameters τ_1, π_1, and ϵ_1

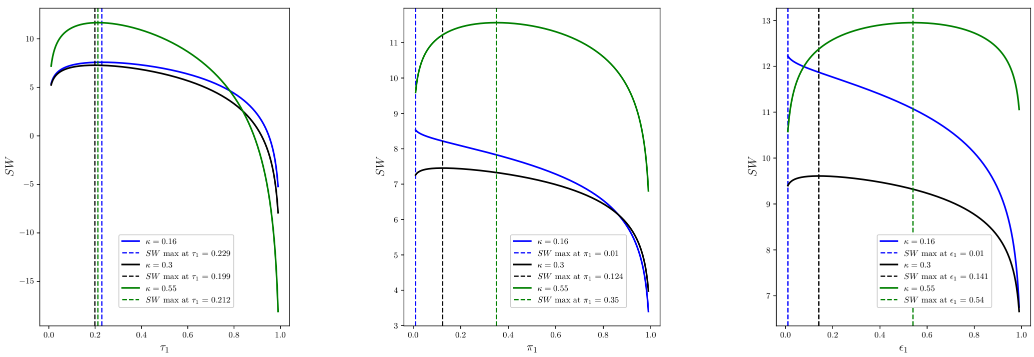

Figure 4: The relationship between long-run social welfare and policy parameters τ_1, π_1, and ϵ_1 under different values of social discount factor κ

We also find that the effects of social discount factor κ differ across various welfare-maximizing policy parameters. On the one hand, according to the left panel of Fig. 4, when the social discount factor is relatively low, the tax rate that maximizes social welfare is the highest. As the social discount factor increases, the welfare-maximizing tax rate initially decrease. Yet, a further increase in the social discount factor leads to a rise in welfare-maximizing tax rate again. Mechanically, the result can be explained by the way that social discount factor affects intergenerational tradeoff. When future generations are heavily discounted, the government aims to prioritize the welfare of the initial old. Therefore, what the government can do is to raise the tax rate to increase revenue, which in turn will increase the public health expenditure on elderly people, and ultimately improve their health status and welfare. With an increasing concern for future generations, the government will lower the tax rate so that individuals will have more disposable income and invest more in children's education. While the public education and health expenditures on children will be cut down at the same time, the marginal effect of private investment in human capital outweighs that of public expenditures, leading to an increase in human capital, and consequently greater welfare for future generations. However, with the government attaching much greater weights to future generations, the tax rate will be raised again because of the diminishing marginal effect of private education investment on children's human capital. In other words, the government has to resume increasing public expenditures on children's education and health to enhance the efficiency of human capital accumulation. As a result, a non-monotonic relationship between welfare-maximizing tax rate and social discount factor is found, suggesting that higher social discount factor does not always lead to more resource for future generations, and that intergenerational tradeoff plays a complex role in determining fiscal policy. On the other hand, according to the middle and right panels of Fig. 4, the optimal shares of revenue allocated to children's education π_1 and health ϵ_1 increase monotonically with social discount factor κ, because greater concerns for future generations will prompt the government to enhance human capital accumulation by increasing public expenditures on children.

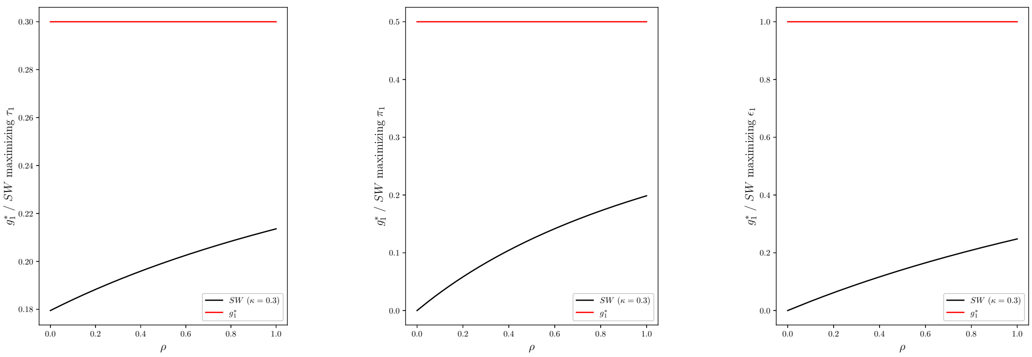

Figure 5: The relationship between policies and preference for children ρ

Moreover, we find that growth-maximizing policies do not depend on preference for children ρ in Subsection 2.6. Here, we explore the correlation between welfare-maximizing policies and ρ in Fig. 5. Evidently, all welfare-maximizing policy parameters respond positively to an increase in ρ. Higher ρ will push up the fertility, resulting in lower per child private education investment due to both decreasing disposable income and increasing children. Consequently, human capital of future generations will decline, and so will their welfare. To counteract the loss in human capital, the government will increase public expenditures on each child's education and health – either by raising the tax rate τ_1, or by raising the shares of revenue allocated to children's education π_1 and health ϵ_1. Although increasing τ_1 also directly reduces the welfare of initial old, the welfare increasing effect for future generations dominates, thereby improving social welfare. This implication reveals the fundamental difference between growth-maximizing and welfare-maximizing objectives. Growth-maximizing policies only focus on the long-run output efficiency, which is independent of individual preference. In contrast, welfare-maximizing policies take individual preference into account, and involve intergenerational tradeoff.

5 Conclusions

This paper aims at investigating the impacts of government policies on fertility and economic growth when considering both private and public investments in health and education. We also intend to discuss and compare the growth-maximizing policies with welfare-maximizing policies.

We have examined that there exists a unique and stable equilibrium growth path. Then we found that, in our specified model, fertility is not affected by income tax rate. We also showed that if there is an excessive allocation of public funds towards education and an insufficient allocation towards public health, it may weaken the increase in human capital stock, accompanied by a decrease in savings and physical capital stock. Next, we calculated the income tax rate and revenue allocation rates that maximize economic growth rate. Another implication is that public health expenditure on children should be sufficiently substantial to prevent economic contraction resulting from a reduction in private education investment and human capital stock, and in order to promote the growth rate, government should allocate public health expenditure to children as much as possible.

The numerical welfare analysis revealed that growth-maximizing policies are realized by sacrificing the welfare of elderly people. The value of social discount factor plays a critical role in determining the welfare-maximizing policies, as it determines how the government balances welfare across generations. Furthermore, unlike growth-maximizing policies, which are independent of individual preference for children, welfare-maximizing policies take it into account because such preference affects the utility of individuals.

Declarations

The author has no competing interests to declare that are relevant to the content of this article.

Appendix A

To confirm the shape of the dynamic equation of k, we calculate the first-order and second-order partial derivatives of k_{t+1} with respect to k_t, and the results are as follows:

$$\begin{aligned}\frac{\partial k_{t+1}}{\partial k_t} =&\theta^{-1}\left[\frac{1}{n_t}A(1-\alpha)\right]^{1-\phi-\eta-\mu}(1-qn_t)^{-(\phi+\eta+\mu)}(\rho\phi)^{-\phi}(1-\tau_1)^{1-\phi}\\ &\times\left(\frac{1}{1+\beta+\beta\gamma_1+\rho\phi}\right)^{1-\phi}(\beta+\beta\gamma_1)\epsilon_1{}^{-\mu}\pi_1{}^{-\eta}(1-\pi_1)^{-\mu}\tau_1{}^{-(\eta+\mu)}\\ &\times\alpha(1-\phi-\eta-\mu)k_t{}^{\alpha(1-\phi-\eta-\mu)-1}>0,\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 k_{t+1}}{\partial k_t{}^2} =&\theta^{-1}\left[\frac{1}{n_t}A(1-\alpha)\right]^{1-\phi-\eta-\mu}(1-qn_t)^{-(\phi+\eta+\mu)}(\rho\phi)^{-\phi}(1-\tau_1)^{1-\phi}\\ &\times\left(\frac{1}{1+\beta+\beta\gamma_1+\rho\phi}\right)^{1-\phi}(\beta+\beta\gamma_1)\epsilon_1{}^{-\mu}\pi_1{}^{-\eta}(1-\pi_1)^{-\mu}\tau_1{}^{-(\eta+\mu)}\\ &\times\alpha(1-\phi-\eta-\mu)\overbrace{[\alpha(1-\phi-\eta-\mu)-1]}^{-}k_t{}^{\alpha(1-\phi-\eta-\mu)-2}<0.\end{aligned}$$

From the results above, we infer that k_{t+1} is a monotonically increasing function of k_t with a decreasing slope.

Appendix B

Dividing both sides of the Cobb-Douglas production function Eq. (11) by N_t, we get:

$$\frac{Y_t}{N_t}=A\frac{K_t{}^{\alpha}H_t{}^{1-\alpha}}{N_t{}^{\alpha}N_t{}^{1-\alpha}},$$

then using $k_t\equiv K_t/H_t$, $h_t\equiv H_t/L_t$ and Eq. (15), the equation above can be written as:

$$y_t=Ak_t{}^{\alpha}h_t(1-qn_t),$$

since the birth rate is a constant value, when k takes on the value of steady state, $Ak^{*\alpha}(1-qn_t)$ is also constant, which means the growth rates of production per capita and human capital per unit of effective labor have to be the same according to the following equation:

$$\frac{y_t{}^*}{h_t{}^*} = Ak^{*\alpha}(1 - qn),$$

thus, we finally get:

$$g_y{}^* = g_h{}^* = g_1{}^*.$$

Appendix C

In the model of Section 3, the lifetime utility of an individual changes to:

$$U_t = \ln c_t v_t + \beta \ln d_{t+1} + \rho \ln n_t h_{t+1},$$

where v_t is the health status of an agent when she is in adulthood.

The young and old budget constraints change to:

$$c_t + s_t + x_t + e_t n_t = (1 - \tau_2) w_t h_t (1 - q n_t),$$

$$d_{t+1} = (1 + r_{t+1}) s_t,$$

respectively, where x_t denotes the private investment in health when young.

The formation of health status changes to:

$$v_t = B x_t{}^{\gamma_2} (\overline{G}_t^{v,y})^{1-\gamma_2},$$

where $\overline{G}_t^{v,y} \equiv G_t^{v,y}/N_t$, and $G_t^{v,y}$ denotes the aggregate government health expenditure on the young age in period t.

The budget constraint of government change to:

$$T_t = \tau_2 w_t h_t (1 - q n_t) N_t = G_t^{e,c} + G_t^{v,c} + G_t^{v,y},$$

where $G_t^{e,c} = \pi_2 T_t$, $G_t^{v,c} = \epsilon_2 (1 - \pi_2) T_t$, and $G_t^{v,y} = (1 - \epsilon_2)(1 - \pi_2) T_t$.

Appendix D

We compare the numerator and denominator of Eqs. (8) and (25) when $\gamma_1 = \gamma_2$:

$$\rho - \rho\phi = \rho - \rho\phi,$$

$$(1 + \beta + \beta\gamma_1 + \rho) q < (1 + \beta + \gamma_2 + \rho) q,$$

it is obvious that $\frac{\rho - \rho\phi}{(1+\beta+\beta\gamma_1+\rho)q} > \frac{\rho - \rho\phi}{(1+\beta+\gamma_2+\rho)q}$.

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